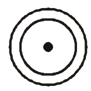
CIRCLE

A. (1) DEFINITION

A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point remains constant. The fixed point is called the centre of the circle and the constant distance is called the radius of the circle.

(2) BASIC THEOREMS AND RESULTS OF CIRCLES

(a) Concentric circles : Circles having same centre.



(b) Congruent circles : Iff their radii are equal.

(c) Congruent arcs : Iff they have same degree measure at the centre.

Theorem 1 :

(i) If two arcs of a circle (or of congruent circles) are congruent, the corresponding chords are equal.
Converse : If two chords of a circle are equal then their corresponding arcs are congruent.

(ii) Equal chords of a circle (or of congruent circles) subtend equal angles at the centre.
Converse : If the angle subtended by two chords of a circle (or of congruent circle) at the centre are equal, the chords are equal.

Theorem 2 :

(i) The perpendicular from the centre of a circle to a chord bisects the chord.
Converse : The line joining the mid point of a chord to the centre of a circle is perpendicular to the chord.

(ii) Perpendicular bisectors of two chords of a circle intersect at tis centre.

Theorem 3 :

(i) There is one and only one circle passing through three non collinear points.

(ii) If two circles intersects in two points, then the line joining the centres is perpendicular bisector of common chords.

Theorem 4 :

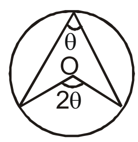
(i) Equal chords of a circle (or of congruent circles) are equidistant from the centre.
Converse : Chords of a circle (or of congruent circles) which are equidistant from the centre are equal.

(ii) If two equal chords are drawn from a point on the circle, then the centre of circle will lie on angle bisector of these two chords.

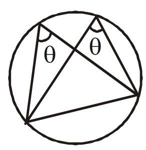
(iii) Of any two chords of a circle larger will be near to centre.

Theorem 5 :

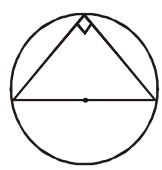
(i) The degree measure of an arc or angle subtended by an arc at the centre is double the angle subtended by it at any point of alternate segment.



(ii) Angle in the same segment of a circle are equal.



(iii) The angle in a semi circle is right angle.



Converse : The arc of a circle subtending a right angle in alternate segment is semi circle.

Theorem 6 :
An angle subtended by a minor arc in the alternate segment is acute and any angle subtended by a major arc in the alternate segment is obtuse.

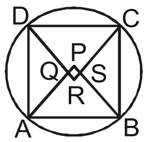
Theorem 7 :
If a line segment joining two points subtends equal angles at two other points lying on the same side of the line segment, the four points are concyclic, i.e. lie on the same circle.

(d) Cyclic Quadrilaterals : A quadrilateral is called a cyclic quadrilateral if its all vertices lie on a circle.

Theorem 1 :
The sum of either pair of opposite angles of a cyclic quadrilateral is 180º.
OR
The opposite angles of a cyclic quadrilateral are supplementary.
Converse : If the sum of any pair of opposite angle of a quadrilateral is 180º, then the quadrilateral is cyclic.

Theorem 2 :
If a side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle.

Theorem 3 :
The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.



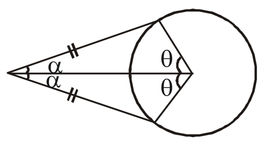
Theorem 4 :
If two sides of cyclic quadrilateral are parallel then the remaining two sides are equal and the diagonals are also equal.
OR
A cyclic trapezium is isosceles and its diagonals are equal.
Converse : If two non-parallel sides of a trapezium are equal then it is cyclic.
OR
An isosceles trapezium is always cyclic,

Theorem 5 :
The bisectors of the angles formed by producing the opposite sides of a cyclic quadrilateral (provided they are not parallel), intersect at right angle.

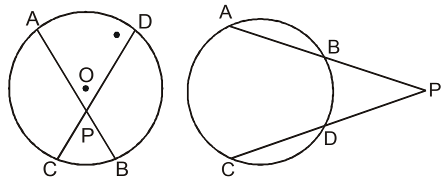
(3) TANGENTS TO A CIRCLE

Theorem 1 :
A tangent to a circle is perpendicular to the radius through the point of contact.
Converse : A line drawn through the end point of a radius and perpendicular to it is a tangent to the circle.

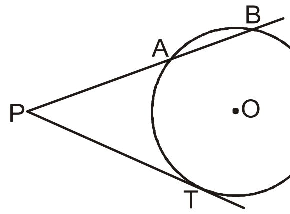
Theorem 2 :
If two tangents are drawn to a circle from an external point, then :
(i) they are equal
(ii) the subtend equal angles at the centre,
(iii) they are equally inclined to the segment, joining the centre to that point.



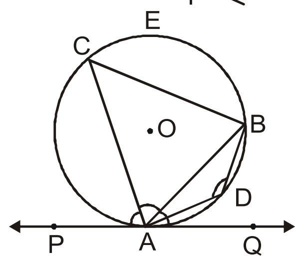
Theorem 3 :
If two chords of a circle intersect inside or outside the circle when produced, the rectangle formed by the segments of one chord is equal in area to the rectangle formed by the two segments of the other chord
PA × PB = PC × PD



Theorem 4 :
If PAB is a secant to a circle intersecting the circle at A and B and PT is tangent segment, the PA × PB = PT^2
OR
Area of the rectangle formed by the two segments of a chord is equal to the area of the square of side equal to the length of the tangent from the point on the circle.



