Algebraic Expressions

Factors, Coefficients and Terms of Algebraic Expressions

Shiva and Somesh are brothers. Shiva's age is 3 years less than Somesh's age.

Now, how can we represent this situation with an algebraic expression?

Let us assume Somesh's age as x years. Therefore, Shiva's age = $(x - 3)$ years

Here,$(x - 3)$ is an algebraic expression that represents Shiva's age.

Here, we can notice one thing. The ages of both Somesh and Shiva can vary, but the difference between the ages, i.e. 3 years, is always constant. In this algebraic expression $(x - 3)$, x can vary but the number 3 does not. Hence, x is known as a **variable (or algebraic number)** and 3 is called a **constant (absolute term)**.

Let us consider some algebraic expressions given below.

(i) $3x + 5$

(ii) $4x^2 - 21$

Here, the first expression $(3x + 5)$ is formed by adding $3x$ and 5. In this case, $3x$ and 5 are called **algebraic terms or simply terms** of the expression.The terms are always added to form an algebraic expression. They are never subtracted to form an algebraic expression. However, an expression may have positive or negative terms. In the expression $(3x - 5)$, the terms of the expression are $3x$ and (-5), and not $3x$ and 5. Thus, we added the terms $3x$ and (-5) to get the expression $(3x - 5)$.

In expression (ii), $4x^2$ and (-21) are added to form $4x^2 - 21$. Therefore, $4x^2$ and (-21) are terms of the expression $4x^2 - 21$.

Let us again consider the expression $3x + 5$. Here, the term $3x$ is a product of 3 and x. We cannot factorise 3 and x further. Hence, 3 and x are called **factors** of the term $3x$.

The term 5 cannot be expressed as the product of variables and constant. Therefore, 5 is itself a factor of 5.

In expression (ii), the term $4x^2$ can be written as

$4x^2 = 4 \times x^2$

$4x^2 = 4x \times x$

But x^2 and $4x$ cannot be the factors of $4x^2$ as they can be factorised further.

$x^2 = x \times x$ and $4x = 4 \times x$

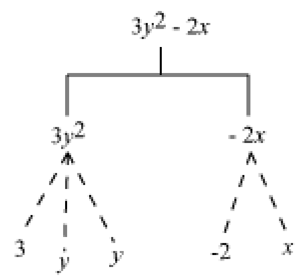
We can write $4x^2$ as the product of 4, x, and x as shown below.

$4x^2 = 4 \times x \times x$

Therefore, 4, x and x are called factors of $4x^2$.

We can also represent the factors and terms of an algebraic expression by a tree diagram.

Similarly, we can represent the tree diagram of an expression ($3y^2 - 2x$) as shown below.



In the expression $3y^2 - 2x$, we can see that the term $3y^2$ is the product of a numerical, i.e. 3, and other variables. This numerical, i.e. 3, is known as the **numerical coefficient** of the term, $3y^2$. Similarly, the coefficient of the term $-2x$ is -2. Generally, we define the numerical coefficient or efficient as

The numerical factor of a term is called the numerical coefficient (or constant coefficient) of the term.

Using this definition, we can say that the numerical coefficient of $-15xy$ is -15, since

$-15xy = -15 \times x \times y$

We can also write $-15xy$ as $-15xy = x \times (-15y) = y \times (-15x)$

Thus, we can say that the coefficient of x is $-15y$ and the coefficient of y is $-15x$.

Can we find the numerical coefficients of x in the expression $(x - 5)$ and that of xy in the expression $(7 - xy)$?

In case of $(x - 5)$, 1 is the numerical coefficient of x. In case of $(7 - xy)$, −1 is the numerical coefficient of xy.

Note: In the expression $-15xy$, xy is said to be the algebraic coefficient.

Thus, we can say that

If the coefficient of a term is 1, then it is not written before the term. If the coefficient of the term is −1, then only the '−' sign is put before the term.

