ALL INDIA TEST SERIES

FULL TEST – 22

JEE (Main)

Time Allotted: 3 Hours **Maximum Marks: 300**

General Instructions:

- The test consists of total 90 questions.
- Each subject (PCM) has 30 questions.
- This question paper contains **Three Parts.**
- **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics.
- Each part has only two sections: **Section-A and Section-B**.
- **Section – A** : Attempt all questions.
- **Section – B :** Do any five questions out of 10 Questions.

Section-A (01 – 20, 31 – 50, 61 – 80) *contains 60 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

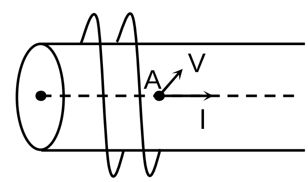
Section-B (21 – 30, 51 – 60, 81 – 90) *contains 30 Numerical answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

Physics PART – A

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. A slightly divergent beam of non-relativistic charged particles accelerated by a potential difference V passes through a point A at the axis of a long straight solenoid. The beam is brought to focus at a distance l from A at two successive values of magnetic induction B_1 and B_2. Find the specific charge q/m of particles.



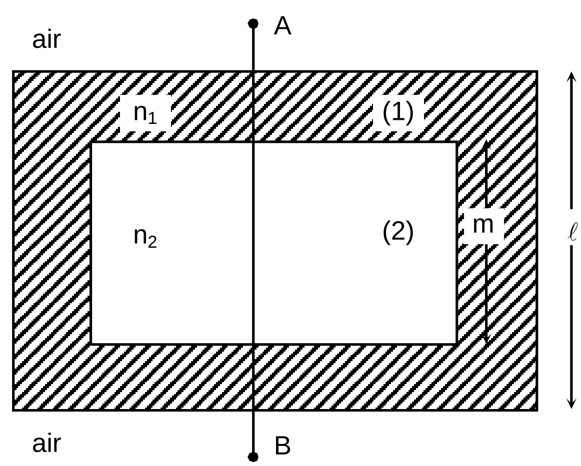
(A) $\dfrac{2\pi^2 V}{\ell^2 (B_2 - B_1)^2}$

(B) $\dfrac{4\pi^2 V}{\ell^2 (B_2 - B_1)^2}$

(C) $\dfrac{8\pi^2 V}{\ell^2 (B_2 + B_1)^2}$

(D) $\dfrac{8\pi^2 V}{\ell^2 (B_2 - B_1)^2}$

2. In a thick glass slab of thickness ℓ and refractive index n_1, a cuboidal cavity of thickness m is carved as shown in the figure and is filled with a liquid of refractive index n_2 ($n_1 > n_2$). The ratio of $\dfrac{\ell}{m}$, so that shift produced by this slab is zero when an observer A observes an object B with the paraxial rays is



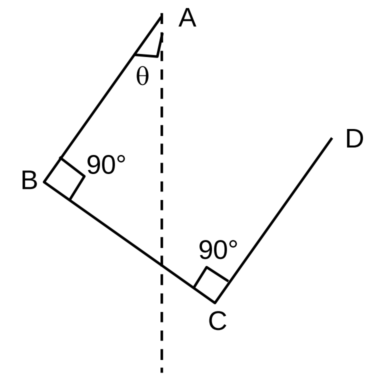
(A) $\dfrac{n_1 - n_2}{n_2 - 1}$

(B) $\dfrac{n_1 - n_2}{n_2 (n_1 - 1)}$

(C) $\dfrac{n_1 - n_2}{n_1 - 1}$

(D) $\dfrac{n_1 - n_2}{n_1 (n_2 - 1)}$

3. Three identical rods are rigidly joined and hinged at point A as shown in the figure. The angle θ made by the rod AB with the vertical is



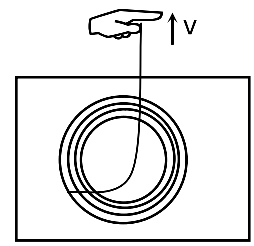
(A) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(B) $\tan^{-1}\left(\frac{3}{4}\right)$

(C) $\tan^{-1}(1)$

(D) $\tan^{-1}\left(\frac{4}{3}\right)$

4. A uniform rope of linear mass density λ and length ℓ is coiled on a smooth horizontal surface. One end is pulled up with a constant velocity v. Then the average power delivered by the external agent in pulling the entire rope just off the horizontal surface is



(A) $\frac{1}{2}\lambda\ell v^2 + \frac{\lambda\ell^2 g}{2}$

(B) $\lambda\ell g v$

(C) $\lambda v^3 + \frac{\lambda\ell v g}{2}$

(D) $\lambda\ell v g + \frac{1}{2}\lambda v^3$

5. A capillary tube with inner cross-section in the form of a square of side 'a' is dipped vertically in a liquid of density ρ and surface tension σ which wet the surface of capillary tube with angle of contact θ. The approximate height to which liquid will be raised in the tube is (Neglect the effect of surface Tension at the corners of capillary tube)

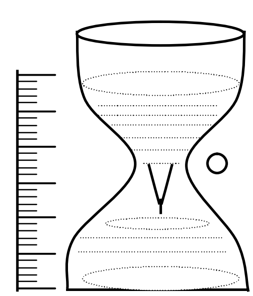
(A) $\frac{2\sigma\cos\theta}{a\rho g}$

(B) $\frac{4\sigma\cos\theta}{a\rho g}$

(C) $\frac{8\sigma\cos\theta}{a\rho g}$

(D) None of these

6. A water clock [clepsydra] used in ancient Greece is designed as a vessel with a small orifice O of area a. The water level falls at a constant rate v_1 in the vessel. The time is determined according to the level of water in the vessel. What should be equation corresponding to the shape of the vessel for the time scale to be uniform?



(A) $y = Kx^4$ where $K = \frac{\pi^2 v_1^2}{2ga^2}$

(B) $y = Kx^3$ where $K = \frac{\pi^2 v_1^2}{2ga^2}$

(C) $y = Kx^4$ where $K = \dfrac{\pi^2 v_1^2}{ga}$

(D) $y = Kx^4$ where $K = \dfrac{\pi^2 v_1^2}{ga^2}$

7. A pipe of length ℓ_1, closed at one end is kept in a chamber of gas of density ρ_1. A second pipe open at both ends is placed in a second chamber of gas of density ρ_2. The compressibility of both the gases are equal. Calculate the length of the second pipe if frequency of first overtone in both the cases are equal.

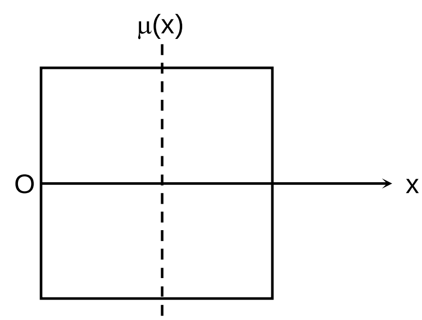
(A) $\dfrac{4}{3}\ell_1\sqrt{\dfrac{\rho_2}{\rho_1}}$

(B) $\dfrac{4}{3}\ell_1\sqrt{\dfrac{\rho_1}{\rho_2}}$

(C) $\ell_1\sqrt{\dfrac{\rho_2}{\rho_1}}$

(D) $\ell_1\sqrt{\dfrac{\rho_1}{\rho_2}}$

8. A composite glass slab is manufactured so that its refractive index varies along its thickness according to the relation $\mu(x) = \left[1 + \dfrac{\alpha x}{t}\right]$ where t is the thickness of the slab.
The optical path length introduced by the slab when it is placed in the path of light passing normally through it, is given by



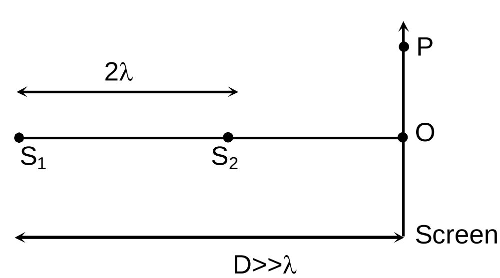
(A) $(1 + \alpha)t$

(B) $\left(1 + \dfrac{\alpha}{2}\right)t$

(C) $\dfrac{t}{1+\alpha}$

(D) $\dfrac{t}{\alpha}\left[\ln(1+\alpha)\right]$

9. Two sources S_1 and S_2 emitting coherent light waves of wavelength λ in the same phase are situated as shown. The distance OP, so that the light intensity detected at P is equal to that at O is



(A) $D\sqrt{2}$

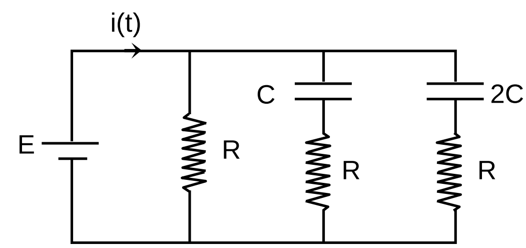
(B) $D/2$

(C) $D\sqrt{3}$

(D) $D/\sqrt{3}$

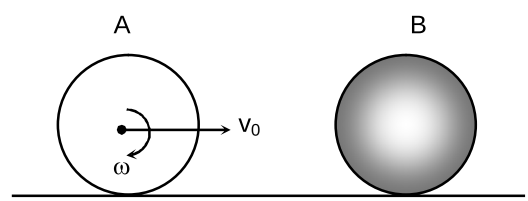
10. The voltage of AC source is E = 220 sin ($\omega t + \pi/6$) and the AC in the circuit is I = 10 sin ($\omega t - \pi/6$). The average power dissipated is
(A) 150 W
(B) 550 W
(C) 250 W
(D) 50 W

11. Two capacitors, shown in the circuit, are initially uncharged and the cell is ideal. The switch S is closed at t = 0. Which of the following functions represents the current i(t), through the cell as a function of time?



