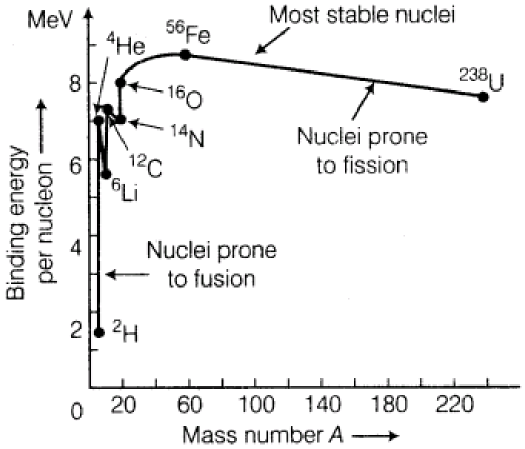
Mass Defect & Binding Energy

2 Marks Questions

1.Using the curve for the binding energy per nucleon as a function of mass number A, state clearly how the release in energy in the processes of nuclear fission and nuclear fusion can be explained. [All India 2011]

Ans.

The binding energy per nucleon curve is shown as below:



Binding energy per nucleon as a function of mass number A **(1)**

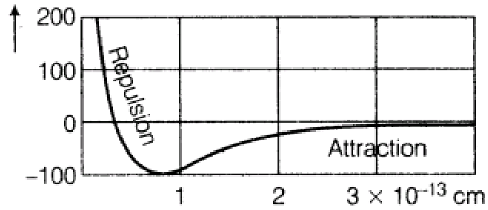
Explanation of Release of Energy in Nuclear Fission and Fusion The curve reveals that binding energy per nucleon is smaller for heavier nuclei than the middle level nuclei. This shows that heavier nuclei are less stable than middle level nuclei. In nuclear fission, binding energy per nucleon of reactants (heavier nuclei) changes from nearly 7.6 MeV to 8.4 MeV (for nuclei of middle level mass).Higher value of the binding energy of the nuclear product results in the liberation of energy during the phenomena of nuclear fission

In nuclear fusion, binding energy per nucleon of lighter nuclei into heavier one changes from low value of binding energy per nucleon to high value and release of energy takes place in fusion e.g., two $_1H^2$ (Binding energy per nucleon $\simeq$ 1.5 MeV/nucleon) combine to form $_2He^4$ (Binding energy per nucleon $\simeq$ 7 MeV/nucleon) and therefore the energy is liberated during nuclear fusion. **(1/2)**

2.Draw a plot of potential energy of a pair of nucleons as a function of their separation. Write two important conclusions which you can draw regarding the nature of nuclear forces. [All India 2010]

Ans.

Graph manifests that



Graph between the potential energy of a pair of nucleons as a function of

(1)

The conclusions drawn from the graph are given as below:

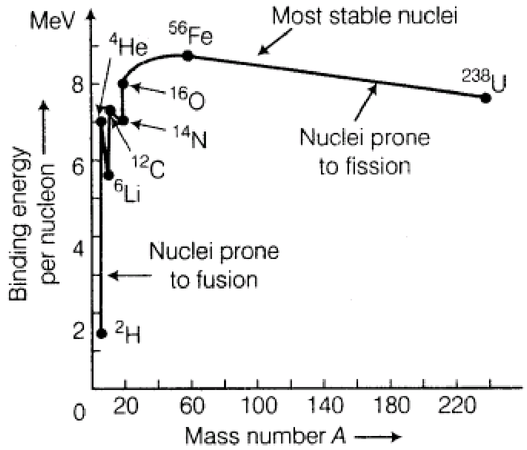
(*i*) Nuclear force is a short range force.

(*ii*) Nuclear force is of attractive nature when separation between the nuclei greater than 1 fm and of repulsive nature when separation is less than 1 fm. **(1)**

3.Draw a plot of the binding energy per nucleon as a function of mass number for a large number of nuclei 20 > A > 240 . How do you explain the constancy of binding energy per nucleon in the range of 30<A<170 using the property that nuclear force is short-ranged? [All India 2010]

Ans.

