Linear Inequalities

In this Chapter...

- Linear Inequality
- Solution of an Inequality
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- Linear Inequations in Two Variables and their Graphical Solution

An **inequality** is said to be linear, if the variable (s) occurs in first degree only and there is no term involving the product of the variables. e.g. $\mathbf{ax + b \le 0}$, $\mathbf{ax + by + c > 0}$, $\mathbf{ax \le 4}$.

Linear Inequality in One Variable

A linear inequality which has only one variable, is called linear inequality in one variable. e.g. $\mathbf{ax + b < 0}$, where $\mathbf{a \ne 0}$.

An inequality in one variable, in which degree of variable is 2, is called quadratic inequality in one variable.

e.g. $\mathbf{ax^2 + bx + c \ge 0,\ a \ne 0;\ 3x^2 + 2x + 1 \le 0}$

Linear Inequality in Two Variables

A linear inequality which have only two variables, is called linear inequality in two variables. e.g. $\mathbf{3x + 11y \le 0,\ 4t + 3y > 0}$

Concept of Intervals

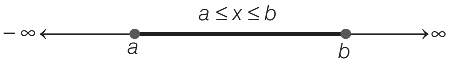
On number line or real line, various types of infinite subsets, known as intervals, are defined below

Closed Interval

If $\mathbf{a}$ and $\mathbf{b}$ are real numbers, such that $\mathbf{a < b}$, then the set of all real numbers $\mathbf{x}$, such that $\mathbf{a \le x \le b}$, is called a closed interval and is denoted by $\mathbf{[a, b]}$.

$\therefore \quad \mathbf{[a, b] = \{x: a \le x \le b, x \in R\}}$

On the number line, $\mathbf{[a, b]}$ may be represented as follows



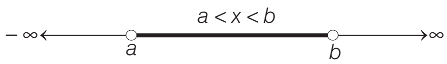
Here, end points of the interval i.e. **a** and **b** are included in the interval. So, on number line, draw filled circle (•) at **a** and **b**.

Open Interval

If $\mathbf{a}$ and $\mathbf{b}$ are real numbers, such that $\mathbf{a < b}$, then the set of all real numbers $\mathbf{x}$, such that $\mathbf{a < x < b}$, is called an open interval and is denoted by $\mathbf{(a, b)}$ **or** $\mathbf{]a, b[}$.

$\therefore \quad \mathbf{(a, b) = \{x : a < x < b, x \in R\}}$

On the number line, $\mathbf{(a, b)}$ may be represented as follows



Here, end points of the interval i.e. **a** and **b** are not included in the interval. So, on number line, draw open circle (o) at **a** and **b**.

Semi-open or Semi-closed Intervals

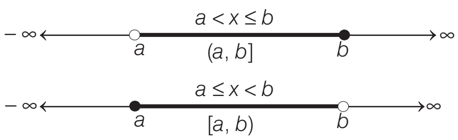
If $\mathbf{a}$ and $\mathbf{b}$ are real numbers, such that $\mathbf{a < b}$.

Then, $\mathbf{(a, b] = \{x : a < x \le b, x \in R\}}$

and $\quad \mathbf{[a, b) = \{x : a \le x < b, x \in R\}}$

are known as semi-open or semi-closed intervals.

On the number line, these intervals may be represented as follows



Solution of an Inequality

Any solution of an inequality is the value(s) of variable(s) which makes it a true statement.

We can find the solutions of an inequality by hit and trial method but it is not very efficient because this method is time consuming and sometimes not feasible. So, we solve inequalities with systematic technique.

Some properties or rules which are used to solve the inequalities, are given below

Addition or Subtraction

Some number may be added (or subtracted) to (from) both sides of an inequality i.e. if $\mathbf{a > b}$, then for any number $\mathbf{c}$,

$$\mathbf{a + c > b + c} \text{ or } \mathbf{a - c > b - c}$$

e.g. (i) $\mathbf{10 > 5} \Rightarrow 10 + 7 > 5 + 7$ [adding 7 both sides]

$\Rightarrow \mathbf{17 > 12}$, which is true.

(ii) $\mathbf{-8 > -13} \Rightarrow -8 - 2 > -13 - 2$

[subtracting 2 from both sides]

$\Rightarrow \mathbf{-10 > -15}$, which is true.

Multiplication or Division

If both sides of an inequality are multiplied (or divided) by the same positive number, then the sign of inequality remains the same. But when both sides are multiplied (or divided) by the same negative number, then the sign of inequality is reversed.

Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be three real numbers, such that $\mathbf{a > b}$ and $\mathbf{c \neq 0}$.

(i) If $\mathbf{c > 0}$, then $\dfrac{\mathbf{a}}{\mathbf{c}} > \dfrac{\mathbf{b}}{\mathbf{c}}$ and $\mathbf{ac > bc}$.

(ii) If $\mathbf{a > b}$ and $\mathbf{c < 0}$, then $\dfrac{\mathbf{a}}{\mathbf{c}} < \dfrac{\mathbf{b}}{\mathbf{c}}$ and $\mathbf{ac < bc}$.

Method to Solve a Linear Inequality in One Variable

Step I Collect all terms involving the variable ($\mathbf{x}$) on one side and constant terms on other side with the help of above rules and then reduce it in the form $\mathbf{ax < b}$ or $\mathbf{ax \le b}$ or $\mathbf{ax > b}$ or $\mathbf{ax \ge b}$.

Step II Divide this inequality by the coefficient of variable ($\mathbf{x}$). This gives the solution set of given inequality.

Step III Write the solution set.

Representation of solution of Linear Inequality in One Variable on Number Line

To represent the solution of a linear inequality in one variable on a number line, use the following rules

(i) To represent $\mathbf{x < a}$ (or $\mathbf{x > a}$) on a number line, put a circle (o) on the number $\mathbf{a}$ and dark the line to the left (or right) of the number $\mathbf{a}$.

(ii) To represent $\mathbf{x \le a}$ (or $\mathbf{x \ge a}$) on a number line, put a dark circle (•) on the number $\mathbf{a}$ and dark the line to the left (or right) of the number $\mathbf{a}$.

System of Inequalities in One Variable

Two or more inequalities taken together comprise a system of inequalities and the solution of the system of inequalities are the solutions common to all the inequalities comprising the system.

e.g. $\mathbf{x = 10}$ is the solution of the system of inequalities

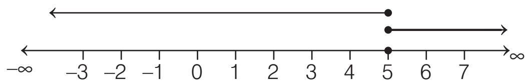
$$\mathbf{4x + 3 \le 91} \text{ and } \mathbf{2x \ge x + 8}$$

Solution of System of Linear Inequalities in One Variable

We know that, the solution set of a linear inequality in one variable is the set of all points on the number line satisfying the given inequality.

Therefore, the solution set of a system of linear inequalities in one variable is defined as the **intersection of the solution set** of the linear inequalities in the system.

e.g. If the solution sets of linear inequalities in the system are $(-\infty, \mathbf{5}]$ and $[\mathbf{5}, \infty)$, then the solution of system of linear inequalities in one variable is 5 only. Because, if we represent the solution sets on the number line, we see that the value which are common to both is 5 only.



The process of finding solution of system of linear inequalities of different types are given below.

Type I

When Two Separate Linear Inequalities are Given

If the given system of inequalities comprise by two separate linear inequalities, then to solve these we use the following working steps

Step I Solve each inequality separately and obtain their solution sets.

Step II Represent the solution sets on a number line and then find the values of the variable which are common to them.

Or Find the intersection of the solution sets obtained in step I.

Type 2

When Inequalities of the form $a \le \dfrac{cx + d}{e} \le b$, where $a, b, c, d \in R$

This type of inequalities will be formed by combining the inequalities $\mathbf{a} \le \dfrac{\mathbf{cx + d}}{\mathbf{e}}$ and $\dfrac{\mathbf{cx + d}}{\mathbf{e}} \le \mathbf{b}$.

To solve such type of inequalities, make the middle term free from constant (i.e. write the given inequalities as $\mathbf{f \le x \le g}$, where $\mathbf{f}$ and $\mathbf{g}$ are some real numbers by using the rule of addition, subtraction, multiplication, division in each term of given inequalities.)

Linear Inequations in Two Variables and their Graphical Solution

An inequality of the form $\mathbf{ax + by + c > 0}$ or $\mathbf{ax + by + c < 0}$ or $\mathbf{ax + by + c \geq 0}$ or $\mathbf{ax + by + c \leq 0}$, where $\mathbf{a \neq 0}$ and $\mathbf{b \neq 0}$, is called a linear inequality in two variables x and $\mathbf{y}$.

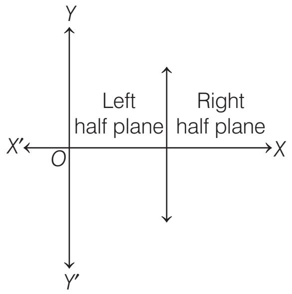
The region containing all the solutions of an inequality, is called the **solution region.**

Concept of Half Planes

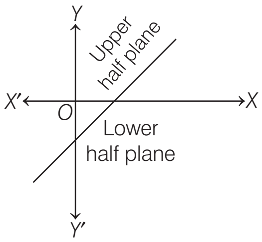
The graph of $\mathbf{ax + by + c = 0}$ is a straight line which divides the **cartesian plane or XY-plane** into two parts. Each part is known as half plane.

Types of Half Planes

1. **Left and right half planes** A vertical line will divide the **XY**-plane in two parts, left half plane and right half plane.



2. **Lower and upper half planes** A non-vertical line will divide the **XY**-plane into two parts, lower half plane and upper half plane.



3. **Closed half plane** A half plane in **XY**-plane is called a closed half plane, if the line separating the plane is also included in the half plane.

   Therefore, the graph of a linear inequality involving sign $\leq$ or $\geq$ is always closed half plane.

4. **Open half plane** A half plane in **XY**-plane is called an open half plane, if the line separating the plane is not included in the half plane.

   Therefore, the graph of a linear inequality involving sign $<$ or $>$ is always an open half plane.

Graph of Linear Inequality in Two Variables

Working rule for drawing the graph of linear inequalities in two variables are discussed below

Step I Consider the equation $\mathbf{ax + by = c}$, in place of given inequality $\mathbf{ax + by \leq c}$ or $\mathbf{ax + by \geq c}$ or $\mathbf{ax + by < c}$ or $\mathbf{ax + by > c}$, which represents a straight line in **XY**-plane**.**

Step II Put $\mathbf{x = 0}$ in the equation obtained in step I to get the point, where the line meets **Y**-axis. Similarly, put $\mathbf{y = 0}$ to obtain a point, where the line meets the **X**-axis.

Step III Draw a line joining the points obtained in step II. If the inequality is of the form of $<$ or $>$, then draw dotted line to indicate that the points on the line are excluded. Otherwise, draw a thick or dark line to indicate that the points on this line are included.

Step IV Take any point (preferable origin, i.e. (0, 0)), not lying on the line, and check whether this satisfies the given linear inequality or not.

Step V If the inequality is satisfied, by this point then shade that portion of the plane which contains the chosen point. The shaded region obtained in step V, represents the graph of linear inequality in two variable.

Graphical Solutions of System of Linear Inequalities in Two Variables

A system of linear inequalities in two variables can be solved by graphical method.

For finding the solution, we use the following steps

Step I Draw the graph of all the given inequalities.

Step II Find the common shaded region, which satisfies all the given linear inequalities.

Step III This common region is the required solution region of the system of given inequalities.

If there is no common region, then the system of inequalities has no solution.

Inequality between Coordinate Axes

The inequality $\mathbf{x \geq 0}$ consist of **Y**-axis and the plane on the right side of **Y**-axis. Also, $\mathbf{y \geq 0}$ consist of X-axis and the plane above the **X**-axis.

$\therefore$ The inequalities $\mathbf{x \geq 0}$ and $\mathbf{y \geq 0}$, together represent the first quadrant including the point on the axes.

Solved Examples

Example 1. Solve $\mathbf{24x < 100}$, when

(i) **x** is a natural number (ii) **x** is an integer.

Sol. We have $\mathbf{24x < 100}$, dividing both sides by 24,

$$\Rightarrow \quad \frac{24x}{24} < \frac{100}{24} \qquad \text{(using rule 2)}$$

$$\Rightarrow \quad x < \frac{50}{12} \Rightarrow x < \frac{25}{6}$$

$$\Rightarrow \quad x < 4\frac{1}{6} \qquad \left(\text{i. e. } x \text{ is less than } 4\frac{1}{6}\right)$$

(i) When **x** is a natural number (only positive integer) in this case, the solution set of the inequality is {1, 2, 3, 4}.

(ii) When **x** is an integer, the solution set of the given inequality is $\{\ldots\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4\}$

Example 2. Solve $\mathbf{-12x > 30}$, when

(i) **x** is a natural number (ii) **x** is an integer.

Sol. We have $\mathbf{-12x > 30}$, dividing both sides by -12

$$\frac{-12x}{-12} < \frac{30}{-12}$$

$$\Rightarrow \quad x < -\frac{5}{2}$$

(i) When **x** is a natural number, then there is no solution of given inequality. As natural numbers are positive numbers and there is no positive number which is less than negative number.

(ii) When **x** is an integer, the solution of given inequality is $\{\ldots -4, -3\}$ as there are infinite number which are less then $-\frac{5}{2}$.

Example 3. Solve $\mathbf{5x - 3 < 7}$, when

(i) **x** is an integer (ii) **x** is a real number.

Sol. We have, $\mathbf{5x - 3 < 7}$, adding 3 on both sides,

$$\Rightarrow \quad 5x - 3 + 3 < 7 + 3 \qquad \text{(using rule 1)}$$

$$5x < 10$$

On dividing both side by 5, we get

$$\frac{5x}{5} < \frac{10}{5} \qquad \text{(using rule 2)}$$

$$\Rightarrow \quad x < 2$$

(i) When **x** is an integer, the solution of the given inequality is $\{\ldots, -1, 0, 1\}$.

