

Chapterwise Practise Problems (CPP) for JEE (Main & Advanced)

Chapter - Trigonometric Functions

Level-1

SECTION - A

Straight Objective Type

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

- If $\sin \theta + \sqrt{3} \cos \theta = 6x - x^2 - 11$, $0 \leq \theta \leq 2\pi$, $x \in \mathbb{R}$ holds for
(A) no values of x and θ
(B) one value of x and one value of θ
(C) two values of x and two values of θ
(D) two pairs of values of (x, θ)
- If $\sin \theta = 3\sin(\theta + 2\alpha)$, then the value of $\tan(\theta + \alpha) + 2\tan \alpha$ is
(A) 3 (B) 2
(C) 1 (D) 0
- If $\sin \alpha + \sin \beta = a$, $\cos \alpha + \cos \beta = b$, then $\tan \frac{\alpha - \beta}{2}$ is equal to
(A) $\sqrt{\frac{4 - a^2 - b^2}{a + b^2}}$ (B) $\pm \sqrt{\frac{4 - a^2 - b^2}{a^2 + b^2}}$
(C) $-\sqrt{\frac{4 - a^2 - b^2}{a^2 + b^2}}$ (D) none of these
- $\frac{1}{\sin 3\alpha} \left[\sin^3 \alpha + \sin^3 \left(\frac{2\pi}{3} + \alpha \right) + \sin^3 \left(\frac{4\pi}{3} + \alpha \right) \right]$ is equal to
(A) $\frac{4}{3}$ (B) $\frac{3}{4}$
(C) $-\frac{3}{4}$ (D) none of these
- The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$, where x is a variable, has real roots. Then the interval of p may be
(A) $(0, 2\pi)$ (B) $(-\pi, 0)$
(C) $\left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$ (D) $(0, \pi)$
- The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ ($0 \leq x \leq 2\pi$) is
(A) 0 (B) 1
(C) 2 (D) 3
- The number of solutions of the equation $\sin \left(\frac{\pi x}{2\sqrt{3}} \right) = x^2 - 2\sqrt{3} \cdot x + 4$
(A) Forms an empty set
(B) is only one
(C) is only two
(D) is greater than 2
- The set of all x in $\left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$ satisfying $|4\sin x - 1| < \sqrt{5}$ is given by
(A) $\left(-\frac{\pi}{10}, \frac{3\pi}{10} \right)$ (B) $\left(\frac{\pi}{10}, \frac{3\pi}{10} \right)$
(C) $\left(\frac{\pi}{10}, -\frac{3\pi}{10} \right)$ (D) none of these
- The values of a for which the equation $\sqrt{a} \sin x - 2 \cos x = \sqrt{2} + \sqrt{2 - a}$ has solutions are
(A) $a > 0$ (B) $a \leq 3$
(C) $0 \leq a \leq 2$ (D) $\sqrt{5} - 1 \leq a \leq 2$
- The minimum value of $27^{\cos 2x} \cdot 81^{\sin 2x}$ is
(A) 1 (B) 1/9
(C) 1/81 (D) 1/243

11. In $[-2\pi, 2\pi]$ number of root(s) of $5^{\sec^2 x} = 1 + 5^{\tan^2 x}$ is

- (A) 0 (B) 2
(C) 4 (D) 8

12. Identify the correct combination of True (T) and False(F) of the given three statements

STATEMENT-1 : If $\sin x = a$, where $a \in (-1, 1)$ then we get exactly two values of x in $[0, 2\pi]$

STATEMENT-2 : $\tan x + \cot x$ cannot be equal to 1

STATEMENT-3 : $|x - 3|^2 - 4|x - 3| - 5 = 0$ has 2 real roots.

