CONTENT

▸ FUNDAMENTALS OF MATHEMATICS - I

JEE (ADVANCED) SYLLABUS

Rational inequality, properties of Log, Exponential and Log equations and inequations.

JEE (MAIN) SYLLABUS

Sets : Sets and their representation; Union, intersection and complement of sets and their algebraic properties; Power set; Rational inequality, properties of Log, Exponential and Log equations and inequations.

Fundamentals of Mathematics-I

SETS

A set is a collection of well defined objects which are distinct from each other. Sets are generally denoted by capital letters A, B, C, ... etc. and the elements of the set by small letters a, b, c .... etc.

If a is an element of a set A, then we write $a \in A$ and say a belongs to A.

If a does not belong to A then we write a A,

e.g. the collection of first five prime natural numbers is a set containing the elements 2, 3, 5, 7, 11.

METHODS TO WRITE A SET :

(i) **Roster Method or Tabular Method :** In this method a set is described by listing elements, separated by commas and enclose then by curly brackets. Note that while writing the set in roster form, an element is not generally repeated e.g. the set of letters of word SCHOOL may be written as {S, C, H, O, L}.

(ii) **Set builder form (Property Method) :** In this we write down a property or rule which gives us all the element of the set.

$A = \{x : P(x)\}$ where P(x) is the property by which $x \in A$ and colon (:) stands for 'such that'

Example # 1 : Express set $A = \{x : x \in N \text{ and } x = 2^n \text{ for } n \in N\}$ in roster form

Solution : $A = \{2, 4, 8, 16, ...\}$

Example # 2 : Express set $B = \{x^3 : x < 5, x \in W\}$ in roster form

Solution : $B = \{0, 1, 8, 27, 64\}$

Example # 3 : Express set A = {0, 7, 26, 63, 124} in set builder form

Solution : $A = \{x : x = n^3 - 1, n \in N, 1 \le n \le 5\}$

TYPES OF SETS

Null set or empty set : A set having no element in it is called an empty set or a null set or void set, it is denoted by ϕ or { }. A set consisting of at least one element is called a non-empty set or a non-void set.

Singleton set : A set consisting of a single element is called a singleton set.

Finite set : A set which has only finite number of elements is called a finite set.

Order of a finite set : The number of distinct elements in a finite set A is called the order of this set and denoted by O(A) or n(A). It is also called cardinal number of the set.

e.g. A = {a, b, c, d} $\Rightarrow$ n(A) = 4

Infinite set : A set which has an infinite number of elements is called an infinite set.

Equal sets : Two sets A and B are said to be equal if every element of A is member of B, and every element of B is a member of A. If sets A and B are equal, we write A = B and if A and B are not equal then

$A \neq B$

Equivalent sets : Two finite sets A and B are equivalent if their cardinal number is same i.e. n(A) = n(B)

e.g. A = {1, 3, 5, 7}, B = {a, b, c, d} $\Rightarrow$ n(A) = 4 and n(B) = 4

$\Rightarrow$ A and B are equivalent sets

Note - Equal sets are always equivalent but equivalent sets may not be equal

Example # 4 : Identify the type of set :

(i) $A = \{x \in W : 3 \le x < 10\}$ (ii) $A = \{\alpha, \beta, \gamma, \delta\}$

(iii) $A = \{1, 0, -1, -2, -3, ...\}$ (iv) $A = \{1, 8, -2, 6, 5\}$ and $B = \{1, 8, -2, 6, 5\}$

(v) A = {x : x is number of students in a class room}

Solution : (i) finite set (ii) finite set

(iii) infinite set (iv) equal sets

(v) singleton set

Self Practice Problem :

(1) Write the set of all integers 'x' such that $-2 < x - 4 < 5$.

(2) Write the set {1, 2, 5, 10} in set builder form.

(3) If $A = \{x : x^2 < 9, x \in Z\}$ and $B = \{-2, -1, 1, 2\}$ then find whether sets A and B are equal or not.

Answers (1) {3, 4, 5, 6, 7, 8}

(2) {x : x is a natural number and a divisor of 10}

(3) Not equal sets

SUBSET AND SUPERSET :

Let A and B be two sets. If every element of A is an element of B then A is called a subset of B and B is called superset of A. We write it as $A \subset B$.

e.g. $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$ $\Rightarrow$ $A \subset B$

If A is not a subset of B then we write $A \not\subset B$

PROPER SUBSET :

If A is a subset of B but $A \neq B$ then A is a proper subset of B. Set A is not proper subset of A so this is improper subset of A

Note : (i) Every set is a subset of itself

(ii) Empty set ϕ is a subset of every set

(iii) $A \subset B$ and $B \subset A \Leftrightarrow A = B$

(iv) The total number of subsets of a finite set containing n elements is 2^n.

(v) Number of proper subsets of a set having n elements is $2^n - 1$.

(vi) Empty set ϕ is proper subset of every set except itself.

POWER SET :

Let A be any set. The set of all subsets of A is called power set of A and is denoted by P(A)

Example # 5 : Examine whether the following statements are true or false :

(i) $\{a\} \not\subset \{b, c, a\}$

(ii) $\{x, p\} \not\subset$ {x : x is a consonant in the English alphabet}

(iii) $\{\alpha, \beta, \gamma, \delta\} \subset \{\alpha, \beta, \phi, \psi\}$

(iv) $\{a, b\} \in \{a, \{a\}, b, c\}$

Solution : (i) False as {a} is subset of {b, c, a}

(ii) False as x, p are consonant

(iii) False as element γ, δ is not in the set $\{\alpha, \beta, \phi, \psi\}$

(iv) False as $a, b \in \{a, \{a\}, b, c\}$ and $\{a, b\} \subset \{a, \{a\}, b, c\}$

Example # 6 : Find power set of set A = {1, 2, 3}

Solution : $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

Example # 7 : If ϕ denotes null set then find

(a) $P(\phi)$ (b) $P(P(\phi))$

(c) $n(P(P(P(\phi))))$ (d) $n(P(P(P(P(\phi)))))$

Solution : (a) $P(\phi) = \{\phi\}$ (b) $P(P(\phi)) = \{\phi, \{\phi\}\}$

(c) $n(P(P(P(\phi)))) = 2^2 = 4$ (d) $n(P(P(P(P(\phi))))) = 2^4 = 16$

Self Practice Problem :

(4) State true/false : $A = \{p, q, r, s\}$, $B = \{p, q, r, p, t\}$ then $A \subset B$.

(5) State true/false : $A = \{p, q, r, s\}$, $B = \{s, r, q, p\}$ then $A \subset B$.

(6) State true/false : $[4, 15) \subset [-15, 15]$

Answers (4) False (5) True (6) True

UNIVERSAL SET :

A set consisting of all possible elements which occur in the discussion is called a universal set and is denoted by U.

e.g. if $A = \{1, 2, 3\}$, $B = \{2, 4, 5, 6\}$, $C = \{1, 3, 5, 7\}$ then $U = \{1, 2, 3, 4, 5, 6, 7\}$ can be taken as the universal set.

SOME OPERATION ON SETS :

(i) **Union of two sets :** $A \cup B = \{x : x \in A \text{ or } x \in B\}$

e.g. $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ then $A \cup B = \{1, 2, 3, 4\}$

(ii) **Intersection of two sets :** $A \cap B = \{x : x \in A \text{ and } x \in B\}$

e.g. $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ then $A \cap B = \{2, 3\}$

(iii) **Difference of two sets :** $A - B = \{x : x \in A \text{ and } x \notin B\}$. It is also written as $A \cap B'$.

Similarly $B - A = B \cap A'$ e.g. $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$; $A - B = \{1\}$

(iv) **Symmetric difference of sets :** It is denoted by $A \Delta B$ and $A \Delta B = (A - B) \cup (B - A)$

(v) **Complement of a set :** $A' = \{x : x \notin A \text{ but } x \in U\} = U - A$

e.g. $U = \{1, 2, ..., 10\}$, $A = \{1, 2, 3, 4, 5\}$ then $A' = \{6, 7, 8, 9, 10\}$

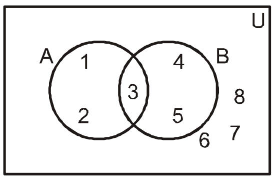
(vi) **Disjoint sets :** If $A \cap B = \phi$, then A, B are disjoint sets.

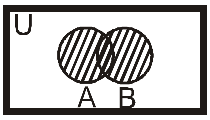
e.g. If $A = \{1, 2, 3\}$, $B = \{7, 8, 9\}$ then $A \cap B = \phi$

VENN DIAGRAM :

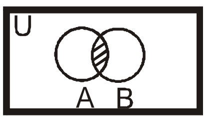
Most of the relationships between sets can be represented by means of diagrams which are known as venn diagrams. These diagrams consist of a rectangle for universal set and circles in the rectangle for subsets of universal set. The elements of the sets are written in respective circles.