(ii) When **x** is a real number, the solution of given inequality is $(-\infty, 2)$ i.e. all the numbers lying between $-\infty$ **and 2** but ∞ and 2 are not included as $\mathbf{x < 2}$.

Example 4. $\mathbf{2(2x + 3) - 10 < 6(x - 2)}$.

Sol. We have, $2(2x + 3) - 10 < 6(x - 2)$

$$\Rightarrow \quad 4x + 6 - 10 < 6x - 12$$

$$\Rightarrow \quad 4x - 4 < 6x - 12$$

Transferring the term **6x** to LHS and (-4) to RHS,

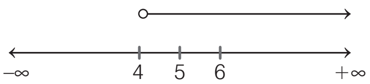
$$4x - 6x < -12 + 4$$

$$\Rightarrow \quad -2x < -8$$

On dividing both sides by -2, we get

$$\Rightarrow \quad \frac{-2x}{-2} > \frac{-8}{-2} \qquad \text{(using rule 2)}$$

$$\Rightarrow \quad x > 4$$



$\therefore$ Solution set = $(4, \infty)$.

Example 5. $\mathbf{37 - (3x + 5) \geq 9x - 8(x - 3)}$.

Sol. We have, $37 - (3x + 5) \geq 9x - 8(x - 3)$

$$(37 - 3x - 5) \geq 9x - 8x + 24$$

$$\Rightarrow \quad 32 - 3x \geq x + 24$$

Transferring the term **24** to LHS and the term $(-3x)$ to RHS,

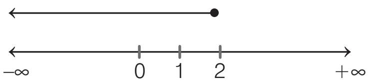
$$32 - 24 \geq x + 3x$$

$$\Rightarrow \quad 8 \geq 4x \Rightarrow 4x \leq 8$$

On dividing both sides by 4, we get

$$\Rightarrow \quad \frac{4x}{4} \leq \frac{8}{4} \qquad \text{(using rule 2)}$$

$$\Rightarrow \quad x \leq 2$$



$\therefore$ Solution set = $(-\infty, 2]$.

Example 6. Find the solutions of the system of inequalities $\mathbf{3x - 7 < 5 + x}$ and $\mathbf{11 - 5x \leq 1}$ on the number line.

Sol. Given, inequalities are

$$3x - 7 < 5 + x \qquad \ldots \text{(i)}$$

and $$11 - 5x \leq 1 \qquad \ldots \text{(ii)}$$

From inequality (i), we have

$$3x - 7 < 5 + x$$

or $$x < 6 \qquad \ldots \text{(iii)}$$

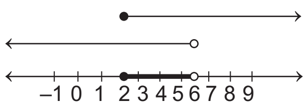
Also, from inequality (ii), we have

$$11 - 5x \leq 1$$

or $$-5x \leq -10$$

i.e. $$x \geq 2 \qquad \ldots \text{(vi)}$$

If we draw the graph of inequalities (iii) and (vi) on the number line, we see that the values of **x**, which are common to both, are shown by bold line in figure.



Example 7. The solution of the inequality $-8 \le 5x - 3 < 7$ is

Sol. In this case, we have two inequalities, $-8 \le 5x - 3$ and $5x - 3 < 7$, which we will solve simultaneously.

We have, $-8 \le 5x - 3 < 7$

or $\quad -5 \le 5x < 10$

or $\quad -1 \le x < 2$ i.e. $x \in [-1, 2)$.

Example 8. In an experiment, a solution of hydrochloric acid is to be kept between 30°C and 35° C. The range of temperature in degree Fahrenheit, if conversion formula is given by $C = \frac{5}{9}(F - 32)$, where C and F represent temperature in degree Celsius and degree Fahrenheit respectively, is between …A… and …B… . Here, A and B refer to

Sol. It is given that $30 < C < 35$.

Putting $C = \frac{5}{9}(F - 32)$, we get

$$30 < \frac{5}{9}(F - 32) < 35,$$

Multiplying $\frac{9}{5}$ to each term,

or $\quad \frac{9}{5} \times (30) < (F - 32) < \frac{9}{5} \times (35)$

or $\quad 54 < (F - 32) < 63$

or $\quad 86 < F < 95$.

Thus, the required range of temperature is between 86° F and 95° F.

Example 9. Find the graphical solution of the inequality $2x + y \ge 6$.

Sol. We have the given inequality $2x + y \ge 6$ …(i)

Step 1. Consider the inequation as a strict equation

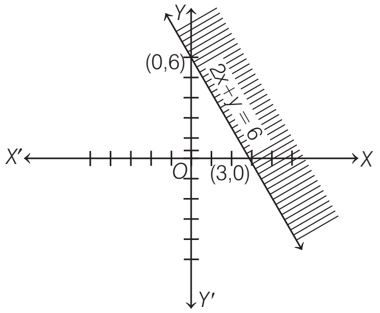
i.e. $\quad 2x + y = 6$

Step 2. Find the points on X-axis and Y-axis i.e.

x	3	0
y	0	6

Step 3. Plot the graph using the above table.

Step 4. Take a point $(0, 0)$ and put it in the given inequation (i), we get $0 + 0 \ge 6$ which is false, so shaded region will be away from the origin.



Example 10. Find the graphical solution of $3x + 4y \le 12$.

Sol. We have the given inequality $3x + 4y \le 12$ …(i)

Step 1. Consider the inequation as a strict equation i.e.

$$3x + 4y = 12$$

Step 2. Find the points on the X-axis and Y-axis i.e.

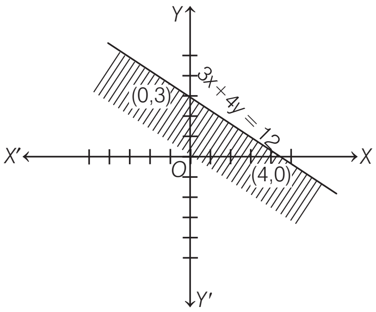
x	4	0
y	0	3

Step 3. Plot the graph using the above table.

Step 4. Take a point (0, 0) and put it in the given inequation (i), we get

$$0 + 0 \le 12$$

which is true, so the shaded region will be towards the origin.



Here, shaded region shows the inequality.

Example 11. Represent the inequality $y + 8 \ge 2x$ graphically.

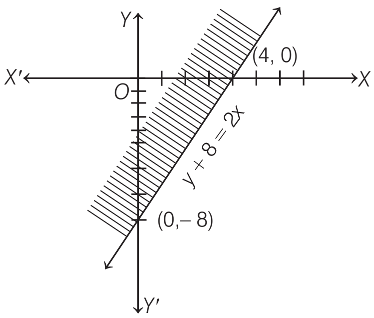
Sol. The given inequation is $y + 8 \ge 2x$ …(i)

Step 1. Consider the inequation as strict equation i.e.

$$y + 8 = 2x$$

Step 2. Find the points on the X-axis and Y-axis i.e.

x	0	4
y	− 8	0



Step 3. Plot the graph using the above table.

Step 4. Take a point (0, 0) and put it in the given inequation (i), we get

$$0 + 8 \ge 0$$

which is true, so the shaded region will be towards the origin.

Thus, shaded region shows the inequality.

Example 12. Solve the following inequality graphically $x - y \le 2$.

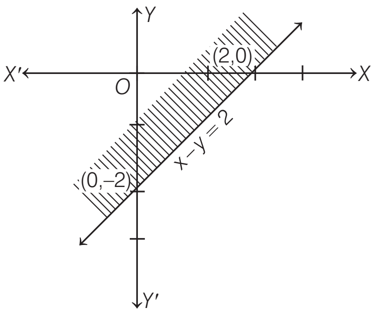
Sol. The given inequation is $\mathbf{x - y \le 2}$ ...(i)

Step 1. Consider the inequation as a strict equation i.e.

$$x - y = 2$$

Step 2. Find the points on the *X*-axis and *Y*-axis i.e.

x	2	0
y	0	– 2



Step 3. Plot the graph using the above table.

Step 4. Take a point **(0, 0)** and put it in the given inequation (i), we get

$$0 - 0 \le 2 \Rightarrow 0 \le 2$$

which is true, so the shaded region will be towards the origin.

Thus, shaded region shows the inequality.

Example 13. Solve the system of inequality graphically $x + y \le 6, x + y \ge 4$.

Sol. The given system of inequalities

$$x + y \le 6 \quad \text{...(i)}$$

$$x + y \ge 4 \quad \text{...(ii)}$$

Step 1. Consider the inequations as strict equations

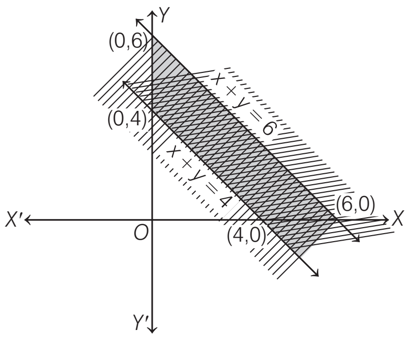
i.e. $\mathbf{x + y = 6}$ and $\mathbf{x + y = 4}$

Step 2. Find the points on the **X**-axis and **Y**-axis for $\mathbf{x + y = 6}$

x	0	6
y	6	0

and $\mathbf{x + y = 4}$

x	0	4
y	4	0



Step 3. Plot the graph using the above tables.

Step 4. Take a point (0, 0) and put it in the inequations (i) and (ii),

$$0 + 0 \le 6$$

$$0 \le 6 \quad \text{(True)}$$

So, the shaded region will be towards the origin.

and $\quad 0 + 0 \ge 4 \Rightarrow 0 \ge 4$ (False)

So, the shaded region will be away from the origin.

Thus, common shaded region shows the solution of the inequalities.

Example 14. Solve the system of equations graphically

$$x + y \ge 4, 2x - y > 0.$$

Sol. The given system of inequalities

$$x + y \ge 4 \quad \text{...(i)}$$

$$2x - y > 0 \quad \text{...(ii)}$$

Step 1. Consider the given inequations as strict equations.

$$x + y = 4$$

$$2x - y = 0$$

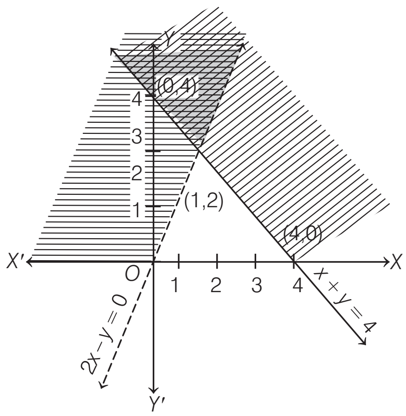
Step 2. Find the points on the **X**-axis and **Y**-axis for

$\mathbf{x + y = 4}$

x	0	4
y	4	0

and $\mathbf{2x - y = 0}$

x	0	1
y	0	2



Step 3. Plot the graph using the above tables.

Step 4. Take a point **(0, 0)** and put it in the inequation (i)

$$0 + 0 \ge 4 \quad \text{(False)}$$

So, the shaded region will be away from the origin.

Take a point **(1, 0)** and put it in the inequation (ii)

$$2 - 0 > 0 \quad \text{(True)}$$

So, the shaded region will be towards the origin.

Thus, common shaded region shows the solution of the inequalities.

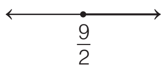
Chapter Practice

PART 1

Objective Questions

• Multiple Choice Questions

1. The solution set is



(a) $x \in \left(\frac{9}{2}, \infty\right)$ (b) $x \in \left[\frac{9}{2}, \infty\right)$

(c) $x \in -\left[\infty, \frac{9}{2}\right)$ (d) $x \in \left(-\infty, \frac{9}{2}\right]$

2. The solution set of the inequality $4x + 3 < 6x + 7$ is

(a) $[-2, \infty)$ (b) $(-\infty, -2)$

(c) $(-2, \infty)$ (d) None of these

3. If $-3x + 17 < -13$, then

(a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$

(c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10)$

4. The solution set of $5x - 3 < 7$, where x is an integer is

(a) $\{\ldots\ldots, -1, 0, 1\}$ (b) $\{\ldots\ldots, -3, -2, -1\}$

(c) $(-\infty, 2)$ (d) None of these

5. If $3x + 8 > 2$, then which of the following is true?

(a) $x \in \{-1, 0, 1, 2, \ldots\}$, when x is an integer

(b) $x \in [-2, \infty)$, when x is a real number

(c) Both (a) and (b)

(d) None of the above

6. The solution set of $3(1 - x) < 2(x + 4)$ is

(a) $[-1, \infty)$ (b) $(-1, \infty)$

(c) $(-\infty, -1]$ (d) $(-\infty, 1)$

7. The set of real x satisfying the inequality $\frac{5 - 2x}{3} \le \frac{x}{6} - 5$ is

(a) $(-\infty, 8)$ (b) $(8, \infty)$

(c) $[8, \infty)$ (d) $(-\infty, 8]$

8. The solution set of $5x - 3 \ge 3x - 5$ is

(a) $(-1, \infty)$ (b) $(1, \infty)$

(c) $[-1, \infty)$ (d) None of these

9. Which of the following is the solution set of the inequality $\frac{x}{4} < \frac{(5x - 2)}{3} - \frac{(7x - 3)}{5}$?

(a) $(4, \infty)$ (b) $(-\infty, 4)$

(c) $[4, \infty)$ (d) $(-\infty, 4]$

10. The solution set of $-12 < 4 - \frac{3x}{-5} \le 2$ is

(a) $\left(\frac{-80}{3}, \frac{-10}{3}\right]$ (b) $\left[-\frac{80}{3}, \frac{10}{3}\right)$

