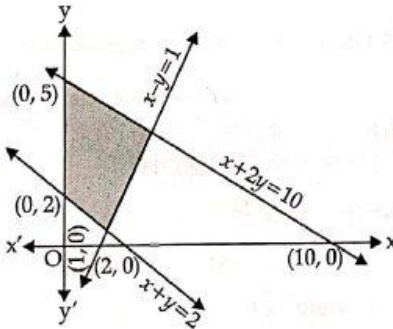


Linear Programming

MULTIPLE CHOICE QUESTIONS (SOLVED)

1. The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below.



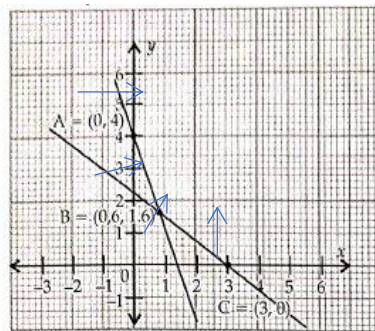
Which of the following is **not** a constraint to the above Linear Programming Problem?

- (a) $x + y \geq 2$ (b) $x + 2y \leq 10$ (c) $x - y \geq 1$ (d) $x - y \leq 1$

Ans. Option (c) is correct.

Solution: We observe, (1,2) does not satisfy the inequality $x - y \geq 1$ ((You can take any point in the feasible region). Therefore, it will not contain the shaded feasible region.

2. The corner points of the shaded unbounded feasible region of LPP are (0,4), (0.6,1.6) and (3,0) as shown in the figure. The minimum value of the objective function $Z = 4x + 6y$ occurs at



- (a) (0.6,1.6) only
- (b) (3,0) only
- (c) (0.6,1.6) and (3,0) only
- (d) At every point of the line-segment joining the points (0.6,1.6) and (3,0)

Ans. Option (d) is correct.

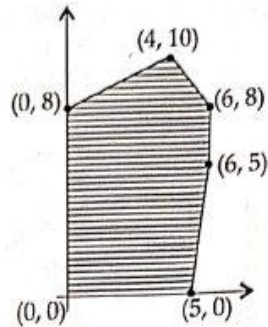
Solution: The minimum value of the objective function occurs at two adjacent corner points (0.6, 1.6) and (3,0) and there is no point in the half plane $4x + 6y < 12$ in common with the feasible region. So the minimum value occurs at every point of the line segment joining the two points.

- 3) If the corner points of the feasible region are A(0,10), B(5,5), C(15,15) and D (0,20), then at which point(s) is the objective function $Z = 3x + 9y$ maximum?
- (a) Point B
 - (b) Point C
 - (c) Point D
 - (d) every point on the line segment CD

Ans. Option (d) is correct.

Solution: Z is maximum 180 at points C(15,15) and D(0,20)
 $\Rightarrow Z$ is maximum at every point on the line segment CD

Q.4. In the given graph, the feasible region for an LPP is shaded. The objective function $Z = 2x - 3y$, will be minimum at:



- (a) (4,10)
- (b) (6,8)
- (c) (0,8)
- (d) (6,5)

Ans. Option (c) is correct

Solution: Z is minimum -24 at (0,8)

Q. 5. A linear programming problem is as follows:

Minimize $Z = 30x + 50y$
 Subject to the constraints,

$$\begin{aligned} 3x + 5y &\geq 15 \\ 2x + 3y &\leq 18 \\ x \geq 0, y &\geq 0 \end{aligned}$$

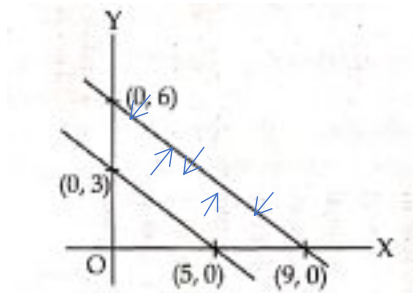
In the feasible region, the minimum value of Z occurs at

- (a) A unique point (b) No point (c) In finitely many points (d) Two points only

Ans. Option (d) is correct

Solution: Let the equation of constraints be

$$\begin{aligned} 3x + 5y &= 15 \\ 2x + 3y &= 18 \\ x = 0, y &= 0 \end{aligned}$$



Comer points of feasible region	$Z = 30x + 50y$
(5,0)	150
(9,0)	270
(0,3)	150
(0,6)	300

Minimum value of Z occurs at two points.

Q. 6. For an objective function $Z = ax + by$, where $a, b > 0$; the corner points of the feasible region determined by a set of constraints (linear inequalities) are $(0,20)$, $(10,10)$, $(30,30)$ and $(0,40)$. The condition on a and b such that the maximum Z occurs at both end points $(30,30)$ and $(0,40)$ is:

- (a) $b - 3a = 0$
 (b) $a = 3b$

(c) $a + 2b = 0$

(d) $2a - b = 0$

Ans. Option (a) is correct

Solution: As Z is maximum at (30,30) and (0,40)

$$\Rightarrow 30(a + b) = 40b$$

$$\Rightarrow b - 3a = 0$$

Q 7. A mining company owns two different mines that produce ore which is graded in two classes: high and medium grade. The two mines have different operating characteristics as detailed below:

Mine	Cost per day (in lakhs)	Production (tons/day)	
		High grade	Medium grade
X	105	5	3
Y	90	1	2

For a particular week, the company contracted to provide 10 tons of high-grade and 8 tons of medium grade ore to a factory. The company wants to work out how many days per week it should operate each mine to fulfill the contract keeping the mining cost as low as possible.

Taking x to be the number of days per week that mine X is operated, and y to be the number of days per week mine Y is operated, what will be the objective function and constraints for this LPP (Assume $0 \leq x, y \leq 7$)

(a) Objective function: $105x + 90y$; Constraints $5x + y \geq 10$ & $3x + 2y \geq 8$

(b) Objective function $105x + 90y$; Constraints. $5x + 3y \geq 10$ & $x + 2y \geq 8$

(c) Objective function $5x + 3y$; Constraints: $5x + y \geq 105$ & $x + 2y \geq 90$

(d) Objective functions $x + 2y$ Constrains: $5x + 3y \leq 105$ & $x + 2y \leq 90$

Ans. Option (a) is correct.

Solution : The objective function is to minimize the total cost, with minimum number of high-grade ores as 10 and minimum number of medium grade ore as 8. As per the contract, the company must provide 10 tons of high-grade and 8 tons of medium grade. So, it must produce at least 10 tons of high grade and 8 tons of medium grade.

Option (b) the constraints are incorrect with numbers from the table processed incorrectly

Option (c) the objective function is incorrect and constrains are also incorrect. Numbers taken from the table only.

Option (d) the objective function is incorrect, and constraints are also incorrect. Numbers taken from the table only.

Q. 8 The comer points of the bounded feasible region determined by a system of linear constraints are (0, 3) (1, 1) and (3, 0). Let $Z = px + qy$, where $p, q > 0$

The condition on p and q so that the minimum of Z occurs at $(3, 0)$ and $(1, 1)$ is

- (a) $p = 2q$ (b) $p = \frac{q}{2}$ (c) $p = 3q$ (d) $p = q$

Ans. Option (b) is correct.

Solution: $Z = px + qy$, At $(3,0), Z = 3p$

and at $(1,1), Z = p + q$

From (ii) & (iii), $3p = p + q \Rightarrow 2p = q$.

Q. 9. The Solution set of the inequality $3x + 5y < 4$ is

- (a) an open half-plane not containing the origin
 (b) an open half-plane containing the origin.
 (c) the whole xy -plane not containing the line $3x + 5y = 4$
 (d) A closed half plane containing the origin

Ans. Option (b) is correct.

Solution: The strict inequality represents an open half plane and it contains the origin as $(0, 0)$ satisfies it.

Q. 10. The corner points of the feasible region determined by the system of linear constraints are $(0,0), (0,40), (20,40), (60,20), (60,0)$. The objective function is $Z = 4x + 3y$.

Compare the quantity in Column A and Column B

Column A	Column B
Maximum of Z	325

- (a) the quantity in column A is greater. (b) the quantity in column B is greater.
 (c) the two quantities are equal.
 (d) The relationship cannot be determined on the basis of the information supplied.

Ans. Option (b) is correct.

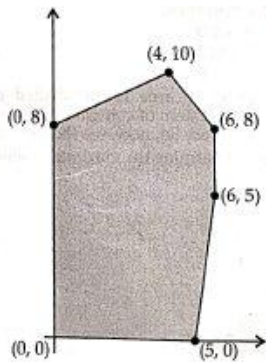
Solution:

Corner points	Corresponding value of $Z = 4x + 3y$
$(0,0)$	0
$(0,40)$	120
$(20,40)$	200
$(60,20)$	300 → Maximum
$(60,0)$	240

Hence, maximum value of $Z = 300 < 325$

So, the quantity in column B is greater.

Q. 11. The feasible Solution for a LPP is shown in given figure. Let $Z = 3x - 4y$ be the objective function. Maximum of Z occurs at



- (a) (0,0) (b) (0,8) (c) (5,0) (d) (4,10)

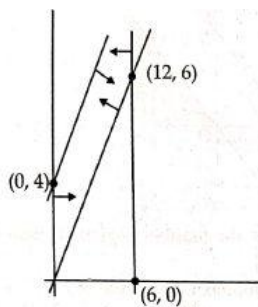
Ans. Option (c) is correct.

Solution:

Corner points	Corresponding value of $Z = 3x - 4y$
(0,0)	0
(5,0)	15 → Maximum
(6,5)	-2
(6,8)	-14
(4,10)	-28
(0,8)	-32 → Minimum

Hence, the maximum of Z occurs at (5,0) and its maximum value is 15.

Q. 12. The feasible region for an LPP is shown in the given Figure.



Let $F = 3x - 4y$ be the objective function. Maximum value of F is
 (a) 0 (b) 8 (c) 12 (d) -18

Ans. Option (c) is correct

Solution: The feasible region as shown in the figure, has objective function $F = 3x - 4y$.