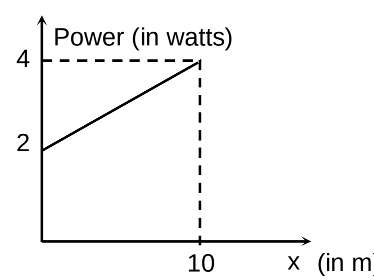
(A) $i(t) = i_0 + i_1 e^{-t/\tau};\ \tau = 3C \times \frac{R}{3}$
(B) $i(t) = i_0 + i_1 e^{-t/\tau} + i_2 e^{-t/2\tau};\tau = RC$
(C) $i(t) = i_1 + i_1 e^{-t/\tau};\tau = 3C \times \frac{R}{3}$
(D) $i(t) = i_0 + i_1 e^{-t/\tau};\tau = 3RC$
Where i_0, i_1 and i_2 are constants.

12. A hollow smooth uniform sphere A of mass m rolls without sliding on a smooth horizontal surface. It collides head on elastically with another stationary smooth solid sphere B of the same mass m and same radius. The ratio of kinetic energy of B to that of A after the collision is



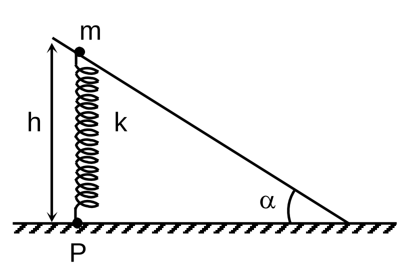
(A) 1:1
(B) 2:3
(C) 3:2
(D) None of these

13. A particle A of mass $\frac{10}{7}$ Kg is moving in the positive direction of x-axis. Its initial position is x = 0 and initial velocity is 1 m/s. The velocity of the particle at x = 10 m is: (use the graph given).



(A) 4 m/s
(B) 2 m/s
(C) $3\sqrt{2}$ m/s
(D) $\frac{100}{3}$ m/s

14. A bead of mass m is sliding down the fixed inclined rod without friction. It is connected to a point P on the horizontal surface with a light spring of spring constant k. The bead is initially released from rest and the spring is non deformed and vertical. The bead just stops at the bottom of the inclined rod. Find the angle which the inclined rod makes with the horizontal.



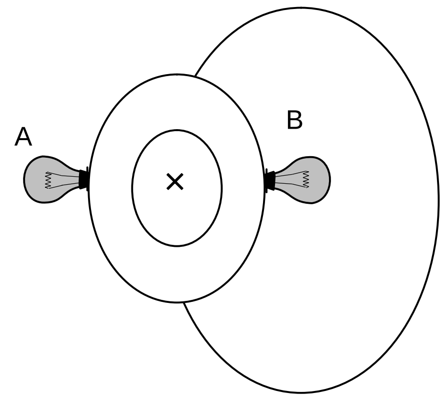
(A) $\cot^{-1}\left[1+\sqrt{\frac{2mg}{kh}}\right]$

(B) $\tan^{-1}\left[1+\sqrt{\frac{2mg}{kh}}\right]$

(C) $\cot^{-1}\left[1+\sqrt{\frac{mg}{kh}}\right]$

(D) $\tan^{-1}\left[1+\sqrt{\frac{mg}{kh}}\right]$

15. A very long solenoid perpendicular to the page generates a downward magnetic field whose magnitude increases with the time. A conducting wire loop around the solenoid contains two identical bulbs A and B which are glowing. Two points diametrically opposite on the wire loop are shorted with another wire lying to the right of bulb B in the plane of the page as shown in the figure. After the shorting wire is inserted.



(A) Bulb A goes out and bulb B dims.
(B) Bulb A goes out and bulb B gets brighter.
(C) Bulb B goes out and bulb A dims.
(D) Bulb B goes out and bulb A gets brighter

16. A ray of light traveling along positive Z-axis is reflected twice:

(i) for the first time, by a mirror whose normal is along $-\left(\hat{i}+\hat{k}\right)$

(ii) for the second time, by a mirror whose normal is along $\left(\hat{i}+\hat{k}+\hat{j}\right)$, where the symbols have their usual meanings. The final ray is along

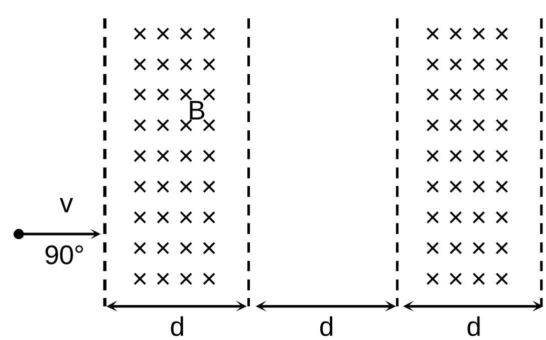
(A) $\hat{j}+\hat{k}$

(B) $\hat{k}+\hat{i}$

(C) $2\hat{j}+2\hat{k}-\hat{i}$

(D) $\hat{j}+\hat{k}-2\hat{i}$

17. A positive charged particle of mass m and charge q is projected with a velocity v as shown in the figure. If radius of curvature of charged particle in magnetic field is R(2d < R < 3d), then time elapsed by charged particle in magnetic field regions is



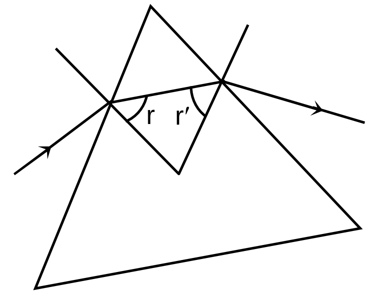
(A) $\frac{m}{2qB}$

(B) $\frac{m}{qB}\sin^{-1}\left[\frac{2d}{R}\right]$

(C) $\frac{m}{qB}$

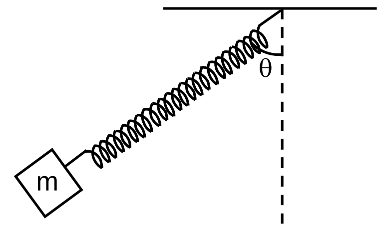
(D) $\frac{m}{qB}\sin^{-1}\left[\frac{d}{R}\right]$

18. r and r′ denote the angles inside an equilateral prism, as usual, in degrees. Consider that during some time interval from t = 0 to t = t, r′ varies with time as $r' = 10 + t^2$, during this time r will vary as (assume that r and r′ are in degree)



(A) $50 - t^2$
(B) $50 + t^2$
(C) $60 - t^2$
(D) $60 + t^2$

19. A block of mass m is suspended by means of an ideal spring of force constant K from ceiling of a car which is moving along a circular path of radius r with acceleration 'a'. The time period of oscillation of the block when it is displaced along the spring, will be



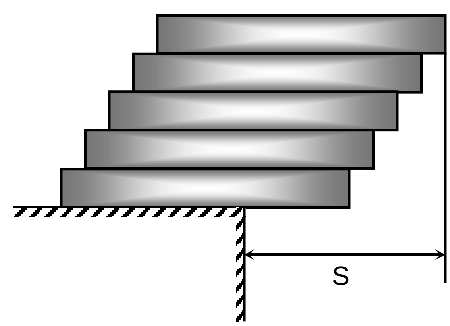
(A) $2\pi\sqrt{\dfrac{mg+ma}{K}}$

(B) $2\pi\sqrt{\dfrac{m}{K\sqrt{g^2+a^2}}}$

(C) $2\pi\sqrt{\dfrac{m}{K}}$

(D) $2\pi\sqrt{\dfrac{m}{K^2+g^2+a^2}}$

20. Five identical bricks each of length L are piled with one on the top of the other on a table as shown in the figure. The maximum distance S the top brick can overhang the table with the system still balanced is



(A) $\dfrac{7}{8}L$

(B) $\dfrac{2}{3}L$

(C) $\dfrac{21L}{32}$

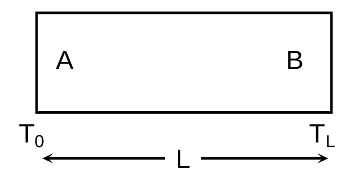
(D) $\dfrac{137L}{120}$

SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

21. The binding energy of deuteron $\left({}_1^2H\right)$ is 1.15 MeV per nucleon and an alpha particle $\left({}_2^4He\right)$ has a binding energy of 7.1 MeV per nucleon. Then in the reaction.
$${}_1^2H + {}_1^2H \rightarrow {}_2^4He + Q$$
What is the energy released Q (in MeV)?

