Question Bank

Standard:- 12th (**Arts and Science**)

Subject:- **MATHEMATICS AND STATISTICS**

सूचना

१. फक्त विद्यार्थ्यांना प्रश्नप्रकारांचा सराव करून देण्यासाठीच

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QUESTION BANK

STD XII   Arts and Science Stream

MATHEMATICS AND STATISTICS   (40)

Part-I

1. MATHEMATICAL LOGIC

Q1) Select and write the most appropriate answer from the given alternatives:

i)   Which of the following statement is true
a) 3 + 7 =4 or 3 − 7 = 4
b) If Pune is in Maharashtra, then Hyderabad is in Kerala
c) It is false that 12 is not divisible by 3
d) The square of any odd integer is even.

ii)   Which of the following is not a statement
a) 2+2 =4
b) 2 is the only even prime number
c) Come here
d) Mumbai is not in Maharashtra

iii) If p is any statement then (p ∨~p) is a
a) Contingency
b) Contradiction
c) Tautology
d) None of these

iv)   If p and q are two statements , then (p → q) ↔ (~q →~p) is
a) Contradiction
b) Tautology
c) Neither (i) nor (ii)
d) None of these

v)   Negation of p → (p∨~q) is
a) ~p → (~p ∨ q)
b) p ∧ (~p ∧ q)
c) ~p ∨ (~p ∨~q)
d) ~p → (~p → q)

vi) If p : He is intelligent
     q : He is strong
     Then, symbolic form of statement "It is wrong that, he is intelligent or strong " is
     a) ~p ∨ ~q
     b) ~(p ∧ q)
     c) ~(p ∨ q)
     d) p ∨ ~q

vii) A biconditional statement is the conjunction of two -------------- statements
     a) Negative
     b) Compound
     c) Connective
     d) Conditional

viii) If p → q is an implication , then the implication ~q → ~ p is called its
     a) Converse
     b) Contrapositive
     c) Inverse
     d) Alternative

ix) The negation of the statement (p ∧ q) → (r ∨ ~p)
     a) p ∧ q∧ ~ r
     b) (p ∧ q) ∨ r
     c) p ∨ q ∨ ~ r
     d) (p v q) ∧ (r ∨ s)

x) The false statement in the following is
     a) p ∧ (~p) is contradiction
     b) (p → q) ↔ (~ q → ~p) is a contradiction
     c) ~ (~p) ↔ p is a tautology
     d) p ∨ (~p) ↔ p is a tautology

Q 2) Attempt the following   1 marks

     i)   Find the negation of $10 + 20 = 30$
     ii)   State the truth Value of $x^2 = 25$
     iii)   Write the negation of p → q
     iv)   State the truth value of $\sqrt{3}$ is not an irrational number
     v)   State the truth value of (p ∨ ~p)
     vi)   State the truth value of (p ∧ ~p)

Q3) Attempt the following   2 marks

i) : If statements p, q are true and r, s are false, determine the truth values of the following.

a) $\sim p \wedge (q \vee \sim r)$ b) $(p \wedge \sim r) \wedge (\sim q \vee s)$

ii) Write the following compound statements symbolically.

a) Nagpur is in Maharashtra and Chennai is in Tamilnadu.

b) Triangle is equilateral or isosceles.

iii) . Write the converse and contrapositive of the following statements.

"If a function is differentiable then it is continuous".

iv) Without using truth table prove that :

$\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

Answers

i)   a) F   b) F   ii) a) $p \wedge q$ b) $p \vee q$
ii)   converse: If function is continuous then it is differentiable.
      Contrapositive: If function is not continuous then it is not differentiable.

Q4) Answer the following questions

i) Write the negation of the statement " An angle is a right angle if and only if it is of measure 90^0"

ii) Write the following statements in symbolic form
   a) Milk is white if and only if the sky is not blue
   b) If Kutab – Minar is in Delhi then Taj- Mahal is in Agra
   c) Even though it is not cloudy , it is still raining

iii) Use quantifiers to convert the given open sentence defined on N into a true statement

a) $n^2 \geq 1$

b) $3x - 4 < 9$

c) $Y + 4 > 6$

iv) Examine whether the statement pattern is a tautology, contradiction or contingency

$(p \wedge {\sim}q) \rightarrow ({\sim}p \wedge {\sim}q)$

v) Using truth table prove that ${\sim}p \wedge q \equiv (p \vee q) \wedge {\sim}p$

vi) Write the dual of the following

a) 13 is prime number and India is a democratic country

b) $(p \wedge {\sim}q) \vee ({\sim}p \wedge q) \equiv (p \vee q) \wedge {\sim}(p \wedge q)$

vii) Write the converse, inverse and contrapositive of the statement "If it snows, then they do not drive the car"

Q5) Answer the following questions

i) Examine whether the statement pattern $[p \rightarrow ({\sim}q \vee r)] \leftrightarrow {\sim}[p \rightarrow (q \rightarrow r)]$ is a tautology, contradiction or contingency.

ii) Using truth table prove that $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

iii) Without using truth table show that $(p \vee q) \wedge ({\sim}p \vee {\sim}q) \equiv (p \vee {\sim}q) \wedge ({\sim}p \vee q)$

iv) With proper justification state the negation of $(p \leftrightarrow q) \vee ({\sim}q \rightarrow {\sim}r)$

v) Prepare truth table for $(p \wedge q) \vee {\sim}r$

2. MATRICES

I. MCQ (2 marks each)

Q. 1. The adjoint matrix of $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ is

a) $\begin{bmatrix} 4 & 8 & 3 \\ 2 & 1 & 6 \\ 0 & 2 & 1 \end{bmatrix}$
b) $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$
c) $\begin{bmatrix} 11 & 9 & 3 \\ 1 & 2 & 8 \\ 6 & 9 & 1 \end{bmatrix}$
d) $\begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 3 \\ -2 & 3 & -3 \end{bmatrix}$

Q. 2. $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is

a) A 　　　　　　b) $-A$ 　　　　　　c) adj (A) 　　　　　　d) -adj (A)

Q. 3. The solution (x, y, z) of the equation

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \text{ is } (x, y, z) =$$

a)$(1, 1, 1)$ 　　　　b) $(0, -1, 2)$ 　　　　c) $(-1, 2, 2)$ 　　　　d) $(-1, 0, 2)$

Q. 4. If ω is a complex cube root of unity, then the matrix $A = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{bmatrix}$ is

a) Singular matrix 　　　　　　　　b) Non-symmetric matrix

c) Skew-symmetric matrix 　　　　　　d) Non- Singular matrix

Q. 5. If $A = \begin{bmatrix} 4 & -1 \\ -1 & k \end{bmatrix}$ such that $A^2 - 6A + 7I = 0$, then k $=$...

a) 1 　　　　b) 3 　　　　c) 2 　　　　d) 4

Q. 6. $\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} =$.....

a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 　　　b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ 　　　c) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ 　　　d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q. 7. If $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$, then the only correct statement about the matrix A

is....

a) $A^2 = I$ 　　b) A is a zero matrix 　　c) A^{-1} does not exit

d) $A = (-1) I$, where I is a unit matrix.

Q. 8. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then $A^{10} =$.....

a) $\begin{bmatrix} \cos10\alpha & -\sin10\alpha \\ \sin10\alpha & \cos10\alpha \end{bmatrix}$ 　　　　b) $\begin{bmatrix} \cos10\alpha & \sin10\alpha \\ -\sin10\alpha & \cos10\alpha \end{bmatrix}$

c) $\begin{bmatrix} \cos10\alpha & \sin10\alpha \\ -\sin10\alpha & -\cos10\alpha \end{bmatrix}$ 　　　　d) $\begin{bmatrix} \cos10\alpha & -\sin10\alpha \\ -\sin10\alpha & -\cos10\alpha \end{bmatrix}$

Q. 9. The element of second row and third column in the inverse of $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is...

    a) -2　　　　　b) -1　　　　　c) 1　　　　　d) 2

Q. 10. If $A = \begin{bmatrix} 4 & 5 \\ 2 & 5 \end{bmatrix}$, then $|(2A)^{-1}| = ...$

    a) $\frac{1}{30}$　　　　b) $\frac{1}{20}$　　　　c) $\frac{1}{60}$　　d) $\frac{1}{40}$

Q. 11. If $\begin{bmatrix} x - y - z \\ -y + z \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$, then the value of x, y and z are respectively...

    a) 0, -3, 3　　　　b) 1, -2 , 3　　c) 5, 2, 2　　d) 11, 8, 3

Q. 12 The value of x, y, z for the following system of equations

    x + y + z = 6,　x - y+ 2z = 5, 2x + y -z = 1 are...

    a) x = 1, y = 2, z = 3　　　b) x = 2, y = 1, z = 3

    c) x = -1, y = 2, z = 3　　　d) x = y = z = 3

Q. 13. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then $|A||adjA| = ...$

    a) 3^3　　　b) 3^9　　　　c) 3^6　　　　a) 3^{27}

Q. 14. System of equations x + y = 2 , 2x + 2y = 3 has....

    a) no solution　　　　　　　b) only one solution

    c) many finite solutions.　　　d) infinite solutions.

Q. 15.　If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$ and B is the inverse of matrix A, then $\alpha = ...$

    a) -2　　　　　b) -1　　　　　c) 2　　　　　d) 5

II. Very Short Answers (1 mark)

Q. 1. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$, then find the value of $a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$

Q. 2. For an invertible matrix A, if $A \cdot (adjA) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then find the value of $|A|$.

Q. 3. If the inverse of the matrix $\begin{bmatrix} \alpha & 14 & -1 \\ 2 & 3 & 1 \\ 6 & 2 & 3 \end{bmatrix}$ does not exists then find the value of α.

Q. 4. If $A = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then find the matrix $(B^{-1}A^{-1})^{-1}$

Q. 5. $A = \begin{bmatrix} cos\theta & -sin\theta \\ -sin\theta & -cos\theta \end{bmatrix}$ then find A^{-1}.

Q. 6. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then find the value of $|A|^{-1}$

Q. 7. If $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$, and $AB = BA = I$, then find the matrix B.

Q. 8. If $A(\alpha) = \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}$ then prove that $A^2(\alpha) = A(2\alpha)$

Q. 9. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -2 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 2 \\ 4 & -1 & 3 \end{bmatrix}$ then find the order of AB.

Q. 10. $A + I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$ then find the value of $(A+I)(A-I)$

Q. 11. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$ then find A^2

Q. 12. If $A = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$ then find A^2

Q. 13. If $A = \begin{bmatrix} 0 & 3 & 3 \\ -3 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, find the matrix $B'(AB)$

III. Short Answers (2 marks)

Q. 1. If $f(x) = x^2 - 2x - 3$ then find $f(A)$ when $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Q. 2. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 & -2 \end{bmatrix}$, find $B'A'$

Q. 3 If A is invertible matrix of order 3 and $|A| = 5$, then find $|adjA|$

Q. 4. If $A = \begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$ then find $A'B'$

Q. 5. If $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ then find $(A^{-1}B^{-1})$

Q. 6. If $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ then find the matrix X such that $A^{-1}X = B$

Q. 7. Find the matrix X such that $AX = I$ where $A = \begin{bmatrix} 6 & 17 \\ 1 & 3 \end{bmatrix}$

Q. 8. Find A^{-1} using adjoint method, where $A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$

Q. 9. Find A^{-1} using column transformations :

$$\text{i) } A = \begin{bmatrix} 5 & 3 \\ 3 & -2 \end{bmatrix} \qquad \text{ii) } A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

Q. 10. Find the adjoint of matrix $A = \begin{bmatrix} 6 & 5 \\ 3 & 4 \end{bmatrix}$

Q. 11. Transform $\begin{bmatrix} 1 & 2 & 4 \\ 3 & -1 & 5 \\ 2 & 4 & 6 \end{bmatrix}$ into an upper triangular matrix by using suitable row transformations.

IV. Short answers (3 Marks)

Q. 1. If $A = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$, then find A^2 and hence find A^{-1}

Q. 2 If $A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$, then find $(AB)^{-1}$

Q. 3. If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$, find adj(A)

Q. 4. Solve the following by inversion method $2x + y = 5$, $3x + 5y = -3$

Q. 5. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -2 & 5 \end{bmatrix}$, apply $R_1 \leftrightarrow R_2$ and then $C_1 \to C_1 + 2C_3$ on A.

Q. 6. Three chairs and two tables costs ₹ 1850. Five chairs and three tables costs ₹2850. Find the cost of four chairs and one table by using matrices.

Q. 7. If $A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$, show that $A^{-1} = \frac{1}{6}(A - 5I)$

Q. 8. Find the adjoint of matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$

Q. 9. Find the matrix X such that $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix} X = \begin{bmatrix} 2 & 2 & -5 \\ -2 & -1 & 4 \\ 1 & 0 & -1 \end{bmatrix}$,

Q. 10. Find the inverse of $A = \begin{bmatrix} \sec\theta & \tan\theta & 0 \\ \tan\theta & \sec\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q. 11. Transform $\begin{bmatrix} 1 & 2 & 4 \\ 3 & -1 & 5 \\ 2 & 4 & 6 \end{bmatrix}$ into an upper triangular matrix by using suitable row transformations.

Q. 12. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$, then find the matrix X such that XA = B

V. Long answers (4 Marks)

Q. 1. Find the inverse of $A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$ by using elementary row transformations.

Q. 2. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$, find A^{-1} by the adjoint method.

Q. 3. Solve the following equations by using inversion method.

$x + y + z = -1$, $x - y + z = 2$ and $x + y - z = 3$

Q. 4.   If three numbers are added, their sum is 2. If 2 times the second number is subtracted from the sum of first and third numbers, we get 8. If three times the first number is added to the sum of second and third numbers, we get 4. Find the numbers using matrices.

Q. 5. Find the inverse of $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ by elementary column transformations.

Q. 6. If $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is a non-singular matrix, then find A^{-1} by using elementary row transformations. Hence, write the inverse of $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

Q. 7.   Find the inverse of $A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ by using elementary row transformations.

Q. 8. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 24 & 7 \\ 31 & 9 \end{bmatrix}$, then find the matrix X such that AXB = C.

Q. 9. If $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$, verify that A(adj A) = (adj A)A.

Q. 10.   If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$, find AB and $(AB)^{-1}$.

Q. 11. Solve the following system of equations by using inversion method.

$$x + y = 1, \ y + z = \frac{5}{3} \ \text{and} \ z + x = \frac{4}{3}$$

Q. 12. The cost of 4 dozen pencils, 3 dozen pens and 2 dozen erasers is ₹ 60. The cost of 2 dozen pencils, 4 dozen pens and 6 dozen erasers is ₹ 90. Whereas the cost of 6 dozen pencils, 2 dozen pens and 3 dozen erasers is ₹ 70. Find the cost of each item per dozen by using matrices.

3. TRIGONOMETRIC FUNCTIONS

I. MCQ (2 marks each)

1) The principal solutions of $\sqrt{3}\sec x - 2 = 0$ are _____.

    a) $\dfrac{\pi}{3}, \dfrac{11\pi}{6}$ b) $\dfrac{\pi}{6}, \dfrac{11\pi}{6}$ c) $\dfrac{\pi}{4}, \dfrac{11\pi}{4}$ d) $\dfrac{\pi}{6}, \dfrac{11\pi}{3}$

2) In ΔABC, if $\cos A = \dfrac{\sin B}{2\sin C}$, then ΔABC is _____.

    a) an equilateral triangle.   b) a right angled triangle.

    c) an isosceles triangle.   d) an isosceles right angled triangle.

3) $\sin^{-1} x - \cos^{-1} x = \dfrac{\pi}{6}$, then $x =$ _____.

    a) $\dfrac{1}{2}$ b) $\dfrac{\sqrt{3}}{2}$ c) $-\dfrac{1}{2}$ d) $-\dfrac{\sqrt{3}}{2}$

4) The principal value of $\sin^{-1}\left(\dfrac{1}{2}\right)$ is _____.

    a) $\dfrac{\pi}{3}$ b) $\dfrac{\pi}{6}$ c) $\dfrac{2\pi}{3}$ d) $\dfrac{3\pi}{2}$

5) The principal value of $\cos^{-1}\left(-\dfrac{1}{2}\right)$ is _____.

    b) $\dfrac{\pi}{3}$ b) $\dfrac{\pi}{6}$ c) $\dfrac{2\pi}{3}$ d) $\dfrac{3\pi}{2}$

6) In ΔABC, if $\angle A = 30°$, $\angle B = 60°$, then the ratio of sides is _____.

    a) $1:\sqrt{3}:2$ b) $2:\sqrt{3}:1$ c) $\sqrt{3}:1:2$ d) $\sqrt{3}:2:1$

7) In ΔABC, if $b^2 + c^2 - a^2 = bc$, then $\angle A =$ _____.

    a) $\dfrac{\pi}{4}$ b) $\dfrac{\pi}{3}$ c) $\dfrac{\pi}{2}$ d) $\dfrac{\pi}{6}$

8) If polar co-ordinates of a point are $\left(\dfrac{3}{4}, \dfrac{3\pi}{4}\right)$, then its Cartesian co-ordinate are _____.

    a) $\left(\dfrac{3}{4\sqrt{2}}, -\dfrac{3}{4\sqrt{2}}\right)$ b) $\left(\dfrac{3}{4\sqrt{2}}, \dfrac{3}{4\sqrt{2}}\right)$ c) $\left(-\dfrac{3}{4\sqrt{2}}, \dfrac{3}{4\sqrt{2}}\right)$ d) $\left(-\dfrac{3}{4\sqrt{2}}, -\dfrac{3}{4\sqrt{2}}\right)$

9) $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = $ _____.

        a) $-\frac{\pi}{6}$ b) $\frac{\pi}{6}$ c) $\frac{13\pi}{6}$ d) $\frac{5\pi}{6}$

10) If $\sin\left(\sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1}(x)\right) = 1$, then $x = $ _____.

        a) $\frac{1}{5}$ b) $-\frac{1}{5}$ c) 5   d) -5

II. Very Short Answers (1 mark)

1) Evaluate $\cot(\tan^{-1}(2x) + \cot^{-1}(2x))$.

