Introduction to Trigonometry

8

Objective Section (1 mark each)

Fill in the blanks

Q. 1. The value of (tan 1° tan 2° ..... tan 89°) is equal to .... [CBSE OD, Set 1, 2020]

Ans. 1

Explanation :

tan 1° tan 2° .... tan 89°

= tan 1° tan 2° ..... tan 45° ... tan 88° tan 89°

= tan 1° tan 2° .... tan 45° ... tan (90° – 2°) tan (90° – 1°)

= tan 1° tan 2° .... tan 45° ..... cot 2° cot 1° [∵ tan (90° – θ) = cot θ]

$= \tan 1° \tan 2° ..... \tan 45° ... \dfrac{1}{\tan 2°} \dfrac{1}{\tan 1°}$

= tan 45°

= 1 **Ans.**

Q. 2. The value of sin 23° cos 67° + cos 23° sin 67° is .... [CBSE OD, Set 2, 2020]

Ans. 1

Explanation : sin 23° cos 67° + cos 23° sin 67°

= sin 23° cos (90° – 23°) + cos 23° sin (90° – 23°)

= sin 23°. sin 23° + cos 23°. cos 23°

$\begin{bmatrix} \because \ \cos(90° - \theta) = \sin\theta \\ \text{and } \sin(90° - \theta) = \cos\theta \end{bmatrix}$

$= \sin^2 23° + \cos^2 23°$ $[\because \sin^2\theta + \cos^2\theta = 1]$

= 1 **Ans.**

Q. 3. The value of sin 32° cos 58° + cos 32° sin 58° is .... [CBSE OD, Set 3, 2020]

Ans. 1

Explanation : sin 32° cos 58° + cos 32° sin 58°

= sin 32° cos (90° – 32°) + cos 32° sin (90° – 32°)

= sin 32° sin 32° + cos 32° cos 32°

$\begin{bmatrix} \because \ \cos(90° - \theta) = \sin\theta \\ \text{and } \sin(90° - \theta) = \cos\theta \end{bmatrix}$

$= \sin^2 32° + \cos^2 32°$ $[\because \sin^2\theta + \cos^2\theta = 1]$

= 1. **Ans.**

Q. 4. The value of $\dfrac{\tan 35°}{\cot 55°} + \dfrac{\cot 78°}{\tan 12°}$ is .... [CBSE OD, Set 3, 2020]

Ans. 2

Explanation : $\dfrac{\tan 35°}{\cot 55°} + \dfrac{\cot 78°}{\tan 12°}$

$= \dfrac{\tan 35°}{\cot(90° - 35°)} + \dfrac{\cot(90° - 12°)}{\tan 12°}$

$= \dfrac{\tan 35°}{\tan 35°} + \dfrac{\tan 12°}{\tan 12°}$ [∵ cot (90° – θ) = tan θ]

= 1 + 1 = 2 **Ans.**

Q. 5. $\dfrac{\cos 80°}{\sin 10°}$ + cos 59° cosec 31° = ... [CBSE Delhi, Set 1, 2020]

Ans. 2

Explanation : $\dfrac{\cos 80°}{\sin 10°}$ + cos 59° cosec 31°

$= \dfrac{\sin(90 - 80)°}{\sin 10°}$ + cos 59° sec(90 – 31)°

$\begin{bmatrix} \because \ \cos\theta = \sin(90° - \theta) \\ \text{cosec}\,\theta = \sec(90° - \theta) \end{bmatrix}$

$= \dfrac{\sin 10°}{\sin 10°}$ + cos 59°. sec 59°

$= 1 + \cos 59° \times \dfrac{1}{\cos 59°}$ $\left[\because \sec\theta = \dfrac{1}{\cos\theta}\right]$

= 1 + 1 = 2 **Ans.**

Q. 6. The value of $\left(\sin^2\theta + \dfrac{1}{1 + \tan^2\theta}\right)$ = ... [CBSE Delhi, Set 1, 2020]

Ans. 1

Explanation : $\sin^2\theta + \dfrac{1}{1 + \tan^2\theta}$

$= \sin^2\theta + \dfrac{1}{\sec^2\theta}$ $\{\because \sec^2\theta = 1 + \tan^2\theta\}$

$= \sin^2\theta + \left(\dfrac{1}{\sec\theta}\right)^2$

$= \sin^2\theta + \cos^2\theta$

$= 1 \qquad \{\because \sin^2\theta + \cos^2\theta = 1\}$ **Ans.**

Q. 7. The value of

$(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) = \ldots\ldots\ldots\ .$

[CBSE Delhi, Set-I, 2020]

Ans. 1

Explanation : $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$= (1 + \tan^2\theta)[(1)^2 - (\sin\theta)^2]$

$= (\sec^2\theta)(1 - \sin^2\theta) \qquad \{\because \sec^2\theta = 1 + \tan^2\theta\}$

$= \frac{1}{\cos^2\theta} \times \cos^2\theta \qquad [\because \cos^2\theta = 1 - \sin^2\theta]$

$= 1$

Q. 8. $\left(\frac{\sin 35^\circ}{\cos 55^\circ}\right)^2 + \left(\frac{\cos 43^\circ}{\sin 47^\circ}\right)^2 - 2\cos 60^\circ = \ldots\ldots\ldots\ .$

[CBSE Delhi, Set 2, 2020]

Ans. 1

Explanation: $\left(\frac{\sin 35^\circ}{\cos 55^\circ}\right)^2 + \left(\frac{\cos 43^\circ}{\sin 47^\circ}\right)^2 - 2\cos 60^\circ$

$= \left[\frac{\cos(90-35)^\circ}{\cos 55^\circ}\right]^2 + \left[\frac{\sin(90-43)^\circ}{\sin 47^\circ}\right]^2 - \cos 60^\circ$

$\left[\begin{array}{ll}\because & \cos(90^\circ - \theta) = \sin\theta \\ \text{and,} & \sin(90^\circ - \theta) = \cos\theta\end{array}\right]$

$= \left(\frac{\cos 55^\circ}{\cos 55^\circ}\right)^2 + \left(\frac{\sin 47^\circ}{\sin 47^\circ}\right)^2 - 2\cos 60^\circ$

$= (1)^2 + (1)^2 - 2 \times \frac{1}{2} \qquad \left(\because \cos 60^\circ = \frac{1}{2}\right)$

$= 1 + 1 - 1$

$= 1$

Q. 9. $\frac{2\cos 67^\circ}{\sin 23^\circ} - \frac{\tan 40^\circ}{\cot 50^\circ} - \cos 0^\circ = \ldots\ldots\ldots\ldots\ldots$

[CBSE Delhi, Set 3, 2020]

Ans. 0

Explanation : $\frac{2\cos 67^\circ}{\sin 23^\circ} - \frac{\tan 40^\circ}{\cot 50^\circ} - \cos 0^\circ$