The binding energy per nucleon curve is shown as below:



Binding energy per nucleon as a function of mass number A

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Explanation of Release of Energy in Nuclear Fission and Fusion The curve reveals that binding energy per nucleon is smaller for heavier nuclei than the middle level nuclei. This shows that heavier nuclei are less stable than middle level nuclei. In nuclear fission, binding energy per nucleon of reactants (heavier nuclei) changes from nearly 7.6 MeV to 8.4 MeV (for nuclei of middle level mass).Higher value of the binding energy of the nuclear product results in the liberation of energy during the phenomena of nuclear fission

In nuclear fusion, binding energy per nucleon of lighter nuclei into heavier one changes from low value of binding energy per nucleon to high value and release of energy takes place in fusion e.g., two ${}_1H^2$ (Binding energy per nucleon ≃ 1.5 MeV/nucleon) combine to form ${}_2He^4$ (Binding energy per nucleon ≃ 7 MeV/nucleon) and therefore the energy is liberated during nuclear fusion. **(1/2)**

The binding energy per nucleon in the range of 30 < A < 170 has average binding energy per nucleon = 8.5 MeV. The higher value of binding energy per nucleon is due to stability of these nucleons. Neutron-proton ratio is higher in this range of mass number which leads to stability of the nuclei. Also, the nuclear force is strongly attractive enough to overcome the coulombian repulsive force acting between positively charged protons,

4.A heavy nucleus X of mass number 240 and binding energy per nucleon 7.6 MeV is splitted into two fragments Y and Z of mass numbers 110 and 130. The binding energy of nucleons in Y and Z is 8.5 MeV per nucleon. Calculate the energy released per fission in MeV. [hots; Delhi 2010]

Ans.

In these types of questions, we have to keep in mind about difference of mass involved between reactants and product. Energy will be involved accordingly.

Energy released per fission

$= (110 + 130) \times 8.5 \text{ MeV} - 240 \times 7.6 \text{ MeV}$ **(1)**

$= 240 \times (8.5 - 7.6) \text{ MeV}$

$= 240 \times 0.9 = 216.0 \text{ MeV}$ **(1)**

5. If both the numbers of protons and neutrons are conserved in a nuclear reaction like

$$_6C^{12} + _6C^{12} \longrightarrow _{10}N^{20} + _2He^4$$

In what way, is the mass converted into the energy? Explain. **[Delhi 2010]**

Ans.

The sum of masses of nuclei of product element is less than the sum of masses of reactants and hence, loss of mass takes place during the reaction. This difference of mass of product element and reactant converts into energy and liberated in the form of heat. **(1)**

Here, sum of masses of $_{10}Ne^{20}$ and $_2He^4$ is less than the sum of two $_6C^{12}$ and conversion of this mass defect is used to produce energy. **(1)**

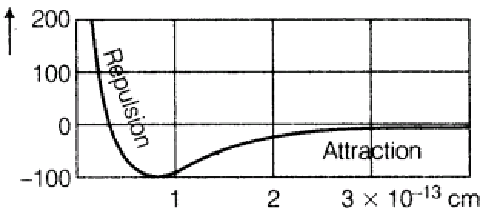
6.(i) The mass of a nucleus in its ground state is always less than the total mass of its constituents neutrons and protons. Explain, (ii) Plot a graph showing the variation of potential energy of a pair of nucleons as a function of their separation. [All India 2009]

Ans.

(*i*) Mass defect occurs in nucleus which converts into energy as per Einstein's mass-energy relation, $E = mc^2$ and produces binding energy. This energy binds nucleons together due to nuclear forces in spite of repulsive coulombian forces. **(1)**

(ii)

Graph manifests that



Graph between the potential energy of a pair of nucleons as a function of

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The conclusions drawn from the graph are given as below :

(*i*) Nuclear force is a short range force.