Corner points	Corresponding value of $F = 3x - 4y$
(0,0)	0
(12,6)	12 → Maximum
(0,4)	-16 → Minimum

Hence, the maximum value of F is 12 .

Q. 13. Corner points of the feasible region for an LPP are (0,2), (3,0), (6,0), (6,8) and (0,5). Let $F = 4x + 6y$ be the objective function.

The minimum value of F occurs at

- (a) (0,2) only
- (b) (3,0) only
- (c) the mid-point of the line segment joining the points (0,2) and (3,0)
- (d) any point on the line segment joining the points (0,2) and (3,0)

Ans. Option (d) is correct

Solution:

Corner points	Corresponding value of $F = 4x + 6y$
(0,2)	12 → Minimum
(3,0)	12 → Minimum
(6,0)	24

Q. 14. The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4,6) and minimum value 19 at (3,2). Which of the following is true ?

- (a) $a = 9, b = 1$
- (b) $a = 5, b = 2$
- (c) $a = 3, b = 5$
- (d) $a = 5, b = 3$

Ans. Option (c) is correct

Solution: $Z = ax + by$

$$\begin{aligned} \therefore 4a + 6b &= 42 \\ 3a + 2b &= 19 \end{aligned}$$

from (i) & (ii) $a = 3, b = 5$

Q. 15. The corner points of the feasible region of a linear programming problem are (0,4), (8,0) and $(\frac{20}{3}, \frac{4}{3})$. If $Z = 30x + 24y$ is the objective function, then, maximum value of Z - minimum value of Z is

equal to:

(a) 40

(b) 96

(c) 144

(d) 136

Ans. Option (c) is correct

Solution: $Z = 30x + 24y$

At $(8,0)$ $Z = 30 \times 8 + 24 \times 0 = 240$ Maximum

At $(0,4)$ $Z = 30 \times 0 + 24 \times 4 = 96$ Minimum

At $(\frac{20}{3}, \frac{4}{3})$ $Z = 30 \times \frac{20}{3} + 24 \times \frac{4}{3} = 232$

= Maximum - Minimum

= $240 - 96 = 144$

Q. 16. A Linear Programming Problem is as follows:

Minimise

subject to the constraints

$$z = 2x + y$$

$$x \geq 3, x \leq 9, y \geq 0$$

$$x - y \geq 0, x + y \leq 14$$

The feasible region has

(a) 5 corner points including $(0,0)$ and $(9,5)$

(b) 5 corner points including $(7,7)$ and $(3,3)$

(c) 5 corner points including $(14,0)$ and $(9,0)$

(d) 5 corner points including $(3,6)$ and $(9,5)$

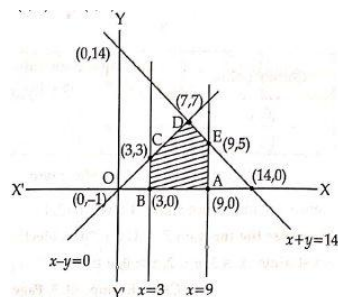
Ans. Option (b) is correct.

Solution: On plotting the constraints $x = 3$, $x = 9$, $x = y$ and $x + y = 14$, we get the following graph.

From the graph given below it clear that feasible region is ABCDEA, including corner points

$A(9,0)$, $B(3,0)$, $C(3,3)$, $D(7,7)$ and $E(9,5)$.

Thus feasible region has 5 corner points including $(7,7)$ and $(3,3)$.



Q.17. The corner points of the feasible region for a Linear Programming problem are

$P(0,5)$, $Q(1,5)$, $R(4,2)$ and $S(12,0)$. The minimum value of the objective function $Z = 2x + 5y$ is at the point

(a) P

(b) Q

(c) R

(d) S

Ans. Option (c) is correct.

Solution:

Corner Points	Value of $Z = 2x + 5y$
$P(0,5)$	$Z = 2(0) + 5(5) = 25$
$Q(1,5)$	$Z = 2(1) + 5(5) = 27$
$R(4,2)$	$Z = 2(4) + 5(2) = 18$ Minimum
$S(12,0)$	$Z = 2(12) + 5(0) = 24$

Thus, minimum value of Z occurs at $R(4,2)$.

Q.18. A Linear Programming Problem is as follows:

Maximise / Minimise objective function

$$Z = 2x - y + 5$$

Subject to the constraints

$$\begin{aligned} 3x + 4y &\leq 60 \\ x + 3y &\leq 30 \\ x &\geq 0, y \geq 0 \end{aligned}$$

If the corner points of the feasible region are $A(0,10)$, $B(12,6)$, $C(20,0)$ and $O(0,0)$, then which of the following is true ?

- (a) maximum value of Z is 40 (b) minimum value of z is -5
- (c) difference of maximum and minimum values of z is 35
- (d) At two corner points, value of Z are equal

Sol. Option (b) is correct.

Solution:

Corner Points	Value of $Z = 2x - y + 5$
$A(0, 10)$	$Z = 2(0) - 10 + 5 = -5$ (Minimum)
$B(12, 6)$	$Z = 2(12) - 6 + 5 = 23$
$C(20, 0)$	$Z = 2(20) - 0 + 5 = 45$ (Maximum)
$O(0,0)$	$Z = 0(0) - 0 + 5 = 5$

So, the minimum value of Z is -5.

Q.19. The corner points of the feasible region determined by a set of constraints (linear inequalities) are $P(0,5)$, $Q(3,5)$, $R(5,0)$ and $S(4,1)$ and the objective function is $Z = ax + 2by$ where $a, b > 0$. The condition on a and b such that the maximum Z occurs at Q and S is

- (a) $a - 5b = 0$ (b) $a - 3b = 0$ (c) $a - 2b = 0$ (d) $a - 8b = 0$

Sol. Option (d) is correct.

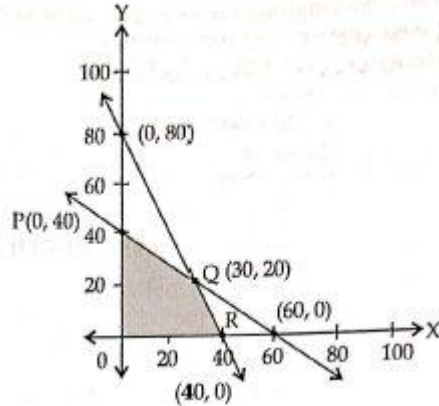
Solution: Given, Max. $Z = ax + 2by$

Max. value of Z on $Q(3,5) =$ Max. value of Z on $S(4,1)$

$$\Rightarrow 3a + 10b = 4a + 2b$$

$$\Rightarrow a - 8b = 0$$

Q.20. For an L.P.P the objective function is $Z = 4x + 3y$, and the feasible region determined by a set of constraints (linear inequations) is shown in the graph.



Which one of the following statements is true?

- (a) Maximum value of Z is at R .
- (b) Maximum value of Z is at Q .
- (c) Value of Z at R is less than the value at P .
- (d) Value of Z at Q is less than the value at R .

Ans. Option (b) is correct.

Solution:

Corner Points of Feasible Region	Value of $z = 4x + 3y$
$O(0,0)$	$Z = 4(0) + 3(0) = 0$
$P(0,40)$	$Z = 4(0) + 3(40) = 120$
$Q(30, 20)$	$Z = 4(30) + 3(20) = 180$ (Maximum)
$R(40,0)$	$Z = 4(40) + 3(0) = 160$

Thus, Maximum value of Z is at Q , which is 180 .

Assertion/Reason Based Questions (Solved)

Direction: In the following question, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices

- (a) Both A and R are true, and R is the correct explanation of A.
- (b) Both A and R are true, but R is not the correct explanation of A
- (c) A is true but R is false.
- (d) A is false and R is true.

Q. 1. Assertion (A): If the corner points of the feasible region for a linear programming problem are $P(0,4)$, $Q(1,4)$, $R(4,1)$ and $S(12, -1)$, then minimum value of objective function $Z = 2x + 4y$ is at the point $R(4,1)$

Reason (R): If the corner points of the feasible region for a linear programming problem $P(0,4)$, $Q(1,4)$, $R(4,1)$ and $S(12, -1)$, then maximum value of objective function $Z = 2x + 4y$ is 20 .

Ans. Option (b) is correct.

Solution:

Corner	Value of $Z = 2x + 4y$
$P(0,4)$	$Z = 2(0) + 4(4) = 16$
$Q(1,4)$	$Z = 2(1) + 4(4) = 18$
$R(4,1)$	$Z = 2(4) + 4(1) = 12$ → Minimum
$S(12, -1)$	$Z = 2(12) + 4(-1) = 20$ → Maximum

Minimum value is 12 at the point $R(4,1)$,

Maximum value is 20 at the point $R(12, -1)$.

Q. 2. The corner points of the feasible region determined by a set constraints (linear inequalities) are $P(1,6)$, $Q(4,5)$, $R(6,1)$ and $S(5,2)$ and the objective function is $Z = ax + 3by$ where $a, b > 0$

Assertion (A): The relation between a and b ,such that the maximum Z occur at P and Q is $a = b$.

Reason (R): The relation between a and b such that the maximum Z occur at P and Q is $a = 3b$

Ans. Option (c) is correct.

Solution: Given Max. $Z = ax + 3by$

Max value of Z on $P(1,6) =$ Max. value of Z on $Q(4,5)$

or, $a + 18b = 4a + 15b$

$$3a = 3b$$

or, $a = b$

Q.3.The corner points of the bounded feasible region for an L.P.P. are $(60,0)$, $(120,0)$, $(60,30)$ and $(40,20)$. The objective function $Z = ax + by$, $a, b > 0$ has maximum value 600 at points $(120,0)$ and $(60,30)$.

Assertion (A): Minimum value of $Z = 300$

Reason (R): $a = 5, b = 10$.

Ans. Option (a) is correct.

Solution: $Z = ax + by$ has maximum value at points $(120,0)$ and $(60,30)$.