Theorem 5 :
If a chord is drawn through the point of contact of tangent to a circle, then the angles which this chord makes with the given tangent are equal respectively to the angles formed in the corresponding alternate segments.
$\angle BAQ = \angle ACB$ and $\angle BAP = \angle ADB$



Converse :
If a line is drawn through an end point of a chord of a circle so that the angle formed with the chord is equal to the angle subtended by the chord in the alternate segment, then the line is a tangent to the circle.

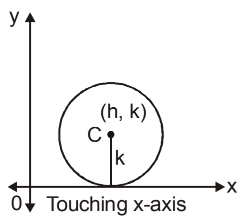
B. STANDARD EQUATIONS OF THE CIRCLE

(a) Central Form :
If (h, k) is the centre and r is the radius of the circle then its equation is $(x - h)^2 + (y - k)^2 = r^2$

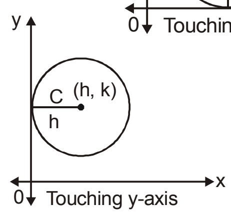
Special Cases :

(i) If centre is origin (0, 0) and radius is 'r' then equation of circle is $x^2 + y^2 = r^2$ and this is called the standard form.

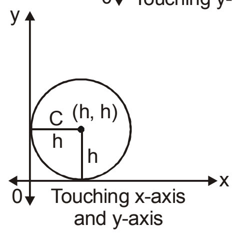
(ii) If radius of circle is zero then equation of circle is $(x - h)^2 + (y - k)^2 = 0$.
Such circle is called zero circle or point circle.



(iii) When circle touches x-axis
then equation of the circle is
$(x - h)^2 + (y - k)^2 = k^2$.
or $x^2 + y^2 - 2hx - 2ky + h^2 = 0$

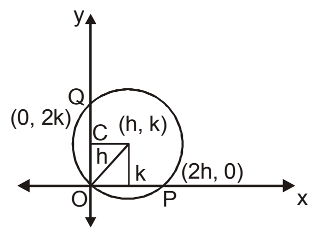


(iv) When circle touches y-axis
then equation of the circle is
$(x - h)^2 + (y - k)^2 = h^2$
or $x^2 + y^2 - 2hx - 2ky + k^2 = 0$



(v) When circle touches both the axes (x-axis and y-axis)
then equation of the circle is
$(x - h)^2 + (y - h)^2 = h^2$
or $x^2 + y^2 - 2hx - 2hy + h^2 = 0$

(vi) When circle passes through the origin and centre of the circle is (h, k) then radius $\sqrt{h^2+k^2}$ = r and intercept cut on x-axis OP = 2h, and intercept cut on y-axis is OQ = 2k and equation of circle is
$(x - h)^2 + (y - k)^2 = h^2 + k^2$ or $x^2 + y^2 - 2hx - 2hy = 0$



Note : Circle may exist in any quadrant hence for general cases use ± sign before h and k.

(b) General equation of circle :

$x^2 + y^2 + 2gx + 2fy + c = 0$. where g, f, c are constants and centre is (–g, –f)

i.e. $\left(-\frac{\text{coefficient of } x}{2}, -\frac{\text{coefficient of } y}{2}\right)$ and radius r = $\sqrt{g^2+f^2-c}$

Notes :

(i) If $(g^2 + f^2 - c) > 0$, then r is real and positive and the circle is a real circle.

(ii) If $(g^2 + f^2 - c) = 0$, then radius r = 0 and circle is a point circle.

(iii) If $(g^2 + f^2 - c) < 0$, then r is imaginary then circle is also an imaginary circle with real centre.

(iv) $x^2 + y^2 + 2gx + 2fy + c = 0$, has three constants and to get the equation of the circle at least three conditions should be known $\Rightarrow$ A unique circle passes through three non collinear points.

(v) **The general quadratic equation in x and y, $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ represents a circle if :**

- coefficient of x^2 = coefficient of y^2 or $a = b \neq 0$
- coefficient of xy = 0 or h = 0
- $(g^2 + f^2 - c) \geq 0$ (for a real circle)

(c) Intercepts cut by the circle on axes :

The intercepts cut by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on :

(i) x-axis = $\left|2\sqrt{g^2-c}\right|$ **(ii)** y-axis = $\left|2\sqrt{f^2-c}\right|$

Notes :

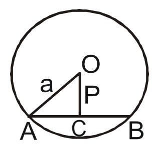
(i) If the circle cuts the x-axis at two distinct point then $g^2 - c > 0$

(ii) If circle touches x-axis then $g^2 = c$.

(iii) If circle touches y-axis $f^2 = c$.

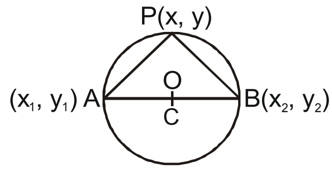
(iv) Circle lies completely above or below the x-axis then $g^2 < c$.