(*ii*) Nuclear force is of attractive nature when separation between the nuclei greater than 1 fm and of repulsive nature when separation is less than 1 fm. **(1)**

7. Calculate the energy released in the following nuclear reaction:

$${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He + Q$$

[Mass of ${}^{238}_{92}U = 238.05079$ u

Mass of ${}^{234}_{90}Th = 234.043630$ u

Mass of ${}^{4}_{2}He = 4.002600$ u

$1u = 931.5$ MeV] **[All India 2008]**

Ans.

Sum of masses of ${}^{234}_{90}Th$ and ${}^{4}_{2}He$

$= 234.043630 + 4.002600$

$= 238.046230$ u

Mass of ${}^{238}_{92}U = 238.05079$ u **(1)**

$\therefore$ Loss of mass (mass defect) in given nuclear reaction $= 238.05078 - 238.046230$

$= 0.00456$ u

$\therefore$ Energy released in nuclear reaction

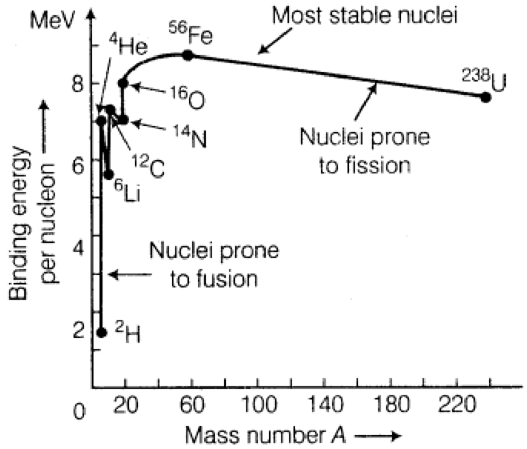
$= 0.00456 \times 931.5$ MeV

$= 4.24764$ MeV **(1)**

8.Sketch a graph showing the variation of binding energy per nucleon as a function of mass number Afor large number of nuclei. State briefly from which region of the graph can release of energy in the process of nuclear fusion be explained.[Foreign 2008]

Ans.

The binding energy per nucleon curve is shown as below:



Binding energy per nucleon as a function of mass number A **(1)**

Explanation of Release of Energy in Nuclear Fission and Fusion The curve reveals that binding energy per nucleon is smaller for heavier nuclei than the middle level nuclei. This shows that heavier nuclei are less stable than middle level nuclei. In nuclear fission, binding energy per nucleon of reactants (heavier nuclei) changes from nearly 7.6 MeV to 8.4 MeV (for nuclei of middle level mass).Higher value of the binding energy of the nuclear product results in the liberation of energy during the phenomena of nuclear fission

In nuclear fusion, binding energy per nucleon of lighter nuclei into heavier one changes from low value of binding energy per nucleon to high value and release of energy takes place in fusion e.g., two $_1H^2$ (Binding energy per nucleon $\simeq$ 1.5 MeV/nucleon) combine to form $_2He^4$ (Binding energy per nucleon $\simeq$ 7 MeV/nucleon) and therefore the energy is liberated during nuclear fusion. **(1/2)**

In the range of mass number, 2 to 20, these are maximum and minimum as the curve. Here $^4_2$He, $_6C^{12}$ and $^{16}_8$O are at maxima and $_1H^2$, Li,N are at minima. This range of mass number may facilitate release of energy in nuclear fusion, e.g. two $_1H^2$ nuclei of low binding energy when combined in a nuclear fusion to form $_2He^4$ of high value of binding energy per nucleon. In this process, energy will release in the form of heat. **(2)**

9. Why is it necessary to slow down the neutrons produced through the fission of $^{235}_{92}$U nuclei

(by neutrons) to sustain a chain reaction? What type of nuclei are (preferably) needed for slowing down fast neutrons? **[All India 2008C]**

Ans.