22. The temperature of a mono-atomic gas in a uniform container of length L varies linearly from T_0 to T_L as shown in the figure. If the molecular weight of the gas is M_0, then the time taken by a wave pulse in traveling from end A to end B is $\left(\frac{2L}{\sqrt{T_L}+\sqrt{T_0}}\right)\sqrt{\frac{kM_0}{R}}$. Find the value of k.

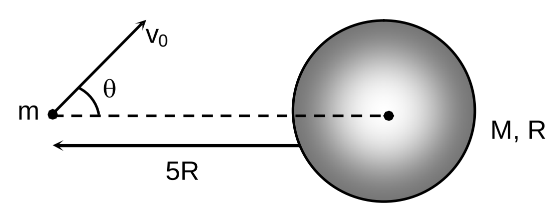


23. In a series LCR circuit, the difference of the frequencies at which current amplitude falls to $\frac{1}{\sqrt{2}}$ times the current amplitude at resonance is $\frac{kR}{\pi L}$. Find the value of k.

24. The relation between internal energy U, pressure P and volume V of an ideal gas in an adiabatic process is U = 2 + 3PV. If $(C_P + C_V) = kR$. Find the value of k.

25. A monoatomic ideal gas is taken through a process whose equation is given by:
$P = KV^{-1/2}$
Where P is the pressure and V is the volume of the gas. The molar heat capacity of the gas in the above process is ($C_V + \alpha R$). Find the value of α.

26. A spaceship is sent to investigate a planet of mass M and radius R. While hanging motionless in space at a distance 5R from the centre of the planet, the spaceship fires an instrument package with speed v_0 as shown in the figures. The package has mass m, which is much smaller than the mass of the spaceship. The angle θ for which the package just grazes the surface of the planet is $\sin^{-1}\left[\frac{1}{5}\sqrt{1+\frac{kGM}{5v_0^2R}}\right]$. Find the value of k.



27. In a resonance tube with tuning fork of frequency 512Hz, first resonance occurs at a length of 30.3 cm and second resonance occurs at a length of 63.7cm. The maximum possible error in the measurement of the speed of the sound is (in cm/s)

28. A uniform rope of length 12m and mass 6kg hangs from a rigid support. A block of mass 2kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06m is produced at the lower end of the rope. What is the wavelength (in m) of the pulse when it reaches the top of the rope?

29. One end of a uniform rod of length 4.5m is placed in boiling water while its other end is placed in melting ice. A point P on the rod is maintained at a constant temperature of 800 °C. The mass of the steam produced per second is equal to the mass of ice melted per second. If latent heat of steam is seven times the latent heat of ice, the distance (in m) of the point P from the steam chamber must be equal to

30. A beam of light of wavelength 600nm from a distant source falls normally on a single slit 1mm wide and a resulting diffraction pattern is observed on a screen 2m away. The distance between the first dark fringes on the either side of the central bright fringe is (in mm)

Chemistry PART – B

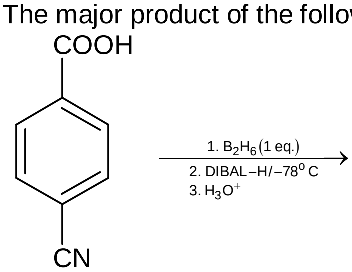
SECTION – A
(One Options Correct Type)

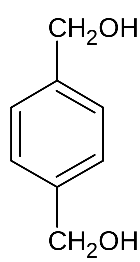
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

31. Which solution will possess the highest freezing point?
    (A) 0.1% NaCl solution
    (B) 0.1% sucrose solution
    (C) 0.1% $Al_2(SO_4)_3$ solution
    (D) 0.1% $K_4[Fe(CN)_6]$ solution

32. Salicyclic acid when heated with Zn dust gives
    (A) Benzene
    (B) Phenol
    (C) Salicylaldehyde
    (D) Benzoic acid

33. The major product of the following reaction is



(A)


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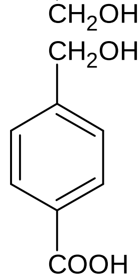

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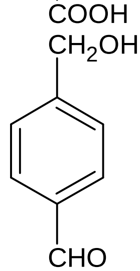
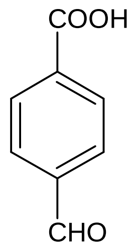
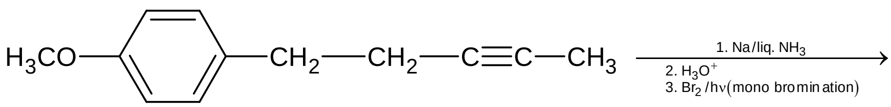
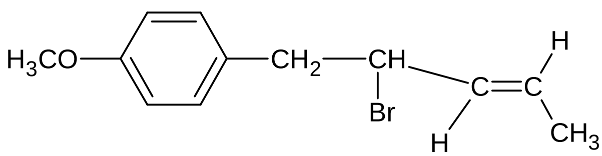
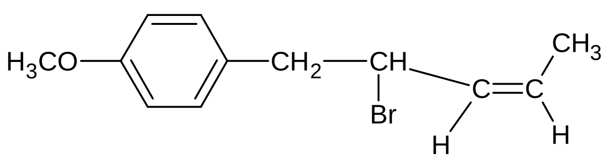


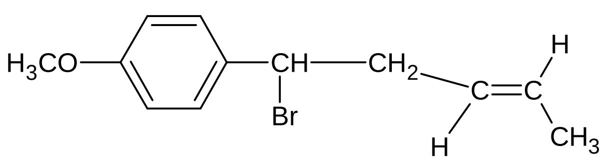
(D) 

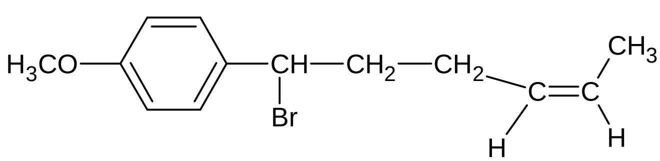
34. The major product of the following reaction is



(A) 

(B) 

(C) 

(D) 

35. Which of the following reagents may be used to distinguish between $\beta-$naphthol and adipic acid?

(A) Aqueous NaOH
(B) Tollen's reagent
(C) Molish reagent
(D) Neutral $FeCl_3$

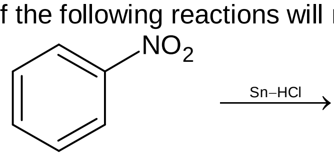
36. A solution containing 0.5 g of $CrCl_3.6H_2O$ was passed through a cation exchange resin and acid coming out of the cation exchange resin required 30.02 ml of 0.125 M NaOH. The correct formula of the complex [Molecular weight of the complex = 226.6 g mol^{-1}].

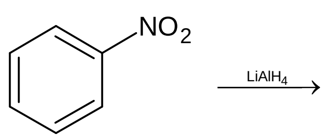
(A) $\left[Cr\left(H_2O\right)_6\right]Cl_3$
(B) $\left[Cr\left(H_2O\right)_5Cl\right]Cl_2.H_2O$
(C) $\left[Cr\left(H_2O\right)_4Cl_2\right]Cl.2H_2O$
(D) $\left[Cr\left(H_2O\right)_3Cl_3\right].3H_2O$

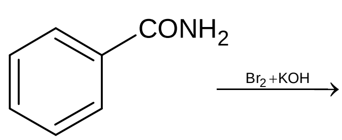
37. Which of the following is not actinoid?
(A) Curium
(B) Californium
(C) Uranium
(D) Terbium

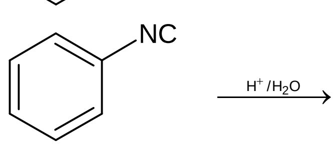
38. Which of the following statements is not correct regarding conformers?
(A) Conformers generally have negligibly small difference in their potential energy
(B) Conformers of ethane can be separated at room temperature
(C) Gauche conformers of ethylene glycol is more stable than its anti-conformer
(D) Anti-conformer of butane is more stable than its Gauche conformer

39. If the ionization potential of Mg 9.48 eV, the ionization of calcium will be
(A) 18.96 eV
(B) 9.90 eV
(C) 6.42 eV
(D) same as that of Mg

40. Which of the following reactions will not give aniline as major product?
(A)


(B)


(C)


(D)


41. Large difference in boiling points is observed in
(A) O and S
(B) S to Se
(C) Se to Te
(D) Te to Po

42. Which of the following species is not aromatic?
(A) Benzene
(B) Cyclooctatetraenyl dianion
(C) Tropylium ion
(D) Cyclopentadienyl cation

43. Which of the following reactions does not take place?

(I) $BF_3 + F^- \longrightarrow BF_4^-$

(II) $BF_3 + 3F^- \longrightarrow BF_6^{3-}$

(III) $AlF_3 + 3F^- \longrightarrow AlF_6^{3-}$

(IV) $SiF_4 + 2F^- \longrightarrow SiF_6^{2-}$

(A) Only (I)
(B) Only (II) and (IV)
(C) Only (II)
(D) Only (I) and (II)

44. Among the following which does not have planar structure?

(A) $N(SiH_3)_3$

(B) $N(CH_3)_3$

(C) CO_3^{2-}

(D) NO_3^{2-}

45. When H_2S gas passed through the HCl containing aqueous solution of $CuCl_2, HgCl_2, BiCl_3$ and $CoCl_2$. It does not precipitate out

(A) CuS
(B) HgS
(C) Bi_2S_3
(D) CoS

46. Which of the following statements is correct regarding defect in crystals?

(A) Schottky defect in crystals is observed when unequal number of cations and anions are missing from the lattice.
(B) Frankel defect in crystals is observed when equal number of cations and anions are missing from the lattice.
(C) Schottky defect is also called dislocation defect.
(D) Frankel defect in crystals is observed when a cation leaves its normal site and occupies an interstitial site.

