# STRUCTURAL ANALYSIS

# Determinacy :-

**Degree of static Indeterminacy ($D_s$)**

$$\boxed{D_s = \underbrace{D_{se}}_{\text{external Indeterminacy}} + \underbrace{D_{si}}_{\text{Internal Indeterminacy}}}$$

$\therefore D_{si} = D_s - D_{se}$

How to get $D_{se}$ :- $\boxed{D_{se} = r - s}$

$r$ → no. of external reaction generate

$s$ → no. of available equilibrium equation

**$r$ table**

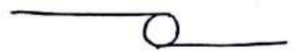
| | 2D | 3D |
|---|---|---|
| roller | only $f_y$ hence ($r=1$) | ($r=1$) |
| Hinge | ($r=2$) $f_x, f_y$ | ($r=3$) $f_x, f_y, f_z$ |
| Fix | ($r=3$) $F_x, f_y$, $M_z$ | ($r=6$) $f_x, f_y, f_z$, $m_x\ m_y\ m_z$ |

**$s$ table**

2D str: $s=3$ — $f_x$, $f_y$, $M_z$

3D str / Space: $s=6$ — $F_x$, $f_y$, $f_z$, $m_x$, $m_y$, $m_z$

## Special :-

① 

only vertical rxn ($f_y$)

hence to make It fix we have to add 2 restrain $fx$, M

② 

only $f_y$, M

hence only 1 restrain ($F_x$) is added to make It fix.

③ 

rxn $f_x$, M

hence only 1 restrain added $f_y$ to make It fix

---

(Type-1) frame ($D_s$)

**2D**

$D_s = 3C - R'$

if Hinge then $R' = m - 1$

**3D**

$D_s = 6C - R'$

if Hinge $R' = 3(m-1)$

m → no. of members meeting at hinge

C → min no. of cut required to open tree structure

$R'$ → Restrain added to make fix Joint

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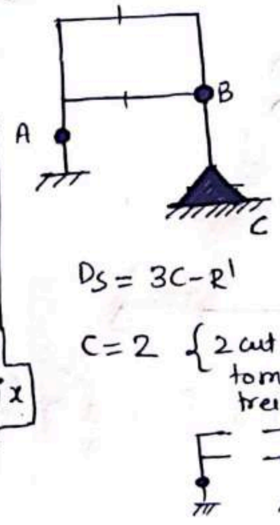
(Type-1) Practice Problem :-

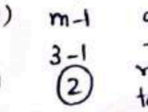
**(2D)**





$D_s = 3C - R'$

$C = 2$ {2 cut req. to make tree str.



$R' = A + B + C = 4$

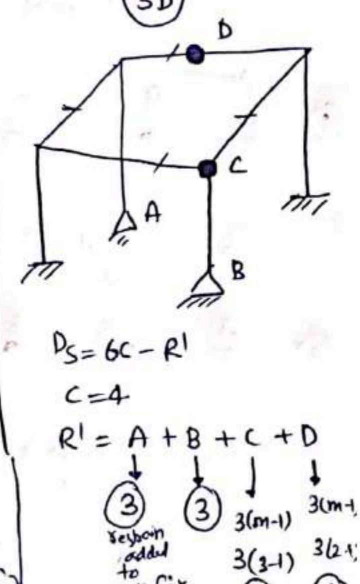
A → $(m-1)$, $2-1$, ①

B → $m-1$, $3-1$, ②

C → already Hinge hence only 1 restrain M added to make It fix ①

$\therefore$ Ans: $D_s = 3 \times 2 - 4 = 2$

**(3D)**





$D_s = 6C - R'$

$C = 4$

$R' = A + B + C + D$

A → ③ restrain added to make fix

B → ③

C → $3(m-1)$, $3(3-1)$, ⑥

D → $3(m-1)$, $3(2-1)$, ③

Ans ⇒ $6 \times 4 - 15$

$= 24 - 15 = 9$

Type-2 Truss case :-

(2D) (3D)

$\boxed{D_s = (m+r) - 2j}$ (2D) $\boxed{D_s = (m+r) - 3j}$ (3D)

- m → no. of truss member
- r → no. of external reactions
- J → no of truss joints

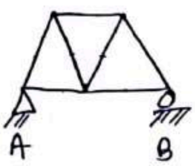
basically $(m+r)$ → no of rxn available (total unknown)

$2j$ or $3j$ → no of eqb eqn available

$2j \rightarrow$ (2D) → $f_x, f_y = 0$

$3j \rightarrow$ (3D) → $f_x, f_y, f_z = 0$

Type-2 In exam 2D truss is asked.



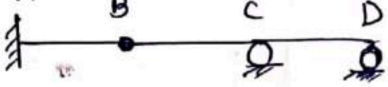

$D_s = (m+r) - 2j$

$m = 7$ $r = (2+1)$ $j = 5$

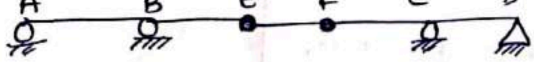
$D_s = 7 + 3 - 2 \times 5 = 0$

Type-3 Beam case :-

$D_s$ = Support reaction removed to make cantilever − (R') restrain added to make fix

① 

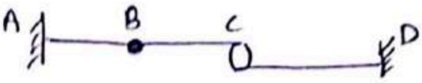

$D_s = (\underset{C}{1} + \underset{D}{1}) - (2 - 1) = 1$ (m-1) B

② 

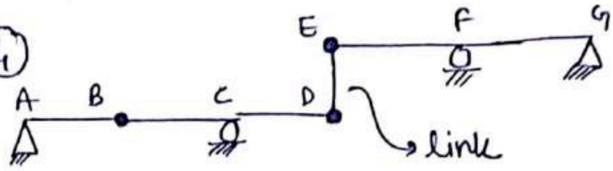

$D_s = (\underset{B}{1} + \underset{C}{1} + \underset{D}{2}) - (\overset{m-1}{\underset{E}{1}} + \overset{m-1}{\underset{F}{1}} + \underset{A}{2})$ to make fix joint at A

$= 0$

②

③ 


$D_s = (\underset{D}{3}) - (\overset{m-1}{\underset{B}{1}} + \underset{C}{2}) = 0$ (vertical only 1 rxn available, 2 restraint need to make fix)

④ 


$D_s = (\underset{1}{C} + \underset{1}{F} + \underset{2}{G}) - (A + B + D + E)$

A → 1 (restr to make fix); B → m-1 → 1; D → 1 (m-1); E → 1 (m-1)

$DS \Rightarrow 0$

(KI)

Kinematic Indeterminacy :- no. of unknown joint displacements like displacement (Δ) & rotation (θ)

Trick for frame (KI) :- (धीरे) ★★★ DHIRe

हतत्व

$\boxed{KI = (D + H) - (I + r_e)}$ or $D + H - I - r_e$

note :- D → Degree of freedom (DOF)

oototal के लिये no of joint का गुणा करें

(2D) rigid joint → 3 $(\Delta x, \Delta y, \theta)$

(3D) rigid joint → 6 $(\Delta x, \Delta y, \Delta z, \theta x, \theta y, \theta z)$

H → hinge DOF (degree of freedom)

member Hinge → (4)

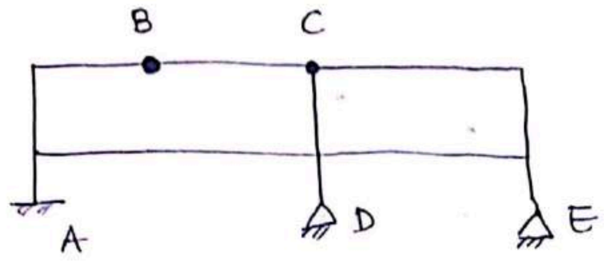
Joint hinge → 2D $(m-1)$, 3D $3(m-1)$

or

I → Inextensible member

$r_e$ → no of external rxn.