(c) $\left[\frac{-80}{3}, \frac{-10}{3}\right]$ (d) None of these

11. If $3 \le 3t - 18 \le 18$, then which one of the following is true?

(a) $15 \le 2t + 1 \le 20$

(b) $8 \le t < 12$

(c) $8 \le t + 1 \le 13$

(d) $21 \le 3t \le 24$

12. The solution set of $2 \le 3x - 4 \le 5$ is

(a) $[2, 3]$ (b) $(2, 3)$

(c) $(2, \infty)$ (d) $(-\infty, 3)$

13. The solution set of the inequality $6 \le -3(2x - 4) < 12$ is

(a) $(0, 1]$ (b) $[0, 1)$ (c) $[0, 1]$ (d) $(0, \infty)$

14. Consider the inequality $40x + 20y \le 120$, where x and y are whole numbers. Then, its solution set is

(a) (0, 0), (5, 5), (1, 1), (2, 2), (3, 0)

(b) (0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5), (0, 6)

(c) (1, 0), (2, 0), (3, 0), (4, 0), (5, 0)

(d) None of the above

15. The length of a rectangle is three times the breadth. If the minimum perimeter of the rectangle is 160 cm, then

(a) breadth $>$ 20 cm

(b) length $<$ 20 cm

(c) breadth ≥ 20 cm

(d) length ≤ 20 cm

16. The marks obtained by a student of class XI in first and second terminal examinations are 62 and 48, respectively. The minimum marks he should get in the annual examination to have an average of atleast 60 marks, are

(a) 70 (b) 50 (c) 74 (d) 48

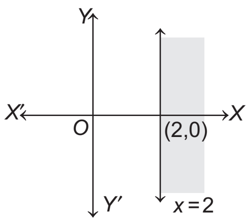
17. The number of pairs of consecutive odd natural numbers both of which are larger than 10, such that their sum is less than 40, is

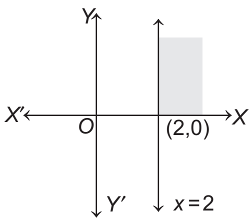
(a) 8 (b) 6 (c) 4 (d) 3

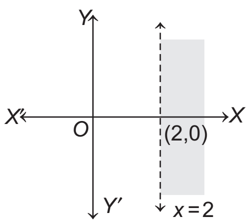
18. In drilling world's deepest hole it was found that the temperature T in degree Celcius, x km below the earth's surface was given by $\mathbf{T = 30 + 25\,(x - 3)}$, $\mathbf{3 \le x \le 15}$. At what depth will the temperature be between 155°C and 205°C?

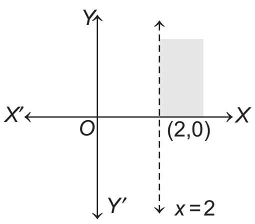
(a) 10 to 12 km (b) 8 to 10 km
(c) 8 to 10 km (d) 15 to 18 km

19. The graphical solution of $\mathbf{3x - 6 \ge 0}$ is

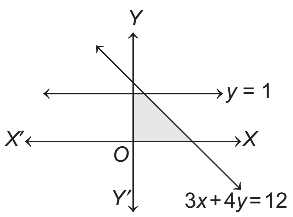
(a) 

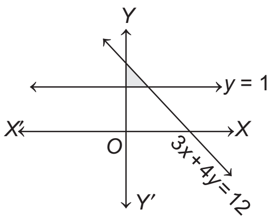
(b) 

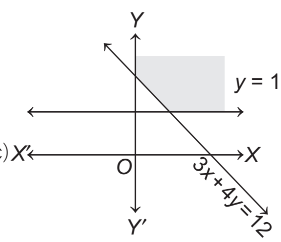
(c) 

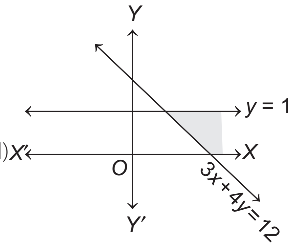
(d) 

20. The graphical solution of the system of linear inequalities, $\mathbf{3x + 4y \ge 12}$, $\mathbf{y \ge 1}$ and $\mathbf{x \ge 0}$, is

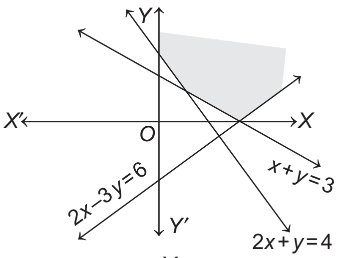
(a) 

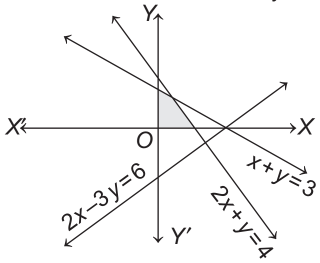
(b) 

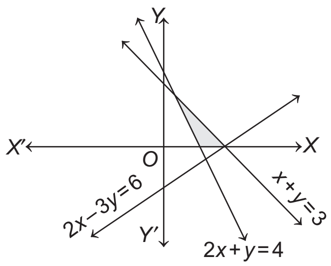
(c) 

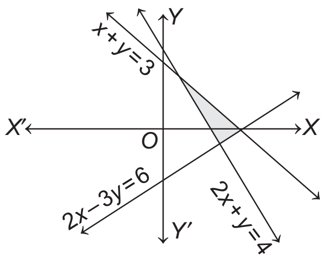
(d) 

21. The solution set of the inequalities $\mathbf{2x + y \ge 4}$, $\mathbf{x + y \le 3}$ and $\mathbf{2x - 3y \le 6}$, is

(a) 

(b) 

(c) 

(d) 

• Case Based MCQs

22. Shweta was teaching "method to solve a linear inequality in one variable" to her daughter.

Step I Collect all terms involving the variable (**x**) on one side and constant terms on other side with the help of above rules and then reduce it in the form $\mathbf{ax < b}$ or $\mathbf{ax \le b}$ or $\mathbf{ax > b}$ or $\mathbf{ax \ge b}$.

Step II Divide this inequality by the coefficient of variable (**x**). This gives the solution set of given inequality.

Step III Write the solution set.

Based on above information, answer the following questions.

(i) The solution set of $24x < 100$, when x is a natural number is
(a) {1, 2, 3, 4} (b) (1, 4)
(c) [1, 4] (d) None of these

(ii) The solution set of $24x < 100$, when x is an integer is
(a) $\{\ldots\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4\}$
(b) $(-\infty, 4]$
(c) $[4, \infty]$
(d) None of the above

(iii) The solution set of $-5x + 25 > 0$ is
(a) $[5, \infty)$ (b) $(-\infty, 5]$ (c) $(5, \infty)$ (d) $(-\infty, 5)$

(iv) The solution set of $3x - 5 < x + 7$ is
(a) $(6, \infty)$ (b) $[6, \infty)$ (c) $(-\infty, 6)$ (d) $(-\infty, 6]$

(v) The solution set of $x + \frac{x}{2} + \frac{x}{3} < 11$ is
(a) $(-\infty, 6]$ (b) $(-\infty, 6)$
(c) $[6, \infty)$ (d) None of these

23. A manufacturing company produces certain goods.The company manager used to make a data record on daily basis about the cost and revenue of these goods separately. The cost and revenue function of a product are given by $C(x) = 20x + 4000$ and $R(x) = 60x + 2000$, respectively, where x is the number of goods produced and sold.

Based on above information, answer the following questions.

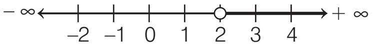
(i) How many goods must be sold to realise some profit?
(a) $x < 50$ (b) $x > 50$ (c) $x \geq 50$ (d) $x \leq 50$

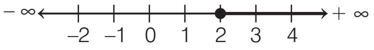
(ii) If the cost and revenue functions of a product are given by $C(x) = 3x + 400$ and $R(x) = 5x + 20$ respectively, where x is the number of items produced by the manufacturer, then how many items must be sold to realise some profit?
(a) $x \leq 190$ (b) $x \geq 190$ (c) $x < 190$ (d) $x > 190$

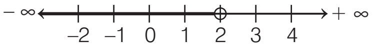
(iii) Let x and b are real numbers. If $b > 0$ and $x < b$, then
(a) x is always positive (b) x is always negative
(c) x is real number (d) None of these

(iv) The solution set of $3x - 5 < x + 7$, when x is a whole number is given by
(a) {0, 1, 2, 3, 4, 5} (b) $(-\infty, 6)$
(c) [0, 5] (d) None of these

(v) Graph of inequality $x > 2$ on the number line is represented by

(a) 

(b) 

(c) 

(d) None of the above

PART 2
Subjective Questions

• Short Answer Type Questions

1. Find the solutions set of $3x - 5 < x + 7$, where x is a natural number.

2. Solve the inequality $\frac{5 - 3x}{3} \leq \frac{x}{6} - 5$.

3. Solve the inequalities $-3 \leq 4 - \frac{7x}{2} \leq 18$.

4. Find the solution set of $\frac{1}{2}\left(\frac{3}{5}x + 4\right) \geq \frac{1}{3}(x - 6)$.

5. Find the solution set of $\frac{(2x - 1)}{3} \leq \frac{(3x - 2)}{4} - \frac{(2 - x)}{5}$.

6. Solve the linear inequalities $-15 < \frac{3(x - 2)}{5} \leq 0$.

7. Solve the following system of inequalities $2x - 3 < 7$ and $2x > -4$. Also, represent the solution graphically on the number line.

8. The sum of three consecutive integers must not be more than 12. What are the integers?

9. A company manufactures cassettes. Its cost and revenue functions are $C(x) = 26000 + 30x$ and $R(x) = 43x$, respectively, where x is the number of cassettes produced and sold in a week. How many cassettes must be sold by the company to realise some profit?

10. The water acidity in a pool is considered normal, when the average pH reading of three daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, then find the range of pH value for the third reading that will result in the acidity level being normal.

11. In the first four examinations, each of 100 marks, Mohan got 94, 73, 72 and 84 marks. If a final average greater than or equal to 80 and less than 90 is needed to obtain a final grade *B* in a course, then what range of marks in the fifth (last) examination will result, if Mohan receiving grade *B* in the course?

12. Find all pairs of consecutive even positive integers, both of which are larger than 5, such that their sum is less than 23.

13. A solution is to be kept between 68°F and 77°F. What is the range of temperature in degree Celsius (C), if the Celsius/Fahrenheit (F) conversion formula is given by $F = \frac{9}{5}C + 32$?

14. The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is atleast 61 cm, find the minimum length of the shortest side.

15. The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm, then find the minimum length of the shortest side.

16. A man wants to cut three lengths from a single piece of board of length 91 cm, the second length is to be 3 cm longer than the shortest and third length is to be twice as long as the shortest. What are the possible lengths for the shortest board, if third piece is to be atleast 5 cm longer than the second?

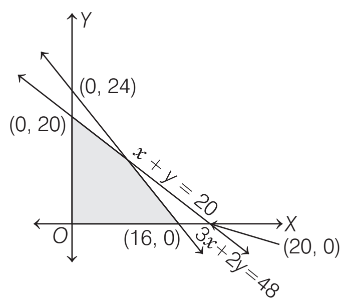
17. A manufacturer has 600 L of 12% solution of acid. The volume of 30% acid solution must be added to it, so that acid content in the resulting mixture will be more than 15% but less than 18%, is between …X… and …Y… . Here, X and Y refer to

18. **IQ** of a person is given by the formula $\mathbf{IQ} = \frac{\mathbf{MA}}{\mathbf{CA}} \times 100$ where, MA is the mental age and CA is chronical age. If $80 \le \mathbf{IQ} \le 140$ for a group of 12 yr old children, find the range of their mental age.

19. Ravi obtained 70 and 75 marks in first two unit tests. Find the maximum marks, he should get in the third test to have an average of atleast 60 marks.

20. To receive grade A in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks.) If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get grade **A** in the course.

21. Find the linear inequalities for which the shaded region in the given figure is the solution set.



22. Solve the inequality $2x + y > 3$ graphically.

23. Solve the inequality $5x + 2y \le 10$ graphically.

24. Draw the graph of the inequality $y + 8 \ge 2x$.

25. Solve $4x - y > 0$ graphically.

• Long Answer Type Questions

26. Solve the following system of inequalities and represent the solution graphically on the number line.

$$5(2x - 7) - 3(2x + 3) \le 0,\ 2x + 19 \le 6x + 47$$

27. Solve the following system of linear inequalities and represent the solution graphically on the number line.

$$2(2x + 3) - 10 < 6(x - 2) \text{ and } \frac{2x - 3}{4} + 6 \ge 2 + \frac{4x}{3}$$

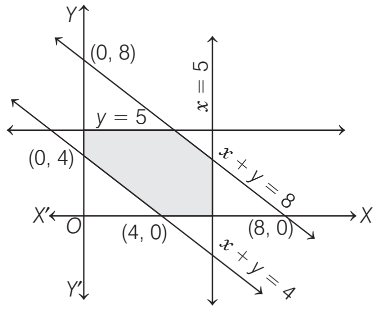
28. A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 L of the 9% solution, how many litres of 3% solution will have to be added?

29. A solution of 8% boric acid is to be diluted by adding 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 L of the 8% solution, how many litres of the 2% solution will have to be added?