For example If $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ then their venn diagram is

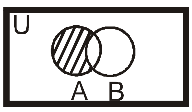




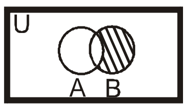
$A \cup B$



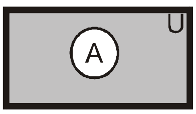
$A \cap B$



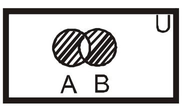
$A - B$



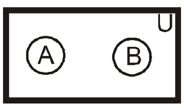
$B - A$



A'



$(A \Delta B) = (A - B) \cup (B - A)$



Disjoint

LAWS OF ALGEBRA OF SETS (PROPERTIES OF SETS):

- **(i) Commutative law :** $(A \cup B) = B \cup A$; $A \cap B = B \cap A$
- **(ii) Associative law :** $(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$
- **(iii) Distributive law :** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- **(iv) De-morgan law :** $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$
- **(v) Identity law :** $A \cap U = A$; $A \cup \phi = A$
- **(vi) Complement law :** $A \cup A' = U$, $A \cap A' = \phi$, $(A')' = A$
- **(vii) Idempotent law :** $A \cap A = A$, $A \cup A = A$

NOTE :

(i) $A - (B \cup C) = (A - B) \cap (A - C)$; $A - (B \cap C) = (A - B) \cup (A - C)$

(ii) $A \cap \phi = \phi$, $A \cup U = U$

Example # 8 : Let A = {1, 2, 3, 4, 5, 6} and B = {4, 5, 6, 7, 8, 9} then find $A \cup B$

Solution : $A \cup B$ = {1, 2, 3, 4, 5, 6, 7, 8, 9}

Example # 9 : Let A = {1, 2, 3, 4, 5, 6}, B = {4, 5, 6, 7, 8, 9}. Find A – B and B – A.

Solution : A – B = {x : $x \in A$ and x B} = {1, 2, 3}

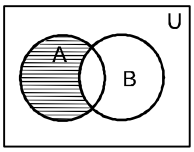
similarly B – A = {7, 8, 9}

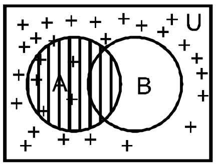
Example # 10 : State true or false :

(i) $A \cup A' = A$ (ii) $U \cap A = A$

Solution : (i) false because $A \cup A' = U$ (ii) true as $U \cap A = A$

Example # 11 : Use Venn diagram to prove that $A - B = A \cap B'$.





Solution : ≣ A – B ; |||| = A, ++ = B' ; = $A \cap B'$

From venn diagram we can conclude that $A - B = A \cap B'$.

Self Practice Problem :

(7) Find $A \cup B$ if A = {x : x = 2n + 1, $n \le 5$, $n \in N$} and B = {x : x = 3n – 2, $n \le 4$, $n \in N$}.

(8) Find A – (A – B) if A = {5, 9, 13, 17, 21} and B = {3, 6, 9, 12, 15, 18, 21, 24}

Answers (7) {1, 3, 4, 5, 7, 9, 10, 11} (8) {9, 21}

SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS IN SETS :

If A, B, C are finite sets and U be the finite universal set then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A - B) = n(A) - n(A \cap B)$

(iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(iv) Number of elements in exactly two of the sets A, B, C

$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(v) Number of elements in exactly one of the sets A, B, C

$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

Example # 12 : In a group of 60 students, 36 read English newspaper, 22 read Hindi newspaper and 12 read neither of the two. How many read both English & Hindi news papers ?

Solution : $n(U) = 60$, $n(E) = 36$, $n(H) = 22$

$n(E' \cap H') = 12 \Rightarrow n(E \cup H)' = 12$

$\Rightarrow \quad n(U) - n(E \cup H) = 12$

$\Rightarrow \quad n(E \cup H) = 48$

$\Rightarrow \quad n(E) + n(H) - n(E \cap H) = 48$

$\Rightarrow \quad n(E \cap H) = 58 - 48 = 10$

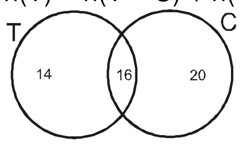
Example#13 : In a group of 50 persons, 14 drink tea but not coffee and 30 drink tea. Find

(i) How many drink tea and coffee both ? (ii) How many drink coffee but not tea ?

Solution : T : people drinking tea

C : people drinking coffee

(i) $n(T) = n(T - C) + n(T \cap C) \Rightarrow 30 = 14 + n(T \cap C) \Rightarrow n(T \cap C) = 16$



(ii) $n(C - T) = n(T \cup C) - n(T) = 50 - 30 = 20$

Self Practice Problem :

(9) Let A and B be two finite sets such that $n(A - B) = 15$, $n(A \cup B) = 90$, $n(A \cap B) = 30$. Find n(B)

(10) A market research group conducted a survey of 1000 consumers and reported that 720 consumers liked product A and 450 consumers liked product B. What is the least number that must have liked both products ?

Answers (9) 75 (10) 170

Intervals :

Intervals are basically subsets of R and are commonly used in solving inequalities or in finding domains. If there are two numbers $a, b \in R$ such that $a < b$, we can define four types of intervals as follows :

Name	Representation	Discription
Open Interval	(a, b)	$\{x : a < x < b\}$ i.e. end points are not included.
Close Interval	[a, b]	$\{x : a \le x \le b\}$ i.e. end points are also included. This is possible only when both a and b are finite.
Open - Closed Interval	(a, b]	$\{x : a < x \le b\}$ i.e. a is excluded and b is included.
Close - Open Interval	[a, b)	$\{x : a \le x < b\}$ i.e. a is included and b is excluded.

Note : (1) The infinite intervals are defined as follows :

(i) $(a, \infty) = \{x : x > a\}$ (ii) $[a, \infty) = \{x : x \ge a\}$

(iii) $(-\infty, b) = \{x : x < b\}$ (iv) $(-\infty, b] = \{x : x \le b\}$

(v) $(-\infty, \infty) = \{x : x \in R\}$

(2) $x \in \{1, 2\}$ denotes some particular values of x, i.e. x = 1, 2

(3) If there is no value of x, then we say $x \in \phi$ (null set)

General Method to solve Inequalities :

(Method of intervals (Wavy curve method)

Let $g(x) = \left(\dfrac{(x-b_1)^{k_1}(x-b_2)^{k_2} --- (x-b_n)^{k_n}}{(x-a_1)^{r_1}(x-a_2)^{r_2} --- (x-a_n)^{r_n}}\right)$ ... (i)

Where k_1, k_2 .....k_n and r_1, r_2 ....r_n $\in$ N and b_1, b_2.....b_n and a_1, a_2 ......a_n are real numbers.

Then to solve the inequality following steps are taken.

Steps : -

Points where numerator becomes zero are called zeros or roots of the function and where denominator becomes zero are called poles of the function.

(i) First we find the zeros and poles of the function.

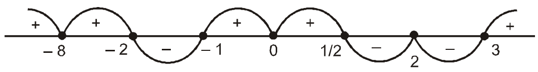
(ii) Then we mark all the zeros and poles on the real line and put a vertical bar there dividing the real line in many intervals.

(iii) Determine sign of the function in any of the interval and then alternates the sign in the neghbouring interval if the poles or zeros dividing the two interval has appeared odd number of times otherwise retain the sign.

(iv) Thus we consider all the intervals. The solution of the g(x) > 0 is the union of the intervals in which we have put the plus sign and the solution of g(x) < 0 is the union of all intervals in whichwe have put the minus sign.

Example# 14 : Solve the inequality if $f(x) = \dfrac{(x-2)^{10}(x+1)^3\left(x-\frac{1}{2}\right)^5(x+8)^2}{x^{24}(x-3)^3(x+2)^5}$ is > 0 or < 0.

Solution. Let $f(x) = \dfrac{(x-2)^{10}(x+1)^3\left(x-\frac{1}{2}\right)^5(x+8)^2}{x^{24}(x-3)^3(x+2)^5}$ the poles and zeros are 0, 3,– 2,– 1, $\frac{1}{2}$,– 8, 2



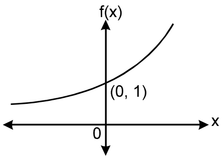
If f(x) > 0, then $x \in (-\infty, -8) \cup (-8, -2) \cup (-1, 0) \cup \left(0, \frac{1}{2}\right) \cup (3, \infty)$

and if f(x) < 0, then $x \in (-2, -1) \cup \left(\frac{1}{2}, 2\right) \cup (2, 3)$ **Ans.**

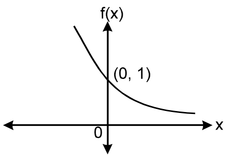
Exponential Function

A function $f(x) = a^x = e^{x \ln a}$ ($a > 0$, $a \neq 1$, $x \in R$) is called an exponential function. Graph of exponential function can be as follows :

Case - I
For a > 1



Case - II
For 0 < a 1



Logarithm of A Number :

The logarithm of the number N to the base ' a ' is the exponent indicating the power to which the base ' a ' must be raised to obtain the number N. This number is designated as $\log_a N$. Hence:

$\log_a N = x \Leftrightarrow a^x = N$, $a > 0$, $a \neq 1$ & $N > 0$

If a = 10, then we write log b rather than $\log_{10} b$.
If a = e, we write $\ell n\, b$ rather than $\log_e b$. Here ' e ' is called as Napier's base & has numerical value equal to 2.7182.