## Practice frame KI :-




$KI = (D + H) - I - r_e$

$D = 3 \times \cancel{\text{no of joints}}^{9} = 27$

$H = B + C = 6$

member hinge ⇓ 4

⇓ joint hinge $(m-1)$ $(3-1) = 2$

$I$ = Inextensible member = 0

[otherwise question में [illegible] रहेगा]

$r_e = \underset{3}{A} + \underset{2}{D} + \underset{2}{E} = 7$

$KI = (27 + 6) - 0 - 7 = 26$

---

Truss (KI)

2D: $\boxed{KI = 2j - r_e}$

3D: $\boxed{KI = 3j - r_e}$

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## Stability of structure :-

### (I) External stability :-

Structure sufficiently restraint so that rigid body movement does not happen.

condition :-

(i) Three reactions in plane structure neither be concurrent & nor parallel

{ अन्यथा unstable हो जायेगा }

(ii) In space structure

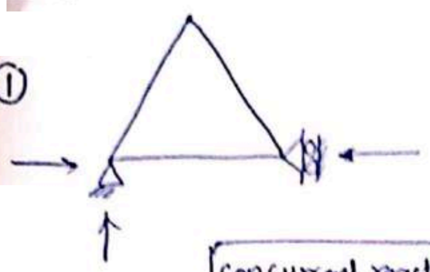
rxn should be
- non parallel
- non concurrent
- non coplanar

### (II) Internal stability :-

- when part of structure moves appreciably with respect to other part.

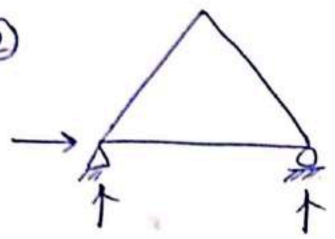
Ex. :-

(1)



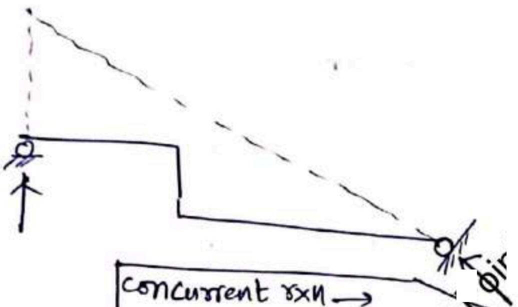
concurrent reactions → unstable structure

(2)



nonconcurrent rxn → stable structure

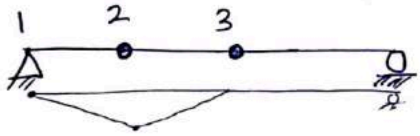
(3)



concurrent rxn → unstable str

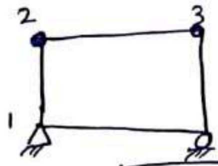
note:- 3 hinge in continuation → mechanism form hence unstable structure

(i)



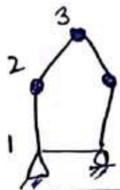

unstable str

(ii)




Unstable

(iii)

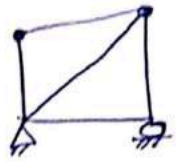



unstable

Special case :- but if there is bracing or additional member . then It will be stable.

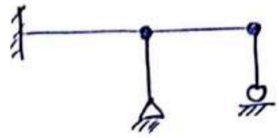
like in Previous (ii) example if 1 & 3 connected by member then It will be stable structure.

(i)



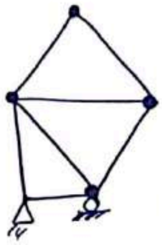
⇒ Stable

(ii)



⇒ Stable

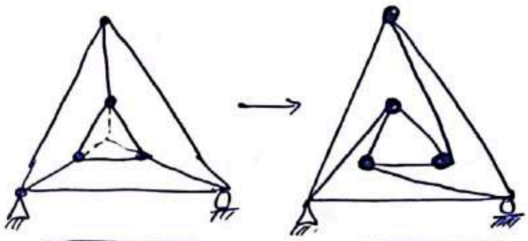
(iii)



⇒ Stable

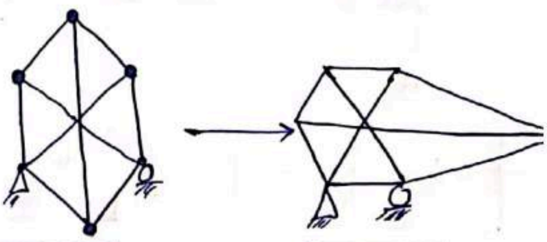
fixing of 1 joint has ensured that appreciable deformation of one part can not take place wrt other

note:- Triangle within Triangle



Unstable → Stable

note:-



Unstable {if all diagonals are equal.}

Stable {diagonals are unequal}

# Truss :-

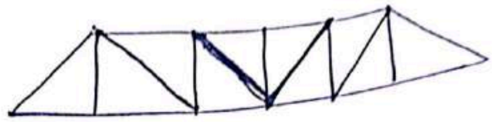
assumption →
- pinjoint
- Loading at joints
- selfwt neglectid

Trussdeflection
- Angle weight method
- Joint displacement method.
- williat mohr method ⇒ Graphical method

## forms of Truss

① Pratt Truss :- for the loads in gravity direction.

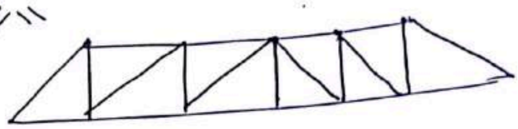
Truss =//



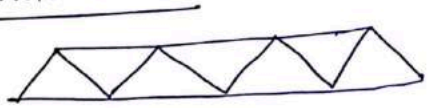
Pratt truss

(II) Howe Truss :- Just opposite of Pratt truss

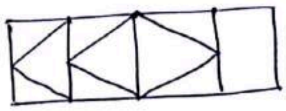
Truss //\\



(III) warren Truss



(IV) K Truss :-



N type force → due to external load

n type force → due to unit ~~load~~ { Apply unit load in that direction in which deflection is needed }

total Deflection $= \frac{nNL}{AE} + nL\alpha\Delta T + n\,dL$

- $\frac{nNL}{AE}$ → due to Force
- $nL\alpha\Delta T$ → due to temp. change
- $n\,dL$ → due to member too short or to long.

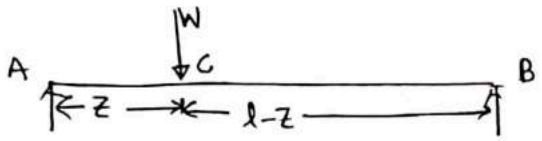
$\boxed{F = P + Kx}$

$$\boxed{x = \frac{-\Sigma PKL/AE}{\Sigma K^2L/AE}}$$

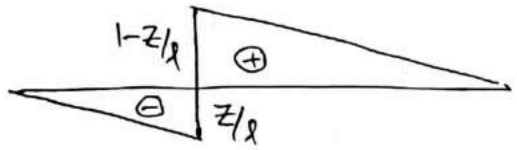
## Types of moving loads on simply supported Beam

- Single point load W
- Uniformly distributed load (udl)
  - udl > span
  - udl < span
- train load


