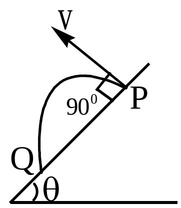
QUICK REVISION TEST

SINGLE CORRECT CHOICE TYPE QUESTIONS

1 If time taken by projectile to reach Q is T, then PQ =



A. $Tv\sin\theta$ **B.** $Tv\cos\theta$ **C.** $Tv\sec\theta$ **D.** $Tv\tan\theta$

Answer :D

Solution : $T = \dfrac{2V}{g\cos\theta}$

$$PQ = 0 + \frac{1}{2}(g\sin\theta)T^2 = \frac{g\sin\theta}{2} \times \frac{4V^2}{g^2\cos^2\theta} = \left(\frac{V\sin\theta}{\cos\theta}\right)\left(\frac{2V}{g\cos\theta}\right)$$

$$PQ = TV\tan\theta$$

2 A body is projected from ground with a velocity $40\sqrt{2}\,ms^{-1}$ at 45^0 to horizontal. After four seconds, it is stopped and again projected horizontally with a speed $40\,ms^{-1}$. Its horizontal displacement from point of projection (by the time it strikes ground) can be ____

A. 400 m **B.** 394 m **C.** 344 m **D.** 294 m

Answer :D

Solution : $\bar{V} = 40\hat{i} + 40\hat{j}$ $t_a = \dfrac{40}{10} = 4s$. After four seconds its at max height and its horizontal displacement is $x = 40 \times 4 = 160m$. From the highest point in its trajectory its again projects horizontally with $40ms^{-1}$. Take that point as centre and assume a horizontal circle. The radius vectors of that circle are horizontal. In which ever direction you throw it the time of descent is 4 sec and from that point the horizontal distance is 160 m, since horizontal velocity is $40ms^{-1}$ however this 160m can be in any direction hence total horizontal displacement is vertical addition of $\bar{R}_1 = 160$ and $\bar{R}_2 = 160$. Range of $\bar{R}_1 + \bar{R}_2$ will be between zero and 320 m. Hence D is correct option.

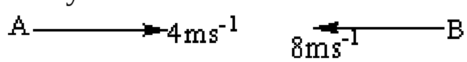
3 From a height 100m, a body is dropped and at the same time another body is projected vertically up with velocity $10\,ms^{-1}$ from the ground. The time after which they meet in air is _______ $[g=10\,ms^{-2}]$

A. 10s **B.** 8s **C.** 6s **D.** They do not meet in air

Answer :D

Solution : The time of descent of falling body is approximately 4.5 seconds. Time of flight of the body projected up is only 2 seconds. Hence they wont meet in air.

4 Velocity of ‘A’ increases at rate of $1ms^{-1}$ every second, and velocity of ‘B’ decreases at rate of $1ms^{-1}$ every second. Their relative acceleration is ________ ms^{-2}



A. 2 **B.** 4 **C.** 6 **D.** Zero

Answer :D

Solution : $a_A = 1ms^{-2} \rightarrow a_B = 1ms^{-2} \rightarrow$

$a_r = 1-1=0$

5 The acceleration experienced by a moving motor boat after its engine is cut off is $a=-Kv^3$. If V_0 is speed at cut off, magnitude of velocity at time ‘t’ after cut off is _______

A. $\frac{V_0}{\sqrt{2V_0^2kt+1}}$ **B.** V_0e^{-kt} **C.** $\frac{V_0}{2}$ **D.** V_0

Answer :A

Solution : $\frac{dv}{dt}=-kv^3$

$$\int_{v_0}^{v}\frac{dv}{v^3}=-\int_0^t k\,dt \Rightarrow \left[-\frac{1}{2v^2}\right]_{v_0}^{v}=-kt \Rightarrow \frac{1}{v_0^2}-\frac{1}{v^2}=(-kt)2$$

$$V=\frac{V_0}{\sqrt{2v_0^2kt+1}}$$

6 A particle moves with a velocity $3\hat{i}+4\hat{j}\,ms^{-1}$ from origin. The displacement of particle along line x=y after two seconds will be ______

A. 10 m **B.** $\frac{7}{\sqrt{2}}m$ **C.** $7\sqrt{2}m$ **D.** 5m

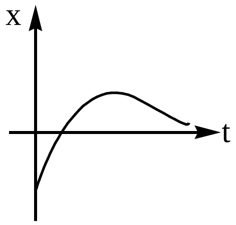
Answer :C

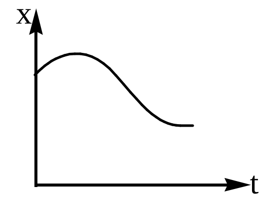
Solution : $|\bar{S}|=\bar{v}t=6\hat{i}+8\hat{j}$

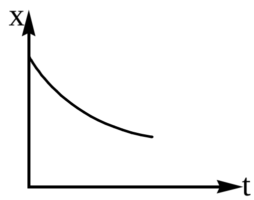
Unit vector along $x=y$ is $\frac{\hat{i}}{\sqrt{2}}+\frac{\hat{j}}{\sqrt{2}}$

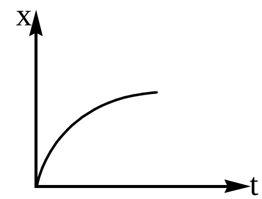
Component of $\bar{S}$ along the unit vector will be $7\sqrt{2}m$

7 Among the four graphs (figure), there is only one graph for which average velocity over the time interval (0,T) can vanish for a suitably chosen T. Which one is it ? x is the position of a particle moving along x-axis.

A. 

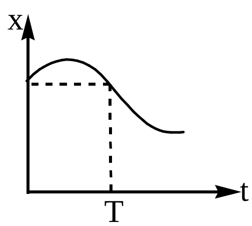
B. 

C. 

D. 

Answer :B

Solution : Average velocity becomes zero when total displacement is zero. It happens if x at t = 0 is equal to x at t = T. It is possible with graph (B) only.



8 The co-ordinates of an insect crawling on a vertical wall changes as $x = e^t$ and $2\log_e y = t$. Find the equation of trajectory of the insect.

A. $y^2 = x$ **B.** $y^2 = -x$ **C.** $x^2 = y$ **D.** $x^2 = -y$

Answer :A

Solution : $x = e^t$

$$2\log_e y = t \Rightarrow \log_e y^2 = t \Rightarrow y^2 = e^t \Rightarrow y^2 = x$$

9 A balloon ascends from rest vertically up relative to air with a constant vertical acceleration of $0.4\, m/s^2$ whereas the air flows due east with a velocity 5 m/s. An insect moves up along the vertical wire fixed with the balloon, with a velocity of 0.1 m/s relative to the wire. Find the magnitude of velocity of the insect w.r.t ground at t = 6 s. Take necessary assumptions.

A. $\frac{3\sqrt{5}}{2}$ m/s **B.** $\frac{5\sqrt{5}}{2}$ m/s **C.** $\frac{5\sqrt{3}}{2}$ m/s **D.** $\frac{3\sqrt{3}}{2}$ m/s

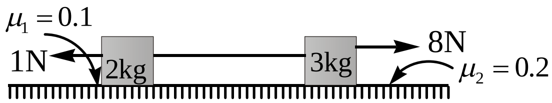
Answer :B

Solution : $\bar{v} = \bar{v}_{ballon} + \bar{v}_{air} + \bar{v}_{insect}$

$$= (at)\hat{j} + 5\hat{i} + 0.1\hat{j} = (0.4 \times 6 + 0.1)\hat{j} + 5\hat{i} = 5i + 2.5\hat{j}$$

$$\bar{v} = \sqrt{25 + \frac{25}{4}} = \frac{\sqrt{125}}{2} = \frac{5\sqrt{5}}{2} m/s$$

10 In the arrangement shown, if f_1 , f_2 and T be the frictional forces on 2 kg block , 3kg block & tension in the string respectively, then their values are:



A. 2 N , 6 N, 3.2 N **B.** 2 N, 6 N, 0 N **C.** 1 N, 6 N , 2 N **D.** 1N,2 N, 3.2 N

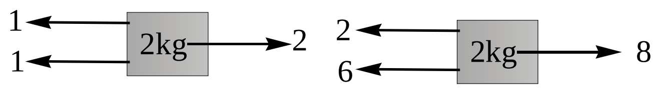
Answer :C

Solution :

Fmax=2N fmax=6N

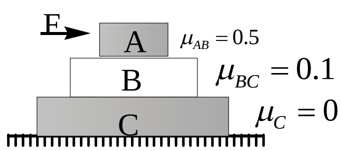
Net force without friction on system is '7N' in right side hence first maximum friction will come on

3 kgblock.



So $f_1 = 1,\ \ f_2 = 6\,N, T = 2N$

11 Three blocks A, B and C of masses 2 kg, 3kg and 4 kg are placed as shown . Coefficient of friction between A and B is 0.5 and that between B and C is 0.1. Maximum force Fthat can be applied horizontally on to A such that the three blocks move together is____$\left(g = 10ms^{-2}\right)$



A. 12.22 N **B.** 13 N **C.** 11.25 N **D.** 15 N

Answer :C

Solution : When the three blocks move together, acceleration of the system= $\dfrac{F}{9}$

This acceleration on $\left(\dfrac{F}{9}\right)$should be less than or equal to maximum possible accelerations of the blocks

for 4kg: $^{a}\max = \dfrac{f_{BC}}{4} = \dfrac{0.1(5)10}{4} = 1.25ms^{-2}$

for $(3+4)kg;\ \ ^{a}\max = \dfrac{f_{AB}}{7} = \dfrac{0.5(20)}{7} = 1.4ms^{-2}$

Now $\frac{F}{9} \le 1.25$

$\Rightarrow F \le 11.25N$

12 A small body starts sliding down from rest on a fixed inclined plane of inclination θ , where base length is equal to l. The coefficient of friction between the body and the surface is μ. If the angle θ is varied keeping l constant, the time of sliding from top to bottom of inclined plane will be least for the relation

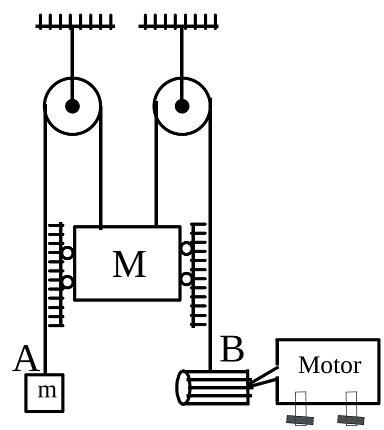
A. $\tan 2\theta = -\frac{1}{\mu}$ **B.** $\tan\theta = +\frac{1}{\mu}$ **C.** $\tan 2\theta = -\frac{1}{2\mu}$ **D.** $\tan\theta = \mu$

Answer :A

Solution : $a = g\sin\theta - \mu g\cos\theta$

$$t = \sqrt{\frac{2\ell\sec\theta}{a}} = \sqrt{\frac{2\ell\sec\theta}{g\sin\theta - \mu g\cos\theta}}\ \frac{dt}{d\theta} = 0 \Rightarrow \tan 2\theta = -\frac{1}{\mu}$$

13 The elevator has a mass M and the counter weight at A has a mass m. The motor supplies a constant force F on the cable at B. Neglecting the mass of the pulley and cable, the speed of the elevator at time t after starting from rest is



A. $\frac{F+(m-M)g}{(M+m)}t$ **B.** $\frac{F-Mg}{(M+m)}t$ **C.** $\frac{F-(M+m)g}{(M+m)}t$ **D.** $\left(\frac{F}{M+m}\right)i$

Answer :A

Solution : $T + F - Mg = Ma$

$mg - T = ma$

$$\frac{F + mg - Mg}{(m+m)} = a$$

$$v = \left(\frac{F + (m-M)g}{M+m}\right)t$$

14 A particle moves in the X-Y plane under the influence of a force such that its linear momentum is

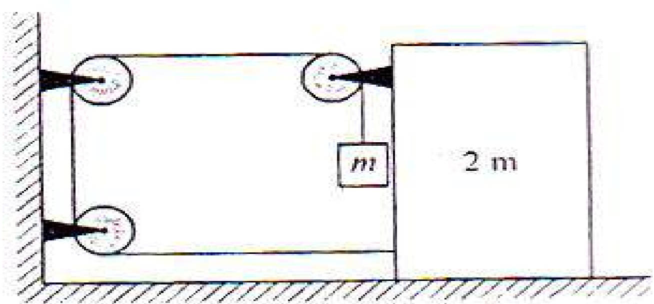
$\vec{p}(t) = A\left[\hat{i}\cos(kt) - \hat{j}\sin(kt)\right]$, where A and k are constants of appropriate dimensions. The angle between the force and the momentum is

A. 0^0 **B.** 30^0 **C.** 45^0 **D.** 90^0

Answer :D

Solution : $F = \frac{dp}{dt}$ and $\overline{F}.\overline{P} = 0$

15 In the system shown in figure, the friction coefficient between ground and bigger block is μ. There is no friction between the two blocks. The string connecting the two blocks is light. All three pulleys are light and frictionless. Then the minimum limiting value of μ so that the system remains in equilibrium is



A. $\frac{1}{2}$ **B.** $\frac{1}{3}$ **C.** $\frac{2}{3}$ **D.** $\frac{3}{2}$

Answer :C

Solution : In equilibrium

T = mg

N = 3 mg

& f = 2T = 2mg

In limiting case $f < f_{\max}$

$\Rightarrow \quad 2mg \leq \mu N$

$\Rightarrow \quad 2mg \leq 3\mu \ mg$

$\Rightarrow \quad \mu \geq \frac{2}{3} Ans.$

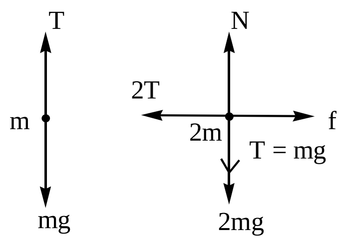
16 Statement: 1) Static friction can be equal to kinetic friction

Statement: 2) Static friction can be less than kinetic friction

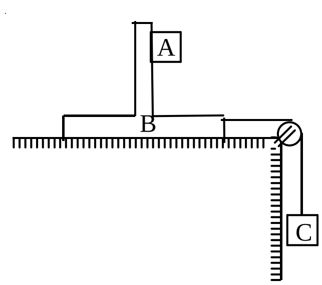
Statement: 3) Static friction can be greater than kinetic friction

A. Statement (1) alone is correct
B. Statement (2) alone is correct
C. Statement (3) alone is correct
D. Statement 1,2,3 all are correct

Answer :D
Solution : Conceptual

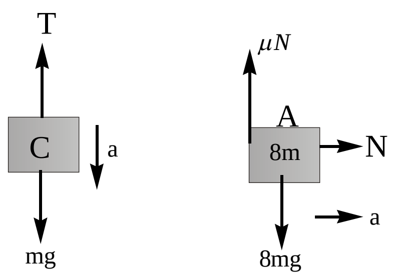


17 In the arrangement shown in the figure,masses of the blocks B and A are 2 m , 8 m respectively . Surface between B and floor is smooth . The block B is connected to block C by means of a pulley. If the whole system is released from rest, then the minimum value of mass of the block C so that the block A remains stationary with respect to B is : (Co-efficient of friction between A and B is μ .)



A. $\frac{m}{\mu}$ **B.** $\frac{2m}{\mu+1}$ **C.** $\frac{10m}{1-\mu}$ **D.** $\frac{10m}{\mu-1}$

Answer :D
Solution : FBD of A



If the acceleration of ‘C’ is a

For block ‘A’ N = 8 ma …………(1)

$8mg = \mu N = \mu 8ma$ ………….(2)

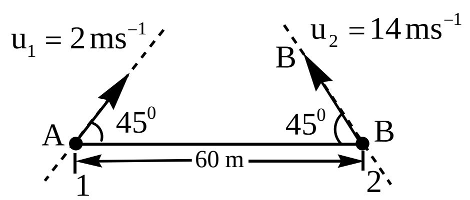
And acceleration a can be written by the equation of system $(A+B+C)$

$$m_1 g = (10\ m + m_1)a$$

$$8mg = \mu 8m\left(\frac{m_1 g}{10m + m_1}\right)$$

$$10\,m + m_1 = \mu m_1 \qquad \Rightarrow m_1 = \frac{10m}{\mu - 1}$$

18 Two particles are located on a horizontal plane at a distance 60m. At t = 0 both the particle are simultaneously projected at angle 45^0 with speeds $2\,ms^{-1}$ and $14\,ms^{-1}$ respectively as shown in figure. Find the time at which the separation between them is minimum?

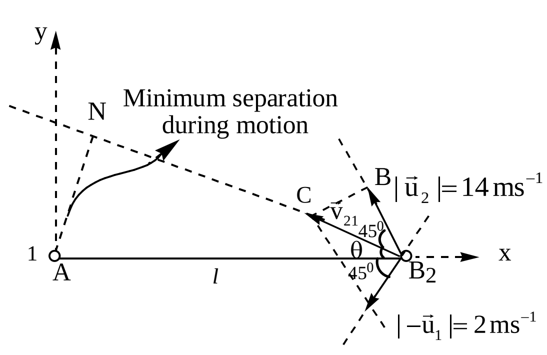


A. $\frac{12\sqrt{2}}{5}s$ **B.** $\frac{12\sqrt{2}}{7}s$ **C.** $\frac{12\sqrt{3}}{5}s$ **D.** $\frac{12\sqrt{3}}{3}s$

Answer :A

Solution : In relative motion, from observer considers himself at rest and observes the motion of object. Graphically, we can drawn the direction of motion of particle 2 w.r.t particle 1

Both the particles are moving in gravitational field with same acceleration ‘g’. Hence, relative acceleration of particle 2 as seen from particle 1 will be zero. It means the relative velocity of particle 2 w.r.t. particle 1 will be constant and will be equal to initial relative velocity. Graphically we can draw the situation as shown.



AN is the minimum separation between the particles and BN is the relative separation between the particle when the distance between 1 and 2 is shortest: From figure we can write

$v_{12}\cos\theta = 14\cos 45^0 + 2\cos 45^0$ ………………….(i)

$v_{12}\sin\theta = 14\sin 45^0 - 2\sin 45^0$ …………………..(ii)

From (i) and (ii) $v_{12} = 10\sqrt{2}\ ms^{-1}$

$\cos\theta = 4/5 \sin\theta = \frac{3}{5}$ as $\theta = 37^0$

Hence, minimum separation between the particles is

$$AN = AB\sin\theta = 60\times\frac{3}{5} = 36\,m$$

The time when separation between the particles in minimum

$$t = \frac{BN}{|\vec{v}_{12}|} \Rightarrow t = \frac{60\cos 37^0}{10\sqrt{2}} = \frac{12\sqrt{2}}{5}s$$

19 A particle moves in the x-y plane whose co-ordinates vary with time t as given by $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$ where 'a' is a constant of appropriate dimensions . Find the distance travelled by the particle in first t seconds.

A. $\frac{at}{2}$ **B.** $\frac{at^2}{2}$ **C.** $\frac{at^2}{3}$ **D.** $\frac{at}{3}$

Answer :B

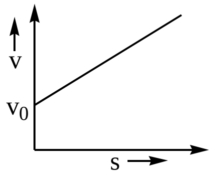
Solution : $v_x = \frac{dx}{dt} = a(-\sin t + t\cos t + \sin t) = at\cos t$

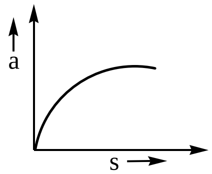
$$v_y = \frac{dy}{dt} = a(\cos t + t\sin t - \cos t) = at\sin t$$

Distance travelled: $D = \int_0^t v dt = \int_0^t \sqrt{v_x^2 + y_y^2}\,dt$

$$\Rightarrow D = \int_0^t \sqrt{(at\cos t)^2 + (at\sin t)^2}\,dt = \int at\,dt = \frac{at^2}{2}$$

20 Figure given below shows the variation of velocity (v) of a particle moving along straight line, with displacement(s). Which of the following graphs best represent the variation in acceleration with displacement?



A.


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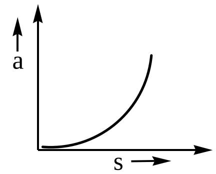

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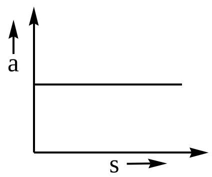
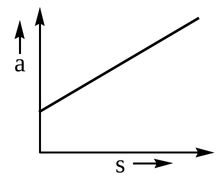


D.


Answer :D

Solution : From the velocity versus displacement graph, $v = ms + v_0$ [where m is the slope of the straight line]

Differentiating eqn. (i) w.r.t. time t, we get

$\frac{dv}{dt} = a = \frac{mds}{dt}$, i.e., a = $mv = m[ms + v_0]$

Or $a = m^2 s + mv_0$

Which is of the from y = mx + C

A straight line with slope $'m^2'$ and y-intercept $'mv_0'$.

21 A particle moves uniformly with speed v along a parabolic path $y = kx^2$ where k is a positive constant of appropriate dimensions. Magnitude of acceleration of the particle at x = 0 is

A. kv^2 **B.** $\frac{k}{2}v^2$ **C.** $2kv^2$ **D.** $3kv^2$

Answer :C

Solution : $\overline{r} = x\hat{i} + y\hat{j} = x\hat{i} + kx^2\hat{j}$

$\overline{v} = \frac{d\overline{r}}{dt} = \frac{dx}{dt}i + \frac{d}{dt}(kx^2)\hat{j} = v_x\hat{i} + 2kxv_x\hat{j}$

At x = 0, $\overline{v} = v_x\hat{i} + 0(\hat{j}) = v_x\hat{i}$ only

As speed is constant, tangential acceleration is zero. Particle possesses normal acceleration only i.e., $\overline{a}$

is always perpendicular to $\overline{v}$. Hence, at x = 0, $\overline{v} = v_x \hat{i}$ only $\Rightarrow \overline{a} = a_y \hat{j}$ only

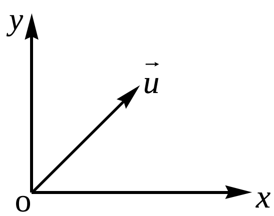
i.e. $a_x = 0$ at x = 0.

$$\overline{a} = \frac{d\overline{v}}{dt} = a_x \hat{i} + 2k[x a_x + v_x^2]\hat{j}$$

At x = 0, $v = v_x$ and $a_x = 0 \Rightarrow \overline{a} = 2kv^2 \hat{j}$

$a = 2kv^2$

22 If a particle is projected from ground with an initial velocity $\vec{u} = \left(\hat{i} + \sqrt{3}\hat{j}\right) m/s$, find the time up to which vertical displacement will be greater than horizontal displacement after projection. Take horizontal direction as x-axis and vertical direction as y-axis and $g = 10\ ms^{-2}$.



A. $\frac{\sqrt{3}}{5} s$ **B.** $\frac{1}{5} s$ **C.** $\frac{\sqrt{3}+1}{5} s$ **D.** $\frac{\sqrt{3}-1}{5} s$

Answer :D

Solution : $y > x [u = \sqrt{3+1} = 2, \theta = 60^0]$

$$(u\sin\theta)t - \frac{1}{2}gt^2 > (u\cos\theta)t$$

$$2\times\frac{\sqrt{3}}{2} - \frac{10}{2}t > 2\times\frac{1}{2} \Rightarrow 2\sqrt{3} - 2 > 10t \Rightarrow t < \frac{\sqrt{3}-1}{5} s$$

23 Given that $y = A\sin\left[\left(\frac{2\pi}{\lambda}(ct - x)\right)\right]$, where y and x are measured in meters. Which of the following statements is true

A. The unit of λ is same as that of x and A

B. The unit of λ is same as that of x but not of A

C. The unit of c is same as that of $\frac{2\pi}{\lambda}$

D. The unit of (ct-x) is same as that of $\frac{2\pi}{\lambda}$

Answer :A

Solution : Conceptual

24 For a particle moving along circular path, the radial acceleration a_r is proportional to time t. If a_t is the tangential acceleration, then which of the following will be independent of time t ?

A. a_t **B.** $a_r.a_t$ **C.** a_r / a_t **D.** $a_r(a_t)^2$

Answer :D

Solution : $a_r = \frac{v^2}{r} \propto t$ ---------(1)

$$\Rightarrow \frac{2v.a_t}{r} = constant$$

$$\Rightarrow a_t \propto t^{-1/2} \text{ ---------(2)}$$

$$\therefore a_r.a_t^2 \propto t^0 .$$

25 A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200N and is directly opposite to the motion. The work

done by the cycle on the road is

A. Zero **B.** – 200 J **C.** + 2000 J **D.** – 2000 J

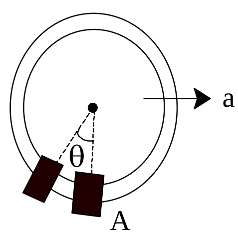
Answer :A

Solution : As the displacement of the road is zero with respect to the ground, work done by the force of cycle on the road is zero.

26 A vertical circular frame start from rest and moves with a constant acceleration a.

A smooth sliding collar A is initially at rest in the bottom position $\theta = 0$. Find the

maximum angular position $\theta_{\max}$ reached by the collar.



