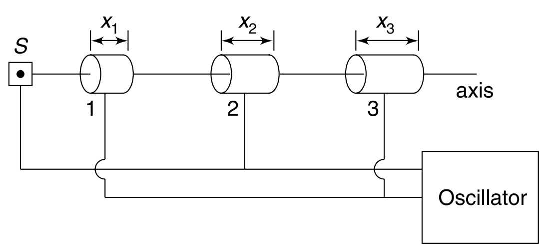
CHAPTER 9

Motion of Charge in Electromagnetic Field

LEVEL 1

Q. 1: A linear oscillator (Linac) is a device which accelerates charged particles in a straight line by means of oscillating electric field. S is a source of ion; emitting ions along a straight line (marked as axis) with negligible kinetic energy. The particles travel along a series of co-axial metallic cylindrical electrodes 1, 2, 3, etc. These electrodes are called drift tubes. These tubes are connected to a high frequency oscillator so that alternate tubes have opposite polarities. Thus in one half cycle if tubes 1, 3, 5… are negative then the source (S), tube 2, 4, 6… are positive. After every half cycle the polarities reverse. Assume that there is no electric field inside the metallic tube and the charged particles maintain a straight line trajectory. Find the ratio of lengths of the tubes $x_1 : x_2 : x_3$... for the device to work.



Q. 2: A particle of mass m and charge q is projected into a uniform magnetic field $\vec{B} = -B_0\hat{k}$ with velocity $\vec{v} = v_0\hat{i}$ from origin. The position vector of the particle at time t is $\vec{r}$. Find the impulse of magnetic force on the particle by the time $\vec{r} \cdot \vec{v}$ becomes zero for the first time.

Q. 3: In a cyclotron the radius of dees is R and the applied magnetic field has a magnitude of B. Particles having charge q and mass m are accelerated using this cyclotron and the maximum kinetic energy that can be imparted to a particle is k.

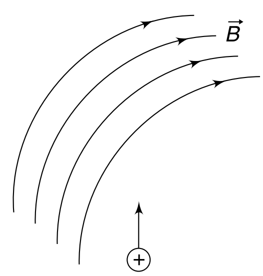
(a) Find k in terms of R, B, q and m.

(b) If protons and alpha particles are accelerated using this cyclotron which particle (proton or alpha) will gain more kinetic energy?

(c) Can we use a cyclotron to impart very high kinetic energy to an electron?

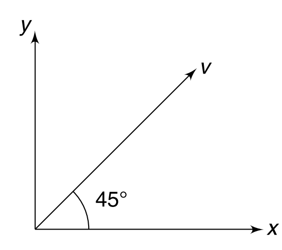
Q. 4: A particle having mass m and charge q is projected at an angle of 75° to the horizontal with a speed u. A uniform electric field $E = \dfrac{mg}{q}$ exists in horizontal direction. Find the time after projection when the velocity of particle makes an angle of 45° with horizontal.

Q. 5: A wide region of space has curved magnetic field lines as shown in the figure. A proton enters into the region as shown. Draw the path of the proton.

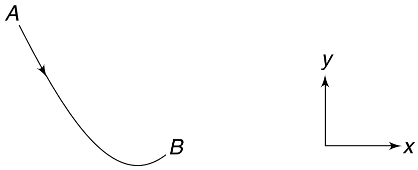


Q. 6: A uniform electric field E exists in a region of space. A charged particle is projected in the plane of electric field making an angle θ with the direction of field. Is it possible to make the particle move in a straight line by applying a uniform magnetic field?

Q. 7: A particle having charge $q = 10\,\mu C$ is moving with a velocity of $v = 10^6$ m/s in a direction making an angle of 45° with positive x axis in the xy plane (see figure). It experiences a magnetic force along negative z direction. When the same particle is projected with a velocity $v' = 10^6$ m/s in positive z direction, it experiences a magnetic force of 10 mN along X direction. Find the direction and magnitude of the magnetic field in the region.



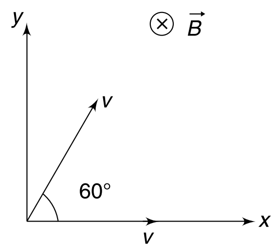
Q. 8: A uniform magnetic field exists perpendicular to the plane of the figure and a uniform electric field exists in the plane of the figure in positive y direction. A charged particle, when projected along xy plane moves along the path shown in the figure. Is the particle positively charged or negatively charged?



Q. 9: Two identical charged particles are projected simultaneously from origin in xy plane. Each particle has charge q and mass m and has been projected with velocity v as shown in the figure. There exists a uniform magnetic field B in negative z direction.

(i) Find $\vec{v}_1 \cdot \vec{v}_2$ at time t where $\vec{v}_1$ and $\vec{v}_2$ are velocities of the particles at time t.

(ii) Find $\vec{r}_1 \cdot \vec{r}_2$ at time $t = \dfrac{\pi m}{qB}$ where $\vec{r}_1$ and $\vec{r}_2$ are the position vectors of the two particles.

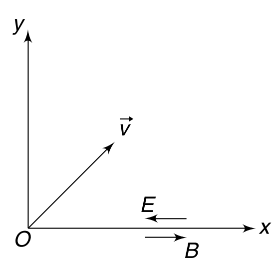


Q. 10: A particle having specific charge σ is projected in xy plane with a speed V. There exists a uniform magnetic field in z direction having a fixed magnitude B_0. The field is made to reverse its direction after every interval of $\dfrac{2\pi}{\sigma B}$. Calculate the maximum separation between two positions of the particle during its course of motion.

Q. 11: A particle having mass m and charge q is projected with a velocity v making an angle θ with the direction of a uniform magnetic field B. Calculate the magnitude of change in velocity of the particle after time $t = \dfrac{\pi m}{2qB}$.

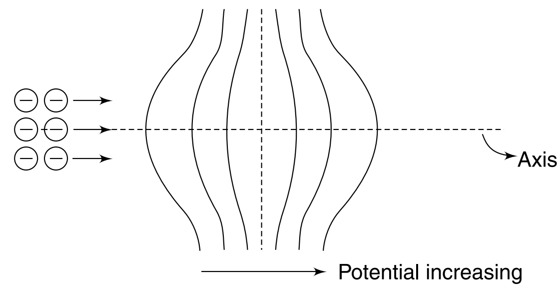
Q. 12: A region has a uniform magnetic field B along positive x direction and a uniform electric field E in negative x direction. A positively charged particle is projected from origin with a velocity $\vec{v} = v_0\hat{i} + v_0\hat{j}$. After some time the velocity of the particle was observed to be $v_0\hat{j}$ while its x co-ordinate was positive.

Write all possible values of $\dfrac{E}{B}$.

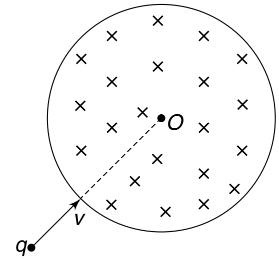


Q. 13: A positively charged particle with specific charge σ leaves the origin with a velocity u directed along positive x direction. The entire space has a uniform electric field (E) and magnetic field (B) directed along the positive y direction. Find the angle that the velocity of the particle makes with y direction at the instant it crosses the y axis for n^{th} time.

Q.14: Euipotential surfaces in a region of electric field are shaped as shown in the figure. The potential of field lines is increasing as one moves to right. A thin beam of electrons enters the region from left along the axis. Give qualitative arguments to show that the region will act as an electrostatic lens trying to focus the beam of electrons.

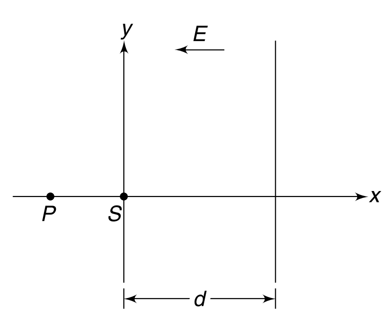


Q. 15: Figure shows a circular region of radius $R = \sqrt{3}\,m$ which has a uniform magnetic field $B = 0.2\ T$ directed into the plane of the figure. A particle having mass $m = 2$ g, speed $v = 0.3$ m/s and charge $q = 1$ mC is projected along the radius of the circular region as shown in figure. Calculate the angular deviation produced in the path of the particle as it comes out of the magnetic field. Neglect any other force apart from the magnetic force.