$= \frac{2\cos(90-23)^\circ}{\sin 23^\circ} - \frac{\tan(90-50)^\circ}{\cot 50^\circ} - \cos 0^\circ$

$= \frac{2\sin 23^\circ}{\sin 23^\circ} - \frac{\cot 50^\circ}{\cot 50^\circ} - \cos 0^\circ$

$\left\{\begin{array}{l}\because \cos(90^\circ - \theta) = \sin\theta \\ \tan(90^\circ - \theta) = \cot\theta\end{array}\right\}$

$= 2(1) - 1 - 1 \qquad \{\cos 0^\circ = 1\}$

$= 2 - 2 = 0$

Very Short Answer Type Questions (1 mark each)

Q. 1. If $\sin A + \sin^2 A = 1$, then find the value of the expression $(\cos^2 A + \cos^4 A)$.

[CBSE OD, Set 1, 2020]

Ans. Given : $\sin A + \sin^2 A = 1$

$\Rightarrow \qquad \sin A = 1 - \sin^2 A$

$\Rightarrow \qquad \sin A = \cos^2 A$

$[\because \sin^2 A + \cos^2 A = 1]$

Squaring both sides, we get

$\sin^2 A = \cos^4 A$

$\Rightarrow \qquad 1 - \cos^2 A = \cos^4 A$

$\Rightarrow \qquad \cos^2 A + \cos^4 A = 1$ **Ans.**

Q. 2. If tan A = cot B, then find the value of (A + B). [CBSE OD, Set 2, 2020]

Ans. Given : $\tan A = \cot B$

$\Rightarrow \qquad \tan A = \tan(90^\circ - B)$

$[\because \tan(90^\circ - \theta) = \cot\theta]$

On comparing both sides, we get

$A = 90^\circ - B$

$\Rightarrow \qquad A + B = 90^\circ.$ **Ans.**

Q. 3. Evaluate: $\sin^2 60^\circ + 2\tan 45^\circ - \cos^2 30^\circ$

[CBSE OD, Set 1, 2019]

Ans. We know,

$\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\tan 45^\circ = 1$ and $\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\therefore \sin^2 60^\circ + 2\tan 45^\circ - \cos^2 30^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{3}{4} + 2 - \frac{3}{4}$

$= 2$

Q. 4. If $\sin A = \frac{3}{4}$, calculate $\sec A$.

[CBSE OD, Set 1, 2019]

Ans. Given, $\sin A = \frac{3}{4}$

$$\cos A = \sqrt{1-\sin^2 A}$$

$$= \sqrt{1-\left(\frac{3}{4}\right)^2} = \sqrt{1-\frac{9}{16}}$$

$$= \sqrt{\frac{7}{16}}$$

$$\Rightarrow \quad \cos A = \frac{\sqrt{7}}{4}$$

$$\Rightarrow \quad \sec A = \frac{1}{\cos A} = \frac{4}{\sqrt{7}}$$

Q. 5. Find A if $\tan 2A = \cot (A - 24°)$

[CBSE Delhi, Set 1, 2019]

Ans. Given, $\tan 2A = \cot (A - 24°)$

or $\cot (90° - 2A) = \cot (A - 24°)$

$[\because \tan\theta = \cot(90° - \theta)]$

or $90° - 2A = A - 24°$

or $3A = 90° + 24°$

or $3A = 114°$

$A = 38°$

Q. 6. Find the value of $(\sin^2 33° + \sin^2 57°)$

[CBSE Delhi, Set 1, 2019]

Ans. $\sin^2 33° + \sin^2 57° = \sin^2 33° + \cos^2 (90° - 57°)$

$= \sin^2 33° + \cos^2 33°$

$= 1 \quad [\because \sin^2\theta + \cos^2\theta = 1]$

Q. 7. If A, B and C are interior angles of ΔABC, then prove that: $\sin \frac{(A+C)}{2} = \cos \frac{B}{2}$.

[CBSE, Term 1, 2016]

Ans. In ΔABC

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \angle A + \angle C = 180° - \angle B$

Divide by 2 on both sides,

$$\frac{\angle A + \angle C}{2} = \frac{180° - \angle B}{2}$$

$$\Rightarrow \quad \frac{\angle A + \angle C}{2} = 90° - \frac{\angle B}{2}$$

Taking sin both sides,

$$\Rightarrow \sin\left(\frac{\angle A + \angle C}{2}\right) = \sin\left(90° - \frac{\angle B}{2}\right)$$

$$\Rightarrow \quad \sin\left(\frac{\angle A + \angle C}{2}\right) = \cos\frac{\angle B}{2}$$

$$\Rightarrow \quad \sin\frac{(A+C)}{2} = \cos\frac{B}{2}$$ **Hence Proved.**

Q. 8. If $x = 3 \sin\theta$ and $y = 4\cos\theta$, find the value of $\sqrt{16x^2 + 9y^2}$.

[CBSE, Term 1, 2016]

Ans. Given, $x = 3\sin\theta$

$\Rightarrow \quad x^2 = 9\sin^2\theta$

$\Rightarrow \quad \sin^2\theta = \frac{x^2}{9}$ ...(i)

And $y = 4\cos\theta$

$\Rightarrow \quad y^2 = 16\cos^2\theta$

$\Rightarrow \quad \cos^2\theta = \frac{y^2}{16}$ ...(ii)

On adding equation (i) and equation (ii),

$$\sin^2\theta + \cos^2\theta = \frac{x^2}{9} + \frac{y^2}{16}$$

$$\Rightarrow \quad 1 = \frac{x^2}{9} + \frac{y^2}{16}$$

$$\Rightarrow \quad 1 = \frac{16x^2 + 9y^2}{144}$$

$$\Rightarrow \quad 16x^2 + 9y^2 = 144$$

Taking square root both sides,

$$\sqrt{16x^2 + 9y^2} = \sqrt{144}$$

$$\Rightarrow \quad \sqrt{16x^2 + 9y^2} = 12$$

Q. 9. If ΔABC is right angled at B, what is the value of $\sin (A + C)$.

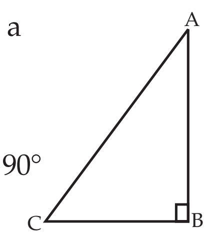
[CBSE, Term 1, Set 1, 2015]

Ans. $\angle B = 90°$ [Given]

We know that in ΔABC,

$\angle A + \angle B + \angle C = 180°$

[Angle sum property of a triangle]



$\Rightarrow \angle A + \angle C + 90° = 180°$

$\Rightarrow \quad \angle A + \angle C = 180° - 90°$

$= 90°$

$\therefore \quad \sin (A + C) = \sin 90° = 1$

Q. 10. If $\sqrt{3}\sin\theta = \cos\theta$, find the value of

$$\frac{3\cos^2\theta + 2\cos\theta}{3\cos\theta + 2}.$$

[CBSE, Term 1, Set 1, 2015]

Ans. $\sqrt{3}\sin\theta = \cos\theta$ [Given]