A. $\tan^{-1}\left(\frac{a}{g}\right)$ **B.** $\tan^{-1}\left(\frac{2a}{g}\right)$ **C.** $2\tan^{-1}\left(\frac{a}{g}\right)$ **D.** $\tan^{-1}\left(\frac{a}{2g}\right)$

Answer :C

Solution : From work-energy theorem w.r.t frame

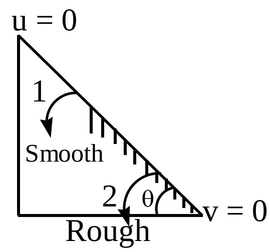
$$W_g + W_N + W_{pseudo} = \Delta K$$

$$\Rightarrow -mgR(1-\cos\theta) + O + maR\sin\theta = 0$$

$$\Rightarrow -g(2\sin^2\frac{\theta}{2}) + 2a\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 0$$

$$\Rightarrow \ Tan\frac{\theta}{2} = \frac{a}{g} \qquad\qquad \Rightarrow \ \theta = 2Tan^{-1}\left(\frac{a}{g}\right)$$

27 The ratio of lengths of smooth and rough part of a fixed inclined plane is 1:2. A small body starts moving from top and again comes to rest at the bottom point. Then the co-efficient of friction between body and rough surface is



A. $\frac{3}{2}\tan\theta$ **B.** $\frac{2}{3}\tan\theta$ **C.** $3\tan\theta$ **D.** $\frac{1}{3}\tan\theta$

Answer :A
Solution : From work energy theorem

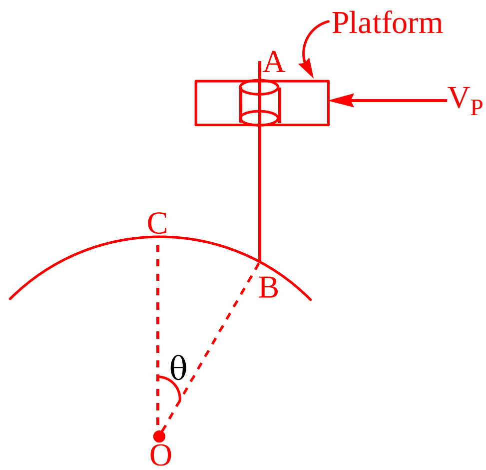
$$\omega_g + \omega_N + \omega_f = \Delta K$$

$$\Rightarrow \ mg\,\ell\sin\theta + 0 - \mu\, mg\cos\theta\frac{2\ell}{3} = 0$$

$$\Rightarrow \ \sin\theta = \mu(\frac{2}{3}\cos\theta)$$

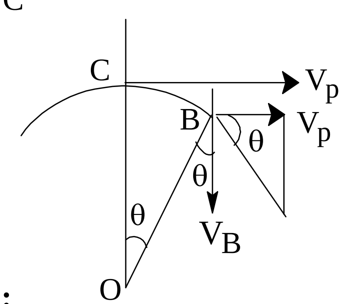
$$\Rightarrow \ \mu = \frac{3}{2}Tan\theta$$

28 A platform P is moving with a velocity υ_p over hemispherical shell. A vertical rod AB passing through a hole in the platform is moving on the shell and remains vertical. There is sufficient friction between rod and shell to stop the slip. C is the crown of the shell and O is its centre $BOC = \theta$ at any instant. Find the velocity of point B in dowanward motion at that instant.



A. $\upsilon_p \sin\theta$ **B.** $\upsilon_p \cos\theta$ **C.** $\upsilon_p \tan\theta$ **D.** $\upsilon_p \cot\theta$

Answer :C

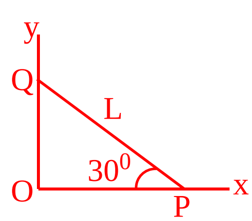


Solution :

$$\tan\theta = \frac{V_B}{V_P}$$

$\Rightarrow V_B = V_P \tan\theta$ hence option (C) is correct.

29 Two bodies P and Q connected through light rigid rod of length L are placed as shown, in the figure. Body P moves toward O with a constant speed V. The velocity of Q is



A. $v\sqrt{3}$ **B.** $\frac{v}{\sqrt{3}}$ **C.** $2\sqrt{v}$ **D.** $\frac{\sqrt{3}}{v}$

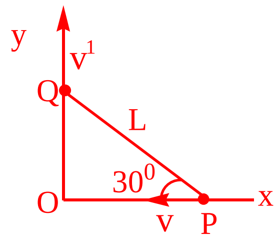
Answer :A

Solution : Let v^1 is the velocity of Q.

$v^1 = v \cot 30^0$ $\cot 30^0 = v$

$v^1 = v\sqrt{3}$.

$x^2 + y^2 = L^2$



$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

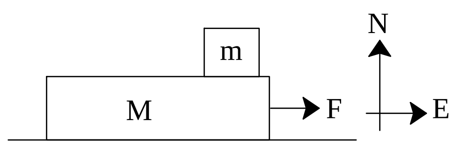
$$x\frac{dx}{dt} = y\frac{dy}{dt}$$

$$xv = yv^1$$

$$\frac{x}{y}v = v^1$$

$$\Rightarrow v^1 = v\sqrt{3}$$

30 A block of mass m is on the smooth horizontal surface of a plank of mass M. The plank is on smooth horizontal surface. Now, a constant horizontal force F acts on M. Now, for a person standing on the ground :



A. The acceleration of m is $\frac{F}{M}$ towards west

B. The acceleration of m is zero

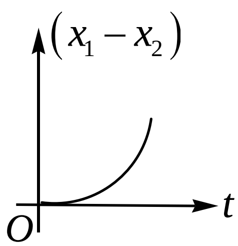
C. The acceleration of m is F/m towards east

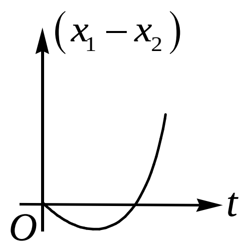
D. The acceleration of m is $\dfrac{F}{M+m}$ towards east

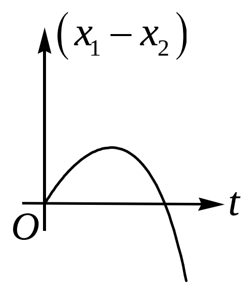
Answer :B
Solution : No horizontal force (external) acts only on the body of mass M

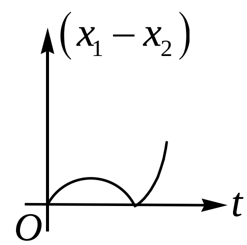
Hence acceleration of m is zero, hence option (B)

31 A body is at rest at x = 0. At t = 0, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through x = 0 moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t ?

A. 

B. 

C. 

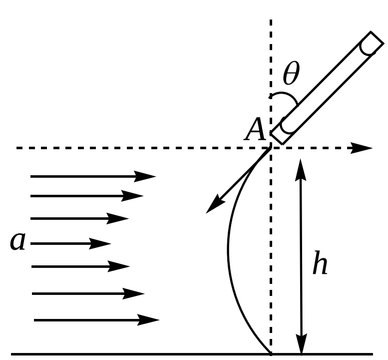
D. 

Answer :B

Solution : $x_1 - x_2 = ut - \frac{1}{2}at^2$, hence option (B) is correct

32 A particle is ejected from the tube at A with a velocity ‘v’ at an angle θ with the vertical y-axis. A strong horizontal wind gives the particle a constant horizontal acceleration a in the x-direction. If the particle strikes the ground at a point directly under its released position and the downward y-acceleration is take

as g then,



A. $h = \dfrac{2v^2 \sin\theta \cos\theta}{a}$

B. $h = \dfrac{2v^2 \sin\theta \cos\theta}{g}$

C. $h=\frac{2v^2}{g}\sin\theta\left(\cos\theta+\frac{a}{g}\sin\theta\right)$ **D.** $h=\frac{2v^2}{a}\sin\theta\left(\cos\theta+\frac{g}{a}\sin\theta\right)$

Answer :D

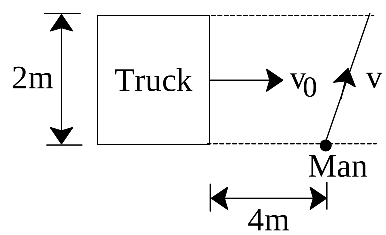
Solution : Since $0=(v\sin\theta)t+\frac{1}{2}(-a)t^2 \Rightarrow t=\frac{2v\sin\theta}{a}$

Also, $h=(v\cos\theta)t+\frac{1}{2}gt^2$

$$\Rightarrow h=\frac{2v^2}{a}\sin\theta\left(\cos\theta+\frac{g}{a}\sin\theta\right)$$

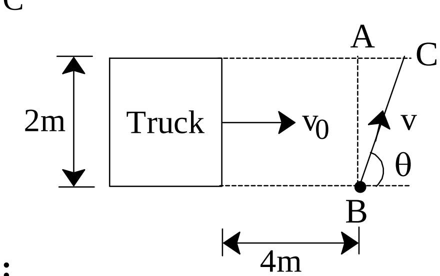
33 A 2m wide truck is moving with a uniform speed $v_0=8\,m/s$ along a

straight horizontal road. A pedestrian starts to cross the road with a uniform speed v when the truck is 4m away from him. The minimum value of v so that he can cross the road safely is



A. 2.62 m/s **B.** 4.6 m/s **C.** 3.57 m/s **D.** 1.414 m/s

Answer :C



Solution :

Let the man starts crossing the road at an angle θ as shown in figure. For safe crossing the condition is that the man must cross the road by the time the truck describes the distance 4 + AC or $4+2\cot\theta$.

$$(vocs\theta)t=AC$$

$$8t=4+AC$$

$$8t=4+(v\cos\theta)t$$

$$(8-v\cos\theta)t=4$$

$$t = \frac{4}{8 - v\cos\theta}$$

$$(v\sin\theta)t = 2 \Rightarrow t = \frac{2}{v\sin\theta}$$

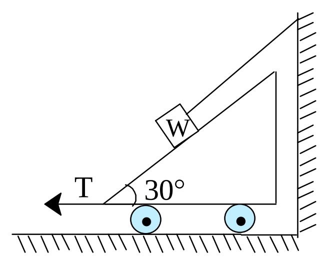
$$\frac{2}{v\sin\theta} = \frac{4}{8 - \cos\theta} \Rightarrow 8 - v\cos\theta = 2v\sin\theta$$

$$v[2\sin\theta + \cos\theta] = 8$$

$$(2\sin\theta + \cos\theta)_{\max} = \sqrt{5}$$

$$v_{\min} = \frac{8}{\sqrt{5}}$$

34 Find the tension T needed to hold the cart equilibrium, if there is no friction.



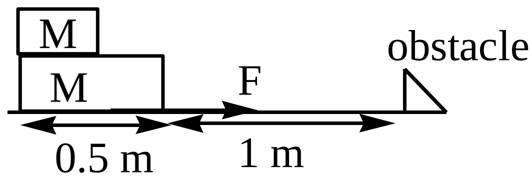
A. $\frac{\sqrt{3}}{4}W$ **B.** $\frac{\sqrt{2}}{2}W$ **C.** $\frac{2}{\sqrt{3}}W$ **D.** $\frac{4}{\sqrt{3}}W$

Answer :A
Solution : $W\cos 30° = N$,

And $T = N\sin 30°$

$$\therefore \quad T = W\cos 30° \sin 30° = \frac{\sqrt{3}}{4}W$$

35 In the diagram shown. The blocks are the same mass M. A force F is applied on the lower block and both the blocks start moving together without any relative motion. Suddenly, the lower block hits a fixed obstacle and comes to rest. The upper block continues to slide on the lower block. The upper block just manages to reach the opposite end of the lower block. The ground is smooth. What is the coefficient of friction between the two blocks ?



A. $\frac{F}{Mg}$ **B.** $\frac{2F}{Mg}$ **C.** $\frac{F}{2Mg}$ **D.** None

Answer :A
Solution : Before striking the obstacle the speed of the block

$$v^2 = 0 + 2a \times 1 = 2\left(\frac{F}{2m}\right) \times 1 = F / m$$

Now by work-energy theorems, we have

$$-f \times 0.5 = \left(0 - \frac{1}{2} mv^2\right)$$

$$-\mu mg \times 0.5 = -\frac{1}{2} m \times \frac{F}{m}$$

$$\therefore \ \mu = \frac{F}{mg}$$

36 A train of mass M is moving on a circular track of radius R with constant speed V. The length of the train is half of the perimeter of the track. The magnitude of linear momentum of the train will be

A. 0 **B.** $\frac{2MV}{\pi}$ **C.** MVR **D.** MV

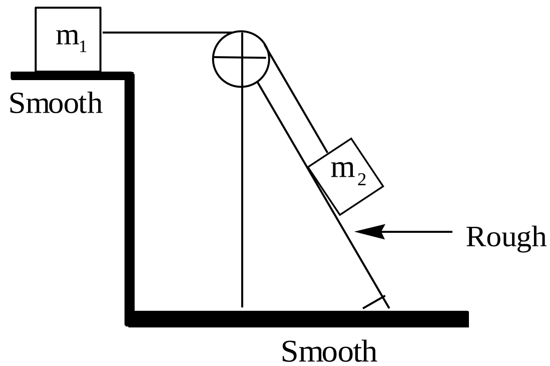
Answer: B

Solution: If we treat as a ring of mass M, then its centre of mass will be at a distance of $\frac{2R}{\pi}$ from the centre of the circle.

Velocity of centre of mass $V_{CM} = (R_{CM})\omega$

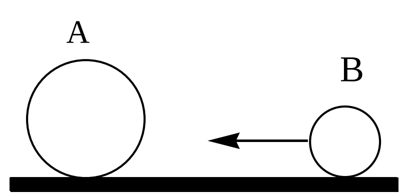
$$\left(\frac{2R}{\pi}\right)\frac{v}{R}=\frac{2V}{\pi}\quad P_{CM}=MV_{CM}=\frac{2MV}{\pi}$$

37 Two blocks of masses m_1 and m_2 are connected by an inextensible light string. The string is passing over a pulley attached with movable wedge. All the fixed surfaces are smooth and the inclined surface of the wedge is rough. The system is released from rest. Consider the two blocks and the wedge as the system and m_2 does not slide on the wedge.



A. The centre of mass of system moves towards right.
B. The centre of mass of the system moves towards left.
C. The centre of mass of the system moves downwards.
D. The centre of mass of the system does not move at all.
Answer: D
Solution: The centre of mass of the system does not move in horizontal direction as net horizontal force is zero and as the mass m_2 does not slide. There will be no vertical displacement.

38 A sphere B of mass m is moving towards a bigger fixed sphere A with velocity v on a smooth horizontal surface, as shown. Sphere B moves and returns back after making an elastic collision and being in contact with sphere A exerts a contact force of magnitude $\left(\frac{mv\sqrt{48}}{3\Delta t}\right)$. Find the angle between contact force and the horizontal at the point of contact. . $\Delta t =$ Time of contact between two spheres.



A. 45^0 **B.** 30^0 **C.** 60^0 **D.** Zero
Answer :B
Solution : Since, collision is elastic the kinetic energy of ball B before collision is equal to kinetic energy of ball A after collision.

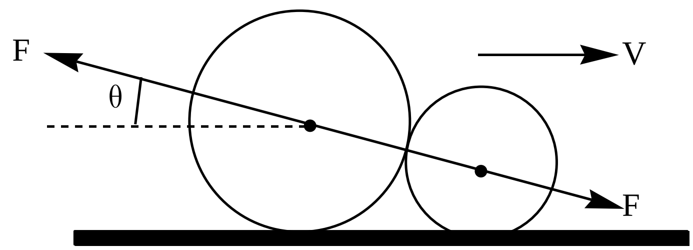
Hence, speed of ball B before and after collision would be same.

From impulse momentum theorem – Linear impulse = change in momentum.

$(F\cos\theta)\Delta t = mv - (-mv) = 2mv$

$F = \frac{2mv}{\Delta t\cos\theta} = \frac{mv\sqrt{48}}{3\Delta t} = \frac{4mv}{\Delta t\sqrt{3}}$

$\cos\theta = \frac{\sqrt{3}}{2}$ or $\theta = 30°$

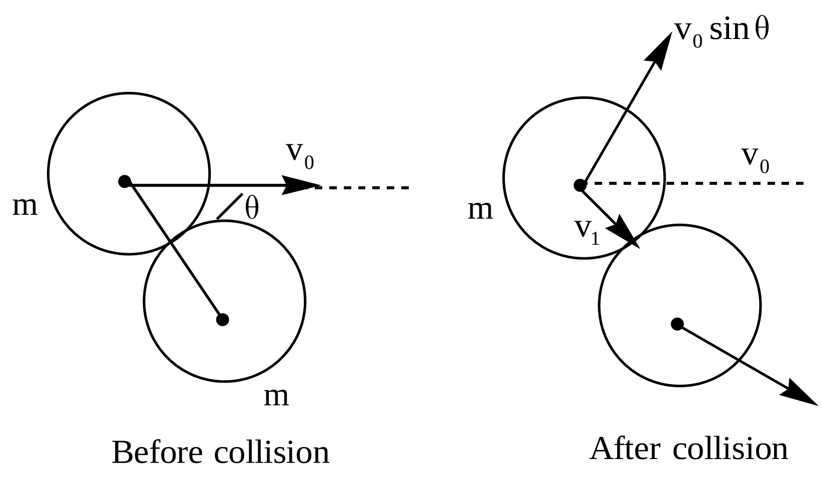


39 A ball collides elastically with another ball of same mass. The collision is oblique and initially one of the ball was at rest. After the collision, the two balls move with same speeds. What will be the angle between the velocities of the balls after the collision?

A. 35^0 **B.** 45^0 **C.** 60^0 **D.** 90^0

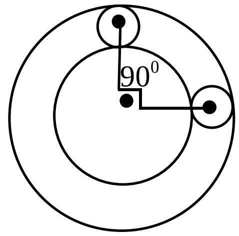
Answer: D

Solution: The initially stationary ball will move along the line of impact after collision. In elastic collision velocities gets interchanged along the line of impact.



If $v_1 = 0$, then velocity of 1st ball is $v_0 \sin\theta$ perpendicular to L_0I and of 2nd ball is $v_0\cos\theta$ along L_0I. So, the required angle is 90^0.

40 A small body A of mass m and B of mass 3m and same size as A move towards each other with speeds V and 2V respectively from the positions as shown, along a smooth horizontal circular track of radius r. After the first elastic collision, they will collide again after the time :



A. $\frac{2\pi r}{V}$ **B.** $\frac{\pi r}{2V}$ **C.** $\frac{\pi r}{V}$ **D.** $\frac{2\pi r}{3V}$

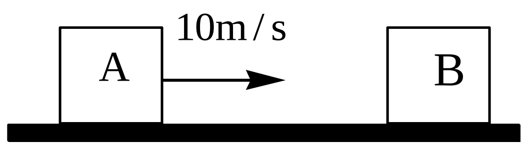
Answer :D

Solution : $e = 1 = \frac{Velocity\ of\ separation}{2V - (-V)}$

Velocity of separation = 3V

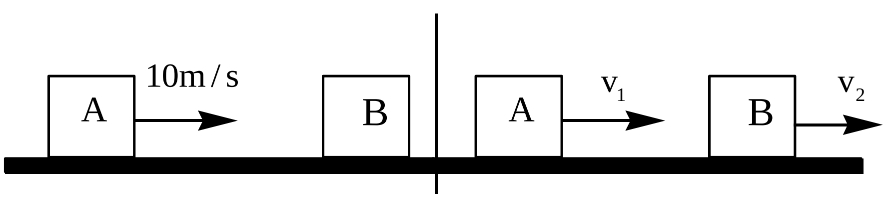
Required time = $\frac{2\pi r}{3V}$

41 As shown in the figure, a block A moving with speed 10 m/s on a horizontal surface collides with another block B at rest initially. The coefficient of restitution is ½. Neglect friction everywhere. The distance between the blocks at 5 s after the collision takes place is



A. 20 m
B. 10 m
C. 25 m
D. cannot be determined because masses are not given

Answer :C

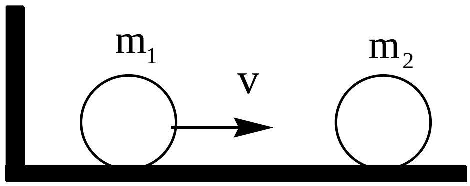


Solution :

$v_{rel} = e(u_{rel}) = \frac{1}{2}(10) = 5$

$S_{rel} = v_{rel} \times \Delta t = 5 \times 5 = 25\,m$

42 Mass m_1 collides elastically with mass m_2 at rest. Out of the four ratios (of mass m_1 and m_2) given in four options, which ratio ensures the second collision between m_1 and m_2. Collision between m_1 and wall is perfectly elastic.



A. $\frac{1}{2}$ **B.** 1 **C.** $\frac{1}{4}$ **D.** 2

Answer :C

Solution : $v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2}.v_1$ should be $-ve \Rightarrow m_1 < m_2$

$$|\overline{v}_1| = \frac{(m_2 - m_1)u_1}{m_1 + m_2}$$

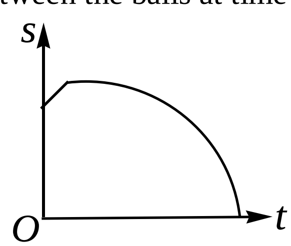
$v_2 = \frac{2m_1u_1}{m_1 + m_2}$ Next collision takes place if $v_1 > v_2 \Rightarrow \frac{(m_2 - m_1)u_1}{m_2 + m_1} > \frac{2m_1u_1}{m_1 + m_2}$

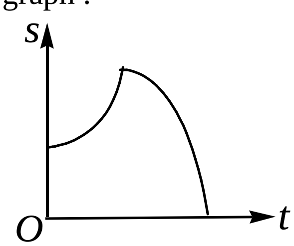
$$m_2 - m_1 > 2m_1$$

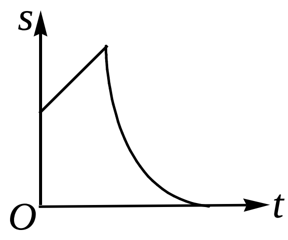
$$m_2 > 3m_1 \Rightarrow \frac{m_1}{m_2} < \frac{1}{3}$$

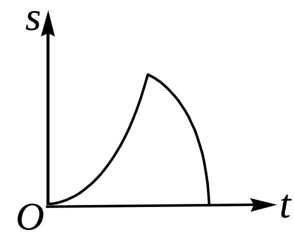
Among the given values, $\frac{1}{4}$ is only valid ratio

43 Two balls are dropped from the top of a tall tower of height 'h' with a time gap interval of t_0 second, where $t_0 < \sqrt{\frac{2h}{g}}$. The floor below is inelastic. Let the second ball be dropped at time t = 0. The distance 's' between the balls at time t is best represented by the graph :

A.


B.


C. 

D. 

Answer :A

Solution : Separation till first ball reaches ground $s=\frac{1}{2}g\,t_0^2+g\,t_0 t$ (straight line)

Afterwards: $s=h-\frac{1}{2}gt^2$ (parabola)

44 Two blocks of masses m_1 and m_2 connected by a non-deformed light spring rest on a rough horizontal surface of coefficient of friction μ . The minimum constant horizontal force to be applied to the block of mass m_1 in order to shift the other block is

A. $\left(\frac{m_1}{2}+m_2\right)\mu g$ **B.** $\frac{1}{2}\left(m_1+m_2\right)\mu g$ **C.** $\left(m_1+m_2\right)\mu g$ **D.** $\left(m_1+\frac{m_2}{2}\right)\mu g$

Answer :D

Solution : work done by $F=Fx=\frac{1}{2}kx^2+\mu m_1 gx$

Where $x=\frac{\mu m_2 g}{k}$

$\Rightarrow F=\frac{1}{2}\mu m_2 g+\mu m_1 g$

45 A particle moves along a circle of radius R such that its kinetic energy (E) on moving through a distance s is given by $E=\lambda s^2$, where λ is a constant. The force acting on the particle is

A. $\frac{2\lambda s^2}{R}$ **B.** $\frac{2\lambda s}{R^2}\sqrt{s^4+R^4}$ **C.** $2\lambda s(\frac{s}{R}+1)$ **D.** $\frac{2\lambda s}{R}\sqrt{s^2+R^2}$

Answer :D

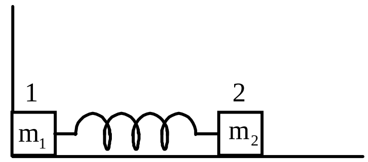
Solution : $\frac{1}{2}mv^2=\lambda s^2\Rightarrow\frac{1}{2}m.2v.\frac{dv}{ds}=2\lambda s$

$\Rightarrow F_t=2\lambda s$ (Tangential force)

$F_c=\frac{mv^2}{R}=\frac{2\lambda s^2}{R}$ (Radial force)

$\Rightarrow F=\sqrt{F_t^2+F_c^2}=\frac{2\lambda s}{R}\left(R^2+s^2\right)^{1/2}$

46 Two bars of masses m_1 and m_2 connected by a weight less spring of stiffness k, rest on a smooth horizontal plane. Bar 2 is shifted by a small distance x_0 to the left and released. The speed of the centre of mass of the system when bar 1 breaks off the wall is



A. $x_0\sqrt{\dfrac{k\,m_2}{m_1+m_2}}$ **B.** $\dfrac{x_0}{m_1+m_2}\sqrt{k\,m_2}$ **C.** $x_0 k\dfrac{m_1+m_2}{m_2}$ **D.** $x_0\dfrac{\sqrt{k\,m_1}}{(m_1+m_2)}$

Answer :B

Solution : At the instant block-I, breaks of the wall its velocity is zero

According to conservation energy

$$\frac{1}{2}K\,x_0^2 = \frac{1}{2}m_2 v_2{}^2 + 0$$

$$v_2 = \sqrt{\frac{K}{m_2}}x_0$$

$$V_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_2 + v_2}$$

$$= \frac{0 + m_2\sqrt{\frac{k}{m_2}}x_0}{m_1 + m_2}$$

$$\frac{x_0}{(m_1+m_2)}\sqrt{Km_2}$$

47 A large circular table with smooth horizontal surface is rotating at a constant angular speed ω about its axis. A groove is made on the surface along a radius and a particle is gently placed inside the groove at a distance *l* from the centre. Find the speed of the particle with respect to the table as its distance from the centre becomes L.

