

Exercise 5.1

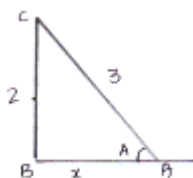
1. In each of the following one of the six trigonometric ratios is given. Find the values of the other trigonometric ratios.

Sol:

(i) $\sin A = \frac{2}{3}$

We know that $\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$

Let us Consider a right angled Δ^{le} ABC.



By applying Pythagorean theorem we get

$$AC^2 = AB^2 + BC^2$$

$$9 = x^2 + 4$$

$$x^2 = 9 - 4$$

$$x = \sqrt{5}$$

We know that $\cos = \frac{\text{adjacent side}}{\text{hypotenuse}}$ and

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\text{So, } \cos \theta = \frac{\sqrt{5}}{3};$$

$$\sec = \frac{1}{\cos \theta} = \frac{3}{\sqrt{5}}$$

$$\tan \theta = \frac{2}{\sqrt{5}};$$

$$\cot = \frac{1}{\tan \theta} = \frac{\sqrt{5}}{2}$$

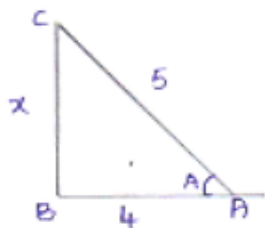
$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{3}{2}$$

(ii)

$$\cos A = \frac{4}{5}$$

We know that $\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$

Let us consider a right angled Δ^{le} ABC.



Let opposite side $BC = x$.

By applying Pythagoras's theorem, we get

$$AC^2 = AB^2 + BC^2$$

$$25 = x^2 + 16$$

$$x^2 = 25 - 16 = 9$$

$$x = \sqrt{9} = 3$$

We know that $\cos A = \frac{4}{5}$

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{3}{5}$$

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{3}{4}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$

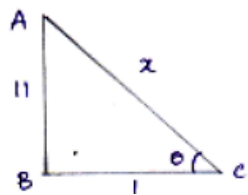
$$\cot A = \frac{1}{\tan A} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

(iii)

$$\tan \theta = 11.$$

$$\text{We know that } \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{11}{1}$$

Consider a right angled $\Delta^{\text{le}} ABC$.



Let hypotenuse $AC = x$, by applying Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$x^2 = 11^2 + 1^2$$

$$x^2 = 121 + 1$$

$$x = \sqrt{122}$$

$$\text{We know that } \sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{11}{\sqrt{122}}$$

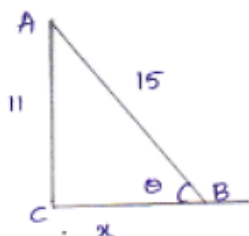
$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{1}{\sqrt{122}}$$

$$\begin{aligned}\operatorname{cosec} \theta &= \frac{1}{\sin \theta} = \frac{\frac{1}{11}}{\frac{1}{\sqrt{122}}} = \frac{\sqrt{122}}{11} \\ \sec \theta &= \frac{1}{\cos \theta} = \frac{1/1}{\frac{1}{\sqrt{122}}} = \sqrt{122} \\ \cot \theta &= \frac{1}{\tan \theta} = \frac{1}{11} = \frac{1}{11}\end{aligned}$$

(iv)

$$\sin \theta = \frac{11}{15}$$

$$\text{We know } \sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{11}{15}$$

Consider right angled $\Delta^{\text{le}} ACB$.Let $x = \text{adjacent side}$

By applying Pythagoras

$$AB^2 = AC^2 + BC^2$$

$$225 = 121 + x^2$$

$$x^2 = 225 - 121$$

$$x^2 = 104$$

$$x = \sqrt{104}$$

$$\cos = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{\sqrt{104}}{15}$$

$$\tan = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{11}{\sqrt{104}}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{15}{11}$$

$$\sec = \frac{1}{\cos \theta} = \frac{15}{\sqrt{104}}$$

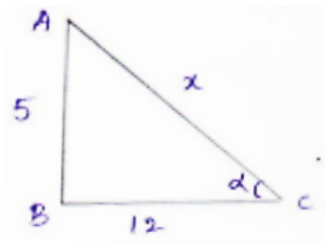
$$\cot = \frac{1}{\tan \theta} = \frac{\sqrt{104}}{11}$$

(v)

$$\tan \alpha = \frac{5}{12}$$

$$\text{We know that } \tan \alpha = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{5}{12}$$

Now consider a right angled $\Delta^{\text{le}} ABC$.



Let x = hypotenuse .By applying Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$x^2 = 5^2 + 12^2$$

$$x^2 = 25 + 144 = 169$$

$$x = 13$$

$$\sin \alpha = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{5}{13}$$

$$\cos \alpha = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{12}{13}$$

$$\cot \alpha = \frac{1}{\tan \alpha} = \frac{12}{5}$$

$$\operatorname{cosec} \alpha = \frac{1}{\sin \alpha} = \frac{13}{5}$$

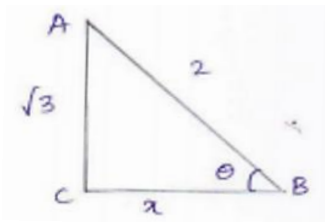
$$\sec \alpha = \frac{1}{\cos \alpha} = \frac{13}{12}$$

(vi)

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\text{We know } \sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$$

Now consider right angled $\Delta^{\text{le}} ABC$.



Let x = adjacent side

By applying Pythagoras

$$AB^2 = AC^2 + BC^2$$

$$4 = 3 + x^2$$

$$x^2 = 4 - 3$$

$$x^2 = 1$$

$$x = 1$$

$$\cos = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{1}{2}$$

$$\tan = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

$$\sec = \frac{1}{\cos \theta} = \frac{1}{\frac{1}{2}} = 2$$

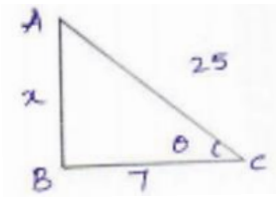
$$\cot = \frac{1}{\tan \theta} = \frac{1}{\sqrt{3}}$$

(vii)

$$\cos \theta = \frac{7}{25}$$

We know that $\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$

Now consider a right angled $\Delta^{\text{le}} ABC$,



Let x be the opposite side.

By applying pythagorn's theorem

$$AC^2 = AB^2 + BC^2$$

$$(25)^2 = x^2 + 7^2$$

$$625 - 49 = x^2$$

$$576 = \sqrt{576} = 24$$

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{24}{25}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{24}{7}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{24}{25}} = \frac{25}{24}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{7}{25}} = \frac{25}{7}$$

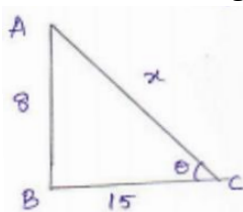
$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{24}{7}} = \frac{7}{24}$$

(viii)

$$\tan \theta = \frac{8}{15}$$

We know that $\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{8}{15}$

Now consider a right angled $\Delta^{\text{le}} ABC$.



By applying Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$x^2 = 8^2 + 15^2$$

$$x^2 = 225 + 64 = 289$$

$$x = \sqrt{289} = 17$$

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{8}{17}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{15}{17}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{8}{15}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{8}{15}} = \frac{15}{8}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{8}{17}} = \frac{17}{8}$$

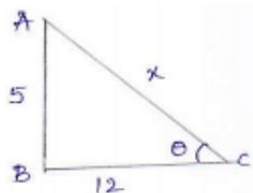
$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{15}{17}} = \frac{17}{15}$$

(ix)

$$\cot\theta = \frac{12}{5}$$

$$\cot\alpha = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{12}{5}$$

Now consider a right angled $\Delta^{\text{le}} ABC$,



By applying Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$x^2 = 25 + 144$$

$$x^2 = 169 = \sqrt{169}$$

$$x = 13$$

$$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$$

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{5}{13}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{12}{13}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{5/13} = \frac{13}{5}$$

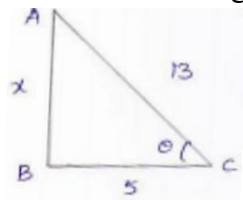
$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{12/13} = \frac{13}{12}$$

(x)

$$\sec\theta = \frac{13}{5}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{13}{5}$$

Now consider a right angled Δ^{le} ABC,



By applying Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$169 = x^2 + 25$$

$$x^2 = 169 - 25 = 144$$

$$x = 12$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\frac{13}{5}} = \frac{5}{13}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{12}{5}$$

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{12}{13}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{12/13} = \frac{13}{12}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{5/13} = \frac{13}{5}$$

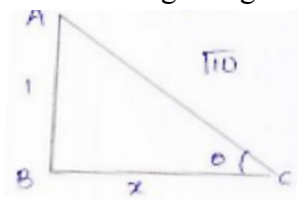
$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{12/5} = \frac{5}{12}$$

(xi)

$$\operatorname{cosec}\theta = \sqrt{10}$$

$$\operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{opposite side}} = \sqrt{10}$$

consider a right angled Δ^{le} ABC, we get



Let x be the adjacent side.

By applying pythagora's theorem

$$AC^2 = AB^2 + BC^2$$

$$(\sqrt{10})^2 = 1^2 + x^2$$

$$x^2 = 10 - 1 = 9$$

$$x = 3$$

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{\sqrt{10}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{3}{\sqrt{10}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{1}{3}$$

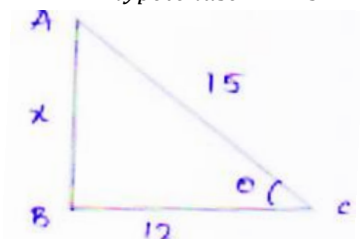
$$\sec \theta = \frac{1}{\cos \theta} = \frac{\sqrt{10}}{3}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{1}{3}} = 3.$$

(xii)

$$\cos \theta = \frac{12}{15}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{12}{15}.$$



Let x be the opposite side.

By applying pythagorn's theorem

$$AC^2 = AB^2 + BC^2$$

$$225 = x^2 + 144$$

$$225 - 144 = x^2$$

$$x^2 = 81$$

$$x = 9$$

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{9}{15}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{9}{12}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{9}{15}} = \frac{15}{9}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{12}{15}} = \frac{15}{12}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{9}{12}} = \frac{12}{9}$$

2. In a $\triangle ABC$, right angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine

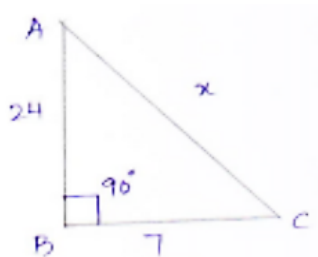
(i) $\sin A$, $\cos A$

(ii) $\sin C$, $\cos C$

Sol:

$\triangle ABC$ is right angled at B

$AB = 24$ cm, $BC = 7$ cm.



Let 'x' be the hypotenuse,

By applying Pythagoras

$$AC^2 = AB^2 + BC^2$$

$$x^2 = 24^2 + 7^2$$

$$x^2 = 576 + 49$$

$$x^2 = 625$$

$$x = 25$$

a. Sin A, Cos A

At $\angle A$, opposite side = 7

adjacent side = 24

hypotenuse = 25

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{7}{25}$$

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{24}{25}$$

b. Sin C, Cos C

At $\angle C$, opposite side = 24

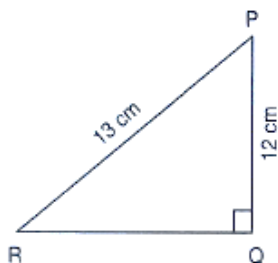
adjacent side = 7

hypotenuse = 25

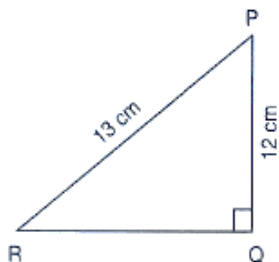
$$\sin C = \frac{24}{25}$$

$$\cos C = \frac{7}{25}$$

3. In Fig below, Find tan P and cot R. Is $\tan P = \cot R$?



Sol:



Let x be the adjacent side.