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$ or $\tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan\theta = \tan 30^\circ \Rightarrow \theta = 30^\circ$

Now,

$$\frac{3\cos^2\theta + 2\cos\theta}{3\cos\theta + 2} = \frac{\cos\theta(3\cos\theta+2)}{(3\cos\theta+2)} = \cos\theta$$

Put $\theta = 30^\circ$

$\Rightarrow \quad \cos 30^\circ = \frac{\sqrt{3}}{2}$

Short Answer Type Questions-I (2 marks each)

Q. 1. Prove that $1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha} = \operatorname{cosec}\alpha$

[CBSE OD, Set 1, 2020]

Ans. Consider, L.H.S. $= 1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha}$

$= 1 + \frac{\operatorname{cosec}^2\alpha - 1}{1+\operatorname{cosec}\alpha}$

$[\because \cot^2\alpha = \operatorname{cosec}^2\alpha - 1]$

$= 1 + \frac{(\operatorname{cosec}\alpha+1)(\operatorname{cosec}\alpha-1)}{1+\operatorname{cosec}\alpha}$

$= 1 + \operatorname{cosec}\alpha - 1$

$= \operatorname{cosec}\alpha =$ R.H.S. **Hence Proved.**

Q. 2. Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

[CBSE OD, Set 1, 2020]

Ans. Consider,

L.H.S. $= \tan^4\theta + \tan^2\theta$

$= \tan^2\theta(\tan^2\theta + 1)$

$= (\sec^2\theta - 1)\sec^2\theta$

$[\because \sec^2\theta - \tan^2\theta = 1]$

$= \sec^4\theta - \sec^2\theta =$ R.H.S.

Hence Proved.

Q. 3. Prove the following identity:

$$\left[\frac{1-\tan A}{1-\cot A}\right]^2 = \tan^2 A : \angle A \text{ is acute}$$

[CBSE, Term 1, 2016]

Ans. L.H.S. $= \left[\frac{1-\tan A}{1-\cot A}\right]^2$

$= \left[\frac{1-\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}\right]^2$

$= \left[\frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}}\right]^2$

$= \left[\frac{(\cos A - \sin A)\sin A}{-(\cos A - \sin A)\cos A}\right]^2$

$= \left[-\frac{\sin A}{\cos A}\right]^2$

$= [-\tan A]^2$

$= \tan^2 A =$ R.H.S.

Hence Proved.

Q. 4. Prove the following identity:

$$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = 1 - \sin\theta\cdot\cos\theta.$$

[CBSE, Term 1, Set 1, 2015]

Ans. L.H.S. $= \frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta}$

$= \frac{(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cdot\cos\theta)}{(\sin\theta + \cos\theta)}$

$[\because a^3 + b^3 = (a+b)(a^2 + b^2 - ab)]$

$= 1 - \sin\theta\cdot\cos\theta =$ R.H.S.

$[\because \sin^2\theta + \cos^2\theta = 1]$

Hence Proved.

Short Answer Type Questions-II (3 marks each)

Q. 1. If $\sin\theta + \cos\theta = \sqrt{2}$, prove that $\tan\theta + \cot\theta = 2$. **[CBSE OD, Set 1, 2020]**

Solution : Given : $\sin\theta + \cos\theta = \sqrt{2}$

Squaring both sides,

$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$

$1 + 2\sin\theta\cos\theta = 2$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow \quad 2\sin\theta\cos\theta = 1$

$$\Rightarrow \quad \sin\theta\cos\theta = \frac{1}{2} \quad \text{...(i)}$$

We have to prove that $\tan\theta + \cot\theta = 2$

Taking L.H.S.,

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$\frac{1}{\sin\theta\cos\theta} = \frac{1}{1/2} = 2 = \text{R.H.S.}$$

Hence Proved.

Q. 2. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or $\frac{1}{2}$. [CBSE OD, Set 2, 2020]

Ans. Given : $1 + \sin^2\theta = 3\sin\theta\cos\theta$

Dividing both sides by $\cos^2\theta$, we get

$$\sec^2\theta + \tan^2\theta = 3\tan\theta$$

$$\Rightarrow (\tan^2\theta + 1) + \tan^2\theta = 3\tan\theta$$

$$[\because \sec^2\theta = \tan^2\theta + 1]$$

$$\Rightarrow \quad 1 + 2\tan^2\theta = 3\tan\theta$$

$$\Rightarrow 2\tan^2\theta - 3\tan\theta + 1 = 0$$

$$\Rightarrow 2\tan^2\theta - 2\tan\theta - \tan\theta + 1 = 0$$

$$\Rightarrow 2\tan\theta(\tan\theta - 1) - 1(\tan\theta - 1) = 0$$

$$\Rightarrow (\tan\theta - 1)(2\tan\theta - 1) = 0$$

$$\Rightarrow \quad \tan\theta = 1 \text{ or } \frac{1}{2}$$

Hence Proved.

Q. 3. Show that :

$$\frac{\cos^2(45° + \theta) + \cos^2(45° - \theta)}{\tan(60° + \theta)\tan(30° - \theta)} = 1.$$

[CBSE OD, Set 3, 2020]

Ans. L.H.S. $= \dfrac{\cos^2(45° + \theta) + \cos^2(45° - \theta)}{\tan(60° + \theta)\tan(30° - \theta)}$

$$= \frac{\cos^2\{90° - (45° - \theta)\} + \cos^2(45° - \theta)}{\tan\{90° - (30° - \theta)\}\tan(30° - \theta)}$$

$$= \frac{\sin^2(45° - \theta) + \cos^2(45 - \theta)}{\cot(30° - \theta)\tan(30° - \theta)}$$

$$\begin{bmatrix} \because \cos(90° - \theta) = \sin\theta \\ \tan(90° - \theta) = \cot\theta \end{bmatrix}$$

$= 1$

$=$ R.H.S. **Hence Proved.**

Q. 4. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$ [CBSE Delhi, Set 1, 2020]

Ans. Given : $\sin\theta + \cos\theta = \sqrt{3}$

To prove : $\tan\theta + \cot\theta = 1$

Proof : We have, $\sin\theta + \cos\theta = \sqrt{3}$

Squaring both the sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$$

$$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$

$$\Rightarrow \quad 1 + 2\sin\theta\cos\theta = 3$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$$\Rightarrow \quad \sin\theta\cos\theta = \frac{2}{2}$$

$$\Rightarrow \quad \sin\theta\cos\theta = 1 \quad \text{...(i)}$$

Now, L.H.S. $= \tan\theta + \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}; \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1}{1} = 1 = \text{R.H.S.} \quad \text{[Using eq. (i)]}$$

Hence Proved.