- (A) F, F, F (B) F, T, T
(C) F, F, T (D) T, F, F

13. The maximum number of solutions of the equation $\cos^8 x - \tan^8 x = 1$, in $[0, 2\pi]$ is

- (A) 1 (B) 3
(C) 2 (D) 4

SECTION - B

Multiple Correct Answer Type

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

14. Which of the following assertions about $\tan 10^\circ$ is/are true

- (A) it is an irrational number
(B) it is less than 2
(C) it is a rational number
(D) it is greater than 2

15. $\sqrt{\cos 2x} + \sqrt{1 + \sin 2x} = 2\sqrt{\sin x + \cos x}$, if

- (A) $\sin x + \cos x = 0$
(B) $x = 2n\pi, n \in \mathbb{I}$
(C) $x = n\pi - \frac{\pi}{4}, n \in \mathbb{I}$
(D) $x = 2n\pi \pm \cos^{-1}\left(-\frac{1}{5}\right), n \in \mathbb{I}$

16. If $|\cos x|^{\sin^2 x - \frac{3}{2}\sin x + \frac{1}{2}} = 1$, then possible values of x are given by

(A) $\frac{n\pi}{3}$ or $n\pi + (-1)^n \frac{\pi}{6}; n \in \mathbb{I}$

(B) $n\pi$ or $2n\pi + \frac{\pi}{2}$ or $n\pi + (-1)^n \frac{\pi}{6}; n \in \mathbb{I}$

(C) $n\pi + (-1)^n \frac{\pi}{6}; n \in \mathbb{I}$

(D) $n\pi; n \in \mathbb{I}$

17. Let $k = \frac{\cot \theta}{\cot 3\theta}, (k \neq 1), \theta \neq n\pi, \frac{n\pi}{3}$, where $n \in \mathbb{Z}$, then

(A) $\frac{\cos \theta}{\cos 3\theta} = \frac{k-1}{2}$ (B) $\frac{\sin 3\theta}{\sin \theta} = \frac{2k}{k-1}$

(C) $k \notin \left[\frac{1}{3}, 3\right]$ (D) $k < \frac{1}{3}$ or $k > 3$

18. Which of the following is/are true?

(A) The maximum value of $2\sin^2 \theta + 3\cos^2 \theta + 4\sin \theta \cos \theta$ is $\frac{5 + \sqrt{17}}{2}$

(B) If $6\sin x + 8\cos x = k$ then total number of possible integral values of k is 21

(C) $\tan 2^\circ > \tan 7^\circ$

(D) $\tan 4^\circ \tan 56^\circ \tan 48^\circ \tan 72^\circ \tan 64^\circ$ is less than 1

19. If $f(x) = \sin^8 x + \cos^8 x$ then

(A) Maximum value of $f(x)$ is 1

(B) Maximum value of $f(x)$ is $\frac{4}{3}$

(C) Minimum value of $f(x)$ is $\frac{1}{8}$

(D) Minimum value of $f(x)$ is $\frac{1}{16}$

20. The solution to equation $|\cos x| = \sin x$ may be

(A) $x = \frac{(8k+1)\pi}{4}$, where $k \in \mathbb{I}$

(B) $x = \frac{(8k+3)\pi}{4}$, where $k \in \mathbb{I}$

(C) $x = n\pi + \frac{\pi}{4}$, where n is odd positive integer

(D) $x = 2n\pi - \frac{\pi}{4}$, where $n \in \mathbb{N}$

21. $10 \cos \theta + 6 \cos \left(\theta + \frac{\pi}{3} \right) + 3 = k$ then possible integral values of k may be
 (A) -10 (B) 17
 (C) 15 (D) 14
22. Let $f(\theta) = 3(\sin \theta - \cos \theta)^4 + 6(\cos \theta + \sin \theta)^2 + 4(\sin^6 \theta + \cos^6 \theta)$ then
 (A) $f\left(\frac{2\pi}{3}\right) = 13$ (B) $f\left(\frac{3\pi}{4}\right) = 13$
 (C) $f(2009) = 13$ (D) $f(2010) = 13$
23. The interval in which λ may lie so that the solutions of the equation $\sin^2 \theta - 2 \sin \theta + \lambda = 0$ lie in $\left(2n\pi + \frac{\pi}{6}, (2n+1)\pi - \frac{\pi}{6} \right), n \in \mathbb{Z}$ is
 (A) $\left[\frac{3}{4}, 1 \right]$ (B) $\left[\frac{4}{5}, 1 \right]$
 (C) $\left[\frac{9}{10}, 1 \right]$ (D) $\left[\frac{3}{4}, 1 \right]$
24. If $\sin x + \sin y + \sin z = 3$, $x, y, z \in [0, 2\pi]$, then
 (A) $x^2 + y^2 + z^2 - xy - yz - zx = 0$
 (B) $x^3 + y^3 - z^3 = xyz$
 (C) $x^3 + y^3 + z^3 = 0$
 (D) $x + y + z = 0$