So, $120a + 0 = 600 \Rightarrow a = 5$.

Also, $60a + 30b = 600 \Rightarrow 60 \times 5 + 30b = 600$

$$\Rightarrow 30b = 300 \Rightarrow b = 10$$

$$\therefore a = 5, b = 10$$

∴ Reason is true.

Now, $Z = 5x + 10y$

Value of Z at $(60,0) = 5 \times 60 + 0 = 300 \rightarrow$ Minimum

Value of Z at $(120,0) = 5 \times 120 + 0 = 600$

Value of Z at $(60,30) = 5 \times 60 + 10 \times 30 = 600$

Value of Z at $(40,20) = 5 \times 40 + 10 \times 20 = 400$

∴ Assertion is true.

Hence, both Assertion & Reason are true and Reason is the correct explanation of Assertion.

Q.4. The corner points of the feasible region for an L.P.P. are $(4,0)$, $(5,0)$, $(5,3)$, $(3,5)$, $(0,5)$ and $(0,4)$.

The objective function $Z = ax - by + 1900$, $a, b > 0$ has maximum value 1950 at $(5,0)$ and minimum value 1550 at $(0,5)$.

Assertion (A): The value of Z at point $(5,3) = 1740$

Reason (R): Maximum and minimum value occurs when $a = 10, b = 70$.

Ans. Option (a) is correct.

Solution: $Z = ax - by + 1900$ has maximum value 1950 at $(5,0)$.

So, $5a + 1900 = 1950 \Rightarrow 5a = 50 \Rightarrow a = 10$

and Z has minimum value 1550 at $(0,5)$.

So, $0 - 5b + 1900 = 1550 \Rightarrow -5b = -350 \Rightarrow b = 70$

$$\therefore a = 10, b = 70$$

Also, value of Z at $(5,3) = 10 \times 5 - 70 \times 3 + 1900 = 1740$.

Hence, both Assertion & Reason are true and Reason is the correct explanation of Assertion

Q.5. Assertion (A): If the feasible region for an L.P.P. is bounded, then the objective function $Z = ax + by$ has both maximum and minimum values.

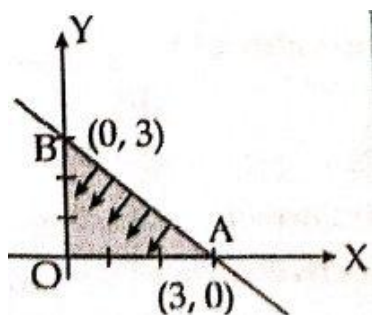
Reason (R): A feasible region of a system of linear inequalities is said to be bounded if it can be enclosed within a circle.

Ans. Option (b) is correct.

Solution: Assertion is true by theorem.

Reason is also true but reason is not the correct explanation of Assertion.

Q.6. Corner points of feasible region are $(0,0)$, $(3,0)$ and $(0,3)$ and objective function $Z = 4x + 7y$.



Assertion (A): Minimum value of Z is 12.

Reason (R): Maximum value of Z is 21.

Ans. Option (d) is correct.

Solution: Thus, corner points are: $O(0,0)$, $A(3,0)$ and $B(0,3)$. Max. value of Z at corner points

$$Z_0 = 4(0) + 7(0) = 0 \rightarrow \text{Min.}$$

$$Z_A = 4(3) + 7(0) = 12$$

$$Z_B = 4(0) + 7(3) = 21 \rightarrow \text{Max.}$$

Q.7. Assertion (A): The point $(4,2)$ does not lie in the half plane of $4x + 6y - 24 < 0$.

Reason (R): The point $(1,2)$ lies in the half plane of $4x + 6y - 24 < 0$.

Ans. Option (b) is correct.

Solution: Put $(4,2)$ in $4x + 6y - 24$, we get

$$\begin{aligned} 4(4) + 6(2) - 24 &= 16 + 12 - 24 \\ &= 28 - 24 \\ &= 4 > 0 \end{aligned}$$

Thus, point $(4,2)$ does not lie in the half plane.

Put $(1,2)$ in $4x + 6y - 24$, we get

$$\begin{aligned} 4(1) + 6(2) - 24 &= 4 + 12 - 24 \\ &= 16 - 24 \\ &= -8 < 0 \end{aligned}$$

Thus, point $(1,2)$ lies in the half plane.

Q. 8. Assertion (A): Feasible region is the set of points which satisfy all of the given constraints and objective function too.

Reason (R): The optimal value of the objective function is attained at the points on X -axis only.

Ans. Option (c) is correct.

Solution: The optimal value of the objective function is attained at the corner points of feasible region.

Q. 9. Assertion (A): *Minimise* $Z = 20x_1 + 20x_2$ subject to

$$x_1 \geq 0, x_2 \geq 2, x_1 + 2x_2 \geq 8, 3x_1 + 2x_2 \geq 15, 5x_1 + 2x_2 \geq 20.$$

Out of the corner points of bounded feasible region $(8,0)$, $(\frac{5}{2}, \frac{15}{4})$, $(\frac{7}{2}, \frac{9}{4})$ and $(0,10)$, the minimum value of Z occurs at $(\frac{7}{2}, \frac{9}{4})$.

Reason (R) :

Corner Points	$Z = 20x_1 + 20x_2$
$(8,0)$	160
$(\frac{5}{2}, \frac{15}{4})$	125

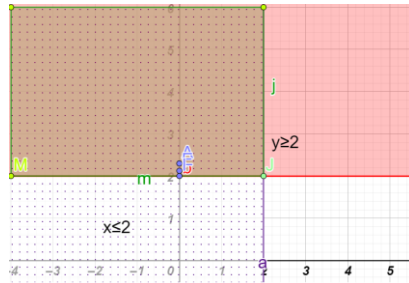
$\left(\frac{7}{2}, \frac{9}{4}\right)$	115 minimum
(0,10)	200

Ans. Option (a) is correct.

Solution: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Q. 10. Assertion (A) : The graph of $x \leq 2$ and $y \geq 2$ is in the first and second quadrants.

Reason (R):



Ans. Option (A) is correct.

Solution: It is clear from the graph given in the Reason (R) that Assertion (A) is true.

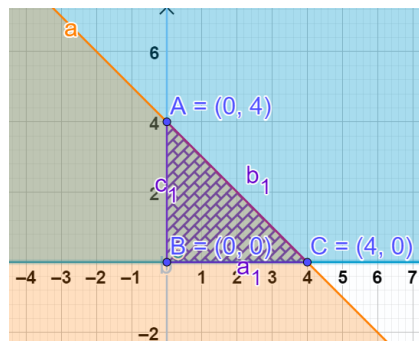
VERY SHORT ANSWER TYPE QUESTIONS (SOLVED)

Q. 1. Solve the following Linear Programming Problem graphically:

Maximize $Z = 3x + 4y$

Subject to $x + y \leq 4, x \geq 0$ and $y \geq 0$.

Sol. The feasible region is a triangle with vertices B(0,0), C(4,0) and A(0,4)



$$Z_0 = 3 \times 0 + 4 \times 0 = 0$$

$$Z_A = 3 \times 4 + 4 \times 0 = 12$$

$$Z_B = 3 \times 0 + 4 \times 4 = 16$$

Thus, maximum of Z is at B(0,4) and the maximum value is 16.

Q. 2. Two tailors, A and B, earn ₹ 300 and ₹ 100 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour

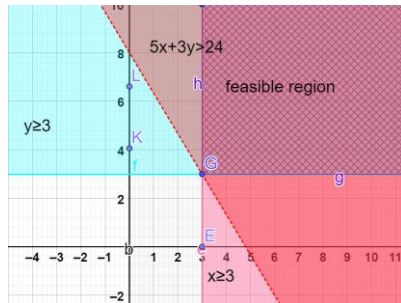
cost. Formulate this as an LPP.

Sol. Let A works for x days and B for y days.

$$\therefore \text{Subject to } \begin{cases} 6x + 10y \geq 60 \\ 4x + 4y \geq 32 \\ x \geq 0, y \geq 0 \end{cases}$$

Q. 3. Find the maximum value of the function $z = 5x + 3y$ subjected to the constraints $x \geq 3$ and $y \geq 3$.

Sol. Here, value of Z at the corner point $(3,3)$ of the feasible region is 24 .



Since, region is unbounded, so we draw the graph of inequality $5x + 3y > 24$.

Since, open half plane of $5x + 3y > 24$ has common points with feasible region.

So, maximum value does not exist.

Q. 4. The feasible region for an LPP is shown in the following figure. Then, find the minimum value of $Z = 11x + 7y$.



Sol. Here, the objective function is $Z = 11x + 7y$, and the corner points of the feasible region are $O(3,2)$, $P(0,3)$ and $N(0,5)$.

$$\therefore \text{At } O(3,2), \quad Z = 11 \times 3 + 7 \times 2 = 33 + 14 = 47 \quad \text{and} \quad \text{At } P(0,3), \quad Z = 11 \times 0 + 7 \times 3 = 21$$

$$\text{At } N(0, 5), \quad Z = 11 \times 0 + 7 \times 5 = 35$$

Hence, the minimum value of Z is 21 which occurs at point $P(0,3)$.

Q. 5. A small firm manufacture necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24 . It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹100 and that on a bracelet is ₹ 300. Formulate an L.P.P. for finding how many of each should be produced daily to

maximize the profit ? It is being given that at least one of each must be produced.

Sol. Let x necklaces and y bracelets are manufactured

\therefore L.P.P is Maximize profit,

$$P = 100x + 300y$$

Subject to constraints

$$x + y \leq 24$$

$$\frac{1}{2}x + y \leq 16 \text{ or } x + 2y \leq 32$$

$$x, y \geq 1$$

Q. 6. A firm has to transport at least 1200 packages daily using large vans which carry 200 packages each and small vans which can take 80 packages each. The cost for engaging each large van is ₹400 and each small van is ₹200. Not more than ₹3,000 is to be spent daily on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimize cost.