2) In ΔABC, prove that $ac\cos B - bc\cos A = a^2 - b^2$.

3) In ΔABC, if $\sin^2 A + \sin^2 B = \sin^2 C$, then show that $a^2 + b^2 = c^2$.

4) Find the polar co-ordinates of point whose Cartesian co-ordinates are $(1, \sqrt{3})$.

5) Prove that $2\tan^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\left(\frac{24}{7}\right)$.

6) Evaluate $\sin[\cos^{-1}\left(\frac{3}{5}\right)]$.

7) In ΔABC, $a = 3, b = 4$ and $\sin A = \frac{3}{4}$, find $\angle B$.

8) Find the principal solutions of $\operatorname{cosec} x = 2$.

9) Find the principal solutions of $\sin x - 1 = 0$.

10) Find the Cartesian co-ordinates of point whose polar co-ordinates are $\left(4, \frac{\pi}{3}\right)$.

III. Short Answer Questions (*2 marks each*):

1) With usual notations, prove that $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2+b^2+c^2}{2abc}$.

2) Find the principal solutions of $\cos 2x = 1$.

3) In ΔABC, prove that $(b-c)^2 \cos^2\left(\frac{A}{2}\right) + (b+c)^2 \sin^2\left(\frac{A}{2}\right) = a^2$.

4) Find the principal solutions of $\sin x = -\frac{1}{2}$.

5) Find the value of $\cos^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

6) In ΔABC, if $a = 13, b = 14, c = 15$, then find the value of $\cos B$.

7) In ΔABC, if $\frac{\cos A}{a} = \frac{\cos B}{b}$, then show that it is an isosceles triangle.

8) Find the principal solution of $\tan x = -\sqrt{3}$.

9) Evaluate $\cos\left[\frac{\pi}{6} + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$.

IV. Short Answer Questions (*3 marks each*):

1) In ΔABC, if $a \cos A = b \cos B$, then prove that ΔABC is either a right angled or an isosceles triangle.

2) In ΔABC, prove that $\frac{\cos 2A}{a^2} - \frac{\cos 2C}{c^2} = \frac{1}{a^2} - \frac{1}{c^2}$.

3) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then show that $\frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx} = 1$.

4) Prove that $\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] = 1$.

5) In ΔABC, if $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$, then show that the triangle is a right angled.

6) In ΔABC, prove that $\sin\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{c}\right)\cos\left(\frac{C}{2}\right)$.

7) If the angles A, B, C of ΔABC are in A.P. and its sides a, b, c are in G.P., then show that a^2, b^2, c^2 are in A.P.

8) Prove that $\cot^{-1}(7) + 2\cot^{-1}(3) = \frac{\pi}{4}$.

V. Long Answer Questions (*4 marks each*):

1) In ΔABC, prove that $\dfrac{\cos^2 A - \cos^2 B}{a+b} + \dfrac{\cos^2 B - \cos^2 C}{b+c} + \dfrac{\cos^2 C - \cos^2 A}{c+a} = 0.$

2) Show that $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{36}{85}\right).$

3) In ΔABC, prove that $\dfrac{a^2 \sin(B-C)}{\sin A} + \dfrac{b^2 \sin(C-A)}{\sin B} + \dfrac{c^2 \sin(A-B)}{\sin C} = 0.$

4) In ΔABC, prove that $\dfrac{b^2-c^2}{a}\cos A + \dfrac{c^2-a^2}{b}\cos B + \dfrac{a^2-b^2}{c}\cos C = 0.$

5) Prove that $2\tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{7}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) = \frac{\pi}{4}.$

6) In ΔABC, if $\angle A = \frac{\pi}{2}$, then prove that $\sin(B - C) = \dfrac{b^2-c^2}{b^2+c^2}.$

7) If $\cos^{-1}x + \cos^{-1}y - \cos^{-1}z = 0$, then show that

$x^2 + y^2 + z^2 - 2xyz = 1.$

4. PAIR OF LINES

I. MCQ (2 marks each)

1.The combined equation of the two lines passing through the origin ,each making angle 45^0 and 135^0 with the positive X axis is …

A. $x^2 + y^2 = 0$ B.   xy=1

C. $x^2 - y^2 = 0$ D . $x^2 + $ xy $=0$

2. The separate equations of the lines represented by $3x^2 - 2\sqrt{3}\,xy - 3y^2 = 0$ are…

A.   x+$\sqrt{3}$y=0 and $\sqrt{3}$x+y=0   B x-$\sqrt{3}$y=0 and $\sqrt{3}$x-y=0

C. x-$\sqrt{3}$y=0 and $\sqrt{3}$x+y=0   D x+$\sqrt{3}$y=0 and $\sqrt{3}$x-y=0

3.The equation $4x^2 + 4xy + y^2 = 0$ represents two….

A.real and distinct lines   B.real and coincident lines
C.imaginary lines   D. perpendicular lines

4.If the lines represented by $kx^2 - 3xy + 6y^2 = 0$ are perpendicular to each other then…..

A.k = 6   B.k= -6   C. k = 3   D.   k = -3

5. Auxillary equation of $2x^2 + 3xy - 9y^2 = 0$ is…..

A. $2m^2 + 3m - 9 = 0$ B. $9m^2 - 3m - 2 = 0$

C. $2m^2 - 3m + 9 = 0$ D. $-9m^2 - 3m + 2 = 0$

6. The combined equation of the lines through origin and perpendicular to the pair of lines $3x^2 + 4xy - 5y^2 = 0$ is…….

A $5x^2 + 4xy - 3y^2 = 0$ B. $3x^2 + 4xy - 5y^2 = 0$

C. $3x^2 - 4xy + 5y^2 = 0$ D. $5x^2 + 4xy + 3y^2 = 0$

7.The acute angle between the lines represented by $x^2 + xy = 0$ is……….

A. $\dfrac{\pi}{2}$ B. $\dfrac{\pi}{4}$ C. $\dfrac{\pi}{6}$ D. $\dfrac{\pi}{3}$

8 .If   2x+y=0 is one of the line represented by $3x^2 + kxy + 2y^2 = 0$

then $k = $ …

A. $\dfrac{1}{2}$.   B. $\dfrac{11}{2}$ C. $\dfrac{2}{3}$ D. $\dfrac{3}{2}$

II. Very Short Answers (2 mark)

1) Find the combine equation of the pair of lines passing through the point   (2,3) and parallel to the coordinate axes.
2) Find the separate equations of the lines given by $x^2 + 2xy\tan\alpha - y^2 = 0$
3) Find k, if the sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their products.
4) Find the measure of acute angle between the lines given by $x^2 - 4xy + y^2 = 0$
5) Find the value of h , if the measure of the angle between the lines $3x^2 + 2hxy + 2y^2 = 0$ is 45^0.

III. Short Answers (3 marks)
1)   Find the combine equation of pair of lines passing through (-1,2), one is parallel to x+3y-1=0 and other is perpendicular to 2x-3y-1=0.
2)   Find the joint equation of pair of lines through the origin which are perpendicular to the lines represented by $5x^2 + 2xy - 3y^2 = 0$
3) Find the condition that the line 4x+5y=0 coincides with one of the lines given by $ax^2 + 2hxy + by^2 = 0$
4) Find the measure of acute angle between the lines represented by $3x^2 - 4\sqrt{3}\,xy + 3y^2 = 0$

IV. Short answers (4 Marks)

1) Show that the combine equation of pair of lines passing through the origin is a homogeneous equation of degree 2 in x and y. Hence find the combined equation of the lines 2x+3y=0 and x-2y=0

2) Show that the homogeneous equation of degree 2 in x and y represents a pair of lines passing through the origin if $h^2 - ab \geq 0$

3) If θ is the acute angle between the lines given by $ax^2 + 2hxy + by^2 = 0$

then prove that $\tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a+b} \right|$. Hence find acute angle between the lines $2x^2 + 7xy + 3y^2 = 0$

4) If the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between the lines $2x^2 - 5xy + 3y^2 = 0$ then show that $100(h^2 - ab) = (a + b)^2$

5. VECTOR AND THREE DIMENSIONAL GEOMETRY

I. MCQ (2 marks each)

1) If $|\bar{a}| = 3$, $|\bar{b}| = 4$, then the value of λ for which $\bar{a} + \lambda \bar{b}$ is perpendicular to

$\bar{a} - \lambda \bar{b}$ is ....

A) $\frac{9}{16}$ B) $\frac{3}{4}$ C) $\frac{3}{2}$ D) $\frac{4}{3}$

2) $(\hat{\imath} + \hat{\jmath} - \hat{k}).(\hat{\imath} - \hat{\jmath} + \hat{k}) =$_____

A) $\hat{\imath} - \hat{\jmath} - \hat{k}$ B) 1   C) -1   D) $-\hat{\jmath} + \hat{k}$

3) The angle θ between two non-zero vectors $\bar{a}$ & $\bar{b}$ is given by $\cos \theta = \cdots$

A) $\frac{\bar{a}.\bar{b}}{|\bar{a}||\bar{b}|}$ B) $\bar{a}.\bar{b}$ C) $|\bar{a}||\bar{b}|$ D) $\frac{|\bar{a}||\bar{b}|}{\bar{a}.\bar{b}}$

4) If sum of two unit vectors is itself a unit vector, then the magnitude of their difference is...
A) $\sqrt{2}$ B) $\sqrt{3}$ C) 1   D) 2

5) If α, β, γ are direction angles of a line and $\alpha = 60°$, $\beta = 45°$, then $\gamma =$ _____

A) 30^0 or 90^0 B) 45^0 or 60^0 C) 90^0 or 30^0 D)

60^0 or 120^0

6) The distance of the point $(3, 4, 5)$ from Y- axis is _____

    A) 3   B) 5   C) $\sqrt{34}$ D) $\sqrt{41}$

7) If $\cos \alpha$, $\cos \beta$, $\cos \gamma$ are the direction cosines of a line then the value of

    $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$ is _____

    A) 1   B) 2   C) 3   D) 4

8) If $|\bar{a}| = 2, |\bar{b}| = 5, and \ \bar{a}.\bar{b} = 8$ then $|\bar{a} - \bar{b}| =$ __

    A) 13   B) 12   C) $\sqrt{13}$ D) $\sqrt{21}$

9) If $\overline{AB} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$, and A(1, 2 ,-1) is given point then coordinates of B are____

    A) (3, 3, -4)   B) (-3, 3 -2)   C) (3, 3, 2)   D) (-3, 3, 4)

10) If l, m, n are direction cosines of a line then $l\hat{\imath} + m\hat{\jmath} + n\hat{k}$ is ____

    A) Null vector   B) the unit vector along the line.

    C) Any vector along the line   D) a vector perpendicular to the line.

11) The values of c that satisfy $|c\,\bar{u}| = 3$, $\bar{u} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is ___

    A) $\sqrt{14}$ B) $3\sqrt{14}$ C) $\frac{3}{\sqrt{14}}$ D) 3

12. The value of $\hat{\imath}.\left(\hat{\jmath} \times \hat{k}\right) + \hat{\jmath}.\left(\hat{\imath} \times \hat{k}\right) + \hat{k}.\left(\hat{\imath} \times \hat{\jmath}\right)$

    A) 0   B) $- 1$ C) 1   D) 3

13. The two vectors $\hat{\jmath} + \hat{k}$ & $3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ represents the two sides AB and AC,

    respectively of   a $\triangle ABC$. The length of the median through A is

    A) $\frac{\sqrt{34}}{2}$ B) $\frac{\sqrt{48}}{2}$ C) $\sqrt{18}$ D) $\sqrt{34}$

II. Very Short Answers (1 mark)

1. Find the magnitude of a vector with initial point : $(1, -3, 4)$; terminal point : $(1, 0, -1)$.
2. **2.** Find the coordinates of the point which is located Three units behind the YZ-plane, four units to the right of the XZ-plane and five units above the XY-Plane.

3. $A(2,3)$, $B(-1,5)$, $C(-1,1)$ and $D(-7,5)$ are four points in the Cartesian plane, Check if , $\overline{CD}$ is parallel to $\overline{AB}$.

4. Find a unit vector in the opposite direction of $\bar{u}$. Where $\bar{u} = 8\hat{\imath} + 3\hat{\jmath} - \hat{k}$.

5. The non zero vectors $\bar{a}$ and $\bar{b}$ are not collinear find the value of λ and μ :

   if $\bar{a} + 3\bar{b} = 2\lambda\bar{a} - \mu\bar{b}$

6. If $\bar{a} = 4\hat{\imath} + 3\hat{k}$ and $\bar{b} = -2\hat{\imath} + \hat{\jmath} + 5\hat{k}$ then find $2\bar{a} + 5\bar{b}$

7. Find the distance from $(4, -2, 6)$ to the XZ- Plane.

8. If the vectors $2\hat{\imath} - q\hat{\jmath} + 3\hat{k}$ and $4\hat{\imath} - 5\hat{\jmath} + 6\hat{k}$ are collinear then find the value of q.

9. Find $\bar{a} \cdot \bar{b} \times \bar{c}$, if $\bar{a} = 3\hat{\imath} - \hat{\jmath} + 4\hat{k}$, $\bar{b} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$, $\bar{c} = -5\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

10. If a line makes angle 90^0 , 60^0 and 30^0 with the positive direction of X, Y and Z axes respectively, find its direction cosines.

III. Short Answers (2 mark)

1. The vector $\bar{a}$ is directed due north and $|\bar{a}| = 24$. The vector $\bar{b}$ is directed due west and $|\bar{b}| = 7$. find $|\bar{a} + \bar{b}|$.

2. Show that following points are collinear $P(4,5,2), Q(3,2,4), R(5,8,0)$

3. If a vector has direction angles 45^0 and 60^0 find the third direction angle.

4. If $\bar{c} = 3\bar{a} - 2\bar{b}$ then prove that $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

5. If $|\bar{a} \cdot \bar{b}| = |\bar{a} \times \bar{b}|$ & $\bar{a} \cdot \bar{b} < 0$, then find the angle between $\bar{a}$ & $\bar{b}$.

6. Find the direction ratios of a vector perpendicular to the two lines whose direction ratios are 1, 3, 2 and -1, 1,2

7. If $\bar{a}$, $\bar{b}$ and $\bar{c}$ are position vectors of the points A, B, C respectively and $5\bar{a} - 3\bar{b} - 2\bar{c} = \bar{0}$, then find the ratio in which the point C divides the line segment BA.

8. If $\bar{a}$ and $\bar{b}$ are two vectors perpendicular each other, prove that
$$(\bar{a} + \bar{b})^2 = (\bar{a} - \bar{b})^2$$

9. Find the position vector of point R which divides the line joining the points P and Q whose position vectors are $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $-5\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ in the ratio $3:2$
   (i) internally (ii) externally.

10. Find a unit vector perpendicular to the vectors $\hat{\jmath} + 2\hat{k}$ & $\hat{\imath} + \hat{\jmath}$

IV. Short Answers (3 mark)

1. If two of the vertices of the triangle are $A(3,1,4)$ and $B(-4,5,-3)$ and the centroid of a triangle is $G(-1,2,1)$, then find the co-ordinates of the third vertex C of the triangle.

2. Find the centroid of tetrahedron with vertices $K(5,-7,0)$, $L(1,5,3), M(4,-6,3), N(6,-4,2)$?

3. If a line has the direction ratios , $4,-12,18$ then find its direction cosines.

4. Show that the points $A(2,-1,0)\ B(-3,0,4), C(-1,-1,4)$ and $D(0,-5,2)$ are non coplanar.

5. Using properties of scalar triple product, prove that

$$\begin{bmatrix} \bar{a}+\bar{b} & \bar{b}+\bar{c} & \bar{c}+\bar{a} \end{bmatrix} = 2 \begin{bmatrix} \bar{a} & \bar{b} & \bar{c} \end{bmatrix}$$

6. The direction ratios of $\overline{AB}$ are $-2,2,1$. If $A = (4,1,5)$ and $l(AB) = 6$ units, Then find B.

7. If $G(a,2,-1)$ is the centroid of the triangle with vertices $P(1,2,3)$,

   $Q(3,b,-4)$ and $R(5,1,c)$ then find the values of a,b and c.

8. If $A(5,1,p)$, $B(1,q,p)$ and $C(1,-2,3)$ are vertices of triangle and

   $G\left(r,-\dfrac{4}{3},\dfrac{1}{3}\right)$ is its centroid then find the values of p,q & r.