Average kinetic energy of neutrons produced in nuclear fission of ${}_{92}U^{235}$ is nearly 2 MeV, whereas the chances of absorption of neutrons of average kinetic energy of nearly 0.024 MeV is high by uranium nuclei. So, there is a need to slow down

the fast neutrons using appropriate substance namely moderator into slow thermal neutrons.Nuclei which have comparable mass to that of neutrons should be preferable be used to slow down fast neutrons. It is due to the fact that the elastic collision between fast neutrons and slow moving protons lead to interchange the velocities

10. (*i*) Write the relation for Binding Energy (BE) (in MeV) of a nucleus of mass ${}_{Z}^{A}M$, atomic number (Z) and mass number (*A*) in terms of the masses of its constituents namely neutrons and protons.

(*ii*) Draw a plot of BE/*A versus* mass number *A* for $2 \leq A \leq 170$. Use this graph to explain the release of energy in the process of nuclear fusion of two light nuclei. **[Delhi 2014]**

Ans.

(*i*) $BE = [Zm_p + (A - Z)m_n - {}_{Z}^{A}M] \times c^2$

where, M is mass of nucleus, m_p is the mass of proton and m_n is the mass of neutron.

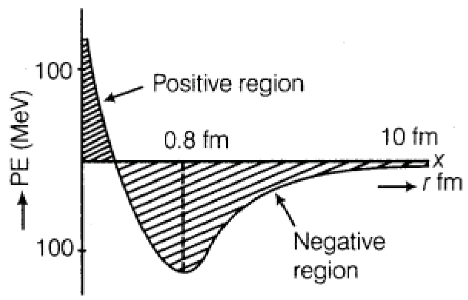
11.Draw a plot of potential energy between a pair of nucleons as a function of their separation. Mark the regions where potential energy is

(i)positive and

(ii)negative.[Delhi 2013]

Ans.

Plot between the potential energy of a pair of nucleons as a function of their separatio**(1)**



(*i*) For distance less than 0.8 fm, negative PE decreases to zero and then becomes positive.

12.Answer the following.

(i)Why is the binding energy per nucleon found to be constant for nuclei in the range of mass number (A) lying between 30 and 170?

(ii)When a heavy nucleus with mass number A = 240 breaks into two nuclei, A = 120, energy is released in the process. [Foreign 2012]

Ans.

(i) The binding energy per nucleon for nucleus of range, $30 < A < 170$ is close to its maximum value. So, the nucleus belongs to this region is highly stable and does not show radioactivity. **(1)**

(ii) Binding energy per nucleon is smaller for heavier nuclei than the middle ones, i.e. heavier nuclei are less stable. When a heavier nucleus such as nucleus of mass number 240 splits into lighter nuclei (mass number 120), the BE/nucleon changes from about 7.6 MeV to 8.4 MeV. Greater BE of the product nuclei result in the liberation of energy. **(2)**

13. (i) In a typical nuclear reaction, e.g.

$${}_1^2\text{H} + {}_1^2\text{H} \longrightarrow {}_2^3\text{He} + n + 3.27$$

although number of nucleons is conserved yet energy is released. How? Explain.

(ii) Show that nuclear density in a given nucleus is independent of mass number A.

Ans.

(i) In a nuclear reaction, the sum of the masses of the target nucleus (${}_1^2$H) and the bombarding particle (${}_1^2$H) may be greater or less than the sum of the masses of the product nucleus (${}_1^3$He) and the outgoing particle (${}_0^1n$). So, from the law of conservation of mass-energy, some energy (3.27 MeV) is evolved or involved in a nuclear reaction. This energy is called Q-value of the nuclear reaction. **(1)**

(ii) Density of nuclear matter is the ratio of mass of the nucleus and its volume.

Density of the nuclear matter

$$= \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} \quad \text{...(i)}$$

If m is average mass of a nucleon and R is the nuclear radius, then mass of nucleus $= mA$, where A is the mass number of the element.

$$\text{Volume of the nucleus} = \frac{4}{3}\pi R^3$$

$$= \frac{4}{3}(\pi R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

$$\text{Thus, density of nucleus} = \frac{mA}{\frac{4}{3}\pi R_0^3 A}$$

where, m = mass of one nucleon

A = mass

$$\text{Number} = \frac{3m}{4\pi R_0^3} \quad \textbf{(1)}$$

As, m and R_0 are constants, therefore density of the nuclear matter is the same for all elements. Now, using $m = 1.66 \times 10^{-27}$ kg.