30. Solve the inequalities graphically

$$3x + 4y \le 60,\ x + 3y \le 30,\ x \ge 0 \text{ and } y \ge 0.$$

31. Find the linear inequalities for which the shaded region in the given figure is the solution set.



32. Solve the following system of linear inequalities $3x + 2y \ge 24,\ 3x + y \le 15$ and $x \ge 4$.

33. Show that the following system of linear inequalities has no solution.

$$x + 2y \le 3, 3x + 4y \ge 12, x \ge 0 \text{ and } y \ge 1.$$

34. The graphical solution of the inequalities

$$x + 2y \le 10,$$
$$x + y \ge 1,$$
$$x - y \le 0,$$
$$x \ge 0$$
and
$$y \ge 0$$

35. Solve the system of inequalities graphically.

$$x + y \le 5,$$
$$4x + y \ge 4,$$
$$x + 5y \ge 5,$$
$$x \le 4$$
and
$$y \le 3$$

SOLUTIONS

Objective Questions

1. (*b*) The given graph represents all the values greater than $\frac{9}{2}$, including $\frac{9}{2}$ on the real line.

$$x \in \left[\frac{9}{2}, \infty\right)$$

2. (c) We have, $4x + 3 < 6x + 7$

or $\quad 4x - 6x < 6x + 4 - 6x$

or $\quad -2x < 4$ or $x > -2$

i.e. all the real numbers which are greater than -2, are the solutions of the given inequality. Hence, the solution set is $(-2, \infty)$.

3. (*a*) Given that, $-3x + 17 < -13$

$\Rightarrow \quad 3x - 17 > 13$ [multiplying by -1 both sides]

$\Rightarrow \quad 3x > 13 + 17$ [adding 17 both sides]

$\Rightarrow \quad 3x > 30$

$\therefore \quad x > 10$

4. (a) We have, $5x - 3 < 7$

On adding 3 both sides, we get

$$5x - 3 + 3 < 7 + 3$$

$\Rightarrow \quad 5x < 10$

On dividing both sides by 5, we get

$$\frac{5x}{5} < \frac{10}{5}$$

$\Rightarrow \quad x < 2$

$\therefore$ When x is an integer, the solution of the given inequality is $\{\ldots\ldots, -1, 0, 1\}$.

5. (*a*) We have, $3x + 8 > 2$

On adding -8 both sides,

$\Rightarrow \quad 3x > -6$

On dividing by 3 both sides,

$\Rightarrow \quad x > -2$

(i) When x is an integer, the solution of the given inequality is $\{-1, 0, 1, 2 \ldots.\}$.

(ii) When x is a real number, the solution of the given inequality is $(-2, \infty)$. i.e. all the numbers lying between -2 and ∞ but -2 and ∞ are not included.

6. (*b*) Again, we have, $3(1 - x) < 2(x + 4)$

$\Rightarrow \quad 3 - 3x < 2x + 8$

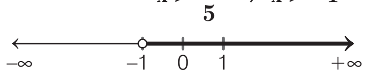
Transferring the term $2x$ to the LHS and the term 3 to RHS,

$$-3x - 2x < 8 - 3$$

$\Rightarrow \quad -5x < 5$

$\Rightarrow \quad \frac{-5x}{-5} > \frac{5}{-5}$

$\Rightarrow \quad x > \frac{-5}{5} \Rightarrow x > -1$



$\therefore$ Solution set is $(-1, \infty)$.

7. (*c*) We have,

$$\frac{5 - 2x}{3} \le \frac{x}{6} - 5$$

or $\quad 2(5 - 2x) \le x - 30$

or $\quad 10 - 4x \le x - 30$

or $\quad -5x \le -40$,

i.e. $\quad x \ge 8$

Thus, all real numbers x which are greater than or equal to 8, are the solutions of the given inequality i.e. $x \in [8, \infty)$.

8. (*c*) We have, $5x - 3 \ge 3x - 5$

Transferring the term $3x$ to LHS and the term (-3) to RHS,

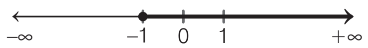
$$5x - 3x \ge -5 + 3$$

$\Rightarrow \quad 2x \ge -2$

$\Rightarrow \quad \frac{2x}{2} \ge -\frac{2}{2}$

$\Rightarrow \quad x \ge \frac{-2}{2}$

$\Rightarrow \quad x \ge -1$



All the numbers on the right side of -1 will be greater than it.

$\therefore$ Solution set is $[-1, \infty)$.

9. (*a*) We have, $\frac{x}{4} < \frac{(5x - 2)}{3} - \frac{(7x - 3)}{5}$

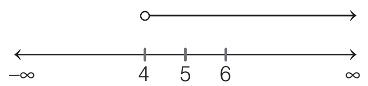
$\Rightarrow \quad 15x < 16x - 4$

Transferring the term $16x$ to LHS.

$$15x - 16x < -4 \Rightarrow -x < -4$$

On multiplying by -1 both sides, we get

$$x > 4$$



$\therefore$ Solution set is $(4, \infty)$.

10. (*a*) The given inequality,

$$-12 < 4 - \frac{3x}{-5} \le 2 \Rightarrow -12 < 4 + \frac{3x}{5} \le 2$$

Adding (-4) to each term,

$$-12 - 4 < 4 + \frac{3x}{5} - 4 \le 2 - 4 \Rightarrow -16 < \frac{3x}{5} \le -2$$

Multiplying by $\frac{5}{3}$ to each term,

$$-16 \times \frac{5}{3} < \frac{3x}{5} \times \frac{5}{3} \le -2 \times \frac{5}{3}$$

$\Rightarrow \quad -\frac{80}{3} < x \le -\frac{10}{3}$

$\therefore$ Solution set is $\left(-\frac{80}{3}, -\frac{10}{3}\right]$ or $\left]-\frac{80}{3}, \frac{-10}{3}\right]$.

11. (*c*) Given, $3 \le 3t - 18 \le 18$

Adding 18 to each term,

$$3 + 18 \le 3t - 18 + 18 \le 18 + 18$$

$$\Rightarrow \quad 21 \le 3t \le 36$$

Dividing by 3 to each term,

$$\frac{21}{3} \le \frac{3t}{3} \le \frac{36}{3}$$

$$\Rightarrow \quad 7 \le t \le 12$$

Adding 1 to each term,

$$7 + 1 \le t + 1 \le 12 + 1$$

$$\Rightarrow \quad 8 \le t + 1 \le 13$$

12. (*a*) The given inequality, $2 \le 3x - 4 \le 5$

$$\Rightarrow \quad 2 + 4 \le 3x \le 5 + 4$$

$$\Rightarrow \quad 6 \le 3x \le 9$$

Dividing by 3 in each term,

$$\frac{6}{3} \le \frac{3x}{3} \le \frac{9}{3}$$

$$\Rightarrow \quad 2 \le x \le 3$$

$\therefore$ Solution set is $[2, 3]$.

13. (*a*) We have,

$6 \le -3(2x - 4) < 12$ or $6 \le -6x + 12 < 12$

On subtracting 12 from each term, we get

$$6 - 12 \le -6x + 12 - 12 < 12 - 12$$

$$\Rightarrow \quad -6 \le -6x < 0$$

On dividing each term by -6, we get

$$\frac{-6}{-6} \ge \frac{-6x}{-6} > \frac{0}{-6}$$

[while dividing each term by the same negative number, then sign of inequalities will get change]

$$\Rightarrow \quad 1 \ge x > 0$$

which can be written as $0 < x \le 1$.

Hence, solution set of given system of inequations is $(0, 1]$.

14. (*b*) Given, $40x + 20y \le 120$, x and y are whole numbers.

To start with, let $x = 0$. Then, LHS of given inequality is

$$40x + 20y = 40(0) + 20y = 20y$$

Thus, we have

$$20y \le 120$$

or $\quad y \le 6$

For $x = 0$, the corresponding values of y can be 0, 1, 2, 3, 4, 5, 6 only. In this case, the solutions of given inequality are (0, 0), (0, 1), (0, 2), (0, 3), (0, 4), (0, 5) and (0, 6).

15. (*c*) Let breadth of rectangle be x cm.

$\therefore$ Length of rectangle $= 3x$

Perimeter of rectangle $= 2$ (Length + Breadth)

$$= 2(x + 3x) = 8x$$

Given, perimeter ≥ 160 cm

$$8x \ge 160$$

Dividing both sides by 8,

$$x \ge 20 \text{ cm}$$

16. (*a*) Let x be the marks obtained by student in the annual examination. Then,

$$\frac{62 + 48 + x}{3} \ge 60$$

or $\quad 110 + x \ge 180$

or $\quad x \ge 70$

Thus, the student must obtain a minimum of 70 marks to get an average of atleast 60 marks.

17. (*c*) Let x be the smaller of the two consecutive odd natural number, so that the other one is $x + 2$. Then, we should have

$$x > 10 \quad \text{...(i)}$$

and $\quad x + (x + 2) < 40 \quad$ ...(ii)

Solving Eq. (ii), we get

$$2x + 2 < 40$$

i.e. $\quad x < 19 \quad$ ...(iii)

From Eqs. (i) and (iii), we get

$$10 < x < 19$$

Since, x is an odd number, x can take the values 11, 13, 15, and 17. So, the required possible pairs will be

(11, 13), (13, 15), (15, 17), (17, 19)

18. (*b*) Given that, $T = 30 + 25(x - 3)$, $3 \le x \le 15$

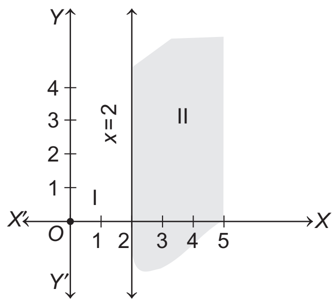
According to the question, $155 < T < 205$

$$\Rightarrow \quad 155 < 30 + 25(x - 3) < 205$$

$$\Rightarrow \quad 8 < x < 10$$

Hence, at the depth 8 to10 km temperature lies between 155°C to 205°C.

19. (*a*) Graph of $3x - 6 = 0$ is given in the figure.



We select a point say (0, 0) and substituting it in given inequality, we see that

$$3(0) - 6 \ge 0$$

or $\quad -6 \ge 0$ which is false.

Thus, the solution region is the shaded region on the right hand side of the line $x = 2$.

Also, all the points on the line $3x - 6 = 0$ will be included in the solution. Hence, a dark line is drawn in the solution region.

20. (*c*) Given inequalities are

$$3x + 4y \ge 12, \quad \text{...(i)}$$

$$y \ge 1 \quad \text{...(ii)}$$

and $\quad x \ge 0 \quad$ ...(iii)

The line corresponding to (i) is

$$3x + 4y = 12 \quad \text{...(iv)}$$

Table for $3x + 4y = 12$

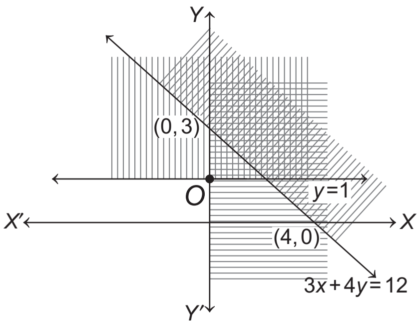
x	4	0
y	0	3

Draw the graph of the line $3x + 4y = 12$.

$\because 3(0) + 4(0) \geq 12$ i.e. $0 \geq 12$, which is not true.

$\therefore$ Inequality (i) represent the half plane made by the line (iv), which does not contain origin.

Inequality $y \geq 1$ represent the half plane made by the line $y = 1$, which does not contain origin. ($\because 0 \geq 1$ is not true)



Inequality $x \geq 0$ represent the right side of the Y-axis.

Hence, the shaded part is common to all the given inequalities.

21. (*d*) The given system of inequalities

$$2x + y \geq 4 \quad \ldots(i)$$
$$x + y \leq 3 \quad \ldots(ii)$$
$$2x - 3y \leq 6 \quad \ldots(iii)$$

Step I Consider the inequations as strict equations i.e.

$$2x + y = 4$$
$$x + y = 3$$
$$2x - 3y = 6$$

Step II Find the points on the **X**-axis and **Y**-axis for,

$2x + y = 4$

x	0	2
y	4	0

$x + y = 3$

x	0	3
y	3	0

and $2x - 3y = 6$

x	0	3
y	−2	0

Step III Plot the graph using the above tables.

Step IV Take a point (0, 0) and put it in the inequations (i), (ii) and (iii), we get

$$0 + 0 \geq 4 \quad \text{(false)}$$

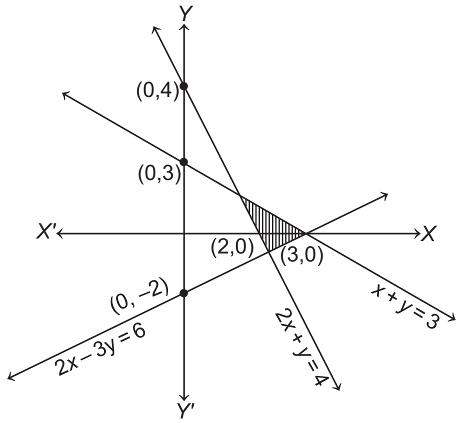
So, the shaded region will be away from origin.

$$0 + 0 \leq 3 \quad \text{(true)}$$

So, the shaded region will be towards origin.

$$0 - 0 \leq 6 \quad \text{(true)}$$

So, the shaded region will be towards origin.



Thus, common shaded region shows the solution of the inequalities.

22. (i) (a) We have, $24x < 100$

On dividing both sides by 24, we get

$$= \frac{24x}{24} < \frac{100}{24} \Rightarrow x < \frac{25}{6}$$

When x is a natural number, then solutions of the inequality $x < \frac{25}{6}$, are all natural numbers, which are less than $\frac{25}{6}$. In this case, the following values of x make the statement true.

$$x = 1, 2, 3, 4$$

Hence, the solution set of inequality is $\{1, 2, 3, 4\}$.

(ii) (a) We have, $24x < 100$

On dividing both sides by 24, we get

$$= \frac{24x}{24} < \frac{100}{24} \Rightarrow x < \frac{25}{6}$$

When x is an integer.