A. $v=\omega\left(L^2-l^2\right)^{\frac{1}{2}}$ **B.** $v=\omega^2\left(l^2-L^2\right)^{\frac{1}{2}}$ **C.** $v=\omega\left(l^2-L^2\right)^{\frac{1}{2}}$ **D.** $v=\omega\left(L^2-l^2\right)^{\frac{3}{2}}$

Answer :A

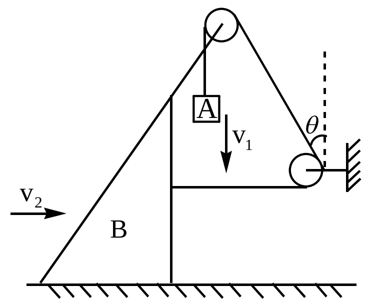
Solution : With respect to observer on platform

$$a = \omega^2 x$$

$$v\frac{dv}{dx} = \omega^2 x$$

Simplifying $v=\omega\left(L^2-l^2\right)^{\frac{1}{2}}$

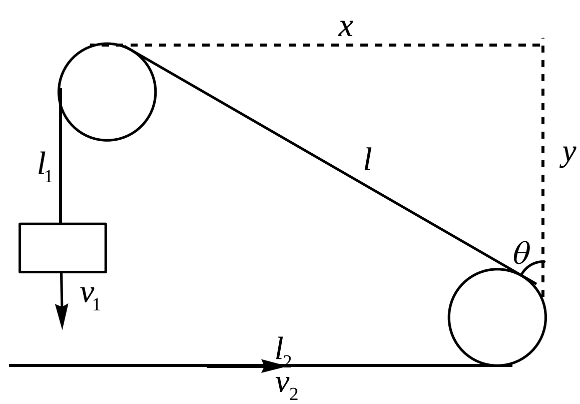
48 In the figure, the block A moves downwards with velocity v_1 and the wedge B moves rightwards with velocity v_2. Correct relation between v_1 and v_2 is



A. $v_2=v_1$ **B.** $v_2=v_1\sin\theta$ **C.** $2v_2\sin\theta=v_1$ **D.** $v_2\left(1+\sin\theta\right)=v_1$

Answer :D

Solution :



Total length of string $L=l_1+l+l_2$

$$\frac{dL}{dt}=\frac{dl_1}{dt}+\frac{dl}{dt}+\frac{dl_2}{dt}$$

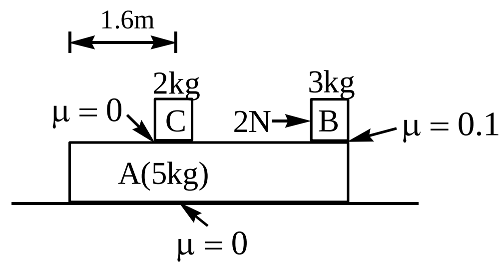
But $l=\sqrt{x^2+y^2}$ and $\frac{-dx}{dt}=\frac{-dl_2}{dt}=v_2$

$$\frac{dl_1}{dt}=v_1$$

$$\Rightarrow O=v_1+\sin\theta(-v_2)-v_2=0$$

$$=v_1=v_2[1+\sin\theta]$$

49 The coefficients of friction and masses of various blocks are shown. A constant horizontal force 2N is applied on block B. What will be the acceleration of block B after 2 seconds? (g = 10m/s^2)



A. $0.20 \text{ m}/s^2$ **B.** $0.25 \text{ m}/s^2$ **C.** $0.15 \text{ m}/s^2$ **D.** $0.67 \text{ m}/s^2$

Answer :B

Solution : Block C does not move with A and B as there is no friction between C and A

Minimum horizontal force, required on B to stip on A is

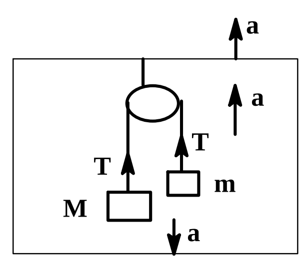
$F=(\mu g)\frac{m}{M}(m+M)=0.1\times10\times\frac{3}{5}(3+5)=\frac{24}{5}=4.8N$. As the applied force is 2N, B does not slip on A. Hence A and B move with common acceleration $a=\frac{2}{3+5}=\frac{2}{8}=0.25\,ms^{-2}$

50 Two blocks of masses M and m are connected with either end of a massless string which passes over a light smooth pulley (M > m). The pulley is connected with the roof of an elevator moving with acceleration a upwards. The magnitude of acceleration of block M relative to block m is 2a. Ratio $\frac{M}{m}$ is equal to

A. $1+\frac{a}{g}$ **B.** $1-\frac{a}{g}$ **C.** $1+\frac{2a}{g}$ **D.** $1-\frac{2a}{g}$

Answer :C

Solution : Relative acceleration of M relative to m = 2a =a –(-a)

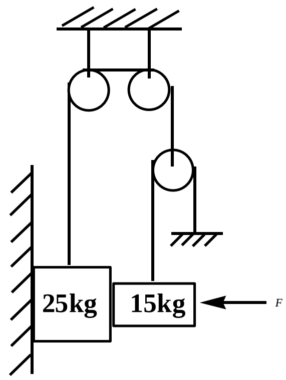


Since, M > m, M is moving downward with acceleration a M(g + a)-T = Ma or T = Mg

T – ma – mg = ma or T = m(2a + g)

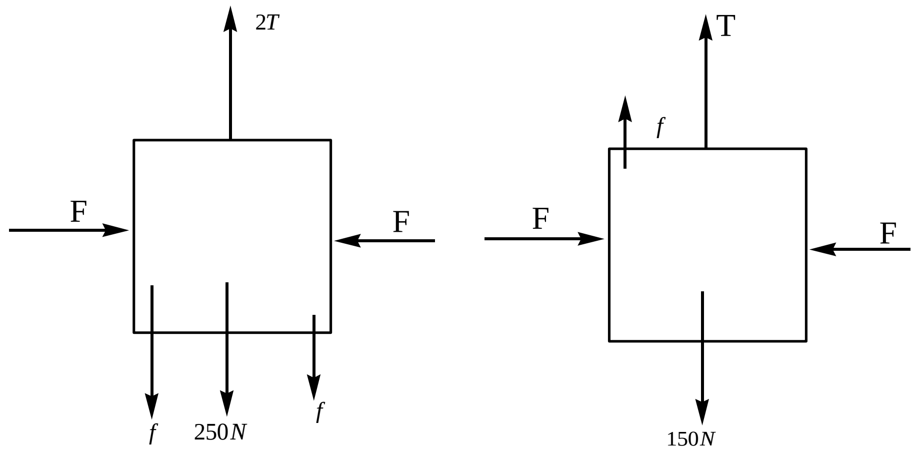
Comparing Mg = m(2a + g) = $\frac{M}{m}=\frac{2a+g}{g}=1+\frac{2a}{g}$

51 Find the minimum value of F to hold the system at equilibrium. Coefficient of friction between all the contact surfaces is 0.4. $(g=10\,ms^{-2})$



A. 125 N **B.** 62.5 N **C.** 31.25 N **D.** 250 N

Answer :C



Solution :

If all the surfaces are assumed as frictionless, it can be proved that 15 kg moves down and 25 kg moves up. Directions of frictions are taken opposite to the relative velocities of blocks as shown.

150 = T + f = T + 0.4F

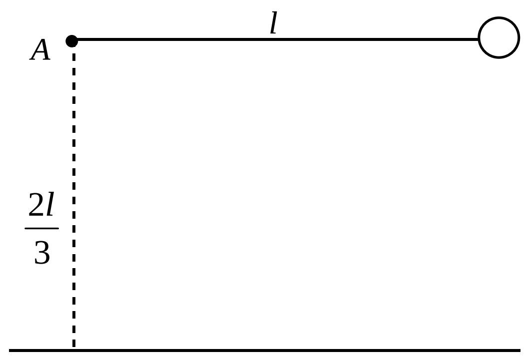
T = 150 – 0.4 F ______ (1)

250 + 2f = 2T $\Rightarrow$ 250 + 0.8F = 2T = 2[150 – 0.4F] = 300 – 0.8F

1.6F = 50

$$F = \frac{500}{16} = \frac{125}{4} = 31.25N$$

52 A is a fixed point at a height $\frac{2l}{3}$ above a perfectly inelastic smooth horizontal plane. A light inextensible string of length l has one end attached to A and other to a heavy particle. The particle is held at the level of A with string just taut and released from rest. The speed of ball just after striking the plane is

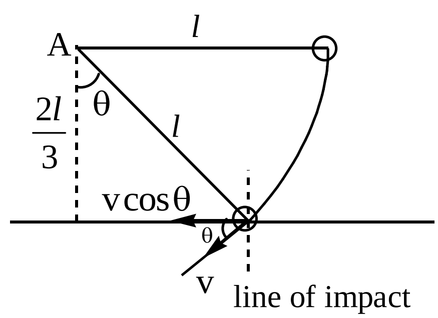


A. $\sqrt{\frac{4gl}{3}}$ **B.** $\sqrt{\frac{2gl}{3}}$ **C.** $\frac{2}{3}\sqrt{\frac{gl}{3}}$ **D.** $\frac{4}{3}\sqrt{\frac{gl}{3}}$

Answer :D

Solution : $v=\sqrt{2gH}$

$$=\sqrt{2g\times\frac{2l}{3}}$$



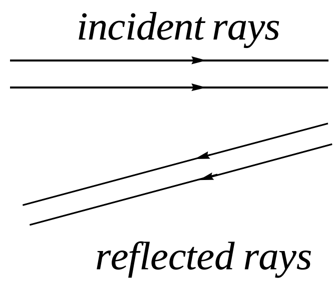
As the collision is perfectly inelastic, the component of velocity perpendicular to the surface becomes zero.

$$v'=v\cos\theta$$

$$=\sqrt{\frac{4gl}{3}}(\frac{2}{3})$$

$$\frac{4}{3}\sqrt{\frac{gl}{3}}$$

53 Fig shows two parallel rays incident on a mirror. They are reflected as parallel rays as shown in the same fig. What is the nature of the mirror?



A. Plane **B.** Concave **C.** Convex **D.** Parabolic

Answer :A
Solution : Parallel rays can be reflected as parallel rays only from a plane mirror.

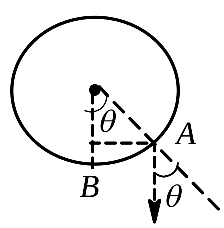
54 A small bob suspended by a thread swings in a vertical plane so that angle made by thread in extreme positions with the vertical is $\theta = \sin^{-1}(4/5)$. The ratio of acceleration of bob in extreme position to its acceleration in lowest position is :

A. 1/2 **B.** 2 **C.** 1 **D.** 1/3

Answer :C

Solution : $\frac{1}{2} m v_B^2 = mgr(1-\cos\theta)$

$\frac{v_B^2}{r} 2g\left(1-\sqrt{1-\sin^2\theta}\right)$



$= 2g\left[1-\sqrt{1-\frac{16}{25}}\right] = \frac{4g}{5}$

$= g.\ \frac{4}{5} = g\sin\theta$ ….(1)

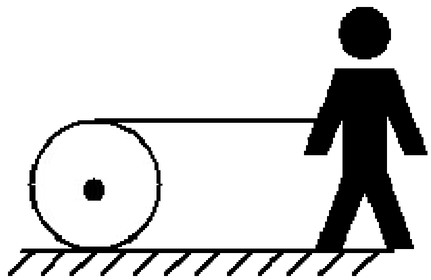
At A. $a_r = 0, a_T = g\sin\theta$ So, $a_A = g\sin\theta$ So, $\frac{a_A}{a_B} = 1.0$

55 The moment of inertia of a thin uniform semicircular wire of mass 'M' and radius 'r' about a line perpendicular to the plane of the wire through the centre is:

A. Mr^2 **B.** $\frac{1}{2} Mr^2$ **C.** $\frac{1}{4} Mr^2$ **D.** $\frac{2}{5} Mr^2$

Answer :A
Solution : $I = MR^2$

56 A string of negligible thickness is wrapped several times around a cylinder kept on a rough horizontal surface. A man standing at a distance ‘l’ from the cylinder holds one end of the string and pulls the cylinder towards him as shown in fig. There is no slipping anywhere. The length of the string passed through the hand of the man while the cylinder reaches his hands is :



A. l **B.** $2\,l$ **C.** $3\,l$ **D.** $4\,l$

Answer :B

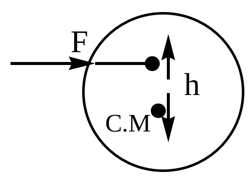
Solution : Since velocity of top end is twice the center so it will travel twice the distance traveled by the centre

57 A thin spherical shell of radius ‘R’ lying on a rough horizontal surface is hit sharply and horizontally by a cue. At what height from the ground should it be hit so that the shell does not slip on the surface

A. $\frac{2}{3}R$ **B.** $\frac{5}{4}R$ **C.** $\frac{5}{3}R$ **D.** $\frac{3}{2}R$

Answer :C

Solution : $\int F dt\, h = I\omega$

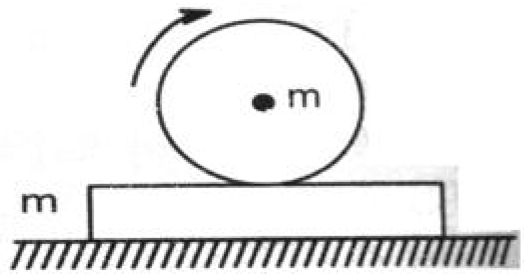


$\int F dt = mv_{cm}$

From above relations: $h = \frac{2}{3}R$

Height from ground $= \frac{5}{3}R$

58 A sphere of mass ‘m’ is given some angular velocity about a horizontal axis through the center, and gently placed on a plank of mass ‘m’. The coefficient of friction between the two is μ . The plank rests on a smooth horizontal surface. The initial acceleration of the sphere relative to the plank will be:



A. Zero **B.** μg **C.** $\frac{7}{5}\mu g$ **D.** $2\mu g$

Answer :D
Solution : Acceleration of sphere = μg (right wards)

Acceleration of plank = μg (left ward)

So relative acceleration = $2\mu g$

59 A circular platform is mounted on a vertical frictionless axle. Its radius is r = 2m and its moment of inertia is I = 200 kg-m^2. It is initially at rest. A 70 kg man stands on the edge of the platform and begins to walk along the edge at speed $v_0 = 1.0\, m/s$ relative to the ground. The angular velocity of the platform is:
A. 1.2 rad/s **B.** 0.4 rad/s **C.** 2.0 rad/s **D.** 0.7 rad/s
Answer :D
Solution : Net external torque is zero. Therefore angular momentum of system will remain conserved i.e.,

$L_i = \mathrm{L}_f$

Initial angular momentum $L_i = 0$

$\therefore$ Final angular momentum should also be zero.

Or angular momentum of man = angular momentum of platform in opposite direction

or $mv_0 r = I\omega$

$$\therefore\ \omega = \frac{mv_0 r}{I} = \frac{(70)(1.0)(2)}{200}\ \therefore \omega = 0.7\ rad/s$$

60 A wheel of radius 'R' rolls on the ground without slipping with a uniform velocity 'v' The relative acceleration of topmost point of the wheel with respect to the bottommost point is:

A. $\frac{v^2}{R}$ **B.** $\frac{2v^2}{R}$ **C.** $\frac{v^2}{2R}$ **D.** $\frac{4v^2}{R}$

Answer :B
Solution : As $a_{CM} = 0\left[v_{CM} = const\right];$ tangential acceleration of each point is zero

$V_{rel} = 2V$

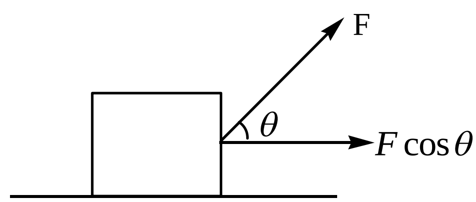
$a = \frac{(2V)^2}{2R}$

61 A particle moves under the action of a force of magnitude F such that the angle θ made by the force with the instantaneous velocity varies with the distance 's' as θ = ks where, k is a positive constant. Find the work done by the force in covering a distance 's'.

A. $\frac{-F}{ks}\sin(ks)$ **B.** $\frac{F}{k}\sin(ks)$ **C.** $-F\sin(ks)$ **D.** $-k\sin(ks)$

Answer :B

Solution : $\theta = ks$



$d\theta = kds$

$dw = F\cos\theta ds$

$W = \int dw = F\int \frac{\cos\theta\, d\theta}{k} = \frac{F}{k}\sin\theta = \frac{F}{k}\sin(kx)$

62 A particle initially at the origin moves in xy plane with a velocity V = ai + bxj. Where a and b are constants. The radius of curvature of the trajectory is _____

A. $\rho = \frac{a}{b}\left[1+\left(\frac{bx}{a}\right)^2\right]^{\frac{3}{2}}$ **B.** $\rho = \frac{a}{b}\left[1+\left(\frac{bx}{a}\right)^2\right]$

C. $\rho = \frac{a}{b}\left[1+\left(\frac{bx}{a}\right)^2\right]^{\frac{1}{2}}$ **D.** $\rho = \frac{a}{b}\left[1-\left(\frac{bx}{a}\right)^2\right]^{\frac{1}{2}}$

Answer :A

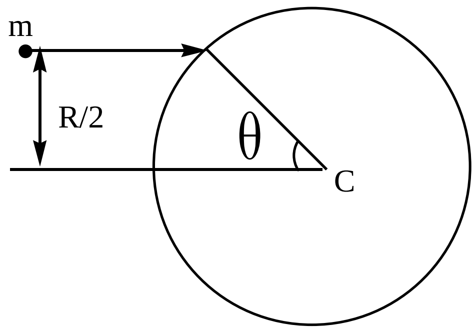
Solution : $\frac{dy}{dx} = \frac{b}{a}.x$

$\frac{d^2y}{dx^2} = \frac{b}{a}$

$$\frac{1}{\rho}=\frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}=\frac{\frac{b}{a}}{\left[1+\left(\frac{bx}{a}\right)^2\right]^{\frac{3}{2}}}$$

or $\rho=\frac{a}{b}\left[1+\left(\frac{bx}{a}\right)^2\right]^{\frac{3}{2}}$

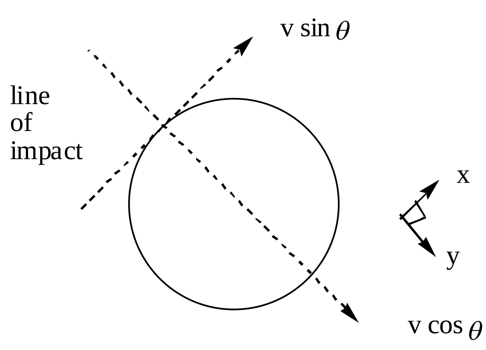
63 A particle of mass m strikes elastically with a disc of radius R, and centre c, with a velocity 'v' as shown in the figure. If the mass of the disc is equal to that of the particle and the surface of contact is smooth, the speed of the disc just after collision is



A. $\frac{2v}{\sqrt{3}}$ **B.** $\frac{\sqrt{3}}{2}v$ **C.** $\frac{v}{2}$ **D.** $2v$

Answer :B

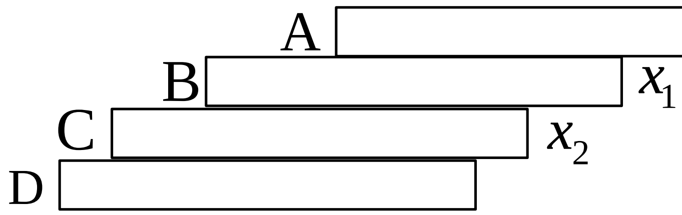
Solution : No external force along the normal line hence we can conserve the linear momentum of the system of the normal.



Then velocity of disc = V1= $(v\cos\theta)\ \hat{j}$

$\sin\theta=\frac{R/2}{R}=\frac{1}{2}$

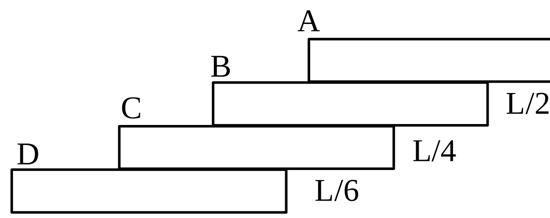
64 Four rectangular blocks A, B, C, D are placed one above the other in such away that each block projects a little beyond the block below it. If length of each block is 'L', the maximum projections of blocks B (x_1) and C (x_2) are......



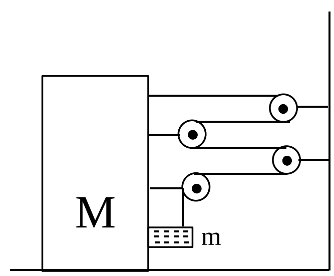
A. $\frac{L}{2}, \frac{L}{4}$ **B.** $\frac{L}{4}, \frac{L}{6}$ **C.** $\frac{L}{6}, \frac{L}{4}$ **D.** $\frac{L}{2}, \frac{L}{6}$

Answer :B

Solution :



65 A block of mass M is connected with a particle of mass m by a light inextensible string as shown in the figure. Assuming all contacting surfaces as smooth, the acceleration of the block after releasing the system is.



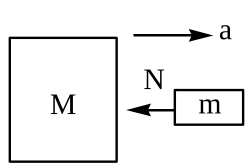
A. $\frac{mg}{M+17m}$ **B.** $\frac{2mg}{M+17m}$ **C.** $\frac{4Mg}{M+17m}$ **D.** $\frac{4mg}{M+17m}$

Answer :D

Solution : x- horizontal distance covered by block ‘M’ with acceleration a

y- vertical distance covered by particle ‘m’ with acceleration a^1

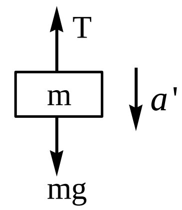
$4x = y \Rightarrow \quad 4a = a'$



N = ma

$4T - N = Ma$

$4T = (m+M)a$ …….. (1)

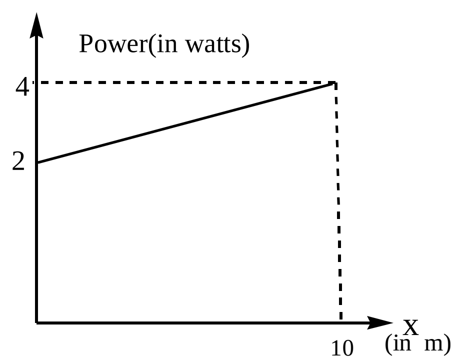


$mg - T = ma'$

$T = m(g-4a)$ ……… (2)

Solving we get $a = \dfrac{4mg}{M+17m}$

66 A particle A of mass $\dfrac{10}{7}kg$ is moving in the positive direction of ‘x’. Its initial position is x = 0 and initial velocity is 1 m/s. The velocity at x = 10 is : (use the graph given)



A. 4 m/s

B. 2 m/s

C. $3\sqrt{2}\, m/s$

D. 100/3 m/s

Answer :A

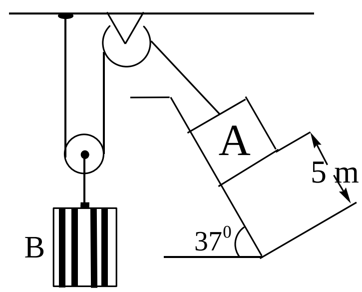
Solution : From graph : area = $\dfrac{1}{2}\times 10\times(4-2)+2\times 10 = 30$

Area under graph = $\int P dx$

$$= \int_1^V F\,V\,dx = \int_1^V m\frac{dV}{dx}\times V^2\,dx = m\left[\frac{V^3}{3}\right]_1^V = \frac{10}{7\times 3}\left[V^3-1\right]$$

$$\therefore 30=\frac{10}{21}\left[V^3-1\right]\Rightarrow V^3=64,\ V=4\ m/s$$

67 The blocks A and B shown in the fig. have masses $M_A=5$ kg and $M_B=4$ kg. The system is released from rest. The speed of B after A has travelled a distance 1 m along the incline is



A. $\frac{\sqrt{3}}{2}\sqrt{g}$ **B.** $\frac{\sqrt{3}}{4}\sqrt{g}$ **C.** $\frac{\sqrt{g}}{2\sqrt{3}}$ **D.** $\frac{\sqrt{g}}{2}$

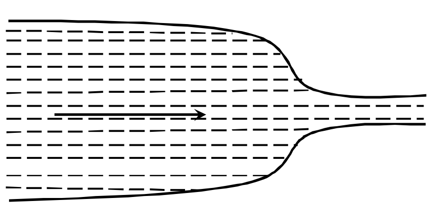
Answer :C

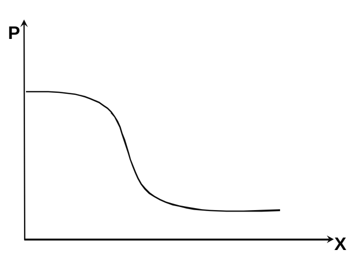
Solution : If A moves down the incline by 1m, B will move up by $\frac{1}{2}m.$ If speed of B is 'V' then speed of A will be 2V.