SECTION-D

Matrix-Match Type

This **Section D** have "match the following" type question. Question contains two columns, **Column-I** and **Column-II**. Match the entries in **Column-I** with the entries in **Column-II**. One or more entries in **Column-I** may match with one or more entries in **Column-II**.

25. Match the Column - I with Column - II

Column - I	Column - II
(A) The value of $\sin^2 5^\circ + \sin^2 10^\circ + \dots + \sin^2 90^\circ$ is	(P) +7
(B) If $4\cos \theta = 3 \cos \alpha$, then the value of $\cot \left(\frac{\theta - \alpha}{2} \right) \cot \left(\frac{\theta + \alpha}{2} \right)$ is	(Q) $\frac{3}{2}$
(C) If $A + B + C = \pi$, then greatest value of $\cos A + \cos B + \cos C$ is	(R) 3
	(S) $\frac{19}{2}$

26. Match the function given in column-I to the number of integers in its range given in column-II

Column - I	Column - II
(A) $f(x) = 2\cos^2 x + \sin x - 8$	(P) 5
(B) $f(x) = \sin^2 x + 3\cos^2 x + 5$	(Q) 4
(C) $f(x) = 4\sin x \cos x - \sin^2 x + 3\cos^2 x$	(R) 3
(D) $f(x) = \cos(\sin x) + \sin(\sin x)$	(S) 2

27. Match column-I with column-II :

Column - I	Column - II
(A) The expression $[\cos^2(\alpha + \beta) + \cos^2(\alpha - \beta) - \cos 2\alpha \cos 2\beta]$, is	(P) independent of α
(B) The expression $\cos^2 \alpha + \cos^2(\alpha + \beta) - 2 \cos \alpha \cos \beta \cos(\alpha + \beta)$, is	(Q) independent of β
(C) The expression $\frac{\sin(\alpha - \beta)\sin(\alpha + \beta)}{1 - \tan^2 \alpha \cot^2 \beta}$ is	(R) independent of α and β
(D) The expression $2 \sin^2 \beta + 4 \cos(\alpha + \beta) \sin \alpha \sin \beta + \cos 2(\alpha + \beta)$, is	(S) dependent on α and β

28. Match Column I with Column II

(A) $\cos^2 52^\circ - \sin^2 22^\circ$ equals	(p) 1
(B) $\cos^2 \frac{3\pi}{5} + \cos^2 \frac{4\pi}{5}$ equals	(q) $\frac{3 - \sqrt{3}}{4\sqrt{2}}$
(C) $\sin 24^\circ + \cos 6^\circ$ equals	(r) $\frac{3}{4}$
(D) $\sin^2 50^\circ + \cos^2 130^\circ$ equals	(s) $\frac{\sqrt{15} + \sqrt{3}}{4}$

Codes

A	B	C	D
(A) q	r	s	p
(B) r	q	p	s
(C) r	q	s	p
(D) q	r	p	s

29. Match the following

Column I

Column II

- (A) If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$
then $\tan (2A + B)$
may be equal to (p) 6
- (B) Period of $\sin \pi x + \cos 4\pi x$ is
(not only fundamental) (q) 5
- (C) If $\sec A + \tan A = \frac{3}{2}$ then
 $13 \sin A$ is equal to (r) 3
- (D) $48 \sin 20^\circ \sin 40^\circ \sin 60^\circ$
 $\sin 80^\circ$ is equal to (s) 9