In this case, solutions of given inequality are

$$\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4$$

Hence, the solution set of inequality is

$$\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

(iii) (d) We have, $-5x + 25 > 0$

On adding $5x$ both sides, we get

$$-5x + 25 + 5x > 0 + 5x \Rightarrow 25 > 5x \Rightarrow 5x < 25$$

On dividing both sides by 5, we get

$$\Rightarrow \frac{5x}{5} < \frac{25}{5} \Rightarrow x < 5$$

Hence, the required solution set is $(-\infty, 5)$.

(iv) (c) We have, $3x - 5 < x + 7$

$\Rightarrow 3x - 5 + 5 < x + 7 + 5$ [adding 5 both sides]

$\Rightarrow 3x < x + 12$

$\Rightarrow 3x - x < x + 12 - x$ [subtracting x from both sides]

$\Rightarrow 2x < 12$

$\Rightarrow \frac{2x}{2} < \frac{12}{2}$ [dividing both sides by 2]

$\Rightarrow x < 6$

The solution set is $\{x : x \in R$ and $x < 6\}$ i.e. any real number less than 6. This can also be written as $(-\infty, 6)$.

(v) (b) We have, $x + \frac{x}{2} + \frac{x}{3} < 11$

$\Rightarrow \quad \frac{6x + 3x + 2x}{6} < 11 \Rightarrow \frac{11x}{6} < 11$

On multiplying both sides by $\frac{6}{11}$, we get

$$\frac{11x}{6} \times \frac{6}{11} < 11 \times \frac{6}{11} \Rightarrow x < 6$$

$\therefore \quad x \in (-\infty, 6)$

23. (i) (b) We know that, Profit = Revenue − Cost

$\therefore$ In order to realise some profit, revenue should be greater than the cost.

Thus, we should have $R(x) > C(x)$

$\Rightarrow \quad 60x + 2000 > 20x + 4000$

$\Rightarrow \quad 60x + 2000 - 20x > 20x + 4000 - 20x$

[subtracting $20x$ from both sides]

$\Rightarrow \quad 40x + 2000 > 4000$

$\Rightarrow \quad 40x + 2000 - 2000 > 4000 - 2000$

[subtracting 2000 from both sides]

$\Rightarrow \quad 40x > 2000$

$\Rightarrow \quad \frac{40x}{40} > \frac{2000}{40}$

[dividing both sides by 40]

$\Rightarrow \quad x > 50$

Hence, the manufacturer must sell more than 50 items to realise some profit.

(ii) (d) In order to realise some profit, we should have

$$R(x) > C(x)$$

$\Rightarrow \quad 5x + 20 > 3x + 400$

$\Rightarrow \quad 5x + 20 - 3x > 3x + 400 - 3x$

$\Rightarrow \quad 2x + 20 > 400$

$\Rightarrow \quad 2x + 20 - 20 > 400 - 20$

$\Rightarrow \quad 2x > 380$

$\Rightarrow \quad x > 190$

(iii) (d) We have, $b > 0$

and $\quad x < b$

Its mean x is always less than some positive quantity.

$\therefore$ x may be a real number.

(iv) (a) We have, $\quad 3x - 5 < x + 7$

$\Rightarrow \quad 3x - 5 + 5 < x + 7 + 5$ [adding 5 both sides]

$\Rightarrow \quad 3x < x + 12$

$\Rightarrow \quad 3x - x < x + 12 - x$

[subtracting x from both sides]

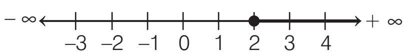
$\Rightarrow \quad 2x < 12$

$\Rightarrow \quad \frac{2x}{2} < \frac{12}{2}$ [dividing both sides by 2]

$\Rightarrow \quad x < 6$

Now, if x is a whole number, then the solution set $\{0, 1, 2, 3, 4, 5\}$.

(v) (b) Graph of $x \geq 2$ on the number line is represented by



Subjective Questions

1. We have, $3x - 5 < x + 7$

$\Rightarrow \quad 3x - 5 + 5 < x + 7 + 5$ [adding 5 both sides]

$\Rightarrow \quad 3x < x + 12$

$\Rightarrow \quad 3x - x < x + 12 - x$ [subtracting x from both sides]

$\Rightarrow \quad 2x < 12$

$\Rightarrow \quad \frac{2x}{2} < \frac{12}{2}$ [dividing both sides by 2]

$\Rightarrow \quad x < 6$

Now, if x is a natural number, then the solution set is $\{1, 2, 3, 4, 5\}$.

2. We have, $\quad \frac{5 - 3x}{3} \leq \frac{x}{6} - 5$

$\Rightarrow \quad \frac{5 - 3x}{3} \leq \frac{x - 30}{6}$

$\Rightarrow \quad \frac{5 - 3x}{3} \times 6 \leq \frac{x - 30}{6} \times 6$

[multiplying both sides by 6]

$\Rightarrow \quad 2(5 - 3x) \leq x - 30$

$\Rightarrow \quad 10 - 6x \leq x - 30$

$\Rightarrow \quad 10 - 6x - 10 \leq x - 30 - 10$

[subtracting 10 from both sides]

$\Rightarrow \quad -6x \leq x - 40$

$\Rightarrow \quad -6x - x \leq x - 40 - x$

[subtracting x from both sides]

$\Rightarrow \quad -7x \leq -40$

$\Rightarrow \quad 7x \geq 40$ [multiplying by -1 both sides]

$\Rightarrow \quad \frac{7x}{7} \geq \frac{40}{7}$ [dividing by 7 both sides]

$\Rightarrow \quad x \geq \frac{40}{7}$

i.e. $\quad x \in \left[\frac{40}{7}, \infty\right)$

Hence, the required solution set is $\left[\frac{40}{7}, \infty\right)$.

3. We have, $-3 \leq 4 - \frac{7x}{2} \leq 18$

On subtracting 4 from each term, we get

$$-3 - 4 \leq 4 - \frac{7x}{2} - 4 \leq 18 - 4 \Rightarrow -7 \leq -\frac{7x}{2} \leq 14$$

On multiplying each term by $\left(\frac{-2}{7}\right)$, we get

$$-7\left(-\frac{2}{7}\right) \geq \frac{-7}{2}x \times \left(\frac{-2}{7}\right) \geq 14 \times \left(\frac{-2}{7}\right)$$

[while multiplying each term by the same negative number, then the sign of inequalities will get change]

$\Rightarrow \quad 2 \geq x \geq -4$ or $-4 \leq x \leq 2$ or $x \in [-4, 2]$

Hence, solution set of given system of inequations is $[-4, 2]$.

4. We have, $\frac{1}{2}\left(\frac{3}{5}x + 4\right) \geq \frac{1}{3}(x - 6)$

$\Rightarrow \quad \frac{1}{2}\left(\frac{3x + 20}{5}\right) \geq \frac{1}{3}(x - 6)$

$\Rightarrow \quad \frac{1}{10}(3x+20) \geq \frac{1}{3}(x-6)$

$\Rightarrow \quad \frac{30}{10}(3x+20) \geq \frac{30}{3}(x-6)$

[multiplying both sides by LCM (10, 3) = 30]

$\Rightarrow \quad 3(3x+20) \geq 10(x-6)$

$\Rightarrow \quad 9x+60 \geq 10x-60$

$\Rightarrow \quad 9x+60-60 \geq 10x-60-60$

[subtracting 60 from both sides]

$\Rightarrow \quad 9x \geq 10x-120$

$\Rightarrow \quad 9x-10x \geq 10x-120-10x$

[subtracting $10x$ from both sides]

$\Rightarrow \quad -x \geq -120 \Rightarrow x \leq 120$

[multiplying both sides by -1]

$\therefore \quad x \in (-\infty, 120]$

5. We have, $\frac{2x-1}{3} \leq \frac{3x-2}{4} - \frac{2-x}{5}$

On multiplying both sides by LCM of 3, 4 and 5 i.e. 60, we get

$$\frac{2x-1}{3} \times 60 \leq \frac{3x-2}{4} \times 60 - \frac{2-x}{5} \times 60$$

$\Rightarrow \quad 20(2x-1) \leq 15(3x-2) - 12(2-x)$

$\Rightarrow \quad 40x-20 \leq 45x-30-24+12x$

$\Rightarrow \quad 40x-20 \leq 57x-54$

$\Rightarrow \quad 40x-20+20 \leq 57x-54+20$ [adding 20 both sides]

$\Rightarrow \quad 40x \leq 57x-34$

$\Rightarrow \quad 40x-57x \leq 57x-34-57x$

[subtracting $57x$ from both sides]

$\Rightarrow \quad -17x \leq -34$

$\Rightarrow \quad \frac{-17x}{17} \leq \frac{-34}{17}$ [dividing both sides by 17]

$\Rightarrow \quad -x \leq -2$

$\Rightarrow \quad x \geq 2$ [multiplying both sides by -1]

Thus, the solution set is $[2, \infty)$.

6. We have,

$$-15 < \frac{3(x-2)}{5} \leq 0$$

$\Rightarrow \quad -15 \times 5 < \frac{3(x-2)}{5} \times 5 \leq 0 \times 5$

[multiplying each term by 5]

$\Rightarrow \quad -75 < 3(x-2) \leq 0$

$\Rightarrow \quad \frac{-75}{3} < \frac{3(x-2)}{3} \leq \frac{0}{3}$ [dividing each term by 3]

$\Rightarrow \quad -25 < x-2 \leq 0$

$\Rightarrow \quad -25+2 < x-2+2 \leq 0+2$

[adding 2 each term]

$\Rightarrow \quad -23 < x \leq 2$

$\therefore \quad x \in (-23, 2]$

Hence, solution set of the given system of inequations is $(-23, 2]$.

7. We have the following inequalities,

$2x-3 < 7$ ...(i)

and $2x > -4$ ...(ii)

From inequality (i), we have

$$2x-3 < 7$$

$\Rightarrow \quad 2x-3+3 < 7+3$ [adding 3 both sides]

$\Rightarrow \quad 2x < 10$

$\Rightarrow \quad \frac{2x}{2} < \frac{10}{2}$ [dividing both sides by 2]

$\Rightarrow \quad x < 5$

$\therefore$ The solution set is $(-\infty, 5)$.

From inequality (ii), we have

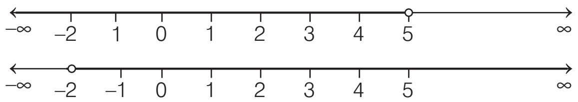
$$2x > -4$$

$\frac{2x}{2} > \frac{-4}{2}$ [dividing both sides by 2]

$\Rightarrow \quad x > -2$

$\therefore$ The solution set is $(-2, \infty)$.

Now, let us draw the graphs of the solutions of both inequalities on number line.



It can be seen that the values of x, which are common to both are lying in the interval $(-2, 5)$.

Hence, the solution set of given system of inequations is $(-2, 5)$ and this can be represented graphically on the number line as

8. Let the three consecutive integers be x, $(x+1)$ and $(x+2)$. Since, sum of these integers must not be more than 12, i.e. $x+(x+1)+(x+2)$ cannot exceed 12, therefore, we have

$\Rightarrow \quad x+(x+1)+(x+2) \leq 12$

$\Rightarrow \quad 3x+3 \leq 12$

$\Rightarrow \quad 3x+3-3 \leq 12-3$

[subtracting 3 from both sides]

$\Rightarrow \quad 3x \leq 9$

$\therefore \quad x \leq 3$ [dividing both sides by 3]

9. Cost function, $C(x) = 26000 + 30x$

and revenue function, $R(x) = 43x$

For profit, $R(x) > C(x)$

$\Rightarrow \quad 26000 + 30x < 43x$

$\Rightarrow \quad 30x - 43x < -26000$

$\Rightarrow \quad -13x < -26000$

$\Rightarrow \quad 13x > 26000$

$\Rightarrow \quad x > \frac{26000}{13}$

$\therefore \quad x > 2000$

Hence, more than 2000 cassettes must be produced to get profit.

10. Given, first pH value = 8.48

and second pH value = 8.35

Let third pH value be x.

Since, it is given that average pH value lies between 8.2 and 8.5.

$$\therefore \quad 8.2 < \frac{8.48 + 8.35 + x}{3} < 8.5$$

$$\Rightarrow \quad 8.2 < \frac{16.83 + x}{3} < 8.5$$

$$\Rightarrow \quad 3 \times 8.2 < 16.83 + x < 8.5 \times 3$$

$$\Rightarrow \quad 24.6 < 16.83 + x < 25.5$$

$$\Rightarrow \quad 24.6 - 16.83 < x < 25.5 - 16.83$$

$$\Rightarrow \quad 7.77 < x < 8.67$$

Thus, third pH value lies between 7.77 and 8.67.

11. Let x be the score obtained by Mohan in the last examination.

Then, $80 \leq \dfrac{94 + 73 + 72 + 84 + x}{5} < 90$

$$\Rightarrow \quad 80 \leq \frac{323 + x}{5} < 90$$

$$\Rightarrow \quad 400 \leq 323 + x < 450$$

[multiplying both sides by 5]

$$\Rightarrow \quad 400 - 323 \leq 323 + x - 323 < 450 - 323$$

[subtracting 323 from each term]

$$\Rightarrow \quad 77 \leq x < 127$$

Since, the upper limit is 100, therefore the required range is $77 \leq x \leq 100$.