By Pythagoras theorem

$$PR^2 = PQ^2 + RQ^2$$

$$169 = x^2 + 144$$

$$x^2 = 25$$

$$x = 5$$

At LP, opposite side = 5

Adjacent side = 12

Hypotenuse = 13

$$\tan P = \frac{\frac{1}{12}}{5} \Rightarrow \frac{5}{12}$$

At LR, opposite side = 12

Adjacent side = 5

Hypotenuse = 13

$$\cot R = \frac{1}{\tan R} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$$

$$[\because \tan R = \frac{\text{opposite side}}{\text{adjacent side}}]$$

$$\therefore \tan P = \cot R$$

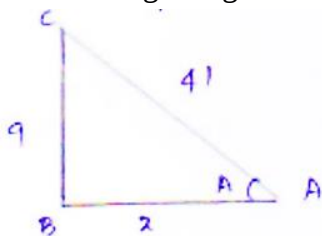
4. If $\sin A = \frac{9}{41}$, compute $\cos A$ and $\tan A$

Sol:

$$\sin A = \frac{9}{41}$$

$$\sin A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{9}{41}$$

Consider right angled triangle ABC,



Let x be the adjacent side

By applying Pythagorean

$$AC^2 = AB^2 + BC^2$$

$$41^2 = 12^2 + 9^2$$

$$x^2 = 41^2 - 9^2$$

$$x = 40$$

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{40}{41}$$

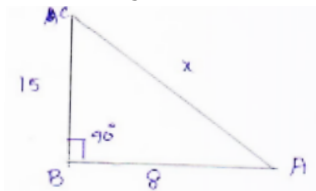
$$\tan A = \frac{\text{opposite side}}{\text{Hypotenuse side}} = \frac{9}{40}$$

5. Given $15 \cot A = 8$, find $\sin A$ and $\sec A$.

Sol:

$15 \cot A = 8$, find $\sin A$ and $\sec A$

$$\cot A = \frac{8}{15}$$



Consider right angled triangle ABC,

Let x be the hypotenuse,

$$AC^2 = AB^2 + BC^2$$

$$x^2 = (8)^2 + (15)^2$$

$$x^2 = 64 + 225$$

$$x^2 = 289$$

$$x = 17$$

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{15}{17}$$

$$\sec A = \frac{1}{\cos A}$$

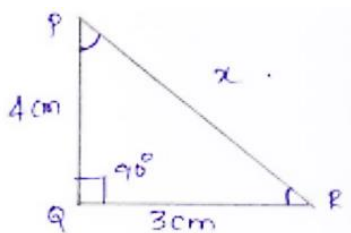
$$\cos A = \frac{\text{adjacent side}}{\text{Hypotenuse}} = \frac{8}{17}$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{8/17} = \frac{17}{8}$$

6. In $\triangle PQR$, right angled at Q, $PQ = 4$ cm and $RQ = 3$ cm. Find the values of $\sin P$, $\sin R$, $\sec P$ and $\sec R$.

Sol:

$\triangle PQR$, right angled at Q.



Let x be the hypotenuse

By applying Pythagoras

$$PR^2 = PQ^2 + QR^2$$

$$x^2 = 4^2 + 3^2$$

$$x^2 = 16 + 9$$

$$\therefore x = \sqrt{25} = 5$$

Find $\sin P$, $\sin R$, $\sec P$, $\sec R$

At P , opposite side = 3 cm

Adjacent side = 4 cm

Hypotenuse = 5

$$\sin P = \frac{\text{opposite side}}{\text{Hypotenuse}} = \frac{3}{5}$$

$$\sec P = \frac{\text{Hypotenuse}}{\text{adjacent side}} = \frac{5}{4}$$

At R , opposite side = 4 cm

Adjacent side = 3 cm

Hypotenuse = 5 cm

$$\sin R = \frac{4}{5}$$

$$\sec R = \frac{5}{3}$$

7. If $\cot \theta = \frac{7}{8}$, evaluate:

(i) $\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$

(ii) $\cot^2 \theta$

Sol:

$$\cot \theta = \frac{7}{8}$$

(i) $\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$

$$= \frac{1-\sin^2 \theta}{1-\cos^2 \theta} \quad [\because (a+b)(a-b) = a^2 - b^2] \quad a = 1, b = \sin \theta$$

$$\text{We know that } \sin^2 \theta + \cos^2 \theta = 1$$

$$1 - \sin^2 \theta = \cos^2 \theta = \cos^2 \theta$$

$$1 - \cos^2 \theta = \sin^2 \theta$$

$$= \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$= \cot^2 \theta$$

$$= (\cot \theta)^2 = \left[\frac{7}{8}\right]^2$$

$$= \frac{49}{64}$$

$$(ii) \cot^2 \theta$$

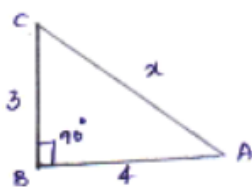
$$\Rightarrow (\cot \theta)^2 = \left[\frac{7}{8}\right]^2$$

$$= \frac{49}{64}$$

8. If $3 \cot A = 4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Sol:

$$3 \cot A = 4, \text{ check } = \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$



$$\cot A = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{4}{3}$$

Let x be the hypotenuse

By Applying Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$x^2 = 4^2 + 3^2$$

$$x^2 = 25$$

$$x = 5$$

$$\tan A = \frac{1}{\cos^2 A} = \frac{3}{4}$$

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{4}{5}$$

$$\sin A = \frac{3}{5}$$

$$\text{LHS} = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{7}{25}$$

$$\text{RHS } \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16-9}{25} = \frac{7}{25}$$

9. If $\tan \theta = \frac{a}{b}$, find the value of $\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$

Sol:

$$\tan \theta = \frac{a}{b} \text{ find } \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \dots (i)$$

Divide equation (i) with $\cos \theta$, we get

$$\begin{aligned} & \Rightarrow \frac{\frac{\cos \theta + \sin \theta}{\cos \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \\ & \Rightarrow \frac{1 + \frac{\sin \theta}{\cos \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \\ & \Rightarrow \frac{1 + \tan \theta}{1 - \tan \theta} \\ & = \frac{1 + \frac{a}{b}}{1 - \frac{a}{b}} \\ & = \frac{b+a}{b-a} \end{aligned}$$

10. If $3 \tan \theta = 4$, find the value of $\frac{4 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta}$

Sol:

$$3 \tan \theta = 4 \text{ find } \frac{4 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta} \quad \dots (i)$$

$$\tan \theta = \frac{4}{3}$$

Dividing equation (i) with $\cos \theta$ we get

$$\begin{aligned} & = \frac{\frac{4 \cos \theta - \sin \theta}{\cos \theta}}{\frac{2 \cos \theta + \sin \theta}{\cos \theta}} = \frac{4 - \tan \theta}{2 + \tan \theta} \left[\because \frac{\sin \theta}{\cos \theta} = \tan \theta \right] \\ & = \frac{4 - \tan \theta}{2 + \tan \theta} \quad \left[\because \frac{\sin \theta}{\cos \theta} = \tan \theta \right] \\ & = \frac{4 - \frac{4}{3}}{2 + \frac{4}{3}} \\ & = \frac{12 - 4}{6 + 4} \\ & = \frac{8}{10} \\ & = \frac{4}{5} \end{aligned}$$

11. If $3 \cot \theta = 2$, find the value of $\frac{4 \sin \theta - 3 \cos \theta}{2 \sin \theta + 6 \sin \theta}$

Sol:

$$3 \cot \theta = 2 \quad \text{find } \frac{4 \sin \theta - 3 \cos \theta}{2 \sin \theta + 6 \cos \theta} \quad \dots (i)$$

$$\cot \theta = \frac{2}{3}$$

$$\begin{aligned} & = \frac{\frac{4 \sin \theta - 3 \cos \theta}{\sin \theta}}{\frac{2 \sin \theta + 6 \cos \theta}{\sin \theta}} \\ & = \frac{4 - 3 \cot \theta}{2 + 6 \cot \theta} \\ & = \frac{4 - 3 \times \frac{2}{3}}{2 + 6 \times \frac{2}{3}} \end{aligned}$$

$$= \frac{4+2}{2+4} = \frac{2}{6}$$

$$= \frac{1}{3}$$

12. If $\tan \theta = \frac{a}{b}$, prove that $\frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} = \frac{a^2 - b^2}{a^2 + b^2}$

Sol:

$$\tan \theta = \frac{a}{b} \quad \text{PT} \quad \frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} = \frac{a^2 - b^2}{a^2 + b^2}$$

$$\text{Let } \frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} \dots (i)$$

Divide both Nr and Dr with $\cos \theta$ of (a)

$$= \frac{\frac{a \sin \theta - b \cos \theta}{\cos \theta}}{\frac{a \sin \theta + b \cos \theta}{\cos \theta}}$$

$$= \frac{a \tan \theta - b}{a \tan \theta + b}$$

$$= \frac{a \times \left(\frac{a}{b}\right) - b}{a \times \left(\frac{a}{b}\right) + b}$$

$$= \frac{a^2 - b^2}{a^2 + b^2}$$

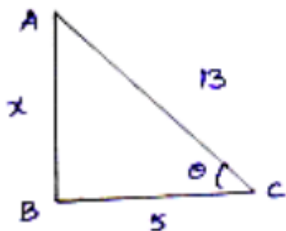
13. If $\sec \theta = \frac{13}{5}$, show that $\frac{2 \cos \theta - 3 \sin \theta}{4 \sin \theta - 9 \cos \theta} = 3$

Sol:

$$\sec \theta = \frac{13}{5}$$

$$\sec \theta = \frac{\text{Hypotenuse}}{\text{adjacent side}} = \frac{13}{5}$$

Now consider right angled triangle ABC



By applying Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$169 = x^2 + 25$$

$$x^2 = 169 - 25 = 144$$

$$x = 12$$

$$\cos \theta = \frac{1}{\sec \theta} = \frac{1}{13} = \frac{5}{13}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{12}{5}$$

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{12}{13}$$

$$\operatorname{Cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{12/13} = \frac{13}{12}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{5/13} = \frac{13}{5}$$

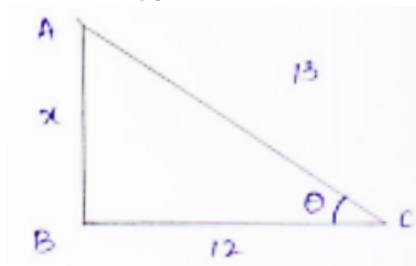
$$\cot \theta = \frac{1}{\tan \theta} = \frac{1}{12/5} = \frac{5}{12}$$

14. If $\cos \theta = \frac{12}{13}$, show that $\sin \theta (1 - \tan \theta) = \frac{35}{156}$

Sol:

$$\cos \theta = \frac{12}{13} \quad \text{S.T.} \quad \sin \theta (1 - \tan \theta) = \frac{35}{156}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{12}{13}$$



Let x be the opposite side

By applying Pythagoras

$$AC^2 = AB^2 + BC^2$$

$$169 = x^2 + 144$$

$$x = 25$$

$$x = 5$$

$$\sin \theta = \frac{AB}{AC} = \frac{5}{13}$$

$$\tan \theta = \frac{AB}{BC} = \frac{5}{12}$$

$$\sin \theta (1 - \tan \theta) = \frac{5}{13} \left(1 - \frac{5}{12}\right)$$

$$= \frac{5}{13} \left[\frac{7}{12}\right] = \frac{35}{156}$$

15. If $\cot \theta = \frac{1}{\sqrt{3}}$, show that $\frac{1 - \cos^2 \theta}{2 - \sin^2 \theta} = \frac{3}{5}$

Sol:

$$\cot \theta = \frac{1}{\sqrt{3}} \quad \frac{1 - \cos^2 \theta}{2 - \sin^2 \theta} = \frac{3}{5}$$

$$\cot \theta = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{1}{\sqrt{3}}$$

Let x be the hypotenuse

By applying Pythagoras

$$AC^2 = AB^2 + BC^2$$

$$x^2 = (\sqrt{3})^2 + 1$$

$$x^2 = 3 + 1$$

$$x^2 = 3 + 1 \Rightarrow x = 2$$

$$\cos \theta = \frac{BC}{AC} = -\frac{1}{2}$$

$$\sin \theta = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$$\frac{1 - \cos^2 \theta}{2 - \sin^2 \theta} \Rightarrow \frac{1 - \left(\frac{1}{2}\right)^2}{2 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \frac{1 - \frac{1}{4}}{2 - \frac{3}{4}} \Rightarrow \frac{\frac{3}{4}}{\frac{5}{4}}$$

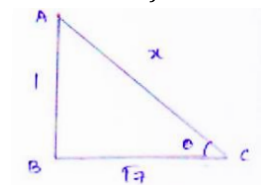
$$= \frac{3}{5}$$

16. If $\tan \theta = \frac{1}{\sqrt{7}}$ $\frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta} = \frac{3}{4}$

Sol:

$$\tan \theta = \frac{1}{\sqrt{7}} \quad \frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta} = \frac{3}{4}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



Let 'x' be the hypotenuse

By applying Pythagoras

$$AC^2 = AB^2 + BC^2$$

$$x^2 = 1^2 + (\sqrt{7})^2$$

$$x^2 = 1 + 7 = 8$$

$$x = 2\sqrt{2}$$

$$\operatorname{Cosec} \theta = \frac{AC}{AB} = 2\sqrt{2}$$

$$\sec \theta = \frac{AC}{BC} = \frac{2\sqrt{2}}{\sqrt{7}}$$

Substitute, $\operatorname{cosec} \theta$, $\sec \theta$ in equation

$$\Rightarrow \frac{(2\sqrt{2})^2 - \left(2\sqrt{\frac{2}{7}}\right)^2}{(2\sqrt{2})^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$$

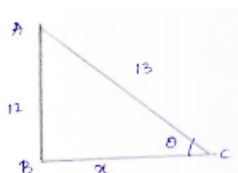
$$\frac{8 - 4 \times \frac{2}{7}}{8 + 4 \times \frac{2}{7}}$$

$$\begin{aligned}
 &\Rightarrow \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}} \\
 &= \frac{\frac{56-8}{7}}{\frac{56+8}{7}} \\
 &= \frac{48}{64} \\
 &= \frac{3}{4}
 \end{aligned}$$

$$L.H.S = R.H.S$$

17. If $\sin \theta = \frac{12}{13}$ find $\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \times \frac{1}{\tan^2 \theta}$

Sol:



Let x be the adjacent side

By applying Pythagoras

$$AC^2 = AB^2 + BC^2$$

$$169 = 144 + x$$

$$x^2 = 25$$

$$x = 5$$

$$\cos \theta = \frac{BC}{AC} = \frac{5}{13}$$

$$\tan \theta = \frac{AB}{BC} = \frac{12}{5}$$

$$\Rightarrow \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{\frac{12}{13} \times \frac{5}{13}} \times \frac{1}{\left[\frac{12}{5}\right]^2}$$

$$\Rightarrow \frac{\frac{144-25}{169}}{\frac{60}{169}} \times \frac{25}{144}$$

$$\Rightarrow \frac{\frac{119}{169}}{\frac{60}{169}} \times \frac{25}{144} = \frac{119}{60} \times \frac{25}{144} = \frac{595}{3456}$$

18. If $\sec \theta = \frac{5}{4}$, find the value of $\frac{\sin \theta - 2 \cos \theta}{\tan \theta - \cot \theta}$

Sol:

Not given

19. If $\cos \theta = \frac{5}{13}$, find the value of $\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} = \frac{3}{5}$

Sol:

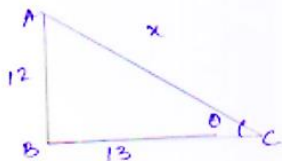
Not given

20. $\tan \theta = \frac{12}{13}$ Find $\frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$

Sol:

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Let x be, the hypotenuse



By Pythagoras we get

$$AC^2 = AB^2 + BC^2$$

$$x^2 = 144 + 169$$

$$x = \sqrt{313}$$

$$\sin \theta = \frac{AB}{AC} = \frac{12}{\sqrt{313}}$$

$$\cos \theta = \frac{BC}{AC} = \frac{13}{\sqrt{313}}$$

Substitute, $\sin \theta$, $\cos \theta$ in equation we get

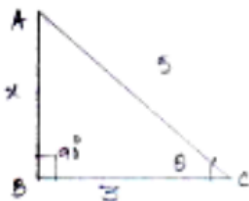
$$\begin{aligned} \frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} &\Rightarrow \frac{2 \times \frac{12}{\sqrt{313}} \times \frac{13}{\sqrt{313}}}{\frac{169}{313} - \frac{144}{313}} \\ &= \frac{\frac{312}{313}}{\frac{25}{313}} = \frac{312}{25} \end{aligned}$$

21. If $\cos \theta = \frac{3}{5}$, find the value of $\frac{\sin \theta - \frac{1}{\tan \theta}}{2 \tan \theta}$

Sol:

$$\cos \theta = \frac{3}{5} \text{ find value of } \frac{\sin \theta - \frac{1}{\tan \theta}}{2 \tan \theta}$$

$$\text{We know that } \cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$



Let us consider right angled Δ ABC

Let x be the opposite side, By applying Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$25 = x^2 + 9$$

$$x^2 = 16 \Rightarrow x = 4$$

$$\sin \theta = \frac{AB}{AC} = \frac{4}{5}$$

$$\tan \theta = \frac{AB}{BC} = \frac{4}{3}$$

Substitute $\sin \theta$, $\tan \theta$ in equation we get

$$\frac{\sin \theta - \frac{1}{\tan \theta}}{2 \tan \theta} = \frac{\frac{4}{5} - \frac{3}{4}}{2 \times \frac{4}{3}}$$

$$= \frac{\frac{16-15}{20}}{\frac{8}{3}} = \frac{\frac{1}{20}}{\frac{8}{3}}$$

$$= \frac{1}{20} \times \frac{3}{8} = \frac{3}{160}$$

22. If $\sin \theta = \frac{3}{5}$, evaluate $\frac{\cos \theta - \frac{1}{\tan \theta}}{2 \cot \theta}$

Sol:

Not given

23. If $\sec A = \frac{5}{4}$, verify that $\frac{3 \sin A - 4 \sin^3 A}{4 \cos^3 A - 3 \cos A} = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

Sol:

Not given

24. If $\sin \theta = \frac{3}{4}$, prove that $\sqrt{\frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{\sec^2 \theta - 1}} = \frac{\sqrt{7}}{3}$

Sol:

Not given

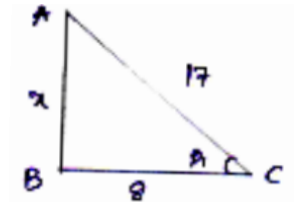
25. If $\sec A = \frac{17}{8}$, verify that $\frac{3 - 4 \sin^2 A}{4 \cos^2 A - 3} = \frac{3 - \tan^2 A}{1 - 3 \tan^2 A}$

Sol:

$$\sec A = \frac{17}{8} \text{ verify that } \frac{3 - 4 \sin^2 A}{4 \cos^2 A - 3} = \frac{3 - \tan^2 A}{1 - 3 \tan^2 A}$$

We know $\sec A = \frac{\text{hypotenuse}}{\text{adjacent side}}$

Consider right angled triangle ABC



Let x be the adjacent side

By applying Pythagoras we get

$$AC^2 = AB^2 + BC^2$$

$$(17)^2 = x^2 + 64$$

$$x^2 = 289 - 64$$

$$x^2 = 225 \Rightarrow x = 15$$

$$\sin A = \frac{AB}{BC} = \frac{15}{17}$$

$$\cos A = \frac{BC}{AC} = \frac{8}{17}$$

$$\tan A = \frac{AB}{BC} = \frac{15}{8}$$

$$\text{L.H.S} = \frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{3-4\times\left(\frac{15}{17}\right)^2}{4\times\left(\frac{8}{17}\right)^2-3} = \frac{3-4\times\frac{225}{289}}{4\times\frac{64}{289}-3} = \frac{867-900}{256-867} = \frac{-33}{-611} = \frac{33}{611}$$

$$\text{R.H.S} = \frac{3-\tan^2 A}{1-3\tan^2 A} = \frac{3-\left(\frac{15}{8}\right)^2}{1-3\times\left(\frac{15}{8}\right)^2} = \frac{3-\frac{225}{64}}{1-3\times\frac{225}{64}} = \frac{\frac{-33}{64}}{\frac{-611}{64}} = \frac{-33}{-611} = \frac{33}{611}$$

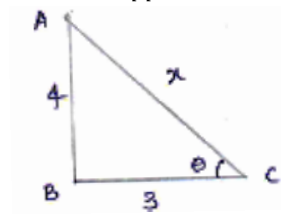
$$\therefore \text{LHS} = \text{RHS}$$

26. If $\cot \theta = \frac{3}{4}$, prove that $\sqrt{\frac{\sec \theta - \operatorname{cosec} \theta}{\sec \theta + \operatorname{cosec} \theta}} = \frac{1}{\sqrt{7}}$

Sol:

$$\cot \theta = \frac{3}{4} \quad \text{P.T} \quad \sqrt{\frac{\sec \theta - \operatorname{cosec} \theta}{\sec \theta + \operatorname{cosec} \theta}} = \frac{1}{\sqrt{7}}$$

$$\cot \theta = \frac{\text{adjacent side}}{\text{opposite side}}$$



Let x be the hypotenuse by applying Pythagoras theorem.

$$AC^2 = AB^2 + BC^2$$

$$x^2 = 16 + 9$$

$$x^2 = 25 \Rightarrow x = 5$$

$$\sec \theta = \frac{AC}{BC} = \frac{5}{3}$$

$$\operatorname{cosec} \theta = \frac{AC}{AB} = \frac{5}{4}$$

On substituting in equation we get

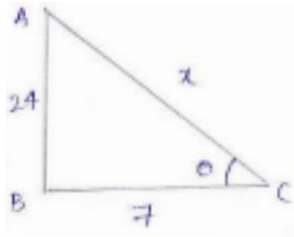
$$\sqrt{\frac{\sec \theta - \operatorname{cosec} \theta}{\sec \theta + \operatorname{cosec} \theta}} = \sqrt{\frac{\frac{5}{3} - \frac{5}{4}}{\frac{5}{3} + \frac{5}{4}}}$$

$$= \sqrt{\frac{\frac{20-15}{12}}{\frac{20+15}{12}}} = \sqrt{\frac{5}{35}} = \frac{1}{\sqrt{7}}$$

27. If $\tan \theta = \frac{24}{7}$, find that $\sin \theta + \cos \theta$

Sol:

$$\tan \theta = \frac{24}{7} \text{ find } \sin \theta + \cos \theta$$



Let x be the hypotenuse. By applying Pythagoras theorem we get

$$AC^2 = AB^2 + BC^2$$

$$x^2 = (24)^2 + (7)^2$$

$$x^2 = 576 + 49 = 625$$

$$x = 25$$

$$\sin \theta = \frac{AB}{AC} = \frac{24}{25}$$

$$\cos \theta = \frac{BC}{AC} = \frac{7}{25}$$

$$\sin \theta + \cos \theta = \frac{24}{25} + \frac{7}{25}$$

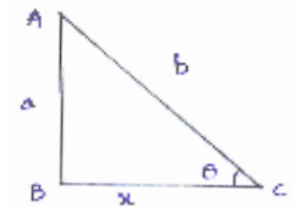
$$= \frac{31}{25}$$

28. If $\sin \theta = \frac{a}{b}$, find $\sec \theta + \tan \theta$ in terms of a and b .

Sol:

$$\sin \theta = \frac{a}{b} \text{ find } \sec \theta + \tan \theta$$

We know $\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$



Let x be the adjacent side

By applying Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$b^2 = a^2 + x^2$$

$$x^2 = b^2 - a^2$$

$$x = \sqrt{b^2 - a^2}$$

$$\sec \theta = \frac{AC}{BC} = \frac{b}{\sqrt{b^2 - a^2}}$$

$$\tan \theta = \frac{AB}{BC} = \frac{a}{\sqrt{b^2 - a^2}}$$

$$\begin{aligned}\sec \theta + \tan \theta &= \frac{b}{\sqrt{b^2-a^2}} + \frac{a}{\sqrt{b^2-a^2}} \\ &= \frac{b+a}{\sqrt{b^2-a^2}} = \frac{b+a}{\sqrt{(b+a)(b-a)}} = \frac{b+a}{\sqrt{b+a}} \cdot \frac{1}{\sqrt{b-a}} = \sqrt{\frac{b+a}{b-a}}\end{aligned}$$

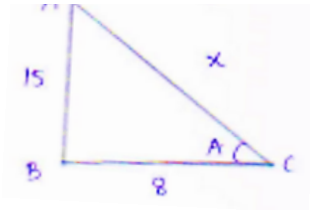
29. If $8 \tan A = 15$, find $\sin A - \cos A$.

Sol:

$8 \tan A = 15$ find. $\sin A - \cos A$

$$\tan A = \frac{15}{8}$$

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}}$$



Let x be the hypotenuse By applying theorem.

$$AC^2 = AB^2 + BC^2$$

$$x^2 = 15^2 + 8^2$$

$$x^2 = 225 + 64$$

$$x^2 = 289 \Rightarrow x = 17$$

$$\sin A = \frac{AB}{AC} = \frac{15}{17}$$

$$\begin{aligned}\sin A - \cos A &= \frac{15}{17} - \frac{8}{17} \\ &= \frac{7}{17}\end{aligned}$$

30. If $3\cos \theta - 4\sin \theta = 2\cos \theta + \sin \theta$ Find $\tan \theta$

Sol:

$$3 \cos \theta - 2 \cos \theta = 4 \sin \theta + \sin \theta \text{ find } \tan \theta$$

$$3 \cos \theta - 2 \cos \theta = \sin \theta + 4 \sin \theta$$

$$\cos \theta = 5 \sin \theta$$

Dividing both side by use we get

$$\frac{\cos \theta}{\cos \theta} = \frac{5 \sin \theta}{\cos \theta}$$

$$1 = 5 \tan \theta$$

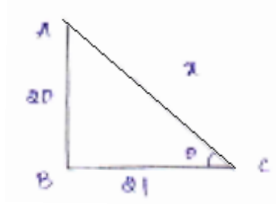
$$\Rightarrow \tan \theta = 1$$

31. If $\tan \theta = \frac{20}{21}$, show that $\frac{1-\sin \theta+\cos \theta}{1+\sin \theta+\cos \theta} = \frac{3}{7}$

Sol:

$$\tan \theta = \frac{20}{21} \quad \text{S.T} \quad \frac{1-\sin \theta+\cos \theta}{1+\sin \theta+\cos \theta} = \frac{3}{7}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{20}{21}$$



Let x be the hypotenuse. By applying Pythagoras we get

$$AC^2 = AB^2 + BC^2$$

$$x^2 = (20)^2 + (21)^2$$

$$x^2 = 400 + 441$$

$$x^2 = 841 \Rightarrow x = 29$$

$$\sin \theta = \frac{AB}{AC} = \frac{20}{29}$$

$$\cos \theta = \frac{BC}{AC} = \frac{21}{29}$$

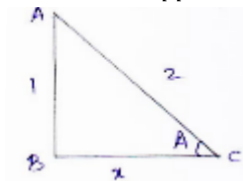
Substitute $\sin \theta$, $\cos \theta$ in equation we get

$$\begin{aligned} &\Rightarrow \frac{1 - \sin \theta + \cos \theta}{1 + \sin \theta + \cos \theta} \\ &\Rightarrow \frac{1 - \frac{20}{29} + \frac{21}{29}}{1 + \frac{20}{29} + \frac{21}{29}} = \frac{\frac{29 - 20 + 21}{29}}{\frac{29 + 20 + 21}{29}} = \frac{30}{70} = \frac{3}{7} \end{aligned}$$

32. If $\operatorname{Cosec} A = 2$ find $\frac{1}{\tan A} + \frac{\sin A}{1 + \cos A}$

Sol:

$$\operatorname{Cosec} A = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{2}{1}$$



Let x be the adjacent side

By applying Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$4 = 1 + x^2$$

$$x^2 = 3 \Rightarrow x = \sqrt{3}$$

$$\sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{2}$$

$$\tan A = \frac{AB}{BC} = \frac{1}{\sqrt{3}}$$

$$\cos A = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$$

Substitute in equation we get

$$\begin{aligned}\frac{1}{\tan A} + \frac{\sin A}{1 + \cos A} &= \frac{1}{\frac{1}{\sqrt{3}}} + \frac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}} \\ &= \sqrt{3} + \frac{\frac{1}{2}}{\frac{2 + \sqrt{3}}{2}} = \sqrt{3} + \frac{1}{2 + \sqrt{3}} = \frac{2\sqrt{3} + 3 + 1}{2 + \sqrt{3}} = \frac{2\sqrt{3} + 4}{2 + \sqrt{3}} = \frac{2(2 + \sqrt{3})}{2 + \sqrt{3}} = 2\end{aligned}$$

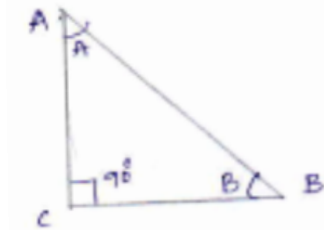
33. If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Sol:

$\angle A$ and $\angle B$ are acute angles.