Codes

A B C D

- (A) r p s q
(B) p r s q
(C) r p q s
(D) p r q s

30. Match the following

Column I

Column II

- (A) If $\sin x + \sin^2 x = 1$, then
 $\cos^2 x + \cos^4 x$ is equal to (p) 0
- (B) If $\sin x + \sin^2 x = 1$, then $\cos^8 x +$
 $2\cos^6 x + \cos^4 x$ is equal to (q) 1
- (C) If $x = y \cos \frac{2\pi}{3} = z \cos \frac{4\pi}{3}$, then
 $xy + yz + zx$ is equal to (r) 2
- (D) If $\sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3$, then
 $\cos^2 \theta_1 + \cos^2 \theta_2 + \cos^2 \theta_3$ is (s) 3

Codes

A B C D

- (A) q p q q
(B) p q q p
(C) q q p p
(D) p q r s

31. Match the equations given in column I with the number of possible real roots given in column II

Column I

Column II

- (A) $\left(x + \frac{1}{x}\right) \operatorname{cosec} x - 1 = 0$ (p) 0
- (B) $(8 + 3\sqrt{7})^{x^2 - 2x} +$
 $(8 - 3\sqrt{7})^{x^2 - 2x} = 15$ (q) 3
- (C) $-\tan x - \tan 2x + \tan 3x =$
 $\tan x \tan 2x \tan 3x$ in $[0, \pi]$ (r) 4
- (D) $\tan^2 x - \tan x = 0$ in $[0, \pi]$ (s) Infinite

Codes

A B C D

- (A) p s r q
(B) p r s q
(C) q s r p
(D) q r s p

SECTION-E

Integer Answer Type

This section contains Integer type questions. The answer to each of the questions is an integer.

32. If (x, y) satisfy the equation $1 + 4x - x^2 =$
 $\sqrt{9 \sec^2 y + 4 \operatorname{cosec}^2 y}$, then the value of $x \cot^2 y$ is
33. The value of $\tan \theta (1 + \sec 2\theta) (1 + \sec 4\theta) (1 + \sec 8\theta)$,
when $\theta = \frac{\pi}{32}$, is
34. The numerical value of $\tan 80^\circ \cdot \tan 60^\circ \cdot \tan 40^\circ \cdot \tan 20^\circ$
is
35. The value of the expression $\frac{\sin 20^\circ (4 \cos 20^\circ + 1)}{\cos 20^\circ \cos 30^\circ}$
is λ then λ^2 is equal to
36. If $S_n = \cos^n \theta + \sin^n \theta$, then value of $3S_4 - 2S_6$ is
37. If $P_n = \cos^n x + \sin^n x$, then $2P_6 - 3P_4 + 1 =$
38. Find the number of solutions of $10 \sin x = |x|$
39. What is the number of solutions of $3x + 2 \tan x =$
 $\frac{5\pi}{2}$ in $[0, 2\pi]$?