(v) Intercept cut by a line on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ or length of chord of the circle = $2\sqrt{a^2-P^2}$ where a is the radius and P is the length of perpendicular from the centre to the chord.



(d) Diametrical form of circle :

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are the end points of the diameter of the circle and $P(x, y)$ is the point other then A and B on the circle then from geometry we know that $\angle APB = 90^\circ$.



$\Rightarrow$ (Slope of PA) × (Slope of PB) = −1

$$\Rightarrow \left(\frac{y-y_1}{x-x_1}\right)\left(\frac{y-y_2}{x-x_2}\right) = -1 \quad \Rightarrow (x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

Note : This will be the circle of least radius passing through (x_1, y_1) and (x_2, y_2)

(e) The parametric forms of the circle :

(i) The parametric equation of the circle $x^2 + y^2 = r^2$ are $x = r\cos\theta$, $y = r\sin\theta$; $\theta \in [0, 2\pi]$

(ii) The parametric equation of the circle $(x - h)^2 + (y - k)^2 = r^2$ is $x = h + r\cos\theta$, $y = k + r\sin\theta$ where θ is parameter.

(iii) The parametric equation of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are $x = -g + \sqrt{g^2 + f^2 - c}\cos\theta$

$y = -f + \sqrt{g^2 + f^2 - c}\sin\theta$ where θ is parameter.

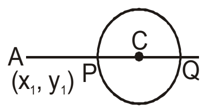
Note that equation of a straight line joining two point α and β on the circle $x^2 + y^2 = a^2$ is

$$x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = a\cos\frac{\alpha-\beta}{2}.$$

C. POSITION OF A POINT W.R.T. CIRCLE

(a) Let the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ and the point is (x_1, y_1) then point (x_1, y_1) lies out side the circle or on the circle or inside the circle according as

$$\Rightarrow \quad x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c >, =, < 0 \text{ or } S_1 >, =, < 0$$



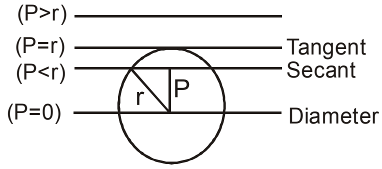
(b) The greatest & the least distance of a point A from a circle with centre C & radius r is AC + r & AC − r respectively.

D. TANGENT LINE OF CIRCLE

When a straight line meet a circle on two coincident points then it is called the tangent of the circle.

(a) Condition of Tangency :

The line L = 0 touches the circle S = 0 if P the length of the perpendicular from the centre to that line and radius of the circle r are equal i.e. P = r.



(b) Equation of the tangent (T = 0) :

(i) Tangent at the point (x_1, y_1) on the circle $x^2 + y^2 = a^2$ is $xx_1 + yy_1 = a^2$.

(ii) (1) The tangent at the point (acos t, asin t) on the circle $x^2 + y^2 = a^2$ is $x\cos t + y\sin t = a$

(2) The point of intersection of the tangents at the points $P(\alpha)$ and $Q(\beta)$ is $\dfrac{a\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, \dfrac{a\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}$

(iii) The equation of tangent at the point (x_1, y_1) on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

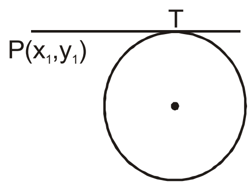
(iv) If line y = mx + c is a straight line touching the circle $x^2 + y^2 = a^2$, then $c = \pm a\sqrt{1+m^2}$ and contact points are $\left(\mp\dfrac{am}{\sqrt{1+m^2}}, \mp\dfrac{a}{\sqrt{1+m^2}}\right)$ or $\left(\mp\dfrac{a^2m}{c}, \pm\dfrac{a^2}{c}\right)$ and equation of tangent is $y = mx \pm a\sqrt{1+m^2}$

(v) The equation of tangent with slope m of the circle $(x-h)^2 + (y-k)^2 = a^2$ is

$(y-k) = m(x-h) \pm a\sqrt{1+m^2}$

Note : To get the equation of tangent at the point (x_1, y_1) on any curve we replace xx_1 in place of x^2, yy_1 in place of y^2, $\dfrac{x+x_1}{2}$ in place of x, $\dfrac{y+y_1}{2}$ in place of y, $\dfrac{xy_1 + yx_1}{2}$ in place of xy and c in place of c.

(c) Length of tangent $(\sqrt{S_1})$:

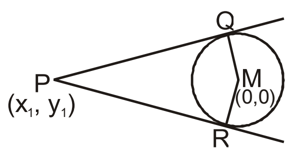


The length of tangent draw from point (x_1, y_1) out side the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is,

$PT = \sqrt{S_1} = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$

Note : When we use this formula the coefficient of x^2 and y^2 must be 1.