LEVEL 2

Q. 16: A source(s) of electrons is kept at origin of the co-ordinate system and it shoots out electrons in xy plane with very small range of velocities. In the region $x = 0$ to $x = d$ there is a uniform electric field $\vec{E} = -E\hat{i}$ which accelerates the electrons to speeds much larger than their original speed when they are ejected by the source. The electrons emerge from the field region, beyond $x = d$, travelling in straight lines. Prove that paths of the electrons, after they emerge out of the field, appear to be diverging from a point P having co-ordinates $(-d, 0)$.



Q. 17: A uniform electric field, $\vec{E} = -E_0\hat{i}$ and a uniform magnetic field, $\vec{B} = B_0\hat{i}$ exist in region $y > 0$. A particle having positive charge q and mass m is projected from the origin with a velocity $\vec{v} = v_0\hat{i} + v_0\hat{j}$. The velocity of the particle when it leaves the region of fields was found to be $-\vec{v}$.

(i) Find the ratio $\frac{E_0}{B_0}$ in terms of v_0.

(ii) Find the co-ordinates of the point where the particle leaves the fields.

(iii) Find the minimum speed of the particle during the course of the motion.

Q. 18: A positively charged ion is released at the origin of a co-ordinate system. The entire space is filled with a uniform electric field directed along positive x axis and a uniform magnetic field directed along positive z direction.

(i) Which (x, y or z) co-ordinate of the particle will always remain zero during its course of motion?

(ii) Which (x, y or z) co-ordinate of the particle will always be negative?

(iii) It is true to say that radius of curvature of the path of the particle is decreasing while its x co-ordinate decreases?

(iv) If the ion is negatively charged will its y co-ordinate ever become negative?

Q. 19: A particle of mass m and charge q is moving in a region where uniform electric field $\vec{E}$ and and uniform magnetic field $\vec{B}$ are present. It is given that $\frac{\vec{E}}{|\vec{E}|} = \frac{\vec{B}}{|\vec{B}|}$. At time $t = 0$, velocity of the particle is $\vec{v}_0$ and $\vec{v}_0 \cdot \vec{E} = 0$. Write the velocity of the particle at time t.

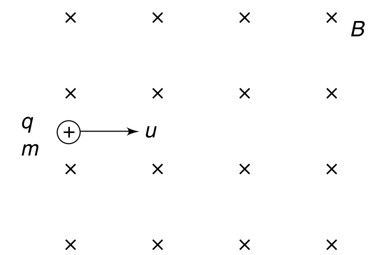
Q. 20: A particle of mass m and charge q is projected into a region having a uniform magnetic field B_0. Initial velocity (v_0) of the particle is perpendicular to the magnetic field. Apart from the magnetic force the particle faces a frictional force which has a magnitude of $f = kv$ where v is instantaneous speed and k is a positive constant.

(a) Find the radius of curvature of the path of the particle after it has travelled through a distance of

$$x_0 = \frac{mv_0}{2k}.$$

(b) Plot the variation of radius of curvature of the path of the particle with time (t).

Q. 21: A uniform magnetic field B_0 exists perpendicular to the plane of the fig. A positively charged particle having charge q and mass m is projected with velocity u into the field. The particle moves in the plane of the fig. During its course of motion the particle is subjected to a friction force which varies with velocity as $\vec{F} = -k\vec{V}$ where k is a positive constant and $\vec{V}$ is instantaneous velocity.



(a) What kind of path the particle will trace? Give qualitative argument to support your answer.

(b) Write the speed of the particle as function of time. Plot the speed–time graph.

(c) Calculate the distance travelled by the particle before it stops.

Assume no other force apart from magnetic force and the friction force.

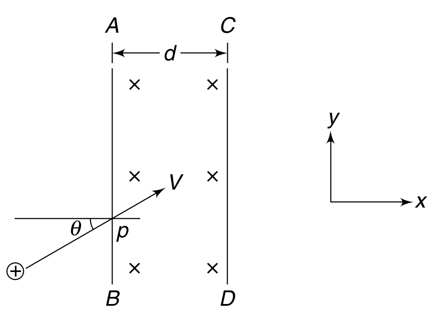
Q. 22: AB and CD are two parallel planes perpendicular to the X axis. There is a uniform magnetic field (B) in the space between them directed in negative Z direction. Width of the region having field is d and rest of the space is having no field. A particle having mass m and charge $+q$ enters the region with a velocity V making an angle θ with the X direction as shown.

(a) Find the values of d for which the particle will come out of the magnetic field crossing CD.

(b) For $d = \left(\frac{\sqrt{2}-1}{2}\right)\frac{mv}{qB}$ and $\theta = \frac{\pi}{6}$ find the angular deviation in the path of the particle.

(c) Find the deviation in path of the particle if

$$d = \frac{5mv}{4qB}(1 - \sin\theta)$$

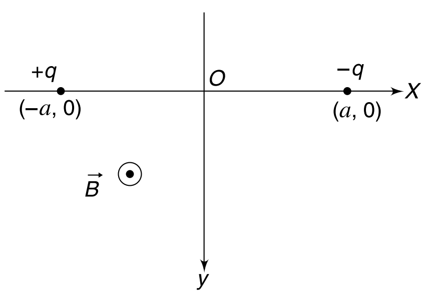


Q. 23: In the last problem, region to the right of CD is filled with a magnetic field of strength twice that in the region between AB and CD. This field exists up to a large distance and both fields are parallel in direction. For $d = \left(\frac{\sqrt{2}-1}{2}\right)\frac{mv}{qB}$

and $\theta = \frac{\pi}{6}$, find the displacement of the particle (measured from its entry point P) by the time it comes out of the magnetic fields. Find the total time spend by the particle in the two fields.

Q. 24: A particle of mass m is thrown along a horizontal surface with speed u and comes to rest after travelling a distance x_0. A uniform magnetic field B_0 is switched on in vertically downward direction and the particle is projected as before after putting a charge q on it. This time it travelled through a distance X_1 before stopping. Now the experiment is repeated with a uniform electric field E_0 in addition to the magnetic field; electric field being parallel to the magnetic field. Particle having mass m and charge q is projected as earlier and found to travel a distance X_2 before stopping. Find X_1 and X_2.

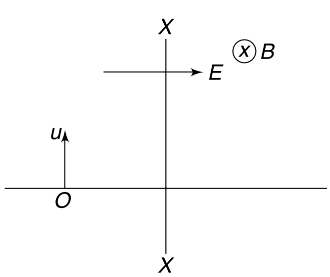
Q. 25: Two particles have equal mass m and electric charge of equal magnitude (q) and opposite sign. The particles are held at rest at co-ordinates $(-a, 0)$ and $(a, 0)$ as shown in the figure. The particles are released simultaneously. Consider only the electrostatic force between the particles and the force applied by the external magnetic field on them.



(a) Find the speed of negatively charged particle as function of its x co-ordinate.

(b) Find the y component of velocity of the negative particle as a function of its x co-ordinate.

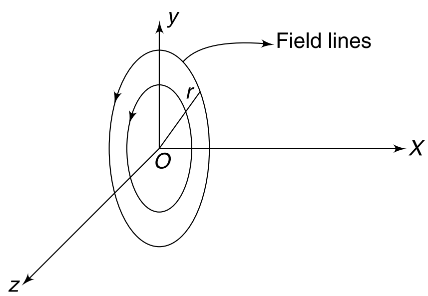
Q. 26: In the fig shown XX represents a vertical plane perpendicular to the plane of the fig. To the right of this plane there is a uniform horizontal magnetic field B directed into the plane of the fig. A uniform electric field E exists horizontally perpendicular to the magnetic field in entire space. A charge particle having charge q and mass m is projected vertically upward from point O. It crosses the plane XX after time T. Find the speed of projection of the particle if it was observed to move uniformly after time T. It is given that $qE = mg$.



Q. 27: A particle of mass m and charge $+q$ is projected from origin with velocity $\vec{V} = V_0\,\hat{i}$ in a magnetic field $\vec{B} = -(B_0 x)\hat{k}$. Here V_0 and B_0 are positive constants of proper dimensions. Find the radius of curvature of the path of the particle when it reaches maximum positive x co-ordinate.