Sol. Let number of large vans = x and number of small vans = y

Minimize cost $Z = 400x + 200y$

Subject to constraints

$$\left. \begin{aligned} 200x + 80y &\geq 1200 \text{ or } 5x + 2y \geq 30 \\ x &\leq y \\ 400x + 200y &\leq 3000 \text{ or } 2x + y \leq 15 \\ x &\geq 0, y \geq 0 \end{aligned} \right\}$$

Q. 7. Determine the maximum value of $Z = 11x + 7y$ subject to the constraints: $2x + y \leq 6$, $x \leq 2$, $x \geq 0$, $y \geq 0$.

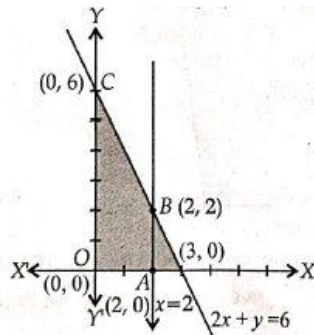
Sol. We have,

Maximise $Z = 11x + 7y$

Subject to the constraints,

$$\begin{aligned} 2x + y &\leq 6 \\ x &\leq 2 \\ x &\geq 0, y \geq 0 \end{aligned}$$

We see that, required area is the shaded region determined by the system of constraints (ii) to (iv) is $OABC$ and is bounded. So, now, we shall use corner point method to determine the maximum value of Z .



Corner points	Corresponding value of $Z = 11x + 7y$
(0,0)	0
(2,0)	22
(2,2)	36
(0,6)	42 → Maximum

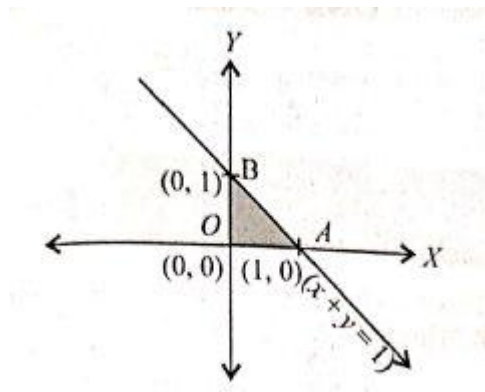
Hence, the maximum value of Z is 42 at (0,6).

Q. 8. Maximize $Z = 3x + 4y$, subject to the constraints: $x + y \leq 1, x \geq 0, y \geq 0$.

Solution. Maximise $Z = 3x + 4y$,
Subject to the constraints,

$$x + y \leq 1, x \geq 0, y \geq 0$$

The shaded region shown in the figure as OAB is bounded and the coordinates of corner points O, A and B are (0,0), (1,0) and (0,1), respectively.

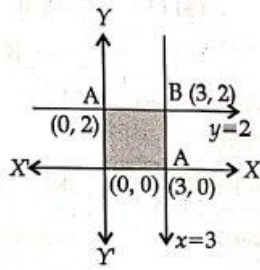


Corner points	Corresponding value of $Z = 3x + 4y$
(0,0)	0
(1,0)	3
(0,1)	4 → Maximum

Hence, the maximum value of Z is 4 at (0,1).

Q.9. Maximize the function $Z = 11x + 7y$, subject to the constraints: $x \leq 3, y \leq 2, x \geq 0, y \geq 0$.

Solution. Maximize $Z = 11x + 7y$,
Subject to the constraints, $x \leq 3, y \leq 2, x \geq 0, y \geq 0$



The shaded region as shown in the figure as $OABC$ is bounded and the coordinates of corner points are $(0, 0)$, $(3,0)$, $(3,2)$ and $(0,2)$, respectively.

Corner points	Corresponding value of $Z = 11x + 7y$
$(0,0)$	0
$(3,0)$	33
$(3,2)$	47 \rightarrow Maximum
$(0,2)$	14

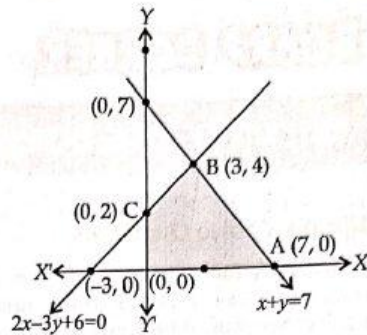
Hence, Z is maximum at $(3,2)$ and its maximum value is 47.

Q.10. Minimise $Z = 13x - 15y$ subject to the constraints: $x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$.

Sol Minimise $Z = 13x - 15y$

Subject to the constraints,

$$x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$$



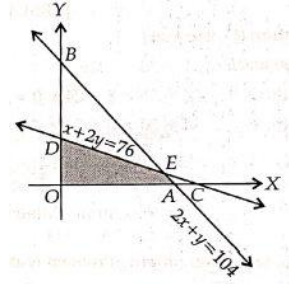
The shaded region shown as $OABC$ is bounded and coordinates of its corner points are $(0,0)$, $(7,0)$, $(3,4)$ and $(0,2)$, respectively.

Corner points	Corresponding value of $Z = 13x - 15y$
$(0,0)$	0
$(7,0)$	91

(3,4)	-21
(0,2)	-30 → Minimum

Hence, the minimum value of Z is -30 at $(0,2)$.

Q. 11. Determine the maximum value of $Z = 3x + 4y$ if the feasible region (shaded) for a LPP is shown in given figure.



Sol. Here, $2x + y = 104$ and $2x + 4y = 152$ intersect at $\Rightarrow E(44,16)$.

As clear from the graph, corner points are O, A, E and D with coordinates $(0,0), (52,0), (144,16)$ and $(0, 38)$, respectively. Also, given region is bounded.

Corner points	Corresponding value of $Z = 3x + 4y$
(0,0)	0
(52,0)	156
(44,16)	196 → Maximum
(0,38)	152

Hence, Z is at $(44,16)$ is maximum and its maximum value is 196.

Q. 12. For the LPP,

$$\text{Maximise } Z = 2x + 3y$$

The coordinates of the corner points of bounded feasible region are $A(3,3), B(20,3), C(20,10), D(18,12)$ and $E(12,12)$. Find the maximum value of Z .

Sol.

Comer Points	$Z = 2x + 3y$
$A(3,3)$	$Z = 15$
$B(20,3)$	$Z = 49$

$C(20,10)$	$Z = 70$
$D(18,12)$	$Z = 72 \rightarrow \text{Max.}$
$E(12,12)$	$Z = 60$

Thus, maximum value of Z is 72 at $D(18,12)$.

Q. 13. The sum of two positive integers is at most 5. The difference between two times of second number and first number is at most 4. If first number is x and second number is y , then for maximizing the product of these two numbers, formulate LPP.

Sol. The required LPP is Max.

Subject to:

$$\begin{aligned} Z &= x \times y \\ x + y &\leq 5 \\ 2y - x &\leq 4 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Q.14. If a 20 year old girl drives her car at 25 km/h, she has to spend ₹ 4/km on petrol. If she drives her car at 40 km/h, the petrol cost increases to ₹5/km. She has ₹ 200 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express the above problem as a Linear Programming Problem.

Sol. Let the distance covered with speed of 25 km/h = x km and the distance covered with speed of 40 km/h = y km

Total distance covered = z km

The L.P.P. of the above problem, therefore, is

$$\text{Maximize } z = x + y$$

Subject to constraints

$$\begin{aligned} 4x + 5y &\leq 200 \\ \frac{x}{50} + \frac{y}{40} &\leq 1 \\ \frac{x}{25} + \frac{y}{40} &\leq 1 \\ x &\geq 0, y \geq 0 \end{aligned}$$

Q.15. A shopkeeper deals in two items-wall hangings and artificial plants. He has ₹15000 to invest and a space to store atmost 80 pieces. A wall hanging costs him ₹300 and an artificial plant ₹150. He can sell a wall hanging at a profit of ₹50 and an artificial plant at a profit of ₹18. Assuming that he can sell all the items that he buys, formulate a linear programming problem in order to maximize his profit.

Solution. Let x be the number of wall hangings and y be the number of artificial plants that the dealer

buys and sells. Then the profit of the dealer is $Z = 50x + 18y$, which is the objective function. As a wall hanging costs ₹300 and an artificial plant cost ₹150, the cost of x wall hangings and y artificial plants is $300x + 150y$. We are given that the dealer can invest atmost ₹15000. Hence, the investment constraint is

$$300x + 150y \leq 15000 \text{ i.e. } 2x + y \leq 100$$

As the dealer has space to store atmost 80 pieces, we have another constraint (space constraint):

$$x + y \leq 80$$

Also the number of wall hangings and artificial plants can't be negative. Thus we have the non-negativity constraints:

$$x \geq 0, y \geq 0$$

Thus the mathematical formulation of the L.P.P. is :

Maximize $Z = 50x + 18y$ subject to the constraints

$$2x + y \leq 100, x + y \leq 80, x \geq 0, y \geq 0$$

Q.16. A retired person wants to invest an amount of upto ₹20000. His broker recommends investing in two types of bonds A and B , bond A yielding 8% return on the amount invested and bond B yielding 7% return on the amount invested. After some consideration, he decides to invest atleast ₹5000 in bond A and no more than ₹8000 in bond B . He also wants to invest atleast as much in bond A as in bond B . Formulate as L.P.P. to maximize his return on investments.

Solution. Let the person invest ₹ x in bonds of type A and ₹ y in bonds of type B , then his earning i.e. return (in ₹)

$$Z = 8\% \text{ of } x + 7\% \text{ of } y = 0.08x + 0.07y$$

As the person can invest upto ₹ 20000, so investment constraint is

$$x + y \leq 20000.$$

Other investment constraints are

$$x \geq 5000, y \leq 8000, x \geq y$$

Non-negativity constraints are $x \geq 0, y \geq 0$.

