

## 8.1

## Some useful short-cut methods

1. If the age of A,  $t$  years ago, was  $n$  times the age of B and at present A's age is  $n_2$  times that of B, then

$$\text{A's present age} = \left( \frac{n_1 - 1}{n_1 - n_2} \right) n_2 t \text{ years}$$

$$\text{and B's present age} = \left( \frac{n_1 - 1}{n_1 - n_2} \right) t \text{ years}$$

**Explanation**

Let the present age of B be  $x$  years.

Then, the present age of A =  $n_2 \times$  years

Given,  $t$  years ago,

$$n_1 (x - t) = n_2 x - t \text{ or } (n_1 - n_2) x = (n_1 - 1) t$$

$$\text{or, } x = \left( \frac{n_1 - 1}{n_1 - n_2} \right) t.$$

$$\text{Therefore, B's present age} = \left( \frac{n_1 - 1}{n_1 - n_2} \right) t \text{ years}$$

$$\text{and A's present age} = \left( \frac{n_1 - 1}{n_1 - n_2} \right) n_2 t \text{ years.}$$

**NUMERICAL CHALLENGE 8.1**

The age of father is 4 times the age of his son. If 5 years ago father's age was 7 times the age of his son at that time, what is father's present age ?

**Solution**

The father's present age

$$= \left( \frac{n_1 - 1}{n_1 - n_2} \right) n_2 t \quad (\text{Here } n_1 = 7, n_2 = 4 \text{ and } t = 5)$$

$$= \left( \frac{7-1}{7-4} \right) 4 \times 5 = \frac{6 \times 4 \times 5}{3} = 40 \text{ years.}$$

2. The present age of A is  $n_1$  times the present age of B. If  $t$  years hence, the age of A would be  $n_2$  times that of B, then

$$\text{A's present age} = \left( \frac{n_2 - 1}{n_1 - n_2} \right) n_2 t \text{ years}$$

$$\text{and B's present age} = \left( \frac{n_2 - 1}{n_1 - n_2} \right) t \text{ years}$$

### Explanation

Let the present age of B be  $x$  years.

Then, the present age of A =  $n_1 x$

Given,  $t$  years hence,

$$(n_1 x + t) = n_2 (x + t)$$

$$\text{or, } (n_1 - n_2)x = (n_2 - 1)t$$

$$\text{or, } x = \left( \frac{n_2 - 1}{n_1 - n_2} \right) t$$

$$\text{Therefore, B's present age} = \left( \frac{n_2 - 1}{n_1 - n_2} \right) n_1 t \text{ years}$$

$$\text{and A's present age} = \left( \frac{n_2 - 1}{n_1 - n_2} \right) n_1 t \text{ years.}$$

## NUMERICAL CHALLENGE 8.2

The age of Mr Gupta is four times the age of his son. After 10 years, the age of Mr Gupta will be only twice the age of his son. Find the present age of Mr Gupta's son.

### Solution

The present age of Mr Gupta's son

$$= \left( \frac{n_2 - 1}{n_1 - n_2} \right) t$$

$$= \left( \frac{2 - 1}{4 - 2} \right) 10$$

(Here  $n_1 = 4$ ,  $n_2 = 2$  and  $t = 10$ )

= 5 years.

3. The age of A,  $t_1$  years ago, was  $n_1$  times the age of B, If  $t_2$  years hence A's age would be  $n_2$  times that of B, then,  
A's age would be  $n_2$  times that of B, then,

$$\text{A's present age} = \frac{n_1(t_1 + t_2)(n_2 - 1)}{n_1 - n_2} + t_1 \text{ years}$$

$$\text{and B's present age} = \frac{t_2(n_2 - 1) + t_1(n_1 - 1)}{n_1 - n_2} \text{ years.}$$

#### Explanation

Let A's present age =  $x$  years and B's present age =  $y$  years.