Remember

$\log_{10} 2 \approx 0.3010$; $\log_{10} 3 \approx 0.4771$
$\ell n\, 2 \approx 0.693$; $\ell n\, 10 \approx 2.303$

Domain of Definition :

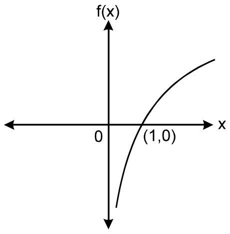
The existence and uniqueness of the number $\log_a N$ can be determined with the help of set of conditions, $a > 0$ & $a \neq 1$ & $N > 0$.

The base of the logarithm 'a' must not equal unity otherwise numbers not equal to unity will not have a logarithm and any number will be the logarithm of unity.

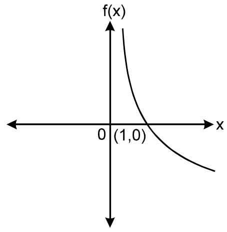
Graph of Logarithmic function :

$f(x) = \log_a x$ is called logarithmic function where $a > 0$ and $a \neq 1$ and $x > 0$. Its graph can be as follows:

Case- I
For $a > 1$



Case- II
For $0 < a < 1$



Fundamental Logarithmic Identity :

$a^{\log_a N} = N$, $a > 0$, $a \neq 1$ & $N > 0$

The Principal Properties of Logarithm:

Let M & N are arbitrary positive numbers, $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$ and α, β are any real numbers, then :

(i) $\log_a (M.N) = \log_a M + \log_a N$; in general $\log_a (x_1\ x_2\ ......x_n) = \log_a x_1 + \log_a x_2 + ...+ \log_a x_n$

(ii) $\log_a (M/N) = \log_a M - \log_a N$

(iii) $\log_a M^\alpha = \alpha. \log_a M$

(iv) $\log_{a^\beta} M = \frac{1}{\beta} \log_a M$

(v) $\log_b M = \frac{\log_a M}{\log_a b}$ (base changing theorem)

NOTE :

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_{1/a} a = -1$
- $\log_b a = \frac{1}{\log_a b}$
- $a^x = e^{x \ln a}$
- $a^{\log_c b} = b^{\log_c a}$

Note : (i) If the number and the base are on the same side of the unity, then the logarithm is positive.

(ii) If the number and the base are on the opposite sides of unity, then the logarithm is negative.

Example#15: Find the value of the followings :

(i) $\log_2 72 + \log_2\left(\frac{32}{81}\right) + \log_2\left(\frac{9}{64}\right)$ **Ans.** 2

(ii) $7^{\frac{1}{\log_{25} 49}}$ **Ans.** 5

Solution. (i) $\log_2 72 + ...$

$= \log_2\left\{2^3.3^2.\frac{2^5}{3^4}.\frac{3^2}{2^6}\right\} = \log_2 4 = 2$

(ii) $7^{\frac{1}{\log_{25}49}} = 7^{\log_{49}25} = 7^{\frac{2}{2}\log_7 5} = 5^{\log_7 7} = 5$

Self practice problem :

(11) Find the value of the followings :

(i) $\log_{49}343$ (ii) $4\log_{27}243$

(iii) $\log_{(1/100)}1000$ (iv) $\log_{(7-4\sqrt{3})}(7+4\sqrt{3})$

(v) $\log_{125}625$

(12) $\log_8 9.\log_9 10$ .... $\log_{63}64$

(13) Find the value of log cot1° + log cot2° + log cot3° + ... + log cot89°

Ans. (11) (i) 3/2 (ii) 20/3 (iii) – 3/2 (iv) –1 (v) 4/3

(12) 2 (13) 0

Logarithmic Equation :

The equality $\log_a x = \log_a y$ is possible if and only if x = y i.e.

$\mathbf{\log_a x = \log_a y \Leftrightarrow x = y}$

Always check validity of given equation, ($x > 0, y > 0, a > 0, a \neq 1$)

Example#16 : $\log_x(4x - 3) = 2$ **Ans.** x = 3

Solution. Domain : $x > 0, 4x - 3 > 0, x \neq 1$

Hence $4x - 3 = x^2 \Rightarrow x^2 - 4x + 3 = 0$

x = 3 or x = 1 (rejected as not in domain)

Exmaple#17 : $\log_2(\log_3\{\log_5(x^2 + 4)\} = 0$ **Ans.** $x = \pm 11$

Solution. $\log_3(\log_5(x^2 + 4)\} = 2° = 1$

$\Rightarrow \log_5(x^2 + 4) = 3^1 = 3$

$\Rightarrow (x^2 + 4) = 5^3 = 125 \Rightarrow x^2 = 121 \Rightarrow x = \pm 11$

Example#18 : $\log_2(x^2) + \log_2(x + 2) = 4$ **Ans.** x = 2

Solution. $\log_2(x^2(x + 2) = 4 \Rightarrow x^3 + 2x^2 - 16 = 0 \Rightarrow (x - 2)\underbrace{(x^2+4x+8)}_{D<0} = 0$

x = 2

Self practice problem

(14) $3^{3\log_3 x} = 27$ (15) $(\log_{10}x)^2 - (\log_{10}x) - 6 = 0$

(16) $3(\log_7 x + \log_x 7) = 10$ (17) $(x+2)^{\log_2(x+2)} = 8(x+2)^2$

Ans. (14) x = 3 (15) $x = 10^3, \frac{1}{10^2}$ (16) $x = 343, \sqrt[3]{7}$ (17) x = 6 or –3/2

Logarithmic Inequality :

Let 'a' is a real number such that

(i) If a > 1, then $\log_a x > \log_a y \Rightarrow x > y$

(ii) If a > 1, then $\log_a x < \alpha \Rightarrow 0 < x < a^\alpha$

(iii) If a > 1, then $\log_a x > \alpha \Rightarrow x > a^\alpha$

(iv) If 0 < a < 1, then $\log_a x > \log_a y \Rightarrow 0 < x < y$

(v) If 0 < a < 1, then $\log_a x < \alpha \Rightarrow x > a^\alpha$

Form - I : $f(x) > 0$, $g(x) > 0$, $g(x) \neq 1$

	Form		**Collection of system**
(a)	$\log_{g(x)} f(x) \geq 0$	$\Leftrightarrow$	$\begin{cases} f(x) \geq 1 \quad , \quad g(x) > 1 \\ 0 < f(x) \leq 1 \quad , \quad 0 < g(x) < 1 \end{cases}$
(b)	$\log_{g(x)} f(x) \leq 0$	$\Leftrightarrow$	$\begin{cases} f(x) \geq 1 \quad , \quad 0 < g(x) < 1 \\ 0 < f(x) \leq 1 \quad , \quad g(x) > 1 \end{cases}$
(c)	$\log_{g(x)} f(x) \geq a$	$\Leftrightarrow$	$\begin{cases} f(x) \geq (g(x))^a \quad , \quad g(x) > 1 \\ 0 < f(x) \leq (g(x))^a \quad , \quad 0 < g(x) < 1 \end{cases}$
(d)	$\log_{g(x)} f(x) \leq a$	$\Leftrightarrow$	$\begin{cases} 0 < f(x) \leq (g(x))^a \quad , \quad g(x) > 1 \\ f(x) \geq (g(x))^a \quad , \quad 0 < g(x) < 1 \end{cases}$

From - II : When the inequality of the form

	Form	**Collection of system**
(a)	$\log_{\phi(x)} f(x) \geq \log_{\phi(x)} g(x) \Leftrightarrow$	$\begin{cases} f(x) \geq g(x),\ \phi(x) > 1, \\ 0 < f(x) \leq g(x); 0 < \phi(x) < 1 \end{cases}$
(b)	$\log_{\phi(x)} f(x) \leq \log_{\phi(x)} g(x) \Leftrightarrow$	$\begin{cases} 0 < f(x) \leq g(x), \phi(x) > 1, \\ f(x) \geq g(x) > 0,\ 0 < \phi(x) < 1 \end{cases}$

Example # 19 : Solve the logarithmic inequality $\log_{1/5} (2x^2 + 7x + 7) \geq 0$.

Solution. Since $\log_{1/5} 1 = 0$, the given inequality can be written as.

$\log_{1/5} (2x^2 + 7x + 7) \geq \log_{1/5} 1$

when the domain of the function is taken into account the inequality is equivalent to the system of inequalities $\begin{cases} 2x^2 + 7x + 7 > 0 \\ 2x^2 + 7x + 7 \leq 1 \end{cases}$

Solving the inequalities by using method of intervals $x \in \left[-2, \frac{-3}{2}\right]$

Example # 20 : Solve the inequality $\log_{1/3} (5x - 1) > 0$.