Thus, mathematically formulation of the L.P.P. is

Maximize $Z = 0.08x + 0.07y$ subject to the constraints

$$x + y \leq 20000, x \geq 5000, y \leq 8000, x \geq y, x \geq 0, y \geq 0$$

Q.17. (Manufacturing problem) A manufacturer produces nuts and bolts for industrial machinery. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts, while it takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹2.50

per package on nuts and ₹1 per package on bolts. Form a linear programming problem to maximize his profit, if he operates each machine for atmost 12 hours

Solution. Suppose that x packages of nuts and y packages of bolts are produced. The objective of the manufacturer is to maximize the profit $Z = 2.50x + 1. y$

Time required on machine A to produce x packages of nuts and y packages of bolts is $1. x + 3. y = x + 3y$, while time required on machine B is $3. x + 1. y = 3x + y$.

From given data, we can formulate the L.P.P. as:

Maximize $Z = 2.5x + y$ subject to the constraints

$$\begin{aligned} x + 3y &\leq 12 && \text{(Machine A constraint)} \\ 3x + y &\leq 12 && \text{(Machine B constraint)} \\ x \geq 0, y &\geq 0 && \text{(Non-negativity constraints)} \end{aligned}$$

Q.18. A manufacturer has three machines M_1, M_2 and M_3 installed in his factory. Machines M_1 and M_2 are capable of being operated for atmost 12 hours, whereas machine M_3 must be operated for atleast 5 hours a day. The manufacturer produces only two items, each requiring the use of these three machines. The following table gives the number of hours required on these machines for producing 1 unit of A or B.

Item	Number of hours required on the machines			
	M_1	M_2	M_3	
A	1	2	1	
B	2	1	$\frac{5}{4}$	

He makes a profit of ₹60 on item A and ₹40 on item B. He wishes to find out how many of each item he should produce to have maximum profit. Formulate it as L.P.P.

Solution. Let the manufacturer produce x units of item A and y units of item B. As he makes a profit of ₹60 on one item of A and ₹ 40 on one item of B, his total profit (in ₹) is

$$Z = 60x + 40y$$

From the given data, we can mathematically formulate the L.P.P. as

Maximize $Z = 60x + 40y$ subject to the constraints

$$\begin{aligned} x + 2y &\leq 12 && \text{(Machine } M_1 \text{ constraint)} \\ 2x + y &\leq 12 && \text{(Machine } M_2 \text{ constraint)} \\ x + \frac{5}{4}y &\geq 5 && \text{(Machine } M_3 \text{ constraint)} \\ x \geq 0, y &\geq 0 && \text{(Non-negativity constraints)} \end{aligned}$$

Q.19. (Diet problem) A diet is to contain atleast 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs ₹4 per kg and F_2 costs ₹5 per kg. One kg of food F_1 contains

3 units of vitamin A and 4 units of minerals. One kg of food F_2 contains 6 units of vitamin A and 3 units of minerals. We wish to find the minimum cost for diet that consists of mixture of these two foods and also meets the minimum nutritional requirements. Formulate this as a linear programming problem.

Solution. Let the mixture consist of x kg of food F_1 and y kg of food F_2 .

We make the following nutritional requirement table from given data :

Resources	Food (in kg)		Requirement (in units)
	F_1	F_2	
Vitamin A (units/kg)	3	6	80
Minerals (units/kg)	4	3	100

Minimum requirement of vitamin A is 80 units, therefore,

$$3x + 6y \geq 80$$

Similarly, the minimum requirement of minerals is 100 units, therefore,

$$4x + 3y \geq 100$$

Also

$$x \geq 0, y \geq 0$$

As cost of food F_1 is ₹4 per kg, and the cost of food F_2 is ₹5 per kg, the total cost of purchasing x kg of food F_1 and y kg of food F_2 is $Z = 4x + 5y$, which is the objective function.

Hence, the mathematical formulation of the L.P.P. is :

Minimize $Z = 4x + 5y$ subject to the constraints

$$3x + 6y \geq 80, 4x + 3y \geq 100, x \geq 0, y \geq 0.$$

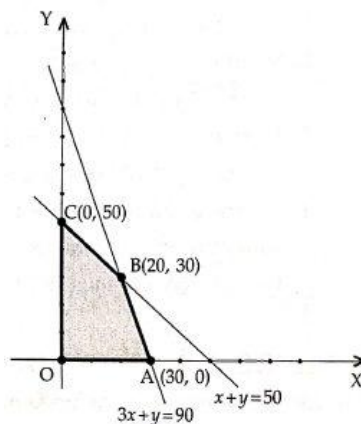
SHORT ANSWER TYPE QUESTIONS (SOLVED)

Q.1. Solve the following linear programming problem graphically:

Maximize and minimize $Z = 60x + 15y$ subject to the constraints

$$x + y \leq 50, 3x + y \leq 90, x, y \geq 0$$

Solution. We draw the lines $x + y = 50, 3x + y = 90$ and shade the region satisfied by the given inequalities. The shaded region in the adjoining figure shows the feasible region determined by the given constraints. We observe that the feasible region OABC is convex polygon and bounded. So we use the corner point method to calculate the maximum and minimum value of Z .



The coordinates of the corner points O, A, B, C are $(0,0), (30,0), (20,30)$ and $(0,50)$ respectively. We evaluate $Z = 60x + 15y$ at each of these points.

Corner point	Value of objective function $Z = 60x + 15y$
(0,0)	$0 \rightarrow$ Smallest
(30,0)	$1800 \rightarrow$ Largest
(20,30)	1650
(0,50)	750

Hence, the minimum value of Z is 0 at the point (0,0) and maximum value of Z is 1800 at the point (30,0).

Q.2.. Find graphically, the maximum value of $Z = 2x + 5y$, subject to constraints given below:

$$2x + 4y \leq 8, 3x + y \leq 6, x + y \leq 4, x \geq 0, y \geq 0$$

Solution. We are required to maximise $Z = 2x + 5y$, subject to the constraints:

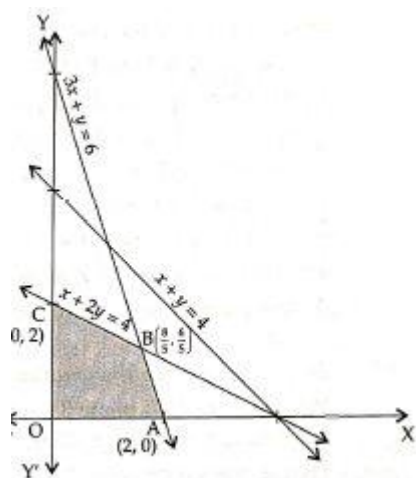
$$\begin{aligned} 2x + 4y &\leq 8 \text{ ie. } x + 2y \leq 4 \\ 3x + y &\leq 6, x + y \leq 4, x \geq 0, y \geq 0 \end{aligned}$$

Draw the lines $x + 2y = 4$ (passes through (4,0), (0,2));

$$3x + y = 6 \text{ (passes through (2,0), (0,6))}$$

$$x + y = 4 \text{ (passes through (4,0), (0,4))}$$

Shade the region satisfied by the given inequalities.



The shaded region in the adjoining figure gives the feasible region determined by the given inequalities. Solving $3x + y = 6$ and $x + 2y = 4$ simultaneously, we get

$$x = \frac{8}{5} \text{ and } y = \frac{6}{5}$$

We observe that the feasible region OABC is a convex polygon and bounded and has corner points

$$O(0,0), A(2,0), B\left(\frac{8}{5}, \frac{6}{5}\right) \text{ and } C(0,2)$$

The optimal Solution occurs at one of the corner points

At $O(0,0), Z = 2.0 + 5.0 = 0;$

at $A(2,0), Z = 2.2 + 5.0 = 4;$

at $B\left(\frac{8}{5}, \frac{6}{5}\right), Z = 2, \frac{8}{5} + 5, \frac{6}{5} = \frac{46}{5};$

at $C(0,2), Z = 2.0 + 5.2 = 10.$

Therefore, Z has maximum value at C and maximum value = 10.

Q.3. Solve the following linear programming problem graphically:

Minimise $Z = 3x + 5y$ subject to the constraints:

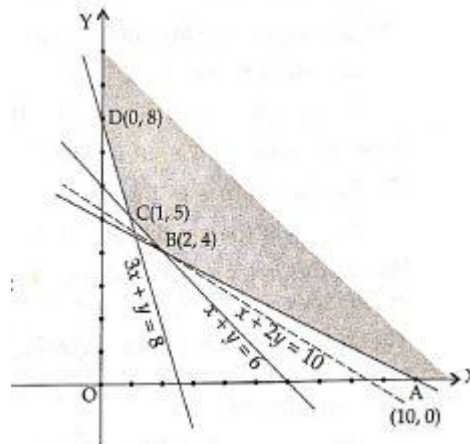
$$x + 2y \geq 10, x + y \geq 6, 3x + y \geq 8, x, y \geq 0.$$

Solution. We are required to minimise $Z = 3x + 5y$, subject to the constraints:

$$x + 2y \geq 10, x + y \geq 6, 3x + y \geq 8, x \geq 0, y \geq 0$$

Draw the lines $x + 2y = 10, x + y = 6$ and $3x + y = 8$.

Shade the region satisfied by the given inequalities.



The shaded region in the adjoining figure gives the feasible determined by the given inequalities.

Solving $x + 2y = 10$ and $x + y = 6$ simultaneously, we get

$$x = 2 \text{ and } y = 4.$$

Solving $x + y = 6$ and $3x + y = 8$

simultaneously, we get $x = 1, y = 5$.

We observe that the feasible region is unbounded and the corner points are

$$A(10,0), B(2,4), C(1,5) \text{ and } D(0,8)$$

At the corner points, the values of Z are :

$$\text{at } A(10,0), Z = 3.10 + 5.0 = 30$$

$$\text{at } B(2,4), Z = 3.2 + 5.4 = 26$$

$$\text{at } C(1,5), Z = 3.1 + 5.5 = 28$$

$$\text{at } D(0,8), Z = 3.0 + 5.8 = 40$$

The smallest value of Z is 26 at $B(2,4)$.

As the feasible region is unbounded, we cannot say whether the minimum value exists or not.