9. Prove by vector method that the angle subtended on semicircle is a right angle.

10. Prove that medians of a triangle are concurrent.

11. Prove that altitudes of a triangle are concurrent.

V. Long Answers (4 mark)

1. Express $-\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ as linear combination of the vectors $2\hat{\imath} + \hat{\jmath} - 4\hat{k}$,

   $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $3\hat{\imath} + \hat{\jmath} - 2\hat{k}$.

2. If Q is the foot of the perpendicular from P(2,4,3) on the line joining the points A(1,2,4) and B(3,4,5), find coordinates of Q.

3. Prove that the angle bisectors of a triangle are concurrent.

4. Using vector method , find the incenter of the triangle whose vertices are

   $A(0,3,0) \quad B(0,0,4)$ and $C(0,3,4)$.

5. Find the angles between the lines whose direction cosines l, m, n satisfy the

   equations $5l + m + 3n = 0$ and $5mn - 2nl + 6lm = 0$

6. Let $A(\bar{a})$ and $B(\bar{b})$ be any two points in the space and $R(\bar{r})$ be a point on the line segment AB dividing it internally in the ratio $m : n$ then prove that $\bar{r} = \frac{m\bar{b}+n\bar{a}}{m+n}$.

7. D and E divides sides BC and CA of a triangle ABC in the ratio $2 : 3$

   respectively. Find the position vector of the point of intersection of AD and BE

   and the ratio in which this point divides AD and BE.

8. If $\bar{u} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\bar{r} = 3\hat{\imath} + \hat{k}$ & $\bar{w} = \hat{\jmath}, \hat{k}$ are given vectors , then find

   $[\bar{u} + \bar{w}] . [(\bar{w} \times \bar{r}) \times (\bar{r} \times \bar{w})]$

9. Find the volume of a tetrahedron whose vertices are

   $A(-1, 2, 3)$ $B(3, -2, 1)$,C(2, 1, 3) and D(-1, -2, 4)

10. If four points A $(\bar{a})$, $B(\bar{b})$,$C(\bar{c})$ & $D(\bar{d})$ are coplanar

   then show that $[\bar{a} \quad \bar{b} \quad \bar{d}] + [\bar{b} \quad \bar{c} \quad \bar{d}] + [\bar{c} \quad \bar{a} \quad \bar{d}] = [\bar{a} \quad \bar{b} \quad \bar{c}]$

6. LINE AND PLANE

I. Multiple choice questions…..**(2 marks)**

1) The equation of X axis is…

A) $x = y = z$ (B) $y = z$ (C) $y = 0, z = 0$ (D) $x = 0, y = 0$

2) If the perpendicular distance of the plane $2x+3y-z = k$ from the origin is $\sqrt{14}$ units , then $k = $ …

A) 14   (B) 196   (C) $2\sqrt{14}$ (D) $\frac{\sqrt{14}}{2}$

3) The equation of the plane passing through the points (1 ,-1 , 1),

(3 ,2 , 4) and parallel to Y axis is…

A)$3x + 2z - 1 = 0$ (B) $3x - 2z = 1$ (C) $3x+2z+1 = 0$ (D)$3x+2z = 2$

4)The direction ratios of the line $3x+1 = 6y -2 = 1 - z$ are

A)2 ,1 ,6   (B)2 ,1 ,-6   (C)2 ,-1 , 6 (D) -2,1 ,6

5)If the planes $2x - my + z = 3$ and $4x - y + 2z = 5$ are parallel then m = …

A)– 2   (B) 2   (C) $\frac{-1}{2}$ (D) $\frac{1}{2}$

6) The direction cosines of the normal to the plane $2x - y + 2z = 3$ are

A)$\frac{2}{3}$,$\frac{-1}{3}$,$\frac{2}{3}$ (B) $\frac{-2}{3}$,$\frac{1}{3}$,$\frac{-2}{3}$ (C) 2 , -1 ,2   (D) -2 ,1 ,-2

7)If the foot of the perpendicular drawn from the origin to the plane is (4 ,-2 ,5) , then the equation of plane is…

A)$4x+y+5z = 14$ (B) $4x-2y-5z = 45$ (C) $x-2y-5z =10$ (D)$4x+y+6z=11$

8)The perpendicular distance of the origin from the plane $x-3y+4z=6$ is….

A) 6   (B) $\frac{6}{\sqrt{26}}$ (C) 36   (D) $\frac{1}{\sqrt{26}}$

9)The coordinates of the foot of perpendicular drawn from the origin to the plane $2x+y-2z = 18$ are…

A)(4 ,2 ,4)   (B) (-4 ,2 ,4)   (C) (-4 ,-2 ,4)   (D) (4 ,2 ,-4)

II. Very Short answers   **(1 marks)**

1)Find the Cartesian equation of a plane passing through A(1 ,2 ,3) and direction ratios of it's normal are 3 ,2 ,5.

2)Find the direction ratios of the normal to the plane $2x+3y+z=7$.

3)Find the vector equation of the line

$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

4)Verify if the point having position vector $4\hat{i}-11\hat{j}+2\hat{k}$ lies on the line

$\bar{r} = (6\hat{i}-4\hat{j}+5\hat{k}) + \mu (2\hat{i} +7\hat{j}+3\hat{k})$,

5)Find the Cartesian equation of the line passing through

A (1 ,2 ,3)and having direction ratios 2,3,7.

6)Find the vector equation of the line passing through the point having position vector $4\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and parallel to the vector $-2\hat{\imath} - \hat{\jmath} + \hat{k}$.

7)Find the Cartesian equation of the plane passing through the points (3 ,2 ,1) and (1 ,3 ,1)

III Short answer questions   (2 marks)

1) Find the direction ratios of the line perpendicular to the lines

$\frac{x-7}{2} = \frac{y+7}{-3} = \frac{z-6}{1}$ and $\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-6}{-2}$

2)Find direction cosines of the normal to the plane $\bar{r}.(3\hat{\imath}+4\hat{k}) = 5$

3)If the normal to the plane has direction ratios 2 ,-1 ,2 and it's perpendicular distance from origin is 6 ,find its equation.

4)Reduce the equation $\bar{r}.(3\hat{\imath} + 4\hat{\jmath} + 12\hat{k})$ 6767 =8 to normal form.

5) Find the Cartesian equation of the line passing through A(1 ,2 ,3) and

  B (2, 3, 4)

 6)Find the perpendicular distance of origin from the plane 6x-2y+3z -7=0

7)Find the acute angle between the lines x=y ; z=0 and x=0 z=0.

IV. Short answer questions   (3 marks)

1)Find Cartesian equation of the line passing through the point A(2 ,1 ,-3) and perpendicular to vectors $\hat{\imath}+\hat{\jmath}+\hat{k}$ and $\hat{\imath}+2\hat{\jmath}-\hat{k}$

2)Find the vector equation of the line passing through the point having position vector $-\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and parallel to the line $\bar{r}=(\hat{\imath}+2\hat{\jmath}+3\hat{k})+ \mu (3\hat{\imath}+2\hat{\jmath}+\hat{k})$;μ is a parameter.

3)Find the Cartesian equation of the line passing through (-1 ,-1 ,2) and parallel to the line 2x-2 = 3y+1 = 6z − 2.

4)Find the Cartesian equation of the plane passing through A(7 ,8 ,6)and parallel to XY plane.

5)Find the coordinates of the foot of perpendicular from the origin to the plane

  2x+6y -3z =63.

6)Find the vector equation of a plane at a distance 6 units from the origin and to which vector $2\hat{\imath}-\hat{\jmath}+2\hat{k}$ is normal.

7)Find the Cartesian equation of the plane passing through the points A(1 ,1 ,2),

B(0 ,2 ,3) C(4 ,5 ,6).

8)Find acute angle between the lines $\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-3}{2}$ and $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z-3}{1}$

9)Find the distance between the parallel lines $\frac{x}{2}=\frac{y}{-1}=\frac{z}{2}$ and $\frac{x-1}{2}=\frac{y-1}{-1}=\frac{z-3}{2}$.

10)Find the equation of the plane passing through the point (7 ,8 ,6) and parallel to the plane $\bar{r}.(6\hat{i}+8\hat{j}+7\hat{k})=0$

11)Find m, if the lines $\frac{1-x}{3}=\frac{7y-14}{2m}=\frac{z-3}{2}$ and $\frac{7-7x}{3m}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles.

V. Long answer questions   (4 marks)

1) show that the lines $\frac{x+1}{-10}=\frac{y+3}{-1}=\frac{z-4}{1}$ and $\frac{x+10}{-1}=\frac{y+1}{-3}=\frac{z-1}{4}$ intersect each other.also find the coordinates of the point of intersection.

2)A(-2 ,3 ,4) B(1 ,1 ,2) C(4 ,-1 ,0) are three points. Find the Cartesian equation of line AB and show that points A, B ,C are collinear.

3)find the Cartesian and vector equation of the line passing through the point having position vector $\hat{i}+2\hat{j}+3\hat{k}$ and perpendicular to vectors $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}-\hat{j}+\hat{k}$.

4)Find the vector equation of the plane which bisects the segment joining A(2 ,3 ,6) and B (4 ,3 ,-2) at right angles.

5)Find vector equation of the plane passing through A(-2 ,7 ,5)and parallel to vectors $4\hat{i}-\hat{j}+3\hat{k}$ and $\hat{i}+\hat{j}+\hat{k}$.

6)Find the Cartesian and vector equation of the plane which makes intercepts 1 ,1 ,1 on the coordinate axes.

7. LINEAR PROGRAMMING PROBLEMS

I. MCQ (2 marks each)

1. The corner points of the feasible solutions are (0,0) (3,0) (2,1) (0,7/3) the maximum value of Z = 4x+5y is

   a) 12   b) 13   c) 35/3   d) 0

2. The half plane represented by 4x+3y >14 contains the point

   a) (0,0)   b) (2,2) c) (3,4)   b) (1,1)

3. The feasible region is the set of point which satisfy

a) The object functions

b) All the given constraints

c) Some of the given constraints

d) Only one constraint

4. Objective function of LPP is

   a) A constraint

   b) A function to be maximized or minimized

   c) A relation between the decision variable

   d) Equation of straight line

5. The value of objective function is maximum under linear constraints

   a) At the center of the feasible region

   b) At (0,0)

   c) At vertex of feasible region

   d) At (-1, -1)

6. If a corner point of the feasible solutions are (0,10) (2,2) (4,0) (3,2) then the point of minimum $Z = 3x + 2y$ is

   a) (2,2)   b) (0,10)   c) (4,0)   b) (3,2)

7. The point of which the maximum value of z= x+y subject to constraints $x+2y \leq 70$, $2x+y \leq 90$, $x \geq 0$, $y \geq 0$ is obtained at

   a) (30,25)   b) (20,35)   c) (35,20)   b) (40,15)

8. A solution set of the inequality $x \geq 0$

   a) Half plane on the Left of y axis

   b) Half plane on the right of y axis excluding the point on y-axis

   c) Half plane on the right of y axis including the point on y axis

   d) Half plane on the upword of x axis

9. Which value of x is in the solution set of inequality $-2X+Y \geq 17$

   a) - 8   b) -6   c) -4   b) 12

10. The graph of the inequality $3X- 4Y \leq 12$, $X \leq 1$, $X \geq 0$, $Y \geq 0$ lies in fully in

    a) I quadrant   b) II quadrant   c) III quadrant   b) IV quadrant

II. Short Answers (2 marks)

1. Solve 4x-18 $\geq$ 0 graphically using xy plane

2. Sketch the graph of inequation x $\geq$ 5y in xoy co-ordinate system

3. Find the graphical solution for the system of linear inequation 2x+y $\leq$ 2, x-y$\leq$ 1

4. Find the feasible solution of linear inequation 2x+3y $\leq$ 12, 2x+y $\leq$ 8,

   x $\geq$ 0, y$\geq$ 0 by graphically

5. Solve graphically x $\geq$ 0, y $\geq$ 0

6. Find the solution set of inequalities 0 $\leq$ x $\leq$ 5, 0 $\leq$ 2y $\leq$ 7

7. Find the feasible solution of in equations 3x+2y $\leq$ 18, 2x+ y $\leq$ 10, X $\geq$ 0,

   Y $\geq$ 0

8. Draw the graph of inequalities x $\leq$ 6, y- 2 $\leq$ 0, x $\geq$ 0, y $\geq$ 0 and indicate the feasible region

9. Check the ordered points (1, - 1), (2, - 1) is a solution of 2x+3y-6 $\leq$ 0

10. Show the solution set of inequations 4x – 5y $\leq$ 20 graphically

III. Long Answers (4 marks)

1. Maximize z = 5x+2y subject to 3x+5y $\leq$ 15, 5x+2y $\leq$ 10, x $\geq$ 0, y $\geq$ 0

2. Maximize z= 7x+11y subject to 3x+5y $\leq$ 26, 5x+3y $\leq$ 30, x $\geq$ 0, y $\geq$ 0

3. Maximize z=10x+25y subject to x+y $\leq$ 5, 0 $\leq$ x $\leq$ 3, 0 $\leq$ y $\leq$ 3

4. Maximize z=3x+5y subject to x+4y $\leq$ 24, 3x+y$\leq$ 21, x+y$\leq$ 9, x $\geq$ 0, y $\geq$ 0 also find the maximum value of z

5. Minimize z=8x+10y subjected to 2x+y $\geq$ 7, 2x+3y$\geq$15, y$\geq$2, x $\geq$ 0, y $\geq$ 0

6. Minimize z=7x+y subjected to 5x+y $\geq$ 5, x+y $\geq$ 3, x $\geq$ 0, y $\geq$ 0

7. Minimize z= 6x+21y subject to x+2y$\geq$ 3, x+ 4y$\geq$ 4, 3x+y $\geq$ 3, x $\geq$ 0, y $\geq$ 0 show that the minimum value of z occurs at more than two points

8. minimize z =2x+4y is subjected to 2x+y$\geq$3, x+2y$\geq$6, x$\geq$ 0, y$\geq$ 0 show that the minimum value of z occurs at more than two points

9. Maximize z=-x+2y subjected to constraints x+y$\geq$ 5, x$\geq$3, x+2y $\leq$ 6, y $\geq$0 is this LPP solvable? Justify your answer

10. x-y $\leq$ 1, x - y $\geq$ 0, x $\geq$ 0, y $\geq$ 0   are the constant for the objective function z = x + y. It is solvable for finding optimum value of z? Justify?

Part 2

1. DIFFERENTIATION

I. MCQ (2 Marks each)

1) If $y = \sec(\tan^{-1} x)$ then $\dfrac{dy}{dx}$ at $x = 1$ is _____

(a) $\dfrac{1}{2}$　　　(b) 1　　　(c) $\dfrac{1}{\sqrt{2}}$　　　(d) $\sqrt{2}$

2) If $f(x) = \log_x (\log x)$ then $f'(e)$ is _____

(a) 1　　　(b) e　　　(c) $\dfrac{1}{e}$　　　(d) 0

3) If $y = 25^{\log_5 \sin x} + 16^{\log_4 \cos x}$ then $\dfrac{dy}{dx} = $ ____

(a) 1　　　(b) 0　　　(c) 9　　　(d) cos x − sin x

4) If $f'(4) = 5, f(4) = 3, g'(6) = 7$ and $R(x) = g[3 + f(x)]$ then $R'(4) = $ _____

(a) 35　　　(b) 12 (c) $\dfrac{7}{5}$　　　(d) 105

5) If $y = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$, $x \in (-1,1)$ then $\dfrac{dy}{dx} = $ _____.

(a) $\dfrac{-2}{1+x^2}$　　　(b) 1　　　(c) $\dfrac{2}{1+x^2}$　　　(d) $\dfrac{1}{1+x^2}$

6) If g is the inverse of f and $f'(x) = \dfrac{1}{1+x^4}$ then $g'(x) = $ _____

(a) $\dfrac{1}{1+[g(x)]^4}$　　　(b) $\dfrac{4x^3}{1+x^4}$　　　(c) $\dfrac{1}{1+[g(x)]^3}$　　　(d) $1+[g(x)]^4$

7) If $\sin^{-1}(x^3 + y^3) = a$ then $\dfrac{dy}{dx} = $ _____

(a) $\dfrac{-x}{\cos a}$　　　(b) $\dfrac{-x^2}{y^2}$　　　(c) $\dfrac{y^2}{x^2}$　　　(d) $\dfrac{\sin a}{y}$

8) If $x = \cos^{-1}(t)$, $y = \sqrt{1-t^2}$ then $\dfrac{dy}{dx} = $ _____

(a) t　　　(b) − t　　　(c) $\dfrac{-1}{t}$　　　(d) $\dfrac{1}{t}$

9) If $x^2 + y^2 = 1$ then $\dfrac{d^2 x}{dy^2} = $ _____.