(d) Equation of Pair to tangents ($SS_1 = T^2$) :



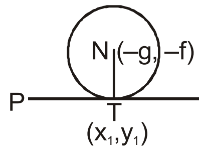
Let the equation of circle $X \equiv x^2 + y^2 = a^2$ and $P(x_1, y_1)$ is any point outside the circle. From the point we can draw two real and distinct tangent PQ & PR and combine equation of pair of tangents is $(x^2 + y^2 - a^2)(x_1^2 + y_1^2 - a^2) = (xx_1 + yy_1 - a^2)^2$ or $SS_1 = T^2$

E. NORMAL OF CIRCLE

Normal at a point of the circle is the straight line which is perpendicular to the tangent at the point of contact and passes through the centre of circle.

(a) Equation of normal at point (x_1, y_1)

or circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

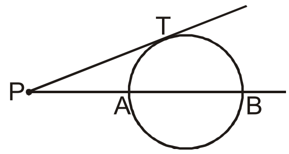


$$y - y_1 = \left(\frac{y_1 + f}{x_1 + g}\right)(x - x_1)$$

(b) The equation of normal on any point (x_1, y_1) of circle $x^2 + y^2 = a^2$ is $\left(\frac{y}{x} = \frac{y_1}{x_1}\right)$

(c) If $x^2 + y^2 = a^2$ is the equation of the circle then at any point 't' of this circle (a cos t, a sin t), the equation of normal is x sin t – y cos t = 0.

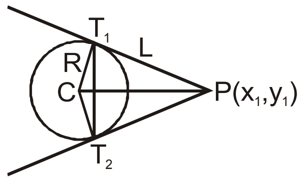
F. POWER OF THE POINT



square of the length of the tangent from the point P is defined as power of the point 'p' w.r. to given circle $\Rightarrow PT^2 = S_1$

Note :- Power of a point remains constant w.r. to a circle $PA \cdot PB = PT^2$

G. CHORD OF CONTACT



A line joining the two points of contacts of two tangents drawn from a point out side the circle, is called chord of contact of that point.
If two tangents PT_1 & PT_2 are drawn from the point $P(x_1, y_1)$ to the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then the equation of the chord of contact T_1T_2 is : $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
(i.e. T = 0 same as equation of tangent).

Remember :

(a) Length of chord of contact $T_1 T_2 = \frac{2LR}{\sqrt{R^2 + L^2}}$.

(b) Area of the triangle formed by the pair of the tangents & its chord of contact $= \frac{RL^3}{R^2 + L^2}$ Where R is the radius of the circle & L is the length of the tangent from (x_1, y_1) on S = 0.

(c) Angle between the pair of tangents from $P(x_1, y_1) = \tan^{-1}\left(\frac{2RL}{L^2 - R^2}\right)$

(d) Equation of the circle circumscribing the triangle PT_1T_2 or quadrilateral CT_1PT_2 is :
$(x - x_1)(x + g) + (y - y_1)(y + f) = 0$.

(e) The joint equation of a pair of tangents drawn from the point A (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is : $SS_1 = T^2$.
Where $S \equiv x^2 + y^2 + 2gx + 2fy + c$; $S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$
$T \equiv xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$.

H. EQUATION OF THE CHORD WITH A GIVEN MIDDLE POINT ($T = S_1$)

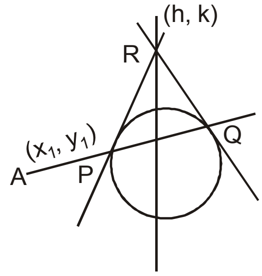
The equation of the chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ in terms of its mid point $M(x_1, y_1)$ is $y - y_1 = -\frac{x_1 + g}{y_1 + f}(x - x_1)$. This on simplification can be put in the form $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ which is designated by $T = S_1$.

Note that : The shortest chord of a circle passing through a point 'M' inside the circle, is one chord whose middle point is M.

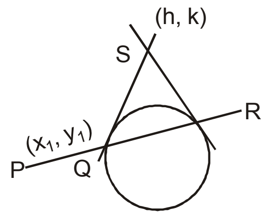
I. DIRECTOR CIRCLE

The locus of point of intersection of two perpendicular tangents to a circle is called director circle. Let P(h, k) is the point of intersection of two tangents drawn on the circle $x^2 + y^2 = a^2$. Then the equation of the pair of tangents is $SS_1 = T^2$. i.e. $(x^2 + y^2 - a^2)(h^2 + k^2 - a^2) = (hx + ky - a^2)^2$
As lines are perpendicular to each then, coefficient of x^2 + coefficient of $y^2 = 0$.
$\Rightarrow$ $[(h^2 + k^2 - a^2) - h_2] + [(h^2 + k^2 - a^2) - k^2] = 0$ $\Rightarrow$ $h^2 + k^2 = 2a^2$
$\therefore$ locus of (h, k) is $x^2 + y^2 = 2a^2$ which is the equation of the director circle.
$\therefore$ director circle is a concentric circle whose radius is $\sqrt{2}$ times the radius of the circle.
Note : The director circle of $x^2 + y^2 + 2gx + 2fy + c = 0$ is $x^2 + y^2 + 2gx + 2fy + 2c - g^2 - f^2 = 0$

J. POLE AND POLAR



Let any straight line through the given point $A(x_1 , y_1)$ intersect the given circle S = 0 in two points P' and Q and if the tangent of the circle at P and Q meet at the point R then locus of point R is called polar of the point A and point A is called the pole, with respect to the given circle.