Let us look at the factorization of the terms $5x^3yz^2$ and $-23x^3yz^2$.

$$5x^3yz^2 = 5\underbrace{x \times x \times x}_{3} \times \underbrace{y}_{1} \times \underbrace{z \times z}_{2}$$

$$-23x^3yz^2 = -23 \times \underbrace{x \times x \times x}_{3} \times \underbrace{y}_{1} \times \underbrace{z \times z}_{2}$$

Here, we can see that the two terms have different numerical factors 5 and −23, but same algebraic factors (each of these term contains the same variable, i.e. x, y, and z. Also, powers of these variables of each term are the same, i.e. power of x, y, and z are 3, 1, and 2 respectively). These terms are known as **like terms.** We can define them as

The terms having the same algebraic factors are called like terms. Like terms may have different numerical factors.

Let us consider the terms $6xy$ and $6x$. Now, $6xy = 6 \times x \times y$ and $6x = 6 \times x$

Here, we can see that the two terms have the same numerical factor 6. Their algebraic factors xy and x are different. Such type of terms having different algebraic factors are said to be **unlike terms.** We define them as

The terms having different algebraic factors are called unlike terms.

Let us discuss some examples to understand these concepts better.

Example 1:

Find the terms in the algebraic expression $\left(-\frac{xy}{7}+14xy^2-3\right)$**.**

Solution:

The terms of the expression are $-\frac{xy}{7}, 14xy^2$, and −3.

Example 2:

Find the factors of $\left(-3x^2yz^3\right)$**.**

Solution:

$-3x^2yz^3 = -3 \times x \times x \times y \times z \times z \times z$

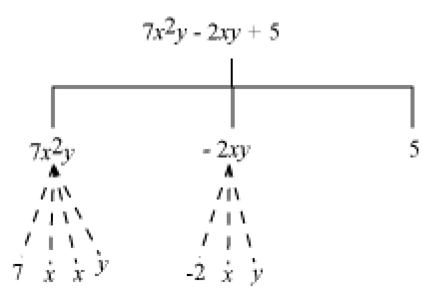
Therefore, the factors of $-3x^2yz^3$ are −3, x, x, y, z, z, and z.

Example 3:

Represent the terms and factors of the algebraic expression $7x^2y - 2xy + 5$ through a tree diagram.

Solution:

The tree diagram representation of the algebraic expression, $7x^2y - 2xy + 5$ is



Example 4:

Find the like terms in the algebraic expression $51x^2y - 21x^2y^2 + \frac{x^2y}{2} + 31 - 6xy - x^2y$.

Solution:

Here, the like terms are $51x^2y$, $\frac{x^2y}{2}$, and $-x^2y$.

Example 5:

Find the coefficients of *pq* in the following terms.

pq^2, $-3pq$, $15p^2q^2$, $-\frac{31}{5}p^2q$

Solution:

Terms	**Coefficients of pq**
pq2	q
−3pq	−3
15p2q2	15pq
$-\frac{31}{5}p^2q$	$-\frac{31}{5}p$

Classification of Expressions Based on the Numbers of Terms

Many times you must have encountered expressions like $5x + y$, $2xyz$, $3x^2$ etc. Do you know what are they called and how are these expressions classified.

Note that monomials, binomials, and trinomials are types of polynomials.

An algebraic expression in which each term contains only the variable(s) with non negative integral exponent(s) is called a polynomial.

A polynomial having four algebraic terms are called four termed polynomial. Example: $x^3 + 5x^2 + 10x - 7$

Let us now consider the following polynomials in variable x.

$x^3 + 5x^2 - 7$, $2x^3 + 4x + 9$, $x^3 - x - 1$, $3x^3 + x^2$

Is there any similarity in the above polynomials?

Yes. In each of the above polynomials, the highest exponent is 3. Can we say that each of the above polynomials is of degree 3?

Let us now discuss some more examples to understand the concept better

Example 1:

Separate monomials, binomials, and trinomials from the following polynomials.

(i) $x + 7$ (ii) 16 (iii) $-21x^2y^2z^2$

(iv) $x^2 - 3$ (v) $7x + 7y - 6xy$ (vi) $6xy^2 - 2x^2y$

(vii) $4x + xy$ (viii) $15xy^2 - 7 - 2x^2y$

Solution:

An expression containing only one term is known as a monomial. Hence, among the given expressions, the monomials are

(ii) 16 **(iii)** $-21x^2y^2z^2$

An expression containing two unlike terms is known as a binomial. Hence, among the given expressions, the binomials are

(i) $x + 7$ **(iv)** $x^2 - 3$ **(vi)** $6xy^2 - 2x^2y$ **(vii)** $4x + xy$

An expression containing three unlike terms is known as a trinomial. Hence, among the given expressions, the trinomials are

(v) $7x + 7y - 6xy$ **(viii)** $15xy^2 - 7 - 2x^2y$

Example 2:

What is the degree of the polynomial $(5x + 6xy^2 - 2y)$?