$\cos A = \cos B$ S.T $\angle A = \angle B$

Let us consider right angled triangle ACB.



We have $\cos A = \frac{\text{adjacent side}}{\text{Hypotenuse}}$

$$= \frac{AC}{AB}$$

$$\cos B = \frac{BC}{AB}$$

$$\cos A = \cos B$$

$$\frac{AC}{AB} = \frac{BC}{AB}$$

$$AC = BC$$

$$\angle A = \angle B$$

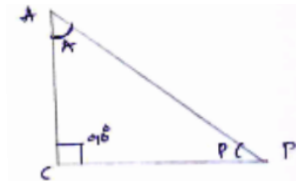
34. If $\angle A$ and $\angle P$ are acute angles such that $\tan A = \tan P$, then show that $\angle A = \angle P$.

Sol:

A and P are acute angle $\tan A = \tan P$

S. T. $\angle A = \angle P$

Let us consider right angled triangle ACP,



We know $\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$$\tan A = \frac{PC}{AC}$$

$$\tan A = \frac{AC}{PC}$$

$$\tan A = \frac{AC}{PC}$$

$$\tan = \tan P$$

$$\frac{DC}{AC} = \frac{AC}{PC}$$

$$(PC)^2 = (AC)^2$$

$$PC = AC \quad [\because \text{Angle opposite to equal sides are equal}]$$

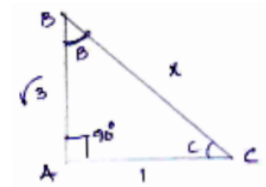
$$\angle P = \angle A$$

35. In a $\triangle ABC$, right angled at A, if $\tan C = \sqrt{3}$, find the value of $\sin B \cos C + \cos B \sin C$.

Sol:

In a $\triangle ABC$ right angled at A $\tan C = \sqrt{3}$

Find $\sin B \cos C + \cos B \sin C$



$$\tan c = \sqrt{3}$$

$$\tan C = \frac{\text{opposite side}}{\text{adjacent side}}$$

Let x be the hypotenuse By applying Pythagoras we get

$$BC^2 = BA^2 + AC^2$$

$$x^2 = (\sqrt{3})^2 + 1^2$$

$$x^2 = 4 \Rightarrow x = 2$$

$$\text{At } \angle B, \sin B = \frac{AC}{BC} = \frac{1}{2}$$

$$\cos B = \frac{\sqrt{3}}{2}$$

$$\text{At } \angle C, \sin = \frac{\sqrt{3}}{2}$$

$$\cos c = \frac{1}{2}$$

On substitution we get

$$\Rightarrow \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{1}{4} + \frac{(\sqrt{3})}{4} \times (\sqrt{3}) = \frac{\sqrt{3} \times \sqrt{3} + 1}{4} = \frac{3+1}{4} = \frac{4}{4} = 1$$

36. State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A .

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A .

(iv) $\sin \theta = \frac{4}{3}$ for some angle θ .

Sol:(a) $\tan A < 1$ Value of $\tan A$ at 45° i.e., $\tan 45 = 1$ As value of A increases to 90° $\tan A$ becomes infinite

So given statement is false.

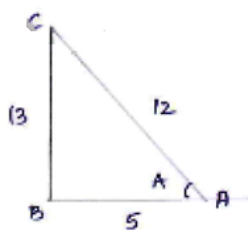
(b) $\sec A = \frac{12}{5}$ for some value of angle of

M-I

 $\sec A = 2.4$ $\sec A > 1$

So given statement is True

M-II

For $\sec A = \frac{12}{5}$ For $\sec A = \frac{12}{5}$ we get adjacent side = 13We get a right angle $\triangle$ ABC

Subtending 90 at B.

So, given statement is true

(c) $\cos A$ is the abbreviation used for cosecant of angle A.The given statement is false. $\therefore \cos A$ is abbreviation used for cos of angle A but not for cosecant of angle A.(d) $\cot A$ is the product of $\cot A$ and A

Given statement is false

 $\therefore \cot A$ is co-tangent of angle A and co-tangent of angle A = $\frac{\text{adjacent side}}{\text{opposite side}}$ (e) $\sin \theta = \frac{4}{3}$ for some angle θ

Given statement is false

Since value of $\sin \theta$ is less than (or) equal to one. Here value of $\sin \theta$ exceeds one, so given statement is false.

Exercise 5.2

Evaluate each of the following (1 – 19):

1. $\sin 45^\circ \sin 30^\circ + \cos 45^\circ \cos 30^\circ$

Sol:

$$\sin 45^\circ \sin 30^\circ + \cos 45^\circ \cos 30^\circ \dots (i)$$

We know that by trigonometric ratios we have,

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \quad \sin 30^\circ = \frac{1}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} \quad \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Substituting the values in (i) we get

$$\begin{aligned} & \frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} \\ &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}+1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}} \end{aligned}$$

2. $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

Sol:

$$\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ \dots (i)$$

By trigonometric ratios we have,

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \cos 60^\circ = \frac{1}{2}$$

Substituting above values in (i), we get

$$\begin{aligned} & \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} \\ &= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1 \end{aligned}$$

3. $\cos 60^\circ \cos 45^\circ - \sin 60^\circ \cdot \sin 45^\circ$

Sol:

$$\cos 60^\circ \cos 45^\circ - \sin 60^\circ \cdot \sin 45^\circ \dots (i)$$

By trigonometric ratios we know that,

$$\cos 60^\circ = \frac{1}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \sin 45^\circ = \frac{1}{\sqrt{2}}$$

By substituting above value in (i), we get

$$\frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} \Rightarrow \frac{1-\sqrt{3}}{2\sqrt{2}}$$

4. $\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$

Sol:

$$\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ \quad \dots(i)$$

By trigonometric ratios we have

$$\sin 30^\circ = \frac{1}{2} \quad \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \sin 90^\circ = 1$$

By substituting above values in (i), we get

$$\begin{aligned} &= \left[\frac{1}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{\sqrt{3}}{2}\right]^2 + [1]^2 \\ &= \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1 \Rightarrow \frac{1+3}{4} + \frac{1+2}{2} \\ &\Rightarrow 1 + \frac{3}{2} = \frac{2+3}{2} = \frac{5}{2} \end{aligned}$$

5. $\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$

Sol:

$$\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ \quad \dots(i)$$

By trigonometric ratios we have

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 60^\circ = \frac{1}{2} \quad \cos 90^\circ = 0$$

By substituting above values in (i), we get

$$\begin{aligned} &\left[\frac{\sqrt{3}}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{1}{2}\right]^2 + [0]^2 \\ &\frac{3}{4} + \frac{1}{2} + \frac{1}{4} = 1 \Rightarrow 1 + \frac{1}{2} = \frac{3}{2} \end{aligned}$$

6. $\tan^2 30^\circ + \tan^2 60^\circ + \tan^2 45^\circ$

Sol:

$$\tan^2 30^\circ + \tan^2 60^\circ + \tan^2 45^\circ \quad \dots(i)$$

By trigonometric ratios we have

$$\tan 30^\circ = \frac{1}{\sqrt{3}} \quad \tan 60^\circ = \sqrt{3} \quad \tan 45^\circ = 1$$

By substituting above values in (i), we get

$$\begin{aligned} &\left[\frac{1}{\sqrt{3}}\right]^2 + [\sqrt{3}]^2 + [1]^2 \\ &\Rightarrow \frac{1}{3} + 3 + 1 \Rightarrow \frac{1}{3} + 4 \\ &\Rightarrow \frac{1+12}{3} = \frac{13}{3} \end{aligned}$$

7. $2 \sin^2 30^\circ - 3 \cos^2 45^\circ + \tan^2 60^\circ$

Sol:

$$2 \sin^2 30^\circ - 3 \cos^2 45^\circ + \tan^2 60^\circ \quad \dots(i)$$

By trigonometric ratios we have

$$\sin 30^\circ = \frac{1}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}} \quad \tan 60^\circ = \sqrt{3}$$

By substituting above values in (i), we get

$$2 \cdot \left[\frac{1}{2}\right]^2 - 3 \left[\frac{1}{\sqrt{2}}\right]^2 + [\sqrt{3}]^2$$

$$2 \cdot \frac{1}{4} - 3 \cdot \frac{1}{2} + 3$$

$$\frac{1}{2} - \frac{3}{2} + 3 \Rightarrow \frac{3}{2} + 2 = 2$$

8. $\sin^2 30^\circ \cos^2 45^\circ + 4 \tan^2 30^\circ + \frac{1}{2} \sin^2 90^\circ - 2 \cos^2 90^\circ + \frac{1}{24} \cos^2 0^\circ$

Sol:

$$\sin^2 30^\circ \cos^2 45^\circ + 4 \tan^2 30^\circ + \frac{1}{2} \sin^2 90^\circ - 2 \cos^2 90^\circ + \frac{1}{24} \cos^2 0^\circ \quad \dots(i)$$

By trigonometric ratios we have

$$\sin 30^\circ = \frac{1}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}} \quad \tan 30^\circ = \frac{1}{\sqrt{3}} \quad \sin 90^\circ = 1 \quad \cos 90^\circ = 0 \quad \cos 0^\circ = 1$$

By substituting above values in (i), we get

$$\left[\frac{1}{2}\right]^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2 + 4 \left[\frac{1}{\sqrt{3}}\right]^2 + \frac{1}{2} [1]^2 - 2 [0]^2 + \frac{1}{24} [1]^2$$

$$\frac{1}{4} \cdot \frac{1}{2} + 4 \cdot \frac{1}{3} + \frac{1}{2} - 0 + \frac{1}{24}$$

$$\frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} = \frac{48}{24} = 2$$

9. $4(\sin^4 60^\circ + \cos^4 30^\circ) - 3(\tan^2 60^\circ - \tan^2 45^\circ) + 5 \cos^2 45^\circ$

Sol:

$$4(\sin^4 60^\circ + \cos^4 30^\circ) - 3(\tan^2 60^\circ - \tan^2 45^\circ) + 5 \cos^2 45^\circ \quad \dots(i)$$

By trigonometric ratios we have

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \cos 30^\circ = \frac{\sqrt{3}}{2} \quad \tan 60^\circ = \sqrt{3} \quad \tan 45^\circ = 1 \quad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

By substituting above values in (i), we get

$$4 \left(\left[\frac{\sqrt{3}}{2}\right]^4 + \left[\frac{\sqrt{3}}{2}\right]^4 \right) - 3([3]^2 - [1]^2) + 5 \left[\frac{1}{\sqrt{2}}\right]^2$$

$$\Rightarrow 4 \left[\frac{9}{16} + \frac{9}{16} \right] - 3[3 - 1] + 5 \left[\frac{1}{2} \right]$$

$$\Rightarrow 4 \cdot \frac{18}{16} - 6 + \frac{5}{2}$$

$$\Rightarrow \frac{1}{4} - 6 + \frac{5}{2}$$

$$= \frac{9}{2} + \frac{5}{2} - 6$$

$$= \frac{14}{2} - 6 = 7 - 6 = 1$$

10. $(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)(\sin^2 30^\circ + 4 \cot^2 45^\circ - \sec^2 60^\circ)$

Sol:

$$(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)(\sin^2 30^\circ + 4 \cot^2 45^\circ - \sec^2 60^\circ) \quad \dots(i)$$

By trigonometric ratios we have

$$\operatorname{Cosec} 45^\circ = \sqrt{2} \quad \sec 30^\circ = \frac{2}{\sqrt{3}} \quad \sin 30^\circ = \frac{1}{2} \quad \cot 45^\circ = 1 \quad \sec 60^\circ = 2$$

By substituting above values in (i), we get

$$\begin{aligned} & \left([\sqrt{2}]^2 \cdot \left[\frac{2}{\sqrt{3}} \right]^2 \right) \left(\left[\frac{1}{2} \right]^2 + 4[1]^2 \cdot [2]^2 \right) \\ & \Rightarrow \left[2 \cdot \frac{4}{3} \right] \left[\frac{1}{4} + 4 - 4 \right] \Rightarrow 3 \cdot \frac{4}{3} \cdot \frac{1}{4} = \frac{2}{3} \end{aligned}$$