Solution. by using the basic property of logarithm.

$$\begin{cases} 5x - 1 < 1 \\ 5x - 1 > 0 \end{cases} \Rightarrow \begin{cases} 5x < 2 \\ 5x > 1 \end{cases} \Rightarrow \begin{matrix} x < \frac{2}{5} \\ x > \frac{1}{5} \end{matrix}$$

$\Rightarrow$ The solution of the inequality is given by $\left(\frac{1}{5}, \frac{2}{5}\right)$ Ans.

Example # 21 : Solve the inequality $\log_{(2x+3)} x^2 < \log_{(2x+3)} (2x + 3)$.

Solution. The given inequality is equivalent to the collection of the systems

$$\left[\begin{array}{l}\begin{cases}0 < 2x+3 < 1 & \text{(i)}\\ x^2 > 2x+3\end{cases}\\ \begin{cases}2x+3 > 1 & \text{(ii)}\\ 0 < x^2 < 2x+3\end{cases}\end{array}\right.$$

Solving system (i) we obtain

$$\begin{cases}\frac{-3}{2} < x < -1\\ (x-3)(x+1) > 0 \quad \text{(iii)}\end{cases}$$

System (iii) is equivalent to the collection of two systems

$$\left[\begin{array}{l}\frac{-3}{2} < x < -1, \quad x > 3; \quad \text{(iv)}\\ \frac{-3}{2} < x < -1, \quad x < -1 \quad \text{(v)}\end{array}\right.$$

system (iv) has no solution. The solution of system (v) is $x \in \left(\frac{-3}{2}, -1\right)$,

solving system (ii) we obtain.

$$\begin{cases}x > -1\\ (x-3)(x+1) < 0\end{cases} \quad \text{or} \quad \begin{cases}x > -1\\ -1 < x < 3\end{cases} \quad \Rightarrow \quad x \in (-1, 3)$$

$$x \in \left(\frac{-3}{2}, -1\right) \cup (-1, 3)$$

Example # 22 : Solve the in equation $\log_{\left(\frac{x^2-12x+30}{10}\right)}\left(\log_2 \frac{2x}{5}\right) > 0$.

Solution. This in equation is equivalent to the collection of following systems.

$$\begin{cases}\frac{x^2-12x+30}{10} > 1,\\ \log_2\left(\frac{2x}{5}\right) > 1,\end{cases} \quad \text{and} \quad \begin{cases}0 < \frac{x^2-12x+30}{10} < 1,\\ 0 < \log_2\left(\frac{2x}{5}\right) < 1\end{cases}$$

Solving the first system we have.

$$\begin{cases}x^2 - 12x + 20 > 0\\ \frac{2x}{5} > 2\end{cases} \Leftrightarrow \begin{cases}(x-10)(x-2) > 0\\ x > 5\end{cases} \Leftrightarrow \begin{cases}x < 2 \text{ or } x > 10\\ x > 5\end{cases}$$

Therefore the system has solution x > 10

Solving the second system we have.

$$\Rightarrow \begin{cases}0 < x^2 - 12x + 30 < 10\\ 1 < \frac{2x}{5} < 2\end{cases} \quad \Rightarrow \begin{cases}x^2 - 12x + 30 > 0 \text{ and } x^2 - 12x + 20 < 0\\ \frac{5}{2} < x < 5\end{cases}$$

$$\Rightarrow \begin{cases}x < 6 - \sqrt{6} \text{ or } x > 6 + \sqrt{6} \text{ and } 2 < x < 10\\ \frac{5}{2} < x < 5\end{cases}$$

∴ The system has solutions $\frac{5}{2} < x < 6 - \sqrt{6}$ combining both systems, then solution of the original inequation is.

$x \in \left(\frac{5}{2}, 6 - \sqrt{6}\right) \cup (10, \infty)$ **Ans.**

Self practice problems :

(18) Solve the following inequalities

(i) $\log_{3x+5}(9x^2 + 8x + 8) > 2$

(ii) $\log_{0.2}(x^2 - x - 2) > \log_{0.2}(-x^2 + 2x + 3)$

(iii) $\log_x (x^3 - x^2 - 2x) < 3$

Answers : (18) (i) $\left(-\frac{4}{3}, -\frac{17}{22}\right)$ (ii) $\left(2, \frac{5}{2}\right)$ (iii) $(2, \infty)$

Characteristic & Mantissa

$[\log_a N]$ is called characteristic of log of N with base 'a'. It is always an integer.

$\{\log_a N\}$ is called mantissa of log of N with base 'a'. Mantissa $\in [0, 1)$

Characteristic of log of 1 with base 10 = 0

characteristic of log of 10 with base 10 = 1

characteristic of log of 100 with base 10 = 2

characteristic of log of 1000 with base 10 = 3

characteristic of log of 83.5609 with base 10 = 1

characteristic of log of 613.0965 with base 10 = 2

Interval,	**Cha.(Base 10)**	**number of digits in no**	**No. of integers in the interval**
[1, 10)	0	1	$9 = 9 \times 10^0$
[10, 100)	1	2	$90 = 9 \times 10^1$
[100, 1000)	2	3	$900 = 9 \times 10^2$
[100, 10000)	3	4	$9000 = 9 \times 10^3$
	|		
	|		
	|		
	n	(n + 1)	9×10^n

Note :

If characteristic of a number (base of log is 10) is found to be n, then there would be (n + 1) digits in that number.

* Characteristic of log of $\frac{1}{10} = 0.1$ with base 10 = – 1

Characteristic of log of $\frac{1}{100} = 0.01$ with base 10 = – 2

Characteristic of log of $\frac{1}{1000} = 0.001$ with base 10 = – 3

Characteristic of log of $\frac{3}{100}$ with base 10 = – 2

Characteristic of log of $\frac{3}{1000}$ with base 10 = – 3

Interval	Characteristic (base 10)	No. of zeros immediately after decimal	No.ofinteger resiprocal of which lies in interval.
[1/10, 1)	–1	0	$9 = 9\times10^{1-1}$
[1/100, 1/10) ≡ [0.01, 0.1)	–2	1	$90 = 9\times10^{2-1}$
$[1/10^3, 1/10^2) \equiv [0.0001, 0.01)$	–3	2	$900 = 9\times10^{3-1}$
[0.0001, 0.001)	–4	3	$9000 = 9\times10^{4-1}$
	|		
	|		
	|		
	|		
	– n	(n – 1)	$= 9 \times 10^{n-1}$

Note :

If characteristic of a number (base of log is 10) is found to be –n, then there would be (n –1) zeros immediately after decimal before first significant digit.

Example # 23 Find the total number of digits in the number 18^{50}.

(Given $\log_{10}2 = 0.3010$; $\log_{10}3 = 0.4771$) **Ans.** 63

Solution. $N = 18^{50}$

$\log_{10}N = 50 \log_{10}18 = 50\ (0.3010 + 0.9542) = 50(1.2552) = 62.76$

Characterstic = $[\log_{10}N] = 62$

No. of digits = 62 + 1 = 63

Self practice problem

(19) Find the total number of zeros immediately after the decimal in 6^{-200} .

Ans. (19) 155

Exercise-1

✍ **Marked questions are recommended for Revision.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Representation of sets, Types of sets, subset and power set

A-1. State wether the following collections is a set or not ?
(i) The collection of natural numbers between 2 and 20
(ii) The collection of numbers which satisfy the equation $x^2 - 5x + 6 = 0$
(iii) The collection of prime numbers between 1 and 100.
(iv) The collection of all intelligent women in Jalandhar.

A-2. Write the following set in tabular form
(i) $A = \{x : x \text{ is a positive prime} < 10\}$
(ii) $B = \{x : x = 3\lambda, x \in I, 1 \le \lambda \le 3\}$

A-3. Write the following set in builder form
(i) set of all rational number
(ii) $\{2, 5, 10, 17, 26, 37, ...\}$

A-4. Identifiy type of set in terms of empty/singleton/finite/infinite
(i) $\{x : x \text{ is a real number and } x^2 - 1 = 0\}$
(ii) $\{x : x \text{ is a real number and } x^2 + 1 = 0\}$
(iii) $\{x : x \text{ is positive real number and } x^2 - 9 = 0\}$
(iv) $\{x : x \text{ is a real number and } x^2 + 2x + 2 \ge 0\}$

A-5. Write power set of the set $A = \{\phi, \{\phi\}\}$.

Section (B) : Operations on sets, Law of Algebra of sets

B-1. Given the sets $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 5, 6\}$, then find the following
(i) $A \cup (B \cap C)$ (ii) $A - (B \cap C)$ (iii) $(B \cup C) - A$

B-2. Find the smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$

B-3. If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where $b, c \in N$, $b \ge 2$, $c \ge 2$ are relatively prime, then write 'd' in terms of b and c.

