Chapter 1

Relations and Functions

Exercise 1.2

Q. 1

Show that the function f: R* → R* defined by f (x) = $\frac{1}{2}$ is one-one and onto, where R* is the set of all non-zero real numbers. Is the result true, if the domain R* is replaced by N with co-domain being same as R*?

Answer:

It is given that f: R* → R* defined by f (x) = $\frac{1}{2}$

check for one-one:

For a function to be one-one, if f(x) = f(y) then x = y. f(x) = f(y)

= $\frac{1}{x} = \frac{1}{y}$

⇒Therefore, f is one – one.

We can see that y ϵ R, there exists x = $\frac{1}{y}$ ∈ R, such that

= f (x) = $\frac{1}{\left(\frac{1}{y}\right)}$ = y

⇒ f is onto.

Therefore, function f is one-one and onto.

Now, let us consider g: N → R* defined by

= g (x) = $\frac{1}{x}$

Then, we get

$\frac{1}{x_1} = \frac{1}{x_2}$

$\Rightarrow x_1 = x_2$

$\Rightarrow$ g is one–one.

It can be observed that g is not onto as for 1.2 ϵ R there does not exist any x in N such that

$= g(x) = \frac{1}{1..2}$

Therefore, function g is one –one but not onto.

Q. 2 A

Check the infectivity and surjectivity of the following functions:

f: N → N given by $f(x) = x^2$

Answer:

It is given that f: N → N given by $f(x) = x^2$

We can see that for x, y ϵ N,

$f(x) = f(y)$

$\Rightarrow x^2 = y^2$

$\Rightarrow x = y$

$\Rightarrow$ f is injective.

Now, let 2 ϵ N. But, we can see that there does not exists any x in N such that

$f(x) = x^2 = 2$

$\Rightarrow$ f is not surjective.

Therefore, function f is injective but not surjective

Q. 2 B

Check the injectivity and surjectivity of the following functions:

f: Z → Z given by f (x) = x2

Answer:

It is given that f: Z → Z given by f (x) = x^2

We can see that f (-1) = f (1) = 1, but -1 ≠ 1

⇒ f is not injective.

Now, let -2 ϵ Z. But, we can see that there does not exists any x in Z such that

f(x) = x^2 = -2

⇒ f is not surjective.

Therefore, function f is neither injective nor surjective.

Q. 2 C

Check the injectivity and surjectivity of the following functions:

f: R → R given by f (x) = x^2

Answer:

It is given that f: R → R given by f (x) = x^2

We can see that f (-1) = f (1) = 1, but -1 ≠ 1

⇒ f is not injective.

Now, let -2 ϵ R. But, we can see that there does not exists any x in R such that

f(x) = x^2 = -2

⇒ f is not surjective.

Therefore, function f is neither injective nor surjective.

Q. 2 D

Check the injectivity and surjectivity of the following functions:

f: N → N given by f (x) = x^3

Answer:

It is given that f: N → N given by f (x) = x^3

We can see that for x, y ϵ N,

f(x) = f(y)

⇒ $x^3 = y^3$

⇒ x = y

⇒ f is injective.

Now, let 2 ϵ N. But, we can see that there does not exists any x in N such that

f(x) = x^3 = 2

⇒ f is not surjective.

Therefore, function f is injective but not surjective.

Q. 2 E

Check the injectivity and surjectivity of the following functions:

f: Z → Z given by f (x) = x^3

Answer:

It is given that f: Z → Z given by f (x) = x^3

We can see that for x, y ϵ N,

f(x) = f(y)

⇒ $x^3 = y^3$

⇒ x = y

⇒ f is injective.

Now, let 2 ϵ Z. But, we can see that there does not exists any x in Z such that

$f(x) = x^3 = 2$

⇒ f is not surjective.

Therefore, function f is injective but not surjective.

Q. 3

Prove that the Greatest Integer Function f: R → R, given by f (x) = [x], is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

Answer:

It is given f: R → R, given by f (x) = [x]

We can see that f (1.2) = [1.2] = 1

f (1.9) = [1.9] = 1

⇒ f (1.2) = f (1.9), but 1.2 ≠ 1.9.