(a) The equation of the polar of point (x_1 , y_1) w.r.t. circle $x^2 + y^2 = a^2$.
Let PQR is a chord which passes through the point $P(x_1, y_1)$ which intersects the circle at point Q and R and the tangents are drawn at points Q and R meet at point S(h, k) then equation of QR the chord of contact is $x_1h + y_1k = a^2$
$\therefore$ locus of point S(h, k) is $xx_1 + yy_1 = a^2$ which is the equation of the polar.

(i) The equation of the polar is the T = 0, so the polar of point (x_1, y_1) w.r.t. circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.
(ii) If point is outside the circle then equation of polar and chord of contact is same. So the chord of contact is polar.
(iii) If point is inside the circle then chord of contact does not exist but polar exists.
(iv) If point lies on the circle then polar, chord of contact and tangent on that point are same.
(v) If the polar of P w.r.t. a circle passes through the point Q, then the polar of point Q will pass through P and hence P and Q are conjugate points of each other w.r.t. the given circle.
(vi) If pole of a line w.r.t. a circle lies on second line. Then pole of second line lies on first line and hence both lines are conjugate lines of each other w.r.t. the given circle.

(b) Pole of a given line with respect to a circle.

To find the pole of a line we assume the coordinates of the pole then from these coordinates we find the polar. This polar and given line represent the same line. Then by comparing the coefficients of similar terms we can get the coordinates of the pole. The pole of $\ell x + my + n = 0$.

w.r.t. circle $x^2 + y^2 = a^2$ will be $\left(\frac{-\ell a^2}{n}, \frac{-ma^2}{n}\right)$.

K. FAMILY OF CIRCLES

(a) The equation of the family of circles passing through the points of intersection of two circles $S_1 = 0$ and $S_2 = 0$ is ; $S_1 + KS_2 = 0$ $(K \neq -1)$.

(b) The equation of the family of circles passing through the point of intersection of a circle S = 0 and line L = 0 is given by S + KL = 0,

(c) The equation of a family of circles passing through two given points (x_1, y_1) and (x_2, y_2) can be written in the form $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + K \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$ where K is a parameter.

(d) The equation of a family of circles touching a fixed line $y - y_1 = m(x - x_1)$ at the fixed point (x_1, y_1) is $(x - x_1)^2 + (y - y_1)^2 + K[y - y_1 - m(x - x_1)] = 0$, where K is a parameter.

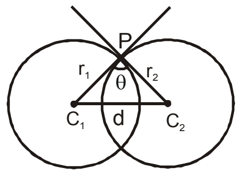
(e) Family of circles cicumscribing a triangle whose sides are given by $L_1 = 0$; $L_2 = 0$ and $L_3 = 0$ is given by ; $L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$ provided coefficient of xy = 0 and coefficient of x^2 = coefficient of y^2

(f) Equation of circle circumscribing a quadrilateral whose side in order are represented by the line $L_1=0$, $L_2 = 0$, $L_3 = 0$ and $L_4 = 0$ are $L_1L_3 + \lambda L_2L_4 = 0$ provided coefficient of x^2 = coefficient of y^2 and coefficient of xy = 0.

L. THE ANGLE OF INTERSECTION OF TWO CIRCLES

Definition : The angle between the tangents of two circles at the point of intersection of the two circles is called angle of intersection of two circles. If two circles are $S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

$S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ and θ is the angle between them

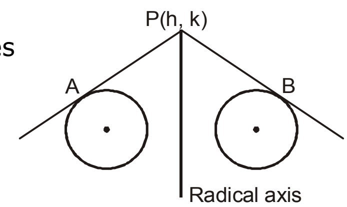


then $\cos\theta = \dfrac{2g_1g_2 + 2f_1f_2 - c_1 - c_2}{\sqrt{g_1^2 + f_1^2 - c_1}\,\sqrt{g_2^2 + f_2^2 - c_2}}$ or $\cos\theta = \left(\dfrac{r_1^2 + r_2^2 - d^2}{2r_1r_2}\right)$

Here a_1 and a_2 are the radii of the circles and d is the distance between their centres.
If the angle of intersection of the two circles is a right angle then such circles are called **"Orthogonal circles"** and conditions for the circles to be orthogonal is $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

M. RADICAL AXIS OF THE TWO CIRCLES ($S_1 - S_2 = 0$)

(a) Definition : The locus of a point, which moves in such a way that the length of tangents drawn from it to the circles are equal and is called the radical axis. It two circles are -



$$S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$$

$$S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$$

Let P(h, k) is a point and PA, PB are length of two tangents on the circles from point P. Then from definition : $\sqrt{h^2 + k^2 + 2g_1h + 2f_1k + c_1} = \sqrt{h^2 + k^2 + 2g_2h + 2f_2k + c_2}$ or $2(g_1 - g_2)h + 2(f_1 - f_2)k + c_1 - c_2 = 0$

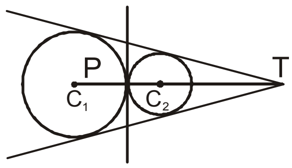
$\therefore$ locus of (h, k) $\Rightarrow$ $2x(g_1 - g_2) + 2y(f_1 - f_2)k + c_1 - c_2 = 0 \Rightarrow$ $\mathbf{S_1 - S_2 = 0}$
which is the equation of radical axis.