Solution:

The degree of the polynomial $5x + 6xy^2 - 2y$ is 3 because the highest sum of the exponents of the variables is 3.

Example 3:

Is the polynomial ($5x + 6xy^2 - 2x$) a trinomial? Justify your answer.

Solution:

The polynomial $5x + 6xy^2 - 2x$ has three terms. However, the terms $5x$ and $-2x$ are like terms. Since the given expression does not contain three unlike terms, it is not a trinomial.

Addition and Subtraction of Polynomials

just like natural numbers, we can even perform mathematical operations on algebraic expressions.

The most important point to remember in this topic is as follows.

We can add and subtract like terms. In case of addition or subtraction of like terms, only their numerical coefficients are added or subtracted. The algebraic part of the terms remains as it is.

In the video given above, we learned adding polynomials by arranging them horizontally. We can add polynomials by arranging them vertically as well.

Adding and subtracting like monomials by arranging them vertically:
In this method, we have to write given like monomials one below other to perform the addition or subtraction. Also, write the coefficients for pure variable terms as 1. To add or subtract like monomials, we just need to add or subtract the coefficients and write the result with the variable.
For example, let us add monomials a^2, $-2a^2$ and $5a^2$. These monomials can be added by arranging vertically as follows:

$$\begin{array}{rr} & 1a^2 \\ + & -2a^2 \\ + & 5a^2 \\ \hline & 4a^2 \\ \hline \end{array}$$

Here, the sum of 1 and −2 is obtained as −1. Further, the sum of −1 and 5 is obtained as 4. So, we wrote 4 with a^2 and obtained the sum of given polynomials as $4a^2$.

Similarly, we can perform subtraction for the given monomials.

$$\begin{array}{rr} & 1a^2 \\ - & -2a^2 \\ - & 5a^2 \\ \hline & -2a^2 \\ \hline \end{array}$$

When we subtracted –2 from 1, we got 3 [1 – (–2) = 3]. Further, on subtracting 5 from 3, we got –2 (3 – 5 = –2). So, we wrote –2 with a^2 and obtained the result as $-2a^2$.

Adding and subtracting polynomials by arranging them vertically:

To add the polynomials, we can arrange them vertically such that each term of lower polynomial is written below its like term in the upper polynomial. Also, write the coefficients for pure variable terms as 1.

Let us add the polynomials $-3x^2 + 4xy - z$ and $2xy + x^2 - 3z$ to learn the concept.

We can observe that $-3x^2$ and x^2 are like terms as they have same variable having same powers. Similarly, $4xy$ and $2xy$, and $-z$ and $-3z$ are other pairs of like terms.

These polynomials can be arranged vertically as follows:

$$\begin{array}{rrrr} & -3x^2 & +4xy & -z \\ + & 1x^2 & +2xy & -3z \\ \hline & & & \\ \hline \end{array}$$

Now, we just need to add the coefficients of like terms and write the variables as they are.

$$\begin{array}{rrrr} & -3x^2 & +4xy & -z \\ + & 1x^2 & +2xy & -3z \\ \hline & -2x^2 & +6xy & -4z \\ \hline \end{array}$$

Thus, the sum of the given polynomials is $-2x^2 + 6xy - 4z$.

Let us now study the concept of subtraction of polynomials by subtracting $2xy + x^2 - 3z$ from $-3x^2 + 4xy - z$.

Let us arrange them vertically first as we have done before.

$$\begin{array}{rrrr} & -3x^2 & +4xy & -z \\ - & 1x^2 & +2xy & -3z \\ \hline & & & \\ \hline \end{array}$$

To subtract a polynomial from other, we add its opposite.

Now, we get

$$\begin{array}{r} -3x^2 + 4xy - z \\ + \quad -1x^2 - 2xy + 3z \\ \hline \\ \hline \end{array}$$

Now, we perform the addition as we have done before.

$$\begin{array}{r} -3x^2 + 4xy - z \\ + \quad -1x^2 - 2xy + 3z \\ \hline -4x^2 + 2xy + 2z \\ \hline \end{array}$$

Thus, the required difference is $-4x^2 + 2xy + 2z$.