Q. 5. Prove that : $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$. [CBSE Delhi, Set 2, 2020]

Ans. To prove : $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

L.H.S. $= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3[(\sin^2\theta)^2 + (\cos^2\theta)^2] + 1$

$= 2[(\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)] - 3(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta] + 1$

$$\left\{\begin{matrix} \because a^3 + b^3 = (a+b)^3 - 3ab(a+b) \\ a^2 + b^2 = (a+b)^2 - 2ab \end{matrix}\right\}$$

$= 2[(1)^3 - 3\sin^2\theta\cos^2\theta(1)] - 3[(1)^2 - 2\sin^2\theta\cos^2\theta] + 1$

$$\{\because \sin^2\theta + \cos^2\theta = 1\}$$

$= 2\,[1 - 3\sin^2\theta\cos^2\theta] - 3[1 - 2\sin^2\theta\cos^2\theta] + 1$

$= 2 - 6\sin^2\theta\cos^2 - 3 + 6\sin^2\theta\cos^2\theta + 1$

$= 2 - 3 + 1$

$= 0$

= R.H.S. **Hence Proved.**

Q. 6. Prove that : $\dfrac{\cot\theta + \text{cosec}\,\theta - 1}{\cot\theta - \text{cosec}\,\theta + 1} = \dfrac{1 + \cos\theta}{\sin\theta}$

[CBSE Delhi, Set 3, 2020]

Ans. $\dfrac{\cot\theta + \text{cosec}\,\theta - 1}{\cot\theta - \text{cosec}\,\theta + 1}$

$= \dfrac{\cot\theta + \text{cosec}\,\theta - (\text{cosec}^2\theta - \cot^2\theta)}{(\cot\theta - \text{cosec}\,\theta + 1)}$

$[\because \text{cosec}^2\theta - \cot^2\theta = 1]$

$= \dfrac{(\text{cosec}\,\theta + \cot\theta) - \dfrac{(\text{cosec}\,\theta + \cot\theta)}{(\text{cosec}\,\theta - \cot\theta)}}{(\cot\theta - \text{cosec}\,\theta + 1)}$

$= \dfrac{(\text{cosec}\,\theta + \cot\theta)[1 - (\text{cosec}\,\theta - \cot\theta)]}{(\cot\theta - \text{cosec}\,\theta + 1)}$

$= \dfrac{(\text{cosec}\,\theta + \cot\theta)[1 - \text{cosec}\,\theta + \cot\theta]}{(1 - \text{cosec}\,\theta + \cot\theta)}$

$= \text{cosec}\,\theta + \cot\theta$

$= \dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{1 + \cos\theta}{\sin\theta}$ **Hence Proved.**

Q. 7. Evaluate:

$$\left(\frac{3\sin 43^\circ}{\cos 47^\circ}\right)^2 - \frac{\cos 37^\circ\ \text{cosec}\ 53^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ}$$

[CBSE OD, Set 1, 2019]

Ans.

$$\left(\frac{3\sin 43^\circ}{\cos 47^\circ}\right)^2 - \frac{\cos 37^\circ\ \text{cosec}\ 53^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ}$$

$$= \left\{\frac{3\cos(90^\circ - 43^\circ)}{\cos 47^\circ}\right\}^2 - \frac{\sin(90^\circ - 37^\circ)\ \text{cosec}\ 53^\circ}{\cot(90^\circ - 5^\circ)\cot(90^\circ - 25^\circ)\tan 45^\circ \tan 65^\circ \tan 85^\circ}$$

$[\because \cos(90^\circ - \theta) = \sin\theta,\ \sin(90^\circ - \theta) = \cos\theta,\ \cot(90^\circ - \theta) = \tan\theta]$

$$= \left\{\frac{3\cos 47^\circ}{\cos 47^\circ}\right\}^2 - \frac{\sin 53^\circ\ \text{cosec}\ 53^\circ}{\cot 85^\circ \cot 65^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ}$$

$$= (3)^2 - \frac{\sin 53^\circ \times \dfrac{1}{\sin 53^\circ}}{\dfrac{1}{\tan 85^\circ} \times \dfrac{1}{\tan 65^\circ} \times \tan 45^\circ \times \tan 65^\circ \times \tan 85^\circ}$$

$= \quad 9 - \dfrac{1}{\tan 45^\circ}$

$= \quad 9 - 1$ $\qquad (\because \tan 45^\circ = 1)$

$= \quad 8$

Q. 8. Find A and B if $\sin(A + 2B) = \dfrac{\sqrt{3}}{2}$ and $\cos(A + 4B) = 0$, where A and B are acute angles. [CBSE OD, Set 2, 2019]

Ans. Given,

$$\sin(A + 2B) = \frac{\sqrt{3}}{2} \text{ and } \cos(A + 4B) = 0$$

$$\sin(A + 2B) = \sin 60^\circ \left(\because \sin 60^\circ = \frac{\sqrt{3}}{2}\right)$$

$A + 2B = 60$ ...(i)

and $\cos(A + 4B) = \cos 90^\circ$ $\quad (\because \cos 90^\circ = 0)$

$A + 4B = 90^\circ$ ...(ii)

On solving equations (i) and (ii), we get

$B = 15^\circ$ and $A = 30^\circ$

Q. 9. Find the value of:

$$\left(\frac{3\tan 41^\circ}{\cot 49^\circ}\right)^2 - \left(\frac{\sin 35^\circ \sec 55^\circ}{\tan 10^\circ \tan 20^\circ \tan 60^\circ \tan 70^\circ \tan 80^\circ}\right)^2$$

[CBSE OD, Set 3, 2019]

Ans.

$$\left(\frac{3\tan 41^\circ}{\cot 49^\circ}\right)^2 - \left(\frac{\sin 35^\circ \cdot \sec 55^\circ}{\tan 10^\circ \cdot \tan 20^\circ \cdot \tan 60^\circ \cdot \tan 70^\circ \cdot \tan 80^\circ}\right)$$

$$= \left(\frac{3\cot(90^\circ - 41^\circ)}{\cot 49^\circ}\right)^2 - \left(\frac{\sin 35^\circ \cdot \text{cosec}\,(90^\circ - 55^\circ)}{\cot(90^\circ - 10^\circ)\cdot\cot(90^\circ - 20^\circ)\cdot\tan 60^\circ \tan 10^\circ \cdot \tan 80^\circ}\right)$$

$$\left[\begin{array}{l}\because \cot(90^\circ - \theta) = \tan\theta \\ \quad \sec(90^\circ - \theta) = \text{cosec}\,\theta\end{array}\right]$$

$$= \left(\frac{3\cot 49^\circ}{\cot 49^\circ}\right)^2 - \left(\frac{\sin 35^\circ \cdot \text{cosec}\ 35^\circ}{\cot 80^\circ \cdot \cot 70^\circ \cdot \tan 60^\circ \cdot \tan 70^\circ \cdot \tan 80^\circ}\right)^2$$

$$= 9 - \left(\frac{1}{\tan 60^\circ}\right)^2 \left(\because \tan 60^\circ = \sqrt{3}\right)$$

$$= 9 - \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 9 - \frac{1}{3}$$

$$= \frac{26}{3}$$

Q. 10. Prove that $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$.

[CBSE Delhi, Set 1, 2019]