N. DIRECT AND TRANSVERSE COMMON TANGENTS

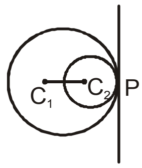
Let two circles having centre C_1 and C_2 and radii, r_1 and r_2 and C_1C_2 is the distance between their centres then

(a) Both circles will touch :

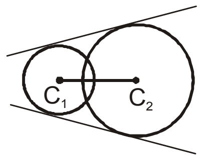
(i) Externally : If $c_1c_2 = r_1 + r_2$ i.e. the distance between their centres is equal to sum of their radii and point P divides C_1C_2 in the ratio $r_1 : r_2$ (internally). In this case there will be **three common tangents.**



(ii) Internally : If $C_1C_2 = |r_1 - r_2|$ i.e. the distance between their centres is equal to difference between their radii and point P divides C_1C_2 in the ratio $r_1 : r_2$ **externally** and in this case there will be **only one common tangent.**

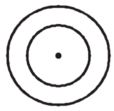


(b) The circles will intersect : when $|r_1 - r_2| < C_1C_2 < r_1 + r_2$ in this case there will be **two common tangents.**

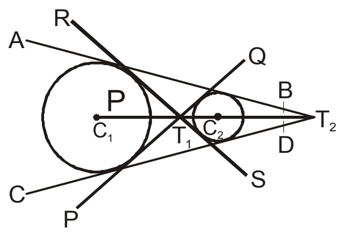


(c) The circles will not intersect :

(i) One circle will lie inside the other circle if $C_1C_2 < |r_1 - r_2|$ In this case there will be **no common tangent.**



(ii) When circle are apart from each other the $C_1C_2 > r_1 + r_2$ and in this case there will be four common tangents. Lines PQ and RS are called **transverse or indirect or internal common tangents** and these lines meet line C_1C_2 on T_1 and T_1 divides the line C_1C_2 in the ratio $r_1 : r_2$ internally and lines AB & CD are called **direct or external common tangents.** These lines meet C_1C_2 produced on T_2. Thus T_2 divides C_1C_2 externally in the ratio $r_1 : r_2$.



Note : Length of direct common tangent = $\sqrt{(C_1C_2)^2 - (r_1 - r_2)^2}$

Length of transverse common tangent = $\sqrt{(C_1C_2)^2 - (r_1 + r_2)^2}$

1. If a be the radius of a circle which touches x-axis at the origin, then its equation is
(A) $x^2 + y^2 + ax = 0$ (B) $x^2 + y^2 \pm 2ya = 0$
(C) $x^2 + y^2 \pm 2xa = 0$ (D) $x^2 + y^2 + ya = 0$

2. The equation of the circle passing through (3, 6) and whose centre is (2, –1) is
(A) $x^2 + y^2 - 4x + 2y = 45$ (B) $x^2+y^2-4x-2y+45=0$
(C) $x^2 + y^2 + 4x - 2y = 45$ (D) $x^2+y^2-4x+2y+45=0$

3. The equation of a circle which passes through the three points (3, 0) (1, –6), (4, –1) is
(A) $2x^2 + 2y^2 + 5x - 11y + 3 = 0$
(B) $x^2 + y^2 - 5x + 11y - 3 = 0$
(C) $x^2 + y^2 + 5x - 11y + 3 = 0$
(D) $2x^2 + 2y^2 - 5x + 11y - 3 = 0$

4. B and C are fixed point having co-ordinates (3, 0) and (–3, 0) respectively. If the vertical angle BAC is 90º, then the locus of the centroid of the ΔABC has the equation
(A) $x^2 + y^2 = 1$ (B) $x^2 + y^2 = 2$
(C) $9(x^2 + y^2) = 1$ (D) $9(x^2 + y^2) = 4$

5. $y = \sqrt{3}x + c_1$ & $y = \sqrt{3}x + c_2$ are two parallel tangents of a circle of radius 2 units, then $|c_1 - c_2|$ is equal to
(A) 8 (B) 4
(C) 2 (D) 1

6. If (6, –3) is the one extremity of diameter to the circle $x^2 + y^2 - 3x + 8y - 4 = 0$ then its other extremity is -
(A) (3/2, –4) (B) (–3, –5)
(C) (3, –5) (D) (3, 5)

7. If $y = 2x + K$ is a diameter to the circle $2(x^2 + y^2) + 3x + 4y - 1 = 0$, then K equals
(A) 0 (B) 1
(C) 2 (D) 1/2

8. $\ell x + my + n = 0$ is a tangent line to the circle $x^2 + y^2 = r^2$, if
(A) $\ell^2 + m^2 = n^2 r^2$ (B) $\ell^2 + m^2 = n^2 + r^2$
(C) $n^2 = r^2(\ell^2 + m^2)$ (D) none of these