Let us discuss some more examples to understand the concept better.

Example 1:

Add the following monomials by arranging them horizontally as well as vertically.

(a) $-3p^2$, $6p^2$ and $-11p^2$

(b) $8x^2y$, $-10x^2y$ and $-2x^2y$

Solution:

Addition by arranging horizontally:

(a) $-3p^2 + 6p^2 + (-11p^2) = (-3 + 6 - 11)p^2 = -8p^2$

(b) $8x^2y + (-10x^2y) + (-2x^2y) = (8 - 10 - 2)x^2y = -4x^2y$

Addition by arranging vertically:

(a)

$$\begin{array}{rr} & -3p^2 \\ + & 6p^2 \\ + & -11p^2 \\ \hline & -8p^2 \\ \hline \end{array}$$

(b)

$$\begin{array}{rr} & 8x^2y \\ + & -10x^2y \\ + & -2x^2y \\ \hline & -4x^2y \\ \hline \end{array}$$

Example 2:

Subtract the following monomials by arranging them horizontally as well as vertically.

(a) 25mn^2 from the sum of 14mn^2 and –mn^2

(b) (–x^2y^2 + 12x^2y^2) from 19x^2y^2

Solution:

Subtraction by arranging horizontally:

(a) $(14mn^2 - mn^2) - 25mn^2 = 13mn^2 - 25mn^2 = -12mn^2$

(b) $19x^2y^2 - (-x^2y^2 + 12x^2y^2) = 19x^2y^2 - 11x^2y^2 = 8x^2y^2$

Subtraction by arranging vertically:

(a)

$$\begin{array}{rr} & 14m^2n^2 \\ + & -1m^2n^2 \\ \hline & 13m^2n^2 \\ \hline \end{array} \qquad \begin{array}{rr} & 13m^2n^2 \\ - & 25m^2n^2 \\ \hline & -12m^2n^2 \\ \hline \end{array}$$

(b)

$$\begin{array}{r} -x^2y^2 \\ +\quad 12x^2y^2 \\ \hline 11x^2y^2 \\ \hline \end{array} \qquad \begin{array}{r} 19x^2y^2 \\ -\ 11x^2y^2 \\ \hline 8x^2y^2 \\ \hline \end{array}$$

Example 3:

Add the expressions $5x^2 + 6xy - 11$, $7\,x^2y - 3y$, and $12x^2y - 3xy + 4$.

Solution:

$(5x^2 + 6xy - 11) + (7x^2y - 3y) + (12x^2y - 3xy + 4)$

$= 5x^2 + 6xy - 11 + 7x^2y - 3y + 12x^2y - 3xy + 4$

$= 5x^2 + 6xy - 3xy + 7x^2y + 12x^2y - 3y - 11 + 4$ (Rearranging the terms)

$=5x^2 + (6 - 3)xy + (7 + 12)x^2y - 3y + (-11 + 4)$

$= 5x^2 + 3xy + 19x^2y - 3y - 7$

Example 4:

Which expression when subtracted from the expression $(7x - 3y + 45xy + 7)$ gives

$(2x - 21y - 42xy)$?

Solution:

To get the required expression, we have to subtract $(2x - 21y - 42xy)$ from $(7x - 3y + 45xy + 7)$.

$(7x - 3y + 45xy + 7) - (2x - 21y - 42xy)$

$= 7x - 3y + 45xy + 7 - 2x + 21y + 42xy$

$= 7x - 2x - 3y + 21y + 45xy + 42xy + 7$

$= (7 - 2)x + (-3 + 21)y + (45 + 42)xy + 7$

$= 5x + 18y + 87xy + 7$

Example 5:

Subtract the sum of ($4y^2 - 6y$) and ($-2y^2 + 3y - 3$) from the sum of ($5y + 7$) and ($3y^2 - 9y + 2$).