47. To 25 ml H_2O_2 solution, excess of acidified solution of KI was added. The iodine liberated required 20 ml of 0.3 M $Na_2S_2O_3$ solution. The volume strength of H_2O_2 solution is

(A) 3.244 V
(B) 1.344 V
(C) 5.4 V
(D) 4.08 V

48. Threshold frequency of a metal is $5\times10^{14}\ s^{-1}$ upon which $1\times10^{15}\ s^{-1}$ frequency light is focused. The maximum kinetic energy of emitted electron is [Given $h = 6.6\times10^{-34}\ J\times s$]

(A) 3.3×10^{-19} J

(B) 3.3×10^{-21} J

(C) 6.6×10^{-21} J

(D) 6.6×10^{-20} J

49. Which of the following is a reducing sugar?
(A) Starch
(B) Sucrose
(C) Cellulose
(D) Lactose

50. Non-stick cookwares generally have a coating of a polymer, whose monomer is
(A) $CH_2 = CH - CN$
(B) $CH_2 = CH - Cl$
(C) $CH_2 = CH - CH_3$
(D) $CF_2 = CF_2$

SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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51. A current of 96.5 A is passed for 18 minutes between copper electrodes in 500 ml aqueous solution of $CuSO_4$ having 1 moles of $CuSO_4$. The molarity of solution after electrolysis would be

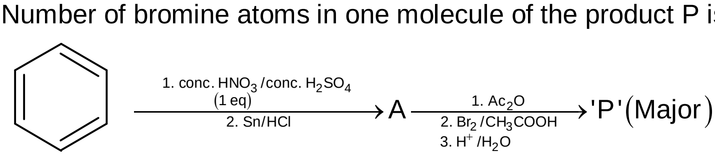
52. The enthalpy of hydrogenation of cyclohexene is $119.5\ \text{kJ mol}^{-1}$. If resonance energy of benzene is $-150.4\ \text{kJ mol}^{-1}$, its enthalpy of hydrogenation would be

53. The value of $\ell n\frac{A_2}{A_1}$ would be if the rate of reactions whose rate constants, activation energies and Arrhenius constants are k_1 and k_2, E_{a_1} and E_{a_2}, A_1 and A_2 respectively at 300 K, are equal at 300 K. Given that activation energy E_{a_2} is 2RT more than activation energy E_{a_1}.

54. The equilibrium constant K_c for the following is
$A(g) + 2B(g) \rightleftharpoons 2C(g)$
If 4 moles of each A and B were present in 10 litre and at equilibrium form 2 moles of C.

55. The value of $\frac{M_B}{M_A}$ would be [M_B and M_A are molar masses of gas B and gas A respectively] if 2g of gas 'B' is added to a container having 1 g of gas A at 4 bar pressure at T K so that total pressure becomes 6 bar at same temperature T K. Assume gases behave an ideal gas.

56. Number of bromine atoms in one molecule of the product P is



57. The percentage of oxygen in compound 'P' formed in the following sequence of reaction is
$$CH_3(CH_2)_5CH_3 \xrightarrow[\substack{2.\ Br_2/h\nu \\ 3.\ Mg/ether \\ 4.\ H_2O}]{1.\ Cr_2O_3/Al_2O_3\ \ 773K,\ 10-20\ atm} P(\text{Major})$$

58. Total number of stereoisomers of the following compound are

$$H_3C-\underset{\displaystyle OH}{\underset{|}{CH}}-CH=CH-CH=CH-Cl$$

59. The bond order in O_2^- is

60. Number of moles of oxygen produced by hydrolysis of 1 mole of XeF_4 is

Mathematics

PART – C

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

61. The relation R defined in N as $aRb \Leftrightarrow b$ is divisible by a is
(A) reflexive but not symmetric
(B) symmetric but not transitive
(C) symmetric and transitive
(D) none of these

62. If $1, \omega, \omega^2$ are three cube roots of unity, then $(a+b\omega+c\omega^2)^3 + (a+b\omega^2+c\omega)^3$ is equal to, if $a+b+c=0$
(A) $27abc$
(B) 0
(C) $3abc$
(D) none of these

63. Given that the equation $z^2+(p+iq)z+r+is=0,$ where p,q,r,s are real and non-zero has a real root, then
(A) $pqr = r^2+p^2s$
(B) $prs = q^2+r^2p$
(C) $qrs = p^2+s^2q$
(D) $pqs = s^2+q^2r$

64. The maximum sum of the series $20+19\frac{1}{3}+18\frac{2}{3}+.........$ is
(A) 310
(B) 300
(C) 320
(D) none of these

65. If the roots of $x^2+x+a=0$ exceed a, then
(A) $2<a<3$
(B) $a>3$
(C) $-3<a<3$
(D) $a<-2$

66. In a certain test there are n questions. In the test 2^{n-i} students gave wrong answers to at least i questions, where $i=1, 2, ...n$. If the total number of wrong answers given is 2047, then n is equal to
(A) 10
(B) 11
(C) 12
(D) 13

67. In the expansion of $(1+x+x^3+x^4)^{10}$, the coefficient of x^4 is

(A) $^{40}C_4$

(B) $^{10}C_4$

(C) 210

(D) 310

68. One of the roots of the given equation $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$ is

(A) $-(a+b)$

(B) $-(b+c)$

(C) $-a$

(D) $-(a+b+c)$

69. If $\tan\theta = \dfrac{x\sin\varphi}{1-x\cos\varphi}$ and $\tan\varphi = \dfrac{y\sin\theta}{1-y\cos\theta}$, then $\dfrac{x}{y}$ is equal to

(A) $\dfrac{\sin\varphi}{\sin\theta}$

(B) $\dfrac{\sin\theta}{\sin\varphi}$

(C) $\dfrac{\sin\varphi}{1-\cos\theta}$

(D) $\dfrac{\sin\theta}{1-\cos\varphi}$

70. If $\begin{vmatrix} \cos(A+B) & -\sin(A+B) & \cos 2B \\ \sin A & \cos A & \sin B \\ -\cos A & \sin A & \cos B \end{vmatrix} = 0$, then B is equal to

(A) $(2n+1)\dfrac{\pi}{2}$

(B) $n\pi$

(C) $(2n+1)\dfrac{\pi}{3}$

(D) $2n\pi$

71. If $a+b+c=0$, a, b, c are distinct non-zero real number and $p \neq 0$, the lines $ax+(b+c)y=p$, $bx+(c+a)y=p$ and $cx+(a+b)y=p$

(A) do not intersect

(B) intersect

(C) are concurrent

(D) none of these

72. Tangents are drawn from the point (4, 3) to the circle $x^2 + y^2 = 9$. The area of the triangle formed by them and the line joining their points of contact is

(A) $\frac{24}{25}$

(B) $\frac{64}{25}$

(C) $\frac{192}{25}$

(D) $\frac{192}{5}$

73. The centre of an ellipse is C and PN is any ordinate and A, A' are the end points of major axis, then the value of $\frac{PN^2}{AN \cdot A'N}$ is

(A) $\frac{b^2}{a^2}$

(B) $\frac{a^2}{b^2}$

(C) $a^2 + b^2$

(D) 1

74. The vector equation of the plane through the point (2, 1, –1) and passing through the line of intersection of the plane $r.(i + 3j - k) = 0$ and $r.(j + 2k) = 0$ is

(A) $r.(i + 9j + 11k) = 0$

(B) $r.(i + 9j + 11k) = 6$

(C) $r.(i - 3j - 13k) = 0$

(D) none of these

75. The moment about the point $M(-2,\ 4,\ -6)$ of the force represented in magnitude and position by $\overrightarrow{AB}$ where the points A and B have the co-ordinates $(1,\ 2,\ -3)$ and $(3,\ -4,\ 2)$ respectively, is

(A) 8i – 9j – 14k

(B) 2i – 6j + 5k

(C) – 3i + 2j – 3k

(D) – 5i + 8j – 8k

76. Solution of $(xy\cos xy + \sin xy)dx + x^2 \cos xy\, dy = 0$ is

(A) $x\sin(xy) = k$

(B) $xy\sin(xy) = k$

(C) $\frac{x}{y}\sin(xy) = k$

(D) $x^2y\sin(xy) = k$

77. The solution of $ye^{-x/y}dx-(xe^{-x/y}+y^3)dy=0$ is

(A) $\frac{y^2}{2}+e^{-x/y}=k$

(B) $\frac{x^2}{2}+e^{-x/y}=k$

(C) $\frac{x^2}{2}+e^{x/y}=k$

(D) $\frac{y^2}{2}+e^{x/y}=k$

78. For a biased die, the probabilities for different faces to turn up are

Face :	1	2	3	4	5	6
Probability :	0.2	0.22	0.11	0.25	0.05	0.17

The die is tossed and you are told that either face 4 or face 5 has turned up. The probability that it is face 4 is