B-4.✍ Sets A and B have 3 and 6 elements respectively. What can be the minimum and maximum number of elements in
(i) $A \cap B$ (ii) $A \cup B$

Section (C) : Cardinal number Problems

C-1. Let $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$, then find $n(A' \cap B')$

C-2.✍ In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. Find the number of newspaper.

C-3.✍ In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3 % buy B and C and 4% buy A and C. If 2% families buy all the three news papers, then find number of families which buy newspaper A only.

C-4. In a survery, it was found that 21 persons liked product A, 26 liked product B and 29 liked product C. If 14 persons liked products A and B, 12 liked products C and A, 13 persons liked products B and C and 8 liked all the three products then

(i) Find the number of persons who liked atleast one product

(ii) The number of persons who like the products A and B but not C

Section (D) : Rational Inequalities

D-1. Solve the following rational in equalities

(i) $\frac{(x-1)(x+2)}{(x-3)(x+3)} < 0$

(ii) $\frac{(1-x)^3(x+2)^4}{(x+9)^2(x-8)} \geq 0$

(iii) $\frac{x^2+4x+4}{2x^2-x-1} > 0$

(iv) $\frac{(2-x^2)(x-3)^3}{(x+1)(x^2-3x-4)} \geq 0$

(v) $\frac{(x+2)(x^2-2x+1)}{4+3x-x^2} \geq 0$

D-2. Solve the following Inequalities

(i) $\frac{7x-5}{8x+3} > 4$

(ii) $\frac{14x}{x+1} < \frac{9x-30}{x-4}$

(iii) $\frac{(x-1)(x-2)(x-3)}{(x+1)(x+2)(x+3)} \leq 1$

(iv) $\frac{x^2+2}{x^2-1} < -2$

D-3. Solve the following rational in equalities

(i) $\frac{(x^2-3x+1)^3}{(x-1)(x+2)} \leq 0$

(ii) $\frac{2x^2-3x-459}{x^2+1} > 1$

(iii) $\frac{x^2-5x+12}{x^2-4x+5} > 3$

(iv) $\frac{x^4+x^2+1}{x^2-4x-5} > 0$

D-4. Solve the following rational in equalities

(i) $x^4 - 5x^2 + 4 \leq 0$

(ii) $x^4 - 2x^2 - 63 \leq 0$

(iii) $(x^2+3x+1)(x^2+3x-3) \geq 5$

D-5. If $1 < \frac{x-1}{x+2} < 7$ then find the range of

(i) x

(ii) x^2

(iii) $\frac{1}{x}$

D-6. Find the number of positive integral value of x satisfying the inequality $\frac{(3^x-5^x)(x-2)}{(x^2+5x+2)} \geq 0$

Section (E) : Logarithmic Properties

E-1. Find the value of

(i) $\log_{10}5.\log_{10}20 + (\log_{10}2)^2$

(ii) $5^{\log_{\sqrt{5}} 2} + 9^{\log_3 7} - 8^{\log_2 5}$

(iii) $\sqrt[3]{5^{\frac{1}{\log_7 5}} + \frac{1}{(-\log_{10} 0.1)}}$

(iv) $\log_{0.75} \log_2 \sqrt{\sqrt{\frac{1}{0.125}}}$

(v) $\left(\frac{1}{49}\right)^{1+\log_7 2} + 5^{-\log_{1/5} 7}$

(vi) $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

E-2. Which of the following numbers are positive/negative

(i) $\log_{\sqrt{3}}\sqrt{2}$ (ii) $\log_{1/7}(2)$ (iii) $\log_{1/3}(1/5)$

(iv) $\log_3(4)$ (v) $\log_7(2.11)$ (vi) $\log_3(\sqrt{7}-2)$

(vii) $\log_4\left(\dfrac{\sqrt{2}+1}{\sqrt{2}-1}\right)$ (viii) $\log_3\left(\dfrac{2\cdot\sqrt[3]{3}}{3}\right)$ (ix) $\log_{10}(\log_{10}9)$

E-3. Let $\log_{10}2 = a$ and $\log_{10}3 = b$ then determine the following logarithms in terms of a and b.

(i) $\log_{10}\left(\sin^2\dfrac{\pi}{3}\right)$ (ii) $\log_{100}4 + 2\log_{100}27$

(iii) $\log_2 9 + \log_3 8$ (iv) $\log_{\sqrt{45}}144$

E-4. (i) Let n = 75600, then find the value of $\dfrac{4}{\log_2 n}+\dfrac{3}{\log_3 n}+\dfrac{2}{\log_5 n}+\dfrac{1}{\log_7 n}$

(ii) If $\log_2(\log_3(\log_4(x)))=0$ and $\log_3(\log_4(\log_2(y)))=0$ and $\log_4(\log_2(\log_3(z)))=0$ then find the sum of x, y and z is

(iii) Suppose n be an integer greater than 1. let $a_n=\dfrac{1}{\log_n 2002}$. Suppose $b = a_2 + a_3 + a_4 + a_5$ and $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then find the value of (b – c)

E-5. Show that the number $\log_2 7$ is an irrational number.

E-6. If $\dfrac{\log a}{b-c}=\dfrac{\log b}{c-a}=\dfrac{\log c}{a-b}$, show that $a^a.\ b^b.\ c^c = 1$.

Section (F) : Logarithmic Equation

F-1. Solve the following equations :

(i) $\log_x(4x-3) = 2$ (ii) $\log_2(\log_3(x^2-1)) = 0$

(iii) $4^{\log_2 x}-2x-3=0$ (iv) $\log_4(\log_2 x) + \log_2(\log_4 x) = 2$.

(v) $\log_3\left(\log_9 x+\dfrac{1}{2}+9^x\right) = 2x$. (vi) $2\log_4(4-x) = 4-\log_2(-2-x)$.

(vii) $x^{(\log_{\sqrt{x}}2x)}=4$ (viii) $x^{0.5\log_{\sqrt{x}}(x^2-x)}=3^{\log_9 4}$.

F-2. (i) Find the product of roots of the equation
$(\log_3 x)^2 - 2(\log_3 x) - 5 = 0$

(ii) Find sum of roots of the equation $4^x - 7.2^x + 6 = 0$

(iii) Solve for x : $x^{\log_{10}x+2}=10^{\log_{10}x+2}$

(iv) Solve for x : $x^{\frac{\log_{10}x+5}{3}} = 10^{5+\log_{10}x}$

Section (G) : Logarithmic inequalities

G-1. Solve the following inequalities

(i) $\log_{\frac{5}{8}}\left(2x^2-x-\dfrac{3}{8}\right)\ge 1$ (ii) $\log_{\frac{1}{2}}(x^2-5x+6)>-1$

(iii) $\log_7\dfrac{2x-6}{2x-1}>0$ (iv) $\log_{1/4}(2-x) > \log_{1/4}\left(\dfrac{2}{x+1}\right)$

(v) $\log_{1/3}(2^{x+2}-4^x)\ge -2$

G-2. Find the number of integers satisfying $\log_{1/5}\dfrac{4x+6}{x} \geq 0$

G-3 Solve the inequalities

(i) $(\log_{.5}x)^2 + \log_{.5}x - 2 \leq 0$ (ii) $15^x - 25.3^x - 9.5^x + 225 \geq 0$

(iii) $8.\left(\dfrac{3^{x-2}}{3^x - 2^x}\right) > 1 + \left(\dfrac{2}{3}\right)^x$

G-4. Solve the following inequalities :

(i) $\log_x(4x - 3) \geq 2$ (ii) $\log_{(3x^2+1)} 2 < \dfrac{1}{2}$

(iii) $\log_{x^2}(2 + x) < 1$

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Representation of sets, Types of sets, subset and power set

A-1. The set of intelligent students in a class is-
(A) a null set (B) a singleton set
(C) a finite set (D) not a well defined collection

A-2. The set A = $\{x : x \in R, x^2 = 16 \text{ and } 2x = 6\}$ is
(A) Null set (B) Singleton set
(C) Infinite set (D) not a well defined collection

A-3. If A = $\{x : -3 < x < 3, x \in Z\}$ then the number of subsets of A is –
(A) 120 (B) 30 (C) 31 (D) 32

A-4. Which of the following are true ?
(A) $[3, 7] \subset (2, 10)$ (B) $(0, \infty) \subset (4, \infty)$ (C) $(5, 7] \subset [5, 7)$ (D) $[2, 7] \subset (2.9, 8)$

A-5. The number of subsets of the power set of set A = {7, 10, 11} is
(A) 32 (B) 16 (C) 64 (D) 256

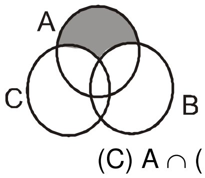
A-6. Which of the folowing sets is an infinite set ?
(A) Set of divisors of 24
(B) Set of all real number which lie between 1 and 2
(C) Set of all humman beings living in India.
(D) Set of all three digit natural numbers