### (I) Single point load :-

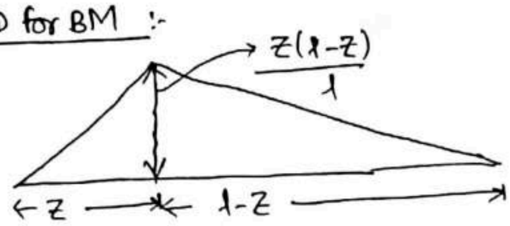



**ILD for shear force (SF)**



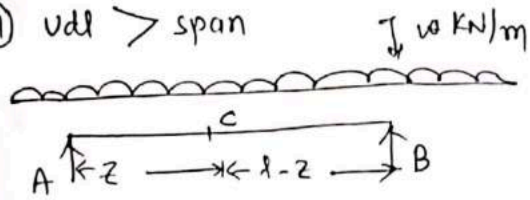

- max SF
  - (+)ve SF: Just right to 'c' — $\boxed{W(1 - Z/l)}$
  - (−)ve SF: Just left of 'c' — $\boxed{WZ/l}$
- absolute max. SF
  - (+)ve: when Point load is at A — $\boxed{+W}$
  - (−)ve: when Point load is at B — $\boxed{-W}$

**ILD for BM :-**

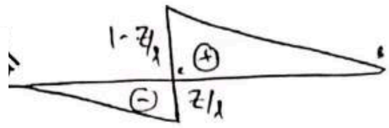



- max BM: at C — $\boxed{\frac{W Z(l-Z)}{l}}$
- absolute max BM: $\frac{dM}{dZ} = 0$, $Z = l/2$ — $\boxed{\frac{Wl}{4}}$

### (II) udl > span

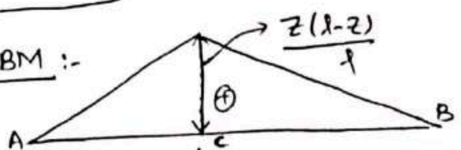



**ILD for sf**




- max sf
  - (+)ve: when udl is on BC portion — $\boxed{w \times \text{area of ILD}_{BC}}$, $\boxed{w\, \frac{1}{2}(l-Z)\left[1-\frac{Z}{l}\right]}$
  - (−)ve: when udl is on AC portion — $\boxed{w \times \frac{1}{2} \times Z \times Z/l}$
- absolute max. sf
  - (+)ve: when $Z = 0$ — $w \times$ ILD area — $\boxed{w \times \left[\frac{1}{2} \times l \times 1\right]}$
  - (−)ve: when $Z = l$ — $w \times$ ILD area — $\boxed{w \times \frac{1}{2} \times l \times 1}$

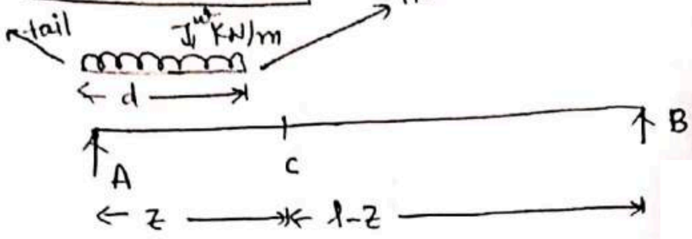
**ILD for BM :-**



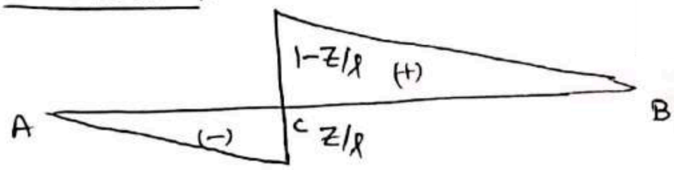

- max BM: at 'c' — $\boxed{w \times \underbrace{\left[\frac{1}{2} \times \left(\frac{Z(l-Z)}{l}\right) \times l\right]}_{\text{area of ILD } ABC}}$
- absolute max. BM: $\frac{dM}{dZ} = 0$, $\boxed{Z = l/2}$ — $\boxed{w \times \underbrace{\left[\frac{1}{2} \times l \times \frac{l}{4}\right]}_{\text{area}}}$

Important

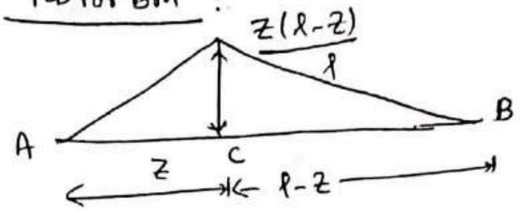
(III) Udl < span :-

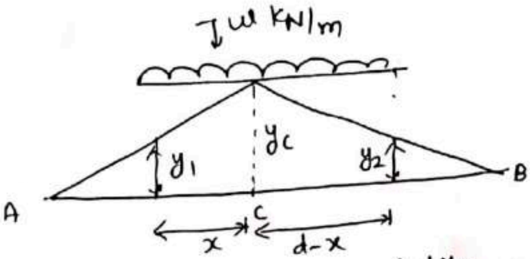



ILD for SF :-




ILD for BM :




for max. BM at C :- $\left(\frac{dM_c}{dx}=0\right)$

$$M_c=\left[\frac{1}{2}\times x\times(y_1+y_c)\right]w+\left[\frac{1}{2}\times(d-x)(y_c+y_2)\right]w$$

$\frac{dM_c}{dx}=0$

$\boxed{y_1=y_2}$ & $\boxed{\frac{z}{l}=\frac{x}{d}}$

- $y_1=y_2$ → ordinates of head & tail are equal
- $\frac{z}{l}=\frac{x}{d}$ → Section divide the load in the same ratio as of span. (v. imp)

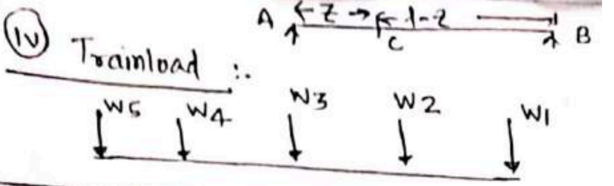
SF

- max SF
  - (+)ve → udl tail touch to the C → w × area → $w\times\frac{1}{2}\times(l-z)\times(1-z/l)$ ⇒ not ∵ whether it covers whole CB or not we don't known.
  - (−)ve → Udl head touch to C
- absolute max SF
  - (+)ve → udl tail touch to A → w × area of ILD
  - (−)ve → Udl head touch to B → w × area of ILD

absolute max. BM :-

when $y_c \rightarrow$ max, $z=l/2$

(iv) Trainload :-

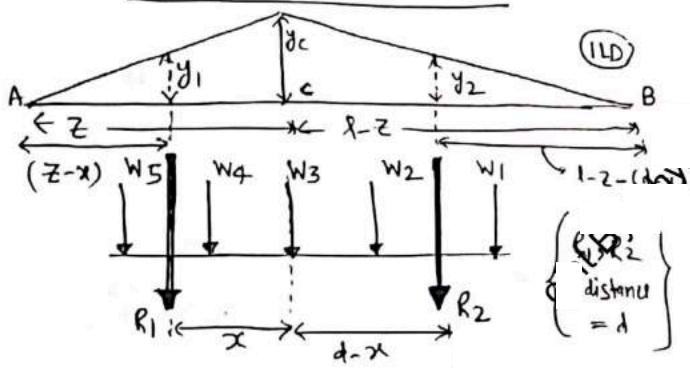



**Case-1 :- max. SF at any section 'c' :-**


max(-)ve SF at C :- Put $W_1$ at c, rest left of c ] do trials
Put $W_2$ at c, " "

max.(+)ve SF at C :- Put $W_5$ at c, rest right of c ] do Trials
Put $W_4$ " "

**Case-2 max. Bm at any section 'c'**




let $W_3$ at 'c' $R_1 \rightarrow$ Resultant of loads left side of c
$R_2 \rightarrow$ ——— right ———

Bm at C :- $M_c = R_1 y_1 + R_2 y_2$ $\begin{cases} y_1 = y_c \times \frac{Z-x}{Z} \\ y_2 = y_c \times \frac{l-Z-(d-x)}{(l-Z)} \end{cases}$

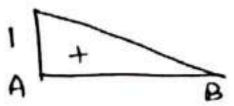
For $Bm_{max}$ at C $\frac{dM_c}{dx} = 0$

$$\boxed{\frac{R_1}{Z} = \frac{R_2}{l-Z}}$$

Conclusion :- average of loads on left and right side of Section should be same
* average of load = $\frac{\text{Resultant load (sum)}}{\text{distance}}$

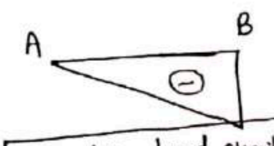
**Case-3 :- absolute max. shearforce :-**

(+)ve




one of the load should be at A, rest do the trials.

