Chapter

1 Rational Numbers

1.1 Introduction

In previous class, experiencing the necessity of rational numbers, we defined them in the form of $\frac{p}{q}$, where p and q are integers and q $\neq 0$. We have studied and practised in detail, about the representation of rational numbers on number line, their equivalency, their simplified (standard) form, their comparison and to find rational numbers between two given rational numbers etc. In this section, continuing the previous class work, we shall study and practise the operations on rational numbers, multiplicative inverse, properties of these rational numbers and insertion of rational numbers between two rational numbers by mean method.

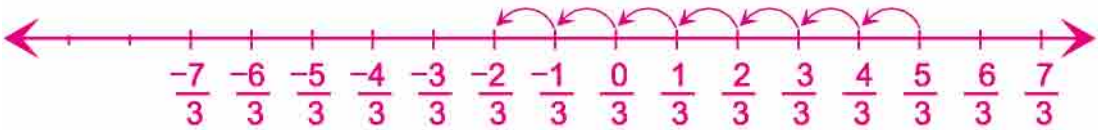
1.1 Operations on Rational Numbers

We know that how the integers and fractions are added, subtracted, multiplied and divided. Let us study these basic operations on rational numbers.

1.2.1 Addition

Vimala added two rational numbers $\frac{5}{3}$ and $-\frac{7}{3}$ which have the same denominators on number line in such a way

$$\frac{5}{3} + \left(-\frac{7}{3}\right)$$



On the above number line, distance between two successive points is $\frac{1}{3}$.

If we add $-\frac{7}{3}$ in to $\frac{5}{3}$ it means we have to move seven steps towards the left of $\frac{5}{3}$ (due to negative sign of $\frac{7}{3}$) Where we reach?

Now we reach on $\frac{-2}{3}$

Therefore, $\frac{5}{3} + \left(-\frac{7}{3}\right) = \frac{-2}{3}$

It can also be done like that

$$\frac{5}{3}+\left(-\frac{7}{3}\right)$$

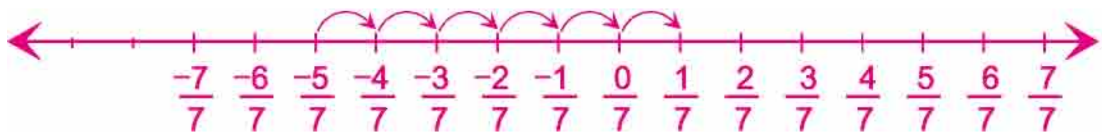
$$=\frac{5+(-7)}{3}$$

$$=\frac{5-7}{3}$$

$$=\frac{-2}{3}$$

Damoder also added two rational numbers $\frac{-5}{7}$ and $\frac{6}{7}$.

$$\frac{-5}{7}+\frac{6}{7}$$



Distance between two successive points on number line is $\frac{1}{7}$.

If we add $\frac{-5}{7}$ in to $\frac{6}{7}$, it means we move six steps towards the right of $\frac{-5}{7}$ $\left(\text{due to positive sign of } \frac{6}{7}\right)$.

We reach on $\frac{1}{7}$ Therefore, $\frac{-5}{7}+\frac{6}{7}=\frac{1}{7}$

It can also be solved as $\frac{-5}{7}+\frac{6}{7}$

$$=\frac{-5+6}{7}$$

$$=\frac{1}{7}$$

Do and Learn

- Find the value

(i) $\frac{-11}{7}+\frac{4}{7}$ (ii) $\frac{3}{5}+\left(\frac{-2}{5}\right)$ (iii) $\frac{-3}{4}+\left(-\frac{5}{4}\right)$

Let us see some other examples-

$$\frac{4}{3}+\frac{1}{3} = \frac{4+1}{3} = \frac{5}{3}$$

$$\frac{-7}{5}+\frac{9}{5} = \frac{-7+9}{5} = \frac{2}{5}$$

$$\frac{4}{7}+\left(\frac{-9}{7}\right) = \frac{4-9}{7} = \frac{-5}{7}$$

$$\frac{-1}{4}+\left(\frac{-2}{4}\right) = \frac{-1-2}{4} = \frac{-3}{4}$$

Thus, we see that while adding the rational numbers having same denominators, numerators are added keeping the denominator same.

Vimala asked Damoder, "How we will add two rational numbers having distinct denominators ?"

Damoder - "Do you remember, we added two fractions with distinct denominators?"

1. Like fractions, first we obtain LCM of these denominators.
2. After that, the equivalent rational numbers are determined with the same LCM as obtained in step -1.
3. Then, both rational numbers are added (with common denominators).

Example 1 Add rational numbers $\frac{-4}{3}$ and $\frac{2}{5}$.

Solution $\frac{-4}{3}+\frac{2}{5}$

LCM of 3 and 5 is 15.

$$-\frac{4}{3} = -\frac{4\times 5}{3\times 5} = -\frac{20}{15}$$

and $$\frac{2}{5} = \frac{2\times 3}{5\times 3} = \frac{6}{15}$$

$$\frac{-4}{3}+\frac{2}{5} = -\frac{20}{15}+\frac{6}{15}$$

$$= \frac{-20+6}{15} = \frac{-14}{15}$$

Do and Learn

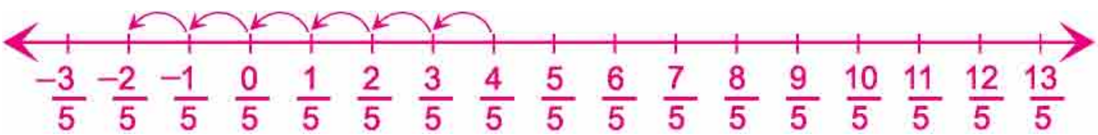
- Find the value

(i) $\frac{2}{5}+\frac{1}{6}$ (ii) $\frac{3}{8}+\left(\frac{-5}{2}\right)$ (iii) $\frac{-7}{20}+\frac{7}{3}$ (iv) $-\frac{5}{7}+\left(\frac{-2}{4}\right)$