Q. 28: A long uniform cylindrical beam of radius R consists of positively charged particles each of charge q, mass m and velocity V along positive x direction. The axis of the beam is the x-axis. The beam is incident on a region having magnetic field in y-z plane. The magnetic field in the region is confined to $0 \le x \le \Delta x$ (Δx is small). The field lines are circular in yz plane as shown. The magnitude of the field is given by

$B = B_0 r$ where B_0 is a constant and r is distance from the origin.



Show that this magnetic field acts as a converging lens for the ion beam and obtain the expression for focal length. Neglect the divergence in the beam caused due to electromagnetic interaction of the charge particles.

LEVEL 3

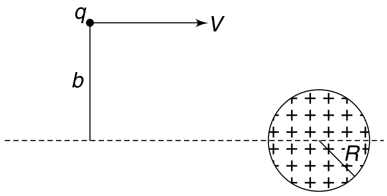
Q. 29: There is a fixed sphere of radius R having positive charge Q uniformly spread in its volume. A small particle having mass m and negative charge $(-q)$ moves with speed V when it is far away from the sphere. The impact parameter (i.e., distance between the centre of the sphere and line of initial velocity of the particle) is b. As the particle passes by the sphere, its path gets deflected due to electrostatics interaction with the sphere.

(a) Assuming that the charge on the particle does not cause any effect on distribution of charge on the sphere, calculate the minimum impact parameter b_0 that allows the particle to miss the sphere. Write the value of b_0 in term of R for the case

$$\frac{1}{2}mV^2 = 100 \cdot \left(\frac{kQq}{R}\right)$$

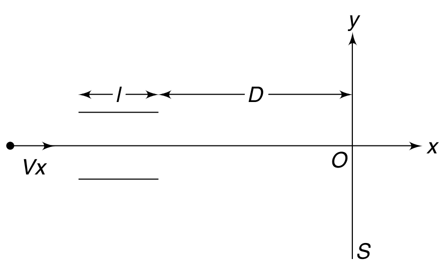
(b) Now assume that the positively charged sphere moves with speed V through a space which is filled with small particles of mass m and charge $-q$. The small particles are at rest and their number density is n [i.e., number of particles per unit volume of space is n].

The particles hit the sphere and stick to it. Calculate the rate at which the sphere starts losing its positive charge $\left(\frac{dQ}{dt}\right)$. Express your answer in terms of b_0.



Q. 30: (a) A charge particle travelling along positive x direction with speed V_x enters a region of width ℓ having a uniform electric field E in positive y direction. A screen is kept, at a distance $D (>> \ell)$ from the region of the field, in yz plane. Find the y co-ordinate of the point where the particle strikes the screen. Charge and mass of the particle is $+q$ and m respectively.

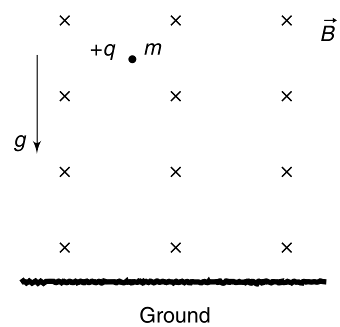
(b) The electric field in the region is replaced with a uniform magnetic field B in negative z direction. Now calculate the y co-ordinate of the point on the screen where the particle hits it. Assume deflection due to field to be small and $D >> \ell$.



(c) Now the field region is filled with a uniform electric (E) and magnetic field (B) both directed in positive y direction. A beam of protons and some other positive ion enters the region travelling along x direction. The particles hit the screen along two curved paths. Explain.

Draw the two curves in yz plane and point out which one represents the protons.

Q. 31: A particle having charge q and mass m is dropped from a large height from the ground. There exists a uniform horizontal magnetic field B in the entire space as shown in the fig. Assume that the acceleration due to gravity remains constant over the entire height involved.



(a) Argue qualitatively that the particle will touch a maximum depth and then start climbing up.

(b) Find the speed of the particle at the moment it starts climbing up.

(c) At what depth from the starting point does the particle starts climbing up?

Q. 32: In a region of space a uniform magnetic field exist in positive z direction and there also exists a uniform electric field along positive y direction. A particle having charge $+q$ and mass m is released from rest at the origin. The particle moves on a curve known as cycloid. If a wheel of radius R were to roll on the X axis, a fixed point on the circumference of the wheel would generate this cycloid. Find the radius R. It is given that strength of magnetic and electric fields are B_0 and E_0 respectively.

ANSWERS

1. $1 : \sqrt{2} : \sqrt{3} : ...$

2. $2mv_0\hat{i}$

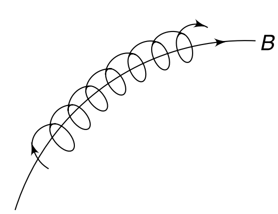
3. (a) $k = \dfrac{R^2q^2B^2}{2m}$

(b) Both will get same energy

(c) No

4. $\dfrac{u}{2\sqrt{2}g}$

5.



6. No

7. 1 mT in negative y direction.

8. Positive.

9. (i) $\frac{v^2}{2}$ (ii) $2\left(\frac{mv}{qB}\right)^2$

10. $\frac{4v}{\sigma B_0}$

11. $\sqrt{2}\, v \sin\theta$

12. $\frac{E}{B} = \frac{v_0}{2\pi n}$ where $n = 1, 2, 3...$

13. $\alpha = \tan^{-1}\left(\frac{uB}{2\pi n E}\right)$

15. 60°

17. (i) $\frac{E_0}{B_0} = \frac{2v_0}{\pi}$ (ii) $\left(0, 0, \frac{2mv_0}{qB_0}\right)$

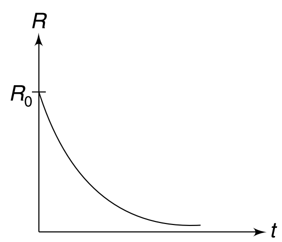
(iii) $v_{min} = v_0$

18. (i) z (ii) y

(iii) yes (iv) It is always negative

19. $\vec{v}\cos\left(\frac{qBt}{m}\right)\vec{v}_0 + \left(\frac{qt}{m}\right)(\vec{E}) + \sin\left(\frac{qBt}{m}\right)\left(\frac{\vec{v}_0 \times \vec{B}}{B}\right)$

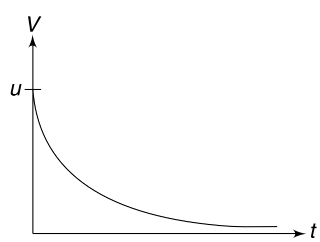
20. (a) $\frac{mv_0}{2qB_0}$ (b)



21. (a) Spiral path

(b) $V = u e^{-\frac{kt}{m}}$

(c) $s = \frac{mu}{k}$



22. (a) $d < \frac{mv}{qB}(1 - \sin\theta)$ (b) 15°

(c) $\pi - 2\theta$

23. $y = \left(\sqrt{3} - \frac{1}{\sqrt{2}}\right)\frac{mv}{qB}$; $t = \frac{5\pi m}{12qB}$

24. $X_1 = X_0$; $X_2 = \frac{mgX_0}{mg + qE_0}$

25. (a) $V = \sqrt{\frac{kq^2}{2m}\left(\frac{1}{x} - \frac{1}{a}\right)}$ (b) $V_y = \frac{qB}{m}(a - x)$

26. $u = 2gT$

27. $R = \sqrt{\frac{mV_0}{2qB_0}}$

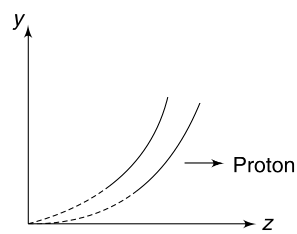
28. $f = \frac{mV}{qB_0 \Delta x}$

29. (a) $b_0 = R\sqrt{1 + \frac{2KQq}{mRV^2}}$ $b_0 \simeq 1.02\, R$

(b) $qn\pi b_0^2 V$

30. (a) $y = \frac{qE\ell D}{mV_x^2}$ (b) $y = \frac{D\ell qB}{mV_x}$

(c)



31. $\frac{2mg}{qB}$ (c) $h = \frac{2m^2 g}{q^2 B^2}$

32. $R = \frac{mE_0}{qB_0^2}$

SOLUTIONS

1. Let a positive ion, having charge q and mass m be accelerated during the half cycle when the drift tube 1 is negative. If the applied potential difference is V, the speed (v_1) of the ion on reaching tube 1 is given by

$$\frac{1}{2} mv_1^2 = qV$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{2qV}{m}}$$

The ions travel with constant speed in the field free space inside the tubes. The length of the tube 1 is so chosen such that when it exits from the tube the tube 1 becomes positive and tube 2 becomes negative. The ion is once again accelerated between the tubes and before entering tube 2 it acquires a speed (v_2) given by

$$\frac{1}{2}mv_2^2 = 2qV$$

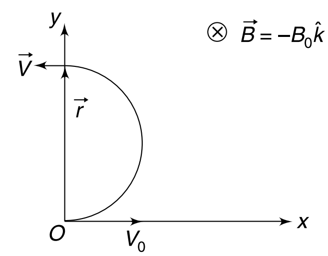
$\Rightarrow$ $$v_2 = \sqrt{2}\sqrt{\frac{2qV}{m}} = \sqrt{2}\,v_1$$

$\Rightarrow$ $$v_2 = \sqrt{2}\,v_1;\ \text{similarly } v_3 = \sqrt{3}\,v_1$$

The ion must spend equal time inside each tube.