(A) $\frac{1}{6}$

(B) $\frac{1}{4}$

(C) $\frac{5}{6}$

(D) none of these

79. The negation of the compound proposition $p\vee(\sim p\vee q)$ is

(A) $(p\wedge\sim q)\wedge\sim p$

(B) $(p\wedge\sim q)\vee\sim p$

(C) $(p\vee\sim q)\vee\sim p$

(D) none of these

80. A tangent to a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ intercepts a length of unity from each of the co-ordinate axes, then the point (a, b) lies on the rectangular hyperbola

(A) $x^2-y^2=2$

(B) $x^2-y^2=1$

(C) $x^2-y^2=-1$

(D) none of these

SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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81. The number of solutions of the equation $\sin^{-1}x=2\tan^{-1}x$ is

82. If $(a+3b)(3a+b)=4h^2$, and the angle between the lines represented by $ax^2+2hxy+by^2=0$ is $\frac{2\pi}{k}$, then k is equal to

83. If $f(x)=\frac{\cos^2 x+\sin^4 x}{\sin^2 x+\cos^4 x}$ for $x \in R$, then $f'\left(\frac{\pi}{3}\right)$ is equal to

84. If $\lim\limits_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$, then the value of $|a-b|$ is equal to

85. If function $f(x)=\frac{\lambda\sin x+6\cos x}{2\sin x+3\cos x}$ is monotonically increasing, then minimum value of $[\lambda]$ is equal to (where [.] denotes greatest integer function)

86. If $P\equiv(0,1,0), Q\equiv(0,0,1)$, then projection of PQ on the plane $x+y+z=3$ is equal to

87. If $\int\frac{dx}{\sin x-\cos x+\sqrt{2}}$ is $\frac{\alpha}{\sqrt{2}}\cot\left(\frac{x}{\beta}+\frac{\pi}{8}\right)+c$, then the value of $\alpha+\beta$ is equal to

88. If $\int_0^{\infty}\frac{x\,dx}{(1+x)(1+x^2)}=\frac{\pi}{k}$, then k is equal to

89. Area bounded by $y=x\sin x$ and x-axis between $x=0$ and $x=2\pi$, is equal to

90. A coin is tossed three times in succession. If E is the event that there are at least two heads and F is the event in which first throw is a head, then $12P\left(\frac{E}{F}\right)$ is equal to

ALL INDIA TEST SERIES

FULL TEST – 22

JEE (Main)

ANSWERS, HINTS & SOLUTIONS

Time Allotted: 3 Hours **Maximum Marks: 300**

General Instructions:

- The test consists of total 90 questions.
- Each subject (PCM) has 30 questions.
- This question paper contains **Three Parts.**
- **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics.
- Each part has only two sections: **Section-A and Section-B**.
- **Section – A** : Attempt all questions.
- **Section – B :** Do any five questions out of 10 Questions.

Section-A (01 – 20, 31 – 50, 61 – 80) *contains 60 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

Section-B (21 – 30, 51 – 60, 81 – 90) *contains 30 Numerical answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

Physics

PART – A

SECTION – A

1. D

Sol. $B_2 - B_1 = \dfrac{2\pi mv}{q\ell}$

$$(B_2 - B_1)^2 = \left(\frac{2\pi m}{q\ell}\right)^2\left(\frac{2qV}{m}\right)$$

2. B

Sol. shift = $(\ell - m)\left[1-\dfrac{1}{n_1}\right] + m\left[1-\dfrac{1}{n_2}\right]$

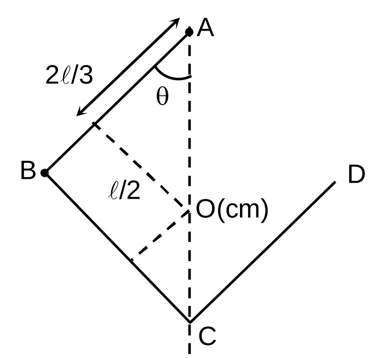
Shift = 0

3. B

Sol. $\tan\theta = \dfrac{OP}{AP} = \dfrac{\ell/2}{2\ell/3}$

$\tan\theta = \dfrac{3}{4}$

$\theta = \tan^{-1}\left(\dfrac{3}{4}\right)$



4. C

Sol. $F = \lambda v^2 + \lambda g y$

Total work done by the applied force = $\int_0^{\ell} F dy = \int_0^{\ell} (\lambda v^2 + \lambda g y)\, dy$

$\Rightarrow \quad \lambda v^2 \ell + \dfrac{\lambda g \ell^2}{2}$

Average power delivered = $\dfrac{\lambda v^2 \ell + \dfrac{\lambda g \ell^2}{2}}{\dfrac{\ell}{v}} = \lambda v^3 + \dfrac{\lambda \ell v g}{2}$

5. B

Sol. Upward force by capillary tube on top surface of liquid is f up = 4σ a $\cos\theta$

If liquid is raised to a height h then we use $4\sigma a\cos\theta = ha^2\rho g$ or $h = \dfrac{4\sigma\cos\theta}{a\rho g}$

6. A

Sol. Torricelli's formula

$v = \sqrt{2gy}$

According to equation of continuity

$Av_1 = av$

$A = \pi x^2$

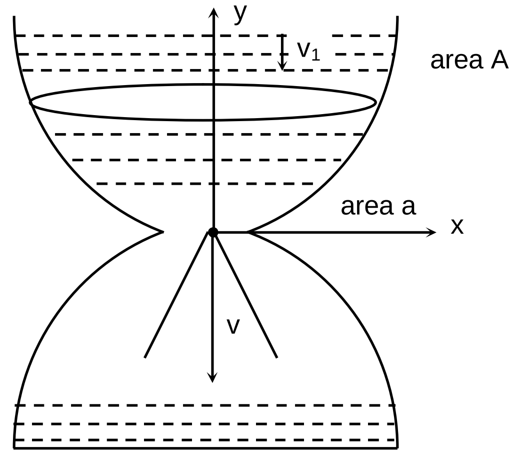
$$\pi x^2 v_1 = a\sqrt{2gy}$$

$$\frac{\pi x^2}{\sqrt{2gy}} = \frac{a}{v_1} = \text{conet} = \text{time}$$

$$\frac{\left(\pi x^2\right)^2}{2gy} = \frac{a^2}{v_1^2}$$

$$y = \left(\frac{\pi^2 v_1^2}{2ga^2}\right)x^4 = kx^4$$

$$\text{Where } K = \frac{\pi^2 v_1^2}{2ga^2}$$



7. B

Sol. Since frequency of first overtone are equal

$$\frac{3}{4\ell_1}\sqrt{\frac{B}{\rho_1}} = \frac{1}{\ell_2}\sqrt{\frac{B}{\rho_2}}$$

$$\ell_2 = \frac{4}{3}\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$$

8. B

Sol. Optical path length = $\int_0^t \mu dx = \int_0^t \left(1+\frac{\alpha x}{t}\right)dx = \left(1+\frac{\alpha}{2}\right)t$

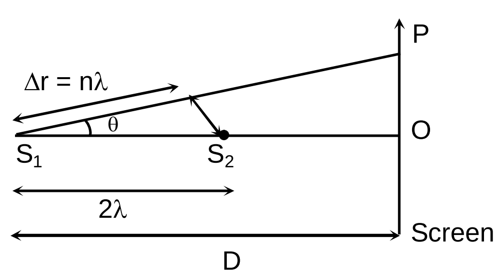
9. C

Sol. As $\cos\theta = \frac{n\lambda}{2\lambda} = \frac{1}{2}$ (for n = 1)

$\theta = 60°$.

$$\tan\theta = \frac{PO}{D}$$

$$PO = D\sqrt{3}$$



10. B

Sol. We know that $Z = \frac{E_0}{I_0}$

Given, $E_0 = 220$ and $I_0 = 10$

So $Z = \frac{220}{10} = 22$ Ohm

$$\phi = \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{3}$$

$$P_0 = \frac{E_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}}\cos\phi = \frac{220}{\sqrt{2}}\frac{10}{\sqrt{2}}\cos\frac{\pi}{3} = 550\text{ W}$$

11. B

Sol. $i(t) = i_0 + i_1e^{-t/RC} + i_2e^{-t/2RC}$

where, $i_0 = i_1 = i_2 = \frac{E}{R}$

12. C

Sol. After collision A stops translation.

13. A

Sol. Area under p–x graph

$$\int P dx = \int\left(\frac{mdv}{dt}\right)v\,dx$$

$$\frac{1}{2}(2+4)10 = \int_{v_i}^{v_f} mv^2 dv$$

14. A

Sol. Loss in gravitational potential energy = Gain in spring potential energy

15. D

Sol. Due to variation in magnetic field there will be an induced current. Due to short circuiting bulb B goes out and resistances of circuit decreases hence current in A increases.