Ans. L.H.S. $= (\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$

$$= \sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\cdot\text{cosec}\,\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta.$$

$$= \sin^2\theta + \cos^2\theta + \text{cosec}^2\theta + \sec^2\theta + 2\sin\theta\cdot\frac{1}{\sin\theta} + 2\cos\theta\cdot\frac{1}{\cos\theta}$$

$$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 4$$

$$\left[\because \begin{array}{l}\text{cosec}^2\theta = 1 + \cot^2\theta \\ \sec^2\theta = 1 + \tan^2\theta\end{array}\right]$$

$= 7 + \tan^2\theta + \cot^2\theta$ (R.H.S.) **Hence Proved.**

Q. 11. Prove that (1 + cot A – cosec A) (1 + tan A + sec A) = 2.

[CBSE Delhi, Set 1, 2019]

Ans. L.H.S. = (1 + cot A – cosec A) (1 + tan A + sec A)

$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$

$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$

$$= \frac{(\sin A + \cos A)^2 - 1}{\sin A\cdot\cos A}$$

$$= \frac{\sin^2 A + \cos^2 A + 2\cdot\sin A\cdot\cos A - 1}{\sin A\cdot\cos A}$$

$$= \frac{1 + 2\sin A\cdot\cos A - 1}{\sin A\cdot\cos A} = \frac{2\sin A\cdot\cos A}{\sin A\cdot\cos A}$$

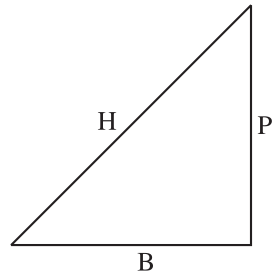
= 2 (R.H.S.) **Hence Proved.**

Q. 12. If $4\tan\theta = 3$, evaluate $\left(\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$

[CBSE, 2018]

Ans. Given, $4\tan\theta = 3$,

$$\Rightarrow \tan\theta = \frac{3}{4}$$



$P = 3K,\ B = 4K$

Now, $H = \sqrt{P^2 + B^2}$

$$= \sqrt{(3K)^2 + (4K)^2}$$

$$= \sqrt{9K^2 + 16K^2}$$

$$= \sqrt{25K^2}$$

$$\Rightarrow H = 5K$$

$$\therefore \sin\theta = \frac{P}{H} = \frac{3K}{5K} = \frac{3}{5}$$

and $\cos\theta = \frac{B}{H} = \frac{4K}{5K} = \frac{4}{5}$

Now, $$\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \frac{4\times\frac{3}{5} - \frac{4}{5} + 1}{4\times\frac{3}{5} + \frac{4}{5} - 1}$$

$$= \frac{\left(\frac{12}{5} - \frac{4}{5} + 1\right)}{\left(\frac{12}{5} + \frac{4}{5} - 1\right)}$$

$$= \frac{\left(\frac{12 - 4 + 5}{5}\right)}{\left(\frac{12 + 4 - 5}{5}\right)}$$

$$= \frac{13/5}{11/5}$$

$$= \frac{13}{11}$$

Q. 13. If $\tan 2A = \cot(A - 18°)$, where $2A$ is an acute angle, find the value of A.

[CBSE, 2018]

Ans. Given, $\tan 2A = \cot(A - 18°)$

$$\Rightarrow \cot(90° - 2A) = \cot(A - 18°)$$

$$[\because \tan\theta = \cot(90° - \theta)]$$

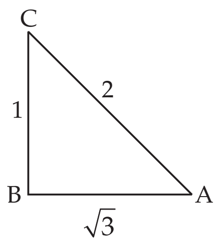
$$\Rightarrow \quad 90° - 2A = A - 18°$$
$$\Rightarrow \quad 90° + 18° = A + 2A$$
$$\Rightarrow \quad 108° = 3A$$
$$\Rightarrow \quad A = \frac{108°}{3}$$
$$\Rightarrow \quad A = 36°$$

Q. 14. If sec $A = \frac{2}{\sqrt{3}}$, find the value of

$$\frac{\tan A}{\cos A} + \frac{1+\sin A}{\tan A}$$

[CBSE Term 1, 2016]

Ans. Given, sec $A = \frac{2}{\sqrt{3}}$



In Δ ABC,

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad 2^2 = (\sqrt{3})^2 + BC^2$$
$$\Rightarrow \quad 4 = 3 + BC^2$$
$$\Rightarrow \quad BC^2 = 4 - 3$$
$$\Rightarrow \quad BC^2 = 1$$
$$\therefore \quad BC = 1$$

So, $\tan A = \frac{1}{\sqrt{3}}$; $\cos A = \frac{\sqrt{3}}{2}$; $\sin A = \frac{1}{2}$

$$\frac{\tan A}{\cos A} + \frac{1+\sin A}{\tan A} = \frac{\frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{2}} + \frac{1+\frac{1}{2}}{\frac{1}{\sqrt{3}}}$$
$$= \frac{2}{3} + \frac{\frac{3}{2}}{\frac{1}{\sqrt{3}}}$$
$$= \frac{2}{3} + \frac{3\sqrt{3}}{2}$$
$$= \frac{4+9\sqrt{3}}{6}$$

Q. 15. Prove that:

$\sec^2 \theta - \cot^2 (90° - \theta) = \cos^2 (90° - \theta) + \cos^2 \theta$

[CBSE Term 1, 2016]

Ans. To prove:

$\sec^2 \theta - \cot^2 (90° - \theta) = \cos^2 (90° - \theta) + \cos^2 \theta$.

$$\text{L.H.S.} = \sec^2 \theta - \cot^2 (90° - \theta)$$
$$= \sec^2 \theta - [\cot (90° - \theta)]^2$$
$$= \sec^2 \theta - (\tan \theta)^2$$
$$= \sec^2 \theta - \tan^2 \theta$$
$$= 1$$
$$\text{R.H.S.} = \cos^2(90° - \theta) + \cos^2 \theta$$
$$= [\cos (90° - \theta)]^2 + \cos^2 \theta$$
$$= (\sin \theta)^2 + \cos^2 \theta$$
$$= \sin^2 \theta + \cos^2 \theta$$
$$= 1$$

Hence, L.H.S. = R.H.S. **Hence Proved**.