SECTION - A

Straight Objective Type

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

- The range of $y = \sin^3 x - 6\sin^2 x + 11\sin x - 6$ is
(A) $[-24, 2]$ (B) $[-24, 0]$
(C) $[0, 24]$ (D) $[24, -2]$
- If $(\tan x - \tan y)^2$, $(\tan y - \tan z)^2$, and $(\tan z - \tan x)^2$ are the A.P. then $(\tan x - \tan y)$, $(\tan y - \tan z)$, and $(\tan z - \tan x)$ are in
(A) A.P. (B) G.P.
(C) H.P. (D) none of these
- If the equation $\sin \theta(\sin \theta + 2\cos \theta) = a$ has a real solution then a belongs to the interval
(A) $\left[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right]$ (B) $\left(-\infty, \frac{1+\sqrt{5}}{2}\right)$
(C) $\left[\frac{1-\sqrt{5}}{2}, \infty\right)$ (D) none of these
- Number of solutions in $[0, 2\pi]$ of $\cos^2(\cos \theta) = \sin^2(\sin \theta)$ is
(A) 0 (B) 1
(C) 2 (D) 4
- The number of solution(s) of the equation $|\sin x| = |\log_5 x|$ is
(A) 1 (B) 2
(C) 3 (D) 4
- The maximum value of $\cos(\cos(\cos(\sin x)))$ for all $x \in \mathbb{R}$ is
(A) 1 (B) $\cos 1$
(C) $\cos(\cos 1)$ (D) $\cos(\cos(\cos 1))$
- The maximum value of $\cos(\cos(\sin x))$, $\forall x \in \mathbb{R}$, is
(A) 1 (B) $\cos(\cos 1)$
(C) $\cos 1$ (D) 0
- The equation $\cos^6 x - \sin^6 x - \frac{1}{16} \cos 6x - \frac{3}{16} \cos 2x - a^2 - a = 0$ will have at least one solution if
(A) $-\frac{3}{2} \leq a \leq \frac{1}{2}$ (B) $\frac{1}{2} \leq a \leq \frac{3}{2}$
(C) $0 \leq a \leq 1$ (D) $-\frac{3}{2} \leq a \leq 1$
- If one of the roots of the equation $x^2 \sec^2 \theta \tan \theta + 2 \tan \theta \sec \theta x + \sec^2 \theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$ is $\cot \theta$ then $2 + 2\sin \theta + \tan \theta$ equals
(A) -1 (B) 1
(C) 0 (D) 2
- If $\frac{1}{\{x\}} = \cos x$ ($\{.\}$ represents fractional part of x) then the number of solution(s) is/are
(A) 0 (B) 1
(C) 2 (D) Infinite
- If $\sin^2 x - a \sin x + b = 0$ has three roots in $(0, \pi)$ then b lies in the interval
(A) $(0, 1)$ (B) $(-1, 0)$
(C) $(1, 2)$ (D) $(-2, -1)$
- The number of possible integral values of 'a' for which $\cos 2x + a \sin x = 2a - 7$ has solutions, is
(A) 6 (B) 7
(C) 5 (D) 4
- The maximum value of $4 \sin^2 x + 3 \cos^2 x + \sin \frac{x}{2} + \cos \frac{x}{2}$ is
(A) $4 + \sqrt{2}$ (B) $4 - \sqrt{2}$
(C) $4\sqrt{2}$ (D) 4
- Number of solutions $(\alpha, \beta) : \alpha, \beta \in [0, 14]$ of $2^{\sin^2 \alpha} \sqrt{2\beta - \beta^2} \geq 2$ is
(A) 1 (B) 2
(C) 3 (D) 4

15. If $|\sin^2 x + 17 - x^2| = |16 - x^2| + 2 \sin^2 x + \cos^2 x$, then x lies in
- (A) $[-8, 8]$ (B) $[-4, 4]$
- (C) $[-\sqrt{17}, \sqrt{17}]$ (D) $(-6, 0)$

SECTION - B

Multiple Correct Answer Type

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

16. If $u = \sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta} + \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$, then which of the following is/are true

- (A) $u_{\max} = \sqrt{2(a^2 + b^2)}$
- (B) $u_{\min} = |a| + |b|$
- (C) $u_{\max} - u_{\min} = 2(a^2 + b^2)$
- (D) $\frac{u_{\max}}{u_{\min}} = \frac{1}{2}$

17. If $\sin^2 x + \sin x = (a + 2)$, then which of the following statement (s) is (are) correct ?

- (A) Number of integral values of a for real solution to exist is 3.
- (B) There exists no solution for $a < \frac{-9}{4}$ or $a > 0$
- (C) The minimum value of a for real solution is - 2.
- (D) Number of prime values of a for real solution to exist is 1.

18. If $\cos x + \cos y = a$, $\cos 2x + \cos 2y = b$, $\cos 3x + \cos 3y = c$, then

- (A) $\cos^2 x + \cos^2 y = 1 + \frac{b}{2}$
- (B) $\cos x \cos y = \frac{a^2}{2} - \left(\frac{b+2}{4}\right)$
- (C) $2a^3 + c = 3a(1+b)$
- (D) $a + b + c = 3abc$