12. Let x be the smaller of the two positive even integers, so that the other one is $x + 2$, then we should have

$$x > 5 \quad \ldots(i)$$

$$x + 2 > 5 \quad \ldots(ii)$$

and their sum, $x + (x + 2) < 23$ ...(iii)

From inequalities (i) and (ii), we get

$$x > 3 \quad \ldots(iv)$$

From inequality (iii), we get

$$2x + 2 < 23$$

$$\Rightarrow \quad 2x + 2 - 2 < 23 - 2$$

[subtracting 2 from both sides]

$$\Rightarrow \quad 2x < 21$$

$$\Rightarrow \quad x < 10.5 \text{ [dividing both sides by 2]} \quad \ldots(v)$$

Now, from inequalities (iv) and (v), we get

$$3 < x < 10.5$$

Since, x is an even number. So, x can take the values 4, 6, 8 and 10.

Hence, the required possible pairs will be $(4, 6)$ $(6, 8)$, $(8, 10)$ and $(10, 12)$.

13. It is given that, $68 < F < 77$

On putting $F = \dfrac{9}{5}C + 32$, we get

$$68 < \frac{9}{5}C + 32 < 77$$

$$\Rightarrow \quad 68 - 32 < \frac{9}{5}C < 77 - 32$$

[subtracting 32 from each term]

$$\Rightarrow \quad 36 < \frac{9}{5}C < 45$$

$$\Rightarrow \quad 36 \times \frac{5}{9} < C < 45 \times \frac{5}{9} \quad \left[\text{multiplying } \frac{5}{9} \text{ in each term}\right]$$

$$\Rightarrow \quad 4 \times 5 < C < 5 \times 5$$

$$\Rightarrow \quad 20 < C < 25$$

Hence, the required range of temperature is between 20°C and 25°C.

14. Let the shortest side be x cm. Then, according to the given condition,

Longest side = $3x$ cm and third side = $(3x - 2)$ cm

Now, perimeter of triangle ≥ 61 i.e. sum of all sides ≥ 61

$$\Rightarrow \quad x + 3x + 3x - 2 \geq 61$$

$$\Rightarrow \quad 2 + 7x - 2 \geq 61 + 2 \quad \text{(using rule 1)}$$

$$\Rightarrow \quad 7x \geq 63$$

$$\Rightarrow \quad \frac{7x}{7} \geq \frac{63}{7} \quad \text{(using rule 2)}$$

$$\Rightarrow \quad x \geq 9$$

$\therefore$ Minimum length of the shortest side = 9 cm

15. Let the length of shortest side be x cm.

According to the given information,

Longest side = $2 \times$ Shortest side

$= 2x$ cm

and third side = $2 +$ Shortest side

$= (2 + x)$ cm

Perimeter of triangle $= x + 2x + (x + 2) = 4x + 2$

According to the question,

Perimeter > 166 cm

$$\Rightarrow \quad 4x + 2 > 166$$

$$\Rightarrow \quad 4x > 166 - 2$$

$$\Rightarrow \quad 4x > 164$$

$$\therefore \quad x > \frac{164}{4} = 41 \text{ cm}$$

Hence, the minimum length of shortest side be 41cm.

16. Let the length of the shortest piece be x cm, so that the lengths of second and third piece are $(x + 3)$ cm and $2x$ cm, respectively.

Then, $\quad x + (x + 3) + 2x \leq 91$ ...(i)

and $\quad 2x \geq (x + 3) + 5$ ...(ii)

From inequality (i), we get

$$4x + 3 \leq 91$$

$$\Rightarrow \quad 4x + 3 - 3 \leq 91 - 3$$

[subtracting 3 from both sides]

$$\Rightarrow \quad 4x \leq 88$$

$$\Rightarrow \quad \frac{4x}{4} \leq \frac{88}{4} \quad \text{[dividing both sides by 4]}$$

$$\Rightarrow \quad x \leq 22 \quad \ldots(iii)$$

From inequality (ii), we get

$$2x \geq x + 8$$

$\Rightarrow$ $2x - x \geq x + 8 - x$ [subtracting **x** from both sides]

$\Rightarrow$ $x \geq 8$ ...(iv)

From inequalities (iii) and (iv), we get

$$8 \leq x \leq 22$$

Hence, the shortest piece must be atleast 8 cm long but not more than 22 cm long.

17. Let **x** L of 30% acid solution is required to be added. Then,

Total mixture = $(x + 600)$ L

Therefore, $30\%\, x + 12\%$ **of** $600 > 15\%$ **of** $(x + 600)$

and $30\%\, x + 12\%$ **of** $600 < 18\%$ **of** $(x + 600)$

or $\frac{30x}{100} + \frac{12}{100}(600) > \frac{15}{100}(x + 600)$

and $\frac{30x}{100} + \frac{12}{100}(600) < \frac{18}{100}(x + 600)$

or $30x + 7200 > 15x + 9000$

and $30x + 7200 < 18x + 10800$

or $15x > 1800$ and $12x < 3600$

or $x > 120$ and $x < 300$

i.e. $120 < x < 300$

$\therefore$ $X = 120$ and $Y = 300$

18. Given that, $80 \leq IQ \leq 140$ ...(i)

Putting $IQ = \frac{MA}{CA} \times 100$ in Eq. (i)

$\Rightarrow$ $80 \leq \frac{MA}{CA} \times 100 \leq 140$

$\Rightarrow$ $80 \leq \frac{MA}{12} \times 100 \leq 140$ ($\because$ given that, CA = 12 yr)

Multiplying by 12 in each term,

$\Rightarrow$ $80 \times 12 \leq \frac{MA}{12} \times 100 \times 12 \leq 140 \times 12$

$\Rightarrow$ $960 \leq MA \times 100 \leq 1680$

Dividing by 100 in each term,

$\Rightarrow$ $\frac{960}{100} \leq \frac{MA \times 100}{100} \leq \frac{1680}{100}$

$\Rightarrow$ $9.6 \leq MA \leq 16.8 \Rightarrow MA \in [9.6, 16.8]$

Hence, mental age should be greater than equal to 9.6 but less than equal to 16.8.

19. Let Ravi got **x** marks in third unit test.

$\therefore$ Average marks obtained by Ravi

$$= \frac{\textbf{Sum of marks in all tests}}{\textbf{Number of tests}}$$

$$= \frac{70 + 75 + x}{3} = \frac{145 + x}{3}$$

Now, it is given that he wants to obtain an average of atleast 60 marks.

Atleast 60 marks means that the marks should be greater than or equal to 60.

i.e. $\frac{145 + x}{3} \geq 60$

$\Rightarrow$ $145 + x \geq 60 \times 3$

$\Rightarrow$ $145 + x \geq 180$

Now, transferring the term **145** to RHS,

$\Rightarrow$ $x \geq 180 - 145 \Rightarrow x \geq 35$

i.e. Ravi should get greater than or equal to 35 marks in third unit test to get an average of atleast 60 marks.

20. Let Sunita got **x** marks in the fifth exam.

$\therefore$ Average marks obtained by Sunita

$$= \frac{\textbf{Sum of marks in all exams}}{\textbf{Number of exams}}$$

$$= \frac{87 + 92 + 94 + 95 + x}{5}$$

$$= \frac{368 + x}{5}$$

Now, it is given that Sunita wants to obtain grade **A** for that her average marks should be greater than or equal to 90.

i.e. $\frac{368 + x}{5} \geq 90$

$\Rightarrow$ $368 + x \geq 450$

Transferring the term 368 to RHS,

$\Rightarrow$ $x \geq 450 - 368$

$\Rightarrow$ $x \geq 82$

i.e. Sunita should got greater than or equal to **82** marks in fifth exam to get grade *A*.

21. Consider the line $3x + 2y = 48$, we observe that the shaded region and the origin are on the same side of the line.

$3x + 2y = 48$ and (0, 0) satisfy the linear constraint $3x + 2y \leq 48$. So, we must have one inequation as $3x + 2y \leq 48$.

Now, consider the line $x + y = 20$. We find that the shaded region and the origin are on the same side of the line $x + y = 20$ and (0, 0) satisfy the constraints $x + y \leq 20$. So, the second inequation is $x + y \leq 20$.

We also notice that the shaded region is above *X*-axis and is on the right side of *Y*-axis, so we must have $x \geq 0$, $y \geq 0$.

Thus, the linear inequations corresponding to the given solution set are $3x + 2y \leq 48$, $x + y \leq 20$ and $x \geq 0$, $y \geq 0$.

22. We have, $2x + y > 3$

In equation form, given inequality can be written as

$$2x + y = 3 \quad ...(i)$$

On putting $x = 0$ in Eq. (i), we get

$$2(0) + y = 3 \Rightarrow y = 3$$

$\therefore$ Line meets **Y**-axis at point *A* (0, 3).

On putting $y = 0$ in Eq. (i), we get

$$2x + 0 = 3 \Rightarrow x = \frac{3}{2}$$

$\therefore$ Line meets **X**-axis at point $B\left(\frac{3}{2}, 0\right)$.

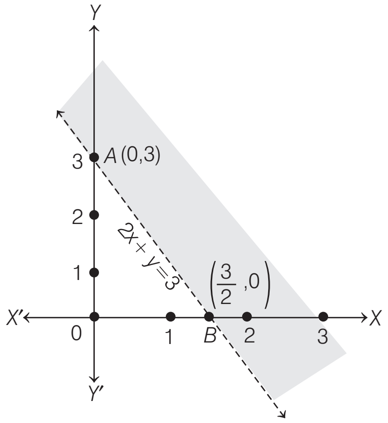
On joining the points **A**(0, 3) and $\mathbf{B}\left(\frac{3}{2}, 0\right)$ by dotted line, we get the line **AB.** [$\because$ given inequality has sign >, so we join the points by a dotted line]

Now, take a point not lying on the line say (0, 0) to check whether it satisfies the given inequality or not.

On putting $\mathbf{x = 0}$ and $\mathbf{y = 0}$ in given inequality, we get

$$\mathbf{2(0) + 0 > 3 \Rightarrow 0 > 3} \text{ which is not correct.}$$

So, we shade the portion which does not contain (0, 0) i.e. the region above the dotted line **AB**.



Here, the shaded region is the required solution region.

23. Given, inequality is $\mathbf{5x + 2y \le 10}$.

Corresponding equation of line is

$$\mathbf{5x + 2y = 10}$$

On putting $\mathbf{x = 0}$, we get

$$\mathbf{5\,(0) + 2y = 10 \Rightarrow y = 5}$$

Thus, the line intersect the **Y**-axis at point **(0, 5)**.

On putting $\mathbf{y = 0}$, we get

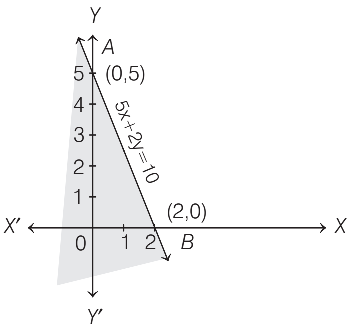
$$\mathbf{5x + 2\,(0) = 10 \Rightarrow x = 2}$$

x	0	2
y	5	0

Thus, the line intersect the **X**-axis at point **(2, 0)**.

Join the points **A (0, 5)** and **B (2, 0)** with a dark line.

[∵ given inequality has sign '≤']



On putting (0, 0) in given inequality, we get

$$\mathbf{5 \times 0 + 2 \times 0 \le 10 \Rightarrow 0 \le 10}, \text{ which is true.}$$

So, the half plane of $\mathbf{5x + 2y \le 10}$ contains origin.

Now, shade the half plane which contain **(0, 0)**.

Here, the shaded region including the points on the line represent the solution region of given inequality.

24. Given, inequality is $\mathbf{y + 8 \ge 2x}$.

In equation form, it can be written as

$$\mathbf{y + 8 = 2x} \text{ or } \mathbf{2x - y = 8}$$

On putting $\mathbf{y = 0}$, we get

$$\mathbf{2x - 0 = 8 \Rightarrow x = 4}$$

∴ Line $\mathbf{2x - y = 8}$ cuts the **X**-axis at *A* (4, 0).

On putting $\mathbf{x = 0}$, we get

$$\mathbf{2 \times 0 - y = 8 \Rightarrow y = -8}$$

∴ Line $\mathbf{2x - y = 8}$ cuts the **Y**-axis at *B*(0, −8).

On joining the points **A (4, 0)** and **B(0, − 8)** by a dark line, we get the line **AB**.

[∵ given inequality has sign ≥, so we draw dark line]

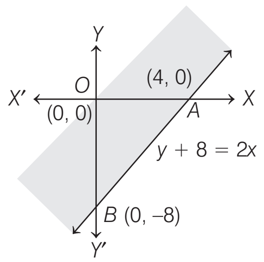
Let us check, whether *O* (0, 0) satisfy the inequality or not and shade the suitable region.

On putting $\mathbf{x = y = 0}$ in $\mathbf{y + 8 \ge 2x}$, we get

$$\mathbf{0 + 8 \ge 2 \cdot 0}$$

$$\Rightarrow \quad \mathbf{8 \ge 0}, \text{ which is true.}$$

So, shade the region containing **(0, 0)** i.e. the region above the line **AB**.



Here, the shaded region including the points on the line represent the graph of the inequality.

25. We have, $\mathbf{4x - y > 0}$

Its equation form is $\mathbf{4x - y = 0}$. ...(i)

On putting $\mathbf{y = 0}$ in Eq. (i), we get

$$\mathbf{4x - 0 = 0 \quad \Rightarrow \quad x = 0}$$

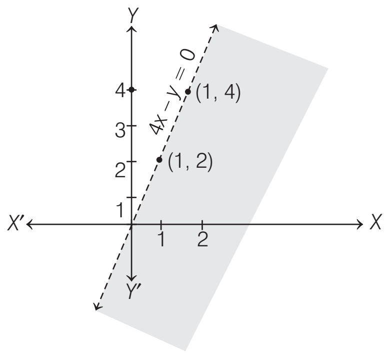
On putting $\mathbf{x = 1}$ in Eq. (i), we get

$$\mathbf{4(1) - y = 0 \quad \Rightarrow \quad y = 4}$$

x	1	0
y	4	0

Thus, the line $\mathbf{4x - y = 0}$ passes through the points **A (0, 0)** and **B (1, 4)**

Since, the inequality is of the form <, so we join the points **A(0, 0)** and **B(1, 4)** by a dotted line.