1.2.2 Subtraction

Manish subtracts two rational numbers having common denominators $\frac{4}{5}$ and $\frac{6}{5}$ on number line like this:

$$\frac{4}{5}-\frac{6}{5}$$



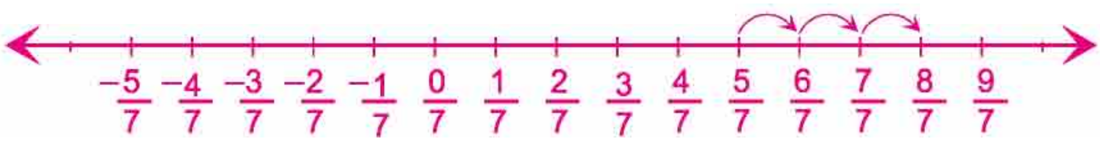
On number line, distance between two successive points is $\frac{1}{5}$.
If we subtract $\frac{6}{5}$ from $\frac{4}{5}$, it means we move six steps towards the left of $\frac{4}{5}$
Where do we reach? We reach on $-\frac{2}{5}$.

$$\text{So, } \frac{4}{5}-\frac{6}{5}=-\frac{2}{5}$$

We can also do like this, $\frac{4}{5}-\frac{6}{5}=\frac{4-6}{5}=-\frac{2}{5}$

Praneeti also subtracted two rational numbers $\frac{5}{7}$ and $\left(-\frac{3}{7}\right)$ like this way,

$$\frac{5}{7}-\left(-\frac{3}{7}\right)$$



On number line, distance between two successive points is $\frac{1}{7}$.

If we subtract $\left(-\frac{3}{7}\right)$ from $\frac{5}{7}$, it means we move three steps towards the right of $\frac{5}{7}$ $\left(\text{by subtracting } -\frac{3}{7}\right)$

Now, we reach on $\frac{8}{7}$.

Thus $\frac{5}{7}-\left(\frac{-3}{7}\right) = \frac{8}{7}$

It can also be written like this,

$$\frac{5}{7}-\left(\frac{-3}{7}\right) = \frac{5-(-3)}{7} = \frac{5+3}{7} = \frac{8}{7}$$

By this practice we can say that while subtracting the two rational numbers having common denominators, numerators are subtracted keeping the denominator the same.

Similarly, $\frac{2}{3}-\frac{1}{3} = \frac{2-1}{3} = \frac{1}{3}$

Other Method

Example 2 Subtract $\frac{-7}{8}$ from $\frac{5}{8}$.

Solution

$$\frac{5}{8}-\left(\frac{-7}{8}\right) = \frac{5}{8}+\left(\frac{7}{8}\right) = \frac{5}{8}+\frac{7}{8} = \frac{5+7}{8} = \frac{12}{8} = \frac{3}{2}$$

Do and Learn

- Find the value-

(i) $\frac{10}{7}-\frac{4}{7}$ (ii) $\frac{-4}{5}-\left(\frac{-2}{5}\right)$ (iii) $\frac{7}{9}-\left(\frac{-4}{9}\right)$

Manish asked Praneeti to tell how to subtract two rational numbers having distinct denominators?

Praneet – Same as we did in addition but at final stage we subtract in place of adding.

Example 3 Subtract $-\frac{3}{8}$ from $-\frac{5}{4}$.

Solution $-\frac{5}{4}-\left(\frac{-3}{8}\right)$

8 is LCM of 4 and 8.

$$-\frac{5}{4} = -\frac{5\times 2}{4\times 2} = -\frac{10}{8}$$

$$-\frac{5}{4}-\left(\frac{-3}{8}\right) = -\frac{10}{8}-\left(\frac{-3}{8}\right)$$

$$= \frac{-10-(-3)}{8}$$

$$= \frac{-10+3}{8}$$

$$= -\frac{7}{8}$$

Do and Learn

- Find the value-

(i) $\frac{4}{3}-\frac{3}{8}$ (ii) $\left(\frac{-3}{7}\right)-\frac{2}{14}$ (iii) $\frac{5}{9}-\left(\frac{-2}{11}\right)$ (iv) $\left(\frac{-2}{9}\right)-\frac{7}{6}$

1.2.3 Multiplication

We have learnt multiplication of fractions.

Now consider the multiplication of rational numbers $\left(2\times-\frac{5}{7}\right)$.

Method-1 (Repetitive addition)

$2\times\left(-\frac{5}{7}\right)$ that is, two time addition of $-\frac{5}{7}$.

i.e. $\left(-\frac{5}{7}\right)+\left(-\frac{5}{7}\right) = \frac{-5}{7}-\frac{5}{7}$

$$= \frac{-5-5}{7}$$

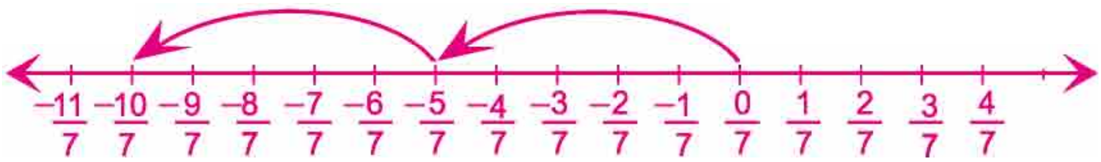
$$= \frac{-10}{7}$$

Method – 2 (By Number Line)

On number line, distance between two successive points is $\frac{1}{7}$.