19. The equality $\sin^6 x + 3 \sin^2 x \cos^2 y + \cos^6 y = 1$
- (A) Is never possible

(B) Is possible when $x = \frac{\pi}{2}$, $y = 0$

(C) Is possible only when $x = y$

(D) Is possible when $x = y = \frac{\pi}{6}$

20. In $[-2\pi, 2\pi]$, number of solutions of $2 \tan x \sec^2 x - 2 \tan^2 x + \tan x - 3 = 0$ is k , then

(A) $k \geq 2$ (B) $k \leq 4$

(C) $k \leq 2$ (D) $k \geq 3$

21. The equation $\sec \theta + \operatorname{cosec} \theta = k$ has

(A) Two roots $\in [0, 2\pi]$ if $k^2 < 8$

(B) Four roots $\in [0, 2\pi]$ if $k^2 > 8$

(C) No root $\in [0, 2\pi]$ if $k^2 > 8$

(D) Four roots $\in [0, 2\pi]$ if $k^2 < 8$

SECTION - C

Linked Comprehension Type

This section contains paragraph. Based upon this paragraph, 2 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Q. Nos. 22 and 23

Sometimes we have quadratic equation having unknown variables/parameter, in terms of some trigonometric functions.

Apparently it is seen while solving such type of equations we have to keep in consideration the domain and range of the corresponding trigonometric functions while finding out the final roots. For example:

$$f(x) = 4 \sin^2 x + 3 \cos^2 x + 3 \sin x \quad \forall x \in \left[0, \frac{\pi}{2}\right]$$

Find the minimum value of $f(x)$. As we know that $\sin x$ is increasing function in $\left[0, \frac{\pi}{2}\right]$

$$\text{Let } f(x) = 4 \sin^2 x + 3 - 3 \sin^2 x + 3 \sin x \Rightarrow f(x) = \sin^2 x + 3 \sin x + 3$$

Now, let us take $\sin x = t$ as $t \in [0, 1]$ as $\sin x > 0$

$$\forall 0 \leq \sin x \leq 1 \quad \forall x \in \left[0, \frac{\pi}{2}\right]$$

This reduces to $t^2 + 3t + 3$. As coefficient of $t^2 > 0$

Hence this will be upward parabola such that minimum value of the function exists at

$$\left(\frac{4ac - b^2}{4a}\right) \Rightarrow t_{\min} = \frac{4 \times 3 \times 1 - 9}{4} = \frac{3}{4}$$

In such a manner we can interrelate these two concepts keeping in mind the domain and co-domain of the respective trigonometric function.

22. If $\sec\theta + \operatorname{cosec}\theta = p$ has four solution between

$$(0, 2\pi) - \left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}, \text{ then}$$

$$(A) p^2 - 8 > 0 \quad (B) p^2 + 8 > 0$$

$$(C) p^2 - 8 < 0 \quad (D) p^2 + 8 < 0$$

23. If in above problem the equation $\sec\theta + \operatorname{cosec}\theta = p$

has only two real solution in $(0, 2\pi) - \left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$, then

$$(A) p^2 - 8 > 0 \quad (B) p^2 - 8 < 0$$

$$(C) p^2 = 8 \quad (D) \text{ none of these}$$

Paragraph for Question Nos. 24 to 25

Let us consider the identity $\cos 3^n \theta (\tan 3^n \theta - \tan 3^{n-1} \theta) = 2 \sin 3^{n-1} \theta$, Now answer the following questions

24. $\frac{\sin \theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta}$ is equal to

$$(A) \frac{1}{2}(\tan 27\theta - \tan \theta) \quad (B) \tan 27\theta - \tan \theta$$

$$(C) 2(\tan 27\theta - \tan \theta) \quad (D) \frac{1}{2}(\tan 27\theta + \tan \theta)$$

25. $\tan \sqrt{3} \theta - \tan \frac{\theta}{\sqrt{3}}$ is equal to

$$(A) \sin\left(\frac{\theta}{\sqrt{3}}\right) \sec(\sqrt{3} \theta)$$

$$(B) 2 \sin\left(\frac{\theta}{\sqrt{3}}\right) \sec(\sqrt{3} \theta)$$

$$(C) \sin\left(\frac{\theta}{\sqrt{3}}\right) \cos(\sqrt{3} \theta)$$

$$(D) 2 \operatorname{cosec}\left(\frac{\theta}{\sqrt{3}}\right) \sec(\theta \sqrt{3})$$