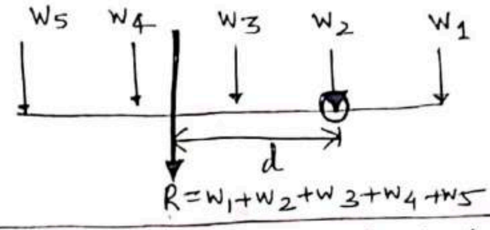
(-)ve



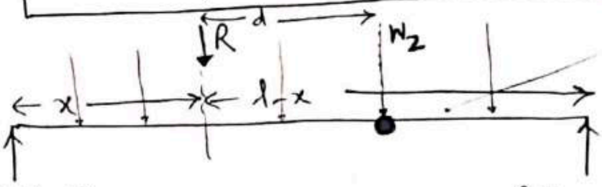

one of the load should be at B, rest do the trials

(3)

**Case-4 :- max. moment under a given load (suppose $W_2$)**




$R \rightarrow$ resultant of all loads which is 'd' distance from $W_2$ and x distance from A.

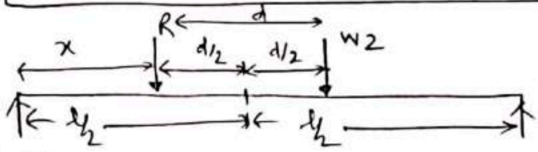



$\frac{R(l-x)}{l} = R_A$ $\frac{Rx}{l}$

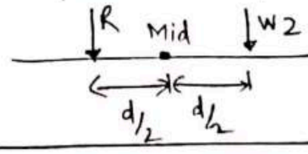
Bm under $W_2$ = $R_A(x+d) - Rd$

for max. $\frac{d(M_{under\,W_2})}{dx} = 0$ $\boxed{x = \frac{l}{2} - \frac{d}{2}}$

hence $W_2$ distance from A $= x + d = \frac{l}{2} + \frac{d}{2}$




Conclusion → Resultant R & $W_2$ should be equidistant from mid span

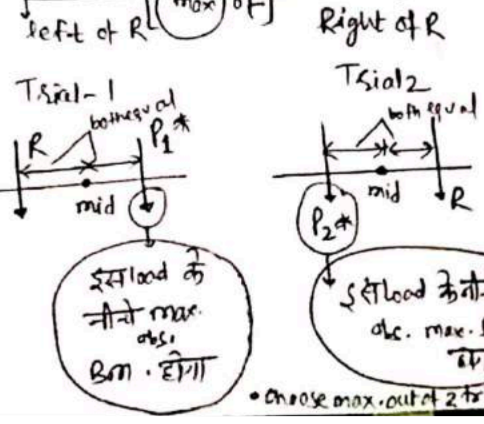



**Case-5 :- absolute max. BM :-**

- when heavier loads near centre
- max. moment under a wheel load

[max of] Left of R Right of R

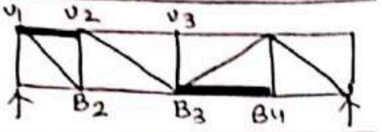
इन दोनों case में से max. वाले को choose करें




• choose max. out of 2 trials.

## ILD for Truss member

### Top chord member or bottom chord member




Ex. ILD for $U_1U_2$ member
ILD for $B_3B_4$ "

Approach :- left that panel & unit load placed in rest of panel, first left of that panel second, right of that panel and then solve.

### Inclined member

Ex. ILD for $U_2B_3$ member

approach → same approach

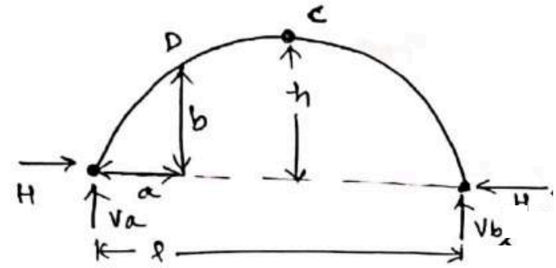
### vertical member

Ex. ILD for $U_2B_2$, $U_3B_3$

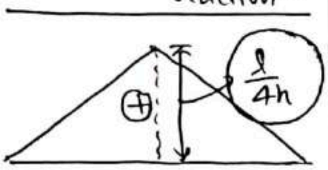
Approach :-
left panel right of this member