⇒ f is not one- one.

Now, let us consider 0.6 ϵ R.

We know that f(x) = [x] is always an integer.

⇒ there does not exist any element x ϵ R such that f(x) = 0.6

⇒ f is not onto.

Therefore, the greatest integer function is neither one-one nor onto.

Q. 4

Show that the Modulus Function f: R → R, given by f (x) = |x|, is neither one-one nor onto, where |x| is x, if x is positive or 0 and |x| is – x, if x is negative.

Answer:

It is given that f: R → R, given by f (x) = | x|

We can see that f (-1) = |-1| = 1, f (1) = |1| = 1

⇒ f (-1) = f (1), but -1 ≠ 1.

⇒ f is not one-one.

Now, we consider -1 ϵ R.

We know that f(x) = |x| is always positive

Therefore, there doesn't exist any element x in domain R such that f(x) = |x| = -1

⇒ f is not onto.

Therefore, modulus function is neither one-one nor onto.

Q. 5

Show that the Signum Function f: R → R, given by

$$f(x) = \begin{cases} 1, & if\ x > 0 \\ 0, & if\ x = 0 \\ -1, & if\ x < 0 \end{cases}$$

is neither one-one nor onto.

Answer:

It is given that f: R → R, given by

$$f(x) = \begin{cases} 1, & if\ x > 0 \\ 0, & if\ x = 0 \\ -1, & if\ x < 0 \end{cases}$$

We can have observed that f (1) = f (2) = but 1 ≠ 2.

Thus, f is not one – one.

Now, as f (x) takes only 3 values (1, 0, -1) for the element -2 in co-domain R, there exists any x in domain R such that f(x) = -2

Thus, f is not onto.

Therefore, the Signum function is neither one-one nor onto.

Q. 6

Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one.

Answer:

It is given that A = {1, 2, 3}, B = {4, 5, 6, 7}

f: A → B is defined as f = {(1, 4), (2, 5), (3, 6)}

Therefore, f (1) = 4, f (2) = 5, f (3) = 6We can see that the images of distinct elements of A under f are distinct.

Therefore, function f is one- one.

Q. 7 A

In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

f: R → R defined by f (x) = 3 – 4x

Answer:

It is given that f: R → R defined by f (x) = 3 – 4x

Let x_1, x_2ϵ R such that $f(x_1) = f(x_2)$

⇒ $3 - 4x_1 = 3 - 4x_2$

⇒ $-4x_1 = -4x_2$

⇒ $x_1 = x_2$

⇒ f is one- one

For any real number (y) in R, there exist $\frac{3-y}{4}$ in R such that

f $\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right)$

⇒ f is onto.

Therefore, function f is bijective.

Q. 7 B

In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

f: R → R defined by f (x) = 1 + x_2

Answer:

It is given that f: R → R defined by f (x) = 1 + x_2

Let x_1, $x_2 \epsilon$ R such that $f(x_1) = f(x_2)$

$= 1 + x_1^2 = 1 + x_2^2$

$= x_1^2 = x_1^2$

$= x_1 = \pm x_2$

Now, f (1) = f (-1) = 2

⇒ f (x1) = f (x2) which does means that x1 = x2

⇒ f is not one – one

Now consider an element -2 in co- domain R.

We can see that f(x) = 1 + x_2 is always positive.

⇒ there does not exist any x in domain R such that f(x) = -2

⇒ F is not onto.

Therefore, function f is neither one-one nor onto.

Q. 8 Let A and B be sets. Show that f: A × B → B × A such that f (a, b) = (b, a) is bijective function.

Answer:

It is given that f: A × B → B × A is defined as f (a, b) = (b, a)

Now let us consider $(a_1, b_1), (a_2, b_2) \in A \times B$

Such that $f(a_1, b_1) = f(a_2, b_2)$

$\Rightarrow (b_1, a_1) = (b_2, a_2)$

$\Rightarrow b_1 = b_2$ and $a_1 = a_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

⇒ f is one-one.