9. If y=c is a tangent to the circle $x^2+y^2-2x+2y-2=0$ at (1, 1), then the value of c is
(A) 1 (B) 2
(C) –1 (D) –2

10. The greatest distance of the point P(10, 7) from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is
(A) 5 (B) 15
(C) 10 (D) none of these

11. The parametric coordinates of any point on the circle $x^2 + y^2 - 4x - 4y = 0$ are
(A) $(-2 + 2\cos\alpha, -2 + 2\sin\alpha)$
(B) $(2 + 2\cos\alpha, 2 + 2\sin\alpha)$
(C) $(2 + 2\sqrt{2}\cos\alpha, 2 + 2\sqrt{2}\sin\alpha)$
(D) none of these

12. Cartesian equations of a circle whose parametric equation are $x = -7 + 4\cos q$, $y = 3 + 4\sin q$ is -
(A) $(x + 7)^2 + (y - 3)^2 = 16$
(B) $(x - 7)^2 + (y - 3)^2 = 16$
(C) $(x - 7)^2 + (y + 3)^2 = 16$
(D) $(x + 7)^2 + (y + 3)^2 = 16$

13. The length of the tangent drawn from the point (2, 3) to the circles $2(x^2 + y^2) - 7x + 9y - 11 = 0$.
(A) 18 (B) 14
(C) $\sqrt{14}$ (D) $\sqrt{28}$

14. The point from which the tangents to the circles $x^2 + y^2 - 8x + 40 = 0$, $5x^2 + 5y^2 - 25x + 80 = 0$ $x^2 + y^2 - 8x + 16y + 160 = 0$ are equal in length is
(A) $\left(8, \frac{15}{2}\right)$ (B) $\left(-8, \frac{15}{2}\right)$
(C) $\left(8, -\frac{15}{2}\right)$ (D) none of these

15. Two perpendicular tangents to the circle $x^2 + y^2 = a^2$ meet at P. Then the locus of P has the equation-
(A) $x^2 + y^2 = 2a^2$ (B) $x^2 + y^2 = 3a^2$
(C) $x^2 = y^2 = 4a^2$ (D) None of these

16. The locus of the mid-points of the chords of the circle $x^2 + y^2 - 2x - 4y - 11 = 0$ which subtend 60º at the centre is
(A) $x^2 + y^2 - 4x - 2y - 7 = 0$
(B) $x^2 + y^2 + 4x + 2y - 7 = 0$
(C) $x^2 + y^2 - 2x - 4y - 7 = 0$
(D) $x^2 + y^2 + 2x + 4y + 7 = 0$

17. Find the locus of mid point of chords of circle $x^2 + y^2 = 25$ which subtends right angle at origin-

(A) $x^2 + y^2 = 25/4$ (B) $x^2 + y^2 = 5$

(C) $x^2 + y^2 = 25/2$ (D) $x^2 + y^2 = 5/2$

18. The locus of the centres of the circles such that the point (2, 3) is the mid point of the chord $5x + 2y = 16$ is

(A) $2x - 5y + 11 = 0$ (B) $2x + 5y - 11 = 0$

(C) $2x + 5y + 11 = 0$ (D) none

19. Pair of tangents are drawn from every point on the line $3x + 4y = 12$ on the circle $x^2 + y^2 = 4$. Their variable chord of contact always passes through a fixed point whose co-ordinates are

(A) $\left(\frac{4}{3}, \frac{3}{4}\right)$ (B) $\left(\frac{3}{4}, \frac{3}{4}\right)$

(C) (1, 1) (D) $\left(1, \frac{4}{3}\right)$

20. Equation of the circle touching the circle $x^2 + y^2 - 15x + 5y = 0$ at the point (1, 2) and passing through the point (0, 2) is

(A) $13x^2 + 13y^2 - 13x - 61y + 70 = 0$

(B) $x^2 + y^2 + 2x = 0$

(C) $13x^2 + 13y^2 - 13x - 61y + 9 = 0$

(D) none of these

21. If the circle $x^2 + y^2 = 9$ touches the circle $x^2 + y^2 + 6y + c = 0$, then c is equal to

(A) –27 (B) 36

(C) –36 (D) 27

22. The equation of three circles are given $x^2 + y^2 = 1$, $x^2 + y^2 - 8x + 15 = 0$, $x^2 + y^2 + 10y + 24 = 0$. Determine the coordinates of the point P such that the tangents drawn from it to the circles are equal in length.

(A) (2, –5/2) (B) (–2, –5/2)

(C) (2, 5/2) (D) (3, –5/3)

23. Two given circles $x^2 + y^2 + ax + by + c = 0$ and $x^2 + y^2 + dx + ey + f = 0$ will intersect each other orthogonally, only when-

(A) $ad + be = c + f$ (B) $a + b + c = d + e + f$

(C) $ad + be = 2c+2f$ (D) $2ad + 2be = c + f$

24. Tangents are drawn to the circle $x^2 + y^2 = 1$ at the points where it is met by the circles. $x^2 + y^2 - (\lambda + 6)x + (8 - 2\lambda)y - 3 = 0$, λ being the variable. The locus of the point of intersection of these tangents is