Moving towards left of Zero ($\frac{5}{7}$ is a negative number).

Take two long jumps of $\frac{5}{7}$ distance (because of twice $\frac{-5}{7}$) . Where do we reach?



We reach on $-\frac{10}{7}$.

Therefore $2 \times \left(-\frac{5}{7}\right) = -\frac{10}{7}$

Method-3 (By Multiplication)

$$2 \times \left(-\frac{5}{7}\right) = \frac{2}{1} \times \left(-\frac{5}{7}\right)$$

$$= \frac{2 \times (-5)}{1 \times 7}$$

$$= -\frac{10}{7}$$

Do and Learn

- Find the Value-

(i) $4 \times \left(-\frac{1}{3}\right)$ (ii) $\left(-\frac{3}{5}\right) \times 7$ (iii) $\left(-\frac{4}{5}\right) \times (-3)$

(iv) $\left(-\frac{3}{7}\right) \times \frac{2}{5}$ (v) $\frac{2}{3} - \left(\frac{-1}{4}\right)$ (vi) $\left(\frac{-3}{2}\right) \times \left(\frac{-9}{7}\right)$

Just think, which procedure you adopted to multiply these numbers.

When we multiply two rational numbers, the product of their numerators is written as a numerator and product of their denominators is written as denominator.

1.2.4 Division

We know the reciprocal of fractions.
Reciprocal (or inverse) of $\frac{3}{7}$ is $\frac{7}{3}$.

This concept is also applied on reciprocal of rational numbers.

Similarly, reciprocal of $-\frac{3}{4}$ is $\frac{4}{-3}$ or $\frac{-4}{3}$ and reciprocal of $-\frac{7}{9}$ is $-\frac{9}{7}$.
We know that, 10 × 5 = 50
In form of division, it can be written in the following two ways:

$50 \div 10 = 5$	$50 \div 5 = 10$
$\frac{50}{10} = 5$	$\frac{50}{5} = 10$
$50 \times \frac{1}{10} = 5$	$50 \times \frac{1}{5} = 10$

By this it is clear that, when dividend is divided by the divisor, the resultant is quotient and if dividend is multiplied by inverse of divisor then resultant number is equal to the quotient. Now it is clear that action of division can be transformed in form of multiplication.

Example 4 Solve $\frac{-21}{8} \div \frac{8}{3}$

Solution $\frac{-21}{8} \div \frac{8}{3} = \frac{-21}{8} \times \frac{3}{8}$

$$= \frac{-21 \times 3}{8 \times 8} = \frac{-63}{64}$$

By the above example, it is clear that to divide any rational number by another rational number, we multiply that rational number by an inverse of another rational number.

Do and Learn

- Solve-

(i) $-\frac{7}{2} \div 4$ (ii) $\frac{-12}{7} \div \frac{3}{4}$ (iii) $\frac{5}{9} \div \left(-\frac{4}{5}\right)$

Division of rational number by the same rational number

$$\frac{3}{7} \div \frac{3}{7} = \frac{3}{7} \times \left(\text{reciprocal of } \frac{3}{7}\right) = \frac{3}{7} \times \frac{7}{3} = 1$$

$$\frac{-4}{5} \div \left(\frac{-4}{5}\right) = \frac{-4}{5} \times \left(\text{reciprocal of } \frac{-4}{5}\right) = \frac{-4}{5} \times \left(-\frac{5}{4}\right) = 1$$

You also consider this type of example.

By the above discussion it is clear that if rational number is divided by the same rational number, result of the division is quotient 1..

In other words, product of any given rational number with its reciprocal is always 1.

Do and Learn

- Solve-

(i) $\frac{5}{7} \div \frac{5}{7}$ (ii) $\frac{-9}{4} \div \frac{-9}{4}$ (iii) $\frac{-7}{11} \div \frac{-7}{11}$

1.3 Properties of Rational Numbers

1.3.1 Closure Property:

(i) Addition Let us consider on sum of any two rational numbers.

$\frac{3}{4} + \frac{5}{3} = \frac{9}{12} + \frac{20}{12} = \frac{9+20}{12} = \frac{29}{12}$ is a rational number.

$\frac{2}{7} + \left(\frac{-6}{11}\right) = \frac{22}{77} + \left(\frac{-42}{77}\right) = \frac{22-42}{77} = \frac{-20}{77}$ is a rational number.

$\frac{5}{11} + \frac{6}{11} = \frac{5+6}{11} = \frac{11}{11} = 1$ is a rational number.

Verify this property on other rational numbers.

It is clear that the sum of any two rational numbers is always a rational number. Thus, **rational numbers are closure under addition** i.e., for any two rational numbers x and y, $(x+y)$ is also a rational number.

(ii) Subtraction

Let us consider the subtraction of any two rational numbers

$\frac{5}{7} - \frac{3}{8} = \frac{40}{56} - \frac{21}{56} = \frac{40-21}{56} = \frac{19}{56}$ is a rational number.

$\frac{7}{8} - \frac{8}{9} = \frac{63}{72} - \frac{64}{72} = \frac{63-64}{72} = \frac{-1}{72}$ is a rational number.

$\frac{1}{4} - \frac{1}{4} = \frac{1-1}{4} = \frac{0}{4} = 0$ is a rational number.