$\therefore$ $$x_1 : x_2 : x_3 = v_1 : v_2 : v_3 : ...$$
$$= 1 : \sqrt{2} : \sqrt{3} : ...$$

2. The particle goes in a circular path in xy plane.



$$\vec{r}\cdot\vec{v} = \text{ when } \vec{r}\perp_r\vec{V}.$$

This happens when the particle has completed half circle (see figure)

Change in momentum of particle $\overrightarrow{\Delta p} = -2mv_0\hat{i}$

$\therefore$ Impulse $= -2mv_0\hat{i}$

3. (a) The maximum radius of circular path of the charge is R.

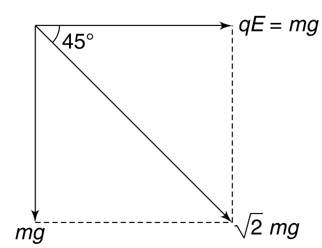
$\therefore$ $$\frac{mV_{\max}}{qB} = R$$

$$k = \frac{1}{2}mV_{\max}^2 = \frac{R^2q^2B^2}{2m} \qquad ...(1)$$

(b) For proton and alpha particle the ratio $\frac{q^2}{m}$ is same. Hence both of them will gain same KE.

(c) Equation (i) suggests that an electron can be imparted a large energy as its mass is too small. But this is not true. As the speed electron becomes high, its mass increases due to relativistic effect. The time period of revolution of the electron $\left(T = \frac{2\pi m}{qB}\right)$ increases because of this. The synchronization between the source frequency and frequency of revolution of charge gets disturbed and the cyclotron fails to accelerate a very light particle like electron to a very high kinetic energy.

4. Electrostatic force is $F_e = qE = mg$.

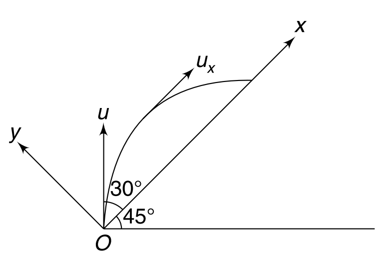


Effective acceleration experienced by the particle is

$$g_{\text{eff}} = \sqrt{2}\,g \quad \text{(In a direction making 45° to the vertical)}$$

Consider the co-ordinate system as shown in the figure. Velocity will be making 45° to the horizontal when it becomes parallel to OX

$\therefore$ $$t = \frac{u\sin 30°}{g_{\text{eff}}} = \frac{u}{2\sqrt{2}\,g}$$



8. When particle moves from A to B, radius of curvature of the path is decreasing. It means the particle is slowing down, as it moves from A to B.

9. (i) The two particles will rotate through same angle in time t. Hence angle between their velocities will remain 60°.

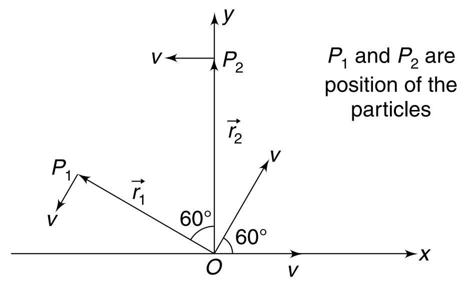
$$\therefore \qquad \vec{v}_1 \cdot \vec{v}_2 = v \cdot v \cdot \cos 60° = \frac{v^2}{2}$$

Note: Speed of charge will remain unchanged in magnetic field.

(ii) Time t given is half the time period of rotation for each charge

$$t = \frac{T}{2}$$

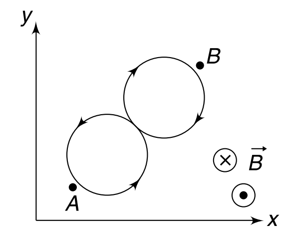
Each charge will complete half circle and will be at a distance $2R = 2\left(\frac{mv}{qB}\right)$ from the origin. Their positions are as shown in the figure.



$$\vec{r}_1 \cdot \vec{r}_2 = (2R)(2R)\cos 60° = 2R^2 = 2\left(\frac{mv}{qB}\right)^2$$

10. Path is as shown. Distance AB is the required maximum separation.

$$\therefore \qquad AB = 4R = 4\frac{mv}{qB_0} = \frac{4v}{\sigma B_0}.$$



11. Path is helical, with time period: $T = \dfrac{2\pi m}{qB}$

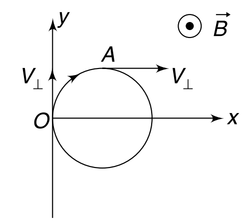
Given time is: $t = \dfrac{T}{4}$

Let the $\vec{B}$ be along + z axis. Velocity component parallel to the field

$$v_{\parallel} = v_z = v\cos\theta$$

This remains unchanged.

Let the component of v perpendicular to $\vec{B}$ be along y direction.



$$v_{\perp} = v_y = v\sin\theta$$

$$\therefore \qquad \text{Initial velocity } \vec{v}_1 = (v\sin\theta)\hat{j} + (v\cos\theta)\hat{k}$$

In time $t = \dfrac{T}{4}$, the particle will rotate through 90° and will be at A. The velocity component in xy plane is always $v_{\perp}$.

$\therefore$ Final velocity at A is

$$\vec{v}_f = (v\sin\theta)\hat{i} + (v\cos\theta)\hat{k}$$

$$\therefore \qquad \overrightarrow{\Delta v} = \vec{v}_f - \vec{v}_i = (v\sin\theta)\hat{i} - (v\sin\theta)\hat{j}$$

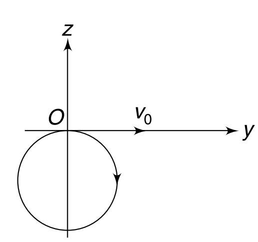
$$\therefore \qquad |\overrightarrow{\Delta v}| = \sqrt{2}v\sin\theta$$

12. If an observer looks at the particle from a point on positive x axis, he will find it rotating in yz plane with speed v_0 as shown in figure.

While rotating in yz plane the particle also advances in x direction. Due to electric force, x component of velocity decreases.

$$a_x = \frac{qE}{m}$$

$$v_x = v_0 - a_x t$$



v_x becomes zero at time: $t_0 = \dfrac{v_0}{a_x} = \dfrac{mv_0}{qE}$

The particle will be exactly at x axis travelling in y direction if time period of circular motion multiplied by an integer gives t_0.

$$\Rightarrow \qquad n \cdot \frac{2\pi m}{qB} = t_0$$

$$\Rightarrow \qquad n \cdot \frac{2\pi m}{qB} = \frac{mv_0}{qE}$$

$$\Rightarrow \qquad E = \frac{v_0 B}{2\pi n}, \qquad n = 1, 2, 3...$$

$$\Rightarrow \qquad \frac{E}{B} = \frac{v_0}{2\pi n}$$

13. Path is helix with increasing pitch. Axis of the helix is parallel to y axis and plane of circle is parallel to xz plane. The particle will cross the y axis for n^{th} time at time given by

$$t = nT = n \cdot \frac{2\pi m}{qB} = \frac{2\pi n}{\sigma B}$$

At this instant, the y component of velocity is

$$v_y = a_y t = (\sigma E)\left(\frac{2\pi n}{\sigma B}\right) = 2\pi n \left(\frac{E}{B}\right)$$

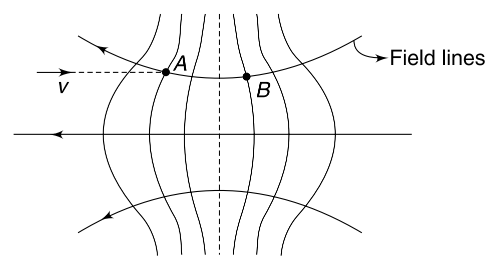
The x component of velocity at this instant will be u and $v_z = 0$

$$\therefore \qquad \tan\alpha = \frac{u}{v_y}$$

$$\therefore \qquad \alpha = \tan^{-1} = \left(\frac{uB}{2\pi n E}\right)$$

14. Electric field lines are perpendicular to the equipotentials as shown in the figure.

An electron at A experiences a force that has a component parallel ($F_{\parallel}$) to the axis and a component perpendicular ($F_{\perp}$) to the axis directed towards the axis.