11. $\operatorname{cosec}^3 30^\circ \cos 60^\circ \tan^3 45^\circ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ$

Sol:

$$\operatorname{cosec}^3 30^\circ \cos 60^\circ \tan^3 45^\circ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ \quad \dots(i)$$

By trigonometric ratios we have

$$\operatorname{Cosec} 30^\circ = 2, \cos 60^\circ = \frac{1}{2}, \tan 45^\circ = 1 \quad \sin 90^\circ = 1 \quad \sec 45^\circ = \sqrt{2} \quad \cot 30^\circ = \sqrt{3}$$

By substituting above values in (i), we get

$$\begin{aligned} & [2]^3 \cdot \frac{1}{2} \cdot (1)^3 \cdot (1)^2 (\sqrt{2})^2 \cdot \sqrt{3} \\ & \Rightarrow 8 \cdot \frac{1}{2} \cdot 1 \cdot 2 \cdot \sqrt{3} \Rightarrow 8\sqrt{3} \end{aligned}$$

12. $\cot^2 30^\circ - 2 \cos^2 60^\circ - \frac{3}{4} \sec^2 45^\circ - 4 \sec^2 30^\circ$

Sol:

$$\cot^2 30^\circ - 2 \cos^2 60^\circ - \frac{3}{4} \sec^2 45^\circ - 4 \sec^2 30^\circ \quad \dots(i)$$

By trigonometric ratios we have

$$\cot 30^\circ = \sqrt{3} \quad \cos 60^\circ = \frac{1}{2} \quad \sec 45^\circ = \sqrt{2} \quad \sec 30^\circ = \frac{2}{\sqrt{3}}$$

By substituting above values in (i), we get

$$\begin{aligned} & (\sqrt{3})^2 - 2 \left[\frac{1}{2} \right]^2 - \frac{3}{4} (\sqrt{2})^2 - 4 \left[\frac{2}{\sqrt{3}} \right]^2 \\ & 3 - 2 \cdot \frac{1}{4} - \frac{3}{4} \cdot 2 - 4 \cdot \frac{4}{3} \\ & 3 - \frac{1}{2} - \frac{3}{2} - \frac{8}{3} \Rightarrow -\frac{5}{3} \end{aligned}$$

13. $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

Sol:

$$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ) \quad \dots(i)$$

By trigonometric ratios we have

$$\cos 0^\circ = 1, \sin 45^\circ = \frac{1}{\sqrt{2}}, \sin 30^\circ = \frac{1}{2}, \sin 90^\circ = 1, \cos 45^\circ = \frac{1}{\sqrt{2}}, \cos 60^\circ = \frac{1}{2}$$

By substituting above values in (i), we get

$$\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right) \left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$$

$$\left[\frac{3}{2} + \frac{1}{\sqrt{2}}\right] \left[\frac{3}{2} - \frac{1}{\sqrt{2}}\right] \Rightarrow \left[\frac{3}{2}\right]^2 - \left[\frac{1}{\sqrt{2}}\right]^2 = \frac{9}{4} - \frac{1}{2} = \frac{7}{4}$$

14. $\frac{\sin 30^\circ - \sin 90^\circ + 2 \cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$

Sol:

$$\frac{\sin 30^\circ - \sin 90^\circ + 2 \cos 0^\circ}{\tan 30^\circ \tan 60^\circ} \quad \dots(i)$$

By trigonometric ratios we have

$$\sin 30^\circ = \frac{1}{2} \quad \sin 90^\circ = 1 \quad \cos 0^\circ = 1 \quad \tan 30^\circ = \frac{1}{\sqrt{3}} \quad \tan 60^\circ = \sqrt{3}$$

By substituting above values in (i), we get

$$\frac{\frac{1}{2} - 1 + 2}{\frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{3}{2} + 1}{1} = \frac{3}{2}$$

15. $\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cos^2 45^\circ$

Sol:

$$\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cos^2 45^\circ \quad \dots(i)$$

By trigonometric ratios we have

$$\cot 30^\circ = \sqrt{3} \quad \sin 60^\circ = \frac{\sqrt{3}}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

By substituting above values in (i), we get

$$\frac{4}{(\sqrt{3})^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\frac{4}{3} + \frac{4}{3} - \frac{1}{2} = \frac{13}{6}$$

16. $4(\sin^4 30^\circ + \cos^2 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) - \sin^2 60^\circ$

Sol:

$$4(\sin^4 30^\circ + \cos^2 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) - \sin^2 60^\circ \quad \dots(i)$$

By trigonometric ratios we have

$$\sin 30^\circ = \frac{1}{2} \quad \cos 60^\circ = \frac{1}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}} \quad \sin 90^\circ = 1 \quad \sin 60^\circ = \frac{\sqrt{3}}{2}$$

By substituting above values in (i), we get

$$4 \left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^2 \right] - 3 \left[\left[\frac{1}{\sqrt{2}}\right]^2 - 1 \right] - \left[\frac{\sqrt{3}}{2}\right]^2$$

$$4 \left[\frac{1}{16} + \frac{1}{4} \right] - 3 \left[\frac{1 - [\sqrt{2}]}{(\sqrt{2})^2} \right] - \frac{3}{4}$$

$$\frac{1}{4} + 1 - 3 \left[\frac{1 - [\sqrt{2}]}{[\sqrt{2}]} \right] - \frac{3}{4}$$

$$= \frac{1}{4} + 1 - \frac{3}{4} + \frac{3}{2} = 2$$

17.
$$\frac{\tan^2 60^\circ + 4 \cos^2 45^\circ + 3 \sec^2 30^\circ + 5 \cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$$

Sol:

$$\frac{\tan^2 60^\circ + 4 \cos^2 45^\circ + 3 \sec^2 30^\circ + 5 \cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ} \quad \dots(i)$$

By trigonometric ratios we have

$$\tan 60^\circ = \sqrt{3} \quad \cos 45^\circ = \frac{1}{\sqrt{2}} \quad \sec 30^\circ = \frac{2}{\sqrt{3}}$$

$$\cos 90^\circ = 0 \quad \operatorname{cosec} 30^\circ = 2 \quad \sec 60^\circ = 2 \quad \cot 30^\circ = \sqrt{3}$$

By substituting above values in (i), we get

$$\begin{aligned} & \frac{(\sqrt{3})^2 + 4 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 + 2 + \left[\frac{2}{\sqrt{3}}\right]^2 + 5(0)^2}{2 + 2\sqrt{2} + (\sqrt{3})^2} \\ &= \frac{3 + 4 \cdot \frac{1}{2} + 3 + \frac{4}{3}}{4 + 3} = \frac{3 + 2 + 4}{1} = 9 \end{aligned}$$

18.
$$\frac{\sin 30^\circ}{\sin 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$$

Sol:

$$\frac{\sin 30^\circ}{\sin 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ} \quad \dots(i)$$

By trigonometric ratios we have

$$\sin 30^\circ = \frac{1}{2} \quad \sin 45^\circ = \frac{1}{\sqrt{2}} \quad \tan 45^\circ = 1 \quad \sec 60^\circ = 2 \quad \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cot 45^\circ = 1 \quad \cos 30^\circ = \frac{\sqrt{3}}{2} \quad \sin 90^\circ = 1$$

By substituting above values in (i), we get

$$\begin{aligned} & \frac{1}{2} \cdot \sqrt{2} + \frac{1}{2} - \frac{\sqrt{3}}{2} \cdot 1 - \frac{\sqrt{3}}{2} \cdot 1 \\ &= \frac{2 + 1 - \sqrt{3} - \sqrt{3}}{2} \end{aligned}$$

19.
$$\frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5 \sin 90^\circ}{2 \cos 0^\circ}$$

Sol:

$$\frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5 \sin 90^\circ}{2 \cos 0^\circ} \quad \dots(i)$$

By trigonometric ratios we have

$$\tan 45^\circ = 1 \quad \operatorname{cosec} 30^\circ = 2 \quad \sec 60^\circ = 2 \quad \cot 45^\circ = 1 \quad \sin 90^\circ = 1 \quad \cos 0^\circ = 1$$

By substituting above values in (i), we get

$$\begin{aligned} & \frac{1}{2} + \frac{2}{1} - 5 \cdot \frac{1}{2} \\ &= \frac{1}{2} + 2 - \frac{5}{2} = -2 + 2 = 0 \end{aligned}$$

20. $2\sin 3x = \sqrt{3}$ $s = ?$

Sol:

$$\sin 3x = \frac{\sqrt{3}}{2}$$

$$\sin 3x = \sin 60^\circ$$

Equating angles we get,

$$3x = 60^\circ$$

$$x = 20^\circ$$

21. $2 \sin \frac{x}{2} = 1$ $x = ?$

Sol:

$$\sin \frac{x}{2} = \frac{1}{2}$$

$$\sin \frac{x}{2} = \sin 30^\circ$$

$$\frac{x}{2} = 30^\circ$$

$$x = 60^\circ$$

22. $\sqrt{3} \sin x = \cos x$

Sol:

$$\sqrt{3} \tan x = 1$$

$$\tan x = \frac{1}{\sqrt{3}}$$

$$\therefore \tan x = \tan 30^\circ$$

$$x = 30^\circ$$

23. $\tan x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

Sol:

$$\tan x = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \quad \left[\because \sin 45^\circ = \frac{1}{\sqrt{2}} \cos 45^\circ = \frac{1}{\sqrt{2}} \sin 30^\circ = \frac{1}{2} \right]$$

$$\tan x = \frac{1}{2} + \frac{1}{2}$$

$$\tan x = 1$$

$$\tan x = \tan 45^\circ$$

$$x = 45^\circ$$

24. $\sqrt{3} \tan 2x = \cos 60^\circ + \sin 45^\circ \cos 45^\circ$

Sol:

$$\sqrt{3} \tan 2x = \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \quad \left[\because \cos 60^\circ = \frac{1}{2} \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}} \right]$$

$$\sqrt{3} \tan 2x = \frac{1}{\sqrt{3}} \Rightarrow \tan 2x = \tan 30^\circ$$

$$2x = 30^\circ$$

$$x = 15^\circ$$

25. $\cos 2x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

Sol:

$$\cos 2x = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2} \quad \left[\because \cos 60^\circ = \sin 30^\circ = \frac{1}{2}, \sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2} \right]$$

$$\cos 2x = 2 \cdot \frac{\sqrt{3}}{4}$$

$$\Rightarrow \cos 2x = \frac{\sqrt{3}}{2}$$

$$\cos 2x = \cos 30^\circ$$

$$2x = 30^\circ$$

$$x = 15^\circ$$

26. If $\theta = 30^\circ$ verify

(i) $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

Sol:

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} \quad \dots (i)$$

Substitute $\theta = 30^\circ$ in (i)

$$\text{LHS} = \tan 60^\circ = \sqrt{3}$$

$$\text{RHS} = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \cdot \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \sqrt{3}$$

$$\therefore \text{LHS} = \text{RHS}$$

(ii) $\sin \theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$

Substitute $\theta = 30^\circ$

$$\sin 60^\circ = \frac{2 \tan 30^\circ}{1 + (\tan 30^\circ)^2}$$

$$= \frac{\sqrt{3}}{2} = \frac{2 \cdot \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\sqrt{3}}{2} = \frac{2}{\sqrt{3}} \cdot \frac{3}{4} \Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$\therefore \text{LHS} = \text{RHS}$$

(iii) $\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$

Substitute $\theta = 30^\circ$

$$\text{LHS} = \cos 60^\circ$$

$$= \cos 2(30^\circ)$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\text{RHS} = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$= \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$$

$$= \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{1}{2}$$

$$\therefore \text{LHS} = \text{RHS}$$

$$(iv) \quad \cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$\text{LHS} = \cos 30^\circ$$

$$\text{RHS} = 4 \cos^3 \theta - 3 \cos \theta$$

$$\text{Substitute } \theta = 30^\circ$$

$$4 \cos^3 30^\circ - 3 \cos 30^\circ$$

$$\cos 3(30^\circ) = \cos 90^\circ \quad 4 \cdot \left[\frac{\sqrt{3}}{2}\right]^3 - 3 \cdot \frac{\sqrt{3}}{2}$$

$$= 0 \quad \Rightarrow \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$$

27. If $A = B = 60^\circ$. Verify

$$(i) \quad \cos(A - B) = \cos A \cos B + \sin A \sin B$$

Sol:

$$\cos(A - B) = \cos A \cos B + \sin A \sin B \quad \dots(i)$$

Substitute A & B in (i)

$$\Rightarrow \cos(60^\circ - 60^\circ) = \cos 60^\circ \cos 60^\circ + \sin 60^\circ \sin 60^\circ$$

$$\cos 0^\circ = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$1 = \frac{1}{4} + \frac{3}{4} = 1 = 1 \quad \text{LHS} = \text{RHS}$$

$$(ii) \quad \text{Substitute A \& B in (i)}$$

$$\sin(60^\circ - 60^\circ) = \sin 60^\circ \cos 60^\circ - \cos 60^\circ \sin 60^\circ$$

$$= \sin 0^\circ = 0 = 0$$

$$\text{LHS} = \text{RHS}$$

$$(iii) \quad \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$A = 60^\circ \quad B = 60^\circ \text{ we get}$$

$$\tan(60^\circ - 60^\circ) = \frac{\tan 60^\circ - \tan 60^\circ}{1 - \tan 60^\circ \tan 60^\circ}$$

$$\tan 0^\circ = 0$$

$$0 = 0$$

$$\text{LHS} = \text{RHS}$$

28. If $A = 30^\circ$ $B = 60^\circ$ verify

$$(i) \quad \sin(A + B) = \sin A \cos B + \cos A \sin B$$

Sol:

$$A = 30^\circ, B = 60^\circ \text{ we get}$$

$$\sin(30^\circ + 60^\circ) = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$$

$$\sin 90^\circ = \frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$\sin 90^\circ = 1 \Rightarrow 1 = 1$$

$$\text{LHS} = \text{RHS}$$

$$(ii) \quad \cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$A = 30^\circ \quad B = 60^\circ$$

$$\cos(90^\circ) = \cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ$$

$$= \cos 90^\circ = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$$

$$0 = 0$$

$$\text{LHS} = \text{RHS}$$

29. $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

Find $\sin 15^\circ \cos 15^\circ$

Sol:

$$\sin(A - B) = \sin A \cos B - \cos A \sin B \quad \dots(i)$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B \quad \dots(ii)$$

Let $A = 45^\circ$ $B = 30^\circ$ we get on substituting in (i)

$$\Rightarrow \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$\sin 15^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$\therefore \sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

(ii) $A = 45^\circ$ $B = 30^\circ$ in equation (ii) we get

$$\cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$\cos 15^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

30. In right angled triangle ABC. $\angle C = 90^\circ$, $\angle B = 60^\circ$. $AB = 15$ units. Find remaining angles and sides.