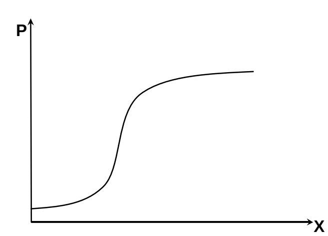
Gain in K.E = loss in P.E, $\frac{1}{2}m_A(2V)^2+\frac{1}{2}m_BV^2=m_Ag\frac{3}{5}-m_Bg\frac{1}{2}$

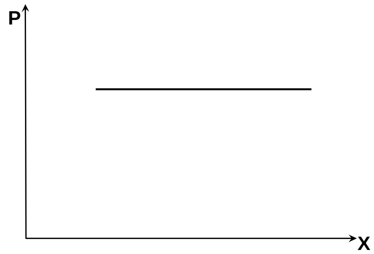
$$V=\frac{1}{2}\sqrt{\frac{g}{3}}$$

68 Water flows through a frictionless horizontal duct with the cross-section varying as shown in figure. Pressure p at points along the axis from left end is best represented by:

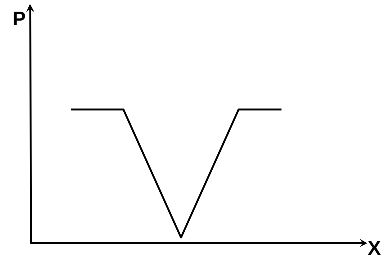


A.


B.




C.

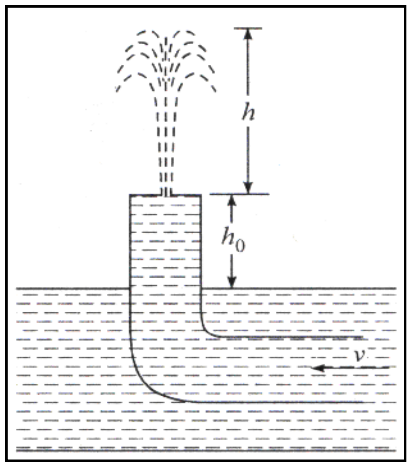


D.

Answer :A

Solution : As cross-sectional area decreases V increases and hence P decreases.

69 A bent tube is lowered into a water stream as shown in fig. The velocity of the stream relative to the tube is equal to v = 2.5 m/s. The closed upper end of the tube located at the height h_0 = 12 cm has a small orifice. To what height 'h' will the water jet rise after coming out of orifice?



A. 0.1 m **B.** 0.44 m **C.** 0.2 m **D.** 0.55 m

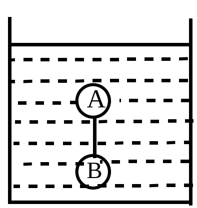
Answer :C

Solution : Let H is the depth of horizontal part of tube from top level of stream. Applying Bernoulli's theorem from entrance of liquid into the tube upto the highest point of water jet,

$$\left(P_0 + H\rho g\right) + \frac{1}{2}\rho v^2 + O = \qquad P_0 + O + \rho g\left(H + h_0 + h\right)$$

$$\Rightarrow v^2 = 2g\left(h_0 + h\right) \quad \Rightarrow h = \frac{v^2}{2g} - h_0 \ = \frac{(2.5)^2}{2\times 9.8} - 0.12 \ = 0.2m$$

70 Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a light string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with some tension in the string. The arrangement is not possible if



A. $d_A < d_F$ **B.** $d_B > d_F$ **C.** $d_A > d_F$ **D.** $d_A + d_B = 2d_F$

Answer :C

Solution : Equilibrium: $B_1 + B_2 = W_1 + W_2$

$d_F\left(V+V\right)g = V_g\left(d_A + d_B\right)$

$2d_F = \left(d_A + d_B\right) \rightarrow (1)$

If $d_B < d_F$, the string becomes slack

$\therefore$ To keep the string tight,

$d_A < d_F$ and $d_B > d_F$

71 The gas inside a soap bubble expands uniformly and slowly so that its radius increases from R to 2R. The atmospheric pressure is P_o and surface tension is s. The work done by the gas in the process is

A. $\dfrac{28\pi R^3 P_o}{3} + 24\pi s R^2$

B. $\dfrac{25\pi R^3 P_o}{3} + 24\pi s R^2$

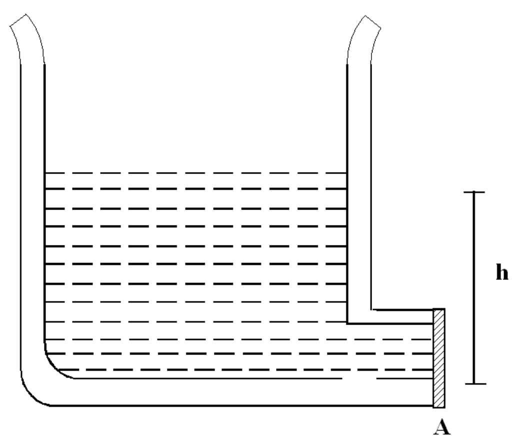
C. $\dfrac{25\pi R^3 P_o}{3} + \dfrac{23\pi s R^2}{2}$

D. $23\pi s R^2$

Answer :A

Solution : $W_{gas} = \int_R^{2R}\left(P_o + \dfrac{4s}{r}\right)4\pi r^2 dr = \dfrac{28\pi R^3 P_o}{3} + 24\pi R^2 s$

72 A small opening near the bottom of the vessel shown in figure has area ‘A’. A disk is held against the opening to keep the liquid from running out. Let F_1 be the net force on the disk applied by liquid and air in this case. Now the disk is moved away from the opening a short distance. The liquid comes out and strikes the disk inelastically. Let F_2 be the force exerted by the liquid in this condition just after the disc is moved. Then $\dfrac{F_1}{F_2}$ is



A. $\dfrac{1}{2}$

B. 1

C. $\dfrac{2}{1}$

D. $\dfrac{1}{4}$

Answer :A

Solution : Let ‘ρ’ be the density of liquid. Then

$F_1 = (\Delta P) A = \rho gh\ A$ …………………(1)

In the second case

$F_2 =$ rate of change of momentum

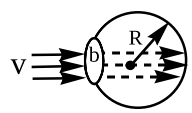
$\rho A v^2 = \rho A\left(\sqrt{2gh}\right)^2$

Or $F_2 = 2\rho gh\ A$ ……………(2)

From equations (1) and (2)

$\frac{F_1}{F_2} = \frac{1}{2}$

73 A soap bubble having surface tension T and radius R is formed on a ring of radius b(b<<R). Air of density ρ is blown into the bubble with velocity v as shown. The air molecules collide perpendicularly with the wall of the bubble and stop. Find the radius R at which the bubble separates from the ring.



A. $\frac{T}{\rho v^2}$ **B.** $\frac{T}{4\rho v^2}$ **C.** $\frac{2T}{\rho v^2}$ **D.** $\frac{4T}{\rho v^2}$

Answer :D

Solution : Excess pressure inside a bubble $= \frac{4T}{R}$

Let area of bubble at wall where air strikes be A.

$\therefore$ Force due to excess pressure $= \frac{4TA}{R}$

Let $\rho =$ density of air, force due to striking air $= \rho A v^2$

For bubble to separate from te ring,

$\rho A v^2 = \frac{4TA}{R}$ or $\rho A v^2 R = 4TA$ or $R = \frac{4T}{\rho v^2}$.

74 A beaker containing water is placed on the platform of a spring balance. The balance reads 15 N. A stone of mass 0.5kg and density 500 kg/m^3 is kept fully immersed in water without touching the walls of beaker and held at rest. What will be the balance reading now? (g = 10 m/ s^2)

A. 20 N **B.** 25 N **C.** 10 N **D.** 15 N

Answer :B

Solution : Reading of weighing machine = weight of the liquid + buoyant force on body

75 A smooth sphere 'A' of mass 'm' is moving with a constant speed 'V' on the smooth horizontal surface collides elastically with an identical sphere at 'B' at rest. After elastic collision speed of sphere 'A' is V/2 then the speed of sphere of 'B' is

A. $\frac{V}{2}$ **B.** $\frac{3V}{2}$ **C.** $\frac{\sqrt{3}V}{2}$ **D.** $\frac{\sqrt{5}V}{2}$

Answer :C

Solution : The given elastic collision is oblique. Hence we apply conservation of K.E

$$\frac{1}{2}mV^2 = \frac{1}{2}m(\frac{V}{2})^2 + \frac{1}{2}mV_B{}^2$$

$$V_B = \frac{\sqrt{3}V}{2}$$

76 Two particles in S.H.M having same time period, same amplitude 'A' and same mean position x = 0 these particles always crossing opposite to each other at $x = \frac{A}{2}$ then the minimum phase difference between the two particles (in rad) is

A. π **B.** $\frac{\pi}{2}$ **C.** $\frac{\pi}{6}$ **D.** $\frac{2\pi}{3}$

Answer :D

Solution : $x = A\sin\theta$

$$\frac{A}{2} = A\sin\theta$$

$$\theta_1 = \frac{\pi}{6}$$

$$\theta_2 = \frac{5\pi}{6}$$

$$\Delta\phi = \frac{2\pi}{3}$$

77 A tunnel is dug in the earth across one of its diameter. Two masses 'm' & '2m' are dropped from the ends of the tunnel. The masses collide and stick to each other and perform S.H.M. Then amplitude of S.H.M. will be: [R = radius of the earth]

A. R **B.** R / 2 **C.** R / 3 **D.** 2R / 3

Answer :C

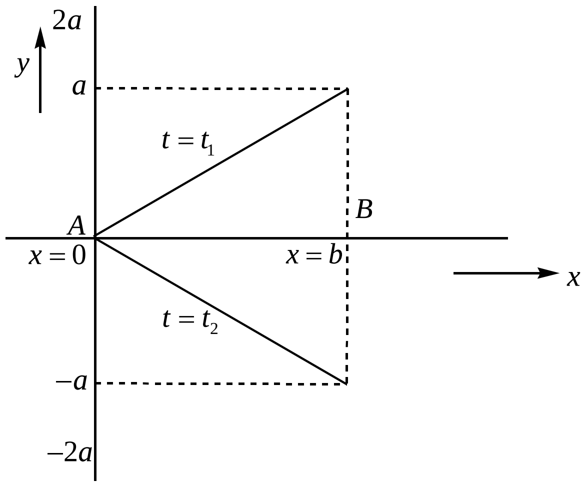
Solution : They collide at the centre of the earth

$$V_c = \frac{2m\sqrt{gR} - m\sqrt{gR}}{2m+m} = A.\omega$$

$$= A_0\sqrt{\frac{g}{R}}$$

$$A = \frac{R}{3}$$

78 A quarter section of a stationary acoustic wave between consecutive node A and anti node B has the graphic profiles as shown in the diagram at two instants of time t = t_1 and t = t_2, where $t_1 \sim t_2 = \frac{T}{2}$ 'T' is the periodic time of the wave. The stationary wave is formed by the superposition of two identical waves. Then the amplitude A and the wavelength λ of each wave are given by :



A. $A = 2a,\ \lambda = 2b$ **B.** $A = \frac{a}{2},\ \lambda = 2b$ **C.** $A = \frac{a}{2},\ \lambda = 4b$ **D.** $A = 2a,\ \lambda = 4b$

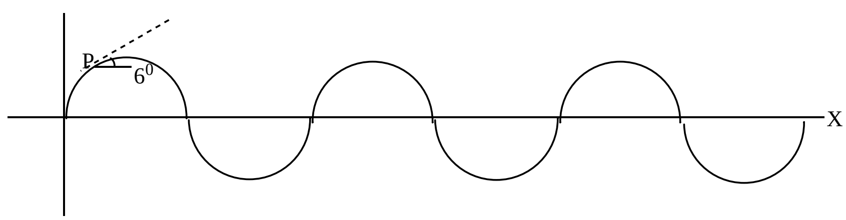
Answer :C

Solution : If $t_1 - t_2 = \frac{T}{2}$, it is quarter section of a full wave $\lambda = 4b$ amplitude maximum at the antinode

$$2A = a \Rightarrow A = \frac{a}{2}$$

Hence 3 is correct option

79 When the plane progressive sinusoidal transverse wave travels on a stretched string along the positive x-axis. At a given instant, the shape of string is as shown in fig. The tangent at the point 'P' on the stretched string makes an angle 6^0. Then the ratio of the kinetic energy of a string element at 'P' to the elastic potential energy stored in the string element at 'P' is

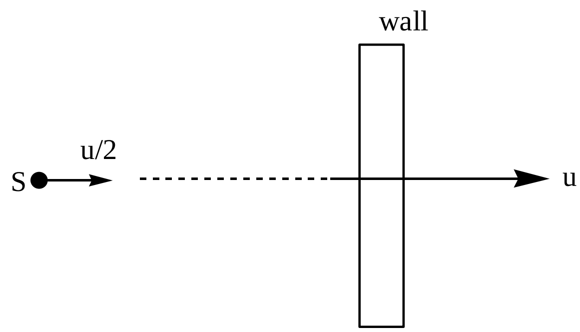


A. $\tan^2 6^0 : 1$ **B.** $\sin^2 6^0 : 1$ **C.** $\cos^2 6^0 : 1$ **D.** 1 : 1

Answer :D

Solution : K.E., P.E are maximum at the mean position and K.E, P.E are zero at highest position

80 A wall is moving with velocity 'u' and a source of sound moves with velocity u/2 in the same direction as shown in the fig. Assuming that the sound travels with velocity 10 u, the ratio of incident sound wavelength on the wall to the reflected sound wavelength by the wall is equal to



A. 19 : 22 **B.** 9 : 11 **C.** 4 : 5 **D.** None

Answer :B

Solution : $\lambda_I = \dfrac{V_{R.V}}{f} = \dfrac{10u - u/2}{f} = \dfrac{19u}{2f}$

$$\lambda_r = \frac{10u+u}{f_r} = \frac{\dfrac{11u}{18f}}{19}$$

$$\frac{\lambda_I}{\lambda_r} = \frac{9}{11}$$

81 A loop of a string of mass per unit length μ and radius 'R' is rotated about an axis passing through centre perpendicular to the plane with an angular velocity ω. A small disturbance is created at a point in the loop. The disturbance travels along string in both sides. The linear speed part of the disturbance which moves in the same sense of rotation as string for a stationary observer is

A. ωR **B.** $2\omega R$ **C.** $3\omega R$ **D.** Zero

Answer :B

Solution : $V_{pulse} = \sqrt{\dfrac{T}{\mu}} = \omega R$

The velocity of disturbance w.r.t ground $\omega R + \omega R = 2\omega R$

82 An acoustic wave given by $y_i = A\sin(\omega t - kx)$ is sent down a string. Upon reflection from one end, the wave becomes $y_r = \dfrac{-A}{2}\sin(\omega t + kx)$. The resultant wave in the string will be

A. A pure standing wave with amplitude A/2

B. A combination of a standing wave with amplitude 2A and a traveling wave of amplitude A

C. A traveling wave with amplitude $\dfrac{3A}{2}$

D. A combination of a standing wave with amplitude A and a traveling wave of amplitude A/2

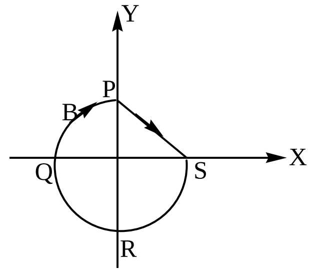
Answer :D

Solution : $y = y_i + y_r$

$$= \frac{A}{2}\sin(\omega t - kx) - A\sin kx \cos \omega t$$

83 A current carrying loop is placed in a uniform magnetic field pointing negative z direction. Branch

PQRS is a three quarter circle, while branch PS is straight. If force on branch PS is F, force on branch PQR is



A. $\sqrt{2}F$ **B.** $\frac{F}{\sqrt{2}}$ **C.** $\frac{\pi F}{\sqrt{2}}$ **D.** $\sqrt{2}\pi F$

Answer :A

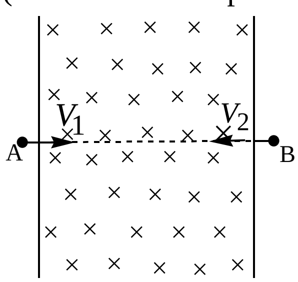
Solution : Force on $PS = F = I(\sqrt{2}R)B$ ____(1)

& Force on $PQR = I(2R)B$ ______(2)

From (1) & (2)

Force on $PQR = \frac{2F}{\sqrt{2}} = \sqrt{2}F$

84 Two particles A and B of same mass and having charges of same magnitude but of opposite nature are thrown into a region of magnetic field (as shown) with speeds v_1 and v_2 $(v_1 > v_2)$. At the time particle A escapes out of the magnetic field, angular momentum of particle B w.r.t particle A is proportional to (assume both the particles escape the region after traversing half circle).



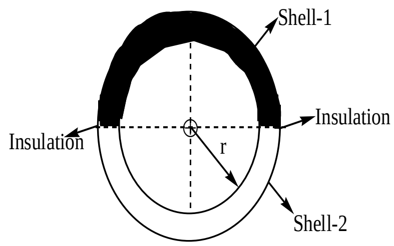
A. $v_1 + v_2$ **B.** $v_1 - v_2$ **C.** $v_1^2 - v_2^2$ **D.** $v_1^2 + v_2^2$

Answer :C

Solution : $R_A = \frac{mV_1}{Bq}$ & $R_B = \frac{mV_2}{Bq}$

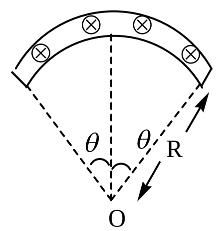
$\left|\vec{L}_{BA}\right| = M\left(R_A - R_B\right)\left(V_1 + V_2\right) \propto \left(V_1 - V_2\right)\left(V_1 + V_2\right)$

85 The figure shows two infinite semi-cylindrical thin shells. Shell-1and shell-2. Shell-1 carries current i_1 in inward direction normal to the plane of paper, while shell-2 carries same current i_1, in opposite direction. A long straight conductor lying along the common axis of the shells is carrying current i_2 in direction same as that of current in shell-1.Force per unit length on the wire is



A. Zero **B.** $\frac{\mu_0 i_1 i_2}{2\pi r}$ **C.** $\frac{2\mu_0 i_1 i_2}{\pi r}$ **D.** $\frac{2\mu_0 i_1 i_2}{\pi^2 r}$

Answer :D



Solution :

Note : $B = \frac{\mu_0 I}{2\pi R}\frac{Sin\theta}{\theta}$

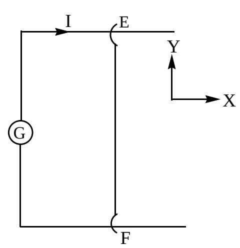
For $\theta = 90^0$ (semi circular)

$\therefore B = \frac{\mu_0 I}{\pi^2 R}$

$\therefore B_{net} = B_1 + B_{2=2_B=}\frac{2\mu_0 I_1}{\pi^2 R}$

$\therefore$ Force per unit length $= \frac{2\mu_0 I_1 I_2}{\pi^2 R}$

86 A conducting slider EF of mass m and length L is placed on two parallel long conducting rails. The generator (G) maintains constant current I in the circuit. The coefficient of friction between the slider (EF) and the conducting rails is 0.50. If a magnetic field $\vec{B} = -B_0\hat{i} - B_0\hat{k}$ exists in entire region, the acceleration of the slider will be (Neglect the gravity)



A. $\frac{ILB_0}{2m}$ **B.** $\frac{ILB_0}{m}$ **C.** $\frac{3ILB_0}{2m}$ **D.** Zero

Answer :A

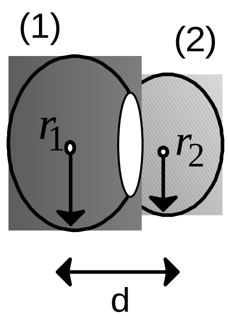
Solution : $\bar{F}$ along x – axis = ILB_0.

Normal reaction $= ILB_0$.

Frictional force $f = \frac{ILB_0}{2}$

$\therefore a = \frac{ILB_0}{2m}$

87 Circular regions (1) and (2) have current densities J and –J respectively normal to the plane of paper , such that their region of intersection carries no current .Magnetic field in the region of inter section is



A. Uniform, proportional to $(r_1 + r_2)$-d

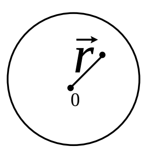
B. Uniform, proportional to d

C. Non – uniform

D. Zero

Answer :B

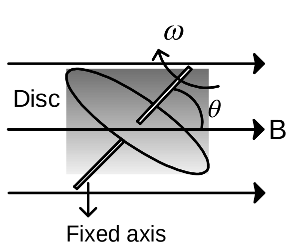
Solution : Since $\overline{B} = \frac{\mu_0}{2}(\vec{J} \times \overline{r})$



$$\overline{B}_{net} = \frac{\mu_0}{2}\left[\vec{J} \times (\overline{r}_1 - \overline{r}_2)\right] = \frac{\mu_0}{2}\left[\vec{J} \times \vec{d}\right]$$

Hence $\overline{B}$ is uniform and proportional to d

88 A disc (of radius r)carrying positive charge q, distributed uniformly, is rotating with angular speed ω (clock wise) in a uniform magnetic field B about a fixed axis (as shown in figure), such that angle made by axis of disc with magnetic field is θ. Torque applied by axis on the disc is



A. $\frac{q\omega r^2 B \sin\theta}{2}$

B. $\frac{q\omega r^2 B \cos\theta}{4}$

C. $\frac{q\omega r^2 B \sin\theta}{2}$

D. $\frac{q\omega r^2 B \sin\theta}{4}$

Answer :D

Solution : $\tau = \vec{M} \times \vec{B}$

$$\tau = \left(\frac{qL}{2m}\right) B \sin(\pi - \theta)$$

$$\tau = \left(\frac{q}{2m}\right)\left(\frac{mR^2}{2}.w\right) B \sin\theta$$

$$\tau = \frac{qR^2 wB}{4} \sin\theta$$

As the axis is fixed and angular momentum is constant, this much amount of torque is also applied by axis on the disc in opposite direction.

89 A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by M, the wire resonates with the same tuning fork forming three antinodes for the same position of bridges. The value of M is

A. 25 kg **B.** 5 kg **C.** 12.5 kg **D.** 1/25 kg

Answer :A

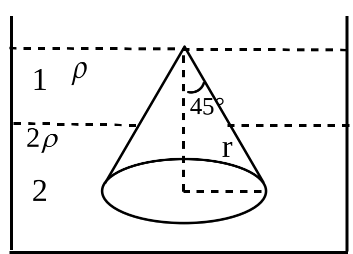
Solution : $f = \frac{p}{2l}\sqrt{\frac{T}{\mu}}$

$$\frac{5}{2l}\sqrt{\frac{9g}{\mu}} = \frac{3}{2l}\sqrt{\frac{Mg}{\mu}}$$

$$5(3) = 3\sqrt{M}$$

$$M = 25kg$$

90 A cone made up of material of density $\frac{3\rho}{2}$ is fully submerged in two liquids as shown in figure. V_1 is volume of cone submerged in liquid 1 and V_2 is volume of cone submerged in liquid 2. The ratio $V_1 : V_2$ is



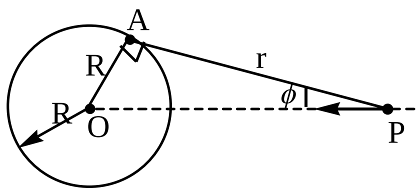
A. 1 : 1 **B.** 2 : 1 **C.** $\frac{3}{2}:1$ **D.** 1 : 2

Answer :A

Solution : $V_1\rho_1 g+V_2\rho_2 g=\left(V_1+V_2\right)\frac{3}{2}\rho g$

$V_1:V_2=1:1$

91 A dipole having dipole moment p is placed in front of a solid neutral conducting sphere as shown in figure. The net potential at point A on the surface of sphere is



A. $\frac{kP\cos\phi}{r^2}$ **B.** $\frac{kP\cos^2\phi}{r^2}$ **C.** $\frac{2kP\cos^2\phi}{r^2}$ **D.** $\frac{kP}{r^2}$

Answer :B

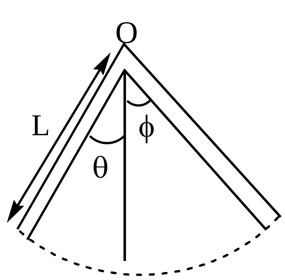
Solution : Due to the placing of dipole in front of conducting sphere the charge gets redistribute on the surface of sphere but the total induced charge on the sphere would be zero. Potential at centre of sphere is equal to sum of potential at centre due to dipole and due to induced charge.

$V_{centre}=V_{induced\,charge}+V_{dipole}$

$=0+\frac{kP}{(OP)^2}=\frac{kP\cos^2\phi}{r^2}$

[$V_{induced\,charge}=0,$ because the total induced charge is on the surface of sphere and is zero] As conductor is an equipotential surface, the potential at all points of conductor would be same. So, potential at $A=\frac{kP\cos^2\phi}{r^2}$.