$F_{\parallel}$ speeds up the electron.

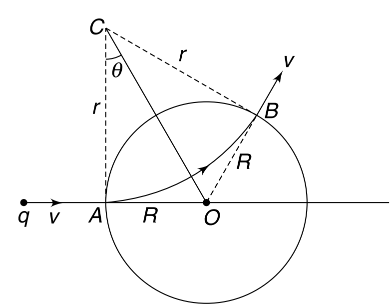
$F_{\perp}$ deflects it towards the axis.

However, at a point like B, $F_{\perp}$ is away from the axis. But the electrons have higher speed at B and their upward deflection will be less.

The overall effect will be that the beam of electrons will converge towards the axis.

15. Radius of circular path of the particle is

$$r = \frac{mv}{qB} = \frac{2 \times 10^{-3} \times 0.3}{10^{-3} \times 0.2} = 3\text{m}$$



C is centre of the circular path.

Particle enters the field at A and leaves at B.

$$\tan\theta = \frac{R}{r} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ.$$

Deviation = $2\theta = 60^\circ$.

[A careful observation tells that direction of velocity at B is along the radius of the given circle].

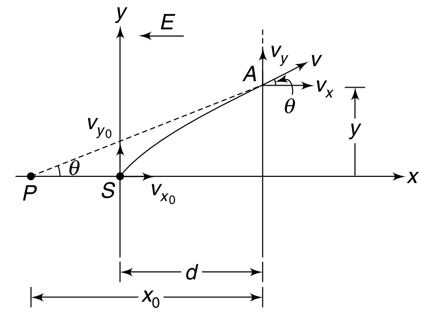
16. Velocity of an electron at the source has x and y components: v_{x_0} and v_{y_0}.

When the electron comes out of the field region–

$$v_y = v_{y_0}$$

and $v_x = v_{x0} + \frac{eE}{m}t$; where t is time of travel.

$$\simeq \frac{eE}{m}t$$



$$\therefore \qquad \tan\theta = \frac{v_y}{v_x} = \frac{v_{y0}\cdot m}{eEt} \qquad \text{...(i)}$$

If tangent to the path at A is extended, it intersects the x axis at P and

$$\tan\theta = \frac{y}{x_0} = \frac{v_{y0}\cdot t}{x_0} \qquad \text{...(ii)}$$

From (i) and (ii) $\frac{m}{eE\cdot t} = \frac{t}{x_0}$

$$\Rightarrow \qquad x_0 = \frac{eE}{m}t^2$$

But $\qquad d \simeq \frac{1}{2}\left(\frac{eE}{m}\right)t^2$

$$\therefore \qquad x_0 = 2d$$

Hence proved.

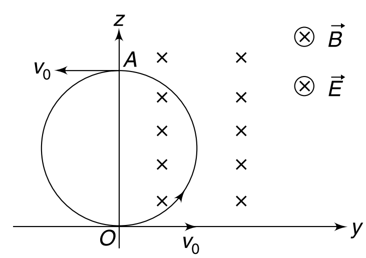
17. When the particle leaves the field, its velocity becomes $-v_0\hat{i} - v_0\hat{j}$.
In $y-z$ plane particle follows a circular path and it moves out of the plane in x direction due to $v_0\,\hat{i}$. The electric force retards it, brings it to rest and then forces it to move in negative x direction.

Since $a_x = -\frac{qE_0}{m}$ = a constant.

$\therefore$ Particle will have $v_x = -v_0$ when it is back to yz plane.

At that moment, its velocity in yz plane must be $-v_0\hat{j}$

$\therefore$ Particle leaves the field at A.



(a) Time of motion in field is $t = \frac{\pi m}{qB_0}$

Also,
$$v_x = u_x + a_x t$$

$$-v_0 = v_0 - \frac{qE_0}{m}t$$

$$\therefore \qquad t = \frac{2v_0 m}{qE_0}$$

$$\therefore \qquad \frac{\pi m}{qB_0} = \frac{2v_0 m}{qE_0}$$

$$\therefore \qquad \frac{E_0}{B_0} = \frac{2v_0}{\pi}$$

(ii) Co-ordinates are

$$x = 0; \quad y = 0$$

$$z = 2R = \frac{2mv_0}{qB_0}$$

(iii) Speed is minimum when $v_x = 0$

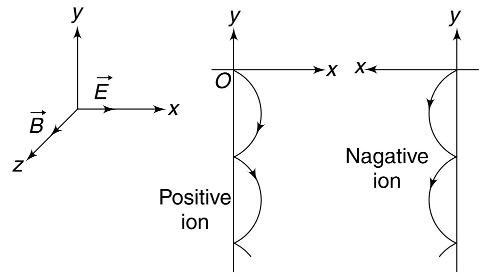
$\therefore$ $$v_{min} = v_0$$

18. Path of the particle is a cycloid in xy plane as shown.

z co-ordinate is always zero, and y co-ordinate is always negative.

As the x co-ordinate increases the particle speeds up due to electric force. Radius of curvature of the path increases. When the ion moves against the electric field, its speed decreases; and hence the radius of curvature of the path decreases.

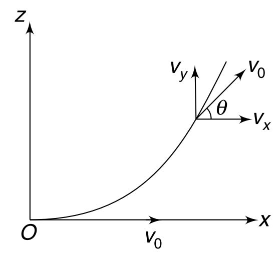
For negative ion, path is as shown in third figure.



19. For simplicity let's assume that initial velocity is along x direction.
Both $\vec{E}$ and $\vec{B}$ (are parallel) are perpendicular to $\vec{v}_0$. Let $\vec{E}$ and $\vec{B}$ be in positive y direction.

The particle will move in circle in xz plane under influence of magnetic field. Under influence of $\vec{E}$ it acquires an increasing velocity in y direction.

Consider the motion in xz plane.



$$\omega = \frac{qB}{m}$$

At time t $$\theta = \omega t = \frac{qB}{m}t$$

$$v_x = v_0 \cos(\omega t) = v_0 \cos\left(\frac{qB}{m}t\right)$$

$$v_z = v_0 \sin\left(\frac{qB}{m}t\right)$$

And $$v_y = a_y t = \frac{qE}{m}t$$

$\therefore$ $$\vec{v} = v_0 \cos\left(\frac{qB}{m}t\right)\hat{i} + \left(\frac{qEt}{m}\right)\hat{j} + v_0 \sin\left(\frac{qBt}{m}\right)\hat{k}$$

As per our assumption

$$\hat{i} = \frac{\vec{v}_0}{v_0},\ \hat{j} = \frac{\vec{E}}{E}$$

And $$\hat{k} = \frac{v_0 \times \vec{B}}{|\vec{v}_0 \times \vec{B}|} = \frac{\vec{v}_0 \times \vec{B}}{v_0 B}$$

$\therefore$ $$\vec{v} = \cos\left(\frac{qBt}{m}\right)\vec{v}_0 + \left(\frac{qt}{m}\right)\vec{E} + \sin\left(\frac{qBt}{m}\right)\left(\frac{\vec{v}_0 \times \vec{B}}{B}\right)$$

20. (a) The magnetic force does not cause any change in speed and the frictional force will always be against the velocity.