Substituting the value of Eq. (ii) and Eq. (iii) in Eq. (i), we get

$$= \frac{A \times 1.66 \times 10^{-27}}{(\frac{4}{3}\pi R_0^3)A}$$

$$= \frac{1.66 \times 10^{-27}}{(\frac{4}{3}\pi R_0^3)}$$

which shows that the density is independent of mass number A.

Using $R_0 = 1.1 \times 10^{-15}$ m and density

$= 2.97 \times 10^{17}$ kg m^{-3}

14.(i)What characteristic property of nuclear force explains the constancy of binding energy per nucleon (BE/A) in the range of mass number A lying 30 < A < 170?

(ii) Show that the density of nucleus over a wide range of nuclei is constant and independent of mass number A.[Delhi 2012]

Ans.(i)The saturation effect of nuclear force explains the constancy of BE/A over wide range of mass number, 70 > A > 30. Saturation effect imply that nucleon interacts only with its neighbouring nucleons and does not interact with nucleons which are not in direct contact with it.

(*ii*) Density of nuclear matter is the ratio of mass of the nucleus and its volume.

Density of the nuclear matter

$$= \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} \quad \ldots\text{(i)}$$

If m is average mass of a nucleon and R is the nuclear radius, then mass of nucleus = mA, where A is the mass number of the element.

$$\text{Volume of the nucleus} = \frac{4}{3}\pi R^3$$

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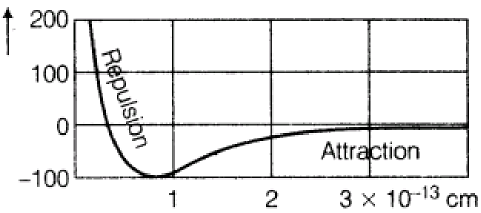
Using $R_0 = 1.1 \times 10^{-15}$ m and density

$= 2.97 \times 10^{17}$kg m^{-3}

15.Draw a plot of potential energy of a pair of nucleons as a function of their separations. Mark the regions where the nuclear force is (i) attractive and (ii) repulsive. Write any two characteristic features of nuclear forces. [All India 2012]

Ans.

Graph manifests that



Graph between the potential energy of a pair of nucleons as a function of

(1)

The conclusions drawn from the graph are given as below :

(*i*) Nuclear force is a short range force.

(*ii*) Nuclear force is of attractive nature when separation between the nuclei greater than 1 fm and of repulsive nature when separation is less than 1 fm. **(1)**

Net interactive force is zero when PE is minimum, i.e. nearly, $r_0 = 1$ fm (in graph).

(*i*) The nuclear force is attractive when separation between the nuclei is greater than $r_0 > 1$ fm. **(1)**

(*ii*) Repulsive when $r_0 < 1$ fm. **(1)**

16.Explain giving necessary reactions, how energy is released during (i)fission (ii) fusion [All India 2011 c]

Ans.

(*i*) **Nuclear Fission** The phenomenon of splitting of heavy nuclei (mass number > 120) into smaller nuclei of nearly equal masses is known as nuclear fission.

In nuclear fission, the sum of the masses of the product is less than the sum of masses of the reactants. This difference of mass gets converted into energy $E = mc^2$ and hence sample amount of energy is released in a nuclear fission.

e.g. ${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n + Q$

Masses of reactant

$$= 235.0439 \text{ amu} + 1.0087 \text{ amu}$$
$$= 236.0526 \text{ amu}$$

Masses of product

$$= 140.9139 + 91.8973 + 3.0261$$
$$= 235.8373 \text{ amu}$$

Mass defect $= 236.0526 - 235.8373$

$$= 0.2153 \text{ amu}$$

$\because$ 1 amu $\equiv$ 931 MeV

$\Rightarrow$ Energy released $= 0.2153 \times 931$

$$= 200 \text{ MeV nearly}$$

Thus, energy is liberated in nuclear fission if ${}^{235}_{92}U$. **(1)**

(*ii*) **Nuclear Fusion** The phenomenon of conversion of two lighter nuclei into a single heavy nucleus is called nuclear fusion. **(1/2)**