---

## ILD for 3-hinge arch :-

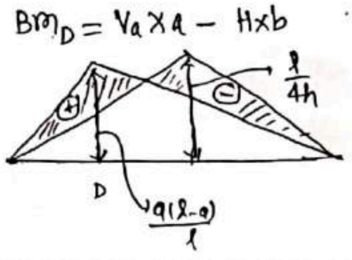



### ILD of Horizontal reaction H



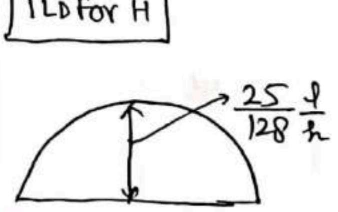

### ILD for BM :

$$BM_D = V_a \times a - H \times b$$

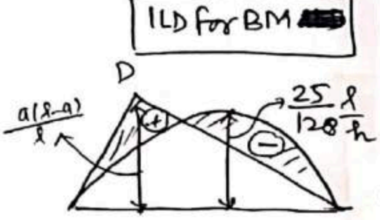



## ILD for 2 hinge arch :-


### ILD for H




### ILD for BM



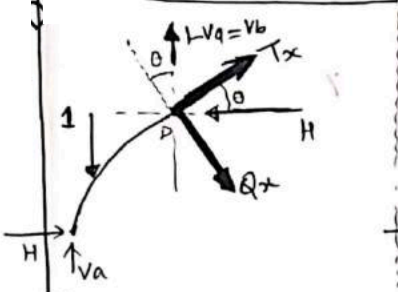

** note :- this shaded area depends on values.

---

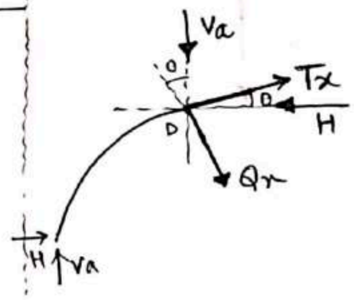
## 3 hinge arch :-

### ILD for normal Thrust & radial shear at 'D' :-

Case-1 :- unit load Before D



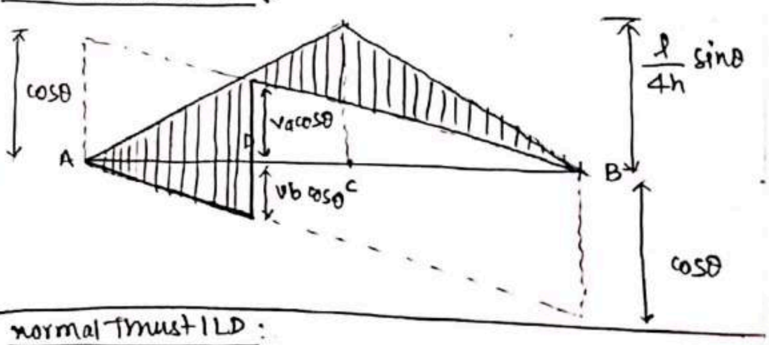

case-2 unit load after D



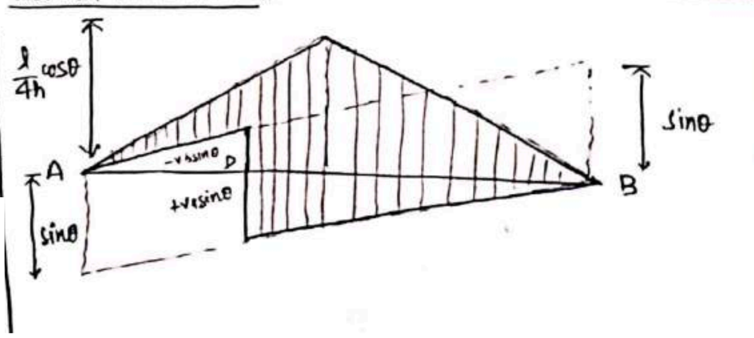

Radial shear at D $Q_x = H\sin\theta + V_b\cos\theta$ {when unit load left of D
$Q_x = H\sin\theta - V_a\cos\theta$ {right of D

normal thrust at D $T_x = H\cos\theta - V_b\sin\theta$ {when unit load left of D}
$T_x = H\cos\theta + V_a\sin\theta$ {right of D

### Radial shear ILD :

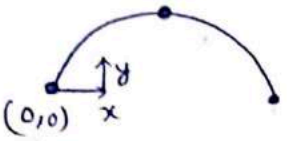



### normal Thrust ILD :



# 3 hinge arch

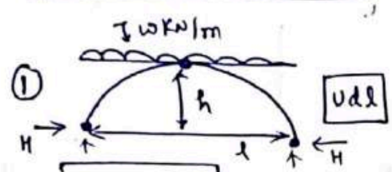
## Parabolic arch




**Parabolic arch**

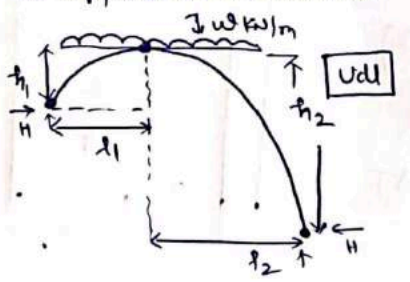
$$y = \frac{4hx(l-x)}{l^2}$$

(I) udl




$$\boxed{H = \frac{wl^2}{8h}}$$

(II) supports not at same level




$$\boxed{l_1 = \frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}}$$

$$\boxed{l_2 = \frac{l\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}}$$

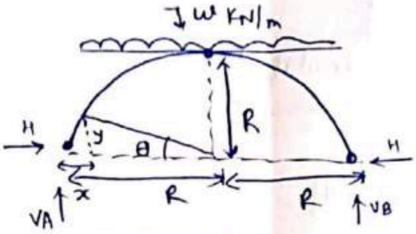
$$\boxed{H = \frac{wl^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}}$$ → same type formula summit cum

$$\frac{Wl^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}$$ **

(III) Supports not same level
But point load at crown


$$\boxed{H = \frac{Wl}{(\sqrt{h_1}+\sqrt{h_2})^2}}$$

## Semicircular arch

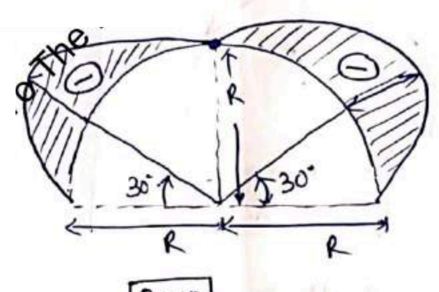



$$\boxed{H = \frac{wR}{2}}$$

$$\boxed{BM_x = V_A \cdot x - \frac{wx^2}{2} - Hy}$$

x, y → components of R से निकालना

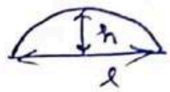
$$\boxed{\therefore BM_{max} = \frac{wR^2}{8} \text{ @ } \theta = 30^\circ}$$




BMD

## effect of temperature change

(ΔT)




$$\boxed{\Delta h = \left(h + \frac{l^2}{4h}\right)\alpha \Delta T}$$

new crown height

$\therefore H = \frac{wl^2}{8h}$ { Parabolic udl (3) hinge

$H \propto 1/h$

$\frac{dH}{H} = -\frac{dh}{h}$

## special case

3 hinge or 2 hinge arch

if fully udl loaded in whole span

then

(i) $BM_x = 0$ (always)

(ii) Radial shear$_x$ = 0 (always)

But

(iii) Normal thrust$_x \neq 0$

$$H = \int \frac{My\,ds}{EI} \Big/ \int \frac{y^2 ds}{EI}$$

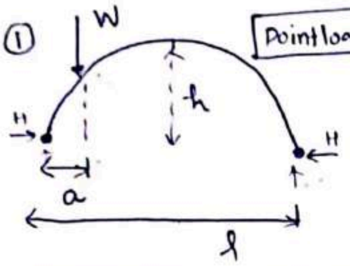
$I = I_0 \sec\theta$ → Moment of Inertia of arch at crown ($\theta = 0 \therefore I = I_0$)

# 2 Hinge arch

$y = \frac{4hx(l-x)}{l^2}$ valid

## Parabolic

① Point load

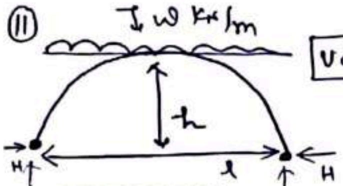


$$\boxed{H = \frac{5}{8} \frac{W a (l-a)(al - a^2 + l^2)}{h l^3}}$$

** if $a = l/2$ ⇒ Point load is in mid

$$\boxed{H = \frac{25}{128} \frac{Wl}{h}}$$

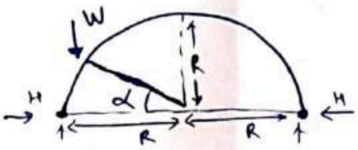
⑪ $w$ kN/m udl



$$\boxed{H = \frac{wl^2}{8h}}$$

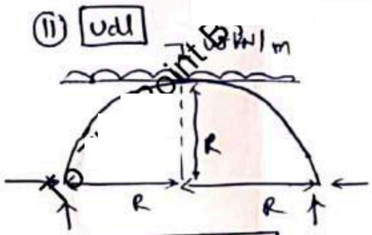
## Semicircular

① point load



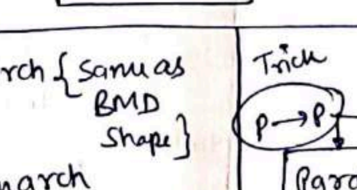
$$\boxed{H = \frac{W}{\pi} \sin^2\alpha}$$

⑪ udl



$$\boxed{H = \frac{4}{3} \frac{wR}{\pi}}$$

⑪ Half udl or triangular loading



$$\boxed{H = \frac{2}{3} \frac{wR}{\pi}}$$

## effect of temperature

$$\boxed{H = \frac{\int L\alpha\Delta T}{\int y^2 ds / EI}}$$

### Parabolic

$$H = \frac{\alpha\Delta T}{\frac{\left(\frac{8}{15}h^2\right)}{EI}}$$

or

$$H = \frac{15 EI \alpha \Delta T}{8 h^2}$$

### Semicircular

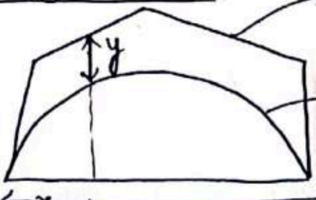
$$H = \frac{\alpha\Delta T}{\frac{\left(\frac{\pi R^2}{4}\right)}{EI}}$$

or $\frac{4 EI\alpha\Delta T}{\pi R^2}$

## Eddy's Theorem :

→ linear arch {Same as BMD Shape}

→ given arch



$\boxed{\text{BM at any section } xx \propto y}$

↓

distance b/w linear arch & given arch.