Solution:

$(5y + 7) + (3y^2 - 9y + 2)$

$= 5y + 7 + 3y^2 - 9y + 2$

$= 3y^2 + 5y - 9y + 7 + 2$ [Rearranging the terms]

$= 3y^2 + (5y - 9y) + (7 + 2)$

$= 3y^2 + (5 - 9)\, y + 9$

$= 3y^2 + (-4)\, y + 9$

$= 3y^2 - 4y + 9$

$(4y^2 - 6y) + (-2y^2 + 3y - 3)$

$= 4y^2 - 6y - 2y^2 + 3y - 3$

$= 4y^2 - 2y^2 - 6y + 3y - 3$ [Rearranging the terms]

$= (4y^2 - 2y^2) + (-6y + 3y) - 3$

$= (4 - 2)\, y^2 + (-6 + 3)\, y - 3$

$= 2y^2 - 3y - 3$

Now, subtracting the sum of ($4y^2 - 6y$)and ($-2y^2 + 3y - 3$) from the sum of ($5y + 7$)and ($3y^2 - 9y + 2$) is the same as subtracting ($2y^2 - 3y - 3$) from ($3y^2 - 4y + 9$).

This can be done as

$(3y^2 - 4y + 9) - (2y^2 - 3y - 3)$

$= 3y^2 - 4y + 9 - 2y^2 + 3y + 3$

$= 3y^2 - 2y^2 - 4y + 3y + 9 + 3$ [Rearranging the terms]

$= (3 - 2)\, y^2 + (-4 + 3)\, y + (9 + 3)$

$= y^2 - y + 12$

Values of Algebraic Expressions at Different Points

A car is moving at a speed of 5 km/h more than twice the speed of a bus. Now, how can we represent the speed of the car with the help of an algebraic expression?

As seen here, the speed of the car changes with a change in the speed of the bus. In this way, we can find the value of an expression at a given point by substituting the value of the variable.

Let us discuss some more examples based on values of algebraic expressions at different points.

Example 1:

Find the value of the following expressions.

(i) $\mathbf{x^3 + \frac{x^2}{25} - 8 \text{ at } x = 5}$

(ii) $\mathbf{4k^2 - 2k + 1}$ **at** $\mathbf{k = \frac{1}{2}}$

(iii) $\mathbf{2x^3y^2 - 4x - 3xy - 5(x^2y^2 - x^2)}$ **when** $\mathbf{x = 5, y = -2}$

Solution:

(i) Putting $x = 5$ in the expression $x^3 + \frac{x^2}{25} - 8$, we obtain

$$x^3 + \frac{x^2}{25} - 8 = 5^3 + \frac{5^2}{25} - 8 = 125 + \frac{25}{25} - 8 = 125 + 1 - 8 = 118$$

(ii) Putting $k = \frac{1}{2}$ in the expression $4k^2 - 2k + 1$, we obtain

$$4k^2 - 2k + 1 = 4\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 1 = 4 \times \frac{1}{4} - 2 \times \frac{1}{2} + 1 = 1 - 1 + 1 = 1$$

(iii) Putting $x = 5, y = -2$ in the expression $2x^3y^2 - 4x - 3xy - 5(x^2y^2 - x^2)$, we obtain

$2x^3y^2 - 4x - 3xy - 5(x^2y^2 - x^2)$

$= 2 \times 5^3 \times (-2)^2 - 4 \times 5 - 3 \times 5 \times (-2) - 5\, [5^2 \times (-2)^2 - 5^2]$

$= 2 \times 125 \times 4 - 20 + 30 - 5(25 \times 4 - 25)$

$= 1000 - 20 + 30 - 5(100 - 25)$

$= 1000 - 20 + 30 - 5 \times 75$

$= 1000 - 20 + 30 - 375$

$= (1000 + 30) - (20 + 375)$

$= 1030 - 395$

$= 635$

Example 2:

At $p = -2$, the value of the expression $3p^2 - \frac{p}{2} + k - 11$ is 12. Find the value of k.

Solution:

Putting $p = -2$ in the given expression, we obtain

$$3p^2 - \frac{p}{2} + k - 11$$
$$= 3(-2)^2 - \frac{(-2)}{2} + k - 11$$
$$= 3 \times 4 + 1 + k - 11$$
$$= 12 + 1 + k - 11$$
$$= 2 + k$$

But the value of the given expression at $p = -2$ is given as 12.

$\therefore 2 + k = 12$

$\Rightarrow k = 12 - 2 = 10$

Therefore, the value of k is 10.