Verify this property on other rational numbers.

It is clear that the subtraction of any two rational numbers is always a rational number. Thus, **rational numbers are closure under subtraction** i.e., for any two rational numbers x and y, $(x-y)$ is also a rational number.

(iii) Multiplication

Let us consider on product of any two rational numbers.

$$\frac{-4}{5} \times \frac{3}{7} = \frac{(-4)\times 3}{5 \times 7} = \frac{-12}{35}$$ is a rational number.

$$\frac{2}{3} + \left(\frac{-4}{9}\right) = \frac{2\times(-4)}{3 \times 9} = \frac{-8}{27}$$ is a rational number.

$$\left(\frac{-2}{7}\right) + \left(\frac{-1}{3}\right) = \frac{(-2)\times(-1)}{7 \times 3} = \frac{2}{21}$$ is a rational number.

Verify this property on other rational numbers.
It is clear that the product of any two rational numbers is always a rational number. Thus, **rational numbers are closure under multiplication** i.e., for any two rational numbers x and y, product $(x \times y)$ is also a rational number.

(iv) Division

Let us consider the division of any two rational numbers.

$$\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{2 \times 4}{3 \times 3} = \frac{8}{9}$$ is a rational number.

$$\frac{-7}{2} \div \frac{3}{5} = \frac{-7}{2} \times \frac{5}{3} = \frac{-7 \times 5}{2 \times 3} = \frac{-35}{6}$$ is a rational number.

$$0 \div \frac{1}{2} = 0 \times \frac{2}{1} = 0$$ is a rational number.

$$5 \div 0 = 5 \times \frac{1}{0} = \frac{5}{0}$$ is not a rational number. (indefinite number).

Thus, we can say that it is not compulsory that division of any two rational numbers is always a rational number. Therefore, **rational numbers are not closure under division.**

Do and Learn Fill in the blanks-

Numbers	Closure under Operations			
	Addition	Subtraction	Multiplication	Division
Natural Number	Yes	-------	-------	-------
Whole Number	------	-------	------	No
Integers	------	Yes	-------	-------
Rational Number	-------	------	Yes	------

1.3.2 Commutative Property

(i) Addition

Consider the sum of two rational numbers $\frac{3}{7}$ and $\frac{-1}{4}$.

$$\frac{3}{7} + \left(\frac{-1}{4}\right) = \frac{12}{28} + \left(\frac{-7}{28}\right) = \frac{12-7}{28} = \frac{5}{28}$$

and $$\left(\frac{-1}{4}\right) + \frac{3}{7} = \left(\frac{-7}{28}\right) + \frac{12}{28} = \frac{-7+12}{28} = \frac{5}{28}$$

So $$\frac{3}{7} + \left(\frac{-1}{4}\right) = \left(\frac{-1}{4}\right) + \frac{3}{7}$$

Therefore $$\frac{-4}{5} + \left(\frac{-2}{3}\right) = \frac{-12}{15} + \left(\frac{-10}{15}\right) = \frac{-12-10}{15} = \frac{-22}{15}$$

$$\frac{-2}{3} + \left(\frac{-4}{5}\right) = \frac{-10}{15} + \left(\frac{-12}{15}\right) = \frac{-10-12}{15} = \frac{-22}{15}$$

So $$\frac{-2}{3} + \left(\frac{-4}{5}\right) = \frac{-4}{5} + \left(\frac{-2}{3}\right)$$

You also verify this commutative property on any other rational numbers.

We can say even if we change the order of rational numbers, sum of any two rational numbers is always remain the same. Therefore, **rational numbers are commutative under the addition** i.e., for any two given rational numbers a and b.

$$a + b = b + a$$

(ii) Subtraction

Consider the subtraction of two rational numbers $\frac{2}{5}$ and $\frac{4}{7}$.

$$\frac{2}{5} - \frac{4}{7} = \frac{14}{35} - \frac{20}{35} = \frac{14-20}{35} = \frac{-6}{35}$$

$$\frac{4}{7} - \frac{2}{5} = \frac{20}{35} - \frac{14}{35} = \frac{20-14}{35} = \frac{6}{35}$$

$$\frac{2}{5} - \frac{4}{7} \neq \frac{4}{7} - \frac{2}{5}$$

Thus, we can say that, **rational numbers are not commutative under the subtraction** i.e., for any two given rational numbers a and b

$$a - b \neq b - a$$

(iii) Multiplication

Consider the multiplication of two rational numbers $\frac{-4}{5}$ and $\frac{3}{7}$.

$$\left(\frac{-4}{5}\right)\times\frac{3}{7} = \frac{(-4)\times 3}{5\times 7} = \frac{-12}{35}$$

and $$\frac{3}{7}\times\left(\frac{-4}{5}\right) = \frac{3\times(-4)}{7\times 5} = \frac{-12}{35}$$

So $$\left(\frac{-4}{5}\right)\times\frac{3}{7} = \frac{3}{7}\times\left(\frac{-4}{5}\right)$$

You also verify this commutative property on any other rational numbers.