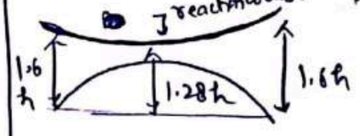


## Trick . Reaction locus of ② Hinge arch

P → P

### Parabolic arch

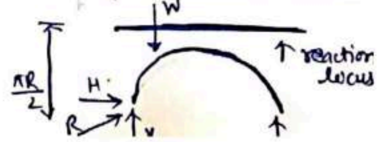
curved (Parabolic) @ 1.6h distance at ends & 1.28h distance at mid.



Trick S → S

### Semicircular arch

Straight line at $\frac{\pi R}{2}$ distance



# method of structure analysis

**force method** or **flexibility method** or **compatability method**

Use when $\boxed{D_s < D_K}$

$D_s \rightarrow$ degree of static indeterminacy.

$D_K \rightarrow$ degree of kinematic indeterminacy

**Displacement method** 'or' **stiffness method** 'or' **equilibrium method**

Use when $\boxed{D_K < D_s}$

Example :-

(I) unit load method / virtual work method — By Bernoulli
(II) consistent deformation method
(III) Three moment theorem method
(IV) column analogy method
(condition — Rigid frame with fix support)
(V) castigliano's theorem of minimum strain energy
(VI) elastic centre method
(VII) maxwell - mohr equation

example :- Trick $\boxed{SMS}$ + $\boxed{KM}$

(I) Slope deflection method (S) → G.A Maney method
(II) moment distribution method (m) → Hardy Cross method
(III) stiffness method (S)
(IV) kani's method
{ (successive approximation, rotational cofficient
Iterative approach for applying slope deflection method }
(V) min potential energy method.

| $D_s < D_K$ force method | $D_K < D_s$ Displacement method |
|---|---|
| • redundant forces or reactions are taken as unknown | • displacements ($\theta$, $\Delta$) are taken as unknown |
| • compatability condition is used to get or find unknown reaction/force | • equilibrium equation at joints are required to find unknown displacement |
| • no of compatability equation ⇒ $D_s$ | • no. of equilibrium condition/eqn $= D_K$ |

**Strain energy stored**

| due to axial load | due to bending | due to shear force | due to torsion |
|---|---|---|---|
| Ex. In truss | Ex. in beam, frame | | |
| $\int \frac{P^2 dx}{2AE}$ | $\int \frac{m^2 dx}{2EI}$ | $\int \frac{S^2 dx}{2AG}$ | $\int \frac{T^2 dx}{2GI_P}$ |

AE → Axial rigidity
EI → Flexural "
$GI_p$ → torsional rigidity

$\frac{AE}{L}$ → axial stiffness
$\frac{EI}{L}$ → Flexural stiffness
$\frac{GI_P}{L}$ → Torsional stiffness

Castigliano's 1st theorem →

$\boxed{\frac{\partial U}{\partial \Delta} = P}$ $\boxed{\frac{\partial U}{\partial \theta} = M}$

U → total strain energy
Δ → displacement in direction of force P
θ → rotation ——— moment M

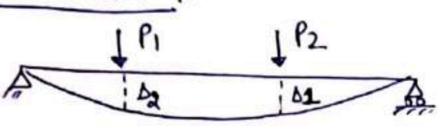
V.Imp

Castigliano's 2nd Theorem :

$\boxed{\frac{\partial U}{\partial P} = \Delta}$ $\boxed{\frac{\partial U}{\partial m} = \theta}$

Apply P to get Δ in P direction.
Apply M to get θ in that sense.(direction)

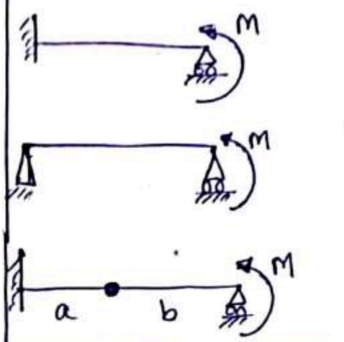
Betti's Law :-




$\boxed{P_1 \Delta_2 = P_2 \Delta_1}$

Special case of Betti's law → maxwell reciprocal Theorem

carryover factor = $\frac{\text{carryover moment}}{\text{Applied moment}}$



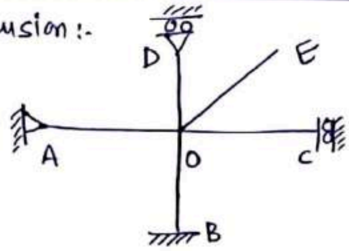

$\boxed{cof = \frac{1}{2}}$

$\boxed{cof = 0}$

$\boxed{cof = a/b}$

Distribution factor Df = $\frac{\text{Stiffness of member}}{\text{Sum of stiffness of all member at that joint}}$

conclusion :-




$\boxed{M_{OA} = \left(\frac{3EI}{L}\right)\theta}$ ∵ far end is hinge

$\boxed{M_{OB} = \left(\frac{4EI}{L}\right)\theta}$ ∵ far end is fix

$\boxed{M_{OC} = \left(\frac{EI}{L}\right)\theta}$

$\boxed{M_{OD} = 0}$ → ∵ far end is roller

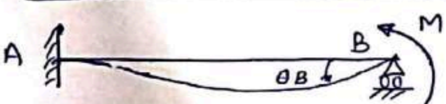
$\boxed{M_{OE} = 0}$ → free

moment distribution method (Hardy cross method) :-

Stiffness (k) = $\frac{\text{force (P)}}{\text{deflection } \Delta} = \frac{\text{moment(m)}}{\text{rotation } (\theta)}$

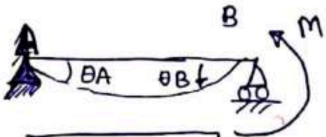
Stiffness of Beam

Stiffness of member BA when far end A is fix




$\boxed{k = \frac{4EI}{L}}$

Stiffness of member BA when far end A is hinge.




$\boxed{k = \frac{3EI}{L}}$

Relative stiffness

when far end is fix

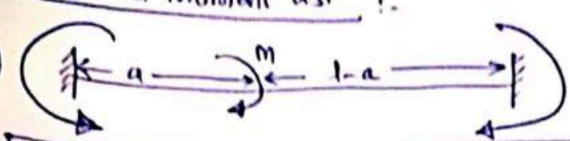
$\boxed{R.S = I/L}$

when far end is hinge

$\boxed{R.S = \frac{3I}{4L}}$

## Fixed end moment list :-

(1)

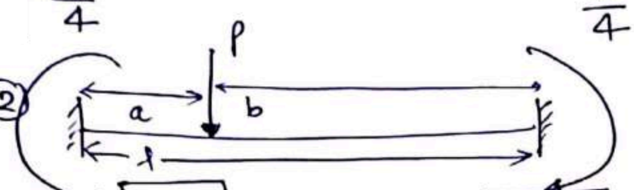



$$\frac{M}{l^2}(l-a)(l-3a) \qquad \frac{M}{l^2}(a)(2l-3a)$$

Special case :- $a = l/2$

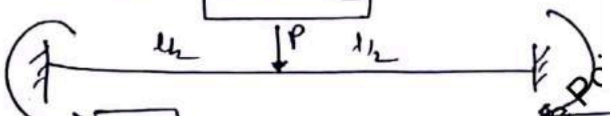

$$\frac{M}{4} \qquad \frac{M}{4}$$

(2)