(a) x^3 (b) y^3 (c) $-y^3$ (d) $-\dfrac{1}{x^3}$

10) If $x^2 + y^2 = t + \dfrac{1}{t}$ and $x^4 + y^4 = t^2 + \dfrac{1}{t^2}$ then $\dfrac{dy}{dx} =$ _____

(a) $\dfrac{x}{2y}$ (b) $-\dfrac{y}{x}$ (c) $-\dfrac{x}{2y}$ (d) $\dfrac{y}{x}$

11) If $x = a\,t^4$ $y = 2a\,t^2$ then $\dfrac{dy}{dx} =$ _____

(a) $\dfrac{1}{t}$ (b) $-\dfrac{1}{t}$ (c) $\dfrac{1}{t^2}$ (d) $-\dfrac{1}{t^2}$

II. Very Short answer questions (1 mark each)

1) Differentiate $y = \sqrt{x^2 + 5}$ w.r. to x

2) Differentiate $y = e^{\tan x}$ w.r. to x

3) If $y = \sin^{-1}(2^x)$, find $\dfrac{dy}{dx}$.

4) If $f(x)$ is odd and differentiable, then $f'(x)$ is

5) If $y = e^{1+\log x}$ then find $\dfrac{dy}{dx}$

III. Short answer questions (2 mark each)

1) If $y = \log[\cos(x^5)]$ then find $\dfrac{dy}{dx}$

2) If $y = \sqrt{\tan \sqrt{x}}$, find $\dfrac{dy}{dx}$

3) Find the derivative of the inverse of function $y = 2x^3 - 6x$ and calculate its value at $x = -2$

4) Let $f(x) = x^5 + 2x - 3$ find $(f^{-1})'(-3)$

5) If $y = \cos^{-1}[\sin(4^x)]$, find $\dfrac{dy}{dx}$

6) If $y = \tan^{-1}\left(\sqrt{\dfrac{1+\cos x}{1-\cos x}}\right)$, find $\dfrac{dy}{dx}$

7) If $x = \sin\theta$, $y = \tan\theta$ then find $\dfrac{dy}{dx}$

8) Differentiate $\sin^2(\sin^{-1}(x^2))$ w.r. to x

IV. Short answer questions (3 mark each)

1) If $y = \log\left[\sqrt{\dfrac{1-\cos\left(3x/2\right)}{1+\cos\left(3x/2\right)}}\right]$, find $\dfrac{dy}{dx}$

2) If $y = \log\left[4^{2x}\left(\dfrac{x^2+5}{\sqrt{2x^3-4}}\right)^{3/2}\right]$, find $\dfrac{dy}{dx}$

3) Differentiate $\cot^{-1}\left(\dfrac{\cos x}{1+\sin x}\right)$ w.r. to x

4) Differentiate $\sin^{-1}\left(\dfrac{2\cos x + 3\sin x}{\sqrt{13}}\right)$ w.r. to x

5) Differentiate $\tan^{-1}\left(\dfrac{8x}{1-15x^2}\right)$ w.r. to x

6) If $\log_5\left(\dfrac{x^4+y^4}{x^4-y^4}\right) = 2$, show that $\dfrac{dy}{dx} = \dfrac{12x^3}{13y^2}$

7) If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + ...\infty}}}$, show that $\dfrac{dy}{dx} = \dfrac{\sin x}{1-2y}$

8) Find the derivative of $\cos^{-1}x$ w.r. to $\sqrt{1-x^2}$

9) If $x\sin(a + y) + \sin a \cos (a + y) = 0$ then show that $\dfrac{dy}{dx} = \dfrac{\sin^2(a+y)}{\sin a}$

10) If $y = 5^x . x^5 . x^x . 5^5$, find $\dfrac{dy}{dx}$

V. Long answer questions (4 mark each)

1) If $y = e^{m\tan^{-1}x}$, show that $(1+x^2)\dfrac{d^2y}{dx^2} + (2x-m)\dfrac{dy}{dx} = 0$

2) If $x^7 . y^5 = (x + y)^{12}$, show that $\dfrac{dy}{dx} = \dfrac{y}{x}$

3) Differentiate $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ w.r. to $\tan^{-1}\left(\dfrac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

4) If $y = \sin^{-1}\left(\dfrac{a\cos x - b\sin x}{\sqrt{a^2+b^2}}\right)$ then find $\dfrac{dy}{dx}$

5) If y = cos (m cos^{-1} x) then show that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + m^2 y = 0$

:: Theorems ::

1) If y = f(u) is a differentiable function of u and u = g(x) is a differentiable function of x such that the composite function y = f[g(x)] is a differentiable function of x then $\dfrac{dy}{dx} = \dfrac{dy}{du}.\dfrac{du}{dx}$ Hence find $\dfrac{dy}{dx}$ if y = sin^2 x

2) Suppose y = f(x) is a differentiable function of x on an interval I and y is one-one, onto and $\dfrac{dy}{dx} \neq 0$ on I. Also if $f'(y)$ is differentiable on f(I) then

$\dfrac{dx}{dy} = \dfrac{1}{\dfrac{dy}{dx}}$, $\dfrac{dy}{dx} \neq 0$

3) If x = f(t) and y = g(t) are differentiable functions of t so that y is a differentiable function of x and it $\dfrac{dx}{dt} \neq 0$ then $\dfrac{dy}{dx} = \dfrac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}}$.

2. APPLICATIONS OF DERIVATIVE

I. MCQ (2 Marks each)

1. The slope of the tangent to the curve x = 2 sin$^3\theta$, y = 3 cos^3 θ at θ = $\dfrac{\pi}{4}$ is

   (A) $\dfrac{3}{2}$ (B) $-\dfrac{3}{2}$ (C) $\dfrac{2}{3}$ (D) $-\dfrac{2}{3}$

2. The slope of the normal to the curve $y = x^2 + 2e^x + 2$ at (0,4) is

   (A) 2   (B) −2   (C) $\dfrac{1}{2}$ (D) $-\dfrac{1}{2}$

3. If the line y = 4x − 5 touches the curve y^2 = ax^3 + b at the point (2, 3) then a + b is

   (A) −5   (B) 2   (C) −7   (D) 9

4. If the tangent at (1, 1) on $y^2 = x(2-x)^2$ meets the curve again at P, then P is

   (A) (4,4)   (B)(-1, 2)   (C) (3, 6)   (D)

   $\left(\dfrac{9}{4}, \dfrac{3}{8}\right)$

5. The displacement of a particle at time t is given by s = 2t^3 − 5t^2 + 4t − 3. The time when the acceleration is 14 ft/sec^2, is

(A) 1 sec   (B) 2 sec   (C) 3 sec   (D) 4 sec

6. Let $f(x) = x^3 - 62 + 9x + 18$, then $f(x)$ is strictly decreasing in …

(A) $(-\infty, 1)$ (B) $[3,\infty)$ (C) $(-\infty, 1) \cup [3,\infty)$ (D) $(1, 3)$

7. A ladder 5 m in length is resting against vertical wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of 1.5 m /sec. The length of the higher point of the when foot of ladder is 4 m away from the wall decreases at the rate of ____

(A)   1   (B) 2   (C) 2.5   D) 3

8. The edge of a cube is decreasing at the rate of 0.6 cm/sec then the rate at which its volume is decreasing when the edge of the cube is 2 cm, is

(A) 1.2 cm^3 /sec (B) 3.6 cm^3 /sec   (C)   4.8 cm^3 /sec   (D) 7.2 cm^3 /sec

9.   A particle moves along the curve $y = 4x^2 + 2$, then the point on the curve at which y – coordinate is changing 8 times as fast as the x – coordinate is

(A) (2,18)   (B) (−1,6)   (C) (1,6)   (D) (0,2)

10. The function $f(x) = x \log x$ is minimum at $x =$

(A) e (B) $\dfrac{1}{e}$ (C)   1   (D) $-\dfrac{1}{e}$

II. Very Short answer questions (1 mark each)

1. Find the slope of tangent to the curve $y = 2x^3 - x^2 + 2$ at $\left(\dfrac{1}{2}, 2\right)$.

2. The displacement of a particle at time t is given by s $= 2t^3 - 5t^2 + 4t - 3$.Find the velocity when $t = 2$ sec.

3. Prove that function f(x) $= x - \dfrac{1}{x}$, x $\in$ R and x$\neq$ 0 is increasing function.

4. Show that f(x) $= x - \cos x$ is increasing for all x.

5. Show that the function $f(x) = x^3 + 10x + 7$ for $x \in R$ is strictly increasing.

III. Short answer questions (2 mark each)

1. Find the slope of normal to the curve $3x^2 - y^2 = 8$ at the point (2,2).

2. Find the slope of tangent to the curve $x = \sin\theta$ and $y = \cos 2\theta$ at $\theta = \dfrac{\pi}{6}$.

3. Find the equation of normal to the curve y $= 2\,x^3 - x^2 + 2$ at $\left(\dfrac{1}{2}, 2\right)$

4. A car is moving in such a way that the distance it covers, is given by the equation

$s = 4t^2 + 3t$,where s is in meters and t is in seconds. What would be the velocity and the acceleration of the car at time $t = 20$ second ?

5. A man of height 2 meters walks at a uniform speed of 6 km/hr away from a lamp post of 6 meters high. Find the rate at which the length of the shadow is increasing

6. Water is being poured at the rate of 36 m³/sec in to a cylindrical vessel of base radius 3 meters. Find the rate at which water level is rising.

7. Test whether the function $f(x) = x^3 + 6x^2 + 12x - 5$ is increasing or decreasing for all $x \in R$.

8. Test whether the following function $f(x) = 2 - 3x + 3x^2 - x^3$, $x \in R$ is increasing or decreasing.

9. Find the values of x for which the function $f(x) = 2x^3 - 6x^2 + 6x + 24$ is strictly increasing.

IV. Short answer questions (3 mark each)

1. Find the point on the curve $y = \sqrt{x - 3}$ where the tangent is perpendicular to the line 6x + 3y − 5 = 0.

2. A spherical soap bubble is expanding so that its radius is increasing at the rate of 0.02 cm/sec. At what rate is the surface area is increasing, when its radius is 5 cm?

3. The surface area of a spherical balloon is increasing at the rate of 2 cm²/sec. At what rate the volume of the balloon is increasing when radius of the balloon is 6 cm?

4. A ladder 10 meter long is leaning against a vertical wall. If the bottom of the ladder is pulled horizontally away from the wall at the rate of 1.2 meters per seconds, find how fast the top of the ladder is sliding down the wall when the bottom is 6 meters away from the wall.

5. Find the values of x for which the function $f(x) = x^3 - 6x^2 -36x +7$ is strictly increasing.

6. Find the values of x, for which the function $f(x) = x^3 + 12x^2 + 36x + 6$ is monotonically decreasing.

7. The profit function *P(x)* of a firm, selling x items per day is given by $P(x) = (150 - x)x - 1625$. Find the number of items the firm should manufacture to get maximum profit. Find the maximum profit.

8. Divide the number 30 in to two parts such that their product is maximum.

9. A wire of length 36meters is bent in the form of a rectangle. Find its dimensions if the area of the rectangle is maximum.

V. Long answer questions (4 mark each)

1. Find points on the curve given by $y = x^3 - 6x^2 + x + 3$ where the tangents are parallel to the line $y = x + 5$.

2. The volume of the spherical ball is increasing at the rate of 4π cc/sec. Find the rate at which the radius and the surface area are changing when the volume is 288πcc.

3. The volume of a sphere increase at the rate of 20 cm³/sec. Find the rate of change of its surface area when its radius is 5 cm.

4. A man of height 180 cm is moving away from a lamp post at the rate of 1.2 meters per second. If the height of the lamp post is 4.5 meters, find the rate at which

   (i) his shadow is lengthening. (ii)the tip of the shadow is moving.

5. Find the values of x for which f(x) =2 x³ – 15x² -144x -7 is

   (a) Strictly increasing   (b) strictly decreasing

6. Find the local maximum and local minimum value of
   $f(x) = x^3 - 3x^2 - 24x + 5$.

7. A wire of length 120cm is bent in the form of a rectangle. Find its dimensions if the area of the rectangle is maximum.

8. An open box is to be cut out of piece of square card board of side 18 cm by cutting of equal squares from the corners and turning up the sides. Find the maximum volume of the box.

9. A rectangular sheet of paper has it area 24 sq. Meters. The margin at the top & the bottom are 75cm each & the sides 50cm each. What are the dimensions of the paper if the area of the printed space is maximum?

10. A box with a square base is to have an open top. The surface area of the box is 192 sq. cm. What should be its dimensions in order that the volume is largest?

11. A wire of length l is cut into two parts. One part is bent into a circle and the other into a square. Show that the sum of the areas of the circle and the square is least, if the radius of the circle is half the side of the square.

3. INDEFINITE INTEGRATION

I. MCQ (2marks)

Q1) $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx = \ldots$

    a) $\frac{1}{2}\sqrt{\tan x} + c$ b) $2\sqrt{\tan x} + c$

    c) $\sqrt{2} \tan x + c$ d) $\frac{1}{\sqrt{2\tan x}} + c$

Q2) $\int \frac{1}{\sqrt{x} + \sqrt{x^3}} dx = \ldots$

    a) $2\tan^{-1}(\sqrt{x}) + c$ b) $\tan^{-1}(2x - 1) + c$

    c) $2\tan \sqrt{x} + c$ d) $2\tan^{-1}(x) + c$

Q3) $\int \frac{1}{\sqrt{(x-3)(x-2)}} dx = \ldots$

    a) $\frac{1}{2}\log[(2x - 1) + \sqrt{x^2 - x - 6}] + c$ b) $\tan^{-1}(2x - 1) + c$

    c) $\log\left[\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 6}\right] + c$ d) $\log\left[\left(x - \frac{1}{2}\right) + \sqrt{x^2 + x + 6}\right] + c$

Q4) $\int \frac{1}{\cos x - \sin x} dx = \ldots$

    a) $\frac{1}{\sqrt{2}}\log\left[\operatorname{cosec}\left(x + \frac{\pi}{4}\right) - \cot\left(x + \frac{\pi}{4}\right)\right] + c$

    b) $\sqrt{2} \log\left[\operatorname{cosec}\left(x + \frac{\pi}{4}\right) + \cot\left(x + \frac{\pi}{4}\right)\right] + c$

    c) $\frac{1}{\sqrt{2}}\log\left[\sec\left(x + \frac{\pi}{4}\right) + \tan\left(x + \frac{\pi}{4}\right)\right] + c$

    d) $\sqrt{2}\log\left[\sec\left(x + \frac{\pi}{4}\right) - \tan\left(x + \frac{\pi}{4}\right)\right] + c$

Q5) $\int \frac{x^2}{\sqrt{1-x^6}} dx = \ldots$

    a) $-\sin^{-1}(x^3) + c$ b) $\cos^{-1}(x^3) + c$

    c) $\sin^{-1}(x^3) + c$ d) $\frac{1}{3}\sin^{-1}(x^3) + c$

Q6)$\int \frac{e^x}{x}[x(\log x)^2 + 2\log x]dx =...$

    a)$e^x \log x + c$ b) $e^x(\log x)^2 + c$

    c) $e^{2x} \log x + c$ d) $e^{2x}(\log x)^2 + c$

Q7)$\int \sqrt{x^2 + 2x + 5}\, dx =...$

    a)$(x + 1)\sqrt{x^2 + 2x + 5} + \log[(x + 1) + \sqrt{x^2 + 2x + 5}] + c$

    b) $(x + 2)\sqrt{x^2 + 2x + 5} + \log[(x + 2) + \sqrt{x^2 + 2x + 5}] + c$

    c) $(\frac{x+2}{2})\sqrt{x^2 + 2x + 5} + \frac{1}{2}\log[(x + 2) + \sqrt{x^2 + 2x + 5}] + c$

    d) $(\frac{x+1}{2})\sqrt{x^2 + 2x + 5} + 2\log[(x + 1) + \sqrt{x^2 + 2x + 5}] + c$

Q8)If $f(x) = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, $g(x) = e^{\sin^{-1}x}$, then $\int f(x)g(x)dx =...$

    a)$e^{\sin^{-1}x}(\sin^{-1}x - 1) + c$

    b) $e^{\sin^{-1}x}(1 - \sin^{-1}x) + c$

    c) $e^{\sin^{-1}x}(\sin^{-1}x + 1) + c$

    d) $-e^{\sin^{-1}x}(\sin^{-1}x + 1) + c$

Q9)$\int \frac{\sin^m x}{\cos^{m+2} x} dx =...$

    a)$(m + 2)\tan^{m+1}x + c$ b)$\frac{\tan^m x}{m} + c$

    c) $(m + 1)\tan^{m+1}x + c$ d) $\frac{\tan^{m+1}x}{m+1} + c$

Q10)$\int \frac{2}{\sqrt{x}-\sqrt{x+3}} dx =...$

    a)$-\frac{2}{3}\left[x^{\frac{3}{2}} + (x + 3)^{\frac{3}{2}}\right] + c$ b)$\frac{2}{3}\left[x^{\frac{3}{2}} - (x + 3)^{\frac{3}{2}}\right] + c$

    c)$\frac{4}{9}\left[x^{\frac{3}{2}} - (x + 3)^{\frac{3}{2}}\right] + c$ d)$-\frac{4}{9}\left[x^{\frac{3}{2}} + (x + 3)^{\frac{3}{2}}\right] + c$

Q11)$\int \cos\sqrt{x}\, dx =...$

    a)$2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + c$ b) $\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x} + c$

    c) $2\left[\sqrt{x}\cos\sqrt{x} + \sin\sqrt{x}\right] + c$ d)$\frac{1}{2}\left[\sqrt{x}\sin\sqrt{x} - \cos\sqrt{x}\right] + c$