92 A thin bar of mass m and length L can freely rotate about a horizontal axis passing through the point O. The bar is deflected from the vertical by an angle ϕ and released. The speed of lower end of the bar when it makes an angle θ $(\theta<\phi)$ from the vertical



A. $\sqrt{3gL(\cos\theta-\cos\phi)}$ **B.** $\sqrt{gL(\cos\theta-\cos\phi)}$

C. $\sqrt{\frac{gL}{3}\left(\cos\phi-\cos\theta\right)}$

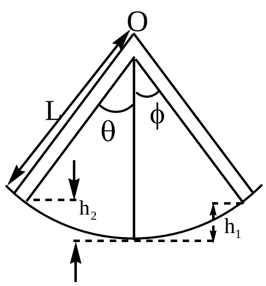
D. $\sqrt{gL\left(\cos\phi-\cos\theta\right)}$

Answer :A

Solution : According to conservation of energy

$$mgh_1 = mgh_2 + \frac{1}{2}I\omega^2$$

$$mg\left[\frac{L}{2}\left(1-\cos\phi\right)-\frac{L}{2}\left(1-\cos\theta\right)\right] = \frac{1}{2}\frac{mL^2}{3}\omega^2$$



$$mg[\cos\theta-\cos\phi] = \frac{mL}{3}\omega^2 \text{ or } \omega = \sqrt{\frac{3g}{L}(\cos\theta-\cos\phi)}$$

$$V = L\omega = \sqrt{3gL(\cos\theta-\cos\phi)}$$

93 A ball is rising towards the surface with a constant velocity in a liquid whose density is four times that of the material of the ball. How many times of its weight is the drag force acting on the rising ball

A. 2 **B.** 3 **C.** 4 **D.** 5

Answer :B

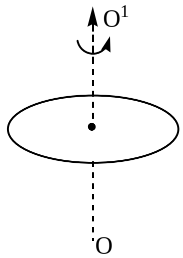
Solution : Let ρ be the density of the material of the ball, the equation of motion is given by

$$\frac{4}{3}\pi r^3\rho g - \frac{4}{3}\pi r^3.4\rho g + F = 0$$; F = force of friction & r = radius of the ball

$$\therefore F = \frac{4}{3}\pi r^3.3\rho g$$

= 3 times the weight of the ball

94 A lead ring of radius r rotates about a vertical axis OO^1 passing through its centre and perpendicular to the plane of the ring. Find the number of rotations per second at which the ring just breaks. The ultimate strength of lead is σ_m and its density is ρ.

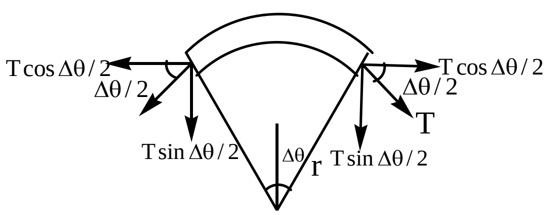


A. $\sqrt{\frac{\sigma_m}{\rho}}\times\frac{1}{8\pi r}$ **B.** $\sqrt{\frac{\sigma_m}{4\rho}}\times\frac{1}{2\pi r}$ **C.** $\sqrt{\frac{\sigma_m}{\rho}}\times\frac{1}{2\pi r}$ **D.** $\sqrt{\frac{\sigma_m}{2\rho}}\times\frac{1}{2\pi r}$

Answer :C

Solution : Consider a part of the ring subtending an angle $\Delta\theta$ at the centre.

Mass of the element $=\frac{m}{l}\times r\Delta\theta;\ m=$ mass of the ring.



Let T be the tension on the ring. Of the two components only T sin $\Delta\theta/2$ is operational. The net force is

$$2T\sin\frac{\Delta\theta}{2}=T\Delta\theta=\frac{m}{l}r\Delta\theta\,\omega^2$$

$$\text{or}\quad T=\frac{m}{l}r^2\omega^2=\frac{m}{2\pi r}r^2\omega^2=\frac{mr\omega^2}{2\pi}$$

Mass of the ring $m=2\pi rA\rho$ where A is the area of cross-section of the wire.

$$T=\frac{2\pi r}{2\pi}A\rho r\omega^2=r^2A\rho\left(2\pi n\right)^2$$

$$\sigma_m=\text{ultimate strength}=\frac{T_{\max}}{A}=\left(r^2\rho\right)4\pi^2n_{\max}^2$$

$$\therefore n=\sqrt{\frac{\sigma_m}{\rho}}\times\frac{1}{2\pi r}$$

95 Two cars A and B are moving towards each other with a speed of 30 m/s. A person sitting in car A fires shots after every 3 seconds and person sitting in car B observes them. What will be the time difference recorded by him between two consecutive shots ? Velocity of sound $\upsilon=330m/s$.

A. 3s **B.** 3.6 s **C.** 2 s **D.** 2.5 s

Answer :D

Solution : The apparent frequency as heard by the person sitting in car B is given by

$$n' = n.\frac{\upsilon+30}{\upsilon-30} = n\frac{330+30}{330-30} = \frac{6n}{5}$$

Let t and t^1 be the time intervals between two shots as recorded by the persons in car A and B respectively.

Then $\frac{t'}{t} = \frac{n}{n'} or\, t' = \frac{t.n}{n'} = 3\times\frac{5}{6} = 2.5\,s$

96 A material of wire having density $\rho = 1.4\,g/cc$ is not wetted by water of surface tension70 dyne/cm. Find the maximum radius of the wire which can float on the surface of water

A. 0.18 cm **B.** 0.9 cm **C.** 0.54 cm **D.** 0.3 cm

Answer :A

Solution : $2LT = L\pi r^2 \rho g$; L = length of the wire

T = surface tension of water

r = radius of wire

$\rho =$ density of the material of wire

$$r = \left(\frac{2T}{\pi\rho g}\right)^{1/2}$$

On putting the values of the various terms we have

$$r = \left(\frac{2\times 70}{3.14\times 1.4\times 980}\right)^{1/2} = 0.18\,cm$$

97 In a region where there is no gravitation field, a small sphere of mass ‘m’ moving with an initial velocity V_0 in a viscous medium experience a drag force $R = -bV$. Determine the velocity of sphere as a function of time

A. $V_0 = V\,e^{-bt/m}$ **B.** $V = V_0 e^{-bt/m}$

C. $V = V_0\,e^{-m/bt}$ **D.** $V = V_0\,e^{m/bt}$

Answer :B

Solution : Equation of motion of sphere is given by

$$m\frac{dV}{dt} = -bV$$

$$\int\frac{dV}{V} = -\frac{b}{m}\int dt$$

$$\log_e V = -\frac{bt}{m} + C$$

At t = 0, $V = V_0$

$\therefore C = \log V_0$ or $V = V_0\, e^{-bt/m}$

98 What will the percentage error be in calculating the atmospheric pressure equal to 760 mm of mercury according to the height of a mercury column if the diameter of the meniscus of the liquid surface is 5 mm. Surface tension of mercury = 0.5 N/m

A. ≈ 0.2 **B.** ≈ 0.7 **C.** ≈ 0.4 **D.** ≈ 0.9

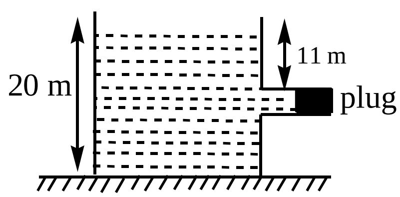
Answer :C

Solution : $\frac{2T}{r} = h\rho g$

$\therefore\ h = \frac{2T}{r\rho g} = \frac{2\times 0.5}{2.5\times 10^{-3}\times 13.6\times 10^{3}\times 9.8}$ = 3.00 mm

% error = $\frac{100\times 3}{760} \approx 0.4$

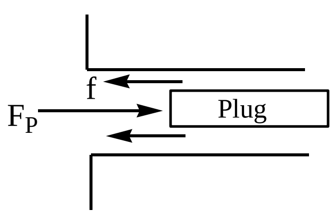
99 The water in a reservoir is 20 m deep. A horizontal pipe 6 cm in diameter passes through the reservoir 11 m below the water surface as shown in figure. A plug secures the pipe opening. (Take g = 10 m/s^2). Find the force of friction between the plug and pipe wall



A. 280 N **B.** 310.86 N **C.** 300 N **D.** 320 N

Answer :B

Solution : F.B.D of plug



Force of friction = Force due to pressure difference

= Pressure difference on the sides of the plug × Area of cross section of the plug

$= (\rho g h)A$

$$= 10^3 \times 10 \times 11 \times \pi \times (3\times 10^{-2})^2 \;\; = 10^3 \times 10 \times 11 \times 3.14 \times 9 \times 10^{-4}$$

$$= 110 \times 3.14 \times 9 \times 10^{-1} \;\; = 990 \times 3.14 \times 10^{-1} = 99 \times 3.14 = 310.86 N$$

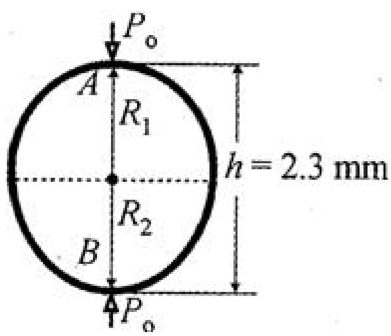
Force of friction is 310.86 N

100 A water drop falls in air with a uniform velocity. Find the difference between the curvature radii of the drop's surface at the upper most and lower most points of the drop which are separated by the distance h is (take proper approximation regarding radii of the surfaces of drop if required i.e., difference in the radii of the surfaces is very small).

A. $\frac{\rho g h^3}{8T}$ **B.** $\frac{3\rho g h^3}{8T}$ **C.** $\frac{\rho g h^3}{4T}$ **D.** $\frac{\rho g h^3}{2T}$

Answer :A

Solution : Suppose R_1 and R_2 be the radii of curvatures at the upper point and lower point of the drop respectively. The pressure inside the drop at the upper end,



$$P_A = P_0 + \frac{2T}{R_1}$$

and pressure at the lower end

$$P_B = P_0 + \frac{2T}{R_2}$$

Where P_0 is the atmospheric pressure.

As the drop is falling with uniform velocity, so

$$P_B = P_A + \rho g h$$

or $$P_0 + \frac{2T}{R_2} = P_0 + \frac{2T}{R_1} + \rho g h$$

or $$2T\left(\frac{1}{R_2} - \frac{1}{R_1}\right) = \rho g h$$

or $\quad 2\frac{(R_1 - R_2)}{R_1 R_2} = \rho g h$

It can be assumed that $R = R_2$ and $R_1 + R_2$ =h

so $\quad R_1 = R_2 = h/2.$ Therefore, we get

$2T(R_1 - R_2) = (\rho g h) R_1 R_2$

$= \rho g h \times \left(\frac{h}{2} \times \frac{h}{2}\right)$

$\therefore \quad R_1 - R_2 = \frac{\rho g h^3}{8T}$

101 A conical glass capillary tube of length 0.1 m has diameters 10^{-3} & $5\times10^{-4}\, m$ at the ends. When it is just immersed in a liquid at $0^0 C$ with larger diameter in contact with it, the liquid rises to $8\times10^{-2}\, m$ in the tube. The density of the liquid is $(1/14)\times10^4\, kg/m^3$ and angle of contact is zero. Effect of temperature on density of liquid and glass is negligible.

A. Surface tension at $0^0 C$ is $6.6\times10^{-2}\, N/m$

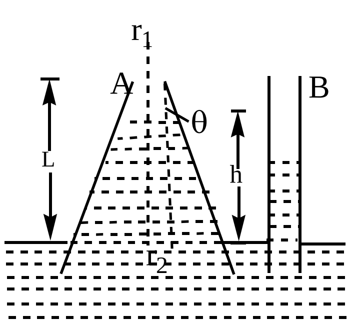
B. Surface tension at $0^0 C$ is $8.4\times10^{-2}\, N/m$

C. Surface tension at $0^0 C$ is $7.7\times10^{-2}\, N/m$

D. Surface tension at $0^0 C$ is $5.4\times10^{-2}\, N/m$

Answer :B

Solution : If r is the radius of the meniscus in the conical tube, then from the geometry of figure we have $\tan\theta = \frac{r - r_1}{L - h} = \frac{r_2 - r_1}{L}$



i.e., $\dfrac{r-2.5\times10^{-4}}{0.1-0.08}=\dfrac{(5-2.5)\times10^{-4}}{0.1}$

i.e., $r=3\times10^{-4}m$;

$$T_{0^0C}=\frac{hr\rho g}{2}=\frac{(8\times10^{-2}\,m)(3\times10^{-4}\,m)\left(\frac{1}{14}\times10^{4}\,kg/m^{3}\right)(9.8\,m/s^{2})}{2}=8.4\times10^{-2}\,N/m$$

102 A closed organ pipe has length '*l*'. The air in it is vibrating in 3rd overtone with maximum amplitude 'a'. The amplitude at a distance of *l*/7 from closed end of the pipe is equal to

A. A **B.** a/2 **C.** $\dfrac{a\sqrt{3}}{2}$ **D.** Zero

Answer :A

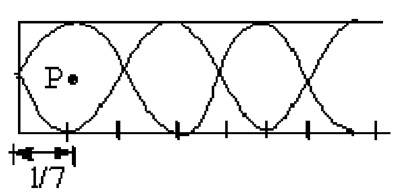
Solution : The figure shows variation of displacement of particle in a closed organ pipe for 3rd overtone.

For third overtone

$l=\dfrac{7\lambda}{4}$ or $\lambda=\dfrac{4l}{7}$ or $\dfrac{\lambda}{4}=\dfrac{l}{7}$

Hence the amplitude at P at a distance $l/7$ from closed end is 'a' because there is an antinode at that point.

Alternate : Because there is node at x=0 the displacement amplitude as function of x can be written as



$A=a\sin kx=a\sin\dfrac{2\pi}{\lambda}x$

For third overtone

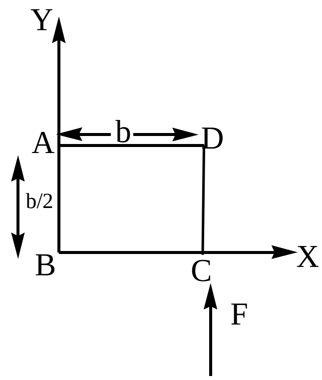
$l=\dfrac{7\lambda}{4}$ or $\lambda=\dfrac{4l}{7}$

$A=a\sin\dfrac{7\pi}{2l}\dfrac{l}{7}=a\sin\dfrac{\pi}{2}=a$

At $x=\dfrac{l}{7}$ $\Rightarrow A=a$

103 A uniform rectangular lamina of mass ‘m’ and size $\left(b\times\frac{b}{2}\right)$ is resting on a smooth horizontal table.

A force F is applied at point ‘C’ perpendicular to side BC.



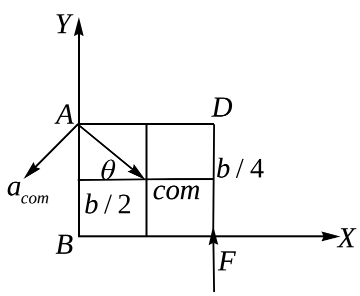
A. angular acceleration 'α' about centre of mass of the lamina is $\frac{12F}{5mb}$

B. acceleration of point ‘A’ w.r.t centre of mass is $\frac{6F}{\sqrt{5}\,m}$

C. acceleration of point ‘A’ w.r.t ground is $\bar{a}=\frac{-6F}{5m}\hat{i}-\frac{7F}{5m}\hat{j}$

D. angular acceleration 'α' about the centre of mass of the lamina is $\frac{4F}{5\,mb}$

Answer :B



Solution :

$a_{COM,ground}=F/m$

$\tau=Fb/2$

$I=\frac{m}{12}[b^2+\frac{b^2}{4}]=\frac{5}{48}mb^2$

$\alpha=\frac{\tau}{I}=\frac{24F}{5mb}$; from fig : $\sin\theta=\frac{1}{\sqrt{5}};\cos\theta=\frac{2}{\sqrt{5}}$

acceleration of ‘A’ w.r.t. COM is $a_{A,COM}=\frac{6F}{\sqrt{5}m}$

acceleration of 'A' w.r.t ground, $\bar{a} = \frac{-6F}{5m}\,\hat{i} - \frac{17F}{5m}\,\hat{j}$

104 Inside a satellite orbiting around the earth, water does not fall out of an inverted glass. This is best explained by the fact that

A. The earth's force of attraction on the water is negligibly small at the height of the satellite
B. The satellite and the earth exert equal and opposite forces on the water
C. The gravitational attraction between the glass and the water balances the earth's attraction on the water
D. The water and the glass have the same acceleration towards the centre of the earth.

Answer :D

Solution : Water and glass have common acceleration. i.e. both are freely falling bodies. Hence one will be relatively at rest with respect to other.

105 A wire of length L and six identical cells of negligible internal resistance are connected in series. The temperature of the wire is raised by ΔT in time t due to the current. N similar cells are now connected in series with a wire of the same material and cross-section, but of length 3L. The temperature of the wire is raised by the same amount ΔT in the same time t. The value of N is

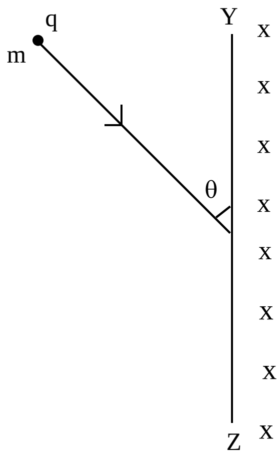
A. 24 **B.** 18 **C.** 12 **D.** 9

Answer :B

Solution : $\left.\begin{matrix} m\,\alpha\,L \\ R\,\alpha\,L \end{matrix}\right\} \Rightarrow \begin{matrix} L\,s\,\Delta T\,\alpha\,\frac{(6E)^2}{L}t \\ 3L\,s\,\Delta T\,\alpha\,\frac{(NE)^2}{3L}t \end{matrix}$

$\Rightarrow \frac{1}{3} = \frac{36}{N^2}\times 3 \Rightarrow N = 6\times 3 = 18$

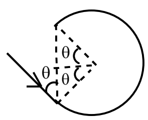
106 A particle with charge +q and mass m enters a magnetic field of magnitude B, existing only to the right of the boundary YZ. The direction of motion of the particle is perpendicular to the direction of B. Let $T = \frac{2\pi m}{qB}$. The time spent by the particle in the field will be



A. $\left(\frac{2\theta}{\pi}\right)T$ **B.** $\left(\frac{\pi-2\theta}{2\pi}\right)T$ **C.** $\left(\frac{\pi+2\theta}{2\pi}\right)T$ **D.** $\left(\frac{\pi-\theta}{\pi}\right)T$

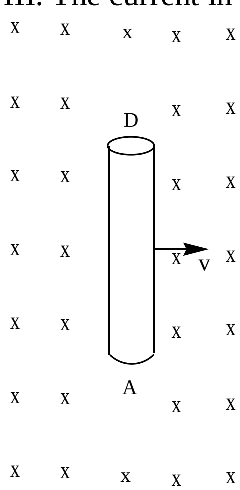
Answer :D

Solution :



$$t=\frac{2\pi-2\theta}{2\pi}\times T=\left(\frac{\pi-\theta}{\pi}\right)T$$

107 The conductor AD in the figure moves to the right in a uniform magnetic field directed into the paper. Then which of the following statements is/are correct?
I. D will acquire a higher potential with respect to A
II. The free electrons in AD will move towards A when A and D are connected externally.
III. The current in AD flows from lower to higher potential when A and D are connected externally.



A. I and II only **B.** I and III only **C.** II only **D.** I, II and III

Answer :D

Solution : Apply Fleming's right hand rule.

108 A parallel plate capacitor, filled with a dielectric of dielectric constant K, is charged to a potential V_0. It is now disconnected from the cell and the dielectric slab is removed. If it now discharges, with time constant τ through a resistance, the potential difference across it will be V_0 after a time

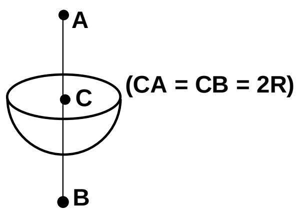
A. $\tau \ln\left(1-\frac{1}{K}\right)$ **B.** $\tau \ln\left(\frac{K}{K-1}\right)$ **C.** $\tau \ln K$ **D.** Zero

Answer :C

Solution : $CV_0 = kCV_0 e^{-t/t}$

$$V = V_0 \Rightarrow e^{-t/\tau} = \frac{1}{k} \Rightarrow t = \tau \ln k$$

109 The diagram shows a uniformly charged hemisphere of radius R. It has volume charge density ρ. If the electric field at a point 2R distance above its centre is E, then what is the electric field at the point which is at 2R distance below its centre ?

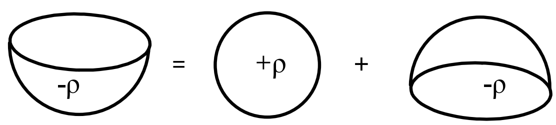


A. $\frac{\rho R}{6\varepsilon_0} + E$ **B.** $\frac{\rho R}{12\varepsilon_0} - E$ **C.** $\frac{-\rho R}{6\varepsilon_0} + E$ **D.** $\frac{\rho R}{24\varepsilon_0} + E$

Answer :B

Solution : Apply principle of superposition

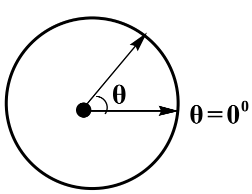
Electric field due to a uniformly charged sphere



$$= \frac{\rho R}{12\varepsilon_0}$$

$$E_{\text{Resultant}} = \frac{\rho R}{12\varepsilon_0} - E$$

110 The linear charge density on a dielectric ring of radius R is varying with θ as $\lambda = \lambda_0 \cos\left(\frac{\theta}{2}\right)$ where λ_0 is a +ve constant of appropriate dimensions. The potential at the centre of the ring is



A. 0 **B.** $\frac{\lambda_0}{2\pi\varepsilon_0}$ **C.** $\frac{\lambda_0}{4\pi\varepsilon_0}$ **D.** $\frac{\lambda_0}{\pi\varepsilon_0}$

Answer :A

Solution : Potential is a scalar quantity. The charge density function suggests that net charge is zero. Net charge on the ring

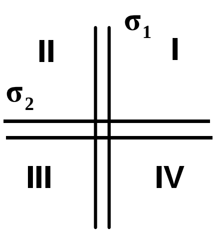
$$Q = \int_0^{2\pi} \lambda R d\theta$$

.

$$= \lambda_0 R \int_0^{2\pi} \cos(\theta/2)\, d\theta = 0$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}\right) = 0$$

111 Two infinite dielectric sheets having charge densities σ_1 and σ_2 (charge per unit area) are placed in two perpendicular planes whose two cross sectional view is shown in the figure. The charges are distributed uniformly on the sheets in electrostatic equilibrium condition. Four points are marked as I, II, III and IV. The electric field intensities at these points are $\vec{E}_1, \vec{E}_2, \vec{E}_3$ and $\vec{E}_4$ respectively. The correct expression for electric field intensities is



A. $|\vec{E}_1| = |\vec{E}_2| = \frac{\sqrt{\sigma_1^2 + \sigma_2^2}}{2\varepsilon_0} \neq |\vec{E}_4|$

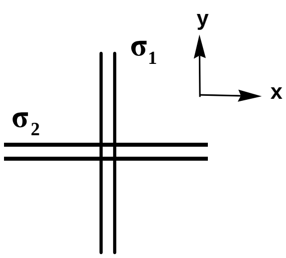
B. $|\vec{E}_2| = |\vec{E}_4| = \frac{\sqrt{\sigma_1^2 - \sigma_2^2}}{2\varepsilon_0}$

C. $|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3| = |\vec{E}_4| = \frac{\sqrt{\sigma_1^2 + \sigma_2^2}}{2\varepsilon_0}$

D. none of the above

Answer :C

Solution : Using principle of superposition $\vec{E}$ due to infinite plane sheet having surface charge density σ on its one face is, $\frac{\sigma}{2\varepsilon_0}$



So, $\vec{E}_1 = \frac{\sigma_1}{2\varepsilon_0}\hat{i} + \frac{\sigma_2}{2\varepsilon_0}\hat{j}$

$$\vec{E}_2 = -\frac{\sigma_1}{2\varepsilon_0}\hat{i} + \frac{\sigma_2}{2\varepsilon_0}\hat{j}$$

$$\vec{E}_3 = -\frac{\sigma_1}{2\varepsilon_0}\hat{i} - \frac{\sigma_2}{2\varepsilon_0}\hat{j}$$

$$\vec{E}_4 = \frac{\sigma_1}{2\varepsilon_0}\hat{i} - \frac{\sigma_2}{2\varepsilon_0}\hat{j}$$

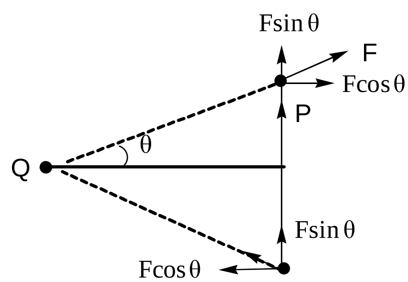
112 $1\mu C$ charge is uniformly distributed on a thin spherical shell given by equation $x^2 + y^2 + z^2 = 25$. What will be intensity of electric field at a point $(1,1,2)$? (All the quantities are in S.I units)

A. $5\,\text{N/C}$ **B.** $45\,\text{N/C}$ **C.** $\frac{5\sqrt{3}}{2}\,\text{N/C}$ **D.** Zero

Answer :D

Solution : $r = \sqrt{1+1+4} = \sqrt{6} < 5$

$\Rightarrow E_{\text{in}} = 0$



113 The density of the core of a planet is ρ_1 and that of the outer shell is ρ_2. The radii of the core and that of the planet are R and 2R respectively. If the gravitational acceleration at the surface of the planet is same at a depth R, then $\frac{\rho_1}{\rho_2}$ is

A. $\frac{7}{4}$ **B.** $\frac{4}{7}$ **C.** $\frac{3}{8}$ **D.** $\frac{7}{3}$

Answer :D

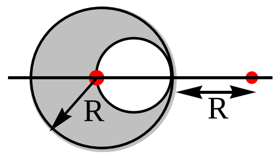
Solution : $\frac{g_{\text{on the shell}}}{g_{\text{on core}}} = \left(\frac{M_{\text{shell}} + M_{\text{core}}}{M_{\text{core}}}\right)\left(\frac{R}{2R}\right)^2$

$$1 = \left[\frac{\rho_1 \frac{4}{3}\pi R^3 + \rho_2 \frac{4}{3}\pi\left(8R^3 - R^3\right)}{\rho_1 \frac{4}{3}\pi R^3}\right]\left[\frac{1}{4}\right]$$

$\Rightarrow \rho_1 + 7\rho_2 = 4\rho_1$

$$\Rightarrow \frac{\rho_1}{\rho_2} = \frac{7}{3}$$

114 A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at a distance 2R from the centre of the sphere. A spherical cavity of radius R/2 is now made in the sphere as shown in the figure. The sphere with the cavity now applies a gravitational force F_2 on the same particle. The ratio F_1 / F_2 is

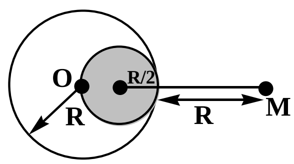


A. $\frac{1}{2}$ **B.** $\frac{3}{4}$ **C.** $\frac{7}{8}$ **D.** $\frac{9}{7}$

Answer :D

Solution : Mass of cavity = M/8 if mass of sphere = M as volume of cavity is 1/8 of sphere.

$$F_1 = \frac{GMm}{4R^2}$$

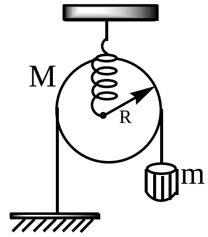


$$F_2 = \frac{GMm}{4R^2} - \frac{GMm}{8\left(\frac{3}{2}R\right)^2}$$

$$= \frac{GMm}{R^2}\left(\frac{1}{4} - \frac{1}{18}\right) = \frac{GMm}{R^2}\left[\frac{9-2}{36}\right] = \frac{GMm}{R^2}\frac{7}{36}$$

$$\therefore \frac{F_1}{F_2} = \frac{36}{4 \times 7} = \frac{9}{7}$$

115 In the arrangement shown in the figure, a pulley of mass M is suspended from a light spring of stiffness constant k and a mass m is suspended form a string tightly wound on the pulley. If the mass m is given a small downward displacement and released, the time period of its small oscillations is (Assume that the string does not slip on pulley)



A. $2\pi\sqrt{\dfrac{8m+3M}{2k}}$ **B.** $2\pi\sqrt{\dfrac{3m+8M}{2k}}$ **C.** $2\pi\sqrt{\dfrac{3M+8m}{k}}$ **D.** $2\pi\sqrt{\dfrac{8M+3m}{k}}$

Answer :A

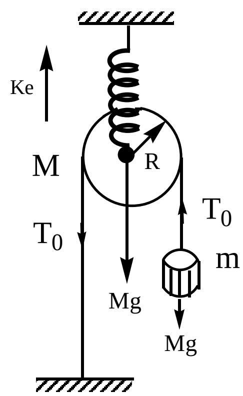
Solution : Let the initial elongation of the spring in the equilibrium state be e. Then $Ke = Mg + 2T_0$

$T_0 = mg$

$\Rightarrow Ke = Mg + 2mg$

When the mass m is pulled and released, let the instantaneous displacement of m be x and that of pulley be x/2.