Example 3:

Time taken by a car to cover a distance is given by the expression $\frac{2}{x}-\frac{3}{x^2}+\frac{1}{k}$. Find the value of *k* when *x* = 2 and the time taken by the car is half an hour.

Solution:

The given expression $\frac{2}{x}-\frac{3}{x^2}+\frac{1}{k}$ represents the time taken by the car to cover a particular distance.

When $x = 2$, we have

Time taken by the car = $\frac{1}{2}$ hour

$$\Rightarrow \frac{2}{2}-\frac{3}{2^2}+\frac{1}{k}=\frac{1}{2}$$
$$\Rightarrow 1-\frac{3}{4}+\frac{1}{k}=\frac{1}{2}$$
$$\Rightarrow \frac{1}{4}+\frac{1}{k}=\frac{1}{2}$$
$$\Rightarrow \frac{1}{k}=\frac{1}{2}-\frac{1}{4}$$
$$\Rightarrow \frac{1}{k}=\frac{1}{4}$$
$$\Rightarrow k=4$$

Example 4:

Numeric value of area of a figure is given by the expression $2\left(\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}\right)$. When $a=\frac{5}{2}$ and $c=\frac{5}{8}$ the numeric value of area is $\frac{8}{5}$. Find the value of *b*.

Solution:

Expression for numeric value of area of a figure **=** $2\left(\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}\right)$

When $a=\frac{5}{2}$ and $c=\frac{5}{8}$, we have

Numeric value of area of figure = $\frac{8}{5}$

$$\Rightarrow 2\left(\frac{1}{\left(\frac{5}{2}\right)b}+\frac{1}{b\left(\frac{5}{8}\right)}+\frac{1}{\left(\frac{5}{8}\times\frac{5}{2}\right)}\right)=\frac{8}{5}$$

$$\Rightarrow 2\left(\frac{2}{5b}+\frac{8}{5b}+\frac{16}{25}\right)=\frac{8}{5}$$

$$\Rightarrow 2\left(\frac{10}{5b}+\frac{16}{25}\right)=\frac{8}{5}$$

$$\Rightarrow \frac{4}{b}+\frac{32}{25}=\frac{8}{5}$$

$$\Rightarrow \frac{4}{b}=\frac{8}{5}-\frac{32}{25}$$

$$\Rightarrow \frac{4}{b}=\frac{40-32}{25}$$

$$\Rightarrow \frac{4}{b}=\frac{8}{25}$$

$$\Rightarrow b=\frac{25}{2}$$

Using Variables in Geometric Formulae

Algebraic formulae have applications in many areas. They are also used extensively in geometry, where they are used to shorten the geometric formulae.

For example, we know that the perimeter of an equilateral triangle is three times the length of its side. If we denote the perimeter of the triangle by variable *P* and the length of the side of the triangle by *l*, then we can restate the formulae as:

$$P = 3l$$

Here, the value of *P* will change with a change in the value of *l*. In this formula, *P* and *l* are variables and 3 is a constant.

Similarly, the perimeter of a square is written as:

$$P = 4l$$

Here, the variables *P* and *l* represent the perimeter and length of the side of the square respectively, whereas 4 is a constant.

The perimeter of a regular pentagon is written as:

$$P = 5l$$

Again, the variables *P* and *l* represent the perimeter and length of the side of the pentagon respectively and 5 is a constant.

Let us now write the formula for the perimeter of a rectangle using variables. We know that the perimeter of a rectangle is twice the sum of its length and breadth.

If we denote variables *P*, *l*, and *b* for the respective perimeter, length, and breadth of the rectangle, then the formula of the perimeter of the rectangle in terms of algebraic expression will be written as:

$$P = 2\,(l + b)$$

Here, *P*, *l*,and *b* are variables and 2 is a constant.

Now, let us try to denote the formulae of areas of some geometrical figures like triangle, rectangle, square, etc in terms of algebraic expression and variables. Let us start with the area of a rectangle.

We know that the area of a rectangle is the product of its length and breadth.

If we denote the variables *A*, *l*, and *b* for the area, length, and breadth of the rectangle, then the formula of area of rectangle in terms of variables is

$$A = l \times b = lb$$

Here, *A*, *l*,and *b* are variables. This expression does not contain any constant.