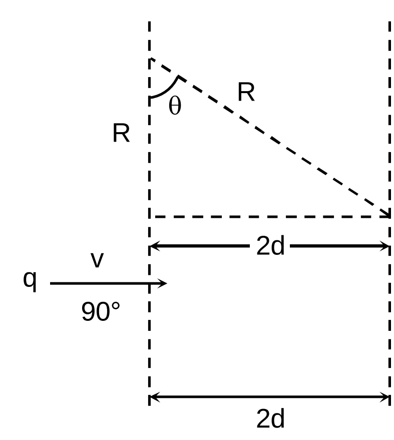
16. C

Sol. When an incident ray $\vec{I}$ is reflected by a mirror whose normal is $\vec{N}$, the reflected ray is given by: $\vec{R} = \frac{-2(\vec{I}.\vec{N})\vec{N}}{\vec{N}.\vec{N}} + \vec{I}$

17. B

Sol. $t = \frac{2\pi m}{qB}\frac{\theta}{2\pi}$

$\sin\theta = \frac{2d}{R}$



18. A

Sol. r + r′ = A

r = A – r′

r = 60° – (10 + t^2) = 50 – t^2

19. C

Sol. Time period of a spring block system

20. D

Sol. S = L/2 + L/4 + L/6 + L/8 + L/10 = $\frac{137L}{120}$

SECTION – B

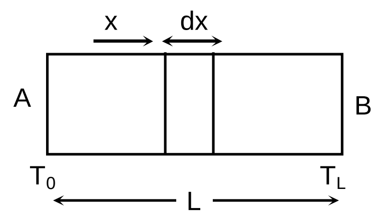
21. 00023.80

Sol. Energy released in the process = difference of energies of alpha particles and deuterons.

22. 00000.60

Sol. $v = \sqrt{\frac{\gamma RT}{M_0}} = \sqrt{\frac{5RT}{3M_0}}$

$$dx = vdt = \sqrt{\frac{5R}{3M_0}\left(T_0 + \frac{(T_L - T_0)x}{L}\right)}dt$$



23. 00000.50

Sol. $I_R = \frac{E_0}{R} = \frac{I_R}{\sqrt{2}} = \frac{E_0}{\sqrt{2}R} = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

$$\Rightarrow \quad \omega_1 L - \frac{1}{\omega_1 C} = -R$$

$$\Rightarrow \quad \omega_2 L - \frac{1}{\omega_2 C} = +R$$

$$L(\omega_1 + \omega_2) = \left(\frac{\omega_1 + \omega_2}{\omega_1\omega_2}\right)\frac{1}{C} \Rightarrow \omega_1\omega_2 = \frac{1}{LC}$$

$$L(\omega_2 - \omega_1) + \left(\frac{\omega_2 - \omega_1}{\omega_1\omega_2}\right)\frac{1}{C} = 2R$$

24. 00007.00

Sol. U + 2 + 3nRT

$dU = n(3R)dT = nC_V dT \quad C_V = 3R$

25. 00002.00

Sol. $TV^{-1/2}$ = Constants

$\left(\frac{dv}{dT}\right)_{process} = \frac{2V}{T}$

$$C_P = C_V + \frac{P}{n}\left(\frac{dv}{dT}\right)$$

$$C_V + \frac{P}{n}\left[\frac{2V}{T}\right] = C_V + \frac{2PV}{nT} = C_V + 2R$$

26. 00008.00

Sol. Conserving angular momentum of the package about the centre of the planet and conserving mechanical energy.

27. 00204.80

Sol. For first resonance $\ell_1 + e = \lambda/4$

For second resonance $\ell_2 + e = 3\lambda/4$

and $\Delta v = 2f(\Delta\ell_2 + \Delta\ell_1)$

28. 00000.12

Sol. Tension at the lower end is 2g

Tension at the upper end is 8g

$V_2/V_1 = \sqrt{(T_2/T_1)}$

Since the frequency is constant

$V_2/V_1 = \lambda_2/\lambda_1 = 2$

29. 00000.50

Sol. $$\frac{KA(800-100)}{x} = \frac{7KA(800-0)}{\ell - x}$$

$$x = \frac{\ell}{9} = \frac{4.5}{9} = 0.50 \text{ m}$$

30. 00002.40

Sol. $\Delta y = (2\lambda D)/a = 2.40$ mm

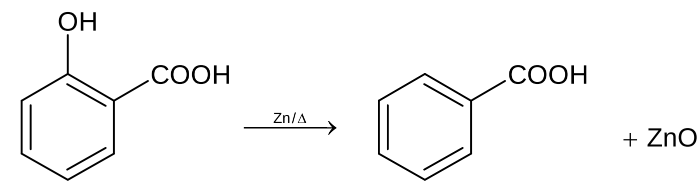
PART – B

SECTION – A

31. B

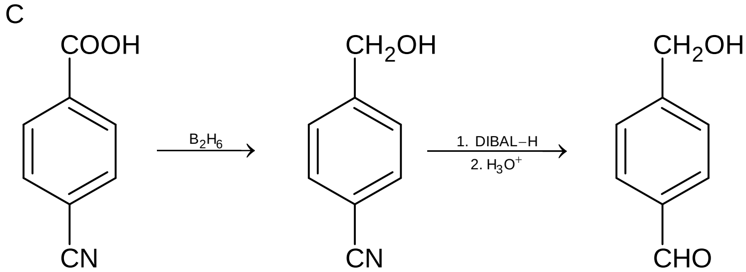
32. D

Sol.



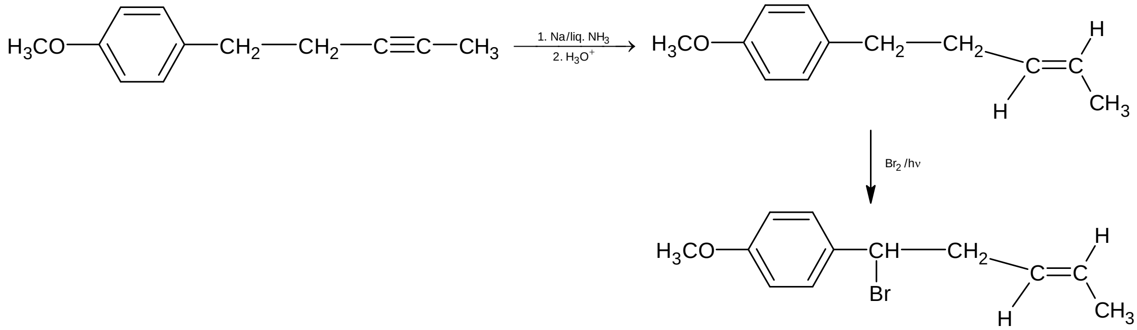
33. C

Sol.



34. C

Sol.



35. D

Sol. $\beta-$naphthol has phenolic group.

36. B

Sol. Moles of complex $=\dfrac{0.5}{266.5}$

$\because$ 1 moles gives = x moles of acid on cation exchange

$\therefore \dfrac{0.5}{266.5}$ moles gives = $\dfrac{x\times1}{266.5}\times0.5$

Hence, $\dfrac{x\times1}{266.5}\times0.5=\dfrac{30.02\times0.125}{1000}$

$x=\dfrac{30.02\times0.125\times266.5}{0.5\times1000}$

x = 2

37. D

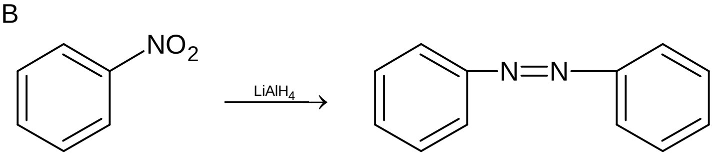
38. B

39. C

Sol. On moving down the group IP decreases.

40. B

Sol.



41. A

42. D

43. C

44. B

Sol. $N(CH_3)_3$ has tetrahedral structure.

45. D

46. D

47. B

Sol. gm eq of H_2O_2 = gm eq of $Na_2S_2O_3$

$N \times 25 = 0.3 \times 20$

$n = \frac{0.3 \times 20}{25} = 0.24$

$V = 5.6 \times 0.24 = 1.344$

48. A

49. D

50. D

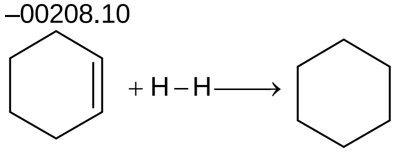
SECTION – B

51. 00002.00

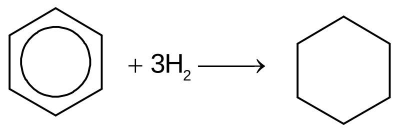
Sol. Conc. of $CuSO_4$ would not change hence $M = \frac{1}{500} \times 1000 = 2\ M$.

52. –00208.10

Sol.