$\therefore$ $$ma = -kv$$

$$v\frac{dv}{dx} = -\frac{k}{m}v$$

$\therefore$ $$\int_{v_0}^{v} dv = -\frac{k}{m}\int_0^{x_0} dx$$

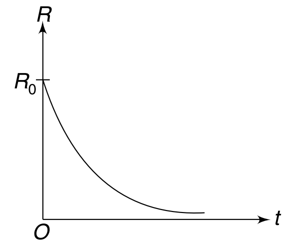
$\Rightarrow$ $$v - v_0 = -\frac{kx_0}{m}$$

$\therefore$ $$v = v_0 - \frac{kx_0}{m}$$

At
$$x_0 = \frac{mv_0}{2k}$$
$$v = \frac{v_0}{2}$$
$$\therefore \quad R = \frac{mv_0/2}{qB_0} = \frac{mv_0}{2qB_0}$$

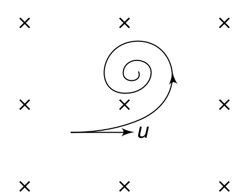
(b) $m\frac{dv}{dt} = -kv$

$$\int_{v_0}^{v} \frac{dv}{dv} = -\frac{k}{m}\int_0^t dt$$
$$\ell n\frac{v}{v_0} = -\frac{kt}{m}$$
$$\therefore \quad v = v_0 e^{-\frac{kt}{m}}$$
$$\therefore \quad R = R_0 e^{-\frac{kt}{m}} \text{ where } R_0 = \frac{mv_0}{qB}$$
$$= \text{initial radius}$$



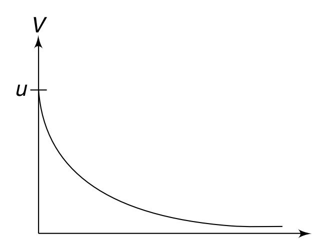
Graph is as given.

21. (a) The particle will follow a spiral path. Magnetic force will always remain perpendicular to the velocity and its only action will be to change the direction of motion.
The frictional force reduces the speed. Due to this the radius of curvature of the path $\left(R = \frac{mV}{qB}\right)$ keeps decreasing.



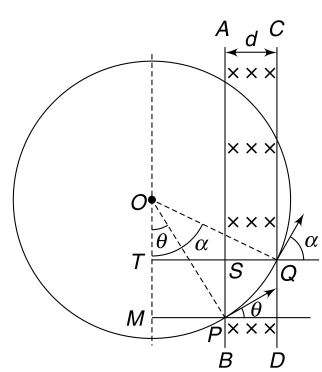
(b)
$$m = \frac{dV}{dt} = -kV$$
$$\int_u^v \frac{dV}{V} = -\frac{k}{m}\int_0^t dt$$
$$\ell n\left(\frac{V}{u}\right) = -\frac{kt}{m}$$
$$V = ue^{-\frac{kt}{m}}$$

(c)
$$V = ue^{-\frac{kt}{m}}$$
$$\frac{ds}{dt} = ue^{-\frac{kt}{m}}$$
$$\int_0^s ds = u\int_0^\infty e^{-\frac{kt}{m}}$$



[particle comes to rest as $t \to \infty$]

$$S = \frac{mu}{k}$$

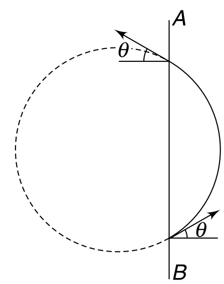


22. The circle shown is the indicative path had there been a magnetic field everywhere. Particle enters the field at P and leaves it at Q. Radius of circle $R = \frac{mv}{qB}$

$$MP = TS = R\sin\theta$$
$$TQ = R\sin\theta + d$$

$\therefore \quad \sin\alpha = \dfrac{TQ}{OQ} = \dfrac{R\sin\theta + d}{R} = \sin\theta + \dfrac{d}{R}$

$$\alpha = \sin^{-1}\left[\sin\theta + \frac{d}{R}\right]$$



The particle will emerge to the right of CD as long as $\alpha < \dfrac{\pi}{2}$

$\Rightarrow \quad \sin\theta + \dfrac{d}{R} < 1$

$$d < R(1 - \sin\theta)$$

For $\quad \theta = 30°$

$$d < \frac{R}{2} \quad \Rightarrow \quad d < \frac{mv}{2qB}$$

For $\quad d = \left(\dfrac{\sqrt{2} - 1}{2}\right) R$ and $\theta = 30°$

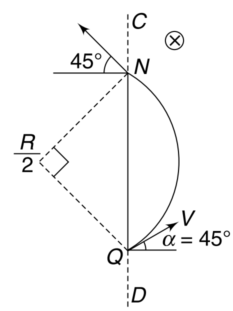
$$\sin\alpha = \sin 30° + \frac{\sqrt{2} - 1}{2} = \frac{1}{2} + \frac{\sqrt{2} - 1}{2} = \frac{1}{\sqrt{2}}$$

$$\alpha = 45°$$

Deviation = 15°

For $d > R(1 - \sin\theta)$, the particle will emerge out of the field crossing AB.
Deviation in this case will be $= \pi - 2\theta$

23. From the solution of last problem, the displacement in y direction during crossing of first field is



$$y_1 = R\cos\theta - R\cos\alpha$$

$$= R(\cos 30° - \cos 45°) = R\left(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right)$$

In the second field the particle enters at Q and leaves at N, while moving on a circle of radius $\dfrac{R}{2}$ $\quad \left[\because R = \dfrac{mv}{qB} \text{ and } B \text{ has been doubled}\right]$

Particle completes a quarter circle in the second field.

$$y_2 = QN = \frac{R}{2}\sqrt{2} = \frac{R}{\sqrt{2}}$$

At N, the particle enters the first field at 45° to negative X. It will get deflected (anticlockwise) by 15° as seen in the previous problem. And

$$y_3 = y_1$$

$\therefore \quad y = y_1 + y_2 + y_3$

$$= 2y_1 + y_2$$

$$= 2R\left[\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right] + \frac{R}{\sqrt{2}}$$

$$= \left(\sqrt{3} - \frac{1}{\sqrt{2}}\right)R$$

Time spent can be calculated as:

$$t = \frac{2\pi m}{qB} \times \left(\frac{15°}{360°}\right) \times 2 + \frac{2\pi m}{q(2B)}\left(\frac{90°}{360°}\right)$$

$$= \frac{\pi m}{6qB} + \frac{\pi m}{4qB} = \frac{5\pi m}{12qB}$$

24. Let coefficient of friction be μ.

$$u^2 = 2(\mu g)X_0 \quad \ldots(1)$$

$$\mu = \frac{u^2}{2gX_0}$$

With magnetic field switched on, the friction force does not change. Magnetic field does not perform any work on the moving charge. Only friction (which is always tangential to the path) eats away the entire KE

$$\Rightarrow \quad X_1 = X_0$$

With both the fields switched on, the normal reaction on the particle becomes.

$$N = mg + qE_0$$

$$\therefore \quad f = \mu(mg + qE_0)$$

$$\therefore \quad \mu(mg + qE_0)\cdot X_2 = \frac{1}{2}mu^2$$

$$\frac{u^2}{2gX_0}(mg + qE_0)X_2 = \frac{1}{2}mu^2$$

$$X_2 = \frac{mgX_0}{mg + qE_0}$$

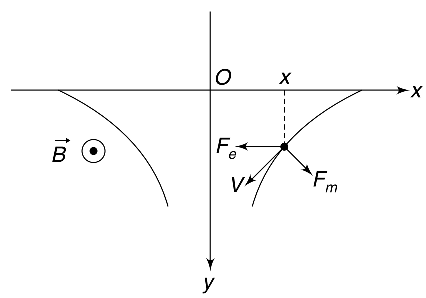
25. (a) The two particles follow symmetrical curved paths. As the electrostatic attraction speeds them, the magnetic force curves the path.