To check whether the smallest value 26 is minimum

We draw the half plane $3x + 5y < 26$ and notice that there is no common point with the feasible region.

Hence, 26 is indeed the minimum value.

Hence, the given objective function $Z = 3x + 5y$ has minimum value 26 at $B(2,4)$.

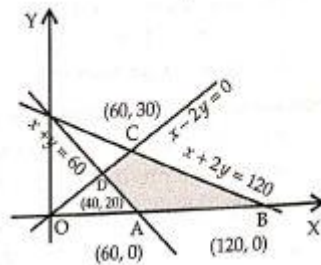
Q.4. Maximize and minimize $Z = 5x + 10y$ subject to the constraints

$$x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x \geq 0, y \geq 0$$

Solution. Maximize and minimize $Z = 5x + 10y$ subject to the constraints

$$x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x \geq 0, y \geq 0$$

Draw the lines $x + 2y = 120$, $x + y = 60$, and $x - 2y = 0$; shade the region satisfied by the given inequalities.



The feasible region is polygon $ABCD$, which is convex and bounded. Corner points of feasible region are $A(60,0)$, $B(120,0)$, $C(60,30)$ and $D(40,20)$.

The values of $Z = 5x + 10y$ at the points A , B , C and D are 300, 600, 600 and 400 respectively.

Minimum value = 300 at $A(60,0)$, maximum value = 600 at $B(120,0)$ and $C(60,30)$.

In fact, all points on the line segment BC give the same maximum value = 600.

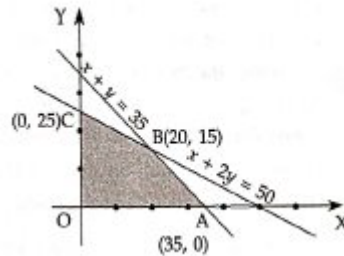
Q.5. A furniture trader deals in only two items-chairs and tables. He has ₹50000 to invest and a space to store atmost 35 items. A chair costs him ₹1000 and a table costs him ₹2000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and a table, respectively. Formulate the above problem as an L.P.P. to maximise the profit and solve it graphically.

Solution. Let x be the number of chairs and y be the number of tables that the dealer buys and sells. The profit on a chair is ₹150 and the profit on a table is ₹250. So, total profit $P = 150x + 250y$. Hence, the problem can be formulated as an L.P.P. as follows:

Maximise $P = 150x + 250y$ subject to the constraints

$$\begin{aligned}
 &x + y \leq 35 \\
 \text{i.e. } &1000x + 2000y \leq 50000 \\
 &x + 2y \leq 50 \\
 &x \geq 0, y \geq 0
 \end{aligned}$$

Draw the lines $x + y = 35$ and $x + 2y = 50$; shade the region satisfied by the given inequalities.



The shaded portion shows the feasible region which is bounded. The point of intersection of the lines $x + y = 35$ and $x + 2y = 50$ is $B(20,15)$.

The four corner points of the feasible region OABC are $O(0,0)$, $A(35,0)$, $B(20,15)$ and $C(0,25)$.

At $(0,0)$, $P = 150 \times 0 + 250 \times 0 = 0$.

At $A(35,0)$, $P = 150 \times 35 + 250 \times 0 = 5250$.

At $B(20,15)$, $P = 150 \times 20 + 250 \times 15 = 6750$.

At $C(0,25)$, $P = 150 \times 0 + 250 \times 25 = 6250$.

We find that P is maximum at $B(20,15)$ and maximum value of $P = 6750$.

Hence, the dealer gets maximum profit of ₹6750 when he buys and sells 20 chairs and 15 tables.

LONG ANSWER TYPE QUESTIONS (SOLVED)

Q. 1. The linear programming problem is as follows:

$$\text{Minimize: } 50x + 20y$$

Subject to the constraint,

$$6x + 2y \geq 1200$$

$$300 \leq x + y \leq 600$$

$$x > 0, y > 0$$

(a) Find the values of x and y for which the value of the objective function $50x + 20y$ is the least. Justify your answer.

(b) Can the objective function have its minimum or maximum value at any of points $E(0,300)$, $F(100, 300)$ or $G(100,200)$ and why?

Point	A(300, 0)	B(0, 600)	C(0, 600)	D(150, 150)	G(100, 200)
Value of objective function at the point	15000	30000	12000	10500	9000

Concept: If the feasible region of a linear programming problem where $Z = ax + by$ is an objective function and constraints are linear inequalities in x and y is bounded, then Z has both a maximum and minimum value in R and each of these occurs at a corner point of R .

Sol. Coverting the given inequations into equations, we get

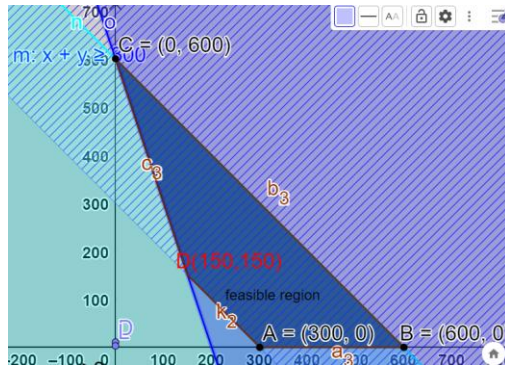
$$6x + 2y = 1200$$

$$x + y = 300$$

$$x + y = 600$$

$$x = 0 \text{ and } y = 0$$

Now on drawing these lines on graph we obtain following graph:



From graph it is clear that we have corner points of the shaded feasible region are $ABCD$ are $A(300,0)$, $B(600,0)$, $C(0,600)$ and $D(150,150)$.

These points have been obtained by solving the equations of the corresponding intersecting lines, simultaneously.

The values of the objective function at these points are given in the following table:

Point (x, y)	Value of the objective function $z = 5x + 2y$
$A(300,0)$	15,000
$B(600,0)$	30,000
$C(0,600)$	12,000
$D(150,150)$	10,500
$G(100,200)$	9,000

Clearly, Z is minimum at point $D(150,150)$.

The minimum value of Z is 10,500 .

Here the point $D(x = 150$ and $y = 150)$ is the optimal feasible Solution of the given LPP specifying the value of the objective function is the least at D among the 4 corner points of the feasible region.

Note: Here point G is out of feasible region.

In Option (B) Constraint $x + 2y > 8$ does not validate the data given in question.

In Option(C) Both Objective function and constraints are incorrect.

In Option (D) both objective function and all constraints are incorrect as these do not validate the data of given LPP.

Q.2. A manufacturing company makes two types of teaching aids A and B of Mathematics for class X . Each type of A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ₹80 on each piece of type A and ₹120 on each piece of type B . How many pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as an LPP and solve graphically.

What is the maximum profit per week?

Solution. Let x and y be the number of teaching aids of type A and type B respectively.

As the profit on the type A of teaching aid is ₹ 80 and on the type B is ₹ 120,

so, the total profit $Z = 80x + 120y$.

Hence, the problem can be formulated as an L.P.P. as follows:

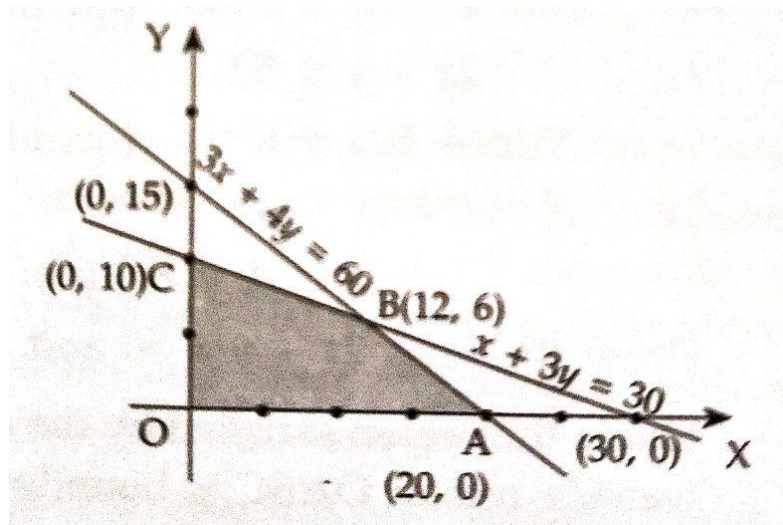
$$\text{Maximise } Z = 80x + 120y$$

Subject to the Constraints ; $9x + 12y \leq 180$ i.e. $3x + 4y \leq 60$

$$x + 3y \leq 30$$

$$x \geq 0, y \geq 0$$

We draw straight lines $3x + 4y = 60$, $x + 3y = 30$ and shade the region satisfied by the above inequalities.



The shaded region shows the feasible region which is bounded. The point of intersection of the lines $3x + 4y = 60$ and $x + 3y = 30$ is $B(12,6)$

The corner points of the feasible region $OABC$ are $O(0,0)$, $A(20,0)$, $B(12,6)$ and $C(0,10)$. The optimal Solution occurs at one of the corner points.

$$\text{At } O(0,0), Z = 80 \times 0 + 120 \times 0 = 0$$

$$\text{At } A(20,0), Z = 80 \times 20 + 120 \times 0 = 1600$$

$$\text{At } B(12,6), Z = 80 \times 12 + 120 \times 6 = 1680$$

At $C(0,10)$, $Z = 80 \times 0 + 120 \times 10 = 1200$.

We find that value of Z is maximum at $B(12,6)$.

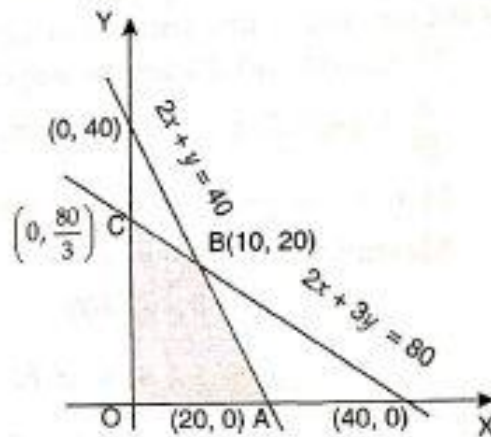
Hence, the manufacturer should produce 12 aids of type A and 6 aids of type B to get a maximum profit of ₹ 1680 .