We know that, area of a square = side × side

If we denote the variables *A* and *l* for the area and length of the side of the square, then the formula of area of the square in terms of variables is

$$A = l \times l = l^2$$

Here, *A* and *l* are variables. This expression also does not contain any constant.

We know that the area of a triangle equals one-half the product of the length of its base and its corresponding height.

If we denote the variables *A*, *b*, and *h* for the area, base, and height of the triangle, then the formula of area of triangle is

$$A = \frac{1}{2} \times b \times h = \frac{bh}{2}$$

Here, *A*, *b*, and *h* are variables and $\frac{1}{2}$ is a constant.

Now, we can easily find the perimeter or area of a polygon by putting in the values of the variables in the given formula. For example, for a rectangle of length 7 cm and breadth 4 cm, the perimeter is obtained by putting the value $l = 7$ cm and $b = 4$ cm in the formula, $P = 2\ (l + b)$.

Now, the perimeter of the given rectangle, $P = 2\ (7 + 4)$ cm = 22 cm

We can also find the area of this rectangle by putting the value $l = 7$ cm and $b = 4$ cm in the formula, $A = lb$.

Now, the area of the given rectangle, $A = lb = (7 \times 4)\ \text{cm}^2 = 28\ \text{cm}^2$

Similarly, we can express the formulae used in different areas in terms of variables.

For example, the formula of loss in any transaction is as follows:

Loss = Cost price – Selling price

If we denote loss by *l*, cost price by *c* and selling price by *s* then the above formula can be expressed as follows:

$$l = c - s$$

Now, let us discuss some examples to have better understanding of use of variables in geometric formulae.

Example 1:

The perimeter of a regular octagon (an eight-sided polygon) is given by the formula

$L = 8s$, where L and s are the perimeter and length of the side of the octagon.

Identify the variables and constant in this formula.

Solution:

The formula for the perimeter of a regular octagon is $L = 8s$.

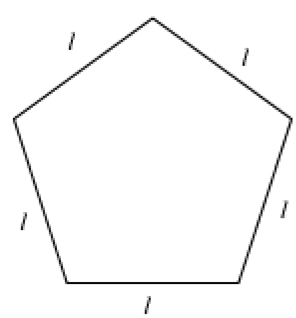
Here, the value of L changes with a change in the value of s. However, the value 8 does not change. Therefore, L and s are variables and 8 is a constant.

Example 2:

Derive the rule to find the perimeter of a regular pentagon by representing the length of one side by a variable l.

Solution:

The length of one side of a regular pentagon is l, where l is a variable.



Perimeter of a regular pentagon = 5 × length of one side

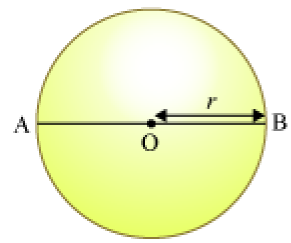
∴ Perimeter of a regular pentagon = $5 \times l = 5l$

Let *p* be the perimeter of the pentagon. Then, the following rule is obtained.

P* = 5*l

Example 3:

Derive the rule to find the diameter of a ball by taking the radius of the ball as the variable *r*.



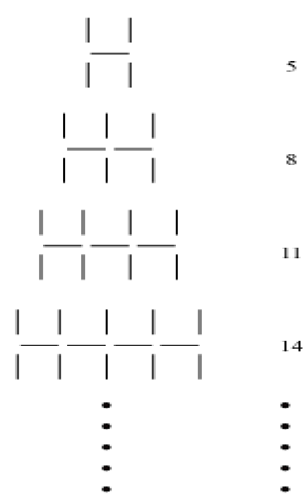
Solution:

We know that the diameter of a ball is twice the radius of the ball.

∴ Diameter of the ball = 2 × radius of the ball = 2 × r = 2r

Expression Of Number Patterns In The Form Of Algebraic Expressions And Vice-Versa

Let us take some matchsticks and join them to form some patterns as shown in the following figure. The number of matchsticks used to make each pattern is written alongside it.



Now, let us try to represent the number of matchsticks used to form each pattern with the help of an algebraic pattern. The given pattern is formed by repeating the shape, which is made of 5 matchsticks.

Thus, we require 5 matchsticks to form the first shape, 8 matchsticks to form the second shape, 11 matchsticks to form the third shape, and so on. Thus, for each extra shape, we require 3 more matchsticks.