$\Delta H = -119.5\ kJ\ mol^{-1}$



$\Delta H = ?$

$$\left(\Delta_{hyd}H\right)_{theo} = \left(\Delta_h H\right)_{actual} + \left(\Delta H\right)_{resonance}$$

$$3x - 119.5 = \left(\Delta_h H\right)_{actual} + \left(-150.4\right)$$

$$\left(\Delta_h H\right)_{actual} = -358.5 + 150.4$$

$$= -208.1$$

53. 00002.00

Sol. $$\ell n\frac{k_2}{k_1} = \ell n\frac{A_2}{A_1} + \left(\frac{E_{a_1} - E_{a_2}}{RT}\right)$$

$$\ell n\frac{A_2}{A_1} = \frac{E_{a_2} - E_{a_1}}{RT} = \frac{2RT + E_{a_1} - E_{a_1}}{RT} = 2$$

54. 00003.33

Sol.
$$A(g) \ + \ 2B(g) \ \rightleftharpoons \ 2C(g)$$

At eq.	$\frac{4-x}{10}$	$\frac{4-2x}{10}$	$\frac{2x}{10}$	$2x = 2$
At eq.	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$x = 1$

$$K_c = \frac{\left(\frac{2}{10}\right)^2}{\left(\frac{3}{10}\right) \times \left(\frac{2}{10}\right)^2} = \frac{10}{3} = 3.33$$

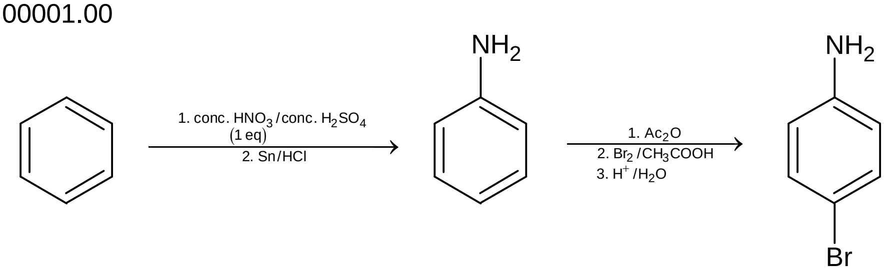
55. 00004.00

Sol. $$\frac{P_A}{P_T} = \frac{n_A}{n_A + n_B}$$

$$\frac{4}{6} = \frac{\frac{1}{M_A}}{\frac{1}{M_A} + \frac{2}{M_B}}, \quad \frac{M_B}{M_A} = 4$$

56. 00001.00

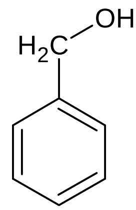
Sol.



57. 00014.81

Sol

$$CH_3(CH_2)_5CH_3 \xrightarrow[\substack{2.\ Br_2/h\nu \\ 3.\ Mg/ether \\ 4.\ H_2O}]{1.\ Cr_2O_3/Al_2O_3\ \ 773K,\ 10-20\ atm}$$



58. 00008.00

Sol. n = 3 there is no symmetry hence total stereoisomers $= 2^n = 2^3 = 8$.

59. 00001.50

Sol. $O_2^- = \sigma_{1s}^2, \sigma_{1s}^{*2}\sigma_{2s}^2, \sigma_{2s}^{*2}, \sigma_{2pz}^2, \sigma_{2px}^2 = \sigma_{2py}^2, \sigma_{2px}^{*2} = \sigma_{2py}^{*1}$

$$\text{Bond order} = \frac{6-3}{2} = 1.5$$

60. 00000.50

Sol. $6XeF_4 + 12H_2O \longrightarrow 4Xe + 2XeO_3 + 24HF + 3O_2$

Mathematics PART – C

SECTION – A

61. A

Sol. For any $a \in N$, we find that a|a, therefore R is reflexive but R is not symmetric, because aRb does not imply that bRa.

62. A

Sol. Use Expansion

63. D

Sol. Given that $z^2 + (p+iq)z + r + is = 0$ ..... (1)

Let $z = \alpha$ (where α is real) be a root of (1), then

$\alpha^2 + (p+iq)\alpha + r + is = 0$

or $\alpha^2 + p\alpha + r + i(q\alpha + s) = 0$

Equating real and imaginary parts, we have $\alpha^2 + p\alpha + r = 0$ and $q\alpha + s = 0$

Eliminating α, we get $\left(\frac{-s}{q}\right)^2 + p\left(\frac{-s}{q}\right) + r = 0$

or $s^2 - pqs + q^2r = 0$ or $pqs = s^2 + q^2r$

64. A

Sol. n^{th} term of the series is $20 + (n-1)\left(-\frac{2}{3}\right)$

For sum to be maximum, n^{th} term ≥ 0

$\Rightarrow 20 + (n-1)\left(-\frac{2}{3}\right) \geq 0 \Rightarrow n \leq 31$

Thus the sum of 31 terms is maximum and is equal to

$\frac{31}{2}\left[40 + 30 \times \left(-\frac{2}{3}\right)\right] = 310$

65. D

Sol. If the roots of the quadratic equation $ax^2 + bx + c = 0$ exceed a number k, then $ak^2 + bk + c > 0$ if $a > 0, b^2 - 4ac \geq 0$ and sum of the roots $> 2k$

Therefore, if the roots of $x^2 + x + a = 0$ exceed a number a, then

$a^2 + a + a > 0,\ 1 - 4a \geq 0$ and $-1 > 2a$

$\Rightarrow a(a+2) > 0, a \leq \frac{1}{4}$ and $a < -\frac{1}{2}$

$\Rightarrow a > 0$ or $a < -2,\ a < \frac{1}{4}$ and $a < -\frac{1}{2}$

Hence, $a < -2$

66. B

Sol. Since the number of students giving wrong answers to at least i question $(i = 1, 2, ..., n) = 2^{n-i}$

The number of students answering exactly $i\ (1 \leq i \leq -1)$ questions wrongly

= {the number of students answering at least i questions wrongly, $i = 1, 2, ...,n$)}

– {the number of students answering at least $(i+1)$ questions wrongly

$$(2 \le i+1 \le n)\} = 2^{n-i} - 2^{n-(i+1)} (1 \le i \le n-1)$$

Now, the number of students answering all the n questions wrongly $= 2^{n-n} = 2^0$

Thus the total number of wrong answers

$= 1(2^{n-1} - 2^{n-2} + 2(2^{n-2} - 2^{n-3}) + 3(2^{n-3} - 2^{n-4}) + ... + (n-1)(2^1 - 2^0) + n(2^0)$

$= 2^{n-1} + 2^{n-2} + 2^{n-3} + ... + 2^0 = 2^n - 1$ ($\because$ Its a G.P.)

67. D

Sol. $(1 + x + x^3 + x^4)^{10} = (1+x)^{10} (1+x^3)^{10}$

$= (1 + {}^{10}C_1 . x + {}^{10}C_2 . x^2 + ...) (1 + {}^{10}C_1 . x^3 + {}^{10}C_2 . x^6 + ...)$

$\therefore$ Coefficient of $x^4 = {}^{10}C_1 . {}^{10}C_1 + {}^{10}C_4 = 310$

68. D

Sol. $\begin{vmatrix} x+a & b & c \\ b & x+c & a \\ c & a & x+b \end{vmatrix} = 0$

$\Rightarrow (x+a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & x+c & a \\ 1 & a & x+b \end{vmatrix} = 0$

$\Rightarrow x = -(a+b+c)$ is one of the root of the equation

69. B

Sol. $x \sin\varphi = \tan\theta - x\cos\varphi\tan\theta$

$\Rightarrow x = \dfrac{\tan\theta}{\sin\varphi + \cos\varphi\tan\theta} = \dfrac{\sin\theta}{\cos\theta\sin\varphi + \cos\varphi\sin\theta} = \dfrac{\sin\theta}{\sin(\theta+\varphi)}$

Similarly, $y = \dfrac{\sin\varphi}{\sin(\theta+\varphi)}$; $\therefore$ $\dfrac{x}{y} = \dfrac{\sin\theta}{\sin\varphi}$

70. A

Sol. On expanding determinant,

$\cos^2(A+B) + \sin^2(A+B) + \cos 2B = 0$

$1 + \cos 2B = 0$ or $\cos 2B = \cos\pi$

or $2B = 2n\pi + \pi$ or $B = (2n+1)\dfrac{\pi}{2}, n \in Z$.

71. A

Sol. By the help of given condition of $a+b+c=0$, the three lines reduce to $x - y = \dfrac{p}{a}$ or $\dfrac{p}{b}$ or $\dfrac{p}{c} (p \ne 0)$.