Q12)$\int \frac{2(\cos^2 x - \sin^2 x)}{\cos^2 x + \sin^2 x} dx =...$

    a)$\sin 2x + c$ b)$\cos 2x + c$

    c)$\tan 2x + c$ d)$2\sin 2x + c$

Q13)$\int \frac{\log x}{(\log ex)^2} dx =...$

   a)$x(1 + \log x) + c$ b)$\frac{x}{1+\log x} + c$

   c)$\frac{1}{1+\log x} + c$ d) $\frac{1}{1-\log x} + c$

Q14)If $\int \frac{1}{x+x^5} dx = f(x) + c$, then $\int \frac{x^4}{x+x^5} dx =...$

   a)$f(x) - \log x + c$ b) $f(x)+\log x + c$

   c)$\log x - f(x) + c$ d)$\frac{1}{5}x^5 f(x) + c$

Q15)$\int \frac{e^x(x-1)}{x^2} dx =...$

   a)$xe^{-x} + c$ b)$\frac{e^x}{x^2} + c$

   c)$\left(x - \frac{1}{x}\right) e^x + c$ d)$\frac{e^x}{x} + c$

II. Very Short Answers (1 mark)

Q1)$\int \sqrt{1 + \sin 2x}\ dx$

Q2) $\int \frac{\sin 4x}{\cos 2x} dx$

Q3) $\int \frac{e^{3x}}{e^{3x}+1} dx$

Q4) $\int \frac{1}{\sqrt{3x^2+8}} dx$

Q5) $\int \frac{\log x}{x} dx$

Q6) $\int (2 + \cot x - \cosec^2 x)e^x\ dx$

Q7) $\int e^x \left[\frac{(x+3)}{(x+4)^2}\right] dx$

Q8) $\int \frac{e^{2x}+e^{-2x}}{e^x} dx$

Q9) $\int x^x(1 + \log x)\ dx$

Q10)$\int \frac{1}{x \sin^2(\log x)} dx$

Q11)$\int \sqrt{x} \sec(x)^{\frac{3}{2}} \tan(x)^{\frac{3}{2}} dx$

Q12)$\int \frac{\cos 2x}{\sin^2 x} dx$

Q13)$\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$

Q14)$\int \cot^2 x \, dx$

Q15)$\int \frac{x}{x+2} dx$

III. Short Answers (2 marks)

Q1)$\int \frac{sinx}{1+sinx} dx$

Q2)$\int \frac{(2x-7)}{\sqrt{4x-1}} dx$

Q3)$\int \frac{1}{4x+5x^{-11}} dx$

Q4)$\int e^{3logx}(x^4+1)^{-1} dx$

Q5)$\int \frac{\sin(x-a)}{\cos(x+b)} dx$

Q6)$\int \cos^7 x \, dx$

Q7)$\int \frac{x^7}{(1+x^4)^2} dx$

Q8)$\int \frac{1}{\sqrt{2x^2-5}} dx$

Q9)$\int \frac{\log(logx)}{x} dx$

Q10)$\int x^2\sqrt{a^2-x^6} \, dx$

Q11)$\int \sqrt{4^x(4^x+4)} \, dx$

Q12)$\int [cosec(logx)][1 - \cot(logx)]dx$

Q13)$\int \frac{\cos2x}{\sin^2 x \cos^2 x} dx$

Q14)$\int \sin4x \cos3x \, dx$

Q15)$\int \frac{e^x\log(sine^x)}{tane^x} dx$

IV. Short answers (3 Marks)

Q1)$\int \frac{1}{x(x^3-1)} dx$

Q2)If $f'(x) = x - \frac{3}{x^3}$, $f(1) = \frac{11}{2}$ find $f(x)$

Q3)$\int \frac{(x^2+2)}{x^2+1} a^{x+tan^{-1}x} \, dx$

Q4) $\int \frac{7+4x+5x^2}{(2x+3)^{\frac{3}{2}}} dx$

Q5) $\int \sqrt{\frac{9+x}{9-x}} dx$

Q6) $\int \frac{1}{4x^2-20x+17} dx$

Q7) $\int \frac{sinx}{sin3x} dx$

Q8) $\int \frac{1}{2+cosx-sinx} dx$

Q9) $\int sec^3x \, dx$

Q10) $\int sin(logx) \, dx$

Q11) $\int sec^2x\sqrt{tan^2x + tanx - 7} \, dx$

Q12) $\int e^{sin^{-1}x} \left[\frac{x+\sqrt{1-x^2}}{\sqrt{1-x^2}}\right] dx$

Q13) $\int e^x \frac{(1+x^2)}{(1+x)^2} \, dx$

Q14) $\int \frac{x^2+x-1}{x^2+x-6} \, dx$

Q15) $\int \frac{6x^3+5x^2-7}{3x^2-2x-7} \, dx$

V. Long answers (4 Marks)

Q1) $\int \frac{dx}{2+3tanx}$

Q2) $\int \sqrt{tanx} + \sqrt{cotx} \, dx$

Q3) $\int \frac{3x+4}{\sqrt{2x^2+2x+1}} dx$

Q4) $\int \sqrt{\frac{e^{3x}-e^{2x}}{e^x+1}} dx$

Q5) $\int x^3 tan^{-1}x \, dx$

Q6) $\int xsin2x \, cos5x \, dx$

Q7) $\int \frac{x+sinx}{1-cosx} dx$

Q8) $\int \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} dx$

Q9) $\int \frac{dx}{x^3-1}$

Q10)$\int \frac{5e^x}{(e^x+1)(e^{2x}+9)} dx$

Q11)$\int \frac{1}{sinx(3+2cosx)} dx$

Q12)$\int xcos^3x \, dx$

Q13)$\int \frac{sin2x}{3sin^4x-4sin^2x+1} dx$

Q14)$\int \frac{3e^{2x}+5}{4e^{2x}-5} dx$

Q15)$\int \frac{(2logx+3)}{x(3logx+2)[(logx)^2+1]} dx$

4. DEFINITE INTEGRATION

I. MCQ (2 marks each)

(1) $\int_1^9 \frac{x+1}{\sqrt{x}} dx$

   (A)$\frac{80}{3}$ (B)$\frac{64}{3}$ (C)$\frac{17}{3}$ (D)$\frac{15}{3}$

(2) $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx =$

   (A)$\frac{\pi}{2}-1$ (B)$\frac{\pi}{2}+1$ (C)$\frac{\pi}{2}-2$ (D)$\frac{\pi}{2}+2$

(3) $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx =$

(A)$2\sqrt{e}(1+\sqrt{e})$ (B) $\sqrt{e}(1-\sqrt{e})$ (C) $\sqrt{e}(\sqrt{e}-1)$ (D) $\sqrt{e}(1+\sqrt{e})$

(4) $\int_0^{\frac{\pi}{4}} \sqrt{1+sin2x} \, dx =$

   (A)$\frac{1}{\sqrt{2}}$ (B)$\sqrt{2}+1$ (C)$2\sqrt{2}$ (D) 1

(5) If $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}} = \frac{k}{3}$ then $k = \cdots$

   (A) $\sqrt{2}(2\sqrt{2}-2)$ (B) $\frac{\sqrt{2}}{3}(2-2\sqrt{2})$ (C)$\frac{2\sqrt{2}-2}{3}$ (D) $4\sqrt{2}$

(6)$\int_{\frac{\pi}{5}}^{\frac{3\pi}{10}} \frac{sinx}{sinx+cosx} dx =$

(A) $\frac{\pi}{10}$ (B) $\frac{\pi}{20}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

(7) $\int_0^1 \frac{x^2-2}{x^2+1} dx =$

(A) $1 - \frac{3\pi}{4}$ (B) $2 - \frac{3\pi}{4}$ (C) $1 + \frac{3\pi}{4}$ (D) $2 + \frac{3\pi}{4}$

(8) Let $I_1 = \int_e^{e^2} \frac{1}{\log x} dx$ and $I_2 = \int_1^2 \frac{e^x}{x} dx$ then

(A) $I_1 = \frac{1}{3} I_2$ (B) $I_1 + I_2 = 0$ (C) $I_1 = 2I_2$ (D) $I_1 = I_2$

(9)$\int_0^4 \frac{1}{\sqrt{4x-x^2}} dx =$

(A) 0   (B)2π (C) π (D)4π

(10) $\int_0^{\frac{\pi}{2}} \log(tanx)\, dx =$

(A) $\frac{\pi}{8}(\log 2)$ (B) 0   (C) $-\frac{\pi}{8}(\log 2)$ (D) $\frac{\pi}{2}(\log 2)$

II. Very Short Answers (1 mark)

(11) *Evaluate:* $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} cosx\, dx$.

(12) *Evaluate:* $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^3 \sin^4 x\, dx$

(13) *Evaluate:* $\int_0^1 \frac{1}{1+x^2} dx$

(14) *Evaluate:* $\int_0^{\frac{\pi}{4}} \sec^2 x\, dx$

(15) *Evaluate:* $\int_0^1 |x| dx$

(16) *Evaluate:* $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$

(17) *Evaluate:* $\int_1^2 \frac{x}{1+x^2} dx$

(18) *Evaluate:* $\int_0^1 \frac{e^x}{\sqrt{e^x-1}} dx$

(19) *Evaluate:* $\int_0^{\frac{\pi}{2}} \frac{sin2x}{1+\sin^2 x} dx$

(20) *Evaluate:* $\int_0^1 (x + 1)^2 dx$

III. Short Answers (2 marks)

(21) Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} sin^2x \, dx$

(22) Evaluate: $\int_0^{\pi/2} \sqrt{1 - cos4x} \, dx$

(23) Evaluate: $\int_0^{\frac{\pi}{2}} cos^3x \, dx$

(24) Evaluate: $\int_0^{\pi} cos^2x . dx$

(25) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{tan^3x}{1+cos\,2x} \, dx$

(26) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{cosx}{4 - sin^2x} \, dx$

(27) Evaluate: $\int_1^3 \frac{cos(logx)}{x} \, dx$

(28) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{sin^2 x}{(1+cos\,x)^2} \, dx$

(29) Evaluate: $\int_0^9 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} \, dx$

IV. Short Answers (3 marks)

(30) Prove that : $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ where $a < c < b$

(31) Prove that: $\int_a^b f(x) \, dx = \int_a^b f(a + b - x) \, dx$

(32) Prove that: $\int_0^a f(x) \, dx = \int_0^a f(a - x) \, dx$. Hence find $\int_0^{\frac{\pi}{2}} sin^2x \, dx$.

(33) Evaluate: $\int_0^{\pi/2} \frac{sin^4x}{sin^4x+ cos^4x} \, dx$

(34) Evaluate: $\int_3^8 \frac{(11-x)^2}{x^2+(11-x)^2} \, dx$

(35) Evaluate: $\int_{-1}^1 |5x - 3| \, dx$

(36) Evaluate: $\int_{-4}^2 \frac{1}{x^2+4x+13} \, dx$

(37) Evaluate: $\int_0^1 \frac{1}{\sqrt{3+2x-x^2}} dx$

(38) Evaluate: $\int_0^1 x . \tan^{-1} x \, dx$

(39) Evaluate: $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx$

(40) Evaluate: $\int_0^{\frac{\pi}{4}} \sec^4 x \, dx$

(41) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{5 + 4 \cos x} dx$

(42) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$

(43) Evaluate: $\int_{-1}^1 \frac{1}{a^2 e^x + b^2 e^{-x}} dx$

(44) Evaluate: $\int_0^a \frac{1}{x+ \sqrt{a^2 - x^2}} dx$

(45) Evaluate: $\int_0^3 x^2 (3 - x)^{\frac{5}{2}} dx$

(46) Evaluate: $\int_0^1 t^2 \sqrt{1 - t} \, dt$

V. Long answers (4 Marks)

(47) *Prove that:* $\int_0^{2a} f(x) \, dx = \int_0^a f(x) \, dx + \int_0^a f(2a - x) \, dx$

(48) Prove that: $\int_{-a}^a f(x) \, dx = 2 \int_0^a f(x) \, dx$, if $f(x)$ is even function.

$$= 0 \qquad \text{, if } f(x) \text{ is odd function}$$

(49) *Evaluate:* $\int_0^{1/2} \frac{1}{(1-2x^2) \sqrt{1-x^2}} \, dx$

(50) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{3 \tan^2 x + 4 \tan x + 1} dx$

(51) Evaluate: $\int_{\frac{1}{\sqrt{2}}}^1 \frac{(e^{\cos^{-1} x})(\sin^{-1} x)}{\sqrt{1- x^2}} dx$

(52) Evaluate: $\int_0^1 \frac{\log(x+1)}{x^2+1} dx$

(53) Evaluate: $\int_0^\pi x \cdot sinx \cdot cos^2x \cdot dx$

(54) Evaluate: $\int_0^{\frac{\pi}{2}} x\ sinx\ dx$

(55) Evaluate: $\int_{-1}^1 \frac{1+x^3}{9-x^2}\ dx$

(56) Evaluate: $\int_0^1 \left(\frac{1}{1+x^2}\right) sin^{-1} \left(\frac{2x}{1+x^2}\right)\ dx$

(57) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{cos2x}{1+cos\ 2x+sin\ 2x}\ dx$

(58) Evaluate: $\int_0^{\pi/4} log\ (1+tanx)\ dx$

(59) Evaluate: $\int_0^\pi \frac{1}{3+2sinx+cosx}\ dx$

5. APPLICATION OF DEFINITE INTEGRATION

I. MCQ (2 marks each)

Ex.1: The area of the region bounded by the curve $y = sinx$, X-axis and

the lines $x = 0, x = 4\pi$ is ...... sq. units.

(A)   2   (B) 4   (C) <u>8</u>   (D) 16

Ex.2: The area of the region bounded by the ellipse $x^2/64 + y^2/100 = 1$,

is .... sq. units.

(A)64 π   (B) <u>80π</u>   (C) π/80   (D) 100π

Ex.3: The area bounded by the parabola $y^2 = x$ along the X- axis &

the lines $x =0$, $x= 2$ is ...... sq. units.

(A)   4/3   (B) <u>(4√2)/3</u>   (C) 2/3   (D) (2√2)/3

Ex.4: The area bounded by the curve $y^2 = x^2$, and the line $x = 8$ is ......

(A)16 sq. units (B) <u>64 sq. units</u> (C) 32 sq. units (D) 4 sq. units

Ex.5: The area of the region included between the parabolas $y^2 = 16x$

and $x^2 = 16y$, is given by .... sq.units

(A)   256   (B) 16/3   (C) <u>256/3</u>   (D) 64/3

Ex.6: The area enclosed between the two parabolas $y^2 = 20x$

and $y = 2x$ is ..... sq. units.

(A)   20/3   (B) 40/3   (C) 10/3   (D) <u>50/3</u>

Ex.7: The area bounded by the parabola $y^2 = 32x$ the X-axis and

the latus rectum is …….. sq. units

   (A)   512/3   (B) 512/5   (C) 512   (D) 64/3

Ex.8: The area bounded by the ellipse $x^2/4 + y^2/25 = 1$ & and

   the line $x/2 + y/5 = 1$ is ……. sq. units

   (A)   5(π-2)   (B) (5/2) (π-2)   (C) (5/3)(π-2)   (D) (5/4)(π-2)

Ex.9: The area of triangle ΔABC whose vertices are A(1,1) ,B(2,1) &

   C(3,3) is …….sq. units.

   (A)   1   (B) 2   (C) 3   (D) 4

Ex.10: The area enclosed by the line $2x + 3y = 6$ along X-axis &

   the lines x = 0 , x = 3 is ……… sq. units.

   (A)   1   (B) 2   (C) 3   (D) 4

II. Very Short Answers (1 mark)

Ex. 1 : Find the area bounded by the curve $y^2 = 36x$, the line x = 2

   in first quadrant .

Ex.2: Find the area bounded by the curve $y = \sin x$, the lines $x = 0$ and $x = \pi/2$.

Ex.3: Find the area enclosed between $y = \cos x$ and X-axis between the

   lines $x = \pi/2$ & $x \le 3\pi/2$

Ex.4: Find the area of the region bounded by the parabola $y^2 = 32x$ and

   its Latus rectum in first quadrant .

Ex.5: Find the area of the region bounded by the curve $y = x^2$, the

   X−axis and the given lines $x = 0$, $x = 3$

Ex.6: Find the area of the region bounded by the curve $y^2 = 8x$, the

   X−axis and the given lines $x = 1, x = 3, y \ge 0$

Ex.7: Find the area of the region bounded by the curve $x^2 = 12y$, the

   Y−axis and the given lines $y = 2, y = 4, x \ge 0$

Ex.8: Find the area of the ellipse $x^2/1 + y^2/4 = 1$, in first quadrant

Ex.9: Find the area of sector bounded by the circle $x^2 + y^2 = 25$,

   in the first quadrant.

Ex.10: Using integration, find the area of the region bounded by the line

   $2y + x = 8$, X−axis and the lines x=2 & x = 4.

III. Short Answers (3 marks)

Ex.1: Find the area enclosed between the X-axis and the curve $y = \sin x$

for values of x between 0 to 2π .

Ex.2: Find the area of the region bounded by the parabola $x^2 = 4y$ and

The X-axis & the line $x = 1$, x $= 4$.