Sol:

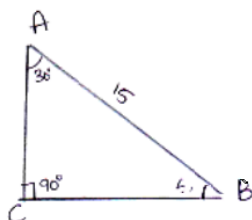
In a Δ the sum of all angles = 180°

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 90^\circ + 60^\circ + \angle A = 180^\circ$$

$$\angle A = 180^\circ - 150^\circ$$

$$\therefore \angle A = 30^\circ$$



From above figure

$$\cos B = \frac{BC}{AB}$$

$$\cos 60^\circ = \frac{BC}{15}$$

$$\frac{1}{2} = \frac{BC}{15}$$

$$BC = \frac{15}{2}$$

$$\sin B = \frac{AC}{15}$$

$$\sin 60^\circ = \frac{AC}{15}$$

$$\frac{\sqrt{3}}{2} = \frac{AC}{15} \Rightarrow AC = \frac{15\sqrt{3}}{2}$$

31. In $\triangle ABC$ is a right triangle such that $\angle C = 90^\circ$, $\angle A = 45^\circ$, $BC = 7$ units find $\angle B$, AB and AC

Sol:

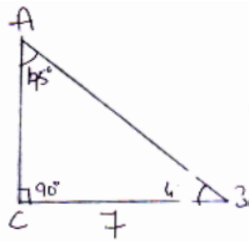
Sum of angles in $\triangle = 180^\circ$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$45^\circ + \angle B + 90^\circ = 180^\circ$$

$$\angle B = 180^\circ - 135^\circ$$

$$\angle B = 45^\circ$$



From figure $\cos B = \frac{BC}{AB}$

$$\cos 45^\circ = \frac{7}{AB}$$

$$\frac{1}{\sqrt{2}} = \frac{7}{AB}$$

$$AB = 7\sqrt{2} \text{ units}$$

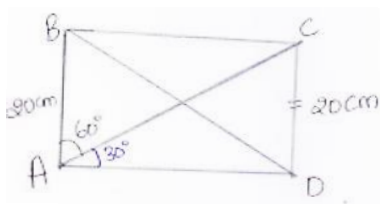
From figure $\sin B = \frac{AC}{AB}$

$$\sin 45^\circ = \frac{AC}{7\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{AC}{7\sqrt{2}} \therefore AC = 7 \text{ units}$$

32. In rectangle ABCD $AB = 20\text{cm}$, $\angle BAC = 60^\circ$, calculate side BC and diagonals AC and BD .

Sol:



Consider $\triangle ABC$ we get

$$\cos A = \frac{AB}{AC}$$

$$\therefore \cos 60^\circ = \frac{20}{AC}$$

$$\frac{1}{2} = \frac{20}{AC} \quad \therefore AC = 40 \text{ cm}$$

$$\therefore AC = 40 \text{ cm}$$

$$\sin A = \frac{BC}{AC}$$

$$\sin 60^\circ = \frac{BC}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{BC}{40}$$

$$\therefore BC = 20\sqrt{3} \text{ cm}$$

Consider $\triangle ACD$ we know $\angle CAD = 30^\circ$

$$\therefore \tan 30^\circ = \frac{CD}{AD} = \frac{1}{\sqrt{3}} = \frac{20}{AC} = AD = 20\sqrt{3}$$

In rectangle diagonals are equal in magnitude

$$\therefore BD = AC = 40 \text{ cm}$$

33. If $\sin(A + B) = 1$ and $\cos(A - B) = 1$, $0^\circ < A + B \leq 90^\circ$, $A \geq B$. Find A & B

Sol:

$$\sin(A + B) = 1$$

$$\therefore \sin(A + B) = \sin 90^\circ$$

$$A + B = 90^\circ \quad \dots(i)$$

$$\cos(A - B) = 1$$

$$\cos(A - B) = \cos 0^\circ$$

$$A - B = 0^\circ \quad \dots(ii)$$

Adding (i) & (ii) we get

$$A + B = 90^\circ$$

$$\underline{A - B = 0^\circ}$$

$$A = 90^\circ \quad A = 45^\circ$$

$$A - B = 0$$

$$A = B \Rightarrow B = 45^\circ$$

34. If $\tan(A - B) = \frac{1}{\sqrt{3}}$ and $\tan(A + B) = \sqrt{3}$, $0^\circ < A + B \leq 90^\circ$, $A \geq B$, Find A & B

Sol:

$$\tan(A - B) = \tan 30^\circ$$

$$\tan(A + B) = \tan 60^\circ$$

$$\therefore A - B = 30^\circ \quad \dots(i)$$

$$A + B = 60^\circ \quad \dots(ii)$$

Add (i) & (ii)

$$A - B = 30^\circ$$

$$\underline{A + B = 60^\circ}$$

$$2A = 90^\circ \quad A = 45^\circ$$

$$A - B = 30^\circ \quad 45^\circ - B = 30^\circ$$

$$B = 45^\circ - 30^\circ = 15^\circ$$

35. If $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$, $0^\circ < A + B \leq 90^\circ$, $A > B$, Find A & B

Sol:

$$\sin(A - B) = \sin 30^\circ$$

$$\cos(A + B) = \cos 60^\circ$$

$$A - B = 30^\circ \quad \dots(i)$$

$$A + B = 60^\circ \quad \dots(ii)$$

Add (i) & (ii) we get

$$2A = 90^\circ, A = 45^\circ.$$

$$A - B = 30^\circ$$

$$45 - B = 30^\circ \quad B = 45 - 30^\circ$$

$$B = 15^\circ$$

36. In right angled triangle $\triangle ABC$ at B, $\angle A = \angle C$. Find the values of

(i) $\sin A \cos C + \cos A \sin C$

Sol:

$$\text{In } \triangle ABC \angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + 90^\circ + \angle A = 180^\circ$$

$$2\angle A = 90^\circ$$

$$\angle A = 45^\circ$$

$$\therefore \angle A = 45^\circ$$

(ii) $\sin 45^\circ \cos 45^\circ + \cos 45^\circ \sin 45^\circ$

$$\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2} + \frac{1}{2} = 1$$

(ii) $\sin A \sin B + \cos A \cos B$

$$\angle A = 45^\circ \sin 90^\circ + \cos 45^\circ \cos 90^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot 1 + 0$$

$$= \frac{1}{\sqrt{2}}$$

37. Find acute angles A & B, if $\sin(A + 2B) = \frac{\sqrt{3}}{2}$ $\cos(A + 4B) = 0$, $A > B$.

Sol:

$$\sin(A + 2B) = \sin 60^\circ$$

$$\cos(A + 4B) = \cos 90^\circ$$

$$A + 2B = 60^\circ \quad \dots(i)$$

$$A + 4B = 90^\circ \quad \dots(ii)$$

Subtracting (ii) from (i)

$$A + 4B = 90^\circ$$

$$-A - 2B = -60$$

$$2B = 30^\circ \quad \therefore B = 15^\circ$$

$$A + 4B = 90^\circ$$

$$4B = 4(15^\circ) = 4B = 60^\circ$$

$$\therefore A + 60^\circ = 90^\circ \therefore A = 30^\circ$$

38. If A and B are acute angles such that $\tan A = \frac{1}{2}$ $\tan B = \frac{1}{3}$ and $\tan (A + B) =$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} A + B = ?$$

Sol:

$$\tan A = \frac{1}{2} \quad \tan B = \frac{1}{3}$$

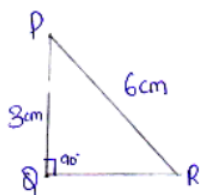
$$\tan (A + B) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} = \frac{\frac{5}{6}}{1 - \frac{1}{6}} = 1$$

$$\tan (A + B) = \tan 45^\circ$$

$$\therefore A + B = 45^\circ$$

39. In ΔPQR , right angled at Q, $PQ = 3\text{cm}$ $PR = 6\text{cm}$. Determine $\angle P = ?$ $\angle R = ?$

Sol:



From above figure

$$\sin R = \frac{PQ}{PR}$$

$$\sin R = \frac{3}{6} = \frac{1}{2}$$

$$\therefore \sin R = \sin 30^\circ$$

$$R = 30^\circ$$

We know in Δ le $\angle P + \angle Q + \angle R = 180^\circ$

$$\angle P + 90^\circ + 30^\circ = 180^\circ$$

$$\angle P = 60^\circ$$

Exercise 5.3

Evaluate the following:

1. $\frac{\sin 20^\circ}{\cos 70^\circ}$

Sol:

(i)

$$\Rightarrow \frac{\sin (90^\circ - 70^\circ)}{\cos 70^\circ} \Rightarrow \frac{\cos 70^\circ}{\cos 70^\circ} \quad [\because \sin (90^\circ - \theta) = \cos \theta]$$

$$\Rightarrow \frac{\cos 70^\circ}{\cos 70^\circ} = 1$$

(ii)

$$\frac{\cos 19^\circ}{\sin 71^\circ}$$

$$\Rightarrow \frac{\cos(90^\circ - 71^\circ)}{\sin 71^\circ} \Rightarrow \frac{\sin 71^\circ}{\sin 71^\circ} [\because \cos(90^\circ - \theta) = \sin \theta]$$

$$= 1$$

(iii)

$$\frac{\sin 21^\circ}{\cos 69^\circ} \Rightarrow \frac{\sin(\cos 69^\circ)}{\cos 69^\circ} = \frac{\cos 69^\circ}{\cos 69^\circ} [\because \sin(90^\circ - \theta) = \cos \theta]$$

$$= 1$$

(iv)

$$\frac{\tan 10^\circ}{\cot 80^\circ} \Rightarrow \frac{\tan(90^\circ - 80^\circ)}{\cot 80^\circ} = \frac{\cot 80^\circ}{\cot 80^\circ} [\because \tan(90^\circ - \theta) = \cot \theta]$$

$$= 1$$

(v)

$$\frac{\sec 11^\circ}{\operatorname{cosec} 79^\circ} \Rightarrow \frac{\sec(90^\circ - 79^\circ)}{\operatorname{cosec} 79^\circ} = \frac{\operatorname{cosec} 79^\circ}{\operatorname{cosec} 79^\circ} [\because \sec(90^\circ - \theta) = \operatorname{cosec} \theta]$$

$$= 1$$

Evaluate the following:

2. (i) $\left[\frac{\sin 49^\circ}{\cos 45^\circ} \right]^2 + \left[\frac{\cos 41^\circ}{\sin 49^\circ} \right]$

Sol:We know that $\sin(49^\circ) = \sin(90^\circ - 41^\circ) = \cos 41^\circ$ similarly $\cos 41^\circ = \sin 49^\circ$

$$\Rightarrow \left[\frac{\cos 41^\circ}{\cos 41^\circ} \right]^2 + \left[\frac{\sin 49^\circ}{\sin 49^\circ} \right] = 1^2 + 1^2 = 2$$

(ii)

$$\cos 48^\circ - \sin 42^\circ$$

Sol:

$$\cos 48^\circ = \cos(90^\circ - 42^\circ) = \sin 42^\circ$$

$$\therefore \sin 42^\circ - \sin 42^\circ = 0$$

(iii)

$$\frac{\cot 40^\circ}{\cos 35^\circ} - \frac{1}{2} \left[\frac{\cos 35^\circ}{\sin 55^\circ} \right]$$

Sol:

$$\cot 40^\circ = \cot(90^\circ - 50^\circ) = \tan 50^\circ$$

$$\cos 35^\circ = \cos(90^\circ - 55^\circ) = \sin 55^\circ$$

$$\Rightarrow \frac{\tan 50^\circ}{\tan 50^\circ} - \frac{1}{2} \left[\frac{\sin 55^\circ}{\sin 55^\circ} \right]$$

$$= 1 - \frac{1}{2} [1]$$

$$= \frac{1}{2}$$

(iv)

$$\left[\frac{\sin 27^\circ}{\cos 63^\circ} \right] - \left[\frac{\cos 63^\circ}{\sin 27^\circ} \right]^2$$