We can say that the product of any two rational numbers is always the same even we change the order of multiples. Therefore, **rational numbers are commutative under the multiplication** i.e., for any two given rational numbers a and b

$$a \times b = b \times a$$

(iv) Division

Consider the division of two rational numbers $\frac{7}{3}$ and $\frac{14}{5}$.

$$\frac{7}{3}\div\frac{14}{5} = \frac{7}{3}\times\frac{5}{14} = \frac{35}{42}$$

and $$\frac{14}{5}\div\frac{7}{3} = \frac{14}{5}\times\frac{3}{7} = \frac{42}{35}$$

So $$\frac{7}{3}\div\frac{14}{5} \neq \frac{14}{5}\div\frac{7}{3}$$

Thus, we can say that , **rational numbers are not commutative under the division** i.e., for any two given rational numbers a and b

$$a \div b \neq b \div a$$

Do and Learn Fill in the blanks-

Numbers	Commutativity			
	Addition	Subtraction	Multiplication	Division
Natural Number	Yes	No	Yes	No
Whole Number	-------	-------	-------	-------
Integers	-------	-------	-------	-------
Rational Number	-------	-------	-------	-------

1.3.3 Associative Property

(i) Addition

Check this property with three rational numbers $\frac{-5}{4}$, $\frac{3}{8}$ and $\frac{-7}{6}$.

$$\frac{-5}{4}+\left(\frac{3}{8}+\frac{-7}{6}\right) = \frac{-5}{4}+\left(\frac{9-28}{24}\right) = \frac{-5}{4}+\left(\frac{-19}{24}\right) = \frac{-5}{4}-\frac{19}{24} = \frac{-30-19}{24} = \frac{-49}{24}$$

$$\left(\frac{-5}{4}+\frac{3}{8}\right)+\frac{-7}{6} = \left(\frac{-10+3}{8}\right)+\frac{-7}{6} = \left(\frac{-7}{8}\right)+\left(\frac{-7}{6}\right) = \frac{-21+(-28)}{24} = \frac{-21-28}{24} = \frac{-49}{24}$$

So $\frac{-5}{4}+\left(\frac{3}{8}+\frac{-7}{6}\right) = \left(\frac{-5}{4}+\frac{3}{8}\right)+\frac{-7}{6}$

Do and Learn

Are the addition on both sides the same?

(i) $\frac{-3}{5}+\left(\frac{2}{3}+\frac{4}{7}\right)=\left(\frac{-3}{5}+\frac{2}{3}\right)+\frac{4}{7}$

(ii) $\frac{1}{2}+\left(\frac{-3}{4}+\frac{-5}{8}\right)=\left(\frac{1}{2}+\frac{-3}{4}\right)+\frac{-5}{8}$

You also verify this associative property on any other rational numbers. We find that **rational numbers are associative under the addition** i.e., for any three given rational numbers a,b and c

$$a+(b+c) = (a+b)+c$$

(ii) Subtraction

Verify this on three rational numbers $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{-5}{4}$.

$$\frac{1}{2}-\left[\frac{3}{4}-\left(\frac{-5}{4}\right)\right] \qquad \left(\frac{1}{2}-\frac{3}{4}\right)-\left(\frac{-5}{4}\right)$$

$$= \frac{1}{2}-\left(\frac{3+5}{4}\right) \qquad = \left(\frac{2-3}{4}\right)-\left(\frac{-5}{4}\right)$$

$$= \frac{1}{2}-\frac{8}{4} \qquad = \frac{-1}{4}-\left(\frac{-5}{4}\right)$$

$$= \frac{2-8}{4} \qquad = \frac{-1}{4}+\frac{5}{4}$$

$$= \frac{-6}{4} \qquad = \frac{4}{4}$$

$$= \frac{-3}{2} \qquad = 1$$

$$\frac{1}{2}-\left[\left(\frac{3}{4}-\frac{-5}{4}\right)\right] \neq \left(\frac{1}{2}-\frac{3}{4}\right)-\left(\frac{-5}{4}\right)$$

We find that **subtraction is not associative for rational numbers** i.e., for any three rational numbers a, b and c

$$a-(b-c) \neq (a-b)-c$$

(iii) Multiplication Consider the three rational numbers $\frac{2}{3}$, $\frac{4}{7}$ and $\frac{-5}{7}$.

$$\frac{2}{3}\times\left[\frac{4}{7}\times\left(\frac{-5}{7}\right)\right] \qquad \left(\frac{2}{3}\times\frac{4}{7}\right)\times\left(\frac{-5}{7}\right)$$

$$= \frac{2}{3}\times\left[\frac{4\times(-5)}{7\times 7}\right] \qquad = \left(\frac{2\times 4}{3\times 7}\right)\times\left(\frac{-5}{7}\right)$$

$$= \frac{2}{3}\times\left(\frac{-20}{49}\right) \qquad = \frac{8}{21}\times\left(\frac{-5}{7}\right)$$

$$= \frac{2\times(-20)}{3\times 49} \qquad = \frac{8\times(-5)}{21\times 7}$$

$$= \frac{2\times -20}{3\times 49} \qquad = \frac{8\times -5}{21\times 7}$$

$$= \frac{-40}{147} \qquad = \frac{-40}{147}$$

So $$\frac{2}{3}\times\left(\frac{4}{7}\times\frac{-5}{7}\right) = \left(\frac{2}{3}\times\frac{4}{7}\right)\times\left(\frac{-5}{7}\right)$$

You also verify this property by taking any other rational numbers we find that **rational numbers are associative under multiplication** i.e., for any three rational numbers a, b and c

$$a\times(b\times c) = (a\times b)\times c$$

Do and Learn

Verify –

(i) $\frac{-4}{3} \times \left[\left(\frac{-2}{5}\right) \times \frac{1}{7}\right] = \left[\frac{4}{3} \times \left(\frac{-2}{5}\right)\right] \times \frac{1}{7}$

(ii) $\frac{-3}{5} \times \left[\frac{4}{11} \times \left(\frac{-3}{22}\right)\right] = \left[\left(\frac{-3}{5}\right) \times \frac{4}{11}\right] \times \frac{-3}{22}$