Ex.3: Find the area of the region bounded by the parabola $y^2 = 16x$ and

the line $x = 4$.

Ex.4: Find the area of the region bounded by the curves

$x^2 = 8y$, $y = 2$, $y = 4$ and the Y-axis, lying in the first quadrant.

Ex.5: Find the area of the region bounded by the curve $y = \sin x$,

the X−axis and the given lines $x = - \pi$, x $= \pi$

Ex.6: Find the area of the ellipse x^2 /36 + y^2 / 64 $= 1$, using integration.

IV. Long answers (4 Marks)

Ex.1 : Find the area of the region bounded by the curves $y^2 = 4ax$

and $x^2 = 4ay$.

Ex.2: Find the area of the region lying between the parabolas

$4y^2 = 9x$ and $3x^2 = 16y$

Ex.3: Find the area of the sector bounded by the circle $x^2 + y^2 = 16$,

& the line $y = x$ in the first quadrant.

Ex.4: Find the area of the region included between $y = x^2 + 5$ and

the line $y = x + 7$

Ex.5: Find the area enclosed between the circle $x^2 + y^2 = 9$,

along X − axis and the line $x = y$, lying in the first quadrant.

Ex.6: Find the area enclosed between the circle $x2 + y2 = 1$ and

the line $x + y = 1$, lying in the first quadrant.

Ex.7: Find the area of the region bounded by the curve

$(y - 1)^2 = 4 (x + 1)$ and the line $y = (x - 1)$.

6. DIFFERENTIAL EQUATIONS

I.   Select & write the correct alternative from the given option for each question (2 Marks)

1. Solution of the equation $x\frac{dy}{dx} = y\,logy$ is

   a) $y = ae^x$ b) $y = be^{2x}$ c) $y = be^{-2x}$ d) $y = e^{ax}$

2. Bacterial increases at the rate proportional to the number present. If original number M doubles in 3 hours, then number of bacteria will be 4M in

       a) 4 hours   b) 6 hours   c) 8 hours   d) 10 hours

3. The general solution of $\frac{dy}{dx} = e^{-x}$ is

   a) $y = e^x + c$ b) $y = e^{-x} + c$

   c) $y = -e^{-x} + c$ d) $y = e^x + c$

4. The differential equation of $y = Ae^{5x} + Be^{-5x}$ is

   a) $\frac{d^2y}{dx^2} = 25y$ b) $\frac{d^2y}{dx^2} = -25y$

   c) $\frac{d^2y}{dx^2} = 5y$ d) $y\frac{d^2y}{dx^2} = -5y$

5. Differential equation of the function $c + 4yx = 0$ is

   a) $xy + \frac{dy}{dx} = 0$ b) $x\frac{dy}{dx} + y = 0$

   c) $\frac{dy}{dx} - 4xy = 0$ d) $x\frac{dy}{dx} + 1 = 0$

6. General solution of $y - x\frac{dy}{dx} = 0$

   a) $3logx + \frac{7}{y} = c$ b) $2logx + \frac{3}{y} = c$

   c) $logx - logy = logc$ d) $3logy + \frac{2}{x} = c$

7. The order and degree of $(\frac{dy}{dx})^3 - \frac{d^3y}{dx^3} + ye^x$ is

       a) 3 , 1   b) 1 , 3   c) 3 , 3   d) 1 , 1

8. The order and degree of $\left(1 + \left(\frac{dy}{dx}\right)^3\right)^{\frac{2}{3}} = 8\frac{d^3y}{dx^3}$ are respectively

       a) 3 , 1   b) 1 , 3   c) 3 , 3   d) 1 , 1

9. The solution of $\frac{dy}{dx} = 1$ is

a) $x + y = c$ b) $xy = c$ c) $x^2 + y^2 = c$ d) $y - x = c$

10.   The solution of $\frac{dy}{dx} + \frac{x^2}{y^2} = 0$ is

a) $x^3 + y^3 = 7$ b) $x^2 + y^2 = c$ c) $x^3 + y^3 = c$ d) $x + y = c$

II   Answer the following   1 marks

1. Find the differential equation of family of lines making equal intercept on coordinate axes.

2. Find the general solution of $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

3. Form the differential equation of family of standard circle

4. State the degree of differential equation $e^{\frac{dy}{dx}} + \frac{dy}{dx} = x$

III.   Attempt the following questions   (2 marks)

1. Form the differential equation of $y = (c_1 + c_2)e^x$

2. Solve the differential equation
   $\sec^2 y \tan x \, dy + \sec^2 x \tan y \, dx = 0$

3. Solve the differential equation $\frac{dy}{dx} = e^{(x+y)} + x^2 e^y$

4. Find the differential equation of family of all ellipse whose major axis is twice the minor axis.

IV.   Attempt the following questions (3 marks)

1. Solve the differential equation $\frac{dy}{dx} + y = e^{-x}$

2. Solve the differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$

3. Solve $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$ when $x = \frac{2}{3}, y = \frac{1}{3}$

4. Solve the differential equation $xdx + 2ydx = 0$

5. Solve the differential equation $(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$

6. Solve the following differential equation
   $\frac{dy}{dx} = x^2 y + y$

7. Find the differential equation by eliminating arbitrary constants from the relation $x^2 + y^2 = 2ax$

8. Find the differential equation by eliminating arbitrary constants from the relation $y = (c_1 + c_2 x)e^x$

9. Verify $y = logx + c$ is the solution of differential equation

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

10. Solve : $\frac{dy}{dx} + \frac{2}{x}y = x^2$

1. For the differential equation, find the particular solution
$$(x - y^2 x)dx - (y + x^2 y)dy = 0 \ when \ x = 2 , y = 0$$

2. Solve the differential equation $\frac{dy}{dx} + \frac{x-2y}{2x-y} = 0$

3. Find the differential equation from the relation $x^2 + 4y^2 = 4b^2$

4. If the population of a town increases at a rate proportional to the

population at that time. If the population increases from 40 thousands to 60 thousands in 40 years, what will be the population in another 20 years?.

(Given $\sqrt{\frac{3}{2}} = 1.2247$)

5. The rate of growth of bacteria is proportional to the number present. If initially, there were 1000 bacteria and the number doubles in 1 hours, find the number of bacteria after 5/2 hours. (Given $\sqrt{2} = 1.414$)

6. Solve the following differential equation
$$yx\frac{dy}{dx} = x^2 + 2y^2$$

7. Solve the following differential equation

$$y \, logy \frac{dy}{dx} = logy - x$$

8. For the differential equation, find the particular solution

$$\frac{dy}{dx} = (4x + y + 1), \text{ when } y = 1, x = 0$$

9. Solve the following differential equation

$$y^2 \, dx + (xy + x^2)dy = 0$$

10. Solve the following differential equation

$$x^2 \frac{dy}{dx} = x^2 + xy - y^2$$

7. PROBABILITY DISTIBUTIONS

I. MCQ (2 marks each)

(i) Let the p.m.f. of a random variable X be

$P(x) = \frac{3-x}{10}$, for x = -1 ,0,1,2

$= 0$, otherwise

Then E (X) is …..

(A) 1   (B) 2   (C) 0   (D) -1

(ii) c.d.f of a discrete random variable X is

(A) an identity function.   (B) a step function.

(B) an even function.   (D) an odd function.

(iii) If X denotes the number on the uppermost face of cubic die when

it is tossed, then E(x) is ….

(A) $\frac{2}{7}$ (B) $\frac{7}{2}$ (C) 1   (D) $\frac{1}{2}$

(iv) A random variable X has the following probability distribution

X	2	3	4
P(x)	0.3	0.4	0.3

Then the variance of this distribution is

(A) 0.6   (B) 0.7

(C) 0.77   (D) 0.66

(v) For the random variable X, if $V(X) = 4$, $E(X) = 3$, then $E(X^2)$ is …

(A) 9   (B) 13

(c) 12   (D) 7

(vi) If a d.r.v. X takes values 0, 1, 2, 3, . . . with probability

$P(X = x) = k(x + 1) \times 5^{-x}$, where k is a constant then $P(X = 0) = $…

(A) $\frac{7}{25}$ (B) $\frac{16}{25}$ (C) $\frac{18}{25}$ (D) $\frac{19}{25}$

(vii) The p.m.f of a d.r.v. X is $P(X = x) = \begin{cases} \frac{\binom{5}{x}}{2^5}, & \text{for } x = 0, 1, 2, 3, 4, 5 \\ 0, & \text{otherwise} \end{cases}$

If $a = P(X \leq 2)$ and $b = P(X \geq 3)$ then

(A) $a < b$ (B) $a > b$ (C) $a = b$ (D) $a + b = 2$

(viii) If the p.m.f of a d.r.v. X is $P(X = x) = \begin{cases} \frac{x}{n(n+1)}, & \text{for } x = 1, 2, 3, …, n \\ 0, & \text{otherwise} \end{cases}$ then

$E(X) = \cdots$

(A) $n + \frac{1}{2}$ (B) $\frac{n}{3} + \frac{1}{6}$ (C) $\frac{n}{2} + \frac{1}{5}$ (D) $n + \frac{1}{3}$

(ix) If the p.m.f of a d.r.v. X is $P(X = x) = \begin{cases} \frac{c}{x^3}, \text{for } x = 1, 2, 3 \\ 0, \text{ otherwise} \end{cases}$

   then $E(X) = \cdots$

   (A) $\frac{343}{297}$ (B) $\frac{294}{251}$ (C) $\frac{297}{294}$ (D) $\frac{294}{297}$

(x) If a d.r.v. X has the following probability distribution :

X	−2	−1	0	1	2	3
P(X=x)	0·1	k	0·2	2k	0·3	k

   then $P(X = -1)$ is …

   (A) $\frac{1}{10}$ (B) $\frac{2}{10}$ (C) $\frac{3}{10}$ (D) $\frac{4}{10}$

(xi) If a d.r.v. X has the following probability distribution:

X	1	2	3	4	5	6	7
P(X=x)	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

   then $k = \dots$

   (A) $\frac{1}{7}$ (B) $\frac{1}{8}$ (C) $\frac{1}{9}$ (D) $\frac{1}{10}$

II. Very Short Answers (1 mark)

1. Let X represent the difference between number of heads and number of tails obtained when a coin is tossed 6 times. What are possible values of X?

2. An urn contains 5 red and 2 black balls. Two balls are drawn at random. X denotes number of black balls drawn. What are possible values of X?

3. State which of the following are not probability mass function of random variable. Give reasons of your answers.

(i)

X	0	1	2
P (X)	0.4	0.4	0.2

(ii)

X	0	1	2	3	4
P (X)	0.1	0.5	0.2	-0.1	0.2

(iii)

X	0	1	2
P (X)	0.1	0.6	0.3

(iv)

Y	-1	0	1
P (Y)	0.6	0.1	0.2

4. Find mean for the following probability distribution

X	0	1	2	3
P(X=4)	$\dfrac{1}{6}$	$\dfrac{1}{3}$	$\dfrac{1}{3}$	$\dfrac{1}{6}$

5. State whether the following is not the probability mass function of random variable. Give reasons for your answer.

X	3	2	1	0	-1

P(X=x)	0.3	0.2	0.4	0	0.05

III. Short Answers (2 marks)

(1) Find the expected value and variance of r.v. X whose p.m.f. is given below.

X	1	2	3
$P(X=x)$	$\dfrac{1}{5}$	$\dfrac{2}{5}$	$\dfrac{2}{5}$

(2)   Find the probability distribution of number of heads in two tosses   of a coin .

(3) The probability distribution of X is as follows:

X	0	1	2	3	4
$P(X=x)$	0·1	k	2k	2k	k

 Find k and P $[X < 2]$.

(4) The following probability distribution of r.v. X

X	−3	−2	−1	0	1	2	3
$P(X=x)$	0.05	0.1	0.15	0.20	0.25	0.15	0.1

Find the probability that (i) X is positive. (ii) X is odd

(5) In the p.m.f. of r.v. X

X	1	2	3	4	5
$P(X=x)$	$\dfrac{1}{20}$	$\dfrac{3}{20}$	a	$2a$	$\dfrac{1}{20}$

Find a and obtain c.d.f. of X.

IV. Short answers (3 Marks)

(1) Find the probability distribution of the number of successes in tosses of a die, where a success is defined as number greater than 4 appears on at least one die.

(2) A coin is biased so that the head is 3 times as likely to occur as a tail. If the coin is tossed twice, find the probability distribution of tails..

(3) A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	2 k^2	$7k^2+k$

Determine : (i) k   ii)$P(X < 3)$ iii)$P(X > 4)$

(4) Find the probability distribution of the number of doublets in three throws of a pair of dice.

(5) Find the mean and variance of the number randomly selected from 1 to 15.

(6) Let the p.m.f. of r.v. X be $f(x) = \begin{cases} \frac{3-x}{10}, & \text{for } x = -1, 0, 1, 2 \\ 0, & \text{otherwise} \end{cases}$

Calculate $E(X)$ and $Var(X)$.

(7) Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as six appears on at least one die.

V. Long answers (4 Marks)

1. Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the mean or expectation of X and variance of X.

2. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled of pack of 52 cards. Find the mean, variance and standard deviation of the number of kings drawn.

3. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E (X).

4. In a meeting,70% of the members favor and 30% oppose a certain proposal. A member is selected at random and we take X=0 if he opposed, and X=1 if he is in favor. Find E(X) and Var (X) .

5. The following is the c.d.f. of r.v. X

X	-3	-2	-1	0	1	2	3	4
F(x)	0.1	0.3	0.5	0.65	0.75	0.85	0.9	1

Find p.m.f. of X. (i) $P\ (-1\ \leq\ X\ \leq\ 2)$ (ii) $P\ (X\ \leq\ 3/\ X\ >\ 0)$.

6. A player tosses two coins he wins Rs.10 if 2 heads appears , Rs. 5 if 1 head appears and Rs.2 if no head appears. Find the expected winning amount and variance of winning amount.

7. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

8. Let X denote the sum of the number obtained when two fair dice are rolled. Find the standard deviation of X.

8. BINOMIAL DISTRIBUTION

Q.1) Multiple Choice Questions (M.C.Q)   **2m**

    1. In a Binomial distribution with n = 4, if 2 P(X=3) = 3 P (X =2), then value of p is ....

    a) $\dfrac{9}{13}$ b) $\dfrac{4}{13}$ c) $\dfrac{6}{13}$ d) $\dfrac{7}{13}$

    2. Given that X ~ B (n, P). If n = 10, P = 0.4 then E(X) = ...

      a) 1   b) 2   c) 3   d) 4

    3. For X ~ B (n, P), If v(x) = 2.4 and P = 0.4 then n = ...

      a) 10   b) 20   c) 30   d) 40

    4. Given that X ~ B (n = 10, P), E (X)= 8 then value of P = ...

      a) 0.4   b) 0.8   c) 0.6   d) 0.7

    5. Bernoulli distribution is a particular case of binomial distribution if n = ...

      a) 4   b) 10   c) 2   d)1

II. Very Short Answers (1 mark)

i) If X ~ B(n,P) , E (X) = 12, V(X) = 4 then find n

ii) For Bernoulli Distribution , state formula for E(X) and V(X)

iii) For X ~ B(n , P) and P(X=x) = $^{8}C_{x}$ $(1/2)^{x}$ $(1/2)^{3-x}$ then state values of n and P

iv) State the formula for p.m.f of Binomial Distribution.

v) A die is thrown . If X denotes the number of positive divisor of the outcomes then find the range of random variable X.

III. Short Answers (2 marks)

    i) A r.v X ~ B(n,P) . If the value of mean and variance of X are 18 and 12 respectively then find total number of positive value of X.

    ii) Given X ~ B(n,P) , If E(X) = 6, V(X) = 4.2 find n and P

    iii) If X ~ B(n, P) with n = 10 , P = 0.4 then find E (X²)

    iv) If X ~ B(6, P) and 2. P(X =3) = P(X=2) then find P.

v) Let the p.m.f of r.v.x be $p(x) = {}^4C_x \left(\frac{5}{9}\right)^x \left(\frac{4}{9}\right)^{4-x}$, x =0,1,2,4 Find E(X) and Var (X)

IV. Short answers (3 Marks)

i) A Fair coin is tossed 5 times , find the probability that a) coin shows exactly three times head b) no head.

ii) The probability that certain kind of component will survive a cheak test is 0.6. Find the probability that exactly 2 of the next 4 tested components survive.

iii) Find the probability of guessing correctly at least nine out of ten answers in a "true" or "false" objective test.

iv) A fair coin is tossed 8 times. Find the probability that it shows heads i) exactly 5 times ii) at least once.

v) The Probability that a person who undergoes a kidney operation will be recovered is 0.5. find the probability that out of 6 patients who undergo similar operation   a) none will recover b) half of them recover.

Answer Key

Part I

1. MATHEMATICAL LOGIC

Q1) i) c   ii) c   iii) c   iv) b   v) b   vi) c   vii) d   viii) b   ix) a   x) b

Q2)

i) $10 + 20 \neq 30$ ii)   not defined   iii) (p ∧~q)

iv)   F   v)   T   vi)   F

Q3)

iii)   a) F   b) F   ii) a) $p \wedge q$ b) $p \vee q$

iii)   converse: If function is continuous then it is differentiable.

Contrapositive: If function is not continuous then it is not differentiable.