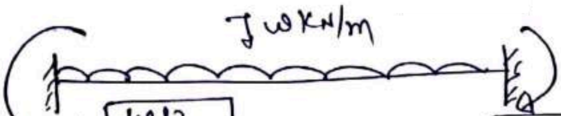

$$\frac{Pab^2}{l^2} \qquad \frac{Pba^2}{l^2}$$

Special case $a = b = l/2$



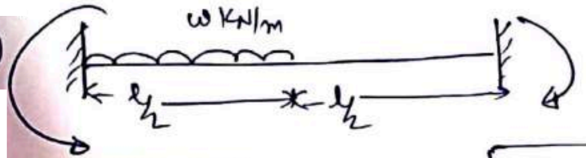

$$Pl/8 \qquad Pl/8$$

(3)



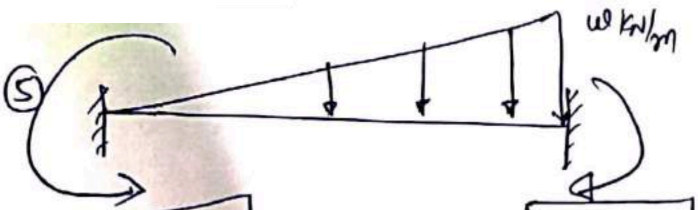

$$\frac{wl^2}{12} \qquad \frac{wl^2}{12}$$

(4)



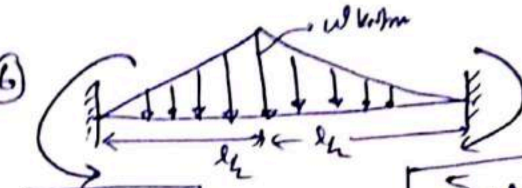

$$\frac{11}{192} wl^2 \qquad \frac{5}{192} wl^2$$

(5)



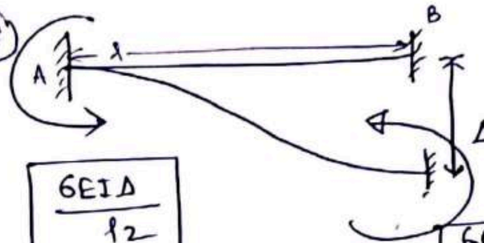

$$\frac{wl^2}{30} \qquad \frac{wl^2}{20}$$

(6)




$$\frac{5}{96} wl^2 \qquad \frac{5}{96} wl^2$$

(7)



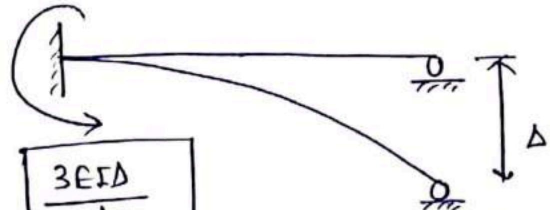

$$\frac{6EI\Delta}{l^2} \qquad \frac{6EI\Delta}{l^2}$$

$$R_A = \frac{12EI\Delta}{l^3} \qquad R_B = \frac{12EI\Delta}{l^3}$$

(8)


$$\frac{6EI(\Delta_1+\Delta_2)}{l^2} \qquad \frac{6EI(\Delta_1+\Delta_2)}{l^2}$$

(9)

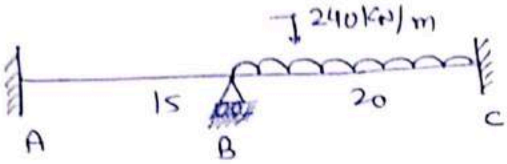



$$\frac{3EI\Delta}{l^2} \qquad M = 0$$

## moment distribution method type :-

(mostly asked in Exam)

**Type-1** :- when both ends of beam is fixed.




$$DF_{BA} = \frac{4EI/15}{4EI/15 + \frac{4EI}{20}} = 0.40$$

$$\therefore DF_{BC} = 1 - DF_{BA} = 0.60$$

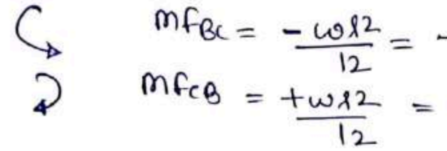
Fixed end moment (anticlockwise = (-)ve)

$$mf_{BC} = -\frac{wl^2}{12} = -8000 \text{ kNm}$$

$$mf_{CB} = +\frac{wl^2}{12} = +8000 \text{ kNm}$$

| | A | B | B | C |
|---|---|---|---|---|
| DF | – | 0.40 | 0.60 | – |
| FEM | 0 | 0 | -8000 | +8000 |
| Balance | | +3200 | +4800 | |
| com | +1600 | – | – | +2400 |
| Balance | – | – | – | |
| end moment | +1600 | +3200 | -3200 | +10400 |

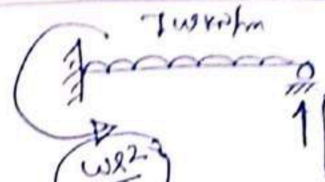
**Type-2** when one end is roller / hinge.




FEm $\quad MF_{AB} = -\frac{wl^2}{12} \quad mF_{BA} = +\frac{wl^2}{12}$

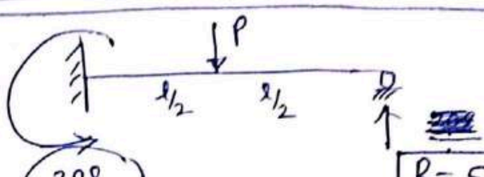
| | A | B |
|---|---|---|
| DF | - | - |
| FEm | $-\frac{wl^2}{12}$ | $+\frac{wl^2}{12}$ |
| | $-\frac{wl^2}{24}$ ← half transfer | $-\frac{wl^2}{12}$ make B mat B = 0 |
| end | $-\frac{wl^2}{8}$ | |

**Remember** :




$M = \frac{wl^2}{8}$ $\quad R = \frac{3wl}{8}$

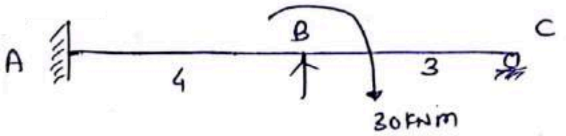
**Remember** :




$M = \frac{3Pl}{16}$ $\quad R = \frac{5P}{16}$

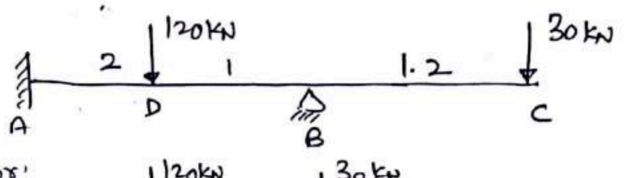
**Type-3**

when There is concentrated moment between the beam.