(iv) Division

Consider any given three rational numbers $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{-2}{7}$.

$$\frac{2}{3} \div \left[\frac{3}{4} \div \left(\frac{-2}{7}\right)\right]$$
$$= \frac{2}{3} \div \left[\frac{3}{4} \times \left(-\frac{7}{2}\right)\right]$$
$$= \frac{2}{3} \div \left[\frac{3 \times (-7)}{4 \times 2}\right]$$
$$= \frac{2}{3} \div \left(\frac{-21}{8}\right)$$
$$= \frac{2}{3} \times \frac{8}{-21}$$
$$= \frac{-16}{63}$$

$$\left(\frac{2}{3} \div \frac{3}{4}\right) \div \left(\frac{-2}{7}\right)$$
$$= \left(\frac{2}{3} \times \frac{4}{3}\right) \div \left(\frac{-2}{7}\right)$$
$$= \left(\frac{2 \times 4}{3 \times 3}\right) \div \left(\frac{-2}{7}\right)$$
$$= \frac{8}{9} \div \frac{-2}{7}$$
$$= \frac{8}{9} \times \left(\frac{7}{-2}\right)$$
$$= \frac{-56}{18}$$

$$\frac{2}{3} \div \left[\frac{3}{4} \div \left(\frac{-2}{7}\right)\right] \neq \left[\left(\frac{2}{3} \div \frac{3}{4}\right)\right] \div \left(\frac{-2}{7}\right)$$

We find that **rational numbers are not associative for division** i.e., for any three rational numbers a, b and c

$$a \div (b \div c) \neq (a \div b) \div c$$

Do and Learn Fill in the blanks -

Numbers	Associativity			
	Addition	Subtraction	Multiplication	Division
Natural Number	Yes	-------	-------	-------
Whole Number	-------	-------	-------	No
Integers	-------	-------	Yes	-------
Rational Number	-------	-------	-------	-------

1.3.4 Operations of Zero with rational numbers

Can you suggest any number which can be added with any number and obtain the same number? When zero (0) is added with any other rational number, then resultant is the same rational number.

$$5 + 0 = 0 + 5 = 5$$

$$(-3) + 0 = 0 + (-3) = -3$$

$$\left(\frac{-5}{7}\right) + 0 = 0 + \left(\frac{-5}{7}\right) = \frac{-5}{7}$$

Therefore, **Zero is called additive identity**, i.e., for any rational number a,

$$a + 0 = 0 + a = a$$

Consider, do natural numbers contain additive identity?

Do and Learn

Fill in the blanks:-

(i) $3 + \square = 3$ (ii) $\square + 0 = -7$ (iii) $\frac{-4}{9} + \square = \frac{-4}{9}$

(iv) $\square + \frac{9}{13} = \frac{9}{13}$ (v) $\frac{-5}{11} + 0 = \square$

1.3.5 Multiplicative Identity:

Fill in the blanks :

$8 \times \square = 8$ and $\square \times 8 = 8$

$(-5) \times \square = -5$ and $\square \times (-5) = -5$

$\left(\frac{2}{3}\right) \times \square = \frac{2}{3}$ and $\square \times \frac{2}{3} = \frac{2}{3}$

$\left(\frac{-4}{7}\right) \times \square = \frac{-4}{7}$ and $\square \times \left(\frac{-4}{7}\right) = \frac{-4}{7}$

By this exercise we can say that if any rational number is multiplied by 1, then product is the same as rational Number, i.e., **1 is called multiplicative identity** for rational numbers. For any rational number a,

$$a \times 1 = 1 \times a = a$$

Consider, what is the multiplicative identity of integers and whole numbers?

1.3.6 Additive Inverse: Fill in the blanks-

$2 + \square = 0$ and $\square + 2 = 0$

$-3 + \square = 0$ and $\square + (-3) = 0$

$\frac{3}{4} + \square = 0$ and $\square + \frac{3}{4} = 0$

$-\frac{5}{7} + \square = 0$ and $\square + \left(\frac{-5}{7}\right) = 0$

By this exercise we can say that when sum of two numbers is zero (additive identity) then these two numbers are additive inverse to each other. E.g. -1 is additive inverse of 1 and 1 is additive inverse of -1. Similarly, we can say that $\frac{-a}{b}$ is additive inverse of $\frac{a}{b}$ and $\frac{a}{b}$ is additive inverse of $\frac{-a}{b}$.

Do and Learn

Write the additive inverse of the following rational numbers-

(i) 4 (ii) $\frac{-1}{3}$ (iii) $\frac{7}{2}$ (iv) $\frac{-3}{5}$ (v) $\frac{9}{2}$

1.3.7. Multiplicative Inverse:

Fil in the blanks -

$5 \times \square = 1$ and $\square \times 5 = 1$

$-7 \times \square = 1$ and $\square \times (-7) = 1$

$\frac{2}{3} \times \square = 1$ and $\square \times \frac{2}{3} = 1$

$-\frac{2}{3} \times \square = 1$ and $\square \times \left(\frac{-2}{3}\right) = 1$

By this exercise we can say that when product of two rational numbers is 1 (multiplicative identity) then these two numbers are multiplicative inverse (reciprocal) to each other. E.g. $\frac{4}{3}$ is the inverse of $\frac{3}{4}$ and $\frac{3}{4}$ is inverse of $\frac{4}{3}$. For any rational number $\frac{a}{b}$, if $\frac{a}{b} \times \frac{b}{a} = 1 = \frac{b}{a} \times \frac{a}{b}$ then we can say that the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$ and the reciprocal of $\frac{b}{a}$ is $\frac{a}{b}$.