Since the total energy during SHM remains conserved.V=speed of m at that instant



$$\therefore \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{v}{2}\right)^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}K\left(\frac{x}{2}+e\right)^2 - Mg\frac{x}{2} - mgx = \text{ constant}$$

$$\because I = \frac{MR^2}{2} \text{ and for no slipping } \frac{V}{2} + R\omega = v \Rightarrow W = \frac{V}{2R}$$

$$\therefore \frac{1}{2}\left[mv^2 + \frac{M}{4}v^2 + \frac{M}{8}v^2\right] + \frac{1}{2}K\left(\frac{x}{2}+e\right)^2 - Mg\frac{x}{2} - mgx = \text{ constant}$$

Differentiating w.r.t. ‘t’. we get

$$\left(m+\frac{3}{8}M\right)v\frac{dv}{dt}+K\left(\frac{x}{2}+e\right).\frac{1}{2}\frac{dx}{dt}-\frac{Mg}{2}\frac{dx}{dt}-mg\frac{dx}{dt}=0 \qquad (2)$$

Using equation (1) and (2) and then rearranging we get

$$\therefore \frac{dv}{dt}=-\frac{K}{\left(\frac{8m+3M}{2}\right)}x=-\omega^2 x$$

$$\therefore T=2\pi\sqrt{\frac{8m+3M}{2K}}$$

116 A uniform ring of m with outer radius r_2 is fitted tightly on a shaft of radius r_1. The shaft is rotated about its axis with a constant angular acceleration $\propto$. The torque due to elastic forces in the ring as a function of the radial distance r from the axis is $(r_1 < r < r_2$ and the ring does not slip on the shaft).

A. $\frac{m\propto\left(r_2^4-r^4\right)}{\left(r_2^2-r_1^2\right)}$ **B.** $\frac{m\propto\left(r_2^4-r^4\right)}{2\left(r_2^2-r_1^2\right)}$ **C.** $\frac{m\propto\left(r_2^4+r^4\right)}{\left(r_2^2-r_1^2\right)}$ **D.** $\frac{m\propto\left(r_2^4+r^4\right)}{2\left(r_2^2-r_1^2\right)}$

Answer :B

Solution : Consider a ring element of radius r and thickness dr

$$dm=\frac{m}{\pi\left(r_2^2-r_1^2\right)}2\pi r\,dr=\frac{2mr\,dr}{\left(r_2^2-r_1^2\right)}$$

M.I of the part of ring between r and r_2 is

$$I=\int_r^{r_2}dm\ r^2=\frac{2m}{r_2^2-r_1^2}\int_r^{r_2}r^3\,dr=\frac{m}{2(r_2^2-r_1^2)}\left(r_2^4-r^4\right)$$

Torque due to tension at a distance r from axis is the torque on part of ring between r and r_2

$$\therefore\ \tau=I\alpha=\frac{m\alpha\left(r_2^4-r^4\right)}{2\left(r_2^2-r_1^2\right)}$$

117 A coin is placed on a rough horizontal platform, which undergoes horizontal simple harmonic motion about a mean position O. The coin does not slip on the platform. The force of friction acting on the coin is F.

A. F is always opposite to restoring force acting on coin.

B. F is directed towards O when the coin is moving away from O and away from O when the coin moves towards O.

C. F=0 when the coin and platform come to rest momentarily at the extreme position of the simple harmonic motion.
D. F is maximum when the coin and platform come to rest momentarily at the extreme position of the simple harmonic motion.
Answer :D
Solution : Conceptual

118 A body of mass M is attached to the lower end of a vertical metal wire, whose upper end is fixed. The elongation of the wire when M is in equilibrium is ℓ.

1) Loss in gravitational potential energy of M is $Mg\ell$.

2) Elastic potential energy stored in the wire is $Mg\ell$.

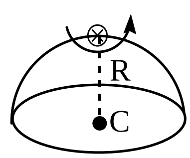
3) Elastic potential energy stored in the wire is $\frac{Mg\ell}{2}$.

4) Heat produced may be $\frac{Mg\ell}{2}$.

A. only 1,2 are true **B.** only 2,3 are true **C.** only 1,4 are true **D.** only 1,3,4 are true

Answer :D
Solution : Conceptual

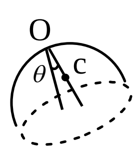
119 A hemispherical shell of mass M and radius R is made to execute small oscillations about the symmetric point at the surface as shown in figure. Find its time period of oscillation.



A. $2\pi\sqrt{\frac{R}{3g}}$ **B.** $2\pi\sqrt{\frac{3R}{4g}}$ **C.** $2\pi\sqrt{\frac{4R}{3g}}$ **D.** $2\pi\sqrt{\frac{R}{g}}$

Answer :C
Solution : Centre of mass of the hemispherical shell is at a distance R/2 form the centre of hemisphere.

$\therefore$ The torque in displaced position is $Mg\frac{R}{2}\sin\theta$



M.I. of the hemispherical shell about the axis passing through O is $\frac{2MR^2}{3}$. (same as the M.I. about a diameter of the base as they are equidistant form the centre of mass)

$$\Rightarrow I\alpha = -Mg\frac{R}{2}\sin\theta$$ (Torque and θ oppositely directed)

$$\Rightarrow \frac{2}{3}MR^2.\alpha = -mg\frac{R}{2}\theta\left(\theta\, is\, small\right)$$

$$\alpha = \frac{3}{4}\frac{g}{R}\theta$$

$$\omega^2 = \frac{3}{4}\frac{g}{R}$$

$$T = 2\pi\sqrt{\frac{4R}{3g}} = 4\pi\sqrt{\frac{R}{3g}}$$

120 Angular frequency of SHM of a particle is ω. There is a point P at a distance 'x' from the mean position 'O'. When the particle passes P, it has velocity is v towards OP. Find the time in which it returns to P again.

A. $\frac{1}{\omega}\tan^{-1}\frac{v}{\omega x}$ **B.** $\frac{2}{\omega}\tan^{-1}\frac{v}{\omega x}$ **C.** $\frac{1}{\omega}\sin^{-1}(\frac{v}{\omega x})$ **D.** $\frac{2}{\omega}\sin^{-1}(\frac{v}{\omega x})$

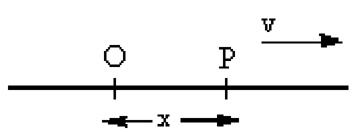
Answer :B

Solution : Let the particle is at P at an instant t starting from mean position and it returns to p again after interval t'

Then $x = a\sin\omega t ...(1)$

Also $x = a\sin\omega(t + t')...(2)$

Form equation (1)



$$\frac{dx}{dt} = a\omega\cos\omega t = v \;\Rightarrow \cos\omega t = v / a\omega ...(3)$$

From equation (2)

$$x = a[\sin\omega t'\cos\omega t + \cos\omega t'.\sin\omega t] \;= a\left[\sin\omega t'\frac{v}{a\omega} + \frac{x}{a}\cos\omega t'\right]$$ (using equation (1) and (3))

$$\Rightarrow x = \frac{v}{\omega}\sin\omega t' + x\cos\omega t'$$

$$\Rightarrow x(1-\cos\omega t') = \frac{v}{\omega}\sin\omega t'$$

$$\Rightarrow x.2.\sin^2\frac{\omega t'}{2} = \frac{v}{\omega}.2.\sin\frac{\omega t'}{2}\cos\frac{\omega t'}{2} \Rightarrow \; t' = \frac{2}{\omega}\tan^{-1}\frac{v}{\omega x}$$

121 The maximum internal pressure (in the absence of an external pressure) that can be sustained by a glass spherical flask of radius 25mm and wall thickness 1mm (Tensile strength of glass is 0.05 Gpa) is nearly

A. 40 atm **B.** 20 atm **C.** 10 atm **D.** 25 atm

Answer :A

Solution : Tensile strength $\sigma = \frac{F}{2\pi r\Delta r} = \frac{\rho\pi r^2}{2\pi r\Delta r}$

$$P = \frac{2\sigma\Delta r}{r} = 40\times10^5 \; pa = 40 atm$$

122 A horizontally oriented thin copper rod of length ℓ is rotated about vertical axis passing through its middle. The breaking stress of copper is σ and density is ρ . The frequency of rotation at which this rod just ruptures is

A. $\sqrt{\frac{\sigma}{\rho\pi^2\ell^2}}\,rpm$ **B.** $\sqrt{\frac{\sigma}{2\pi^2\rho\ell^2}}\,rps$ **C.** $\sqrt{\frac{2\sigma}{\pi^2\rho\ell^2}}\,rps$ **D.** $\sqrt{\frac{3\sigma}{2\pi^2\rho\ell^2}}\,rpm$

Answer :C

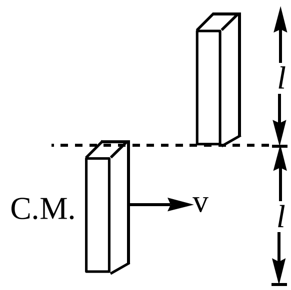
Solution : $T_{ma} = \frac{m\omega^2\ell}{8} = \sigma\times A$

$$\frac{m\omega^2\ell}{8} = \sigma A$$

$$\omega = \sqrt{\frac{8\sigma}{\rho\ell^2}} = 2\pi n$$

$$n = \sqrt{\frac{2\sigma}{\pi^2\rho\ell^2}}rps$$

123 A bar of mass *m*, length *l* is in pure translatory motion with its centre of mass velocity v. It collides with and sticks to another identical bar at rest as shown in figure. Assuming that after collision it becomes one composite bar of length 2l, the angular velocity of the composite bar will be



A. $\frac{3}{4}\frac{v}{l}$, anticlockwise **B.** $\frac{4}{3}\frac{v}{l}$, anticlockwise **C.** $\frac{3}{4}\frac{v}{l}$, clockwise **D.** $\frac{4}{3}\frac{v}{l}$, clockwise

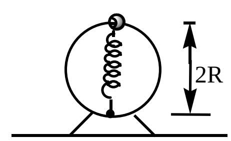
Answer :A

Solution : By Law of Conservation of Angular Momentum $\Sigma mvr = (I_{system})\omega$

$$\Rightarrow mv\frac{\ell}{2} = \frac{(2m)(2\ell)^2}{12}\omega = \frac{2m(4\ell^2)}{12}\omega$$

$$\Rightarrow \omega = \frac{3v}{4\ell}(anticlockwise)$$

124 A bead of mass m slides without friction on a vertical hoop of radius R. The bead moves under the combined influence of gravity and a spring of spring constant k attached to the bottom of the hoop. For simplicity assume, the equilibrium length of the spring to be zero. The bead is released at the top of the hoop with negligible speed as shown. The bead, on passing the bottom point will have a velocity of



A. $2\sqrt{gR}$ **B.** $2\sqrt{gR + \frac{2kR^2}{m}}$ **C.** $2\sqrt{gR + \frac{kR^2}{m}}$ **D.** $\sqrt{2gR + \frac{kR^2}{m}}$

Answer :C

Solution : $\begin{pmatrix} Loss\,in\,elastic \\ potential \\ energy\,of \\ spring\,at \\ lowest\,point \end{pmatrix} + \begin{pmatrix} Loss\,in \\ gravitational \\ potential \\ energy\,of\,bead \\ at\,lowest\,point \end{pmatrix} = \begin{pmatrix} Gain\,in \\ K.E.of\,the \\ bead\,at \\ lowest\,point \end{pmatrix}$

$$\Rightarrow \frac{1}{2}k(2R)^2 + mg(2R) = \frac{1}{2}mv^2$$

$$\Rightarrow v = 2\sqrt{gR + \frac{kR^2}{m}}$$

125 A rocket of mass m exhaust fuel of average density ρ, with a speed v relative to the rocket. The area of cross-section of the opening of the exhaust is A. The minimum value of v to lift the rocket is

A. $\sqrt{\frac{mg}{A\rho}}$ **B.** $\sqrt{\frac{m\rho}{Ag}}$ **C.** $\sqrt{\frac{2mg}{A\rho}}$ **D.** $\sqrt{\frac{2m\rho}{Ag}}$

Answer :A

Solution : $mg = Av^2\rho$

$\Rightarrow v = \sqrt{\frac{mg}{A\rho}}$

126 A sinusoidal wave with amplitude y_m is travelling with speed V on a string with linear density ρ. The angular frequency of the wave is ω. Mark the one which is correct.

A. doubling the frequency doubles the rate at which energy is carried along the string
B. if the amplitude were doubled, the rate at which energy is carried would be halved
C. if the amplitude were doubled, the rate at which energy is carried would be doubled
D. the rate at which energy is carried is directly proportional to the velocity of the wave

Answer :D

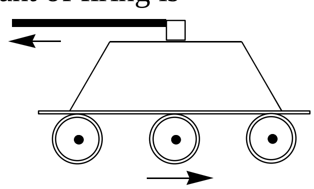
Solution : Power $P = \frac{1}{2}PW^2A^2SV \therefore P\alpha V$

wave $P \rightarrow$ linear density $S \rightarrow$ cross section

$W \rightarrow$ angular frequency $V \rightarrow$ velocity of wave

$A \rightarrow$ amplitude

127 A machine gun is mounted on an armoured car moving with a speed of 20 ms^{-1}. The gun can point against the direction of motion of car. The muzzle speed of bullet is equal to speed of sound in air i.e., 340 ms^{-1}. The time difference between bullets actually reaching and sound of firing reaching at a target 500 m away from car at the instant of firing is



A. 1.2 s **B.** 0.09 s **C.** 0.9 s **D.** 9 s

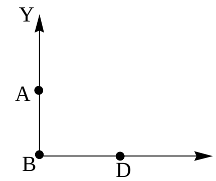
Answer :B

Solution : Time taken by sound to reach target $t_1 = \frac{500}{340}$

Time taken by bullet to reach target $t_2 = \frac{500}{340-20}$

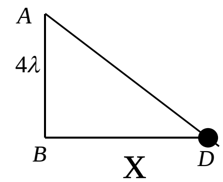
$\therefore \Delta t = t_1 - t_2 = 25\left[\frac{1}{16} - \frac{1}{17}\right] = 0.09\,\text{sec}.$

128 An interference is observed due to two coherent sources 'A'&'B' separated by a distance 4λ along the y-axis where λ is the wavelength of the source. A detector D is moved on the positive x-axis. The number of points on the x-axis excluding the points, x=0 & x=∞ at which maximum will be observed is



A. Three **B.** Four **C.** Two **D.** Infinite

Answer :A



Solution :

Path difference

$$\Delta x=\sqrt{x^2+16\lambda^2}-x\ =x\left[1+\frac{16\lambda^2}{x^2}\right]^{1/2}-x\ =x\left[1+\frac{8\lambda^2}{x^2}\right]-x$$

$$\Delta x=\frac{8\lambda}{x}\quad for\ \max\quad \Delta n=n\lambda$$

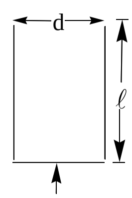
$$\Delta x=\frac{8\lambda}{n}$$

$$n=\frac{8}{x}$$

As n is integer

$\therefore$ Total maximum = 2

129 A tube of diameter d and of length ℓ unit is open at both ends. Its fundamental frequency of resonance is found to be υ_1. The velocity of sound in air is 330 m/sec.One end of tube is now closed. The lowest frequency of resonance of tube is υ_2. Taking into consideration the end correction, $\frac{v_2}{v_1}$ is



A. $\frac{(\ell+0.6d)}{(\ell+0.3d)}$ **B.** $\frac{(\ell+0.3d)}{2(\ell+0.6d)}$ **C.** $\frac{(\ell+0.6d)}{2(\ell+0.3d)}$ **D.** $\frac{(d+0.3\ell)}{2(d+0.6\ell)}$

Answer :C

Solution : for open pipe $f_l=V_1=\frac{V}{2(l+2e)}$

for closed pipe $f_c=V_2=\frac{V}{4(l+e)}$

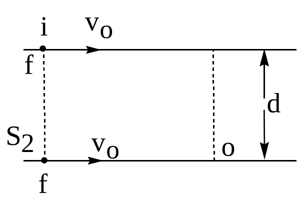
$$\frac{V_2}{V_1} = \frac{\frac{V}{4(l+e)}}{\frac{V}{2(l+2e)}} = \frac{l+2e}{2(l+e)}$$

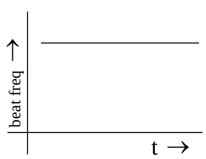
$$= \left(\frac{l+0.6d}{2(l+0.3d)} \right)$$

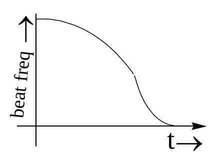
But

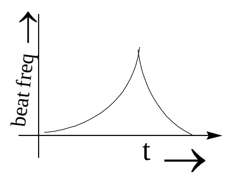
$e = 0.3d$

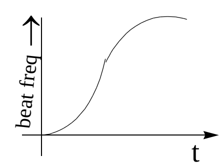
130 Two identical sources moving parallel to each other at separation 'd' are producing sounds of frequency 'f' and are moving with constant velocity v_0. A stationary observer 'o' is on the line of motion of one of the sources. Then the variation of beat frequency heard by O with time is best represented by: (as they come from large distance and go to a large distance)



A. 

B. 

C. 

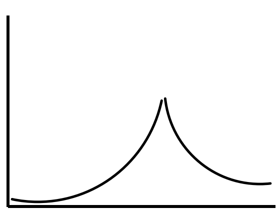
D. 

Answer :C

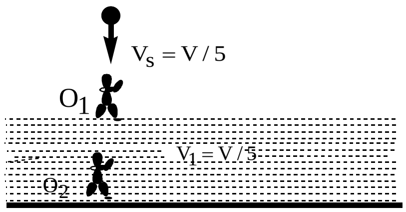
Solution : $\Delta f = f_2 - f_1 = V + \left[\frac{1}{V - VO} - \frac{1}{V - VO\cos\theta} \right]$

$0 < \theta < 90$ Δf increases ; $\theta = 90^0 \Delta f$ is maximum

$90 < \theta < 180^0$ $\Delta +$ decreases



131 In the figure shown an observer O_1 floats (static) on water surface with ears in air while another observer O_2 is moving upwards with constant velocity $V_1 = V/5$ in water . The source moves down with constant velocity $V_s = V/5$ and emits sound of frequency ‘f’. The velocity of sound in air is V and that is water is 4V. For the situation shown in figure :



A. The wavelength of the sound received by O_1 $is \dfrac{4V}{5f}$

B. The wavelength of the sound received by O_1 $is V/f$

C. The frequency of the sound received by O_2 $is \dfrac{4v}{5}$

D. The wavelength of the sound received by O_2 $is \dfrac{V}{5f}$

Answer :A

Solution : $f^1 = \left(\dfrac{v}{v - \frac{v}{5}}\right) f$

$= \dfrac{5}{4} f \qquad \lambda^1 = \dfrac{v}{f^1} = \dfrac{4v}{5f}$

132 Velocity versus displacement of a particle moving in a straight line is $v = \sqrt{9+4s}$, here v is in ms^{-1} and s in meter. Mass of the particle is 2kg. The time in seconds at which average power is $\dfrac{3}{4}$ th of the instantaneous power is

A. 0.92 **B.** 0.67 **C.** 1.5 **D.** 3.0

Answer :C

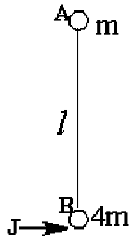
Solution : $P_{av} = \dfrac{3}{4} P_i$ Given $V^2 = 9 + 4s$. On comparing with $V^2 = u^2 + 2as$, we get

$u = 3\,ms^{-1}$ and $a = 2\,ms^{-2}$

$\frac{1}{2}m[\frac{v_f^2 - v_i^2}{t}] = \frac{3}{4}[mav] \Rightarrow \frac{[(3+2t)^2 - 3^2]}{2t} = \frac{3}{4} \times 2(3+2t)$

$\Rightarrow 4t^2 + 12t = 9t + 6t^2 \Rightarrow 2t^2 = 3t \Rightarrow t = 1.5s$

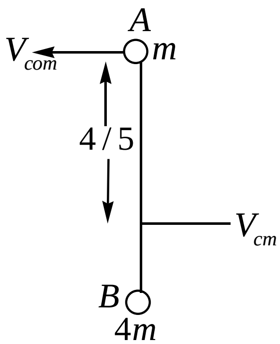
133 An in extensible string of length ℓ connects two masses m and 4m and the system rests on a smooth horizontal floor. An impulse J is imparted to B as shown in figure. The tension in the string in the subsequent motion is



A. $\frac{J^2}{ml}$ **B.** $\frac{J^2}{5ml}$ **C.** $\frac{4J^2}{5ml}$ **D.** $\frac{J^2}{20ml}$

Answer :D

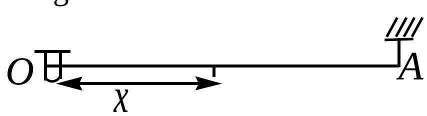
Solution : $j = 5m\,V_{cm}$. In the COM frame, particles will bein circuar motion



$$T = \frac{m(v_{A,cm})^2}{4l/5} = \frac{m(J/5m)^2}{4l/m} = \frac{J^2}{20\,ml}$$

$$\lambda_0 x dx \Rightarrow m = \int_0^L dm = \frac{\lambda_0 L^2}{2}$$

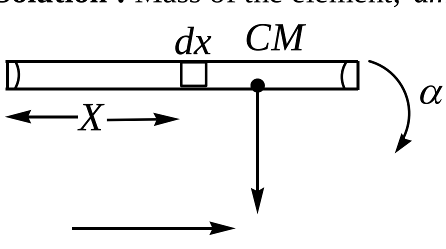
134 A non-uniform rod OA of linear mass density $\lambda = \lambda_0 x(where\,\lambda_0 =$ constant and x is the distance from the point O) and length L is suspended horizontally from the ceiling by a light string at one end A and is hinged at other end O as shown in the figure. The angular acceleration of the rod just after the string is cut will be



A. $2g/L$ **B.** g/L **C.** $4g/3L$ **D.** $3g/4L$

Answer :C

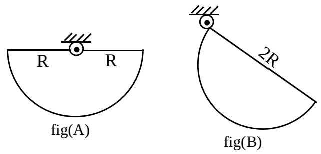
Solution : Mass of the element, $dm =$



$$x_{cm} = \frac{\int x dm}{\int dm} = \frac{\int_0^L \lambda_0 x^2 dx}{\int_0^L \lambda_0 x dx} = \frac{2L}{3} \quad I0\int_0^L dm x^2 = \frac{\lambda_0 L^4}{4}$$

$$\therefore mgx_{cm} = 1\alpha \Rightarrow \alpha = \frac{4g}{3L}$$

135 A uniform semi circular disc hanging freely about a fixed point as shown in fig(A) has frequency of small oscillations f_o. When it is suspended from another point as shown in fig(B), the frequency of small oscillations will be approximately