Now, we can represent the numbers 5, 8, 11, and 14 as expressions involving the numbers 1, 2, 3, and 4 as

$5 = 3 \times 1 + 2$

$8 = 3 \times 2 + 2$

$11 = 3 \times 3 + 2$

$14 = 3 \times 4 + 2$

Thus, we require $(3n + 2)$ number of matchsticks to form n number of shapes.

We can verify this expression by taking $n = 1, 2, 3$ ….

If we take $n = 3$, then $(3n + 2) = 3 \times 3 + 2 = 9 + 2 = 11$, which is indeed the number of line segments required to make the required shape.

Using this expression $(3n + 2)$, we can calculate the number of line segments required to make any number of shapes. For example, if we want to make 25 such shapes, then we require $(3n + 2) = 3 \times 25 + 2 = 75 + 2 = 77$ matchsticks.

If we actually draw the pattern and count the number of matchsticks, then it will be very time consuming. But the use of algebraic expression has made our job much simpler.

We come across these types of number patterns in Algebra.

Let us discuss one such number pattern, which does not involve any figure.

4, 13, 22, 31, 40 … $9n - 5$

The term which occurs at the n^{th} position in the pattern is given by the expression $9n - 5$. If we want to find the number at the 50^{th} position in the pattern, then we can easily calculate it by substituting $n = 50$ in this expression.

Thus, number at the 50^{th} position in the pattern $= 9 \times 50 - 5 = 450 - 5 = 445$

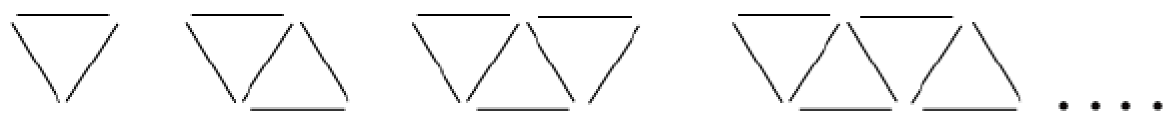
Similarly, the number in 91st position is $9 \times 91 - 5 = 819 - 5 = 814$

Let us discuss some more examples based on number patterns.

Example 1:

Observe the patterns of triangles made from matchsticks. Find the algebraic

expression that gives the number of matchsticks in terms of the number of triangles.



Find the number of matchsticks required to make 29 such triangles. Also find the number of triangles formed, if there are 51 matchsticks.

Solution:

The given pattern is formed by repeating the shape  that is made from 3 matchsticks. The number of matchsticks required to form 1, 2, 3, 4 ... shapes are 3, 5, 7, 9 ... respectively. Here, we can see that for each extra shape, we require 2 more matchsticks.

$3 = 2 \times 1 + 1$

$5 = 2 \times 2 + 1$

$7 = 2 \times 3 + 1$

$9 = 2 \times 4 + 1$

In this way, we require $(2n + 1)$ number of matchsticks to form n number of shapes.

To find the number of matchsticks required to form 29 such shapes, we require to substitute $n = 29$ in the expression, $(2n + 1)$. Therefore, the number of matchsticks required to make 29 shapes is $2 \times 29 + 1 = 58 + 1 = 59$.

To find the number of triangles formed from 51 matchsticks, we have to solve the equation $2n + 1 = 51$.

$2n = 51 - 1 = 50$

$$\therefore\ n = \frac{50}{2} = 25$$

Therefore, 25 triangles can be formed from 51 matchsticks.

Example 2:

Observe the number pattern 8, 13, 18, 23, 28 ………. 5*n* + 3.

Find the 31st and 67th terms of the pattern. Also find the place occupied by the term whose value is 253.

Solution:

It is given that the n^{th} term of the expression is $5n + 3$.

To find the 31st term, we have to substitute $n = 31$ in this expression.

Therefore, 31st term = $5 \times 31 + 3 = 155 + 3 = 158$

Similarly, to find the 67th term, we have to substitute $n = 67$.

Therefore, 67th term = $5 \times 67 + 3 = 335 + 3 = 338$

Now, to find the place occupied by the term whose value is 253, we have to find the value of n which satisfies $5n + 3 = 253$.

$5n = 253 - 3 = 250$

$$\therefore\ n = \frac{250}{5} = 50$$

Therefore, the 50th term of the pattern is 253.