Now, take the point, say (1, 2) not lying on the line, to check whether it satisfies the given linear inequality or not.

At $(1, 2)$, $4(1) - 2 > 0 \Rightarrow 2 > 0$, which is true.

$\therefore$ (1, 2) satisfies the given inequality, so we shade the region which contains (1, 2). Thus, shaded region (excluding all points on the line) gives the solution region.

26. We have the following inequalities

$$5(2x - 7) - 3(2x + 3) \le 0 \quad \ldots(i)$$

and $$2x + 19 \le 6x + 47 \quad \ldots(ii)$$

From inequality (i), we get

$$5(2x - 7) - 3(2x + 3) \le 0$$

$\Rightarrow$ $10x - 35 - 6x - 9 \le 0 \Rightarrow 4x - 44 \le 0$

$\Rightarrow$ $4x - 44 + 44 \le 44$ [adding 44 both sides]

$\Rightarrow$ $4x \le 44$

$\Rightarrow$ $x \le 11$ ...(iii)

[dividing both sides by 4]

$\therefore$ The solution set is $(-\infty, 11]$.

From inequality (ii), we get

$$2x + 19 \le 6x + 47$$

$\Rightarrow$ $2x + 19 - 2x \le 6x + 47 - 2x$

[subtracting $2x$ from both sides]

$\Rightarrow$ $19 \le 4x + 47$

$\Rightarrow$ $19 - 47 \le 4x + 47 - 47$

[subtracting 47 from both sides]

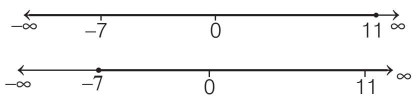
$\Rightarrow$ $-28 \le 4x$ or $4x \ge -28$

$\Rightarrow$ $\frac{4x}{4} \ge \frac{-28}{4}$ [dividing both sides by 4]

$\Rightarrow$ $x \ge -7$

$\therefore$ The solution set is $[-7, \infty)$.

Now, let us draw the graphs of the solutions of both inequalities on number line.



It can be seen that the values of x, which are common to both are lying in the interval $[-7, 11]$.

Hence, the solution set of given system of inequations is $[-7, 11]$ and this can be represented graphically on the number line as

27. Given, system of linear inequalities is

$$2(2x + 3) - 10 < 6(x - 2) \quad \ldots(i)$$

and $$\frac{2x - 3}{4} + 6 \ge 2 + \frac{4x}{3} \quad \ldots(ii)$$

From inequality (i), we get

$$2(2x + 3) - 10 < 6(x - 2)$$

$\Rightarrow$ $4x + 6 - 10 < 6x - 12$

$\Rightarrow$ $4x - 4 < 6x - 12$

$\Rightarrow$ $4x - 4 + 4 < 6x - 12 + 4$ [adding 4 both sides]

$\Rightarrow$ $4x < 6x - 8$

$\Rightarrow$ $4x - 6x < 6x - 8 - 6x$

[subtracting $6x$ from both sides]

$\Rightarrow$ $-2x < -8$

$\Rightarrow$ $2x > 8$ [dividing both sides by -1, then inequality sign will also change]

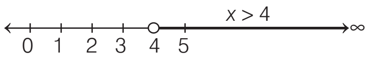
$\Rightarrow$ $\frac{2x}{2} > \frac{8}{2}$ [dividing both sides by 2]

$\therefore$ $x > 4$ ...(iii)

Thus, any value of x greater than 4 satisfies the inequality.

$\therefore$ The solution set is $(4, \infty)$.

The representation of solution of inequality (i) is



From inequality (ii), we get

$$\frac{2x - 3}{4} + 6 \ge 2 + \frac{4x}{3}$$

$\Rightarrow$ $\frac{2x - 3 + 24}{4} \ge \frac{6 + 4x}{3}$

$\Rightarrow$ $\frac{2x + 21}{4} \ge \frac{6 + 4x}{3}$

$\Rightarrow$ $3(2x + 21) \ge 4(6 + 4x)$

[multiplying both sides by LCM $(4, 3) = 12$]

$\Rightarrow$ $6x + 63 \ge 24 + 16x$

$\Rightarrow$ $6x + 63 - 63 \ge 24 + 16x - 63$

[subtracting 63 from both sides]

$\Rightarrow$ $6x \ge 16x - 39$

$\Rightarrow$ $6x - 16x \ge 16x - 39 - 16x$

[subtracting $16x$ from both sides]

$\Rightarrow$ $-10x \ge -39$

$\Rightarrow$ $10x \le 39$

[multiplying both sides by -1, the inequality sign will also change]

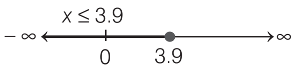
$\Rightarrow$ $\frac{10x}{10} \le \frac{39}{10}$ [dividing both sides by 10]

$\Rightarrow$ $x \le 3.9$ ...(iv)

Thus, any value of x less than or equal to 3.9 satisfies the inequality.

$\therefore$ The solution set is $(-\infty, 3.9]$.

Its representation on number line is,



We see that there is no common value of x, which satisfies both inequalites (iii) and (iv).

Hence, the given system of inequalities has no solution set.

28. Let x L of 3% solution be added to 460 L of 9% solution of acid.

Then, total quantity of mixture $= (460 + x)$ L

Total acid content in the $(460 + x)$ L of mixture

$$= \left(460 \times \frac{9}{100} + x \times \frac{3}{100}\right)$$

It is given that acid content in the resulting mixture must be more than 5% but less than 7% acid.

Therefore, $5\% \text{ of } (460 + x) < 460 \times \frac{9}{100} + \frac{3x}{100} < 7\% \text{ of } (460 + x)$

$\Rightarrow \frac{5}{100} \times (460 + x) < 460 \times \frac{9}{100} + \frac{3}{100} x < \frac{7}{100} \times (460 + x)$

$\Rightarrow \quad 5 \times (460 + x) < 460 \times 9 + 3x < 7 \times (460 + x)$ [multiplying by 100]

$\Rightarrow \quad 2300 + 5x < 4140 + 3x < 3220 + 7x$

Taking first two inequalities, $2300 + 5x < 4140 + 3x$

$\Rightarrow \quad 5x - 3x < 4140 - 2300$

$\Rightarrow \quad 2x < 1840$

$\Rightarrow \quad x < \frac{1840}{2}$

$\Rightarrow \quad x < 920$ ...(i)

Taking last two inequalities,

$4140 + 3x < 3220 + 7x$

$\Rightarrow \quad 3x - 7x < 3220 - 4140$

$\Rightarrow \quad -4x < -920$

$\Rightarrow \quad 4x > 920$

$\Rightarrow \quad x > \frac{920}{4}$

$\Rightarrow \quad x > 230$ ...(ii)

Hence, the number of litres of the 3% solution of acid must be more than 230 L and less than 920 L.

29. Let x be the number of litres of 2% boric acid solution. Then, total mixture $= (640 + x)$ litres

According to the question,

$2\% \text{ of } x + 8\% \text{ of } 640 > 4\% \text{ of } (640 + x)$

$\Rightarrow \frac{2x}{100} + \frac{8}{100} \times 640 > \frac{4}{100} (640 + x)$

$\Rightarrow \quad 2x + 5120 > 2560 + 4x$ [multiplying both sides by 100]

$\Rightarrow \quad 2x + 5120 - 2x > 2560 + 4x - 2x$ [subtracting $2x$ from both sides]

$\Rightarrow \quad 5120 > 2560 + 2x$

$\Rightarrow \quad 5120 - 2560 > 2560 + 2x - 2560$ [subtracting 2560 from both sides]

$\Rightarrow \quad 2560 > 2x \Rightarrow 2x < 2560$

$\Rightarrow \quad x < 1280$ [dividing both sides by 2] ...(i)

Also, $2\% \text{ of } x + 8\% \text{ of } 640 < 6\% \text{ of } (640 + x)$

$\Rightarrow \frac{2}{100} \times x + \frac{8}{100} \times 640 < \frac{6}{100} \times (640 + x)$

$\Rightarrow \quad 2x + 5120 < 3840 + 6x$ [multiplying both sides by 100]

$\Rightarrow \quad 2x + 5120 - 2x < 3840 + 6x - 2x$ [subtracting $2x$ from both sides]

$\Rightarrow \quad 5120 < 3840 + 4x$

$\Rightarrow \quad 5120 - 3840 < 3840 + 4x - 3840$ [subtracting 3840 from both sides]

$\Rightarrow \quad 1280 < 4x \text{ or } 4x > 1280$

$\Rightarrow \quad \frac{4x}{4} > \frac{1280}{4}$ [dividing both sides by 4]

$\Rightarrow \quad x > 320$ ...(ii)

Now, from inequalities (i) and (ii), we get

$320 < x < 1280$

Thus, the number of litres of the 2% boric acid solution will have to be greater than 320 L and less than 1280 L.

30. We have the following inequalities

$3x + 4y \le 60$ ...(i)

$x + 3y \le 30$ ...(ii)

$x \ge 0$ ...(iii)

and $y \ge 0$ ...(iv)

Take inequality (i),

$3x + 4y \le 60$

In equation form, it can be written as $3x + 4y = 60$

Now, let us construct the following table

x	0	20
y	15	0

Thus, the line intersect **X**-axis at **A(20, 0)** and **Y**-axis at **B(0, 15)**. Now, joining the points **A** and **B** by a dark line, we get the line **AB**.

On putting (0, 0) in the given inequality, we get

$0 + 0 \le 60 \Rightarrow 0 \le 60$, which is true.

So, for $3x + 4y \le 60$, shade the half plane which contain (0, 0).

Take inequality (ii), $x + 3y \le 30$

In equation form, it can be written as $x + 3y = 30$

Now, let us construct the following table

x	0	30
y	10	0

Thus, the line intersect **X**-axis at **C(30, 0)** and **Y**-axis at **D(0, 10)**.

Now, joining the points **C** and **D** by a dark line, we get the line **CD**.

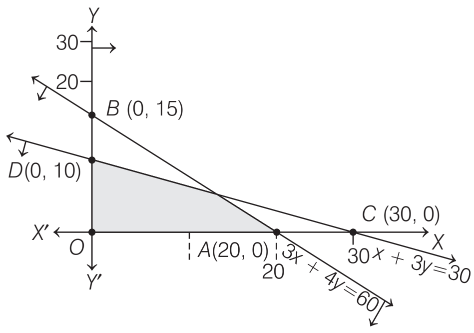
On putting (0, 0) in the given inequality, we get

$0 + 0 \le 30 \Rightarrow 0 \le 30$, which is true.

So, for $x + 3y \le 30$, shade the half plane which contain (0, 0).

Take inequalities (iii) and (iv), $x \ge 0$ and $y \ge 0$

These inequalities represent the region of first quadrant including the points on the axes. So, shade the first quadrant.



Here, common shaded region represent the solution region for system of inequalities.

31. Consider, the line $x + y = 4$.

We observe that the shaded region and the origin lie on the opposite side of this line and (0, 0) satisfies $x + y \le 4$.

Therefore, we must have $x + y \ge 4$, as the linear inequation corresponding to the line $x + y = 4$.

Consider the line $x + y = 8$, clearly the shaded region and origin lie on the same side of this line and (0, 0) satisfies the constraints $x + y \le 8$. Therefore, we must have $x + y \le 8$, as the linear inequation corresponding to the line $x + y = 8$.

Consider the line $x = 5$. It is clear from the graph that the shaded region and origin are on the left of this line and (0, 0) satisfy the constraint $x \le 5$.

Hence, $x \le 5$ is the linear inequation corresponding to $x = 5$.

Consider the line $y = 5$, clearly the shaded region and origin are on the same side (below) of the line and (0, 0) satisfy the constrain $y \le 5$.

Therefore, $y \le 5$ is an inequation corresponding to the line $y = 5$.

We also notice that the shaded region is above the *X*-axis and on the right of the *Y*-axis i.e. shaded region is in first quadrant. So, we must have $x \ge 0$, $y \ge 0$.

Thus, the linear inequations comprising the given solution set are

$$x + y \ge 4,\ x + y \le 8,\ x \le 5,\ y \le 5,\ x \ge 0 \text{ and } y \ge 0.$$

32. Consider the inequation $3x + 2y \ge 24$ as an equation, we have

$$3x + 2y = 24$$

$$\Rightarrow \quad 2y = 24 - 3x$$

x	0	8	4
y	12	0	6

Hence, line $3x + y = 24$ intersects coordinate axes at points $(8, 0)$ and $(0, 12)$.

Now, $(0, 0)$ does not satisfy the inequation $3x + 2y \ge 24$.

Therefore, half plane of the solution set does not contains $(0, 0)$.

Consider the inequation $3x + y \le 15$ as an equation, we have

$$3x + y = 15$$

$$\Rightarrow \quad y = 15 - 3x$$

x	0	5	3
y	15	0	6

Line $3x + y = 15$ intersects coordinate axes at points $(5, 0)$ and $(0, 15)$.

Now, point $(0, 0)$ satisfy the inequation $3x + y \le 15$.

Therefore, the half plane of the solution contain origin.