Q 4) i) An angle is a right angle and it is not of measure 90^0 , or an angle is of measure 90^0 and it is not a right angle.

ii) a) p ↔~q   b) p → q   c)~ p ∧ q

iii) a) $\forall$ n ε N , $n^2 \geq 1$ is a true statement

b) **∃ x ε N** such that $3x - 4 < 9$ is a true statement

c) **∃ y ε N** such that $y + 4 > 6$ is a true statement

iv) Contingency

v) proof

vi) a) 13 is a prime number or India is a democratic country

b)) (p v~q) ∧ (~p v q) ≡ (p ∧q) v~ (p v q)

vii) Converse: If they do not drive the car, then it snows

Inverse: If it does not snow, then they drive the car

Contrapositive: If they drive the car, then it does not snow

Q 5 i) Contradiction

ii) Proof

iii) Proof

iv) [(p ∧~q) v (q ∧~p)] ∧ (~q ∧ r)

v)

P	q	r	~r	P ∧q	(p ∧q) v ~r
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	F	T
F	F	T	F	F	F
F	F	F	T	F	T

2. MATRICES

I. MCQ (2 marks)

1. b) $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$

2. c) adj (A)

3. d) (-1, 0, 2)

4. a) Singular matrix

5. c) 2

6. d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7. a) $A^2 = I$

8. b) $\begin{bmatrix} \cos10\,\alpha & \sin10\,\alpha \\ -\sin10\,\alpha & \cos10\,\alpha \end{bmatrix}$

9. b) -1

10. d) $\frac{1}{40}$

11. b) 1, -2 , 3

12. a) x = 1, y = 2, z = 3

13. b) 3^9

14. a) no solution

15. d) 5

II. Very Short Answers (1 mark)

1. $a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33} = -1$

2. $|A| = 10$

3. $\alpha = -2$

4. $(B^{-1}A^{-1})^{-1} = \begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$

5. $A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$

6. $|A|^{-1} = \frac{1}{ad-bc}$

7. $B = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

8. $A^2(\alpha) = A(2\alpha) = \begin{bmatrix} \cos2\alpha & \sin2\alpha \\ -\sin2\alpha & \cos2\alpha \end{bmatrix}$

9. Order of AB is 3×3

10. $(A+I)(A-I) = \begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$

11. $A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix} = A$

12. $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

13. $B'(AB) = 0$

III. Short Answers (2 marks)

1. $f(A) = A^2 - 2A - 3I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

2. $B'A' = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 9 \\ -1 & 2 & 3 \\ 2 & -4 & -6 \end{bmatrix}$

3. $|adjA| = |A|^{n-1} = 5^{3-1} = 5^2 = 25$

4. $A'B' = \begin{bmatrix} 66 & 55 \\ 55 & 66 \end{bmatrix}$

5. $A^{-1}B^{-1} = \frac{1}{2} \begin{bmatrix} 3 & -7 \\ -1 & 3 \end{bmatrix}$

6. $X = AB = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

7. $X = A^{-1} = \begin{bmatrix} 3 & -17 \\ -1 & 6 \end{bmatrix}$

8. $A^{-1} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$

9. i) $A^{-1} = \frac{-1}{19} \begin{bmatrix} -2 & -3 \\ -3 & 5 \end{bmatrix}$ ii) $A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

10. $adjA = \begin{bmatrix} 4 & -5 \\ -3 & 6 \end{bmatrix}$

11. $A \sim \begin{bmatrix} 1 & 2 & 4 \\ 0 & -7 & -7 \\ 0 & 0 & -2 \end{bmatrix}$ (Operations R_2 -3R_1 , R_3 -2R_1)

IV. Short answers (3 Marks)

1. $A^2 = I$, Hence $A^{-1} = A = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$

2. $AB = \begin{bmatrix} 2 & 1 & 0 \\ 8 & 7 & 2 \\ -1 & 1 & 1 \end{bmatrix}$ $|AB| = 0$, Hence $(AB)^{-1}$ does not exists.

3. $adj(A) = A = = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

4. $x = 4$, $y = -3$

5. $A \sim \begin{bmatrix} 13 & -2 & 5 \\ -1 & 2 & -1 \end{bmatrix}$

6. ₹ 1300

7. $A^{-1} = \frac{1}{6} \begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix} = \frac{1}{6}(A - 5I)$

8. adj $(A) = [A_{ij}]^T_{3x3} = \begin{bmatrix} 0 & -1 & 1 \\ -8 & 3 & -7 \\ 4 & -2 & 2 \end{bmatrix}$

9. $X = \begin{bmatrix} -5 & 2 & 23 \\ 2 & -3 & -18 \\ 1 & 2 & 6 \end{bmatrix}$

10. $A^{-1} = \begin{bmatrix} \sec\theta & -\tan\theta & 0 \\ -\tan\theta & \sec\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

11. $A \sim \begin{bmatrix} 1 & 2 & 4 \\ 0 & -7 & -7 \\ 0 & 0 & -2 \end{bmatrix}$

12. $X = \frac{1}{6} \begin{bmatrix} 4 & 4 & 2 \\ 11 & 8 & -5 \\ 10 & 10 & 2 \end{bmatrix}$

V. Long answers (4 Marks)

1. $A^{-1} = \frac{1}{5} \begin{bmatrix} -2 & 0 & 3 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{bmatrix}$

2. $A^{-1} = \frac{1}{3} \begin{bmatrix} 3 & 0 & 0 \\ -3 & 1 & 0 \\ 9 & 2 & -3 \end{bmatrix}$

3. $x = \frac{5}{2}$, $y = -\frac{3}{2}$, $z = -2$

4. The required numbers are 1 , -2 and 3.

5. $A^{-1} = \frac{1}{6} \begin{bmatrix} 4 & -2 & 2 \\ -3 & 0 & 3 \\ 2 & 2 & -2 \end{bmatrix}$

6. $A^{-1} = \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix}$ hence $A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

7. $A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

8. $X = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$,

9. $A(\text{adj } A) = (\text{adj } A)A = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$.

10. $AB = \begin{bmatrix} 11 & 3 \\ 7 & 2 \end{bmatrix}$, $(AB)^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 11 \end{bmatrix}$

11. $x = \frac{1}{3}$, $y = \frac{2}{3}$, $z = 1$

12. The cost is ₹ 5 per dozen for pencils, ₹ 8 per dozen for pens and ₹ 8 per dozen for erasers.

3. TRIGONOMETRIC FUNCTIONS

Type- I:

1) b) $\frac{\pi}{6}, \frac{11\pi}{6}$

2) c) an isosceles triangle

3) b) $\frac{\sqrt{3}}{2}$

4) d) $\theta = n\pi \pm \frac{\pi}{4}, n \in Z$

5) c) $\frac{2\pi}{3}$

6) a) $1:\sqrt{3}:2$

7) b) $\frac{\pi}{3}$

8) c) $\left(-\frac{3}{4\sqrt{2}}, \frac{3}{4\sqrt{2}}\right)$

9) b) $\frac{\pi}{6}$

10) a) $\frac{1}{5}$

Type- II:

1) 0

4) $\left(2, \frac{\pi}{3}\right)$

6) $\frac{4}{5}$

7) $\angle B = \frac{\pi}{2}$

8) $\frac{\pi}{6}, \frac{5\pi}{6}$

9) $x = n\pi \pm \frac{\pi}{4}, n \in Z$

10) $(2, 2\sqrt{3})$

Type- III:

2) $x = \frac{2n\pi}{5} + \frac{\pi}{10}, n \in Z$ or

   $x = \frac{2n\pi}{3} - \frac{\pi}{6}, n \in Z$

8) $\frac{2\pi}{3}, \frac{5\pi}{3}$

4) $x = \frac{n\pi}{3} + (-1)^n \frac{7\pi}{18}, n \in Z$

9) -1

5) $\frac{\pi}{2}$

6) $\cos B = \frac{33}{65}$

4. PAIR OF LINES

I. MCQ

1-C : $x^2 - y^2 = 0$

2-C: $x-\sqrt{3}y=0$ and $\sqrt{3}x+y=0$

3-B: real and coincident lines

4-B:k=6

5-B: $9m^2 - 3m - 2 = 0$

6-A: $5x^2 + 4xy - 3y^2 = 0$

7-B. : $\frac{\pi}{4}$

8- B: $\frac{11}{2}$

Q2

1. xy+3x-2y+6=0
2. $(\sec\alpha-\tan\alpha)x+y=0$.: $(\sec\alpha+\tan\alpha)x-y=0$
3. k= -2
4. 60^0
5. $h = \pm\frac{7}{2}$

Q3

1.(2 x-3y+5) (3 x+y-9)+0, $6x^2 - 7xy - 3y^2 -3x+32y-45=0$

2.$3x^2 + 2xy - 5y^2 = 0$

3.25a+16b-40h=0

4.30^0

Q4

1. $2x^2 - xy - 6y^2 = 0$

3.45^0

5. VECTORS AND THREE DIMENTIONAL GEOMETRY

I. MCQ

1) B) $\dfrac{3}{4}$ 2) C) -1   3) A) $\dfrac{\bar{a}.\bar{b}}{|\bar{a}||\bar{b}|}$ 4) B) $\sqrt{3}$

5) D) 60^0 or 120^0 6) C) $\sqrt{34}$ 7) B) 2   8) C) $\sqrt{13}$

9) A (3, 3, -4)   10) B) the unit vector along the line

11) C) $\dfrac{3}{\sqrt{14}}$ 12) C) 1   13) A) $\dfrac{\sqrt{34}}{2}$

6. LINES AND PLANE

I. MCQ

1)C   (2) A   (3) B   (4)B   (5) D   (6) A (7) B (8) B (9) D

II. Very Short answer questions (1 marks)

1)3x +2y +5z -22 =0   (2)(2 ,3 ,1)   (3)$\bar{r} = (\hat{j} +2\hat{k}$) +λ $(\hat{i}+ 2\hat{j}+3\hat{k})$

4)Lies on the line

5)$\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{7}$

6)$\bar{r} = (4\hat{i} -\hat{j} +2\hat{k}$) +$\mu(-2\hat{i} - \hat{j} +\hat{k})$ (7) $\dfrac{x-3}{-2}=\dfrac{y-2}{1}$;z = 1

III. Short answer questions (2 marks)

(1) 4 ,5 ,7 (2)$\dfrac{3}{5}$,0 ,$\dfrac{4}{5}$ (3) 2x $-$y +2z = 18   (4)$\bar{r} = \dfrac{3\hat{i}+4\hat{j}+12\hat{k})}{13} =\dfrac{8}{13}$

 (5)x-1 = y-2 = z-3   (6) 1   (7) 45^0

IV. Short answer questions (3 marks)

1)$\dfrac{x-2}{-3}=\dfrac{y-1}{2}=\dfrac{z+3}{1}$ (2) $\bar{r} =(-\hat{i} -\hat{j} +2\bar{k}$)+ $\lambda(3\hat{i} +2\hat{j} +\hat{k}$)

3)$\dfrac{x+1}{3}=\dfrac{y+1}{2}=\dfrac{z-2}{1}$ (4) z = 6   (5) ($\dfrac{18}{7}$,$\dfrac{54}{7}$,$\dfrac{-27}{7}$) (6)$\bar{r}$.$(2\hat{i} - \hat{j}+2\hat{k}$) = 18

7)y $-$z + 1= 0   (8)60^0 (9) $\sqrt{2}$ (10)6x+8y+7z = 148   (11) m = $\dfrac{70}{11}$

V. Long answer questions (4 marks)

1)(-11 ,-4 ,5) (2) $\frac{x+2}{3} = \frac{y-3}{-2} = \frac{z-4}{-2}$ (3)$\frac{x-1}{2} + \frac{y-2}{1} = \frac{z-3}{-3}$

4)$\bar{r} = (\hat{\imath} - 4\hat{k}) = -5$ (5) $\bar{r} \cdot (4\hat{\imath} + \hat{\jmath} - 5\hat{k}) = -26$

6)$\bar{r} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 1$

7. LINEAR PROGRAMMING PROBLEM

Q. 1

1. b) 13
2. c) (3,4)
3. b) All the given constraint
4. b) A function to be maximized or minimized
5. c) At vertex of the feasible region
6. a) (2,2)
7. d) (40,15)
8. c) Half plane on the right of y axis including the point on the y axis
9. a) – 8
10. d) Fourth quadrant

Q.3

1) z is maximum at a point a= (2,0), b= $\left(\frac{20}{19}, \frac{45}{19}\right)$ is 10. z occurs at every point lying on the line segment of ab. hence there are infinite many numbers of optimal solutions
2) z has a maximum value 59 when x = 4.5, y= 2.5
3) z has a maximum value 95 when x = 2, y = 3 4
4) z has maximum value 37 when x = 4, y = 5
5) z has minimum value 52 when x = 1.5, y = 4
6) z has a minimum value 5 when x = 0 y = 5
7) z has a minimum value 22.5 when x = 2, y = 0.5
8) z has a minimum value 12 at vertices c = (6,0) and b = (0,3)
   z has a minimum value 12 at every point at the segment of bc that is minimum value of z occurs at more than two points
9) Shaded region of the graph is unbounded and hence lpp has unbounded solution
10) This lpp has unbounded solution hence optimal solution does not exist

1 : DIFFERENTIATION

MCQ :

1) c	2) c	3) b	4) a	5) c	6) d
7) b	8) a	9) d	10) b	11) c	

1 Mark :

1) $\dfrac{x}{\sqrt{x^2+5}}$

2) $e^{\tan x}.\sec^2 x$

3) $\dfrac{2^x \log 2}{\sqrt{1-(2^x)^2}}$

4) even function

5) e OR $\dfrac{e^{1+\log x}}{x}$

2 Mark :

1) $-5x^4 \tan(x^5)$

2) $\dfrac{\sec^2 \sqrt{x}}{4\sqrt{x}.\sqrt{\tan \sqrt{x}}}$

3) $\dfrac{1}{18}$

4) $\dfrac{1}{2}$

5) $-4^x \log^4$

6) $-\dfrac{1}{2}$

7) $\sec^3 \theta$

3 Mark :

1) $\dfrac{3}{2}\cosec\left(\dfrac{3x}{2}\right)$

2) $2\log^4 + \dfrac{3x}{x^2+5} - \dfrac{9x^2}{2(2x^3-4)}$

3) $\dfrac{1}{2}$

4) 1

5) $\dfrac{3}{1+9x^2} + \dfrac{5}{1+25x^2}$

6) $\dfrac{1}{x}$

8) $\dfrac{1}{x}$

10) $5^x.x^5.x^x.5^5\left[\log 5 + \dfrac{5}{x} + 1 + \log x\right]$

4 Mark :

3) $\dfrac{\sqrt{1-x^2}}{4(1+x^2)}$

4) -1

2. APPLICATION OF DERIVATIVE

1. (B) $-\dfrac{3}{2}$

2. (D) $-\dfrac{1}{2}$

3. (A) -5

4. (D) $\left(\dfrac{9}{4}, \dfrac{3}{8}\right)$

5. (B) 2 sec   6.   (D) $(1, 3)$ 7.   (C) 2.5   8. (D) 7.2 cm^3

/sec

9. (C) $(1,6)$ 10. (B) $\frac{1}{e}$

Solve the following

1 Marks

1. $\frac{1}{2}$ 2. 8 unit/sec

2 Marks

1. $-\frac{1}{3}$ 2. -2 3. $2x + y - 3 = 0$ 4. 163m/s

    5. 3 km/hr

6. $\frac{4}{\pi}$ m /sec 7. Increasing   8. Decreasing.   9. $R - \{1\}$

3 Marks

1. $(4,1)$ 2. 0.8π cm^2/ sec   3. 6 cm^3/ sec

    4. 0.9 m/ sec

5. $x \in (-\infty, -2) \cup (6, \infty)$ 6. $x \in (-6, -2)$ 7. 75, 4000   8.