Given  $x - t_1 = n_1 (y - t_1)$  and  $x + t_2 = n_2 (y + t_2)$

$$\text{i.e., } x - n_1 y = (1 - n_1) t_1 \quad \dots\dots (1)$$

$$\text{and } x - n_2 y = (-1 + n_2) t_2 \quad \dots\dots (2)$$

Solving Eqs. (1) and (2), we get

$$x = \frac{n_1(t_1 + t_2)(n_2 - 1)}{n_1 - n_2} + t_1$$

$$\text{and, } y = \frac{t_2(n_2 - 1) + t_1(n_1 - 1)}{n_1 - n_2}$$

### NUMERICAL CHALLENGE 8.3

10 years ago Amu's mother was 4 times older than her daughter. After 10 years, the mother will be twice older than the daughter. Find the present age of Anu.

#### Solution

Present age of Anu

$$= \frac{t_2(n_2 - 1) + t_1(n_1 - 1)}{n_1 - n_2}$$

(Here  $n_1 = 4$ ,  $n_2 = 2$ ,  $t_1 = 10$  and  $t_2 = 10$ )

$$= \frac{10(2 - 1) + 10(4 - 1)}{4 - 2} = \frac{10 + 30}{2} = 20 \text{ years.}$$

4. The sum of present ages of A and B is  $S$  years. If,  $t$  years ago, the age of A was  $n$  times the age of B, then

$$\text{Present age of A} = \frac{Sn - t(n - 1)}{n + 1} \text{ years}$$

$$\text{and Present age of B} = \frac{S + t(n - 1)}{n + 1} \text{ years.}$$

### Explanation

Let the present age of A and B be x and y years, respectively.

$$\text{Given } x + y = S \quad \dots\dots (1)$$

$$\text{and } x - t = n(y - t)$$

$$\text{or } x - ny = (1 - n)t \quad \dots\dots (2)$$

Solving Eqs. (1) and (2), we get

$$x = \frac{Sn - t(n - 1)}{n + 1}$$

$$\text{and } y = \frac{S + t(n - 1)}{n + 1}$$

## NUMERICAL CHALLENGE 8.4

The sum of the ages of A and B is 42 years. 3 years back, the age of A was 5 times the age of B. Find the difference between the present ages A and B.

### Solution

Here  $S = 42$ ,  $n = 5$  and  $t = 3$

$\therefore$  Present age of A

$$\begin{aligned} &= \frac{Sn - t(n - 1)}{n + 1} = \frac{42 \times 5 - 3 - 3(5 - 1)}{5 + 1} \\ &= \frac{198}{6} = 33 \text{ years} \end{aligned}$$

and present age of B

$$\begin{aligned} &= \frac{S + t(n - 1)}{n + 1} = \frac{42 + 3(5 - 1)}{5 + 1} \\ &= \frac{54}{6} = 9 \text{ years.} \end{aligned}$$

$\therefore$  Difference between the present ages of A and B =  $33 - 9 = 24$  years.

**Note :** If, instead of sum (S), difference (D) of their ages is given, replace S by D and in the denominator  $(n + 1)$  by  $(n - 1)$  in the above formula.

5. **The sum of present ages of A and B is S years. If, t years hence, the age of A would be n times the age of B, then**

$$\text{present age of A} = \frac{Sn + t(n - 1)}{n + 1} \text{ years}$$

$$\text{and present age of B} = \frac{S - t(n - 1)}{n + 1} \text{ years.}$$

**Explanation**

Let the present ages of A and B be  $x$  and  $y$  years, respectively

$$\text{Given } x + y = S \quad \dots\dots (1)$$

$$\text{and } x + t = n(y + t)$$

$$\text{or } x - ny = t(n - 1) \quad \dots\dots (2)$$

Solving Eqs. (1) and (2), we get

$$x = \frac{Sn + t(n-1)}{n+1}$$

$$\text{and } y = \frac{S - t(n-1)}{n+1}$$

## NUMERICAL CHALLENGE 8.5

The sum of the ages of a son and father is 56 years. After four years, the age of the father will be three times that of the son. Find their respective ages.