Can you tell the multiplicative inverse of zero?

Do and Learn

Write the multiplicative inverse of 3, $\frac{1}{5}$ $\frac{-3}{7}$ $\frac{2}{3}$ and $\frac{-5}{6}$ rational numbers.

1.3.8 Distributivity of Multiplication over Addition for Rational Numbers

Consider the following-

$$\frac{5}{4}\times\left[\left(\frac{-2}{8}\right)+\left(\frac{-3}{5}\right)\right]$$
$$= \frac{5}{4}\times\left(\frac{-10-24}{40}\right)$$
$$= \frac{5}{4}\times\left(\frac{-34}{40}\right)$$
$$= \frac{-170}{160}$$
$$= \frac{-17}{16}$$

$$\frac{5}{4}\times\left[\left(\frac{-2}{8}\right)+\left(\frac{-3}{5}\right)\right]$$
$$= \frac{5}{4}\times\left(\frac{-2}{8}\right)+\frac{5}{4}\times\left(\frac{-3}{5}\right)$$
$$= \frac{-10}{32}+\left(\frac{-15}{20}\right)$$
$$= \frac{-50-120}{160}$$
$$= \frac{-170}{160}$$
$$= \frac{-17}{16}$$

So $\frac{5}{4}\times\left[\left(\frac{-2}{8}\right)+\left(\frac{-3}{5}\right)\right] = \frac{5}{4}\times\left(\frac{-2}{8}\right)+\frac{5}{4}\times\left(\frac{-3}{5}\right)$

is $\frac{2}{5}\times\left[\left(\frac{1}{2}\right)+\left(\frac{-3}{4}\right)\right] = \frac{2}{5}\times\frac{1}{2}+\frac{2}{5}\times\left(\frac{-3}{4}\right)$?

This property is called the distributivity of multiplication over addition. Is in rational number disrtibutivity of multiplication is true over subtraction ? If a, b and c are three rational numbers, then

$$a\times(b+c) = a\times b + a\times c$$
$$a\times(b-c) = a\times b - a\times c$$

Do and Learn

(i) $\frac{5}{8}\times\left(\frac{-3}{7}\right)+\frac{5}{8}\times\left(\frac{-7}{6}\right)$ (ii) $\frac{2}{5}\times\left(\frac{-1}{9}\right)+\frac{2}{5}\times\left(\frac{-3}{7}\right)$

(iii) $\left(\frac{-4}{5}\right)\times\frac{2}{9}+\left(\frac{-4}{5}\right)\times\frac{7}{11}$ (iv) $\frac{3}{5}\times\left(\frac{-1}{3}\right)+\frac{3}{5}\times\frac{3}{5}$

1.3.9. Determine the Rational Number between Two Given Rational Numbers:

In previous class, we learnt about finding rational number between two given rational numbers. As we know the Mean method. So, in this section we shall study to find the rational number between two given rational numbers by average (mean) method.

We know that

4,3,2 are natural numbers between 5 and 1.

Is there any natural number between 2 and 3

-2,-1,0,1,2 are integers between -3 and 3.

Is there any integer between two consecutive integers?

There is no integer between two consecutive integers. But we can find rational numbers between two consecutive integers.

Example 5 Find a rational number between 2 and 3.

Solution $\frac{2+3}{2}=\frac{5}{2}$

So $2<\frac{5}{2}<3$

Example 6 Find a rational number between $\frac{3}{5}$ and $\frac{7}{2}$.

Solution

$$\frac{\frac{3}{5}+\frac{7}{2}}{2}$$

$$=\frac{\frac{6+35}{10}}{2}$$

$$=\frac{\frac{41}{10}}{2}$$

$$=\frac{41}{20}$$

By the above examples it is clear that to find the rational numbers between two rational numbers a and b, sum of rational numbers a and b is divided by 2.

$$\text{Mean} = \frac{a+b}{2}$$

Example 7 Find three rational numbers between 3 and 4.

Solution Rational numbers between 3 and 4 is $= \frac{3+4}{2}$

$$= \frac{7}{2}$$

Therefore $3 < \frac{7}{2} < 4$

Rational numbers between 3 and $\frac{7}{2}$ is $= \dfrac{3+\frac{7}{2}}{2} = \dfrac{\frac{6+7}{2}}{2} = \dfrac{\frac{13}{2}}{2} = \dfrac{13}{4}$

So $3 < \frac{13}{4} < \frac{7}{2} < 4$

Rational numbers between $\frac{7}{2}$ and 4 is $= \dfrac{\frac{7}{2}+4}{2} = \dfrac{\frac{7+8}{2}}{2} = \dfrac{\frac{15}{2}}{2} = \dfrac{15}{4}$

So $3 < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < 4$

Thus, rational numbers between 3 and 4 are $\frac{13}{4}$, $\frac{7}{2}$ and $\frac{15}{4}$.

In this manner, we can obtain as many (infinite) rational numbers between two given rational numbers.

Do and Learn

1. Find rational number between -1 and 2.
2. Find rational number between $\frac{2}{3}$ and $\frac{3}{4}$.
3. Find three rational numbers between 2 and 3.