SECTION-D

Single-Match Type

This section contains Single match questions. Each question contains statements given in two columns which have to be matched. The statements in **Column I** are labelled 1, 2, 3 and 4, while the statements in **Column II** are labelled p, q, r, s and t. Four options 1, 2, 3 and 4 are given below. Out of which, only one shows the right matching

26. Match column I with column II

Column I	Column II
(A) Number of solutions of equation $ \tan 2x = \sin x$ in $[0, \pi]$	(P) 1
(B) The value of $4 \tan \frac{\pi}{16} - 4 \tan^3 \frac{\pi}{16} + 6 \tan^2 \frac{\pi}{16} - \tan^4 \frac{\pi}{16} + 1$	(Q) 4
(C) If the equation $\tan(p \cot x) = \cot(p \tan x)$ has a solution in $x \in (0, \pi) - \left\{\frac{\pi}{2}\right\}$, then minimum positive integral value of $\frac{4P}{\pi}$ is equal to	(R) 3
(D) The value of $\frac{2x}{\pi}$ if $5^{\cos^2 2x + 2 \sin^2 x} + 3^{2 \cos^2 x + \sin^2 2x} = 126$ has a solution in $[\pi, 2\pi]$	(S) 2

SECTION-E**Integer Answer Type**

This section contains Integer type questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y and Z(say) are 6, 0 and 9, respectively, then the correct darkening of bubbles will look like the following :

X	Y	Z
0	0	0
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7
8	8	8
9	9	9

27. The number of solution (solutions) of the equation $(\sin 2x + \cos 2x)^{1+\sin 4x} = 2$ in $[-\pi, \pi]$ is _____
28. The number of pairs (x, y) satisfying the equations $\sin x + \sin y = \sin(x + y)$ and $|x| + |y| = 1$ is _____
29. Find the number of solutions of $x \in [0, \pi]$ satisfying the equation $(\log_{\sqrt{3}} \tan x) \left(\sqrt{\log_{\sqrt{3}} 3\sqrt{3} + \log_{\tan x} 3} \right) = -1$ is / are _____
30. The largest value of m , for which the equation $\sqrt{m} \sin x - 2 \cos x = \sqrt{2} + \sqrt{2-m}$ has a solution is _____.
31. The number of ordered pairs (α, β) , where $\alpha, \beta \in (-\pi, \pi)$ satisfying $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = \frac{1}{e}$ is _____.



ANSWERS

LEVEL-1

- | | | | | | |
|--------------------|-------------------------|-----------------------------|---------------|---------------|-------------|
| 1. (B) | 2. (D) | 3. (B) | 4. (C) | 5. (D) | 6. (B) |
| 7. (B) | 8. (A) | 9. (D) | 10. (D) | 11. (A) | 12. (B) |
| 13. (B) | 14. (A,B) | 15. (A,B,C) | 16. (C,D) | 17. (A,B,C,D) | 18. (A,B,D) |
| 19. (A,C) | 20. (A,B) | 21. (A,B,C,D) | 22. (A,B,C,D) | 23. (A,B,C) | 24. (A,B) |
| 25. (A-S, B-P,C-Q) | 26. (A-Q, B-R,C-P, D-S) | 27. (A-P,Q,R, B-P,C-S, D-Q) | | | |
| 28. (A) | 29. (C) | 30. (C) | 31. (B) | 32. (3) | 33. (1) |
| 34. (3) | 35. (4) | 36. (1) | 37. (0) | 38. (6) | 39. (3) |

LEVEL-2

- | | | | | | |
|-----------|-------------------------|-----------|-----------|-----------|-------------|
| 1. (B) | 2. (C) | 3. (A) | 4. (A) | 5. (D) | 6. (C) |
| 7. (B) | 8. (A) | 9. (B) | 10. (A) | 11. (A) | 12. (C) |
| 13. (A) | 14. (D) | 15. (B) | 16. (A,B) | 17. (A,B) | 18. (A,B,C) |
| 19. (C,D) | 20. (A,B,D) | 21. (A,B) | 22. (A) | 23. (B) | 24. (A) |
| 25. (B) | 26. (A-Q, B-S,C-P, D-R) | 27. (3) | 28. (6) | 29. (1) | |
| 30. (2) | 31. (4) | | | | |