Section (B) : Operations on sets, Law of Algebra of sets

B-1. Let A = $\{x : x \in R, -1 < x < 1\}$, B = $\{x : x \in R, x \leq 0 \text{ or } x \geq 2\}$ and $A \cup B = R - D$, then the set D is
(A) $\{x : 1 < x \leq 2\}$ (B) $\{x : 1 \leq x < 2\}$ (C) $\{x : 1 \leq x \leq 2\}$ (D) $\{x : 1 < x < 2\}$

B-2. If A = {2, 3, 4, 8, 10}, B = {3, 4, 5, 10, 12}, C = {4, 5, 6, 12, 14} then $(A \cap B) \cup (A \cap C)$ is equal to
(A) {3, 4, 10} (B) {2, 8, 10} (C) {4, 5, 6} (D) {3, 5, 14}

B-3. The shaded region in the given figure is



(A) $A \cap (B \cup C)$ (B) $A \cup (B \cap C)$ (C) $A \cap (B - C)$ (D) $A - (B \cup C)$

B-4. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 5}, B = {6, 7}, then $A \cap B'$ is
(A) B' (B) A (C) A' (D) B

B-5. If $A = \{x : x = 4n + 1, n \le 5, n \in N\}$ and $B = \{3n : n \le 8, n \in N\}$, then $A - (A - B)$ is :

(A) {9, 21} (B) {9, 12} (C) {6, 12} (D) {6, 21}

B-6. $A \cup B = A \cap B$ iff :

(A) $A \subset B$ (B) $A = B$ (C) $A \supset B$ (D) $A \not\subset B$

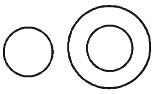
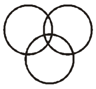
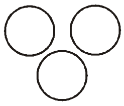
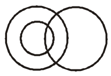
B-7.✍ Consider the following statements :

1. $N \cup (B \cap Z) = (N \cup B) \cap Z$ for any subset B of R, where N is the set of positive integers, Z is the set of integers, R is the set of real numbers.
2. Let $A = \{n \in N : 1 \le n \le 24$, n is a multiple of 3}. There exists no subset B of N such that the number of elemets in A is equal to the number of elements in B.

Which of the above statements is/are correct ?

(A) 1 only (B) 2 only (C) Both 1 and 2 (D) Neither 1 nor 2

B-8.✍Which of the following venn-diagrams best represents the sets of females, mothers and doctors ?

(A)  (B)  (C)  (D) 

Section (C) : Cardinal number Problems

C-1. Let A and B be two sets. Then

(A) $n(A \cup B) \le n(A \cap B)$ (B) $n(A \cap B) \le n(A \cup B)$

(C) $n(A \cap B) = n(A \cup B)$ (D) can't be say

C-2. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is

(A) 80 percent (B) 40 percent (C) 60 percent (D) 70 percent

C-3. A class has 175 students. The following data shows the number of students obtaining one or more subjects : Mathematics 100, Physics 70, Chemistry 40, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics & Physics & Chemistry 18. How many students have offered Mathematics alone ?

(A) 35 (B) 48 (C) 60 (D) 22

C-4.✍ 31 candidates appeared for an examination, 15 candidates passed in English, 15 candidates passed in Hindi, 20 candidates passed in Sanskrit. 3 candidates passed only in English. 4. candidates passed only in Hindi, 7 candidates passed only in Sanskrit. 2 candidates passed in all the three subjects How many candidates passed only in two subjects ?

(A) 17 (B) 15 (C) 22 (D) 14

Section (D) : Rational Inequalities

D-1. The complete solution set of the inequality $\frac{x^4 - 3x^3 + 2x^2}{x^2 - x - 30} \ge 0$ is:

(A) $(-\infty, -5) \cup (1, 2) \cup (6, \infty) \cup \{0\}$ (B) $(-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$

(C) $(-\infty, -5] \cup [1, 2] \cup [6, \infty) \cup \{0\}$ (D) $(-\infty, -5] \cup [1, 2] \cup [6, \infty)$

D-2.✍ Number of positive integral values of x satisfying the inequality

$$\frac{(x-4)^{2017}.\ (x+8)^{2016}\ (x+1)}{x^{2016}(x-2)^3\ .\ (x+3)^5\ .\ (x-6)\ (x+9)^{2018}} \le 0 \text{ is}$$

(A) 0 (B) 1 (C) 2 (D) 3

D-3. The number of prime numbers satisfying the inequality $\frac{x^2 - 1}{2x + 5} < 3$ is

(A) 1 (B) 2 (C) 3 (D) 4

D-4. The complete solution of $\frac{x^2-1}{x+3} \geq 0$ & $x^2 - 5x + 2 \leq 0$ is :

(A) $x \in \left[\frac{5-\sqrt{17}}{2}, \frac{5+\sqrt{17}}{2}\right]$ (B) $x \in \left[1, \frac{5+\sqrt{17}}{2}\right]$

(C) $x \in (-3, -1]$ (D) $x \in (-3, -1] \cup [1, \infty)$

D-5 The solution of the inequality $2x - 1 \leq x^2 + 3 \leq x - 1$ is

(A) $x \in R$ (B) $[-2, 2]$ (C) $(-2, 2)$ (D) $x \in \phi$

D-6. The number of the integral solutions of $x^2 + 9 < (x + 3)^2 < 8x + 25$ is :

(A) 1 (B) 3 (C) 4 (D) 5

D-7. Number of non-negative integral values of x satisfying the inequality $\frac{2}{x^2-x+1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \geq 0$ is

(A) 0 (B) 1 (C) 2 (D) 3

Section (E) : Logarithmic Properties

E-1. If $a^4 \cdot b^5 = 1$ then the value of $\log_a(a^5b^4)$ equals

(A) 9/5 (B) 4 (C) 5 (D) 8/5

E-2. $\frac{1}{1+\log_b a+\log_b c} + \frac{1}{1+\log_c a+\log_c b} + \frac{1}{1+\log_a b+\log_a c}$ has the value equal to

(A) abc (B) $\frac{1}{abc}$ (C) 0 (D) 1

E-3. $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to :

(A) 1/2 (B) 1 (C) 2 (D) 4

E-4. $(\log_2 10) . (\log_2 80) - (\log_2 5) . (\log_2 160)$ is equal to :

(A) $\log_2 5$ (B) $\log_2 20$ (C) $\log_2 10$ (D) $\log_2 16$

E-5. The ratio $\frac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to :

(A) $a^2 - a - 1$ (B) $a^2 + a - 1$ (C) $a^2 - a + 1$ (D) $a^2 + a + 1$

E-6. If $\log_a(ab) = x$, then $\log_b(ab)$ is equal to

(A) $\frac{1}{x}$ (B) $\frac{x}{1+x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x-1}$

E-7. $10^{\log_p(\log_q(\log_r x))} = 1$ and $\log_q\left(\log_r(\log_p x)\right) = 0$ then 'p' equals

(A) $r^{q/r}$ (B) rq (C) 1 (D) $r^{r/q}$

E-8. Which one of the following is the smallest?

(A) $\log_{10}\pi$ (B) $\sqrt{\log_{10} \pi^2}$ (C) $\left(\frac{1}{\log_{10} \pi}\right)^3$ (D) $\left(\frac{1}{\log_{10} \sqrt{\pi}}\right)$

E-9. $\log_{10}(\log_2 3) + \log_{10}(\log_3 4) + \log_{10}(\log_4 5) + ... + \log_{10}(\log_{1023} 1024)$ simplifies to

(A) a composite (B) a prime number

(C) rational which is not an integer (D) an integer

Section (F) : Logarithmic Equation

F-1. The sum of all the solutions to the equation $2\log_{10} x - \log_{10}(2x - 75) = 2$

(A) 30 (B) 350 (C) 75 (D) 200

F-2. If the solution of the equation $\log_x(125x) \cdot \log_{25}^2 x = 1$ are α and β ($\alpha < \beta$). Then the value of $1/\alpha\beta$ is :

(A) 5 (B) 25 (C) 125 (D) 625

F-3. The positive integral solution of the equation $\log_x \sqrt{5} + \log_x 5x = \frac{9}{4} + \log_x^2 \sqrt{5}$ is :

(A) Composite number (B) Prime number
(C) Even number (D) Divisible by 3

F-4. The expression $\log_p \log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\cdots\cdots\sqrt[p]{p}}}}}_{n \text{ radical sign}}$, where $p \ge 2$, $p \in N$; $n \in N$ when simplified is

(A) independent of p (B) independent of p and of n
(C) dependent on both p and n (D) positive

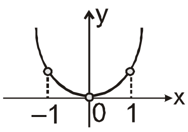
F-5. If $\log_x \log_{18}\left(\sqrt{2} + \sqrt{8}\right) = \frac{1}{3}$. Then the value of 1000 x is equal to

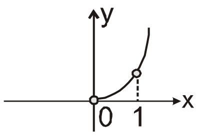
(A) 8 (B) 1/8 (C) 1/125 (D) 125

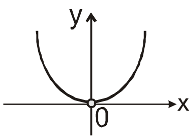
F-6. Number of real solutions of the equation $\sqrt{\log_{10}(-x)} = \log_{10}\sqrt{x^2}$ is :