(A) $2x - y + 10 = 0$ (B) $x + 2y - 10 = 0$

(C) $x - 2y + 10 = 0$ (D) $2x + y - 10 = 0$

25. (6, 0), (0, 6) and (7, 7) are the vertices of a triangle. The circle inscribed in the triangle has the equation

(A) $x^2 + y^2 - 9x + 9y + 36 = 0$

(B) $x^2 + y^2 - 9x - 9y + 36 = 0$

(C) $x^2 + y^2 + 9x - 9y + 36 = 0$

(D) $x^2 + y^2 - 9x - 9y - 36 = 0$

26. The equation to the circle whose radius is 4 and which touches the negative x-axis at a distance 3 units from the origin is

(A) $x^2 + y^2 - 6x + 8y - 9 = 0$

(B) $x^2 + y^2 \pm 6x - 8y + 9 = 0$

(C) $x^2 + y^2 + 6x \pm 8y + 9 = 0$

(D) $x^2 + y^2 \pm 6x - 8y - 9 = 0$

27. If a circle of constant radius 3k passes through the origin 'O' and meets co-ordinate axes at A and B then the locus of the centroid of the triangle OAB is

(A) $x^2 + y^2 = (2k)^2$ (B) $x^2 + y^2 = (3k)^2$

(C) $x^2 + y^2 = (4k)^2$ (D) $x^2 + y^2 = (6k)^2$

28. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq. units. Then the equation of the circle is

(A) $x^2 + y^2 - 2x + 2y = 62$

(B) $x^2 + y^2 + 2x - 2y = 62$

(C) $x^2 + y^2 + 2x - 2y = 47$

(D) $x^2 + y^2 - 2x + 2y = 47$

29. If the pair of line $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then –

(A) $3a^2 - 10ab + 3b^2 = 0$

(B) $3a^2 - 2ab + 3b^2 = 0$

(C) $3a^2 + 10ab + 3b^2 = 0$

(D) $3a^2 + 2ab + 3b^2 = 0$

30. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49p square units, the equation of the circle is -

(A) $x^2 + y^2 + 2x - 2y - 62 = 0$

(B) $x^2 + y^2 - 2x + 2y - 62 = 0$

(C) $x^2 + y^2 - 2x + 2y - 47 = 0$

(D) $x^2 + y^2 + 2x - 2y - 47 = 0$

31. A variable circle passes through the fixed point $A(p, q)$ and touches the x-axis. The locus of the other end of the diameter through A is

(A) $(x - p)^2 = 4qy$ (B) $(x - q)^2 = 4py$

(C) $(y - p)^2 = 4qx$ (D) $(y - q)^2 = 4px$

32. The centre of the circle touching the y-axis at (0, 3) and making an intercept of 2 units on the positive x-axis is

(A) $\left(10, \sqrt{3}\right)$ (B) $\left(\sqrt{3}, 10\right)$

(C) $\left(\sqrt{10}, 3\right)$ (D) $\left(3, \sqrt{10}\right)$

33. A circle touches a straight line $lx + my + n = 0$ and cuts the circle $x^2 + y^2 = 9$ orthogonally, The locus of centres of such circles is

(A) $(lx + my + n)^2 = (l^2 + m^2)(x^2 + y^2 - 9)$

(B) $(lx + my - n)^2 = (l^2 + m^2)(x^2 + y^2 - 9)$

(C) $(lx + my + n)^2 = (l^2 + m^2)(x^2 + y^2 + 9)$

(D) none of these

34. The locus of the centre of a circle which touches externally the circle, $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis is given by the equation

(A) $x^2 - 6x - 10y + 14 = 0$

(B) $x^2 - 10x - 6y + 14 = 0$

(C) $y^2 - 6x - 10y + 14 = 0$

(D) $y^2 - 10x - 6y + 14 = 0$

35. If $\left(a, \frac{1}{a}\right)$, $\left(b, \frac{1}{b}\right)$, $\left(c, \frac{1}{c}\right)$ & $\left(d, \frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units then, abcd =

(A) 4 (B) 1/4

(C) 1 (D) 16

36. The locus of the mid points of the chords of the circle $x^2 + y^2 - ax - by = 0$ which subtend a right angle at $\left(\frac{a}{2}, \frac{b}{2}\right)$ is

(A) $ax + by = 0$

(B) $ax + by = a^2 + b^2$

(C) $x^2 + y^2 - ax - by + \frac{a^2 + b^2}{8} = 0$

(D) $x^2 + y^2 - ax - by - \frac{a^2 + b^2}{8} = 0$

37. The length of the diameter of the circle which touches the x-axis at the pint (1, 0) and passes through the point (2, 3) **[AIEEE-2012]**

(A) 6/5 (B) 5/3

(C) 10/3 (D) 3/5

38. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point : **[JEE-MAIN 2013]**

(A) (5, –2) (B) (–2, 5)

(C) (–5, 2) (D) (2, –5)

39. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to: **[JEE-MAIN 2014]**

(A) $\frac{\sqrt{3}}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

40. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is : **[JEE-MAIN 2015]**

(A) 3 (B) 4

(C) 1 (D) 2

41. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is : **[JEE-MAIN 2016]**

(A) $5\sqrt{3}$ (B) 5

(C) 10 (D) $5\sqrt{2}$