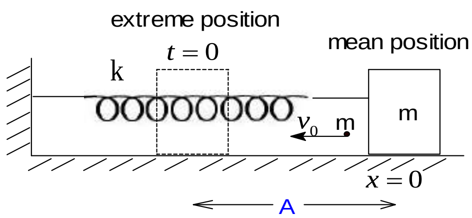
fig(A) fig(B)

A. $3\,f_o$ **B.** $0.2\,f_0$ **C.** $1.8\,f_o$ **D.** $0.9\,f_o$

Answer :D

Solution : $f_0 = \frac{1}{2\pi}\sqrt{\frac{8g}{3\pi R2}}, \& f = \frac{1}{2\pi}\sqrt{\frac{2g\sqrt{16+9\pi^2}}{9\pi R}} \Rightarrow f = 0.9\,f_0$

136 A block of mass 'm' is performing SHM with an amplitude A, on a frictionless surface. When it is at extreme position, a bullet of mass 'm' moving with a velocity v_0 collides and gets embeded into it at time t=0 as shown in figure. The displacement 'x' measured from mean position at any time 't' will be ($mv_0^2 = 2kA^2$)



A. $x = -\sqrt{2}\,A\sin\left[\sqrt{\frac{k}{2m}}t + \frac{\pi}{4}\right]$ **B.** $x = -\sqrt{2}\,A\sin\left[\sqrt{\frac{k}{2m}}t\right]$

C. $x = -\sqrt{2}\,A\sin\left[\sqrt{\frac{k}{2m}}t + \frac{\pi}{3}\right]$ **D.** $x = -A\sin\sqrt{\frac{k}{m}}t$

Answer :A

Solution : $\frac{1}{2}KA^2 + \frac{1}{2}\times 2M\left(\frac{v_0^2}{4}\right) = \frac{1}{2}KA'^2$

(by energy and momentum conservation)

(or) $KA^2 + \frac{1}{2}mv_0^2 = KA'^2 \Rightarrow A^1 = \sqrt{2}A \Rightarrow \phi = \frac{5\pi}{4}\, and\, \omega^1 = \sqrt{\frac{k}{2m}}$

$x = A'\sin\left(\omega^1 i + \phi\right) = -\sqrt{2}A\sin[\sqrt{\frac{k}{2m}}t + \frac{\pi}{4}]$

137 A particle is moving in a circle of radius R in such a way that at any instant the normal and tangential components of the acceleration are equal. If its speed at t = 0 is u_0, the time taken to complete the first revolution is

A. $\frac{R}{u_o}$ **B.** $\frac{u_o}{R}$ **C.** $\frac{R}{u_o}(1-e^{-2\pi})$ **D.** $\frac{R}{u_o}e^{-2\pi}$

Answer :C

Solution : $\frac{dv}{dt}=\frac{v^2}{R}\Rightarrow\int_{u_0}^{v}\frac{dv}{v^2}=\int_0^1\frac{1}{R}dt\Rightarrow R\left[\frac{1}{u_0}-\frac{1}{v}\right]=t$ ----------(1)

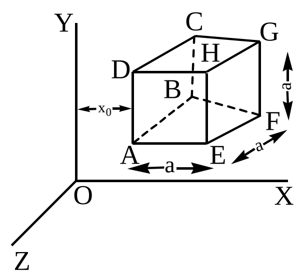
$v\frac{dv}{dt}=\frac{v^2}{R}\Rightarrow\int_{u_0}^{v}\frac{dv}{v^2}=\int_0^{2\pi R}\frac{ds}{R}dt\Rightarrow v=u_0e^{2\pi}$ ----------(2)

And From (1) & (2) we get $t=\frac{R}{u_0}(1-e^{-2\pi})$

138 In a region of space, the electric field is in the *x*-direction and proportional to *x*, i.e., $\vec{E}=E_0x\hat{i}$. Consider an imaginary cubical volume of edge *a*, with its edges parallel to the coordinate axis. The charge inside this volume is

A. Zero **B.** $\varepsilon_0E_0a^3$ **C.** $\frac{1}{\varepsilon_0}E_0a^3$ **D.** $\frac{1}{6}\varepsilon_0E_0a^2$

Answer :B
Solution :



The field at the face $ABCD=E_0x_0\hat{i}$.

$\therefore$ flux over the face $ABCD=-(E_0x_0)a^2$.

The negative sign arises as the field is directed into the cube.

The field at the face $EFGH=E_0(x_0+a)\hat{i}$.

$\therefore$ flux over the face $EFGH=E_0(x_0+a)a^2$.

The flux over the other four faces is zero as the field is parallel to the surfaces.

$\therefore$ total flux over the cube $=E_0a^3=\frac{1}{\varepsilon_0}q$, where q is the total charge inside the cube.

$\therefore q=\varepsilon_0E_0a^3$.

139 A large flat metal surface has a uniform charge density $+\sigma$. An electron of mass *m* and charge *e* leaves the surface at point A with speed *u*, and returns to it at point B. Disregard gravity. The maximum value of AB is

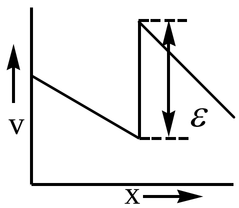
A. $\frac{u^2m\varepsilon_0}{\sigma e}$ **B.** $\frac{u^2e\varepsilon_0}{m\sigma}$ **C.** $\frac{u^2e}{\varepsilon_0\sigma m}$ **D.** $\frac{u^2\sigma e}{\varepsilon_0 m}$

Answer : A

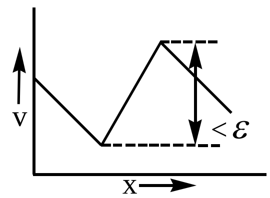
Solution : The force on the electron is $\dfrac{e\sigma}{\varepsilon_0}$ and its acceleration towards the metal sheet is $\dfrac{e\sigma}{m\varepsilon_0}$. The electron will move as a projectile with an effective value of $g=\dfrac{e\sigma}{m\varepsilon_0}$. Its maximum range will then be

$$\frac{u^2}{\frac{e\sigma}{m\varepsilon_0}}=\frac{u^2 m\varepsilon_0}{e\sigma}.$$

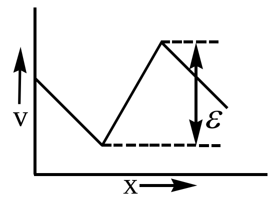
140 The two ends of a uniform conductor are joined to a cell of emf ε and some internal resistance. Starting from the midpoint P of the conductor, we move in the direction of the current and return to P. The potential V at every point on the path is plotted against the distance covered (x). Which of the following best represents the resulting curve?



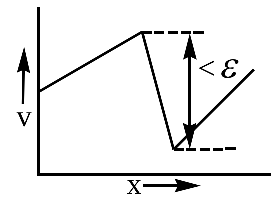
A.



B.



C.

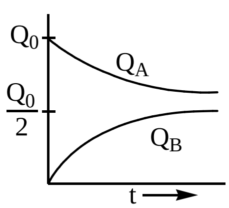


D.

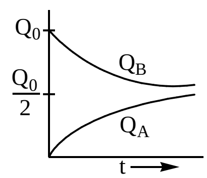
Answer : B

Solution : When we move in the direction of the current in a uniform conductor, the potential decrease linearly. When we pass through the cell, from its negative to its positive terminal, the potential increases by an amount equal to its potential difference. This is less emf, as there is some potential drop across its internal resistance when the cell is driving current.

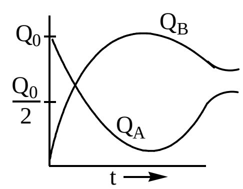
141 A capacitor A with charge Q_0 is connected through a resistance to another identical capacitor B, which has no charge. The charges on A and B after time *t* are Q_A and Q_B respectively, and they are plotted against time *t*. Find the correct curves.



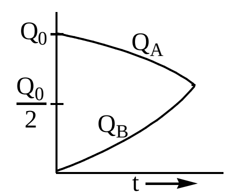
A.



B.



C.



D.

Answer : A

Solution : Conceptual

142 Let S be an imaginary closed surface enclosing mass *m*. Let $\overrightarrow{dS}$ be an element of area on S, the direction of $\overrightarrow{dS}$ being outward from S. Let $\vec{E}$ be the gravitational intensity at $\overrightarrow{dS}$. We define $\phi=\oint_S \vec{E}.\overrightarrow{dS}$, the integration being carried out over the entire surface S.

A. $\phi=-Gm$ **B.** $\phi=-4\pi Gm$

C. $\phi = -\dfrac{Gm}{4\pi}$

D. no relation of the type (a), (b), or (c) can exist

Answer :B

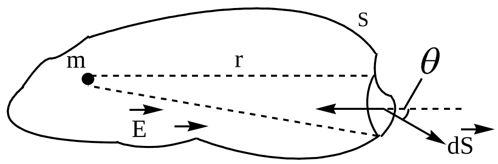
Solution : Follow the method used to prove Gauss's law.

$$E = G.\frac{m}{r^2}$$

$$\vec{E}.\overrightarrow{dS} = EdS\cos(180^0 - \theta) = -EdS\cos\theta$$

$$\phi = \oint_S \vec{E}.\overrightarrow{dS} = \oint_S -G\frac{m}{r^2}dS\cos\theta$$

$$= -Gm.\oint_S \frac{dS\cos\theta}{r^2} = -Gm.\oint_S d\omega = -4\pi Gm.$$



143 A particle of charge per unit mass α is released from origin with velocity $\vec{v} = v_0\hat{i}$ in a magnetic field $\vec{B} = -B_0\hat{k}$ for $x \le \dfrac{\sqrt{3}}{2}\dfrac{v_0}{B_0\alpha}$ and $\vec{B} = 0$ for $x > \dfrac{\sqrt{3}}{2}\dfrac{v_0}{B_0\alpha}$. The x-coordinates of the particle at time $t\left(> \dfrac{\pi}{3B_0\alpha}\right)$ would be

A. $\dfrac{\sqrt{3}}{2}\dfrac{v_0}{B_0\alpha} + \dfrac{\sqrt{3}}{2}v_0\left(t - \dfrac{\pi}{B_0\alpha}\right)$

B. $\dfrac{\sqrt{3}}{2}\dfrac{v_0}{B_0\alpha} + v_0\left(t - \dfrac{\pi}{3B_0\alpha}\right)$

C. $\dfrac{\sqrt{3}}{2}\dfrac{v_0}{B_0\alpha} + \dfrac{v_0}{2}\left(t - \dfrac{\pi}{3B_0\alpha}\right)$

D. $\dfrac{\sqrt{3}}{2}\dfrac{v_0}{B_0\alpha} + \dfrac{v_0 t}{2}$

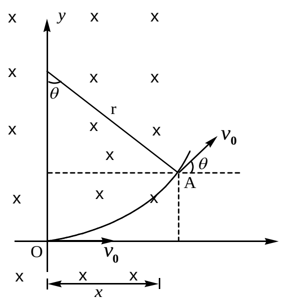
Answer :C

Solution : $r = \dfrac{mv_0}{B_0 q} = \dfrac{v_0}{B_0\alpha}$

$$\frac{x}{r} = \frac{\sqrt{3}}{2} = \sin\theta \qquad \therefore \theta = 60^0$$

$$t_{OA} = \frac{T}{6} = \frac{\pi}{3B_0\alpha}$$

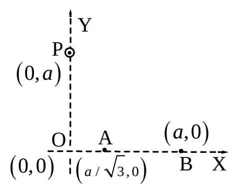
Therefore, x-coordinate of particle at any time $t > \dfrac{\pi}{3B_0\alpha}$ will be



$$x=\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha}+v_0\left(t-\frac{\pi}{3B_0\alpha}\right)\cos 60^0$$

$$=\frac{\sqrt{3}}{2}\frac{v_0}{B_0\alpha}+\frac{v_0}{2}\left(t-\frac{\pi}{3B_0\alpha}\right)$$

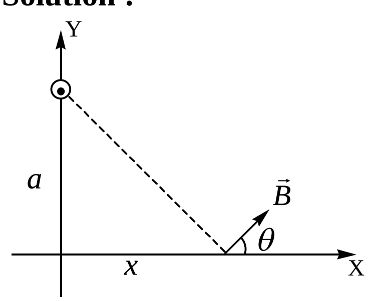
144 An infinite current carrying conductor, parallel to z-axis is situated at point P as shown in the figure. The find $\int_A^B \vec{B}.\overrightarrow{d\ell}$?



A. $\frac{\mu_0 i}{24}$ **B.** $\frac{\mu_0 i}{16}$ **C.** $\frac{\mu_0 i}{12}$ **D.** $\frac{\mu_0 i}{8}$

Answer :A

Solution :



$$B=\frac{\mu_0 i}{2\pi\sqrt{a^2+x^2}}$$

$$\int_A^B \vec{B}.\overrightarrow{d\ell}=\int_{x=\frac{a}{\sqrt{3}}}^{x=a}\frac{\mu_0 i}{2\pi\sqrt{a^2+x^2}}.\frac{a}{\sqrt{a^2+x^2}}.dx$$

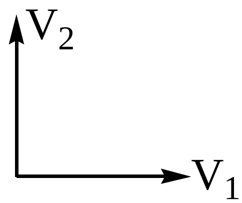
$$=\frac{\mu_0 ia}{2\pi}\int_{x=\frac{a}{\sqrt{3}}}^{x=a}\frac{dx}{\left(x^2+a^2\right)} \quad =\frac{\mu_0 i}{24}$$

145 Two identical non relativistic Particles move at right angle to each other, possessing de Broglie wavelengths λ_1 and λ_2. The deBroglie wavelength of each Particle in their centre of mass frame is

A. $\dfrac{\lambda_1+\lambda_2}{2}$ **B.** $\dfrac{\sqrt{\lambda_1^2+\lambda_2^2}}{2}$ **C.** $\dfrac{\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$ **D.** $\dfrac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$

Answer :D

Solution :



$$\vec{V}_{cm}=\frac{mv_1\hat{i}+mv_2\hat{j}}{2m}=\frac{v_1\hat{i}+v_2\hat{j}}{2}$$

$$\vec{V}_{1c}=\vec{v_1}-\vec{v_c}=v_1\hat{i}-\left[\frac{v_1\hat{i}+v_2\hat{j}}{2}\right]=\frac{v_1\hat{i}-v_2\hat{j}}{2}$$

$$\lambda_1=\frac{h}{mv_1}\quad \lambda_2=\frac{h}{mv_2}$$

$$\lambda_{1c}=\frac{h}{mv_{1c}}=\frac{2h}{m\sqrt{v_1^2+v_2^2}}=\frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$$

Similarly $\lambda_{2c}=\dfrac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$

146 The kinetic energy of the most energetic photoelectrons emitted from a metal surface is doubled when the wavelength of the incident radiation is reduced from λ_1 *to* λ_2. The work function of the metal is

A. $\dfrac{hc}{\lambda_1\lambda_2}(2\lambda_2-\lambda_1)$ **B.** $\dfrac{hc}{\lambda_1\lambda_2}(2\lambda_1-\lambda_2)$ **C.** $\dfrac{hc}{\lambda_1\lambda_2}(\lambda_1+\lambda_2)$ **D.** $\dfrac{hc}{\lambda_1\lambda_2}(\lambda_1-\lambda_2)$

Answer :A

Solution : $KE_{\max}=\dfrac{hc}{\lambda}-W$

147 A parallel beam of light is incident normally on a plane surface absorbing 40% of the light and reflecting the rest. If the incident beam carries 60 watt of power, the force exerted by it on the surface is

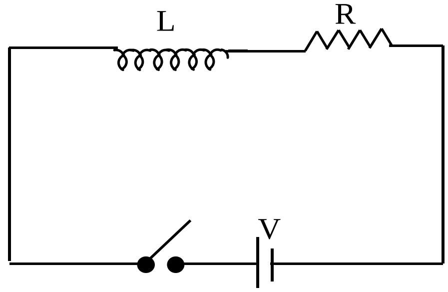
A. $3.2\times10^{-8}\,N$ **B.** $3.2\times10^{-7}\,N$ **C.** $5.12\times10^{-7}\,N$ **D.** $5.12\times10^{-8}\,N$

Answer :B

Solution : Force is rate of change of momentum.
Power absorbed is 0.4 P and power reflected is 0.6 P.

Force $F=\dfrac{0.4P}{C}+\dfrac{2\times0.6P}{C}$ Where C is the velocity of light.

148 A resistor and an inductor in series are connected to a battery through a switch. After the switch has been closed, what is the magnitude of current flowing when the rate of the increase of magnetic energy stored in the coil is at a maximum?



A. $\frac{V}{4R}$ **B.** $\frac{V}{3R}$ **C.** $\frac{V}{2R}$ **D.** $\frac{V}{R}$

Answer :C

Solution : The rate of increase of magnetic energy $\left(E = \frac{LI^2}{2}\right)$ is the difference between the power output of the battery and the power dissipated in the resistor.

$\frac{dE}{dt} = VI - RI^2 = -R\left(I - \frac{V}{2R}\right)^2 + \frac{V^2}{4R} \le \frac{V^2}{4R}$. The rate of increase is maximum when $I = \frac{V}{2R}$

149 Two particles X and Y having equal charges, after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 *and* R_2 respectively. The ratio of mass of X to the mass of Y is

A. $\left(\frac{R_2}{R_1}\right)^{1/2}$ **B.** $\frac{R_2}{R_1}$ **C.** $\left(\frac{R_1}{R_2}\right)^2$ **D.** $\frac{R_1}{R_2}$

Answer :C

Solution : Let the masses of X and Y be m_1 *and* m_2 and let their velocities after being accelerated be υ_1 *and* υ_2 respectively. Since the particles have equal charges and have been accelerated through the same potential difference, their kinetic energies are equal,

i.e., $\frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_2 v_2^2$

In a uniform magnetic field B, the radii of the circular paths are given by

$\frac{m_1 v_1^2}{R_1} = qBv_1 \text{ or } qB = \frac{m_1 v_1}{R_1}$

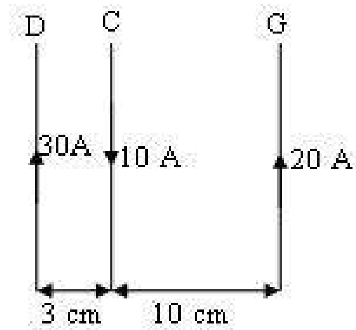
$\text{and } \frac{m_2 v_2^2}{R_2} = qBv_2 \text{ or } qB = \frac{m_2 v_2}{R_2}$

Thereforce $\text{and } \frac{m_1 v_1}{R_1} = \frac{m_2 v_2}{R_2} \text{ or } \frac{m_1^2 v_1^2}{R_1^2} = \frac{m_2^2 v_2^2}{R_2^2}$

But $m_1 v_1^2 = m_2 v_2^2$. Therefore, we have

$$\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$$

150 Three long, straight and parallel wires C, D and G carrying currents are arranged as shown in figure. The force experienced by a 25 cm length of wire C is



A. 0.4 N **B.** 0.04 N **C.** $4\times10^{-3}\,N$ **D.** $4\times10^{-4}\,N$

Answer :D

Solution : The magnetic field due to wire D at wire C is

$$B_D = \left(\frac{\mu_0}{4\pi}\right)\frac{2I}{r} = \frac{10^{-7}\times2\times30}{0.03} = 2\times10^{-4}T$$

Which is directed into the page.

Similarly, the field due to wire G at C is

$$B_G = \frac{10^{-7}\times2\times20}{0.1} = 0.4\times10^{-4}T$$

Which is directed out of the page.

Therefore, the field at the position of the wire C is

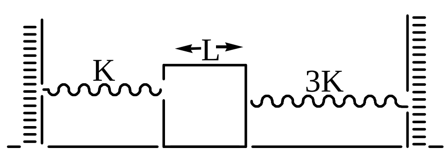
$$B = B_D - B_G = 2\times10^{-4} - 0.4\times10^{-4} = 1.6\times10^{-4}\,T$$

And is directed into the page.

The force on 25 cm of wire C is

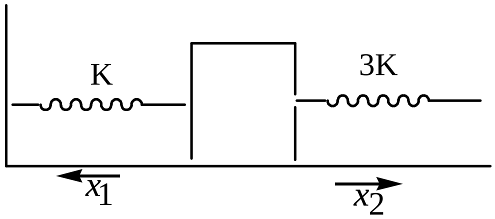
$$F = BIl\,\sin 90^0 = 1.6\times10^{-4}\times10\times0.25 = 4\times10^{-4}\,N$$

151 The system shown consist of two springs. If the temperature of the rod is increased by ΔT. The coefficient of linear expansion of the material of rod is α. If thermal stress in rod is zero, compression in left spring is



A. $\frac{L\alpha\Delta T}{4}$ **B.** $\frac{3L\alpha\Delta T}{4}$ **C.** $\frac{L\alpha\Delta T}{3}$ **D.** Zero

Answer :B



Solution :

$\Delta l = x_1 + x_2$

$\Rightarrow l \propto \Delta\theta = x_1 + x_2$ ______________(1)

under Equilibrium of rol $Kx_1 = 3Kx_2$

$\Rightarrow x_2 = x_{1/3}$ __________________(2)

$\therefore$ *From* (1) & (2)

$$x_1 = \frac{3l \propto \Delta\theta}{4}$$

152 A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is

A. virtual and at a distance of 16 cm from the mirror
B. real and at a distance of 16 cm from the mirror
C. virtual and at a distance of 20 cm from the mirror
D. real and at a distance of 20 cm from the mirror

Answer :B
Solution : For biconvex lens, we have

$$\frac{1}{\upsilon} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{\upsilon} - \frac{1}{-30} = \frac{1}{+15}$$

$\Rightarrow \quad \upsilon = +30cm$

For mirror

$$\frac{1}{\upsilon} + \frac{1}{u} = \frac{1}{f}$$

$\upsilon = -20 \qquad (\therefore \textit{plane mirror})$

Again for lens,

$u = +10$

$\upsilon = +6cm$

$\therefore$ from mirror it is at 16 cm

Hence, option (b) is correct.

153 A vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is

A. 0.02 mm **B.** 0.05 mm **C.** 0.1 mm **D.** 0.2 mm

Answer :D
Solution : The least count Q is equal to 1 M.S.D – 1 V.S.D
Here 20 V.S.D = 16 M.S.D

$$\therefore 1V.S.D = \frac{16}{20} M.S.D = \frac{4}{5} M.S.D$$

Hence, least count = 1 M.S.D-$\frac{4}{5} M.S.D$

Least count = (1mm) x $\frac{1}{5} = 0.2mm$

154 The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

A. 0.9% **B.** 2.4% **C.** 3.1% **D.** 4.2%

Answer :C

Solution : Least count of screw guage $= \frac{Pitch}{No.of\ circular\ divisions}$

$$= \frac{0.5mm}{50} = \frac{1}{100} mm$$

$\therefore$ Diameter D = PSR + CSR

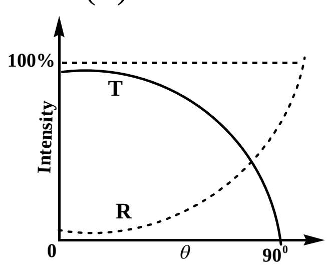
$$= 2.5 + 20 \times \frac{1}{100} = 2.7mm$$

The density $\rho = \frac{m}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}$

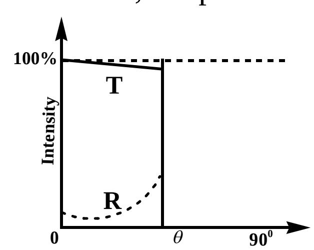
% error in density $= \left(\frac{\Delta m}{m} \times 100 + 3\frac{\Delta D}{D} \times 100\right)\%$

$$= 3.1\%$$

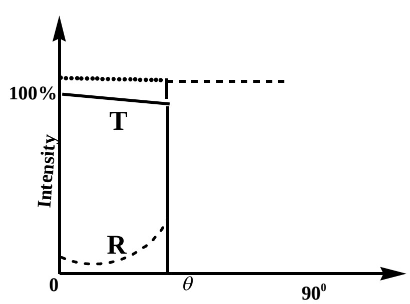
155 A light ray traveling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is



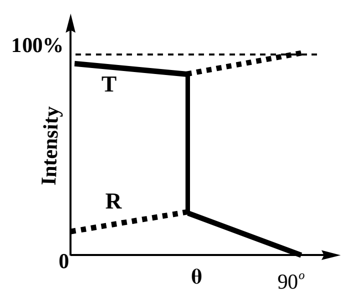
A.



B.



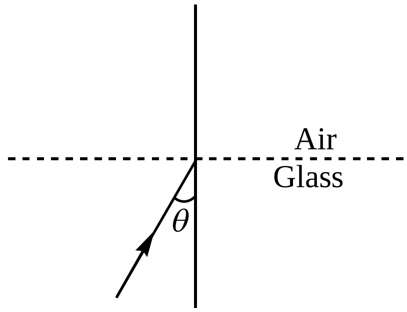
C.



D.

Answer :C

Solution : The incident ray will undergo in TIR, when $\theta \geq \theta_c$.Hence no transmission take place.