**Solution**

The age of father

$$= \frac{Sn + t(n-1)}{n+1} = \frac{56 \times 3 + 4(3-1)}{3+1}$$

(Here  $S = 56$ ,  $t = 4$  and  $n = 3$ )

$$= \frac{176}{4} = 44 \text{ years.}$$

$$\text{The age of son} = \frac{S - t(n-1)}{n+1}$$

$$= \frac{56 - 4(3-1)}{3+1}$$

$$= \frac{48}{4} = 12 \text{ years.}$$

6. If the ratio of the present ages of A and B is  $a : b$  and  $t$  years hence, it will be  $c : d$ , then

$$\text{A's present age} = \frac{at(c-d)}{ad-bc}$$

$$\text{and, B's present age} = \frac{bt(c-d)}{ad-bc}$$

## NUMERICAL CHALLENGE 8.6

The ratio of the age of father and son at present is 6 : 1. After 5 years, the ratio will become 7 : 2. Find the present age of the son.

### Solution

$$\text{The present age of the son} = \frac{bt(c-d)}{ad-bc}$$

$$(\text{Here } a = 6, b = 1, c = 7, d = 2 \text{ and } t = 5)$$

$$= \frac{1 \times 5(7-2)}{6 \times 2 - 1 \times 7} = 5 \text{ years.}$$

**Note :** If, with the ratio of present ages, the ratio of ages  $t$  years ago is given, then replace  $t$  by  $(-t)$  in the above formula.

## NUMERICAL CHALLENGE 8.7

Six years ago Mahesh was twice as old as Suresh. If the ratio of their present ages is 9 : 5 respectively, what is the difference between their present ages ?

### Solution

Present age of Mahesh

$$= \frac{-at(c-d)}{ad-bc}$$

$$= \frac{-9 \times 6(2-1)}{1 \times 9 - 5 \times 2}$$

$$(\text{Here } a = 9, b = 5, c = 2, d = 1 \text{ and } t = 6)$$

$$= 54 \text{ years}$$

Present age of Suresh

$$= \frac{-bt(c-d)}{ad-bc}$$

$$= \frac{-5 \times 6(2-1)}{1 \times 9 - 5 \times 2} = 30 \text{ years.}$$

$\therefore$  Difference of their ages =  $54 - 30 = 24$  years.

## PROBLEMS ON AGES

1. A's age is one-sixths of B's age. B's age will be twice of C's age after 10 years. If C's eighth birthday was celebrated two years ago, then the present age of A must be

- (1) 5 years                      (2) 10 years  
(3) 15 years                    (4) 20 years

**Sol.**  $A = \frac{1}{6} B$ ,  $B + 10 = 2(C + 10)$ ,  $C = 10$

$\therefore B = 30$ ,  $A = 5$  years

2. Sachin was twice as old as Ajay 10 years back. How old is Ajay today if Sachin will be 40 years old 10 years hence ?

- (1) 20 years                      (2) 10 years  
(3) 30 years                      (4) None of these

**Sol.** Sachin's age today = 30 years

Sachin's age 10 years back = 20 years

Ajay's age 10 years back = 10 years

Ajay's age today = 20 years.

3. Two groups of student, whose average ages are 20 years and 30 years, combine to form a third group whose average age is 23 years. What is the ratio of the number of students in the first group to the number of students in the second group ?

- (1) 5 : 2                          (2) 2 : 5  
(3) 7 : 3                          (4) None of these

**Sol.** Let the number of students in the two groups be x and y respectively.

$\therefore 20x + 30y = 23(x + y) \Rightarrow 3x = 7y \Rightarrow \frac{x}{y} = \frac{7}{3}$

4. A years ago, a father was four times his son's age. In six years, his age will be 9 more than twice his son's age. What is the present age of the son ?

- (1) 10 years                      (2) 9 years  
(3) 20 years                      (4) None of these

**Sol.**  $(F - 1) = 4(S - 1)$  ..... (1)

where F and S are the Father's and the Son's ages respectively at present.