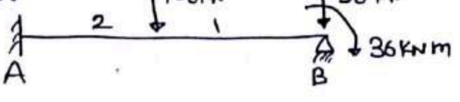



| | A | B | B | C |
|---|---|---|---|---|
| DF | – | 2/3 | 1/3 | – |
| FEM | 0 | 0 | 0 | 0 |
| balance | | +30×2/3 | +30×1/3 | |
| com | 10 | – | – | 0 {∵ roller |
| bal. | – | – | – | – |
| | +10 | +20 | +10 | 0 |

check (net +30) kNm




'or'




| | A | B |
|---|---|---|
| DF | – | – |
| FEm | -26.67 | +53.33 → to make $M_B = +36$ kNm |
| | $-\frac{17.33}{2}$ ← half transfer | -17.33 |
| final fix end moment | -35.33 | +36 |

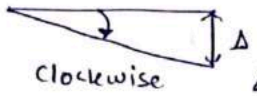
REDMI NOTE 5 PRO

## Slope deflection method :-

$$\boxed{M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B\right] - \frac{6EI\Delta}{l^2}}$$

$$\boxed{M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A\right] - \frac{6EI\Delta}{l^2}}$$

**note:-**




$\Delta = (+)ve$

## modified slope deflection equation

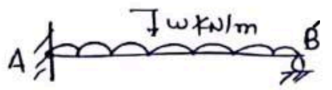
↳ only use when farend is hinge/roller.

$$\boxed{M_{AB} = M_{FAB}^{*} + \frac{3EI}{l}\left[\theta_A - \Delta/l\right]}$$

$M_{FAB}^{*} \longrightarrow$ end moment

(not a ~~fixed~~ end moment)

Ex.




$$M_{FAB} = -\frac{wl^2}{12}$$

$$M_{FAB}^{*} = -\frac{wl^2}{8}$$

## unit load method

$$\Delta = \int \frac{M \cdot m\,dx}{EI}$$

$M \longrightarrow$ due to external load

$m \longrightarrow$ due to unit load

apply unit load in the direction in which deflection is needed

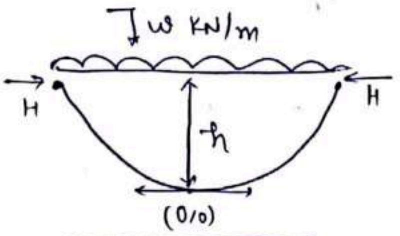
## cable

- Cable shape due to self weight ⇒ $\boxed{\text{catenary}}$

- cable
  - Perfectly flexible (BM at every section = zero)
  - Inextensible

- cable
  - min. Tension ⇒ where sag. max. $\boxed{T_{min} = H}$
  - max. tension $= \sqrt{V_{max}^2 + H^2}$ [at support] where $V_{max}$ exist {$V_{max}$ → vertical reaction}

- symmetrical cable subjected to udl on horizontal span ⟶ profile parabolic




$$\boxed{H = \frac{w l^2}{8h}}$$

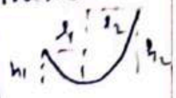
$$\boxed{y = \left(\frac{w}{2H}\right) x^2} \qquad \therefore \frac{dy}{dx} = \frac{8hx}{l^2}$$

$$\therefore \quad T = H\sqrt{1+\left(\frac{dy}{dx}\right)^2} = H\sqrt{1+\left(\frac{8hx}{l^2}\right)^2}$$

- min: $x = 0$ ↳ max. sag; $T_{min} = H$
- max: $x = l/2$ (supports)

Cable length $(S) = \int_0^l \sqrt{1+\left(\frac{dy}{dx}\right)^2}$

- when supports are not at same level




$$\boxed{S = (l_1 + l_2) + \frac{2}{3}\frac{h_1^2}{l_1} + \frac{2}{3}\frac{h_2^2}{l_2}}$$

- when support at same level

$$\boxed{S = l + \frac{8}{3}\frac{h^2}{l}}$$

### varieties of Problems in symmetric parabolic cable

(i) elastic stretch $\boxed{ds = \frac{HL}{AE}\left(1 + \frac{16}{3}\frac{h^2}{l^2}\right)}$

(ii) temp. change

$$\boxed{ds = S\alpha\Delta T = \left(l + \frac{8}{3}\frac{h^2}{l}\right)\alpha\Delta T}$$

(iii) change in sag (dh) due to change in length (ds)

$\because S = l + \frac{8}{3}\frac{h^2}{l} \qquad \therefore h^2 = \frac{3l}{8}(S - l)$

$\therefore 2h\frac{dh}{ds} = 3l/8 \qquad \boxed{\frac{dh}{ds} = \frac{3l}{16h}}$

(iv) change in max. tension ($dT_{max}$) due to change in sag (dh)

$\because T_{max}^2 = V_{max}^2 + H^2$, $V_{max} = \left(\frac{wl}{2}\right)$

$2T_{max}\frac{dT_{max}}{dH} = 2H \qquad \boxed{\frac{dT_{max}}{dH} = \frac{H}{T_{max}}}$

(v) change in Horizontal rxn (dH) due to change in sag (dh)

$\because H = \frac{wl^2}{8h} \qquad \therefore \boxed{\frac{dH}{dh} = -\frac{H}{h}}$

(vi) change in sag (dh) due to support slip (dl)

$\because S = l + \frac{8}{3}\frac{h^2}{l}$

$\therefore h = \frac{3l}{8}(S - l) \Rightarrow \frac{dh}{dl} = 3/8(S - 2l)$, $S \to l + \frac{8}{3}\frac{h^2}{l}$

$$\therefore \boxed{dh = \frac{3l}{16h}\left(1 - \frac{8}{3}\frac{h^2}{l^2}\right)(-dl)}$$

(vii) $\boxed{\text{\% reduction in max. Tension} = \frac{dT_{max}}{T_{max}} \times 100}$

$T_{max} = H\sqrt{1+\left(\frac{dy}{dx}\right)^2}$, $\frac{dy}{dx} = \frac{8hx}{l^2}$, $x = l/2$

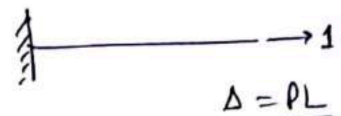
## flexibility & stiffness matrix :-

① $f_{ij} \longrightarrow$ due to unit load/moment at $j$, displacement in the direction of cordinate.
↓
unit

② $k_{ij} \longrightarrow$ force required/developed at $i$ for unit displacement at $j$
↓
unit

### Example :-

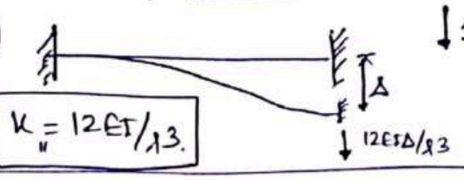
(i)



$\Delta = \frac{PL}{AE}$

$\boxed{K_{11} = \frac{AE}{L}}$

(ii)




$\boxed{K_{11} = 12EI/l^3}$

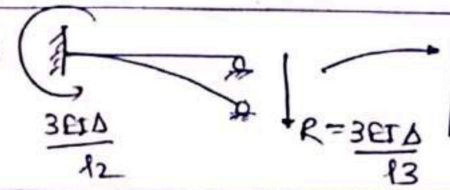
③ $\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}^T = \begin{bmatrix} K_{11} & K_{21} \\ K_{12} & K_{22} \end{bmatrix}$ & $\begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^T = \begin{bmatrix} f_{11} & f_{21} \\ f_{12} & f_{22} \end{bmatrix}$

[ Explained by maxwell reciprocal Theorem ]

④ diagonal terms of matrix $\neq 0$, $\neq (-)ve$ } always (+)ve

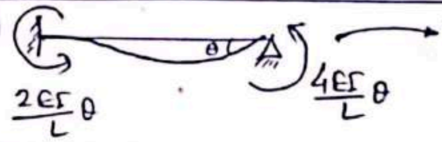
⑤ $\boxed{C_{AB} K_A = C_{BA} K_B}$

(iii)



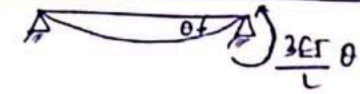

$\boxed{k = \frac{3EI}{l^3}}$

(iv)




$\boxed{k = \frac{4EI}{L}}$

(v)




$\boxed{k = \frac{3EI}{L}}$

### flexibility matrix :-

(i) square symmetric matrix.

(ii) order of matrix $(n \times n)$ = $D_s$

(iii) each flexibility matrix element = displacement

(iv) diagonal elements = always (+)ve { ∵ $\neq 0$, $\neq (-)ve$ }

(v) valid for stable structure

Stiffness matrix $\longrightarrow$ same property except order $(n \times n) = D_k$
↓
Degree of kinematic indeterminacy