Since, the mass of the heavier product nucleus is less than the sum of masses of reactant nuclei and therefore certain mass defect occurs which converts into energy as per Einstein's mass-energy relation. Thus, energy is released during nuclear fusion.

e.g.
${}_1H^1 + {}_1H^1 \longrightarrow {}_1H^2 + e^+ + \nu + 0.42 \text{ MeV}$

Also, ${}_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + {}_1H^1 + 4.03 \text{ MeV}$

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5 Marks Questions

17.(i) Draw the plot of binding energy per nucleon (BE/A) as a function of mass number A.

Write two important conclusions that Can be drawn regarding the nature of nuclear force.
(ii)Use this graph to explain the release of energy in both the processes of nuclear fusion and fission.

(*iii*) Write the basic nuclear process of neutron undergoing β-decay. Why is the detection of neutrinos found very difficult? **[HOTS; All India 2013]**

Ans.

While drawing the plot, we have to keep in mind that first binding energy will increase sharply and then it will be constant almost.

(*i*) For plot of binding energy per nucleon as the function of mass number *A*
Refer to ans 1. **(1)**

Following are the two conclusions that can be drawn regarding the nature of the nuclear force.

(a) The force is attractive and strong enough to produce a binding energy of few MeV per nucleon. **(1/2)**

(b) The constancy of the binding energy in the range of $30 < A < 170$ is a consequence of the fact that the nuclear force is short range force. **(1/2)**

(*ii*) **Nuclear Fission** A very heavy nucleus (say $A = 240$) has lower binding energy per nucleon as compared to the nucleus with $A = 120$. Thus, if the heavier nucleus breaks into the lighter nucleus with high binding energy per nucleon, nucleons are tightly bound. This implies that energy will be released in the process which justifies the energy released in fission reaction

Nuclear Fusion When two light nuclei (A < 10) are combined to form a heavier nuclei, the binding energy of the fused heavier nuclei is more that the binding energy per nucleon of the lighter nuclei. Thus, the final system is more tightly bound than the initial system. Again the energy will be released in fusion reaction.

(*iii*) The basic nuclear process of neutron undergoing β-decay is given as below:

$$n \rightarrow p + e^{-} + \bar{\nu}$$

Neutrinos interact very weakly with matter so, they have a very high penetrating power. That's why the detection of neutrinos is found very difficult. **(1)**

18.Define the O-value of a nuclear process. When can a nuclear process not proceed spontaneously? If both the number of protons and the number of neutrons are conserved in a nuclear reaction in what way is mass converted into energy (or vice-versa) in the nuclear reaction? [All India 2010 c]
Ans.

The Q-value of a nuclear process refers the energy release in the nuclear process which can be determined using Einstein's mass-energy relation, $E = mc^2$. The Q-value is equal to the difference of mass of products and reactant nuclei multiplied by square of velocity of light. (2)

The nuclear process does not proceed spontaneously when Q-value of a process is negative or sum of masses of product in greater than sum of masses of reactant.

19.Draw a plot of binding energy per nucleon (BE/A) versus mass number (A) for a large number of nuclei lying between 2 < A < 240. Using this graph, explain clearly how the energy is released in both the process of nuclear fission and fusion? [All India 2009 C]

Ans.

While drawing the plot, we have to keep in mind that first binding energy will increase sharply and then it will be constant almost.

(i) For plot of binding energy per nucleon as the function of mass number A

Refer to ans 1. **(1)**

Following are the two conclusions that can be drawn regarding the nature of the nuclear force.

(a) The force is attractive and strong enough to produce a binding energy of few MeV per nucleon. **(1/2)**

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