Q. 16. If $\sin \theta = \frac{12}{13}$, $0° < \theta < 90°$, find the value of:

$$\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cdot \cos \theta} \times \frac{1}{\tan^2 \theta}$$

[CBSE Term 1, Set 1, 2015]

Ans. Given, $\sin \theta = \frac{12}{13}$

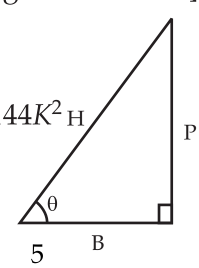
$$\Rightarrow \quad \frac{P}{H} = \frac{12}{13}$$

Let, $P = 12K$, $H = 13K$

$$P^2 + B^2 = H^2$$

[Pythagoras theorem]

$$(12K)^2 + B^2 = (13K)^2$$
$$144K^2 + B^2 = 169K^2$$
$$B^2 = 169K^2 - 144K^2$$
$$= 25K^2$$
$$\therefore \quad B = 5K$$



$$\therefore \quad \cos \theta = \frac{B}{H} = \frac{5K}{13K} = \frac{5}{13}$$

and $\quad \tan \theta = \frac{P}{B} = \frac{12K}{5K} = \frac{12}{5}$

Now, $\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta . \cos \theta} \times \frac{1}{\tan^2 \theta}$

$$= \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

$$= \frac{\frac{144-25}{169}}{\frac{120}{169}} \times \frac{25}{144}$$

$$= \frac{119}{120} \times \frac{25}{144} = \frac{595}{3456}$$

Q. 17. If sec θ + tan θ = *p*, prove that

$$\sin\theta = \frac{p^2-1}{p^2+1}$$ **[CBSE, Term 1, Set 1, 2015]**

Ans. $$\text{R.H.S.} = \frac{p^2-1}{p^2+1}$$

$$= \frac{(\sec\theta+\tan\theta)^2-1}{(\sec\theta+\tan\theta)^2+1}$$

$$= \frac{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta-1}{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta+1}$$

$[\because (a+b)^2 = a^2+b^2+2ab]$

$$= \frac{(\sec^2\theta-1)+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+(1+\tan^2\theta)+2\sec\theta\tan\theta}$$

$$= \frac{\tan^2\theta+\tan^2\theta+2\sec\theta\tan\theta}{\sec^2\theta+\sec^2\theta+2\sec\theta\tan\theta}$$

$\left[\begin{array}{l}\because \sec^2\theta-1=\tan^2\theta \\ \Rightarrow \sec^2\theta = 1+\tan^2\theta\end{array}\right]$

$$= \frac{2\tan^2\theta+2\sec\theta\tan\theta}{2\sec^2\theta+2\sec\theta\tan\theta}$$

$$= \frac{2\tan\theta(\tan\theta+\sec\theta)}{2\sec\theta(\sec\theta+\tan\theta)} = \frac{\tan\theta}{\sec\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$$

= sin θ = L.H.S. **Hence Proved.**

Long Answer Type Questions (4 marks each)

Q. 1. Prove that $\frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\sec^2\theta}+\frac{1}{1+\csc^2\theta}=2$ **[CBSE, 2019]**

Topper's Answers

30. To prove: $\frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\sec^2\theta}+\frac{1}{1+\csc^2\theta}=2.$

Taking from LHS,

= LHS

$= \frac{1}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\sec^2\theta}+\frac{1}{1+\csc^2\theta}$

$= \frac{1}{1+\sin^2\theta}+\frac{1}{1+\csc^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\sec^2\theta}$ [Re-arranging]

$= \frac{1}{1+\sin^2\theta}+\frac{1}{1+\left(\frac{1}{\sin^2\theta}\right)}+\frac{1}{1+\cos^2\theta}+\frac{1}{1+\left(\frac{1}{\cos^2\theta}\right)}$ $\left[\csc\theta = \frac{1}{\sin\theta},\ \sec\theta = \frac{1}{\cos\theta}\right]$

$= \frac{1}{1+\sin^2\theta}+\frac{1\times\sin^2\theta}{1+\sin^2\theta}+\frac{1}{1+\cos^2\theta}+\frac{1\times\cos^2\theta}{1+\cos^2\theta}$

$= \frac{1+\sin^2\theta}{1+\sin^2\theta}+\frac{1+\cos^2\theta}{1+\cos^2\theta}$

= 1 + 1

= 2

= RHS

LHS = RHS

Hence, proved!

Q. 2. Prove that:

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\text{cosec}\,\theta$$

[CBSE OD, Set 1, 2019]

Ans. L.H.S. =

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$=\frac{1}{\sin\theta-\cos\theta}\left[\frac{\sin^2\theta}{\cos\theta}-\frac{\cos^2\theta}{\sin\theta}\right]$$

$$=\frac{1}{\sin\theta-\cos\theta}\left[\frac{\sin^3\theta-\cos^3\theta}{\cos\theta\cdot\sin\theta}\right]$$

$$=\frac{[\sin\theta-\cos\theta][\sin^2\theta+\cos^2\theta+\sin\theta\cdot\cos\theta]}{(\sin\theta-\cos\theta)\cdot(\cos\theta\cdot\sin\theta)}$$

$$=\frac{\sin^2\theta+\cos^2\theta+\sin\theta\cdot\cos\theta}{(\cos\theta\cdot\sin\theta)}$$

$$=\frac{1+\sin\theta\cdot\cos\theta}{\cos\theta\cdot\sin\theta}\qquad[\because \sin^2\theta+\cos^2\theta=1]$$

$$=\frac{1}{\cos\theta\cdot\sin\theta}+\frac{\sin\theta\cdot\cos\theta}{\sin\theta\cdot\cos\theta}$$

$= 1 + \sec\theta.\ \text{cosec}\,\theta$ [R.H.S.]

Hence Proved.

Q. 3. Prove that:

$$\frac{\sin\theta}{\cot\theta+\text{cosec}\,\theta}=2+\frac{\sin\theta}{\cot\theta-\text{cosec}\,\theta}$$

[CBSE OD, Set 1, 2019]

Ans. L.H.S.

$$\frac{\sin\theta}{\cot\theta+\text{cosec}\,\theta}=\frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}}$$

$$=\frac{\sin\theta}{\frac{\cos\theta+1}{\sin\theta}}$$

$$=\frac{\sin^2\theta}{\cos\theta+1}$$

$$=\frac{\sin^2\theta}{1+\cos\theta}\times\frac{(1-\cos\theta)}{(1-\cos\theta)}$$

$$=\frac{\sin^2\theta(1-\cos\theta)}{1-\cos^2\theta}$$

$$=\frac{\sin^2\theta(1-\cos\theta)}{1-\cos^2\theta}$$

$$=\frac{\sin^2\theta(1-\cos\theta)}{\sin^2\theta}$$

$= 1 - \cos\theta$ ...(i)

R.H.S.

$$2+\frac{\sin\theta}{\cot\theta-\text{cosec}\,\theta}=2+\frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}}$$

$$=2+\frac{\sin^2\theta}{\cos\theta-1}$$

$$=2-\frac{\sin^2\theta}{(1-\cos\theta)}$$

$$=2-\frac{\sin^2\theta\times(1+\cos\theta)}{(1-\cos\theta)\times(1+\cos\theta)}$$

$$=2-\frac{\sin^2\theta(1+\cos\theta)}{1-\cos^2\theta}$$

$$=2-\frac{\sin^2\theta\,(1+\cos\theta)}{\sin^2\theta}$$

$= 2 - (1 + \cos\theta)$

$= 1 - \cos\theta$ ...(ii)