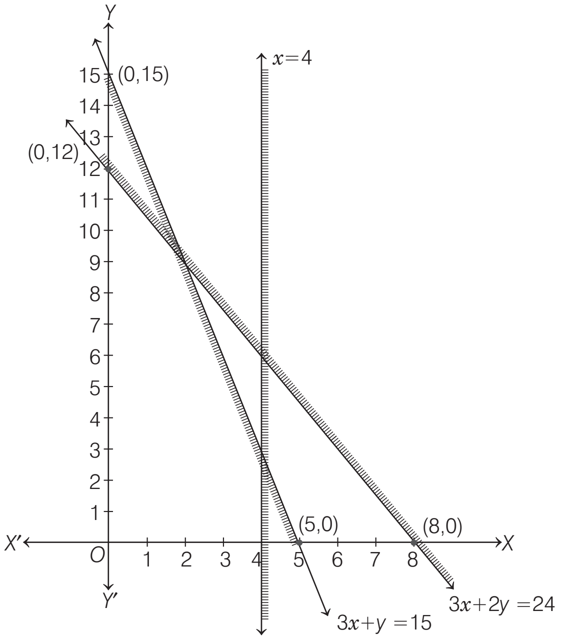
Consider the inequality $x \ge 4$ as an equation, we have

$$x = 4$$

It represents a straight line parallel to *Y*-axis passing through $(4, 0)$. Now, point $(0, 0)$ does not satisfy the inequation $x \ge 4$.

Therefore, half plane does not contains $(0, 0)$,

The graph of the above inequations is given below.



It is clear from the graph that there is no common region corresponding to these inequality.

Hence, the given system of inequalities have no solution.

33. Consider the inequation $x + 2y \le 3$ as an equation, we have

$$x + 2y = 3$$

$$\Rightarrow \quad x = 3 - 2y$$

$$\Rightarrow \quad 2y = 3 - x$$

x	3	1	0
y	0	1	1.5

Now, (0, 0) satisfy the inequation $x + 2y \le 3$.

So, half plane contains (0, 0) as the solution and the line $x + 2y = 3$ intersect the coordinate axis at $(3, 0)$ and (0, 3/2).

Consider the inequation $3x + 4y \ge 12$ as an equation, we have,

$$3x + 4y = 12 \Rightarrow 4y = 12 - 3x$$

x	0	4	2
y	3	0	3/2

Thus, coordinate axis, intersected by the line $3x + 4y = 12$ at points (4, 0) and (0, 3).

Now, (0, 0) does not satisfy the inequation $3x + 4y \ge 12$.

Therefore, half plane of the solution does not contained (0, 0).

Consider the inequation $y \ge 1$ as an equation, we have

$$y = 1.$$

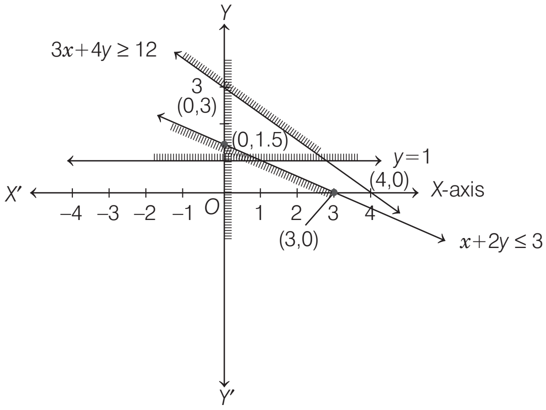
It represents a straight line parallel to *X*-axis passing through point (0, 1).

Now, (0, 0) does not satisfy the inequation $y \ge 1$.

Therefore, half plane of the solution does not contains (0, 0).

Clearly $x \ge 0$ represents the region lying on the right side of *Y*-axis.

The solution set of the given linear constraints will be the intersection of the above region.



It is clear from the graph the shaded portions do not have common region.

So, solution set is null set.

34. The given system of inequalities

$$\mathbf{x + 2y \leq 10} \quad \ldots(i)$$
$$\mathbf{x + y \geq 1} \quad \ldots(ii)$$
$$\mathbf{x - y \leq 0} \quad \ldots(iii)$$
$$\mathbf{x \geq 0, y \geq 0} \quad \ldots(iv)$$

Step I Consider the given inequations as strict equations i.e.

$$\mathbf{x + 2y = 10, x + y = 1, x - y = 0}$$
and $\mathbf{x = 0, y = 0}$

Step II Find the points on the **X**-axis and **Y**-axis for

$\mathbf{x + 2y = 10}$

x	0	10
y	5	0

and $\mathbf{x + y = 1}$

x	0	1
y	1	0

For $\mathbf{x - y = 0}$

x	1	2
y	1	2

Step III Plot the graph of $\mathbf{x + 2y = 10, x + y = 1}$ **and** $\mathbf{x - y = 0}$ using the above tables.

Step IV Take a point (0, 0) and put it in the inequations (i) and (ii),

$$\mathbf{0 + 0 \leq 10} \quad \text{(true)}$$

So, the shaded region will be towards origin.

And $\mathbf{0 + 0 \geq 1}$ (false)

So, the shaded region will be away from the origin.

Again, take a point $\mathbf{(2, 2)}$ and put it in the inequation (iv), we get

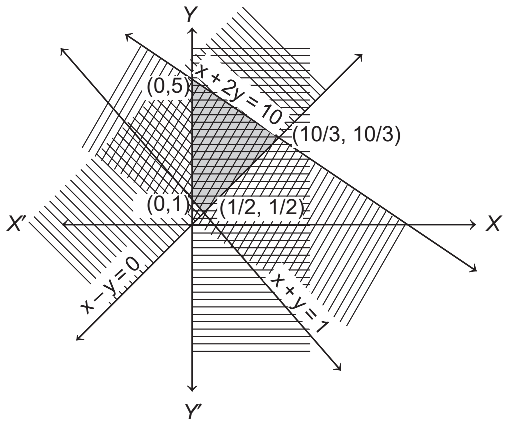
$$\mathbf{2 \geq 0, 2 \geq 0} \quad \text{(true)}$$

So, the shaded region will be towards point (2, 2).

And take a point (0, 1) and put it in the inequation (iii), we get

$$\mathbf{0 - 1 \leq 0} \quad \text{(true)}$$

So, the shaded region will be towards point (0, 1).



Thus, common shaded region shows the solution of the inequalities.

35. We have,

$$\mathbf{x + y \leq 5} \quad ...(i)$$
$$\mathbf{4x + y \geq 4} \quad ...(ii)$$
$$\mathbf{x + 5y \geq 5} \quad ...(iii)$$
$$\mathbf{x \leq 4} \quad ...(iv)$$
and $$\mathbf{y \leq 3} \quad ...(v)$$

Take inequality (i), $\mathbf{x + y \leq 5}$

In equation form, it can be written as $\mathbf{x + y = 5}$

Now, let us construct the following table

x	0	5
y	5	0

Thus, the line $\mathbf{x + y = 5}$ passes through the points **A**(0,5) and *B*(5, 0).

Now, on joining the points **A** and **B** by a dark line, we get the line **AB.**

On putting $\mathbf{x = 0}$ and $\mathbf{y = 0}$ in inequality (i), we get $\mathbf{0 \leq 5}$, which is true.

So, for inequality $\mathbf{x + y \leq 5}$, shade the half plane which contains origin.

Take inequality (ii), $\mathbf{4x + y \geq 4}$

In equation form, it can be written as $\mathbf{4x + y = 4}$

Now, let us construct the following table

x	0	1
y	4	0

Thus, the line $\mathbf{4x + y = 4}$ passes through points **C**(0, 4) and *D*(1, 0).

Now, joining the points **C** and **D** by a dark line, we get the line **CD**.

On putting $\mathbf{x = 0}$ and $\mathbf{y = 0}$ in inequality (ii), we get

$$\mathbf{4(0) + 0 \geq 4}$$

$\Rightarrow$ $\quad \mathbf{0 \geq 4}$, which is false.

So, for inequality $\mathbf{4x + y \geq 4}$, shade the half plane which does not contain origin.

Take inequality (iii), $\mathbf{x + 5y \geq 5}$

In equation form, it can be written as

$$\mathbf{x + 5y = 5}$$

Now, let us construct the following table

x	0	5
y	1	0

Thus, the line $\mathbf{x + 5y = 5}$ passes through the points **E**(0,1) and *B* (5,0).

Now, joining the points **E** and **B** by a dark line, we get the line **EB**.

On putting $\mathbf{x = 0}$ and $\mathbf{y = 0}$ in inequality (iii), we get

$\mathbf{0 \geq 5}$, which is false.

So, for inequality $\mathbf{x + 5y \geq 5}$, shade the half plane which does not contain origin.

Take inequality (iv), $\mathbf{x \leq 4}$

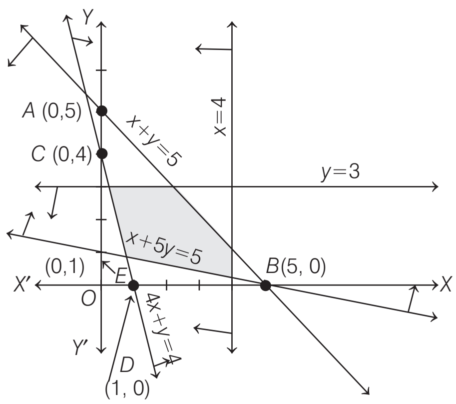
Linear equation corresponding to inequality (iv) is $\mathbf{x = 4}$. This is a line parallel to **Y**-axis at a distance 4 units to the right of **Y**-axis.

For this inequality, shade the half plane which contains origin. [$\because \mathbf{0 \leq 4}$, is true]

Take inequality (v), $\mathbf{y \leq 3}$

Linear equation corresponding to inequality (v) is $\mathbf{y = 3}$. This is a line parallel to **X**-axis at a distance of 3 units above the **X**-axis.

For this inequality, shade the half plane which contains origin. [$\because \mathbf{0 \leq 3}$, is true]



The common shaded region represent the solution region for system of inequalities.

Chapter Test

• Multiple Choice Questions

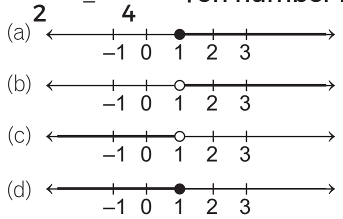
1. The solution set of the inequality
$37-(3x+5)\geq 9x-8(x-3)$ is
(a) $(-\infty, 2)$ (b) $(-\infty, -2)$ (c) $(-\infty, 2]$ (d) $(-\infty, -2]$

2. The solution set of the inequalities
$6\leq -3(2x-4)<12$ is
(a) $(-\infty, 1]$ (b) $(0, 1]$ (c) $(0, 1]\cup[1, \infty)$ (d) $[1, \infty)$

3. If $-5\leq\frac{5-3x}{2}\leq 8$, then
(a) $-\frac{11}{3}<x<5$ (b) $-\frac{11}{3}\leq x\leq 5$
(c) $-11\leq x\leq 5$ (d) $-\frac{11}{3}<x<5$

4. The solution set of $-15<\frac{3(x-2)}{5}\leq 0$ is
(a) $(-23, 2]$ (b) $[-23, 2]$
(c) $[-23, 2)$ (d) None of these

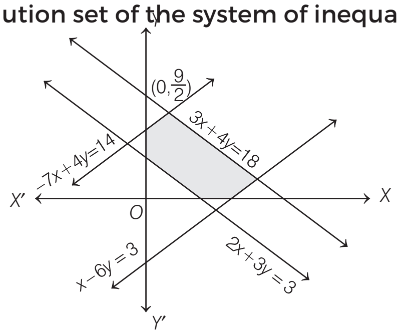
5. The graph of the solutions of inequality
$\frac{3x-4}{2}\geq\frac{x+1}{4}-1$ on number line is



6. The solution set of $7\leq\frac{3x+11}{2}\leq 11$ is
(a) $\left[1, \frac{11}{3}\right)$ (b) $\left[1, \frac{11}{3}\right]$ (c) $\left(1, \frac{11}{3}\right)$ (d) $(1, \infty)$

• Case Based MCQs

7. The shaded region in the given figure is the solution set of the system of inequalities.



On the basis of the above information, solve the following questions.

(i) As the shaded region represents in the first quadrant, then the inequalities satisfies
(a) $x\geq 0, y\leq 0$ (b) $x\geq 0, y\geq 0$
(c) $x\leq 0, y\leq 0$ (d) $x\leq 0, y\geq 0$

(ii) A vertical line will divide the xy-plane in two parts, which is
(a) left and upper half planes
(b) left and right half planes
(c) right and lower half planes
(d) None of the above

(iii) Line $3x+4y=18$ represents the inequality in the given system is
(a) $3x+4y\geq 18$ (b) $3x+4y>18$
(c) $3x+4y\leq 18$ (d) None of these

(iv) In the given graphical solution system, line $2x+3y=3$ represents the solution set
(a) towards the origin
(b) away from the origin
(c) can't say anything
(d) None of the above

(v) In the given shaded region, the solution set also lies
(a) on the inequalities
(b) inside the inequalities
(c) can't say anything
(d) None of the above

• Short Answer Type Questions

8. Find the solution set of $3x-5<x+7$, when x is an integer.

9. Find all the pairs of consecutive odd positive integers, both of which are smaller than 10, such that their sum is more than 11.

10. A solution is to be kept between 40°C and 45°C. What is the range of temperature in degree Fahrenheit, if the conversion formula is

$$F=\frac{9}{5}C+32?$$

• Long Answer Type Questions

11. How many litres of water will have to be added to 1125 L of the 45% solution of acid, so that the resulting mixture will contain more than 25% but less than 30% acid content?

12. Find the solution set of the inequalities $4x+3y\leq 60$, $y\geq 2x$, $x\geq 3$, x and $y\geq 0$.

Answers

1. (c) *2.* (b) *3.* (b) *4.* (a) *5.* (a) *6.* (b)
7. (i) (b) (ii) (b) (iii) (c) (iv) (b) (v) (a) *8.* $\{\ldots, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$
9. (5, 7) and (7, 9) *10.* 104°F and 113°F *11.* more than 562.5 but less than 900 L