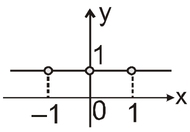
(A) zero (B) exactly 1 (C) exactly 2 (D) 4

F-7. The correct graph of $y = x^{\log_x x^2}$ is

(A) 

(B) 

(C) 

(D) 

Section (G) : Logarithmic inequalities

G-1. The solution set of the inequality $\log_{\sin\left(\frac{\pi}{3}\right)}(x^2 - 3x + 2) \ge 2$ is

(A) $\left(\frac{1}{2}, 2\right)$ (B) $\left(1, \frac{5}{2}\right)$ (C) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$ (D) $(1, 2)$

G-2. If $\log_{0.3}(x - 1) < \log_{0.09}(x - 1)$, then x lies in the interval

(A) $(2, \infty)$ (B) $(1, 2)$ (C) $(-2, -1)$ (D) $\left(1, \frac{3}{2}\right)$

G-3. Solution set of the inequality $2 - \log_2(x^2 + 3x) \ge 0$ is :

(A) $[-4, 1]$ (B) $[-4, -3) \cup (0, 1]$
(C) $(-\infty, -3) \cup (1, \infty)$ (D) $(-\infty, -4) \cup [1, \infty)$

G-4. If $\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1$, then 'x' lies in the interval

(A) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3)$ (B) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 2)$
(C) $(\sqrt{5}, 3\sqrt{5})$ (D) ϕ

G-5. The set of all solutions of the inequality $(1/2)^{x^2 - 2x} < 1/4$ contains the set

(A) $(-\infty, 0)$ (B) $(-\infty, 1)$ (C) $(1, \infty)$ (D) $(3, \infty)$

G-6. The number of positive integers not satisfying the inequality $\log_2(4^x - 2.2^x + 17) > 5$.

(A) 2 (B) 3 (C) 4 (D) 1

G-7. The set of all the solutions of the inequality $\log_{1-x}(x-2) \geq -1$ is

(A) $(-\infty, 0)$ (B) $(2, \infty)$ (C) $(-\infty, 1)$ (D) ϕ

PART - III : MATCH THE COLUMN

1. Match the set P in column one with its super set Q in column II

Column – I (set P)		**Column – II** (set Q)	
(A)	$[3^{2n} - 8n - 1 : n \in N\}$	(p)	$\{49(n-1) : n \in N\}$
(B)	$\{2^{3n} - 1 : n \in N\}$	(q)	$\{64(n-1) : n \in N\}$
(C)	$\{3^{2n} - 1 : n \in N\}$	(r)	$\{7n : n \in N\}$
(D)	$\{2^{3n} - 7n - 1 : n \in N\}$	(s)	$\{8n : n \in N\}$

2.✍

Column-I		**Column-II**	
(A)	If $a = 3\left(\sqrt{8+2\sqrt{7}} - \sqrt{8-2\sqrt{7}}\right)$, $b = \sqrt{(42)(30)+36}$ then the value of $\log_a b$ is equal to	(p)	-1
(B)	If $a = \sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}$, $b = \sqrt{11+6\sqrt{2}} - \sqrt{11-6\sqrt{2}}$, then the value of $\log_a b$ is equal to	(q)	1
(C)	If $a = \sqrt{3+2\sqrt{2}}$, $b = \sqrt{3-2\sqrt{2}}$ then the value of $\log_a b$ is equal to	(r)	2
(D)	If $a = \sqrt{7+\sqrt{7^2-1}}$, $b = \sqrt{7-\sqrt{7^2-1}}$, then the value of $\log_a b$ is equal to	(s)	$\frac{3}{2}$
(E)	The number of zeroes at the end of the product of first 20 prime numbers, is	(t)	None
(F)	The number of solutions of $2^{2x} - 3^{2y} = 55$, in which x and y are integers, is		

3.

Column-I		**Column-II**	
(A)	When the repeating decimal 0.363636............. is written as a rational fraction in the simplest form, the sum of the numerator and denominator is	(p)	4
(B)	Given positive integer p, q and r with $p = 3^q \cdot 2^r$ and $100 < p < 1000$. The difference between maximum and minimum values of $(q + r)$, is	(q)	0
(C)	If $\log_8 a + \log_8 b = (\log_8 a)(\log_8 b)$ and $\log_a b = 3$, then the value of 'a' is	(r)	15
(D)✍	If $P = 3^{\sqrt{\log_3 2}} - 2^{\sqrt{\log_2 3}}$ then value of P is	(s)	16

4.

Column-I		**Column-II**	
(A)	Anti logarithm of $(0.\overline{6})$ to the base 27 has the value equal to	(p)	5
(B)	Characteristic of the logarithm of 2008 to the base 2 is		
(C)	The value of b satisfying the equation, $\log_e 2 \cdot \log_b 625 = \log_{10} 16 \cdot \log_e 10$ is	(q)	7
(D)	Number of naughts after decimal before a significant figure comes in the number $\left(\frac{5}{6}\right)^{100}$, is (Given $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$)	(r)	9
		(s)	10

Exercise-2

✍ Marked questions are recommended for Revision.

*** Marked Questions may have more than one correct option.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1.✍ Let A_1, A_2 and A_3 be subsets of a set X. Which one of the following is correct ?

(A) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containing elements of each of A_1, A_2 and A_3

(B) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containing either A_1 or $A_2 \cup A_3$ but not both

(C) The smallest subset of X containing $A_1 \cup A_2$ and A_3 equals the smallest subset of X containing both A_1 and $A_2 \cup A_3$ only if $A_2 = A_3$

(D) None of these

2. Let A, B, C be distinct subsets of a universal set U. For a subset X of U, let X ' denote the complement of X in U.

Consider the following sets :

1. $(((A \cap B) \cup C)' \cap B')' = B \cap C$
2. $(A' \cap B') \cap (A \cup B \cup C') = (A \cup (B \cup C))'$

Which of the above statements is/are correct ?

(A) 1 only (B) 2 only (C) Both 1 and 2 (D) Neither 1 nor 2

3.✍ In an examination of a certain class, at least 70% of the students failed in Physics, at least 72% failed in Chemistry, at least 80% failed in Mathematics and at least 85% failed in English. How many at least must have failed in all the four subjects ?

(A) 9% (B) 7% (C) 15%

(D) Cannot be determined due to insufficient data

4. Let X and Y be two sets.

Statement-1 $X \cap (Y \cup X)' = \phi$

Statement-2 If $X \cup Y$ has m elements and $X \cap Y$ has n elements then symmetric difference $X \Delta Y$ has m – n elements.

(A) Both the statements are true. (B) Statement-I is true, but Statement-II is false.

(C) Statement-I is false, but Statement-II is true. (D) Both the statements are false.

5. If $\dfrac{6x^2 - 5x - 3}{x^2 - 2x + 6} \le 4$, then the least and the highest values of $4x^2$ are:

(A) 36 & 81 (B) 9 & 81 (C) 0 & 81 (D) 9 & 36

6. Sum of all the real solutions of the inequality $\dfrac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x^2-9)} \le 0$ is

(A) 5 (B) 4 (C) 8 (D) 0

7. If $\log_a b = 2$; $\log_b c = 2$ and $\log_3 c = 3 + \log_3 a$ then (a + b + c) equals

(A) 90 (B) 93 (C) 102 (D) 243

8. Let $x = \left(\log_{1/3} 5\right)\left(\log_{125} 343\right)\left(\log_{49} 729\right)$ and y = $25^{3\log_{289} 11\ \log_{28} \sqrt{17}\ \log_{1331} 784}$, then value of $\dfrac{y}{x}$ is

(A) $\dfrac{5}{3}$ (B) $-\dfrac{5}{3}$ (C) $-\dfrac{4}{5}$ (D) $\dfrac{3}{7}$

9. The expression: $\dfrac{\left(\dfrac{x^2+3x+2}{x+2}\right)+3x-\dfrac{x(x^3+1)}{(x+1)(x^2-x+1)}\log_2 8}{(x-1)(\log_2 3)(\log_3 4)(\log_4 5)(\log_5 2)}$ reduces to

(A) $\dfrac{x+1}{x-1}$ (B) $\dfrac{x^2+3x+2}{(\log_2 5)x-1}$ (C) $\dfrac{3x}{x-1}$ (D) x

10. If a, b, c are positive real numbers such that $a^{\log_3 7} = 27$; $b^{\log_7 11} = 49$ and $c^{\log_{11} 25} = \sqrt{11}$. The value of $\left(a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}\right)$ equals

(A) 489 (B) 469 (C) 464 (D) 400

11. Consider the statement : $x(\alpha - x) < y(\alpha - y)$ for all x, y with $0 < x < y < 1$. The statement is true

(A) if and only if $\alpha \geq 2$ (B) if and only if $\alpha > 2$

(C) if and only if $\alpha < -1$ (D) for no values of α

12. The set of values of x satisfying simultaneously the inequalities $\dfrac{\sqrt{(x-8)\ (2-x)}}{\log_{0.3}\left(\dfrac{10}{7}\ (\log_2 5-1)\right)} \geq 0$ and $2^{x-3} - 31 > 0$ is :

(A) a unit set (B) an empty set

(C) an infinite set (D) a set consisting of exactly two elements.