From equation (i) and (ii), we get

L.H.S. = R.H.S. **Hence Proved.**

Q. 4. Prove that $\frac{\sin A-\cos A+1}{\sin A+\cos A-1}=\frac{1}{\sec A-\tan A}$

[CBSE Delhi, Set 1, 2019]

Ans. L.H.S. = $\frac{\sin A-\cos A+1}{\sin A+\cos A-1}$

Dividing the numerator and denominator by $\cos A$

$$= \frac{\tan A - 1 + \sec A}{\tan A + 1 - \sec A}$$

$$= \frac{(\tan A + \sec A) - 1}{(\tan A - \sec A) + 1}$$

$$= \frac{(\tan A + \sec A) - (\sec^2 A - \tan^2 A)}{\tan A - \sec A + 1}$$

$[\because \sec^2 A - \tan^2 A = 1]$

$$= \frac{(\tan A + \sec A)\ (1 - \sec A + \tan A)}{(\tan A - \sec A + 1)}$$

$= (\tan A + \sec A)$

$$= (\tan A + \sec A) \times \frac{(\tan A - \sec A)}{(\tan A - \sec A)}$$

$$= \frac{\sec^2 A - \tan^2 A}{\sec A - \tan A}$$

$$= \frac{1}{\sec A - \tan A} \quad \text{(R.H.S.)}$$

$\Rightarrow$ L.H.S = R.H.S **Hence Proved.**

Q. 5. Prove that

$$\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A} = \frac{1}{1 - 2\cos^2 A}$$

[CBSE Delhi, Set 2, 2019]

Ans. L.H.S. $= \dfrac{\tan^2 A}{\tan^2 A - 1} + \dfrac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A}$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A}} + \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\frac{1}{\sin^2 A}}{\frac{\sin^2 A - \cos^2 A}{\sin^2 A \cdot \cos^2 A}}$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin^2 A - \cos^2 A} = \frac{1}{\sin^2 A - \cos^2 A}$$

$$= \frac{1}{1 - \cos^2 A - \cos^2 A} \quad [\because \sin^2 A = 1 - \cos^2 A]$$

$$= \frac{1}{1 - 2\cos^2 A}$$

= R.H.S. **Hence Proved.**

Q. 6. If sec $\theta = x + \dfrac{1}{4x}$, $x \neq 0$, **find (sec** θ **+ tan** θ**).**

[CBSE Delhi, Set 3, 2019]

Ans. Given,

$$\sec \theta = x + \frac{1}{4x} \quad \text{...(i)}$$

Squaring both sides, we get

$$\sec^2 \theta = \left(x + \frac{1}{4x}\right)^2$$

$$= x^2 + \frac{1}{16x^2} + 2 \times x \times \frac{1}{4x}$$

$$= x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

Also, $\sec^2 \theta = 1 + \tan^2 \theta$

$\therefore$ $1 + \tan^2 \theta = x^2 + \dfrac{1}{16x^2} + \dfrac{1}{2}$

or $\tan^2 \theta = x^2 + \dfrac{1}{16x^2} - \dfrac{1}{2}$

$$= \left(x - \frac{1}{4x}\right)^2$$

$\Rightarrow$ $\tan \theta = \left(x - \dfrac{1}{4x}\right)$ or $-\left(\dfrac{1}{4x} - x\right)$ ...(ii)

Now, from equation (i) and (ii)

$$\sec \theta + \tan \theta = x + \frac{1}{4x} + x - \frac{1}{4x}$$

$$= 2x$$

or $\sec \theta + \tan \theta = x + \dfrac{1}{4x} + \dfrac{1}{4x} - x$

$$= \frac{1}{2x}.$$

Hence,

$$\sec \theta + \tan \theta = 2x \text{ or } \frac{1}{2x}$$

Q. 7. Prove that: $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A.$

[CBSE, 2018]

Ans.

Topper's Answers

27) To prove: $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A.$

Simplifying LHS:

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$$

$$= \frac{\sin A\,(1 - 2\sin^2 A)}{\cos A\,(2\cos^2 A - 1)}$$

$$= \frac{\sin A}{\cos A}\left[\frac{1-(2\sin^2 A)}{2\cos^2 A - 1}\right]$$

$$= \frac{\sin A}{\cos A}\left[\frac{\sin^2 A + \cos^2 A - 2\sin^2 A}{2\cos^2 A - (\sin^2 A + \cos^2 A)}\right] \quad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{\sin A}{\cos A}\left[\frac{\cos^2 A - \sin^2 A}{\cos^2 A - \sin^2 A}\right]$$

$$= \frac{\sin A}{\cos A} \times 1$$

$$= \tan A. \quad \left[\because \frac{\sin A}{\cos A} = \tan A\right]$$

LHS = RHS

hence proved.

$$\text{L.H.S.} = \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$$

$$= \frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

$$= \frac{\sin A}{\cos A} \times \frac{(1-2\sin^2 A)}{[2(1-\sin^2 A)-1]}$$

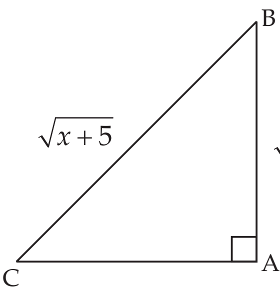
$[\because \cos^2 A = 1 - \sin^2 A]$

$$= \frac{\sin A}{\cos A} \times \frac{(1-2\sin^2 A)}{(2-2\sin^2 A - 1)}$$

$$= \frac{\sin A}{\cos A} \times \frac{(1-2\sin^2 A)}{(1-2\sin^2 A)}$$

$= \tan A =$ R.H.S. **Hence Proved.**

Q. 8. In the ΔABC (see figure), $\angle A =$ right angle, $AB = \sqrt{x}$ and $BC = \sqrt{x+5}$. Evaluate $\sin C . \cos C . \tan C + \cos^2 C . \sin A$

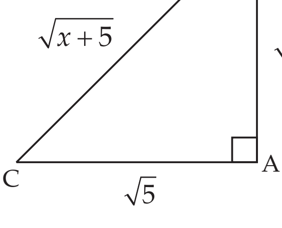


[CBSE Term 1, 2016]

Ans. In $\Delta\, ABC$, by Pythagoras theorem,

$$\left(\sqrt{x+5}\right)^2 = \left(\sqrt{x}\right)^2 + AC^2$$

$$\Rightarrow \quad x + 5 = x + AC^2$$



$\Rightarrow \quad 5 = AC^2$

$\Rightarrow \quad AC = \sqrt{5}$

$\therefore \quad \sin C = \frac{\sqrt{x}}{\sqrt{x+5}}; \cos C = \frac{\sqrt{5}}{\sqrt{x+5}};$