Hence for the range of θ

$0 \leq \theta < \theta_e \begin{cases} \text{Transmission will be more} \\ \text{Reflection will be little} \end{cases}$

For $\theta > \theta_e \begin{cases} \text{No transmission} \\ \text{will take place} \end{cases}$

156 5.6 litre of helium gas at STP is adiabatically compressed to 0.7 litre. Taking the initial temperature to be T_1, the work done in the process is

A. $\frac{9}{8}RT_1$ **B.** $\frac{3}{2}RT_1$ **C.** $\frac{15}{8}RT_1$ **D.** $\frac{9}{2}RT_1$

Answer :A

Solution : $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

or $T_1(5.6)^{2/3} = T_2(0.7)^{2/3} \therefore\ T_2 = 4T_1$

The work $W = \dfrac{nR(\Delta T)}{\gamma - 1}$

$W = \dfrac{nR \times (3T_1)}{2/3} = \dfrac{9}{2}nRT_1$

$P_1V_1 = nRT_1 \Rightarrow \qquad n = \dfrac{1}{4}$

$\therefore \qquad W = \dfrac{9}{8}R.T_1$

157 In a Young's double slit experiment, the separation between the slits is 1.0 mm and the distance between the slits and the screen is 1.0 m. The light falling on the slits contains mainly two wavelengths 600 nm and 500 nm. The least distance from the centre of the fringe pattern where the intensity corresponding to one of these wavelengths is zero, is

A. 0.30 mm **B.** 0.75 mm **C.** 0.25 mm **D.** 1.20 mm

Answer :C

Solution : $y = \dfrac{\beta}{2} = \dfrac{o\lambda}{2d}$

$$\frac{1 \times 500 \times 10^{-4} 10^{-6}}{2 \times 1 \times 10^{-3}} = 250 \times 10^{-6} = 0.25 \times 10^{-3} m = 0.25mm$$

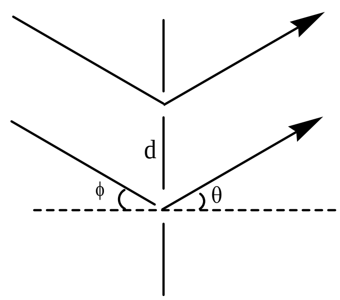
158 An ideal gas undergoes two successive process A and B. In the process A, the values of the increase ΔU in internal energy and the work W done by the gas are $\Delta U = 72J$ and $W = -72J$ respectively. For the process B, $\Delta U = 0$

A. Process A is adiabatic, process B is isochoric
B. Process A is adiabatic, process B is isothermal
C. Process A is isothermal, process B is adiabatic
D. Process A is isobaric, process B is adiabatic

Answer :B

Solution : Conceptual dc=0 adiabatic, d_4=0 isothermal

159 Light is incident at an angle ϕ with the normal to a plane containing two slits of separation d. Select the expression that correctly describe the positions of the interference maxima in terms of the incoming angle ϕ and outgoing angle θ. (m = order of maxima)

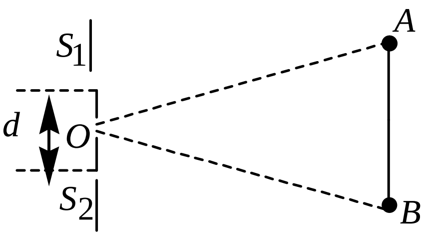


A. $\sin\phi + \sin\theta = \left(\theta > \theta_e \begin{cases}\text{Notransmission}\\ \text{will take place}\end{cases}\right)\frac{\lambda}{d}$
B. $d\sin\theta = m\lambda$
C. $\sin\phi - \sin\theta = (m+1)\frac{\lambda}{d}$
D. $\sin\phi + \sin\theta = m\frac{\lambda}{d}$

Answer :D

Solution : Path difference $= d\sin\phi + d\sin\theta$ for maxima, $\Delta x = m\lambda \Rightarrow \sin\phi + \sin\theta = \frac{m\lambda}{d}$

160 Figure shows two coherent point sources S_1 and S_2 vibrating in same phase. AB is an irregular wire lying parallel to S_1S_2 and at a far distance from the sources S_1 and S_2. Let $\frac{\lambda}{d} = 10^{-3} \angle BOA = 0.12°$. Totally, how many bright spots can be seen on the wire, including pointsA and B ?



A. 2
B. 3
C. 4
D. more than 4

Answer :B

Solution : B) Angular width $= \frac{\lambda}{d} = 10^{-3}$ (given)

No.of fringes within $0.12°$ will be

$$n = \frac{0.12 \times 2\pi}{360 \times 10^{-3}} = [2.09]$$

The number of bright spots will be three including central bright

161 Samples of two radioactive nuclides, X and Y, each have equal activity A_0 at time $t=0$. X has a half-life of 24 years and Y a half-life of 16 years. The samples are mixed together. What will be the total activity of the mixture at $t=48$ years?

A. $\frac{1}{2}A_0$ **B.** $\frac{1}{4}A_0$ **C.** $\frac{3}{16}A_0$ **D.** $\frac{3}{8}A_0$

Answer :D

Solution : Given X has activity A_0 at $t=0$ and its half-life is 24 years

Y has activity A_0 at $t=0$ and its half-life is 16 years

At $t=48$ years, activity of $X=\frac{1}{4}A_0$

At $t=16$ years, activity of $Y=\frac{1}{2}A_0$

At $t=32$ years, activity of $Y=\frac{1}{4}A_0$

At $t=48$ years, activity of $Y=\frac{1}{8}A_0$

Thus, total activity of the mixture of X and Y at $t=48$ years is $\frac{1}{4}A_0+\frac{1}{8}A_0=\frac{3}{8}A_0$

162 A $5\times10^{-4}\mathring{A}$ photon produces an electron – positron pair in the vincinity of a heavy nucleus. Rest energy of electron is $0.511MeV$. If they have the same kinetic energies, the kinetic energy of each particle is nearly

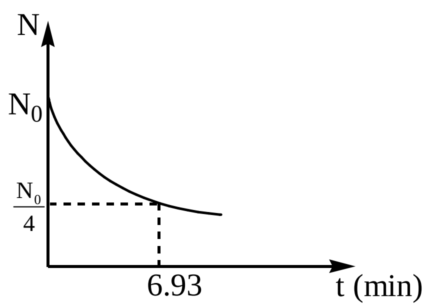
A. $1.2MeV$ **B.** $12MeV$ **C.** $120MeV$ **D.** $1200MeV$

Answer :B

Solution : If the kinetics energy of each particle is k, then

$$2k+2(0.51MeV)=\frac{hc}{\lambda}=\frac{12.4\times10^{-3}MeVA}{5\times10^{-4}A}\Rightarrow k=\frac{24.8-1.022}{2}=11.9MeV$$

163 A radioactive sample undergoes decay as per the following graph. At time $t=0$, the number of undecayed nuclei is N_0. Calculate the number of nuclei left after $1h$.



A. N_0/e^8 **B.** N_0/e^{10} **C.** N_0/e^{12} **D.** N_0/e^{14}

Answer :C

Solution : $t=0, N=N_0$ $t=6.93, N=N_0/4$

$N_0/4$ is the sample left after two half-lives

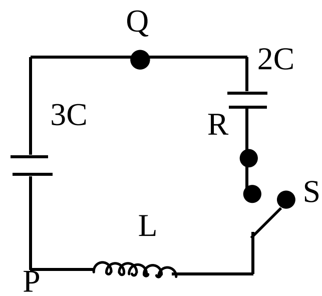
$\therefore 2t_{1/2} = 6.93$

$\Rightarrow 2 \times \dfrac{0.693}{\lambda} = 6.93 \Rightarrow \lambda = 0.2\,\text{min}^{-1}$

$\Rightarrow t = 60\,\text{min}$

$\therefore N = N_0 e^{-\lambda t} = N_0 e^{-0.2 \times 60} = \dfrac{N_0}{e^{12}}$

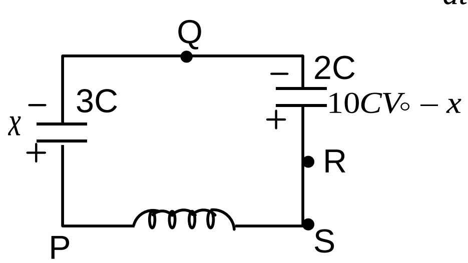
164 In the circuit shown in fig, when switch S is open, the potential difference across the capacitor is $V_P - V_Q = V_o$ *and* $V_R - V_Q = \dfrac{7V_o}{2}$. If the switch 'S' is closed, then energy in the capacitor of capacitance 3C, when the current in the inductor is maximum is



A. Zero **B.** $\dfrac{3}{2}CV_0^2$ **C.** $3CV_0^2$ **D.** $6CV_0^2$

Answer :D

Solution : For maximum current $\dfrac{di}{dt} = o$

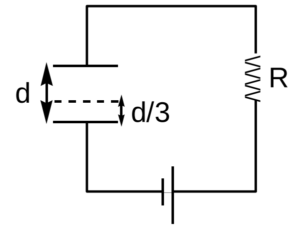


Let x be the maximum charge on 3C. *so* $\dfrac{x}{3c} = \dfrac{10CV_\circ - x}{2c}$

$\Rightarrow x = 6CV_\circ$

$\therefore$ energy $= \dfrac{x^2}{2 \times 3C} = 6CV_\circ^2$

165 A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant K = 2. The level of liquid is $\dfrac{d}{3}$ initially. Suppose the liquid level decreases at a constant speed v, the time constant as a function of time $'t'$ is



A. $\frac{6\in_o R}{5d+3vt}$ **B.** $\frac{(15d+q\ vt)\in_o R}{3d^2-3dvt-9v^2t^2}$

C. $\frac{6\in_o R}{5d-3vt}$ **D.** $\frac{(15d+q\ vt)\in_o R}{2d^2-3dvt-9v^2t^2}$

Answer :A

Solution : The liquid level falls by ϑt in time 't'. The capacitor is a combination of two capacitors connected in series with capacitances as , $C_1=\frac{K\in_\circ A}{\left(\frac{d}{3}-\vartheta t\right)}$ and $C_2=\frac{\in_\circ A}{\left(\frac{2d}{3}+\vartheta t\right)}$

Here K = 2 and A = 1 unit

$\therefore C_e=\frac{C_1C_2}{C_1+C_2}=\frac{6\in_\circ}{5d+3\vartheta t}$ $\therefore$ time constant, $\tau=CR=\frac{6\in_\circ R}{5d+3\vartheta t}$

166 A stationary hydrogen atom of mass m in the ground state achieve minimum excitation energy after head-on, inelastic collision with a moving hydrogen atom. Find the velocity of moving hydrogen atom

A. $\left[\frac{10.2(ev)}{m}\right]^{1/2}$ **B.** $\left[\frac{40.8(ev)}{m}\right]^{1/2}$ **C.** $\left[\frac{20.4(ev)}{m}\right]^{1/2}$ **D.** $\left[\frac{40.8(ev)}{1.0078\,m}\right]^{1/2}$

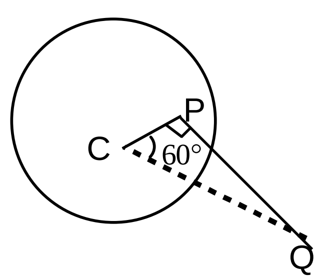
Answer :B

Solution : $mU=2m\vartheta \Rightarrow \vartheta=\frac{u}{2}$. $\Delta E=\frac{1}{2}mu^2-\frac{1}{2}.2m\vartheta^2=\frac{1}{4}mu^2$

Minimum excitation energy is to excite a hydrogen atom from ground state to first excited state $=10.2ev$

$\Rightarrow \frac{1}{4}mu^2=10.2ev \Rightarrow u=\left(\frac{40.8(ev)}{m}\right)^{\frac{1}{2}}$

167 A magnetic field, confined in a cylindrical region of radius R, is changing at the rate of $4Ts^{-1}$. A conducting rod PQ of length $\sqrt{\frac{3}{2}}R$ is placed in the region as shown. The induced emf across the rod will be

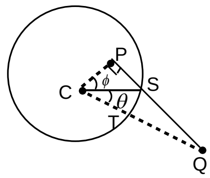


A. $\frac{R^2}{24}(\pi+6)$ B. $\frac{\pi R^2}{24}$ C. $\frac{\pi R^2}{6}$ D. $\frac{R^2}{6}(\pi+6)$

Answer :D

Solution : $\tan 60^0 = \frac{PQ}{CP}$

$\Rightarrow \sqrt{3}\ CP = \sqrt{\frac{3}{2}}\ R$



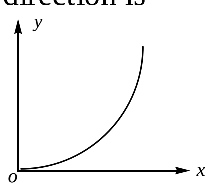
$\Rightarrow CP = \frac{R}{\sqrt{2}}\ and\ CQ = \sqrt{\frac{3R^2}{2}+\frac{R^2}{2}} = \sqrt{2}R$

$\therefore \cos\phi = \frac{CP}{CS} = \frac{R}{\sqrt{2}R} = \frac{1}{\sqrt{2}}, \phi = 45^0$

total flux , $\phi = B\left(area\ of\ \ CPS + CST\right)$

$\sum \frac{dB}{dt}\left(\frac{R^2}{4}+\frac{\pi R^2}{24}\right) = 4\left(\frac{\pi+6}{24}\right)R^2 = \frac{R^2}{6}\left(\pi+6\right)$

168 In the diagram shown, a non – uniform magnetic field $B = B_o x$ has been applied in the direction shown. A particle of mass m and charge – q is projected with a velocity 'U' from origin towards positive x-axis. The displacement of the charged particle along x-direction when its velocity becomes parallel to y – direction is



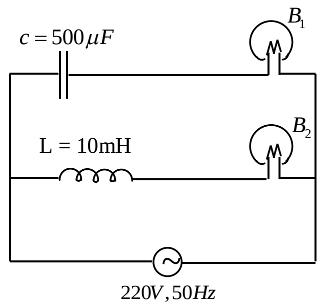
A. $\sqrt{\frac{mu}{qB_o}}$ B. $\sqrt{\frac{2mu}{qB_o}}$ C. $\sqrt{\frac{mu}{2qB_o}}$ D. $\sqrt{\frac{4mu}{qB_o}}$

Answer :B

Solution : Since the magnetic field is a linear function of x, $So\ B_{are} = B = \frac{O + B_o R}{2} = \frac{B_o R}{2}$

$\Rightarrow$ R = maximum horizontal distance $= \frac{mu}{Bq}$ $\Rightarrow R = \frac{mu}{\frac{B_o R}{2}q}$ $\Rightarrow R = \sqrt{\frac{2mu}{B_o q}}$

169 In the circuit shown in fig, if both the bulbs B_1 and B_2 are identical



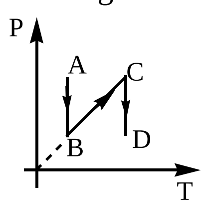
A. Their brightness will be the same
B. B_2 will be brighter than B_1
C. As frequency increases brightness of B_1 will decrease
D. Only B_2 will glow because the capacitor has infinite impedance
Answer :B
Solution : *Let* i_1 *and* i_2 be currents through B_1 and B_2 then,

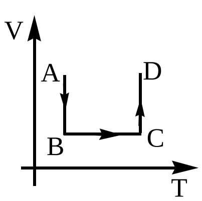
$$i_1\sqrt{R^2 + X^2L} = 220, \quad i_2\sqrt{R^2 + \left(\frac{1}{CW}\right)^2} = 220$$

$$\Rightarrow \frac{i_2}{i_1} = \frac{\sqrt{R^2 + \left(\frac{1}{CW}\right)^2}}{\sqrt{R^2 + (Lw)^2}} = \frac{\sqrt{R^2 + \left(\frac{1}{500 \times 10^{-6} \times 100\pi}\right)^2}}{\sqrt{R^2 + \left(10^{-2} \times 100\pi\right)^2}} = \frac{\sqrt{R^2 + 40}}{\sqrt{R^2 + 9.87}} \Rightarrow i_2 > i_1$$

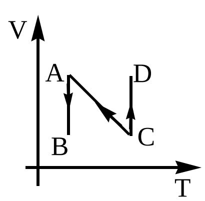
$\therefore$ Bulb B_2 will be brighter. As frequency increases, X_C decreases, X_L increases. I_2 becomes less, i_1 increases.
$\therefore$ Brightness of B_1 will increase and that of B_2 decrease.

170 P-T diagram is shown below. Then choose the corresponding V-T diagram

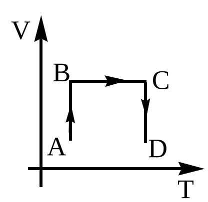




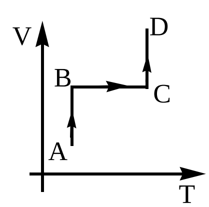
A.



B.



C.



D.

Answer :D
Solution : AB, CD are isothermal and BC is isochoric. $V_B > V_A, V_B = V_C, V_D > V_C$

171 Which of the following will have maximum total kinetic energy at temperature 300 K?

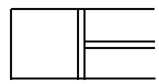
A. 1kg H_2 **B.** 1kg He **C.** $\frac{1}{2}kgH_2 + \frac{1}{2}kg\ He$ **D.** $\frac{1}{4}kg\ H_2 + \frac{3}{4}kg\ He$

Answer :A

Solution : Total kinetic energy = internal energy $U = \frac{f}{2} nRT$

In case of H_2 degrees of freedom is greatest (5) and number of moles n is highest. So, this is the case of maximum kinetic energy.

172 n moles of a gas filled in a container with a frictionless piston at temperature T is in thermodynamic equilibrium initially. If the gas is compressed slowly and isothermally to half of its initial volume, work done by the atmosphere on the piston is



A. $\frac{nRT}{2}$ **B.** $-\frac{nRT}{2}$ **C.** $nRT\left(\ln 2 - \frac{1}{2}\right)$ **D.** $-nRT \ln 2$

Answer :A

Solution : Work done by atmosphere = $P_{atm}\Delta V \ = P_{atm}\frac{V}{2}$ .....(i)

Initially, gas in container is in thermodynamic equilibrium with its surroundings.

$\therefore$ Pressure inside cylinder $= P_{atm}$ from $PV = nRT$

$\Rightarrow P_{atm}V = nRT$ or $V = \frac{nRT}{P_{atm}}$

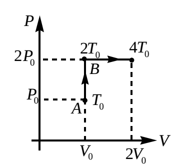
Putting in (i), $W = \frac{nRT}{2}$

173 A diatomic ideal gas is heated at constant volume until the pressure is doubled and then heated at constant pressure until volume is doubled. The average molar heat capacity for whole process is

A. $\frac{13}{6}R$ **B.** $\frac{19}{6}R$ **C.** $\frac{23}{6}R$ **D.** $\frac{17}{6}R$

Answer :B

Solution : Let intial presure, volume, temperature be P_0, V_0, T_0 indicated by state A in P-V diagram. The gas is then isochorically taken to state $B(2P_0, V_0 2T_0)$ and then taken from state B to state $C(2P_0, 2V_0, 4T_0)$ isobarically.



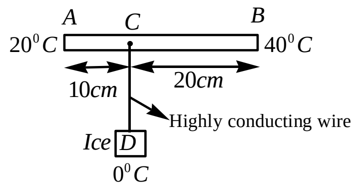
Total heat absorbed by 1 mole of gas

$$\Delta Q = C_v(2T_0 - T_0) + C_P(4T_0 - 2T_0) \ = \frac{5}{2}RT_0 + \frac{7}{2}R \times 2T_0 = \frac{19}{2}RT_0$$

Total change in temperature from state A to C is $\Delta T = 3T_0$

$$\therefore \text{ Molar heat capacity } = \frac{\Delta Q}{\Delta T} = \frac{\frac{19}{2}RT_0}{3T_0} = \frac{19}{6}R$$

174 In the figure shown, AB is a rod of length 30 cm and area of cross-section $1.0 cm^2$ and thermal conductivity 336 SI units. The ends A and B are maintained at temperatures 20^0C and 40^0C respectively. A point C of this rod is connected to a box D, containing ice at 0^0C, through a highly conducting wire of negligible heat capacity. The rate at which ice melts in the box is [Assume latent heat of fusion for ice, $L_f = 80\,\text{cal}.g^{-1}$]



A. $84\,mg.s^{-1}$ **B.** $84\,g.s^{-1}$ **C.** $20\,mg.s^{-1}$ **D.** $40\,mg.s^{-1}$

Answer :D

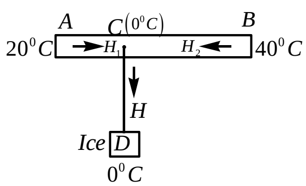
Solution : Thermal resistance of $AC = \dfrac{L}{KA}$

$= \dfrac{0.1}{336 \times 1 \times 10^{-4}} = \dfrac{10^{-3}}{336} = R(\text{suppose})$

Thermal resistance of $BC = \dfrac{0.2}{336 \times 10^{-4}} = 2R$

Temperature of $C = 0^0C$

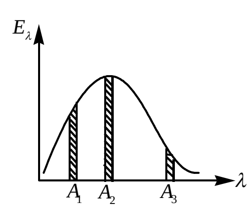
$\therefore H_1 = \dfrac{20}{R}; H_2 = \dfrac{40}{2R} = \dfrac{20}{R}$



$\therefore H = H_1 + H_2 = \dfrac{40}{R} = \dfrac{40 \times 336}{10^3} = \dfrac{13440}{10^3} = 13.44W$

Rate of melting of ice $= \dfrac{H}{L_f} = \dfrac{13.44/4.2}{80} g.s^{-1} = 40mg.s^{-1}$

175 Three separate segments of equal area A_1, A_2 and A_3 are shown in the energy distribution curve of a blackbody radiation. If n_1, n_2 and n_3 are number of photons emitted per unit time corresponding to each area segment respectively, then



A. $n_2 > n_1 > n_3$ **B.** $n_3 > n_1 > n_2$ **C.** $n_1 = n_2 = n_3$ **D.** $n_3 > n_2 > n_1$

Answer :D

Solution : Area $A = E_\lambda (d\lambda)$

But $E_\lambda d\lambda = \frac{dQ}{dt} \times \frac{1}{a}$

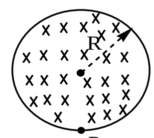
a=surface area of body

$A = \frac{(dn)hc/\lambda}{dt} \times \frac{1}{a}$

$\frac{dn}{dt} = \frac{A\lambda a}{hc} \Rightarrow \frac{dn}{dt} \alpha \lambda$

$\lambda_3 > \lambda_2 > \lambda_1 \Rightarrow n_3 > n_2 > n_1$

176 A uniform magnetic field of induction B is confined to a cylindrical region of radius R. The magnetic field is increasing at a constant rate of $\frac{dB}{dt}$ (tesla/second). An electron of change q, placed at the point P on the periphery of the field experiences an acceleration:



A. $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ toward left

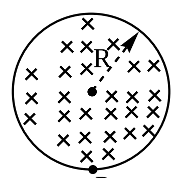
B. $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ toward right

C. $\frac{eR}{m}\frac{dB}{dt}$ toward left

D. Zero

Answer :A

Solution : If we consider the cylindrical surface to be a ring of radius R, there will be an induced emf due to changing field.



$\int \overline{E}.\overrightarrow{dl} = \frac{d\phi}{dt} = -A\frac{dB}{dt}$

$\Rightarrow E(2\pi R) \qquad = -A\frac{dB}{dt} = -\pi R^2 \frac{dB}{dt} \qquad \Rightarrow E = \frac{R}{2}\frac{dB}{dt}$

$\therefore$ Force on the electron.

$F = -Ee = -\frac{eR}{2}\frac{dB}{dt} \qquad = \frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$

As the field is increasing being directed inside the paper, hence there will be anticlockwise induced current (in order to oppose the cause) in the ring (assumed). Hence there will be a force towards left on the electron.

177 One of the lines in the emission spectrum of Li^{2+} has the same wavelength as that of the 2^{nd} line of Balmer series in hydrogen spectrum. The electronic transition corresponding to this line is:

A. $n = 4 \to n = 2$ **B.** $n = 8 \to n = 2$ **C.** $n = 8 \to n = 4$ **D.** $n = 12 \to n = 6$

Answer :D

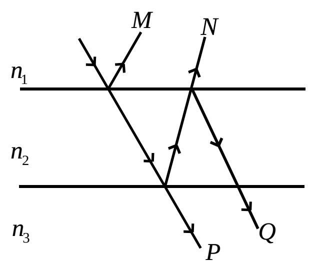
Solution : For 2^{nd} line of Balmer seires in hydrogen specturm

$$\frac{1}{\lambda} = R(1)\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3}{16}R$$

For Li^{2+} :

Which is satisfied by only (D).

178 A ray of light is incident on a thin film. As shown in figure M,N are two reflected rays and P, Q are two transmitted rays. Rays N and Q undergo a phase change of π. Correct ordering of the refracting indices is:



A. $n_2 > n_3 > n_1$

B. $n_3 > n_2 > n_1$

C. $n_3 > n_1 > n_2$

D. None of these, the specified changes can not occur

Answer :B

Solution : Ray N undergoes reflection at surface II with phase change of π

$\Rightarrow\ n_3 > n_2$

Ray Q undergoes a phase-change of π at II, but there is no phase change when it is reflected from surface I.

$\Rightarrow\ n_1 < n_2$

179 In which of the following process the number of protons in the nucleus increases.

A. α - decay **B.** β^- - decay **C.** β^+ - decay **D.** k – capture

Answer :B

Solution : For α - decay : ${}_xA^y \rightarrow {}_{x-2}B^{y-4} + \alpha$

For β^- – decay : ${}_xA^y \rightarrow {}_{x+1}B^y + {}_{+1}\beta^0$

For β^+ – decay : ${}_xA^y \rightarrow {}_{x-1}B^y + {}_{+1}\beta^0$

For k – capture : there will be on change in the number of protons.

Hence, only case in which no of protons increases is β^- – decay Hence (B).