Exercise 1

1. Add the following rational numbers (solve any two on number line)-

(i) $\frac{5}{2}+\left(\frac{-3}{4}\right)$ (ii) $\frac{-2}{3}+\left(\frac{-4}{5}\right)+\frac{5}{6}$ (iii) $0+\frac{-2}{3}$

(iv) $-2\frac{1}{3}+4\frac{3}{5}$ (v) $\frac{-6}{5}+\left(\frac{-13}{7}\right)$ (vi) $\frac{-8}{19}+\frac{(-4)}{57}$

2. Find the value (solve any two on number line)-

(i) $\frac{2}{3}+\frac{5}{4}$ (ii) $-2\frac{1}{9}+7$ (iii) $\frac{-7}{16}+\frac{(-3)}{48}$

(iv) $\frac{-7}{63}+\left(\frac{-5}{21}\right)$ (v) $\frac{-2}{13}+\frac{(-1)}{7}$ (vi) $4\frac{3}{5}-\left(-2\frac{1}{3}\right)$

3. Multiply the following rational numbers-

(i) $\frac{13}{15}\times 5$ (ii) $\frac{4}{-5}\times\frac{-5}{4}$ (iii) $\frac{-2}{5}\times\left(\frac{-3}{7}\right)$

(iv) $\frac{15}{18}\times\frac{5}{6}\times\frac{21}{5}$ (v) $\frac{9}{4}\times\left(\frac{-7}{5}\right)\times\left(\frac{-6}{21}\right)$ (vi) $2\frac{1}{9}\times\left(-3\frac{1}{2}\right)$

4. Find the value-

(i) $(-6)\div\frac{3}{5}$ (ii) $\frac{-27}{5}\div\left(\frac{-54}{10}\right)$ (iii) $\frac{21}{36}\div\left(\frac{-7}{18}\right)$

(iv) $\frac{-7}{12}\div\left(\frac{-3}{13}\right)$ (v) $-2\frac{1}{9}\div 6\frac{1}{9}$ (vi) $\frac{2}{15}\div\left(\frac{-8}{45}\right)$

5. Find the value-

(i) $\frac{3}{5}+\frac{7}{10}+\left(\frac{-8}{12}\right)+\frac{4}{3}$ (ii) $2\frac{1}{2}+\left(-3\frac{1}{2}\right)+\left(-2\frac{1}{3}\right)+\left(2\frac{1}{9}\right)$

(iii) $\left(\frac{-7}{5}\right)\times\frac{2}{3}\times\frac{15}{16}\times\left(\frac{-8}{9}\right)$ (iv) $\frac{1}{2}\div\left[\left(\frac{-1}{3}\right)\div\frac{2}{7}\right]$

6. Find the value of the following using the appropriate properties-

(i) $\frac{3}{5}\times\left(\frac{-3}{7}\right)-\frac{2}{7}\times\frac{3}{2}+\frac{3}{15}\times\frac{5}{9}$ (ii) $\frac{5}{2}-\frac{3}{5}\times\frac{7}{2}+\frac{3}{5}\times\left(\frac{-2}{3}\right)$

7. Find the additive inverse of the following rational numbers –

(i) $\frac{7}{19}$ (ii) $\frac{-9}{5}$ (iii) $\frac{-3}{-7}$

(iv) $\frac{5}{-9}$ (v) $\frac{-13}{-17}$ (vi) $\frac{21}{-31}$

8. Find the multiplicative inverse of the following rational numbers-

(i) -17 (ii) $\frac{-11}{17}$ (iii) $-1\times\frac{-3}{5}$ (iv) $\frac{13}{-19}$

9. Multiply the rational number $\frac{5}{7}$ to inverse of $\frac{-7}{15}$.

10. Fill in the blanks-

(i) Product of two rational numbers is always...............(rational/integers).

(ii) Additive inverse of any negative rational number is .... (negative/positive).

(iii) Inverse of zero is...(zero/ indetermined).

(iv) Additive identity of rational number is.......................(zero/one).

(v) Multiplicative identity for rational number is....................(zero/one).

(vi) Reciprocal of rational number is............of that (inverse/same).

(vii) Negative rational number on number line is always lies on .... of zero (right/left).

(viii) Positive rational number on number line is always lies on .... of zero (right/left).

(ix) When rational number is added with its additive inverse then result is always...............(zero/same).

(x) When rational number is divided by same rational number then result is always...............(zero/one).

11.By mean method-

(i) Write any five rational numbers between -3 and 0.

(ii) Write any four rational numbers larger than 0 and smaller than $\frac{5}{6}$.

(iii) Find any three rational numbers between $\frac{-3}{4}$ and $\frac{5}{6}$.

We Learnt

1. When two rational numbers having common denominators are added or subtracted then their numerators are added while keeping the denominator same.
2. To add or subtract rational numbers with distinct denominators , LCM are taken of their denominators.
3. For product of rational numbers, numerators are multiplied by numerators and denominators are multiplied by numerators.
4. To divide rational number by any other rational number, that rational number is multiplied by inverse of any other rational number.
5. Product of any rational number with its inverse is always 1.
6. Rational numbers are closure under addition, subtraction and multiplication operations.
7. Addition and multiplication operations are commutative and associative for rational numbers.
8. For rational numbers, 0(zero) is additive identity and 1(one) is multiplicative identity.
9. Additive inverse of rational number $\frac{a}{b}$ is $\frac{-a}{b}$ and vice-versa. Similarly, multiplicative inverse of rational number $\frac{a}{b}$ is $\frac{b}{a}$ and vice-versa.
10. Distributive law-
    If a, b and c are rational numbers then-

    $$a \times (b + c) = a \times b + a \times c$$
    $$a \times (b - c) = a \times b + a \times c$$

11. There are so many rational numbers between any two rational numbers. Rational numbers between two rational numbers can be determined by the mean method.