Now, let (b, a) ϵ B × A be any element.

Then, there exists (a, b) ϵ A × B such that f (a, b) = (b, a)

⇒ f is onto.

Therefore, f is bijective.

Q. 9

Let f: N → N be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, if\ n\ is\ odd \\ \qquad for\ all\ n\ \in N \\ \frac{n}{2}, if\ n\ is\ even \end{cases}$$

State whether the function f is bijective. Justify your answer.

Answer:

It is given that

f: N → N be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, if\ n\ is\ odd \\ for\ all\ n\ \in N \\ \frac{n}{2}, if\ n\ is\ even \end{cases}$$

We can have observed that:

f (1) = $\frac{1+1}{2}$ = 1 and f (2) = $\frac{2}{2}$ = 1 (by using the definition of f)

Thus, f (1) = f (2), where 1 ≠ 2.

Therefore, f is not one-one.

Now, let us consider a natural number (n) in co domain N.

Case I: When n is odd.

Then, n = 2r +1 for some r ϵ N.

⇒ there exist 4r + 1 ϵ N such that f(4r+1) = $\frac{4r+1+1}{2}$ = 2r + 1

Case II: When n is even.

Then, n = 2r for some r ϵ N.

⇒ there exist 4r ϵ N such that f(4r) = $\frac{4r}{2}$ = 2r

Therefore, f is onto.

⇒ Function f is not one-one but it is onto.

Thus, Function f is not bijective function.

Q. 10

Let A = R – {3} and B = R – {1}. Consider the function f: A → B defined by f (x) = $\left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

Answer:

It is given that A = R – {3} and B = R – {1}

f: A → B defined by

f (x) = $\left(\frac{x-2}{x-3}\right)$

Now, let x, y ϵ A such that f(x) = f(y)

$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$

⇒ (x - 2) (y - 3) = (y - 2) (x - 3)

⇒ xy – 3x - 2y + 6 = xy -3y - 2x +6

⇒ -3x - 2y = -3y - 2x

⇒ x = y

⇒ f is one –one.

Let y ϵ B = R – {1}

Then, y ≠ 1.

The function f is onto if there exist x ϵ A such that f(x) = y

Now, f(x) = y

$\frac{x-2}{x-3}$ = y

⇒ x – 2 = xy - 3y

⇒ x(1-y) = -3y + 2

$\Rightarrow x = \frac{2-3y}{1-y} \in A$

⇒ y ϵ B, there exists $\frac{2-3y}{1-y}$ ϵ A such that

$\Rightarrow \left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right)}{\left(\frac{2-3y}{1-y}\right)} = \frac{2-3y-3+2y}{2-3y-3+3y} = \frac{-y}{-1}$

⇒ f is onto.

Therefore, function f is one- one and onto.

Q. 11

Let f: R → R be defined as $f(x) = x^4$. Choose the correct answer.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto.

Answer:

f: R → R be defined as $f(x) = x^4$.

Let x, y ϵ R such that f(x) = f(y)

$\Rightarrow x^4 = y^4$

⇒ x = y

Therefore, $f(x_1) = f(x_2)$ which does not implies $x_1 = x_2$.

For instance, f (1) = f (-1) = 1

Therefore, f is not one-one.

Now, an element 2 in co-domain R.

We can see that there does not exist any x in domain R such that

f (x) = 2

Therefore, f is not onto.

Therefore, function f is neither one-one nor onto.

Q. 12

Let f: R → R be defined as f (x) = 3x. Choose the correct answer.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto.

Answer:

It is given that f: R → R be defined as f (x) = 3x.

Let x, y ϵ R such that f(x) = f(y).

⇒ 3x = 3y

⇒ x = y

⇒ f is one–one.

Also, for any real number (y) in co–domain R, there exists $\frac{y}{3}$ in R such that

$f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right)$

Therefore, f is onto.

Therefore, function f is one-one and onto.