Magnetic force does not perform any work, therefore conservation of energy gives-

$$\frac{1}{2}mV^2 \times 2 - k\frac{q^2}{(2x)} = -k\frac{q^2}{(2a)}$$

$$\Rightarrow \quad 2mV^2 = kq^2\left(-\frac{1}{a} + \frac{1}{x}\right)$$

$$\Rightarrow \quad V = \sqrt{\frac{kq^2}{2m}\left(\frac{1}{x} - \frac{1}{a}\right)}$$



(b) Force on $-q$ is

$$\vec{F} = -F_e\hat{i} + (-q)\,(V_x\hat{i} + V_y\hat{j}) \times (-B\hat{k})$$

$$= (-F_e + qV_yB)\hat{i} - qBV_x\hat{j}$$

$$\therefore \quad m\frac{dV_y}{dt} = -qBV_x \;\Rightarrow\; m\frac{dV_y}{dt} = -qB\frac{dx}{dt}$$

$$\Rightarrow \quad \int_0^{V_y} dV_y = -\frac{qB}{m}\int_a^x dx$$

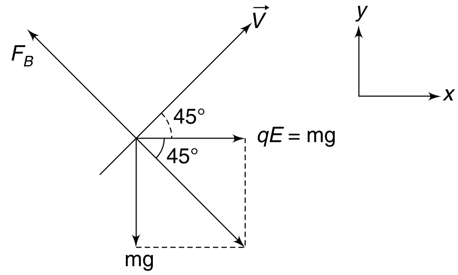
$$\Rightarrow \quad V_y = \frac{qB}{m}(a - x)$$

26. After crossing the XX plane, the forces on the particle are

(1) mg ($\downarrow$)

(2) $qE(\rightarrow)$

(3) Magnetic force perpendicular to velocity ($F_B = qVB$)



For particle to move uniformly, the magnetic force must balance the resultant of mg and qE. Hence, direction of velocity must be as shown in figure.

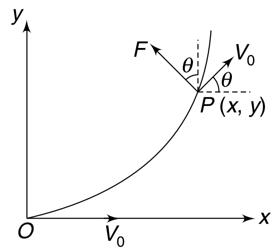
Component of velocity in X and Y direction must be same.

$$V_x = V_y$$

Since acceleration in y direction before entering magnetic field is $a_y = \frac{qE}{m} = \frac{mg}{m} = g$; we have

$$u - gT = gT$$
$$u = 2gT$$

27. Particle moves in xy plane. Its path is not a circle because B is not constant. However, speed of the particle remains constant at V_0.



Let velocity at point $P(x, y)$ be inclined to x axis at an angle θ.

Force will be perpendicular to velocity, inclined at θ with y direction.

$$\therefore \quad ma_y = F\cos\theta$$

$$\Rightarrow \quad m\frac{dV_y}{at} = (B_0x)qV_0 \cos\theta$$

$$\Rightarrow \quad m\frac{dV_y}{dx}\cdot\frac{dx}{dt} = B_0qx(V_x) \qquad [\therefore\ V_0\cos\theta = V_x]$$

But $$\frac{dx}{dt} = V_x$$

$$\therefore \quad \frac{dV_y}{dx} = \frac{B_0q}{m}x$$

$$\Rightarrow \quad \int_0^{v_0} dV_y = \frac{B_0q}{m}\int_0^{x_0} x\,dx$$

Where x_0 = maximum x co-ordinate of the particle where velocity becomes parallel to y axis.

$$\therefore \quad V_0 = \frac{B_0q}{2m}x_0^2$$

$$\Rightarrow \quad x_0 = \sqrt{\frac{2mV_0}{B_0q}}$$

Magnetic force at $x = x_0$ is $F = qV_0(B_0x_0)$

$$\therefore \quad F = qV_0B_0\sqrt{\frac{2mV_0}{B_0q}} = V_0\sqrt{2mqB_0V_0}$$

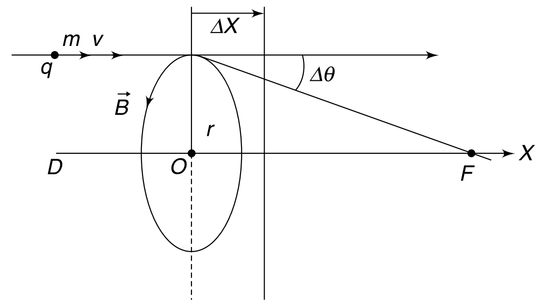
If radius of curvature of path if R then $\frac{mV_0^2}{R} = F$

$$\Rightarrow \quad \frac{mV_0^2}{R} = V_0\sqrt{2mqB_0V_0}$$

$$\Rightarrow \quad R = \sqrt{\frac{mV_0}{2qB_0}}$$

$\Rightarrow$

28. Consider a charge particle at a distance r from the x-axis.



Magnetic force on it is towards O (perpendicular) to its original direction of motion. This will cause the path of the charge to deviate by a small angle $\Delta\theta$.

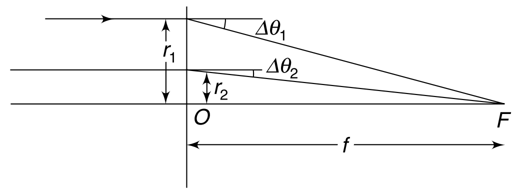
Time required to cross the field is: $\Delta t \simeq \dfrac{\Delta x}{V}$

Impulse of magnetic force $= qV(B_0 r)\dfrac{\Delta x}{V} = qB_0 r\Delta x$

Charge in momentum of the charge in crossing the field

$$\Delta p = qB_0 r\Delta x$$

This change is perpendicular to original direction of momentum.



$$\therefore \quad \Delta p \simeq p\Delta\theta$$

$$qB_0 r\Delta x \simeq mV\Delta\theta$$

$$\Rightarrow \quad \Delta\theta = \frac{qB_0 r\Delta x}{mV}$$

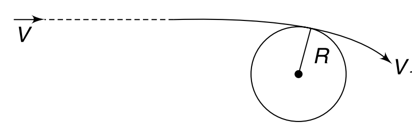
Because $\Delta\theta \;\alpha\; r$, all ions will get focused at one point F or the axis. (see figure.)

$$f = \frac{r_1}{\Delta\theta_1} = \frac{r_2}{\Delta\theta_2}$$

$$f = \frac{r}{\Delta\theta} = \frac{mV}{qB_0\Delta x}$$

29. (a) Angular momentum of the particle remains conserved about the centre of the sphere.

The particle just misses the sphere means that it grazes the surface of the sphere. We will use angular momentum and energy conservation.



Angular momentum conservation:

$$mbV - mRV_1 \quad \Rightarrow \quad V_1 = \frac{bV}{R}$$

Energy conservation:

$$\frac{1}{2}mV^2 - \frac{kQq}{\infty} = \frac{1}{2}mV_1^2 - K\frac{Qq}{R} \quad \left[K = \frac{1}{4\pi\epsilon_0}\right]$$

$$V_1^2 = V^2 + \frac{2KQq}{mR}$$

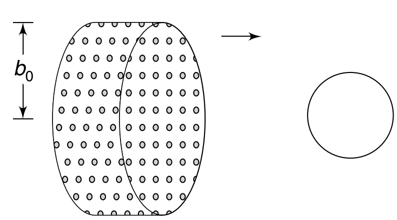
$$\frac{b^2}{R^2}V^2 = V^2 + 2\frac{KQq}{mR}$$

$$b_0 = R\sqrt{1 + \frac{2KQq}{mRV^2}}$$

for $\dfrac{1}{2}mV^2 = 100\dfrac{KQq}{R}$

$$\frac{1}{100} = \frac{2KQq}{mRV^2}$$

$$\therefore \quad b_0 = R[1 + 0.01]^{1/2} \simeq 1.02\,R$$



(b) In the reference frame of the sphere, the situation is identical to that in part (a). As the sphere moves, in a period Δt it will sweep all the particles in a volume:

$$V = \pi b_0^2 \, V \Delta t$$

$\therefore$ Total negative charge deposited on the sphere in interval Δt is

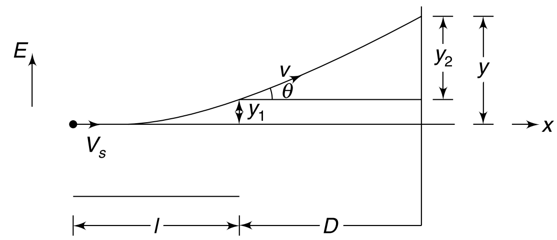
$$\Delta q = q \cdot n \cdot V = q n \pi b_0^2 V \Delta t$$

$$\frac{\Delta q}{\Delta t} = q \; n \; b_0^2 \; V$$

$\therefore$ Charge on sphere decreases at rate

$$\left|\frac{\Delta Q}{\Delta t}\right| = q n \pi b_0^2 V.$$

30. (a) Time to cross the region of electric field is



$$t = \frac{\ell}{V_x}$$

$$\therefore \quad y_1 = \frac{1}{2} a_y t^2 = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{\ell}{V_x}\right)^2$$