Q.3. A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on item of model A is ₹15 and on an item of model B is ₹10. How many items of each model should be made per day in order to maximize daily profit? Formulate the above LPP and solve it graphically and find the maximum profit.

Solution. Since each man skilled or semi-skilled works for at most 8 hours per day and 5 skilled men and 10 semi-skilled men are employed, so the maximum number of skilled working hours available = $5 \times 8 = 40$ and maximum number of semi-skilled working hours available = $10 \times 8 = 80$. If x items of model A and y items of model B are made, then the LPP is maximise $Z = 15x + 10y$ subject to constraints

$$2x + y \leq 40, 2x + 3y \leq 80, x \geq 0, y \geq 0$$

We draw the straight lines $2x + y = 40$ and $2x + 3y = 80$ and shade the region satisfied by the above inequalities.



The shaded region shows the feasible region which is bounded. The point of intersection of the lines $2x + y = 40$ and $2x + 3y = 80$ is $B(10,20)$.

The corner points of the feasible $OABC$ are $O(0,0)$, $A(20,0)$, $B(10,20)$ and $C\left(0, \frac{80}{3}\right)$. The optimal

Solution occurs at one of the corner points.

At $O(0,0)$, $Z = 15 \times 0 + 10 \times 0 = 0$;

at $A(20,0)$, $Z = 15 \times 20 + 10 \times 0 = 300$;

at $B(10,20)$, $Z = 15 \times 10 + 10 \times 20 = 350$;

at $C\left(0, \frac{80}{3}\right)$, $Z = 15 \times 0 + 10 \times \frac{80}{3} = \frac{800}{3} = 266\frac{2}{3}$.

We find that the value of Z is maximum at $B(10,20)$.

Hence, the manufacturer should produce 10 items of model A and 20 items of model B to get maximum profit of ₹350.

Q.4. A company manufactures two types of cardigans: type A and type B. It costs ₹360 to make a type A cardigan and ₹120 to make a type B cardigan. The company can make atmost 300 cardigons and spend atmost ₹ 72000 a day. The number of cardigons of type B cannot exceed the number of cardigons of type A by more than 200. The company makes a profit of ₹ 100 for each cardigan of type A and ₹50 for every cardigan of type B. Formulate this problem as a linear programming problem to maximise the profit of the company. Solve it graphically and find maximum profit.

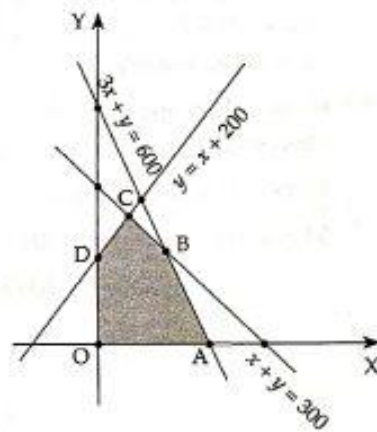
Solution. Let x cardigons of type A and y cardigons of type B be manufactured per day by the company, then profit P (in ₹) = $100x + 50y$.

Hence, the problem can be formulated as an L.P.P. as follows:

$$\begin{aligned} \text{Maximize } P &= 100x + 50y \\ 360x + 120y &\leq 72000 \text{ i.e. } 3x + y \leq 600 \end{aligned}$$

$$\begin{aligned} \text{subject to the constraints } x + y &\leq 300 \\ y &\leq x + 200 \\ x &\geq 0, y \geq 0 \end{aligned}$$

Draw the lines $3x + y = 600$, $x + y = 300$ and $y = x + 200$ and shade the region satisfied by the above inequalities.



The shaded portion OABCD shows the feasible region, which is bounded.

The corner points of the feasible region are $O(0,0)$, $A(200,0)$, $B(150,150)$, $C(50,250)$ and $D(0,200)$.

The values of P at the corner points are :

$$\text{at } O(0,0), P = 100 \times 0 + 50 \times 0 = 0;$$

$$\text{at } A(200,0), P = 100 \times 200 + 50 \times 0 = 20000;$$

$$\text{at } B(150,150), P = 100 \times 150 + 50 \times 150 = 22500;$$

$$\text{at } C(50,250), P = 100 \times 50 + 50 \times 250 = 17500;$$

$$\text{at } D(0,200), P = 100 \times 0 + 50 \times 200 = 10000.$$

We note that the value of P is maximum at the point B .

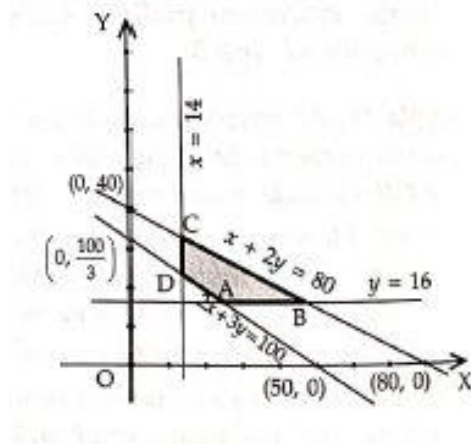
Hence, maximum profit = ₹22500 by manufacturing 150 cardigans of type A and 150 cardigans of type B.

Q.5.. A manufacturer makes two products, A and B. Product A sells at ₹200 each and takes $\frac{1}{2}$ hour to make. Product B sells at ₹300 each and takes 1 hour to make. There is a permanent order for 14 units of product A and 16 units of product B. A working week consists of 40 hours of production and the weekly turnover must not be less than ₹10000. If the profit on each of product A is ₹20 and on product B is ₹30, then how many of each should be produced so that the profit is maximum? Also find the maximum profit.
 Solution. Let x and y be the number of units of products A and B respectively manufactured (and sold), then total profit $P = 20x + 30y$ (in ₹).

Hence, the problem can be formulated as an L.P.P. as follows:

$$\begin{aligned} & \text{Maximize } P = 20x + 30y \\ & \text{subject to the constraints } \quad 200x + 300y \geq 10000 \text{ i.e. } 2x + 3y \geq 100 \\ & \quad \quad \quad \quad \quad \quad \quad \frac{1}{2}x + 1 \cdot y \leq 40 \text{ i.e. } x + 2y \leq 80 \\ & \quad \quad \quad \quad \quad \quad \quad x \geq 14, y \geq 16, x \geq 0, y \geq 0 \end{aligned}$$

We draw the straight lines $2x + 3y = 100$, $x + 2y = 80$, $x = 14$ and $y = 16$ and shade the region satisfied by the above inequalities.



The shaded portion shows the feasible region $ABCD$, which is bounded. The corner points of the feasible region $ABCD$ are $A(26,16)$, $B(48,16)$, $C(14,33)$ and $D(14,24)$.

The optimal Solution occurs at one of the corner points.

At A, $P = 20 \times 26 + 30 \times 16 = 1000$.

At B, $P = 20 \times 48 + 30 \times 16 = 1440$.

At C, $P = 20 \times 14 + 30 \times 33 = 1270$.

At D, $P = 20 \times 14 + 30 \times 24 = 1000$.

Hence, the manufacturer should produce 48 units of product A and 16 units of product B. Maximum profit is ₹ 1440 .

Q.6. A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for atmost 12 hours whereas machine III must be operated at least 5 hours a day. He

produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of ₹600 and ₹400 on each item of M and N respectively. How many units of each item should he produce so as to maximize his profit assuming that he can sell all the items that he produced? What will be the maximum profit?

Solution. Let x units of item M and y units of item N are produced by the manufacturer, then the problem can be formulated as an L.P.P. as follows:

Maximize the profit (in ₹) $Z = 600x + 400y$ subject to the constraints

$$x + 2y \leq 12 \quad (\text{machine I constraint})$$

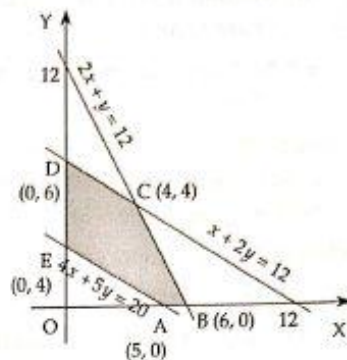
$$2x + y \leq 12 \quad (\text{machine II constraint})$$

$$1x + 1.25y \geq 5 \quad (\text{machine III constraint})$$

$$\text{i.e. } 4x + 5y \geq 20,$$

$$x \geq 0, y \geq 0 \quad (\text{non-negativity constraints})$$

Draw the lines $x + 2y = 12$, $2x + y = 12$ and $4x + 5y = 20$, and shade the region satisfied by the above inequalities.



The feasible region is the polygon ABCDE, which is convex and bounded.

The corner points are A(5,0), B(6,0), C(4,4), D(0,6) and E(0,4).

The values of Z at the corner points are :

$$\text{at } A(5,0), Z = 600 \times 5 + 400 \times 0 = 3000;$$

$$\text{at } B(6,0), Z = 600 \times 6 + 400 \times 0 = 3600;$$

$$\text{at } C(4,4), Z = 600 \times 4 + 400 \times 4 = 4000;$$

$$\text{at } D(0,6), Z = 600 \times 0 + 400 \times 6 = 2400;$$

$$\text{at } E(0,4), Z = 600 \times 0 + 400 \times 4 = 1600.$$

\therefore Maximum profit = ₹4000, when 4 items of M and 4 items of N are produced.

CASE-STUDY BASED QUESTIONS (SOLVED)

Q.1. Read the following text and answer the following questions on the basis of the same:



A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B to go into each bottle of the drink. The chemicals are available in a prepared mix from two different suppliers. Supplier *S* has a mix of 4 units of A and 2 units of B that costs ₹10, the supplier *T* has a mix of 1 unit of A and 1 unit of B that costs ₹ 4. Suppose x units of mix are purchased from supplier *S* and y units are purchased from supplier *T*.