Sol:

$$\sin 27^\circ = \sin (90^\circ - 63^\circ) = \cos 63^\circ \quad [\because \sin (90^\circ - \theta) = \cos \theta]$$

$$\Rightarrow \sin 27^\circ = \cos 63^\circ$$

$$\left[\frac{\sin 27^\circ}{\sin 27^\circ} \right]^2 - \left[\frac{\cos 63^\circ}{\cos 63^\circ} \right]^2 = 1 - 1 = 0$$

(v)

$$\frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 63^\circ}{\cos 63^\circ} - 1$$

Sol:

$$\tan 35^\circ = \tan (90^\circ - 55^\circ) = \cot 55^\circ$$

$$\cot 78^\circ = \cot (90^\circ - 12^\circ) = \tan 12^\circ$$

$$\Rightarrow \frac{\cot 55^\circ}{\cot 55^\circ} + \frac{\tan 12^\circ}{\tan 12^\circ} - 1$$

$$= \tan 1 - 1 = 1$$

(vi)

$$\frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ}$$

Sol:

$$\sec 70^\circ = \sec (90^\circ - 20^\circ) = \operatorname{cosec} 20^\circ \quad [\because \sec (90^\circ - \theta) = \operatorname{cosec} \theta]$$

$$\sin 59^\circ = \sin (90^\circ - 31^\circ) = \cos 31^\circ \quad [\because \sin (90^\circ - \theta) = \cos \theta]$$

$$\Rightarrow \frac{\operatorname{cosec} 20^\circ}{\operatorname{cosec} 20^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ} = 1 + 1 = 2$$

(vii)

$$\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$$

Sol:

$$\sec 50^\circ = \sec (90^\circ - 40^\circ) = \operatorname{cosec} 40^\circ$$

$$\cos 40^\circ = \cos (90^\circ - 50^\circ) = \sin 50^\circ$$

$$\therefore \sin \theta \operatorname{cosec} \theta = 1$$

$$\Rightarrow \operatorname{cosec} 40^\circ \sin 40^\circ + \sin 50^\circ \operatorname{cosec} 50^\circ$$

$$1 + 1 = 2$$

3. Express each one of the following in terms of trigonometric ratios of angles lying between 0° and 45°

(i) $\sin 59^\circ + \cos 56^\circ$

Sol:

$$\sin 59^\circ = \sin (90^\circ - 31^\circ) = \cos 31^\circ$$

$$\cos 56^\circ = \cos (90^\circ - 34^\circ) = \sin 34^\circ$$

$$\Rightarrow \cos 31^\circ + \sin 34^\circ$$

(ii)

$$\tan 65^\circ + \cot 49^\circ$$



Sol:

$$\tan 65^\circ = \tan (90^\circ - 25^\circ) = \cot 25^\circ$$

$$\cot 49^\circ = \cot (90^\circ - 41^\circ) = \tan (41^\circ)$$

$$\Rightarrow \cot 25^\circ + \tan 41^\circ$$

(iii)

$$\sec 76^\circ + \operatorname{cosec} 52^\circ$$

Sol:

$$\sec 76^\circ = \sec (90^\circ - 14^\circ) = \operatorname{cosec} 14^\circ$$

$$\operatorname{Cosec} 52^\circ = \operatorname{cosec} (90^\circ - 38^\circ) = \sec 38^\circ$$

$$\Rightarrow \operatorname{Cosec} 14^\circ + \sec 38^\circ$$

(iv)

$$\cos 78^\circ + \sec 78^\circ$$

Sol:

$$\cos 78^\circ = \cos (90^\circ - 12^\circ) = \sin 12^\circ$$

$$\sec 78^\circ = \sec (90^\circ - 12^\circ) = \operatorname{cosec} 12^\circ$$

$$\Rightarrow \sin 12^\circ + \operatorname{cosec} 12^\circ$$

(v)

$$\operatorname{Cosec} 54^\circ + \sin 72^\circ$$

Sol:

$$\operatorname{Cosec} 54^\circ = \operatorname{cosec} (90^\circ - 36^\circ) = \sec 36^\circ$$

$$\sin 72^\circ = \sin (90^\circ - 18^\circ) = \cos 18^\circ$$

$$\Rightarrow \sec 36^\circ + \cos 18^\circ$$

(vi)

$$\cot 85^\circ + \cos 75^\circ$$

Sol:

$$\cot 85^\circ = \cot (90^\circ - 5^\circ) = \tan 5^\circ$$

$$\cos 75^\circ = \cos (90^\circ - 15^\circ) = \sin 15^\circ$$

$$= \tan 5^\circ + \sin 15^\circ$$

(vii)

$$\sin 67^\circ + \cos 75^\circ$$

Sol:

$$\sin 67^\circ = \sin (90^\circ - 23^\circ) = \cos 23^\circ$$

$$\cos 75^\circ = \cos (90^\circ - 15^\circ) = \sin 15^\circ$$

$$= \cos 23^\circ + \sin 15^\circ$$

4. Express $\cos 75^\circ + \cot 75^\circ$ in terms of angles between 0° and 30° .

Sol:

$$\cot 75^\circ = \cot (90^\circ - 15^\circ) = \tan 15^\circ$$

$$\cot 75^\circ = \cot (90^\circ - 15^\circ) = \tan 15^\circ$$

$$= \sin 15^\circ + \tan 15^\circ$$

5. If $\sin 3A = \cos (A - 26^\circ)$, where $3A$ is an acute angle, find the value of $A = ?$

Sol:

$$\cos \theta = \sin (90^\circ - \theta)$$

$$\Rightarrow \cos (A - 26) = \sin (90^\circ - (A - 26^\circ))$$

$$\Rightarrow \sin 3A = \sin (90^\circ - (A - 26))$$

Equating angles on both sides

$$3A = 90^\circ - A + 26^\circ$$

$$4A = 116^\circ \quad A = \frac{116}{4} = 29^\circ$$

$$\therefore A = 29^\circ$$

6. If A, B, C are interior angles of a triangle ABC , prove that (i) $\tan \left(\frac{C+A}{2} \right) = \cot \frac{B}{2}$

Sol:

$$(i) \quad \tan \left[\frac{C+A}{2} \right] = \cot \frac{B}{2}$$

Sol:

$$\text{Given } A + B + C = 180^\circ$$

$$C + A = 180^\circ - B$$

$$\Rightarrow \tan \left[\frac{180^\circ - B}{2} \right] \Rightarrow \tan \left[90^\circ - \frac{B}{2} \right]$$

$$\Rightarrow \cot \frac{B}{2} \quad [\because \tan(90^\circ - \theta) = \cot \theta]$$

$$\therefore \text{LHS} = \text{RHS}$$

$$(ii) \quad \sin \left[\frac{B+C}{2} \right] = \cos \frac{A}{2}$$

Sol:

$$A + B + C = 180^\circ$$

$$B + C = 180^\circ - A$$

$$\text{LHS} = \sin \left[\frac{180^\circ - A}{2} \right] \Rightarrow \sin \left[90^\circ - \frac{A}{2} \right]$$

$$\cos \frac{A}{2} \quad [\because \sin(90^\circ - \theta) = \cos \theta]$$

$$\therefore \text{LHS} = \text{RHS}$$

7. Prove that

(i)

$$\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ = 1$$

Sol:

$$\tan 20^\circ = \tan (90^\circ - 70^\circ) = \cot 70^\circ$$

$$\tan 35^\circ = \tan (90^\circ - 55^\circ) = \cot 55^\circ$$

$$\tan 45^\circ = 1$$

$$\Rightarrow \cot 70^\circ \tan 70^\circ \times \cot 55^\circ \tan 55^\circ \times \tan 45^\circ \cdot \cot \theta = \tan \theta = 1$$

$$\Rightarrow 1 \times 1 \times 1 = 1 \quad \text{Hence proved.}$$



(ii)

$$\sin 48^\circ \sec 42^\circ + \operatorname{cosec} 42^\circ = 2$$

Sol:

$$\sin 48^\circ = \sin (90^\circ - 42^\circ) = \cos 42^\circ$$

$$\cos (45^\circ) = \cos (90^\circ - 42^\circ) = \sin 42^\circ$$

$$\sec \theta \cdot \cos \theta = 1 \cdot \sin \theta \operatorname{cosec} \theta = 1$$

$$\Rightarrow \cos 42^\circ \sec 42^\circ + \sin 42^\circ \operatorname{cosec} 42^\circ$$

$$\Rightarrow 1 + 1 = 2$$

$$\therefore \text{LHS} = \text{RHS}$$

(iii)

$$\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \operatorname{cosec} 20^\circ = 0$$

Sol:

$$\sin (70^\circ) = \sin (90^\circ - 20^\circ) = \cos 20^\circ$$

$$\operatorname{Cosec} 20^\circ = \operatorname{cosec} (90^\circ - 70^\circ) = \sec 70^\circ$$

$$\cos 70^\circ = \cos (90^\circ - 20^\circ) = \sin 20^\circ$$

$$\Rightarrow \frac{\cos 20^\circ}{\cos 20^\circ} + \frac{\sec 70^\circ}{\sec 70^\circ} - 2 \sin 20^\circ \operatorname{cosec} 20^\circ$$

$$1 + 1 - 2(1) = 0$$

$$\therefore \text{LHS} = \text{RHS} \quad \text{Hence proved}$$

(iv)

$$\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ = 2$$

Sol:

$$\cos 80^\circ = \cos (90^\circ - 10^\circ) = \sin 10^\circ$$

$$\cos 59^\circ = \cos (90^\circ - 31^\circ) = \sin 31^\circ$$

$$\Rightarrow \frac{\sin 10^\circ}{\sin 10^\circ} + \sin 31^\circ \operatorname{cosec} 31^\circ$$

$$= 1 + 1 = 2 \quad [\because \sin \theta \operatorname{cosec} \theta = 1]$$

Hence proved

8. Prove the following:

$$(i) \quad \sin \theta \sin (90 - \theta) - \cos \theta \cos (90 - \theta) = 0$$

Sol:

$$\sin (90 - \theta) = \cos \theta$$

$$\cos (90 - \theta) = \sin \theta$$

$$= 0$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved

$$(ii) \quad \frac{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan \theta}{\operatorname{cosec} (90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot \theta} = 2$$

Sol:

$$\cos (90^\circ - \theta) = \sin \theta \quad \operatorname{cosec} (90^\circ - \theta) = \sec \theta$$



$$\sec(90^\circ - \theta) = \operatorname{cosec} \theta \quad \sin(90^\circ - \theta) = \cos \theta$$

$$\cot(90^\circ - \theta) = \tan \theta$$

$$\Rightarrow \frac{\sin \theta \operatorname{cosec} \theta \tan \theta}{\sec \theta \cos \theta \tan \theta} = \frac{\sin \theta \operatorname{cosec} \theta}{\sec \theta \cos \theta} \quad [\because \sin \theta \operatorname{cosec} \theta = 1]$$

$$= 1 \quad [\sec \theta \cos \theta = 1]$$

$$\frac{\tan(90^\circ - \theta)}{\cot \theta} = \frac{\cot \theta}{\cot \theta} = 1$$

$$\Rightarrow 1 + 1 = 2$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved

$$(iii) \frac{\tan(90^\circ - A) \cot A}{\operatorname{cosec}^2 A} - \cos^2 A = 0$$

Sol:

$$\tan(90^\circ - A) = \cot A$$

$$\Rightarrow \frac{\cot A \cdot \cot A}{\operatorname{cosec}^2 A} - \cos^2 A$$

$$\Rightarrow \frac{\cot^2 A}{\operatorname{cosec}^2 A} - \cos^2 A$$

$$= \frac{\cos^2 A}{\sin^2 A} - \cos^2 A \Rightarrow \cos^2 A \cos^2 A = 0$$

Hence proved

$$(iv) \frac{\cos(90^\circ - A) \sin(90^\circ - A)}{\tan(90^\circ - A)} - \sin^2 A = 0$$

Sol:

$$\cos(90^\circ - A) = \sin A \quad \tan(90^\circ - A) = \cot A$$

$$\sin(90^\circ - A) = \cos A$$

$$\frac{\sin A \cos A}{\cot A} - \sin^2 A = 0$$

$$\frac{\sin A \cdot \cos A}{\cos A} \sin A - \sin^2 A$$

$$\sin^2 A - \sin^2 A = 0$$

$$\text{LHS} = \text{RHS}$$

Hence Proved

$$(v) \sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ = 1$$

Sol:

$$\sin(50^\circ + \theta) = \cos(90^\circ - (50^\circ + \theta)) = \cos(40^\circ - \theta)$$

$$\tan 1^\circ = \tan(90^\circ - 89^\circ) = \cot 89^\circ$$

$$\tan 10^\circ = \tan(90^\circ - 80^\circ) = \cot 80^\circ$$

$$\tan 20^\circ = \tan(90^\circ - 70^\circ) = \cot 70^\circ$$

$$\Rightarrow \cos(40^\circ - \theta) - \cos(40^\circ - \theta) = \cot 89^\circ \tan 89^\circ \cdot \cot 80^\circ \cdot \cot 70^\circ \tan 70^\circ$$

$$\cot \cdot \tan \theta = 1$$

$$= 1 \cdot 1 \cdot 1 = 1$$



LHS = RHS

Hence proved

9. Evaluate:

$$(i) \frac{2}{3} (\cos^4 30^\circ - \sin^4 45^\circ) - 3(\sin^2 60^\circ - \sec^2 45^\circ) + \frac{1}{4} \cot^2 30^\circ$$