At the point of leaving the field region velocity makes angle θ with x direction given by

$$\tan\theta = \frac{V_y}{V_x} = \frac{a_y t}{V_x} = \frac{\left(\frac{qE}{m}\right)\left(\frac{\ell}{V_x}\right)}{V_x} = \frac{qE\ell}{mV_x^2}$$

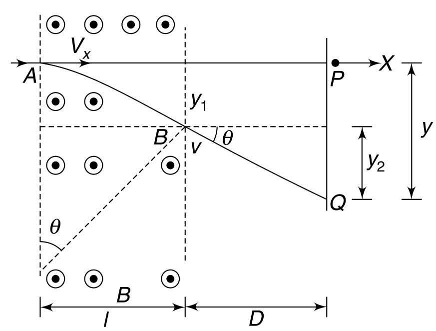
$$y_2 = D \tan\theta = \frac{qE\ell D}{mV_x^2}$$

$$\therefore \quad y = y_1 + y_2 = \frac{qE\ell}{mV_x^2}\left[\frac{\ell}{2} + D\right]$$

If $D >> \ell$

$$y = \frac{qE\ell D}{mV_x^2}$$

(b) Radius of circular path in magnetic field is



$$R = \frac{mV_x}{qB}$$

The deflection angle θ is given by

$$\sin\theta = \frac{\ell}{R} = \frac{\ell qB}{mV_x}$$

For small ℓ, θ can be assumed to be small and y_1 is also small.

$$y \simeq y_2 = D\tan\theta = D\sin\theta = \frac{D\ell qB}{mV_x}$$

(d) Let the initial direction of travel of charge be along x and the two fields be in y direction. The deflection caused due to two fields will be -

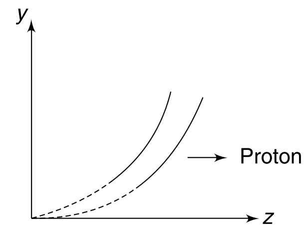
$$y = \frac{qE\ell D}{mV_x^2} \text{ and } z = \frac{qB\ell D}{mV_x}$$

For different values of V_x the ions hit the screen at different points.

Eliminating V_x between the above two expressions gives

$$\frac{z^2}{y} = \frac{D\ell B^2}{E}\left(\frac{q}{m}\right)$$

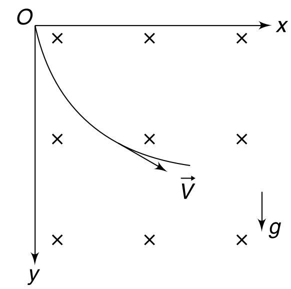
$$z^2 = k\left(\frac{q}{m}\right)y \text{ where } k = \frac{D\ell B^2}{E} = \text{a constant}$$



This represents an equation of a parabola. For different values of $\frac{q}{m}$ we get different curves.

For hydrogen $\frac{q}{m}$ is highest hence curve 1represents hydrogen and curve 2 represents the ion.

31. (b) Consider the origin at the starting point and the x and y axes as shown. Let the velocity of the particle at any time be $\vec{V} = V_x\hat{i} + V_y\hat{j}$



Force on the particle is:

$$F = \text{mg}\hat{j} + q(V_x\hat{i} + V_x\hat{j}) \times B\hat{k}$$

$$= (\text{mg} - qBV_x)\,\hat{j} + (qBV_y)\hat{i}$$

Hence, $$m\frac{dV_y}{dt} = \text{mg} - qBV_x \qquad ...(1)$$

and $$m\,\frac{dV_x}{dt} = qBV_y \qquad ...(2)$$

Differentiating equation (1) with respect to time

$$m\frac{d^2V_y}{dt^2} = -qB\,\frac{dV_x}{dt}$$

using (2)

$$\frac{d^2V_y}{dt^2} = -\frac{q^2B^2}{m^2}V_y$$

Solution to this differential equation is (as learnt in chapter of SHM)

$$V_y = V_{y0}\sin\left(\frac{qB}{m}t + \delta\right) \qquad ...(3)$$

V_{y0} and δ are constants.

It is known that $V_y = 0$ at $t = 0$

$\therefore$ $$\delta = 0$$

$\therefore$ $$V_y = V_{y0}\sin\left(\frac{qB}{m}t\right)$$

Differentiating wrt time

$$a_y = \frac{dV_y}{at} = V_{y0}\,\frac{qB}{m}\cos\left(\frac{qB}{m}t\right)$$

at time $t = 0$; $a_y = g$

$\therefore$ $$g = \frac{V_{y0}qB}{m}$$

$$V_{y0} = \frac{\text{mg}}{qB}$$

$\therefore$ $$V_y = \frac{\text{mg}}{qB}\sin\left(\frac{qB}{m}t\right) \qquad ...(4)$$

Put in (1)

$$\frac{dV_y}{dt} = g\cos\left(\frac{qB}{m}t\right)$$

$$V_x = \frac{mg}{qB}\left[1 - \cos\left(\frac{qB}{m}t\right)\right] \qquad ...(5)$$

Time after which the particle starts climbing up is after V_y becomes zero (for the first time after release).

$$V_y = 0$$

$$\Rightarrow \quad \frac{qB}{m}t = \pi \quad \Rightarrow \quad t = \frac{\pi m}{qB}$$

At this time V_x is

$$V_x = \frac{2mg}{qB}$$

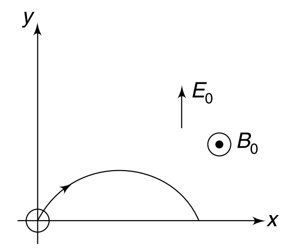
This is the required speed.

(c) Magnetic force does not perform any work during the motion of the charge particle.

$$\therefore \quad mgh = \frac{1}{2}mV_x^2$$

$$h = \frac{2m^2 g}{q^2 B^2}$$

32. The particle moves in xy plane. Let its velocity at any instant be



$$\vec{V} = V_x\hat{i} + V_y\hat{j}$$

$$\vec{F} = q\vec{E} + q(\overline{V} \times \overline{B})$$

$$= qE_0\hat{j} + q\left(V_x\hat{i} + V_y\hat{j}\right) \times (B_0\,\hat{k})$$

$$= q(E_0 - V_xB_0)\hat{j} + qB_0V_y\hat{i}$$

$$\frac{m\,dV_x}{dt} = qB_0V_y \qquad ...(1)$$

And

$$m\frac{dV_y}{dt} = (E_0 - V_xB_0) \qquad ...(2)$$

Differentiating (2) wrt time and substituting for $\frac{dV_x}{dt}$ from (1), we get

$$\frac{d^2V_y}{dt^2} = \left(\frac{qB_0}{m}\right)^2 V_y$$

This equation is of the form $\frac{d^2x}{dt^2} = -\omega^2 x$

$$\therefore \quad V_y = V_{y0}\sin(\omega t + \delta) \qquad \left[\omega = \frac{qB_0}{m}\right]$$

at

$$t = 0;\ V_y = 0 \quad \Rightarrow \quad \delta = 0$$

$$\therefore \quad V_y = V_{y0}\sin\left(\frac{qB_0}{m}t\right)$$

$$a_y = \frac{dV_y}{dt} = V_{y0}\frac{qB_0}{m}\cos\left(\frac{qB_0}{m}t\right)$$

at

$$t = 0;\ a_y = \frac{qE_0}{m}$$

$$\therefore \quad \frac{qE_0}{m} = \frac{qB_0}{m} V_{y0} \quad \Rightarrow \quad V_{y0} = \frac{E_0}{B_0}$$

$$V_y = \frac{E_0}{B_0} \sin\left(\frac{qB_0}{m} t\right)$$

$$\frac{dy}{dt} = \frac{E_0}{B_0} \sin\left(\frac{qB_0}{m} t\right)$$

$$\int_0^y dy = \frac{E_0}{B_0} \int_0^t \sin\left(\frac{qB_0}{m} t\right) dt$$

$$y = \frac{mE_0}{qB_0^2}\left[1 - \cos\left(\frac{qB_0}{m} t\right)\right]$$

$$\therefore \quad y_{\max} = \frac{2mE_0}{qB_0^2}$$

$$\therefore \quad 2R = \frac{2mE_0}{qB_0^2}$$

$$R = \frac{mE_0}{qB_0^2}$$