On the basis of this information, answer the following questions.

(i) What is the total cost of purchase from both the Suppliers, represent this in equation form?

Sol. Suppose x units of mix are purchased from supplier *S* and y units are purchased from supplier *T*.

Total cost $Z = 10x + 4y$.

(ii) Write the minimum requirement of chemical A in equation form?

Sol. The minimum requirement of chemical A per bottle is given by: $4x + y \geq 80$.

(iii) Write the minimum requirement of chemical B in equation form?

OR

How many mixes from *S* and *T* should the company purchase to honour contract requirement and yet minimize cost?

Sol. The minimum requirement of chemical B per bottle is given by : $2x + y \geq 80$.

OR

The given data may be put in the following tabular form:

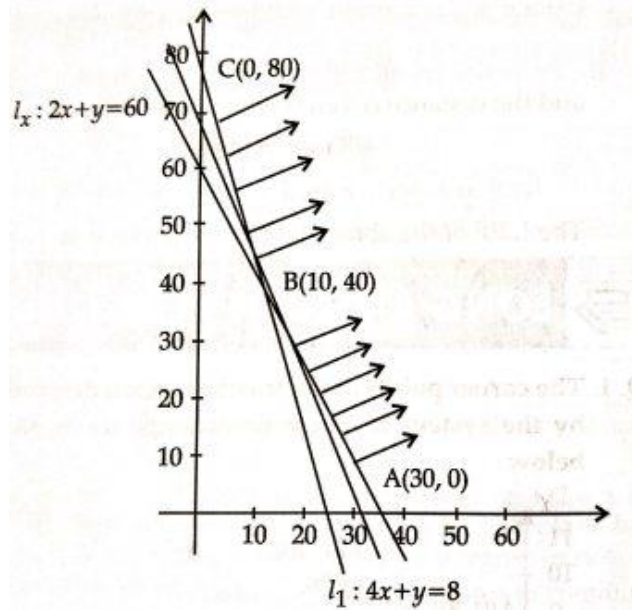
Supplier/	S	T	Minimum Requirement
Chemical A	4	1	80
Chemical B	2	1	60
Cost per unit	₹10	₹4	

$$\begin{aligned}
 4x + y &\geq 80 \\
 2x + y &\geq 60 \\
 x &\geq 0, y \geq 0 \\
 L_1: 4x + y &= 80
 \end{aligned}$$

x	0	25
y	50	0

x	30	0
y	0	60

$$L_2: 2x + y = 60$$



Vertices of feasible region are $A(30,0)$, $B(10,40)$ and $C(0,80)$.

Point (x, y)	Value of objective function $Z = 10x + 4y$
$A(30,0)$	$Z = 10 \times 30 + 4 \times 0 = 300$
$P(10,40)$	$Z = 10 \times 10 + 40 \times 4 = 260$
$C(0,80)$	$Z = 10 \times 0 + 4 \times 80 = 320Z$

Clearly, Z is minimum at $(10,40)$. The feasible region is unbounded and the open half plane represented by $10x + 4y < 260$ does not have points in common with the feasible region. So, Z is minimum at $x = 10, y = 40$. Hence, $x = 10, y = 40$ is the optimal Solution of the given LPP. Hence, the cost per bottle is minimum when the company purchases 10 mixes from supplier 5 and 40 mixes from supplier T.

Q.2. Read the following text and answer the following questions on the basis of the same:

Ramesh rides her bike at 25 km/h. She has to spend ₹2 per km on diesel and if he rides it at faster speed of 40 km/h, the diesel cost increases to ₹ 5 per km. He has ₹ 100 to spend on diesel. Let he travels x km with speed 25 km/h and y km with speed 40 km/h. It is given that objective function is Max. $Z = x + y$.



(i) Write the objective function for given LPP.

Sol. Objective function for given LPP is: $\text{Max } Z = x + y$

(ii) Write the set of constraints for given LPP.

Sol. Subject to constraints:

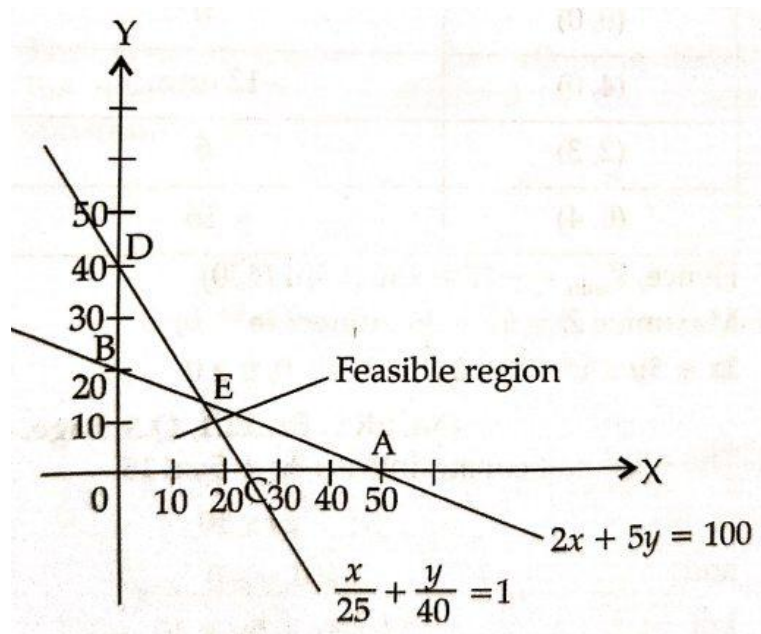
$$2x + 5y \leq 100$$

$$\frac{x}{25} + \frac{y}{40} \leq 1$$

$$x, y \geq 0$$

(iii) Draw the graph for feasible region. OR

Write the corner points of feasible region and find maximum value of objective function.



Intersection point of $2x + 5y = 100$ and $\frac{x}{25} + \frac{y}{40} \leq 1$ is E .

On solving the equations, we get the coordinates of E are $\left(\frac{50}{3}, \frac{40}{3}\right)$

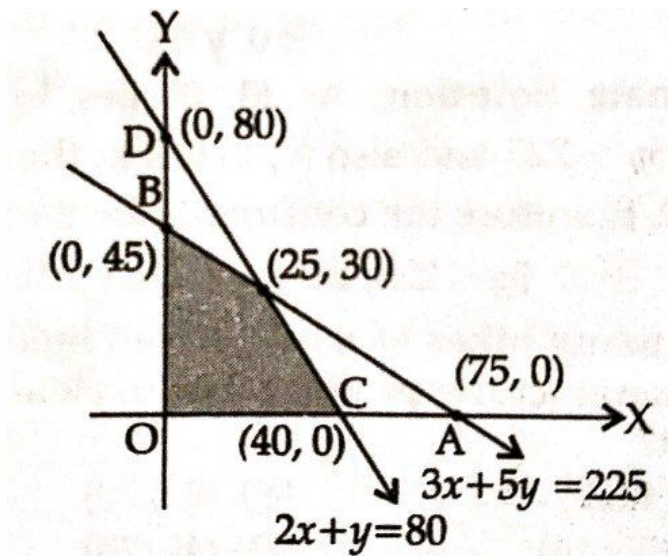
OR

So corner points of LPP are: $O(0,0)$, $B(0,20)$, $C(25,0)$, $(\frac{50}{3}, \frac{40}{3})$

Corner points	value of $x = x + y$
$(0,0)$	0
$(0,20)$	20
$(25,0)$	25
$(\frac{50}{3}, \frac{40}{3})$	$30 \rightarrow$ Maximum

Q.3. A manufacturer produces two Models of bikes Model X and Model Y. Model X takes 6 manhours to make per unit, while Model Y takes 10 man-hours per unit. There is a total of 450 man-hours available per week. Handling and Marketing costs are ₹2,000 and ₹1,000 per unit for Models X and Y respectively. The total funds available for these purposes are ₹80,000 per week. Profits per unit for Models X and Y are ₹1,000 and ₹500, respectively.

The feasible region of LPP is shown in the following graph.



(i) Write the equation of line AB.

Solution: From the given graph $OA = 75$ and $OB = 45$.

The equation of line AB is $\frac{x}{75} + \frac{y}{45} = 1$

i.e., $3x + 5y = 225$

(ii) Write the equation of line CD.

Solution: From the given graph $OC = 40$ and $OD = 80$.

The equation of line CD is $\frac{x}{40} + \frac{y}{80} = 1$

i.e., $2x + y = 80$

(iii) Write the coordinates of point E.

Solution. On solving the equations of lines AB and CD, we get the coordinates of point E i.e., (25,30).

(iv) Write the constraints for L.P.P.

Solution: Let the manufacturer produces x number of model X and y number of model Y bikes. Model X takes 6 man-hours to make per unit and model Y takes 10 man-hours to make per unit.

There is total of 450 man-hours available per week.

$$\begin{aligned} \therefore 6x + 10y &\leq 450 \\ \Rightarrow 3x + 5y &\leq 225 \end{aligned}$$

For models X and Y , handling and marketing costs are ₹2,000 and ₹1,000, respectively, total funds available for these purposes are ₹ 80,000 per week.

$$\begin{aligned} \therefore 2,000x + 1,000y &\leq 80,000 \\ \Rightarrow 2x + y &\leq 80 \\ \text{Also, } x &\geq 0, y \geq 0 \end{aligned}$$

(v) How many bikes of model X and model Y should the manufacturer produce so as to yield a maximum profit?

Solution: The objective function for given L.P.P. is

$$Z = 1000x + 500y$$

From the shaded feasible region, it is clear that coordinates of corner points are (0,0), (40,0), (25,30) and (0,45).

Corner points	Value of $Z = 1000x + 500y$
(0,0)	0
(40,0)	40,000 ← Maximum
(25,30)	25,000 + 15,000 = 40,000 ← Maximum
(0,45)	22,500

So, the manufacturer should produce 25 bikes of model X and 30 bikes of model Y to get a maximum profit of ₹ 40,000.