$\tan C = \frac{\sqrt{x}}{\sqrt{5}}$

and $\quad \sin A = \sin 90°$

$= 1$

Then, $\sin C \cos C \tan C + \cos^2 C \sin A$

$$= \frac{\sqrt{x}}{\sqrt{x+5}} \frac{\sqrt{5}}{\sqrt{x+5}} \frac{\sqrt{x}}{\sqrt{5}} + \left(\frac{\sqrt{5}}{\sqrt{x+5}}\right)^2 .1$$

$$= \frac{x}{x+5} + \frac{5}{x+5}$$

$$= \frac{x+5}{x+5}$$

$= 1$

Q. 9. If $\frac{\cos B}{\sin A} = n$ and $\frac{\cos B}{\cos A} = m$, then show that $(m^2 + n^2) \cos^2 A = n^2$.

[CBSE Term 1, 2016]

Ans. Given, $n = \frac{\cos B}{\sin A}$; $m = \frac{\cos B}{\cos A}$

So, $n^2 = \frac{\cos^2 B}{\sin^2 A}$; $m^2 = \frac{\cos^2 B}{\cos^2 A}$

L.H.S. $= (m^2 + n^2) \cos^2 A$

$$= \left(\frac{\cos^2 B}{\cos^2 A} + \frac{\cos^2 B}{\sin^2 A}\right)\cos^2 A$$

$$= \frac{(\sin^2 A \cos^2 B + \cos^2 A \cos^2 B)}{\cos^2 A \sin^2 A} \times \cos^2 A$$

$$= \frac{\cos^2 B(\sin^2 A + \cos^2 A)}{\sin^2 A}$$

$$= \frac{\cos^2 B}{\sin^2 A}$$

$= n^2 =$ R.H.S. **Hence Proved.**

Q. 10. Prove that:

$$\frac{\sec A - 1}{\sec A + 1} = \left(\frac{\sin A}{1 + \cos A}\right)^2 = (\cot A - \operatorname{cosec} A)^2$$

[CBSE Term 1, 2016]

Ans. L.H.S. $= \frac{\sec A - 1}{\sec A + 1}$

$$= \frac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1} = \frac{\frac{1 - \cos A}{\cos A}}{\frac{1 + \cos A}{\cos A}}$$

$$= \frac{1 - \cos A}{1 + \cos A}$$

$$= \frac{(1 - \cos A)(1 + \cos A)}{(1 + \cos A)(1 + \cos A)}$$

$$= \frac{1 - \cos^2 A}{(1 + \cos A)^2}$$

$$= \frac{\sin^2 A}{(1 + \cos A)^2}$$

$$= \left(\frac{\sin A}{1 + \cos A}\right)^2$$

And, $\left(\frac{\sin A}{1 + \cos A}\right)^2 = \left[\left(\frac{\sin A}{1 + \cos A}\right) \times \frac{(1 - \cos A)}{(1 - \cos A)}\right]$

$$= \left[\frac{\sin A(1 - \cos A)}{1 - \cos^2 A}\right]^2$$

$$= \left[\frac{\sin A(1 - \cos A)}{\sin^2 \text{A}}\right]^2$$

$$= \left[\frac{1 - \cos A}{\sin A}\right]^2$$

$$= \left[\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right]^2$$

$= (\operatorname{cosec} A - \cot A)^2$

$= (-1)^2 [\cot A - \operatorname{cosec} A]^2$

$= [\cot A - \operatorname{cosec} A]^2 =$ R.H.S.

Hence Proved.

Q. 11. If $\tan (A + B) = \sqrt{3}$ and $\tan (A - B) = \frac{1}{\sqrt{3}}$, where $0 < A + B < 90°$, $A > B$, find A and B. Also calculate $\tan A. \sin (A + B) + \cos A. \tan (A - B)$.

[CBSE Term 1, Set 1, 2015]

Ans. Given,

$\tan (A + B) = \sqrt{3}$, $\tan (A - B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan (A + B) = \tan 60°$

$\Rightarrow \quad (A + B) = 60° \quad$ ...(i)

And, $\quad \tan (A - B) = \tan 30°$

$\Rightarrow \quad (A - B) = 30° \quad$ ...(ii)

On adding eqs. (i) & (ii), we get

$$2A = 90°$$

$$\Rightarrow \quad A = \frac{90°}{2} = 45°$$

From eq. (i), $A + B = 60°$

$$\Rightarrow \quad 45° + B = 60°$$

$$\Rightarrow \quad B = 15°$$

$$\therefore \quad A = 45°, B = 15°$$

Now, tan A. sin $(A + B)$ + cos A. tan $(A - B)$

$$= \tan 45°. \sin (60°) + \cos 45° \cdot \tan (30°)$$

$$= 1 \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{\sqrt{3}}{2} + \frac{\sqrt{6}}{6}$$

$$= \frac{3\sqrt{3} + \sqrt{6}}{6}$$

Q. 12. Prove that:

$$\mathbf{(1 + \cot A + \tan A) \cdot (\sin A - \cos A) = \frac{\sec^3 A - \mathrm{cosec}^3 A}{\sec^2 A \cdot \mathrm{cosec}^2 A}}$$

[CBSE Term 1, Set 1, 2015]

Ans. L.H.S. = $(1 + \cot A + \tan A)(\sin A - \cos A)$

$$= \left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)(\sin A - \cos A)$$

$$= \left(\frac{\sin A \cos A + \cos^2 A + \sin^2 A}{\sin A \cdot \cos A}\right)(\sin A - \cos A)$$

$$= \frac{\sin^3 A - \cos^3 A}{\sin A \cdot \cos A}$$

[Using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$$= \frac{\dfrac{\sin^3 A}{\sin^3 A \cdot \cos^3 A} - \dfrac{\cos^3 A}{\sin^3 A \cdot \cos^3 A}}{\dfrac{\sin A \cos A}{\sin^3 A \cdot \cos^3 A}}$$

[Dividing Num. & Denom. by $\sin^3 A \cdot \cos^3 A$]

$$= \frac{\sec^3 A - \mathrm{cosec}^3 A}{\sec^2 A \cdot \mathrm{cosec}^2 A} = \text{R.H.S.}$$

Hence Proved.

Q. 13. Prove the identity:

$$\mathbf{\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{1 - 2\cos^2 A}}$$

[CBSE Term 1, Set 1, 2015]

Ans. L.H.S. = $\dfrac{\sin A + \cos A}{\sin A - \cos A} + \dfrac{\sin A - \cos A}{\sin A + \cos A}$

$$= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}$$

$$= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A + \sin^2 A + \cos^2 A - 2\sin A \cos A}{\sin^2 A - \cos^2 A}$$

$$= \frac{1 + 1}{1 - \cos^2 A - \cos^2 A}$$

$$\left[\begin{array}{l} \because \sin^2 A + \cos^2 A = 1 \\ \Rightarrow \sin^2 A = 1 - \cos^2 A \end{array}\right]$$

$$= \frac{2}{1 - 2\cos^2 A} = \text{R.H.S.}$$

Hence Proved.