Sol:

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \sin 60^\circ = \frac{\sqrt{3}}{2} \quad \cot 30^\circ = \sqrt{3} \quad \sin 45^\circ = \frac{1}{\sqrt{2}} \quad \sec 45^\circ = \frac{1}{\sqrt{2}}$$

Substituting above values in (i)

$$\begin{aligned} & \frac{2}{3} \left[\left(\frac{\sqrt{3}}{2} \right)^4 - \left(\frac{1}{\sqrt{2}} \right)^4 \right] - 3 \left[\left(\frac{\sqrt{3}}{2} \right)^2 \cdot \left[\frac{1}{\sqrt{2}} \right]^2 \right] + \frac{1}{4} (\sqrt{3})^2 \\ & \frac{2}{3} \left[\frac{9}{16} - \frac{1}{4} \right] - 3 \left[\frac{3}{4} - \frac{1}{2} \right] \frac{1-3}{4} \\ & \frac{2}{3} \left[\frac{9-4}{16} \right] - 3 \left[\frac{3-2}{4} \right] - \frac{3}{4} \\ & \Rightarrow \frac{2}{3} \cdot \frac{5}{16} - \frac{3}{4} + \frac{3}{4} \Rightarrow \frac{5}{24} \end{aligned}$$

$$(ii) 4 (\sin^2 30^\circ + \cos^4 60^\circ) - \frac{2}{3} 3 \left[\left(\frac{\sqrt{3}}{2} \right)^2 \cdot \left[\frac{1}{\sqrt{2}} \right]^2 \right] + \frac{1}{4} (\sqrt{3})^2$$

Sol:

$$\begin{aligned} \sin 30^\circ &= \frac{1}{2} \quad \cos 60^\circ = \frac{1}{2} \quad \sin 60^\circ = \frac{\sqrt{3}}{2} \quad \cos 45^\circ = \frac{1}{\sqrt{2}} \quad \tan 60^\circ = \sqrt{3} \\ & \Rightarrow 4 \left[\left[\frac{1}{2} \right]^4 + \left[\frac{1}{2} \right]^4 \right] - \frac{2}{3} \left[\left(\frac{\sqrt{3}}{2} \right)^2 - \left(\frac{1}{\sqrt{2}} \right)^2 \right] + \frac{1}{2} (\sqrt{3})^2 \\ & 4 \left[2 \cdot \frac{1}{16} \right] - \frac{2}{3} \left[\frac{3}{4} - \frac{1}{2} \right] + \frac{3}{2} \\ & = \frac{1}{2} - \frac{2}{3} \cdot \frac{1}{4} + \frac{3}{2} = \frac{11}{6} \end{aligned}$$

$$(iii) \frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\cos \sec 40^\circ}{\sec 50^\circ} - 4 \cos 50^\circ \operatorname{cosec} 40^\circ$$

Sol:

$$\sin 50^\circ = \sin (90^\circ - 40^\circ) = \cos 40^\circ$$

$$\operatorname{Cosec} 40^\circ = \operatorname{cosec} (90^\circ - 50^\circ) = \sec 50^\circ$$

$$\cos 50^\circ = \cos (90^\circ - 40^\circ) = \sin 40^\circ$$

$$\Rightarrow \frac{\cos 40^\circ}{\cos 40^\circ} + \frac{\sec 50^\circ}{\sec 50^\circ} - 4 \sin 40^\circ \operatorname{cosec} 40^\circ$$

$$1 + 1 - 4 = -2$$

$$[\because \sin 40^\circ \operatorname{cosec} 40^\circ = 1]$$

$$(iv) \tan 35^\circ \tan 40^\circ \tan 50^\circ \tan 55^\circ$$

Sol:

$$\tan 35^\circ = \tan (90^\circ - 55^\circ) = \cot 55^\circ$$

$$\tan 40^\circ = \tan (90^\circ - 50^\circ) = \cot 50^\circ$$

$$\tan 65^\circ = 1$$

$$\cot 55 \tan 55 \cdot \cot 50 \tan 50 \cdot \tan 45$$

$$1 \cdot 1 \cdot 1 = 1$$

(v) $\operatorname{Cosec} (65 + \theta) - \sec (25 - \theta) - \tan (55 - \theta) + \cot (35 + \theta)$

Sol:

$$\operatorname{Cosec} (65 + \theta) = \sec (90 - (65 + \theta)) = \sec (25 - \theta)$$

$$\tan (55 - \theta) = \cot (90 - (55 - \theta)) = \cot (35 + \theta)$$

$$\Rightarrow \sec (25 - \theta) - \sec (25 - \theta) \tan (55 - \theta) + \tan (55 - \theta) = 0$$

(vi) $\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$

Sol:

$$\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan (90^\circ - 23^\circ) \tan (90^\circ - 7^\circ)$$

$$\Rightarrow \tan 7^\circ \tan 23^\circ \tan 60^\circ \cot 23^\circ \tan 60^\circ$$

$$1 \cdot 1 \cdot \sqrt{3} = \sqrt{3}$$

(vii) $\frac{2 \sin 68^\circ}{\cos 22^\circ} - \frac{2 \cot 15^\circ}{5 \tan 75^\circ} - \frac{8 \tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5}$

Sol:

$$\sin 68^\circ = \sin (90 - 22) = \cos 22$$

$$\cot 15^\circ = \tan (90 - 75) = \tan 75$$

$$2 \cdot \frac{\cos 22^\circ}{\cos 22^\circ} - \frac{2 \tan 75^\circ}{5 \tan 75^\circ} - \frac{3 \tan 45^\circ \tan 20^\circ \tan 40^\circ \cot 40^\circ \cot 20^\circ}{5}$$

$$= 2 - \frac{2}{5} - \frac{3}{5} = 2 - 1 = 1$$

(viii) $\frac{3 \cos 55^\circ}{7 \sin 35^\circ} - \frac{4(\cos 70^\circ \operatorname{cosec} 20^\circ)}{7(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ)}$

Sol:

$$\cos 55^\circ = \cos (90^\circ - 35^\circ) = \sin 35^\circ$$

$$\cos 70^\circ = \cos (90 - 20) = \sin 20^\circ$$

$$\tan 5^\circ = \cot 85^\circ \tan 25^\circ = \cot 65^\circ$$

$$\Rightarrow \frac{3 \sin 35^\circ}{7 \sin 35^\circ} - \frac{4 (\sin 20^\circ \operatorname{cosec} 20^\circ)}{7(\cot 85^\circ \tan 85^\circ \cot 65^\circ \tan 65^\circ \tan 45^\circ)}$$

$$= \frac{3}{7} - \frac{4}{7} = -\frac{1}{7}$$

(ix) $\frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3} [\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ]$

Sol:

$$\sin 18^\circ = \sin (90^\circ - 72) = \cos 72^\circ$$

$$\tan 10^\circ = \cot 80^\circ \tan 50^\circ = \cot 40^\circ$$

$$\Rightarrow \frac{\sin 18^\circ}{\sin 18^\circ} + \sqrt{3} \left[\tan 80^\circ \cos 30^\circ \cdot \tan 40^\circ \cot 40^\circ \cdot \frac{1}{\sqrt{3}} \right]$$

$$= 1 + \sqrt{3} \cdot \frac{1}{\sqrt{3}} = 2$$

$$(x) \frac{\cos 58^\circ}{\sin 32^\circ} + \frac{\sin 22^\circ}{\cos 68^\circ} - \frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 65^\circ}$$

Sol:

$$\cos 58^\circ = \cos (90^\circ - 32^\circ) = \sin 32^\circ$$

$$\sin 22^\circ = \sin (90^\circ - 68^\circ) = \cos 68^\circ$$

$$\cos 38^\circ = \cos (90 - 52) = \sin 52^\circ$$

$$\tan 18^\circ = \cot 72 \tan 35^\circ = \cot 55^\circ$$

$$\Rightarrow \frac{\sin 32^\circ}{\sin 32^\circ} + \frac{\cos 68^\circ}{\cos 68^\circ} - \frac{\sin 52 \operatorname{cosec} 52}{\tan 72 \cdot \cot 72 \tan 55 \cot 55 \cdot \tan 60}$$

$$= 1 + 1 - \frac{1}{\sqrt{3}} = \frac{2\sqrt{3}-1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6-\sqrt{3}}{3}$$

10. If $\sin \theta = \cos (\theta - 45^\circ)$, where $\theta - 45^\circ$ are acute angles, find the degree measure of θ .

Sol:

$$\sin \theta = \cos (\theta - 45^\circ)$$

$$\cos \theta = \cos (90 - \theta)$$

$$\cos (\theta - 45^\circ) = \sin (90^\circ - (\theta - 45^\circ)) = \sin (90 - \theta + 45^\circ)$$

$$\sin \theta = \sin (135 - \theta)$$

$$\theta = 135 - \theta$$

$$2\theta = 135$$

$$\therefore \theta = 135^\circ/2$$

11. If A, B, C are the interior angles of a $\triangle ABC$, show that:

$$(i) \sin \left(\frac{B+C}{2} \right) = \cos \frac{A}{2} \quad (ii) \cos \left[\frac{B+C}{2} \right] = \sin \frac{A}{2}$$

Sol:

$$A + B + C = 180$$

$$B + C = 180 - \frac{A}{2}$$

$$(i) \sin \left[90 - \frac{A}{2} \right] = \cos \frac{A}{2}$$

$$\therefore \text{LHS} = \text{RHS}$$

$$(ii) \cos \left[90 - \frac{A}{2} \right] = \sin \frac{A}{2}$$

$$\therefore \text{LHS} = \text{RHS}$$

12. If $2\theta + 45^\circ$ and $30^\circ - \theta$ are acute angles, find the degree measure of θ satisfying $\sin (2\theta + 45^\circ) = \cos (30 - \theta^\circ)$

Sol:

Here $2\theta + 45^\circ$ and $30 - \theta^\circ$ are acute angles:

We know that $\sin (90 - \theta) = \cos \theta$

$$\sin (2\theta + 45^\circ) = \sin (90 - (30 - \theta))$$

$$\sin (2\theta + 45^\circ) = \sin (90 - 30 + \theta)$$

$$\sin (20 + 45^\circ) = \sin (60 + \theta)$$

On equating sin of angle of we get

$$2\theta + 45 = 60 + \theta$$

$$2\theta - \theta = 60 - 45$$

$$\theta = 15^\circ$$

13. If θ is a positive acute angle such that $\sec \theta = \operatorname{cosec} 60^\circ$, find $2 \cos^2 \theta - 1$

Sol:

We know that $\sec (90 - \theta) = \operatorname{cosec} \theta$

$$\sec \theta = \sec (90 - 60^\circ)$$

On equating we get

$$\sec \theta = \sec 30^\circ$$

$$\theta = 30^\circ$$

Find $2\cos^2 \theta - 1$

$$\Rightarrow 2 \times \cos^2 30^\circ - 1 \quad \left[\cos 30 = \frac{\sqrt{3}}{2} \right]$$

$$\Rightarrow 2 \times \left(\frac{\sqrt{3}}{2} \right)^2 - 1$$

$$\Rightarrow 2 \times \frac{3}{4} - 1$$

$$\Rightarrow \frac{3}{2} - 1$$

$$= \frac{1}{2}$$

14. If $\cos 2\theta = \sin 4\theta$ where $2\theta, 4\theta$ are acute angles, find the value of θ .

Sol:

We know that $\sin (90 - \theta) = \cos \theta$

$$\sin 2\theta = \cos 2\theta$$

$$\sin 4\theta = \sin (90 - 2\theta)$$

$$4\theta = 90 - 2\theta$$

$$6\theta = 90$$

$$\theta = \frac{90}{6}$$

$$\theta = 15^\circ$$

15. If $\sin 3\theta = \cos (\theta - 6^\circ)$ where 3θ and $\theta - 6^\circ$ are acute angles, find the value of θ .

Sol:

$3\theta, \theta - 6$ are acute angle

We know that $\sin (90 - \theta) = \cos \theta$

$$\sin 3\theta = \sin (90 - (\theta - 6^\circ))$$

$$\sin 3\theta = \sin (90 - \theta + 6^\circ)$$

$$\sin 3\theta = \sin (96^\circ - \theta)$$

$$3\theta = 96^\circ - \theta$$

$$4\theta = 96^\circ$$

$$\theta = \frac{96^\circ}{4}$$

$$\theta = 24^\circ$$

16. If $\sec 4A = \operatorname{cosec} (A - 20^\circ)$ where $4A$ is acute angle, find the value of A .

Sol:

$$\sec 4A = \sec [90 - A - 20] \quad [\because \sec(90 - \theta) = \operatorname{cosec} \theta]$$

$$\sec 4A = \sec (90 - A + 20)$$

$$\sec 4A = \sec (110 - A)$$

$$4A = 110 - A$$

$$5A = 110$$

$$A = \frac{110}{5} \Rightarrow A = 22$$

17. If $\sec 2A = \operatorname{cosec} (A - 42^\circ)$ where $2A$ is acute angle. Find the value of A .

Sol:

$$\text{We know that } (\sec (90 - \theta)) = \operatorname{cosec} \theta$$

$$\sec 2A = \sec (90 - (A - 42))$$

$$\sec 2A = \sec (90 - A + 42)$$

$$\sec 2A = \sec (132 - A)$$

Now equating both the angles we get

$$2A = 132 - A$$

$$3A = \frac{132}{3}$$

$$A = 44$$