$\therefore (F + 6) = 2(S + 6) + 9$  ..... (2)

From Eqs. (1) and (2).  $S = 9$ .

## SOLVED EXAMPLES

5. 5 years ago his mother's age was thrice that of Amit. Amit's present age is 20. What will be the ratio of their ages 10 years from now ?

- (1) 30 : 70                      (2) 1 : 3  
(3) 5 : 2                          (4) 1 : 2

**Sol.** Amit's present age is 20. 5 years ago he was 15; therefore his mother was 45. 10 years from now his mother will be 60; Amit will be 30. Hence the ratio of Amit's age to mother's is 1 : 2.

6. The average age of a class is 15.8 years. The average age of the boys in the class is 16.5 years and that of the girls is 15.4 years. What is the ratio of boys to girls in the class ?

- (1) 1 : 2                          (2) 3 : 4  
(3) 2 : 3                          (4) None of these

**Sol.** Let number of boys = x

Let number of girls = y

$\therefore$  Total number of students = x + y

$\Rightarrow (x + y) \times 15.8 = 16.4x + 15.4y \Rightarrow 0.6x = 0.4y$

$\Rightarrow \frac{x}{y} = \frac{0.4}{0.6} = \frac{2}{3}$

7. The ratio of the ages of the father and the son at present is 7 : 1. After 4 years, the ratio will become 4 : 1. What is the sum of the present ages of the father and the son ?

- (1) 29 years                      (2) 35 years  
(3) 32 years                      (4) None of these

**Sol.** Let the present age of father and son be 7x and x years, respectively.

After 4 years,

age of father = (7x + 4) years

age of son = (x + 4) years

Given,  $\frac{7x + 4}{x + 4} = \frac{4}{1}$

$\Rightarrow 7x + 4 = 4x + 16$

$\Rightarrow 3x = 12$

$\therefore x = 4$

$\therefore 7x + x = 28 + 4 = 32$  years

- 8.** The product of the present ages of Sarita and Gauri is 320. Eight years from now, Sarita's age will be three times the age of Gauri. What was the age of Sarita when Gauri was born ?

- (1) 40 years                      (2) 32 years  
(3) 48 years                      (4) 36 years

**Sol.** Let the present ages of Sarita and Gauri are  $x$  and  $y$ .

Then  $xy = 320$

and  $(x + 8) = 3(y + 8)$

$\Rightarrow x - 3y = 16$

$\Rightarrow x - 3\left(\frac{320}{x}\right) = 16$

$\Rightarrow x^2 - 16x - 960 = 0$

$\Rightarrow (x - 40)(x + 24) = 0$

$\Rightarrow x = 40$  and  $y = 8$

At the time of Gauri born, the age of Sarita is 32 years.

- 9.** In a class, there are 20 boys whose average age is decreased by 2 months, when one boy age 18 years is replaced by a new boy. The age of the new boy is

- (1) 14 years 8 months  
(2) 16 years 4 months  
(3) 15 years  
(4) 17 years 10 months

**Sol.** Total age decreases  $= 20 \times 2 = 40$  months  
 $= 3$  years 4 months

$\therefore$  The age of new boy  $= 18$  years  $- 3$  years  
4 months  
 $= 14$  years 8 months

- 10.** Ashu's mother was three times as old as Ashu 5 years ago. After 5 years, she will be twice as old as Ashu. How old is Ashu today ?

- (1) 35 years                      (2) 10 years  
(3) 20 years                      (4) 15 years

**Sol.** Let the age of Ashu at present be  $x$  years and her mother be  $y$  years.

Now, according to the question,

$3(x - 5) = (y - 5)$

or,  $3x - 15 = y - 5$

or,  $3x - y = 10$  ..... (1)

Again, according to the question,

$2(x + 5) = (y + 5)$

or,  $2x + 10 = y + 5$

or,  $2x - y = -5$

Subtracting Eq. (2) from (1), we get ..... (2)

$x = 15$

Hence, Ashu's today's age is 15 years.