    15,15

9. 9 cms, 9 cms

4 Marks

1. $(0,3)$ and $(4 - 25)$ 2. $\frac{1}{36}$ cm/ sec, $\frac{4\pi}{3}$ cm^2/ sec   3. 8 cm^2

    / sec

4.(i) 0.8 m/ sec (ii) 2 m/ sec   5.(i) $x \in (-\infty, -3) \cup (8, \infty)$ (ii) $x \in$

    $(-3,8)$

6. Max Value $= 33$ at $x = -2$, Min value $= -75$ at $x = 4$ 7. 30 cms ,

    30cms

8. 432 cc   9. 4 cms, 6cms   10. 8 cms, 8 cms, 4 cms

3. INDEFINITE INTEGRATION

Type-I]

1. b) $2\sqrt{\tan x} + c$
2. a) $2\tan^{-1}(\sqrt{x}) + c$
3. c) $\log\left[\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 6}\right] + c$
4. c) $\frac{1}{\sqrt{2}}\log\left[\sec\left(x + \frac{\pi}{4}\right) + \tan\left(x + \frac{\pi}{4}\right)\right] + c$
5. d) $\frac{1}{3}\sin^{-1}(x^3) + c$
6. b) $e^x(\log x)^2 + c$
7. d) $\left(\frac{x+1}{2}\right)\sqrt{x^2 + 2x + 5} + 2\log\left[(x + 1) + \sqrt{x^2 + 2x + 5}\right] + c$
8. a) $e^{\sin^{-1}x}(\sin^{-1}x - 1) + c$
9. d) $\frac{\tan^{m+1}x}{m+1} + c$
10. d) $-\frac{4}{9}\left[x^{\frac{3}{2}} + (x + 3)^{\frac{3}{2}}\right] + c$
11. a) $2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + c$
12. a) $\sin 2x + c$

13. b) $\frac{x}{1+\log x} + c$

14. c) $\log x - f(x) + c$

15. d) $\frac{e^x}{x} + c$

Type-II]

1. $\sin x - \cos x + c$

2. $-\cos 2x + c$

3. $\frac{1}{3}\log(e^{3x} + 1) + c$

4. $\frac{1}{\sqrt{3}}\log\left[\sqrt{3}x + \sqrt{3x^2 + 8}\right] + c$

5. $\frac{(\log x)^2}{2} + c$

6. $e^x(2 + \cot x) + c$

7. $\frac{e^x}{x+4} + c$

8. $e^x - \frac{1}{3e^{3x}} + c$

9. $x^x + c$

10. $-\cot(\log x) + c$

11. $\frac{2}{3}\sec\left(x^{\frac{3}{2}}\right) + c$

12. $-\cot x - 2x + c$

13. $\log[\cos x + \sin x] + c$

14. $-\cot x - x + c$

15. $x - 2\log(x + 2) + c$

Type-III]

1. $\sec x - \tan x + c$

2. $\frac{1}{12}(4x - 1)^{\frac{3}{2}} - \frac{13}{4}\sqrt{4x - 1} + c$

3. $\frac{1}{48}\log(4x^{12} + 5) + c$

4. $\frac{1}{2}\log(x^4 + 1) + c$

5. $[\cos(a + b)\log[\sec(x + b)] - x\sin(a + b) + c$

6. $\sin x - \sin^3 x + \frac{3}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c$

7. $\frac{1}{4}\left[\log(1 + x^4) + \frac{1}{1+x^4}\right] + c$

8. $\frac{1}{\sqrt{2}}\log\left(x + \sqrt{x^2 - \frac{5}{2}}\right) + c$

9. $(\log x)[\log(\log x) - 1] + c$

10. $\frac{1}{6}\left[x^3\sqrt{a^2 - x^6} + a^2\sin^{-1}\left(\frac{x^3}{a}\right)\right] + c$

11. $\frac{1}{\log 2}\left[\frac{2^x}{2}\sqrt{4^x + 4} + 2\log(2^x + \sqrt{4^x + 4})\right] + c$

12. $x\csc(\log x) + c$

13. $-\cot x - \tan x + c$

14. $-\frac{1}{14}\cos 7x - \frac{1}{2}\cos x + c$

15. $[\log\sin(e^x)]^2 + c$

Type-IV]

1. $\frac{1}{3}\log\left(\frac{x^3 - 1}{x^3}\right) + c$

2. $f(x) = \frac{x^2}{2} + \frac{3}{2x^2} + \frac{7}{2}$

3. $\frac{a^{x+\tan^{-1}x}}{\log a} + c$

4. $\frac{5}{12}(2x + 3)^{\frac{3}{2}} - \frac{11}{2}\sqrt{2x + 3} - \frac{49}{4}\frac{1}{\sqrt{2x+3}} + c$

5. $9\sin^{-1}\left(\frac{x}{9}\right) - \sqrt{81 - x^2} + c$

6. $\frac{1}{8\sqrt{2}}\log\left(\frac{2x-5-2\sqrt{2}}{2x-5+2\sqrt{2}}\right)+c$

7. $\frac{1}{2\sqrt{3}}\log\left(\frac{\sqrt{3}+tanx}{\sqrt{3}-tanx}\right)+c$

8. $\sqrt{2}tan^{-1}\left[\frac{tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right]+c$

9. $\frac{1}{2}[secx\,tanx+log(secx+tanx)+c$

10. $\frac{x}{2}[sin(\log x)]-\cos(\log x)+c$

11. $\left(\frac{2tanx+1}{4}\right)\sqrt{tan^2x+tanx-7}-\frac{29}{8}\log\left[\left(tanx+\frac{1}{2}\right)+\sqrt{tan^2x+tanx-7}\right]+c$

12. $xe^{sin^{-1}x}+c$

13. $e^x\left(\frac{x-1}{x+1}\right)+c$

14. $x+\log\left(\frac{x-2}{x+3}\right)+c$

15. $x^2+3x+\log(x-1)+\frac{5}{3}\log(3x+1)+c$

Type-V]

1. $\frac{2x}{13}+\frac{3}{13}\log(2cosx+3sinx)+c$

2. $\sqrt{2}tan^{-1}\left(\frac{tanx-1}{\sqrt{2}\,tanx}\right)+c$

3. $\frac{3}{2}\sqrt{2x^2+2x+1}+\frac{5}{2\sqrt{2}}\log\left[\left(x+\frac{1}{2}\right)+\sqrt{x^2+x+\frac{1}{2}}\right]+c$

4. $\sqrt{e^{2x}-1}-\log\left(e^x+\sqrt{e^{2x}-1}\right)+c$

5. $\frac{1}{4}(tan^{-1}x)(x^4-1)-\frac{x}{12}(x^2-3)+c$

6. $-\frac{xcos7x}{14}+\frac{xcos3x}{6}+\frac{sin7x}{98}-\frac{sin3x}{18}+c$

7. $-xcot\left(\frac{x}{2}\right)+4log\left[sin\left(\frac{x}{2}\right)\right]+c$

8. $\frac{1}{6}tan^{-1}x+\frac{1}{15\sqrt{2}}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)-\frac{\sqrt{3}}{10}tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+c$

9. $\frac{1}{3}\log(x-1)-\frac{1}{6}\log(x^2+x+1)-\frac{1}{\sqrt{3}}tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c$

10. $\frac{1}{2}\log(e^x+1)-\frac{1}{4}\log(e^{2x}+9)+\frac{1}{6}tan^{-1}\left(\frac{e^x}{3}\right)+c$

11. $\frac{1}{10}\log(1-cosx)-\frac{1}{2}\log(1+cosx)+\frac{2}{5}\log(3+2cosx)+c$

12. $\frac{1}{4}\left[\frac{x}{3}sin3x+\frac{1}{9}cos3x+3xsinx+3cosx\right]+c$

13. $\frac{1}{2}\log\left(\frac{3sin^2x-3}{3sin^2x-1}\right)+c$

14. $\frac{7}{8}\log(4e^{2x}-5)-x+c$

15. $\frac{5}{13}\log(3logx+2)-\frac{5}{26}\log[(logx)^2+1]+\frac{12}{13}tan^{-1}(logx)+c$

4. DEFINITE INTEGRATION

1	B	2	A
3	C	4	D
5	D	6	B
7	A	8	D
9	C	10	B
11	$\dfrac{\sqrt{3}-1}{2}$	12	0
13	$\dfrac{\pi}{4}$	14	1
15	$\dfrac{1}{2}$	16	$\dfrac{\pi}{2}$
17	$\dfrac{1}{2}\log\left(\dfrac{5}{2}\right)$	18	$2\sqrt{e}-1$
19	$\log 2$	20	$\dfrac{7}{3}$
21	$\dfrac{\pi}{12}$	22	$\sqrt{2}$
23	$\dfrac{2}{3}$	24	$\dfrac{\pi}{2}$
----	----	25	$\dfrac{1}{8}$
26	$\dfrac{1}{4}\log\left(\dfrac{2\sqrt{2}+1}{2\sqrt{2}-1)}\right)$	27	$\sin(\log 3)$
28	$\dfrac{4-\pi}{2}$	29	$\dfrac{9}{2}$
30	Theorem	31	Theorem

32	$\dfrac{\pi}{4}$	33	$\dfrac{\pi}{4}$
34	$\dfrac{5}{2}$	35	$\dfrac{34}{5}$
36	$\dfrac{1}{3}[\tan^{-1}\dfrac{4}{3} + \tan^{-1}\dfrac{2}{3}]$	37	$\dfrac{\pi}{6}$
38	$\dfrac{\pi}{4} - \dfrac{1}{2}$	39	$\dfrac{\pi}{4} - \dfrac{1}{2}log2$
40	$\dfrac{4}{3}$	41	$\dfrac{2}{3}\tan^{-1}\left(\dfrac{1}{3}\right)$
42	$log\left(\dfrac{4}{3}\right)$	43	$\dfrac{1}{ab}[\tan^{-1}\dfrac{ae}{b} - \tan^{-1}\dfrac{a}{be}]$
44	$\dfrac{\pi}{4}$	45	$\dfrac{16}{77}(3)^{\frac{7}{2}}$
46	$\dfrac{16}{105}$	47	**Theorem**
48	**Theorem**	49	$\dfrac{1}{2}log(2+\sqrt{3})$
50	$\dfrac{1}{2}log2$	51	$e^{\frac{\pi}{4}}\left(\dfrac{\pi}{4}+1\right) - (\dfrac{\pi}{2}+1)$
52	$\dfrac{\pi}{8}log2$	53	$\dfrac{\pi}{3}$
54	1	55	$\dfrac{1}{3}log2$
56	$\dfrac{\pi^2}{16}$	57	$\dfrac{1}{2}\left[\dfrac{\pi}{4} - log2\right]$

58	$\dfrac{\pi}{8}(log2)$	59	$\dfrac{\pi}{4}$

5. APPLICATION OF DEFINITE INTEGRATION

Two mark questions:-

Ex. 1 : The required area A = 8√2 sq. units

Ex.2: Required area = 1 Sq. units.

Ex.3: Required area = 3 sq.units.

Ex.4: Required area = 128/3 sq.units.

Ex.5: Required area = 9 Sq.units.

Ex.6: Required area = (4√2/3)[3√3-1] Sq.units.

Ex.7: Required area = (8/√3)[4-√2] Sq.units.

Ex.8: Required area = π/2 sq.units.

Ex.9: Required area = 25π/4 sq.units.

Ex.10: Required area = 5 sq.units.

Three marks questions:-

Ex.1: Required area = 4 sq.units.

Ex.2: Required area = 21/4 sq.units.

Ex.3: Required area = 128/3 sq.units.

Ex.4: Required area = (16√2/3)(4-√2) sq.units.

Ex.5: Required area =4 Sq.units

Ex.6: Required area = 48π sq.units.

Four marks questions:-

Ex.1 : Required area A = (16/3) a^2 sq. units.

Ex.2: Required area = 4 sq.units.

Ex.3: Required area = 2π sq.units.

Ex.4: Required area = 1 sq.units.

Ex.5: Required area = (9π/8) sq.units.

Ex.6: Required area = (π/4 – 1/2) sq.units.

Ex.7: Required area = 64/3 sq.units.

6. DIFFERENTIAL EQUATIONS

I. **Select & write the correct alternative from the given option for each question (2 Marks)**

Ques. No	Answer	Ques. No	Answer
1	d) $y = e^{ax}$	6	c) $logx - logy = logc$
2	b) 6 hours	7	a) 3 , 1
3	c) e^x	8	c) 3 , 3
4	a) $ye^{-x} = x + c$	9	d) y $-$ x $=$ c
5	b) $x\frac{dy}{dx} + y = 0$	10	c) $x^3 + y^3 = c$

II. **Attempt the following questions (1 marks)**

1. $1 + \frac{dy}{dx} = 0$ 2. $\tan^{-1} x - \tan^{-1} y = c$

3. $x + y\frac{dy}{dx} = 0$ 4.   Not defined

III.   Attempt the following questions (2 marks)

1. $\frac{d^2y}{dx^2} - 2y + 1 = 0$ 2. $tanx \, tany = c$

3. $e^{-y} + e^x + \frac{x^3}{3} = c$ 4. $x + 4y\frac{dy}{dx} = 0$

IV. **Attempt the following questions (3 marks)**

Ques. No	Answer
1	$ye^x = x + c$

2	$x^2 y = \dfrac{x^4}{16}(logx - 1) + c$
3	$log|x + y| = y - x + \dfrac{1}{3}$
4	$x^2 + 2y^2 = c$
5	$logx - logy = \dfrac{1}{x} + \dfrac{1}{y} + c$
6	$logy = \dfrac{x^3}{3} + x + c$
7	$2xy\dfrac{dy}{dx} = y^2 - x^2$
8	$\dfrac{d^2 y}{dx^2} - \dfrac{2dy}{dx} + y = 0$
9	Verified
10	$5x^2 y = x^5 + c$

V. Attempt the following questions (4 marks)

Ques. No	Answer
1	$(1 - x^2)(1 - y^2) = 5$
2	$log\left(\dfrac{x + y}{x - y}\right) - \dfrac{1}{2}log(x^2 - y^2) + 2logx = logc$
3	$x + 4y\dfrac{dy}{dx} = 0$
4	73482
5	45248
6	$x^2 + y^2 = c^2 x^4$
7	$2xlogy = (logy)^2 + c$
8	$log\left|\dfrac{4x + y + 5}{6}\right| = x + c$
9	$logx + \dfrac{1}{4}log\left|\dfrac{2y^2 + xy}{x^2}\right| + \dfrac{3}{4}log\left|\dfrac{2y}{x + 2y}\right| = c$
10	$\dfrac{x + y}{x - y} = cx^2$

7. PROBABILITY DISTIBUTIONS

Q.1.MCQs

(i) (C) 0

(ii) (B) a step function

(iii) (B) $\frac{7}{2}$

(iv) (A) 0.6

(v) (B) 13

(vi) (B) $\frac{16}{25}$

(vii) (C) a=b

(viii) (B) $\frac{n}{3} + \frac{1}{6}$

(ix) (B) $\frac{294}{251}$

(x) (A) $\frac{1}{10}$

(xi) (D) $\frac{1}{10}$

Q.2
(1) $X = \{-6, -4, -2, 0, 2, 4, 6\}$

(2) $X = \{0, 1, 2\}$

(3) (i) P(x) can be regarded as p.m.f.of the random variable X.

(ii) P(x) cannot be regarded as p.m.f.of the random variable X.

(iii) P(x) can be regarded as p.m.f.of the random variable X.

(iv) P(x) cannot be regarded as p.m.f.of the random variable X.

(4) mean = 3/2
(5) P(x) cannot be regarded as p.m.f.of the random variable X.

Section-B

 (1) E(X) = 11/5 and V(X)= 14/25

(2)

X	0	1	2

P(X=x)	0.25	0.5	0.25

(3) $k = 0 \cdot 15.$ *and* $P[X < 2] = 0 \cdot 25$

(4) (i) P(X is positive) = 0.50

   (ii) P (X is odd) = 0.55

(5) a = $\frac{1}{4}$ and c.d.f.is

X	1	2	3	4	5
F(X=x)	$\frac{1}{20}$	$\frac{1}{5}$	$\frac{9}{20}$	$\frac{19}{20}$	1

Section -C

(1)

X	0	1	2
P(X=x)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(2)

X	0	1	2
P(X=x)	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

(3) k = $\frac{1}{10}$, P (X < 3) = $\frac{3}{10}$, P (X > 4) = $\frac{1}{5}$

(4)

X	0	1	2	3
P(X=x)	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

(5) Mean = 8, Var(X)=18.67

(6) E(X)= 0 and Var(X)=1

(7)

X	0	1	2
P(X=x)	$\dfrac{25}{36}$	$\dfrac{10}{36}$	$\dfrac{1}{36}$

Section -D

(1) E(X) = 7 ,Var(X) = 5.83

(2) E(X) = $\dfrac{34}{221}$,Var(X) = 0.1392, S.D.(X) = 0.3730

(3) E(X) = 4.67

(4) E(X) = 0.7 , Var(X) = 0.21

(5) p.m.f. of r.v. X is

X	−3	−2	−1	0	1	2	3	4
P(x)	0.1	0.2	0.2	0.15	0.10	0.10	0.05	0.10

(i) $P\left(-1 \leq X \leq 2\right) = 0.55$, (ii) $P\left(X \leq 3/X > 0\right) = 0.25$.

(6) Expected winning amount = Rs.5.5 and

variance of winning amount = Rs.8.25

(7) The required probability distribution is

X	0	1	2	3	4
P(x)	$\dfrac{256}{625}$	$\dfrac{256}{625}$	$\dfrac{96}{625}$	$\dfrac{16}{625}$	$\dfrac{1}{625}$

(8) Standard deviation of X = 2.41

8. BINOMIAL DISTRIBUTION

Q1) 1) a) $\frac{9}{13}$ 2) d) 4   3) a) 40   4) b) 0.8   5) d) 1

Q2) i) n = 18,   ii) E(X) = P, V(x) = pq   iii) n = 8, P = 1/2,

iv) P(X=x) = $^{n}C_x \, p^x \, q^{n-x}$ for x = 0,1,2...... $0< p < 1 \ \& \ q = 1 - p$

v) range = { 1, 2, 3, 4}

Q.3) i) 55   ii) n = 20, p = 0.3   iii) 18.4 iv) p = $\frac{3}{11}$ v) E(X)= 2.22, V(X) = 0.9876

Q.4) i) $\frac{5}{16}, \frac{1}{32}$ ii) 0.3456 iii) $\frac{11}{1024}$ iv) $\frac{7}{32}, \frac{255}{256}$ v) $\frac{1}{64}, \frac{5}{16}$