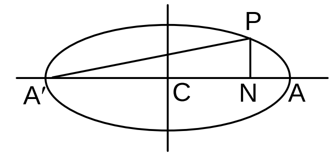
All these lines are parallel. Hence they do not intersect in finite plane

72. C

Sol. Required area $= \dfrac{a}{h^2+k^2}(h^2+k^2-a^2)^{3/2} = \dfrac{3}{4^2+3^2}(4^2+3^2-9)^{3/2} = \dfrac{192}{25}$

73. A

Sol. Let ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



$P=(a\cos\theta, b\sin\theta), A$ and $A' \equiv (\pm a,0), N \equiv (a\cos\theta, 0)$,

$PN = b\sin\theta,\ AN = a(1-\cos\theta)$,

$A'N = a(1+\cos\theta)$

$$\frac{(PN)^2}{AN\,A'N}=\frac{b^2\sin^2\theta}{a^2(1-\cos\theta)(1+\cos\theta)}=\frac{b^2}{a^2}$$

74. A

Sol. The vector equation of a plane through the line of intersection of the planes $r.(i+3j-k)=0$ and $r.(j+2k)=0$ can be written as

$(r.(i+3j-k))+\lambda(r.(j+2k))=0$ ..... (1)

This passes through $2i+j-k$

$\therefore (2i+j-k).(i+3j-k)+\lambda(2i+j-k).(j+2k)=0$

or $(2+3+1)+\lambda(0+1-2)=0 \Rightarrow \lambda=6$

Put the value of λ in (1), we get

$r.(i+9j+11k)=0$, which is the required plane

75. A

Sol. Force $F=\overrightarrow{AB}=(3-1)\mathbf{i}+(-4-2)\mathbf{j}+(2+3)\mathbf{k}\ =2\mathbf{i}-6\mathbf{j}+5\mathbf{k}$

Moment of Force $\vec{F}$ w.r.t $M=\overrightarrow{MA}\times\vec{F}$

$\because\ \overrightarrow{MA}=(1+2)\mathbf{i}+(2-4)\mathbf{j}+(-3+6)\mathbf{k}\ =3\mathbf{i}-2\mathbf{j}+3\mathbf{k}$

Now, $\overrightarrow{MA}\times\vec{F}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 3 & -2 & 3\\ 2 & -6 & 5\end{vmatrix}\ =\mathbf{i}(-10+18)+\mathbf{j}(6-15)+\mathbf{k}(-18+4)=8i-9j-14k$

76. A

Sol. $[xy\cos(xy)+\sin(xy)]dx+x^2\cos(xy)dy=0$

$xy\cos(xy)dx+x^2\cos(xy)dy+\sin(xy)dx=0$

$x\cos(xy)(ydx+xdy)+\sin(xy)dx=0$

$\cot(xy)dxy+\frac{dx}{x}=0$

$\log\sin(xy)+\log x=k \Rightarrow x\sin(xy)=k$

77. A

Sol. $ye^{-x/y}dx-(xe^{-x/y}+y^3)dy=0$

$e^{-x/y}(ydx-xdy)=y^3dy\ \Rightarrow\ e^{-x/y}\frac{(ydx-xdy)}{y^2}=ydy$

$e^{-x/y}d\left(\frac{x}{y}\right)=ydy$. Integrating both sides, we get $k-e^{-x/y}=\frac{y^2}{2}\Rightarrow\frac{y^2}{2}+e^{-x/y}=k$

78. C

Sol. Let A be the event that face 4 turns up and B be the event that face 5 turns up then $P(A) = 0.25, P(B) = 0.05$

Since A and B are mutually exclusive, so $P(A \cup B) = P(A) + P(B) = 0.25 + 0.05 = 0.30$

We have to find $P\left(\frac{A}{A \cup B}\right)$, which is equal to $P\frac{[A \cap (A \cup B)]}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = \frac{0.25}{0.30} = \frac{5}{6}$

79. A

Sol. $\sim [p \vee (\sim p \vee q)] \equiv \sim p \wedge \sim (\sim p \vee q)$

$\equiv \sim p \wedge (\sim (\sim p) \wedge \sim q)$

$\equiv \sim p \wedge (p \wedge \sim q)$

80. B

Sol. Tangent at $(a\sec\theta, b\tan\theta)$ is, $\frac{x}{(a/\sec\theta)} - \frac{y}{(b/\tan\theta)} = 1$ or $\frac{a}{\sec\theta} = 1, \frac{b}{\tan\theta} = 1$

$\Rightarrow a = \sec\theta,\ b = \tan\theta$ or (a,b) lies on $x^2 - y^2 = 1$

SECTION – B

81. 00003.00

Sol. $\sin^{-1} x = 2\tan^{-1} x \Rightarrow \sin^{-1} x = \sin^{-1} \frac{2x}{1+x^2}$

$\Rightarrow \frac{2x}{1+x^2} = x \Rightarrow x^3 - x = 0$

$\Rightarrow x(x+1)(x-1) = 0 \Rightarrow x = \{-1, 1, 0\}$

82. 00006.00

Sol. $\theta = \tan^{-1}\left(\frac{2\sqrt{h^2 - ab}}{a+b}\right) = \tan^{-1}\left(\frac{\sqrt{4h^2 - 4ab}}{a+b}\right) = \tan^{-1}\left(\frac{\sqrt{3a^2 + 3b^2 + 10ab - 4ab}}{a+b}\right) = 60°$

83. 00000.00

Sol. $f(x) = \frac{\cos^2 x + \sin^4 x}{\sin^2 x + \cos^4 x} \Rightarrow f(x) = \frac{\cos^2 x + \sin^2 x(1-\cos^2 x)}{\sin^2 x + \cos^2 x(1-\sin^2 x)}$

$\Rightarrow f(x) = \frac{\sin^2 x + \cos^2 x - \sin^2 x \cos^2 x}{\sin^2 x + \cos^2 x - \sin^2 x \cos^2 x}$

$\Rightarrow f(x) = 1$

84. 00001.00

Sol. $\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$

$$\Rightarrow \lim_{x \to 0} \frac{x\left\{1 + a\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots\right)\right\} - b\left\{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right\}}{x^3} = 1$$

$$\Rightarrow \lim_{x\to 0}\frac{(1+a-b)+x^2\left(\frac{b}{3!}-\frac{a}{2!}\right)+x^4\left(\frac{a}{4!}-\frac{b}{5!}\right)+...}{x^2}=1 \qquad ..... (1)$$

If $1+a-b\neq 0$, then L.H.S. $\to\infty$ as $x\to 0$ while R.H.S. = 1, therefore $1+a-b=0$

Now from equation (1), $\lim_{x\to 0}\frac{x^2\left(\frac{b}{3!}-\frac{a}{2!}\right)+x^4\left(\frac{a}{4!}-\frac{b}{5!}\right)+...}{x^2}=1$

$\Rightarrow \frac{b}{3!}-\frac{a}{2!}=1 \Rightarrow b-3a=6$

Solving $1+a-b=0$ and $b-3a=6$, we get $a=-5/2, b=-3/2$

85. 00004.00

Sol. The function is monotonic increasing, if $f'(x)>0$

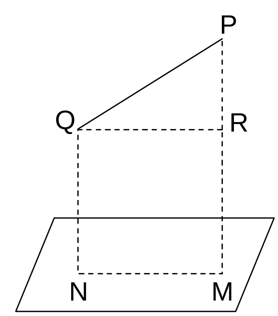
$\Rightarrow \frac{(2\sin x+3\cos x)(\lambda\cos x-6\sin x)}{(2\sin x+3\cos x)^2}-\frac{(\lambda\sin x+6\cos x)(2\cos x-3\sin x)}{(2\sin x+3\cos x)^2}>0$

$\Rightarrow 3\lambda(\sin^2 x+\cos^2 x)-12(\sin^2 x+\cos^2 x)>0$

$\Rightarrow 3\lambda-12>0 \Rightarrow \lambda>4$

86. 00001.41

Sol. Given plane is $x+y+z-3=0$. From point P and Q draw PM and QN perpendicular on the given plane and QR $\perp$MP



$|MP|=\frac{0+1+0-3}{\sqrt{1^2+1^2+1^2}}=\frac{-2}{\sqrt{3}}$, $|NQ|=\frac{-2}{\sqrt{3}}$

$|PQ|=\sqrt{(0-0)^2+(0-1)^2+(1-0)^2}=\sqrt{2}$

$|RP|=|MP|-|MR|=|MP|-|NQ|=0$

$\therefore |NM|=|QR|=\sqrt{PQ^2-RP^2}=\sqrt{(\sqrt{2})^2-0}=\sqrt{2}$

87. 00001.00

Sol. $I=\int\frac{dx}{\sin x-\cos x+\sqrt{2}}$

$=\int\frac{dx}{\sqrt{2}(\sin x.\sin\frac{\pi}{4}-\cos x\cos\frac{\pi}{4}+1)}$

$=\frac{1}{\sqrt{2}}\int\frac{dx}{1-\cos\left(x+\frac{\pi}{4}\right)}=\frac{1}{\sqrt{2}}\int\frac{dx}{1-\cos 2\left(\frac{x}{2}+\frac{\pi}{8}\right)}$

$=\frac{1}{\sqrt{2}}\int\frac{dx}{2\sin^2\left(\frac{x}{2}+\frac{\pi}{8}\right)}=\frac{1}{2\sqrt{2}}\int \operatorname{cosec}^2\left(\frac{x}{2}+\frac{\pi}{8}\right)dx$

$=\frac{1}{2\sqrt{2}}\frac{-\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)}{1/2}+c=\frac{-1}{\sqrt{2}}\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)+c$

88. 00004.00

Sol. $I = \int_0^{\infty} \frac{x dx}{(1+x)(1+x^2)}$

Put $x = \tan\theta$, we get

$I = \int_0^{\pi/2} \frac{\tan\theta}{1+\tan\theta} d\theta = \int_0^{\pi/2} \frac{\sin\theta}{\cos\theta+\sin\theta} d\theta = \frac{\pi}{4}$

89. 00012.56

Sol. Required area $= \int_0^{\pi} y\, dx + \left|\int_{\pi}^{2\pi} y\, dx\right| = 4\pi$ sq.unit

90. 00009.00

Sol. $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT$

$n(E) = 4, n(F) = 4$ and $n(E \cap F) = 3$