## EXERCISE

- 1.** Seven years ago the ratio of the ages of P and Q (in years) was 7 : 6. Which of the following cannot be the ratio of their ages 6 years from now?
    - (1) 13 : 11
    - (2) 15 : 14
    - (3) 13 : 12
    - (4) 16 : 15
  - 2.** The ratio of the present ages of Anand and Bala is 8 : 3. When Anand was 30 years old, Bala was 5 years old. Find the present age of Bala. (In years)
    - (1) 10
    - (2) 12
    - (3) 15
    - (4) 20
  - 3.** Nine years ago A's age and B's age were in the ratio 5 : 7. Which of the following cannot be the ratio of their ages 5 years from now?
    - (1) 11 : 13
    - (2) 13 : 19
    - (3) 21 : 25
    - (4) 15 : 16
  - 4.** The ratio of the present ages of Ram and Shyam is 3 : 2. Which of the following cannot be the ratio of their ages 20 years ago?
    - (1) 8 : 5
    - (2) 17 : 10
    - (3) 9 : 5
    - (4) 7 : 5
  - 5.** Amit is as much younger to Barkha as he is older to Chaman. If the sum of the ages of Barkha and Chaman is 48 years, what is the present age of Amit?
    - (1) 18 years
    - (2) 36 years
    - (3) 24 years
    - (4) 28 years
  - 6.** Bipin is 6 times old as Alok. Bipin's age will be twice of Chandan's age after 10 years. If Chandan's 7<sup>th</sup> birthday was celebrated 3 years ago, what is Alok's present age?
    - (1) 15 years
    - (2) 12 years
    - (3) 5 years
    - (4) none of these
  - 7.** Renuka got married 8 years ago. Today her age is  $1\frac{1}{3}$  times her age at the time of marriage. Her daughter's age is  $\frac{1}{8}$  times her age. Her daughter's age is :
    - (1) 3 years
    - (2) 4 years
    - (3) 6 years
    - (4) 8 years
  - 8.** Ten years ago B was twice of A in age. If the ratio of their present ages is 4 : 3, what is the sum of their present ages?
    - (1) 25 years
    - (2) 30 years
    - (3) 40 years
    - (4) 35 years
  - 9.** The sum of the ages of Aryabhata and Shridhar is 45 years. Five years ago the product of their ages was 4 times the Aryabhata's age at that time. The present ages of Aryabhata and Shridhar respectively are :
    - (1) 25 and 20
    - (2) 35 and 10
    - (3) 36 and 9
    - (4) 40 and 5
  - 10.** The ages of A, B and C together total 185 years. B is twice as old as A and C is 17 years older than A. Then, the respective ages of A, B and C are
    - (1) 40, 86 and 59 years
    - (2) 42, 84 and 59 years
    - (3) 40, 80 and 65 years
    - (4) None of these
  - 11.** The sum of ages of a father and son is 45 years. Five years ago, the product of their ages was four times the father's age at that time. The present age of the father is
    - (1) 39 years
    - (2) 36 years
    - (3) 25 years
    - (4) None of these
  - 12.** If 1 is added to the age of the elder sister, then the ratio of the ages of two sisters becomes 0.5 : 1, but if 2 is subtracted from the age of the younger one, the ratio becomes 1 : 3. Find the age of the two sisters.
    - (1) 8 and 5 years
    - (2) 11 and 6 years
    - (3) 9 and 5 years
    - (4) 8 and 6 years
  - 13.** The age of the father 5 years ago was 5 times the age of his son. At present the father's age is 3 times that of his son. What is the present age of the father ?
    - (1) 33 years
    - (2) 30 years
    - (3) 45 years
    - (4) None of these
  - 14.** Sonu is 4 years younger than Manu while dolly is 4 years younger than Sumit but one-fifth times as old as Sonu. If Sumit is eight years old, how many times as old is Manu as Dolly ?
    - (1) 6
    - (2)  $1\frac{1}{2}$
    - (3) 3
    - (4) None of these

- ## ANSWER KEY

[illegible]