13. The solution set of the inequality $\dfrac{(3^x - 4^x)\cdot \ell n(x+2)}{x^2-3x-4} \leq 0$ is

(A) $(-\infty, 0] \cup (4, \infty)$ (B) $(-2, 0] \cup (4, \infty)$

(C) $(-1, 0] \cup (4, \infty)$ (D) $(-2, -1) \cup (-1, 0] \cup (4, \infty)$

14. If $\sqrt{\log_4\{\log_3\{\log_2(x^2-2x+a)\}\}}$ is defined $\forall x \in R$, then the set of values of 'a' is

(A) $[9, \infty)$ (B) $[10, \infty)$ (C) $[15, \infty)$ (D) $[2, \infty)$

15. If $\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$ then value of x is equal to

(A) $\dfrac{3}{4}$ (B) $-\dfrac{3}{4}$ (C) $-\dfrac{1}{4}$ (D) $\dfrac{3}{2}$

PART-II: NUMERICAL VALUE QUESTIONS

INSTRUCTION :

- The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto two digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.

1. Let U be set with number of elements in it is 2009 and A, B are subsets of U with n $(A \cup B) = 280$. If $n(A' \cap B') = x_1^3 + x_2^3 = y_1^3 + y_2^3$ for some positive integers $x_1 < y_1 < y_2 < x_2$, then find value of $\dfrac{x_2 + y_2}{x_1 + y_1}$

2. Let U be set with number of elements in it is 2009. A is a subset of U with n (A) = 1681 and out of these 1681 elements, exactly 1075 elements belong to a subset B of U. If n (A – B) = $m^2 + p_1 p_2 p_3$ for some positive integer m and distinct primes $p_1 < p_2 < p_3$ then for least m find $\dfrac{p_1 p_3}{p_2}$

3. Let A = {(x, y) : $x \in R$, $y \in R$, $x^3 + y^3 = 1$], B = {(x, y) : $x \in R$, $y \in R$, $x - y = 1$} and C = {(x, y) : $x \in R$, $y \in R$, $x + y = 1$}. If $A \cap B$ contains 'p' elements and $A \cap C$ contains 'q' elements then find (q – p).

4. In a class of 42 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. Then find number of students who have taken exactly one subject.

5. If c(a – b) = a(b – c) then find the value of $\frac{\log(a+c)+\log(a-2b+c)}{\log(a-c)}$ (Assume all terms are defined)

6. If $\log_b a . \log_c a + \log_a b . \log_c b + \log_a c . \log_b c = 3$ (where a, b, c are different positive real numbers $\neq 1$), then find the value of a b c.

7. If $4^A + 9^B = 10^C$, where $A = \log_{16}4$, $B = \log_3 9$ & $C = \log_x 83$, then find x.

8. Let a, b, c, d are positive integers such that $\log_a b = \frac{3}{2}$ and $\log_c d = \frac{5}{4}$. If (a – c) = 9, find the value of $\frac{b+d}{a+c}$.

9. Find the positive number, x , which satisfies the equation $\log_{10}(2x^2 - 21x + 50) = 2$

10. Find the value of x satisfying the equation $\log_{\frac{1}{2}}(x-1) + \log_{\frac{1}{2}}(x+1) - \log_{\frac{1}{\sqrt{2}}}(7-x) = 1$

11. Find sum of roots of equation $\log^2_{10} x + \log_{10}x^2 = \log^2_{10} 2 - 1$

12. If the product of all solutions of the equation $\frac{(2009)x}{2010} = (2009)^{\log_x(2010)}$ can be expressed in the lowest form as $\frac{m}{n}$ then the value of (m – n) is

13. If the complete solution set of the inequality $(\log_{10} x)^2 \geq \log_{10} x + 2$ is $(0, a] \cup [100, \infty)$ then find the value of a.

14. The complete solution set of the inequality $\frac{1}{\log_4 \frac{x+1}{x+2}} < \frac{1}{\log_4(x+3)}$, is $(-a, \infty)$, then determine 'a'.

15. If complete solution set of inequality $\log_{1/2}(x+5)^2 > \log_{1/2}(3x-1)^2$ is $(-\infty, p) \cup (q, r) \cup (s, \infty)$ then find $\frac{p^2+q^2+r^2}{s^2}$

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. Let a > 2, $a \in N$ be a constant. If there are just 18 positive integers satisfying the inequality $(x - a)(x - 2a)(x - a^2) < 0$ then which of the option(s) is/are correct?

(A) 'a' is composite
(B) 'a' is odd
(C) 'a' is greater than 8
(D) 'a' lies in the interval (3, 11)

2. Let N = $\frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$. Then N is :

(A) a natural number (B) a prime number (C) a rational number (D) an integer

3.✍ Values of x satisfying the equation $\log_5^2 x + \log_{5x}\left(\frac{5}{x}\right) = 1$ are

(A) 1 (B) 5 (C) $\frac{1}{25}$ (D) 3

4. The equation $\log_{x^2} 16 + \log_{2x}64 = 3$ has :

(A) one irrational solution (B) no prime solution
(C) two real solutions (D) one integral solution

5.✍ The equation $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$ has

(A) exactly three real solution (B) at least one real solution
(C) exactly one irrational solution (D) complex roots.

6. The solution set of the system of equations $\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_{27}(x + y) = \frac{2}{3}$ is :

(A) {6, 3} (B) {3, 6} (C) {6, 12} (D) {12, 6}

7. Consider the quadratic equation, $(\log_{10}8)x^2 - (\log_{10}5)x = 2(\log_2 10)^{-1} - x$. Which of the following quantities are irrational.

(A) sum of the roots (B) product of the roots
(C) sum of the coefficients (D) discriminant

8.✍ If $\log_a x = b$ for permissible values of a and x then identify the statement(s) which can be correct?

(A) If a and b are two irrational numbers then x can be rational.
(B) If a rational and b irrational then x can be rational.
(C) If a irrational and b rational then x can be rational.
(D) If a rational and b rational then x can be rational.

9. Which of the following statements are true

(A) $\log_2 3 < \log_{12} 10$ (B) $\log_6 5 < \log_7 8$
(C) $\log_3 26 < \log_2 9$ (D) $\log_{16}15 > \log_{10}11 > \log_7 6$

10. If $\frac{1}{2} \le \log_{0.1} x \le 2$, then

(A) maximum value of x is $\frac{1}{\sqrt{10}}$ (B) x lies between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$
(C) minimum value of x is $\frac{1}{10}$ (D) minimum value of x is $\frac{1}{100}$

PART - IV : COMPREHENSION

Comprehension # 1 (1 to 3)

In a group of 1000 people, there are 750 people, who can speak Hindi and 400 people, who can speak Bengali.

1. Number of people who can speak Hindi only is

(A) 300 (B) 400 (C) 500 (D) 600

2. Number of people who can speak Bengali only is

(A) 150 (B) 250 (C) 50 (D) 100

3. Number of people who can speak both Hindi and Bengali is

(A) 50 (B) 100 (C) 150 (D) 200

Comprehension # 2 (4 to 6)

Let A denotes the sum of the roots of the equation $\frac{1}{5-4\log_4 x} + \frac{4}{1+\log_4 x} = 3$.

B denotes the value of the product of m and n, if $2^m = 3$ and $3^n = 4$.

C denotes the sum of the integral roots of the equation $\log_{3x}\left(\frac{3}{x}\right) + (\log_3 x)^2 = 1$.

4. The value of A + B equals

(A) 10 (B) 6 (C) 8 (D) 4

5. The value of B + C equals

(A) 6 (B) 2 (C) 4 (D) 8

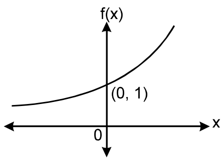
6. The value of A + C ÷ B equals

(A) 5 (B) 8 (C) 7 (D) 4

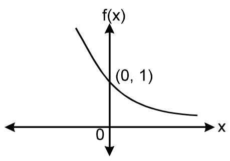
Comprehension # 3 (Q.7- to Q.9)

A function $f(x) = a^x (a > 0, a \neq 1, x \in R)$ is called an exponential function. Graph of exponential function can be as follows :

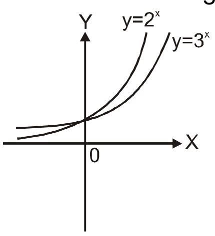
Case - I
For a > 1

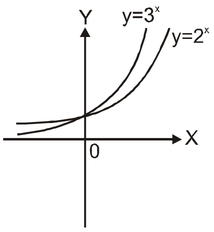


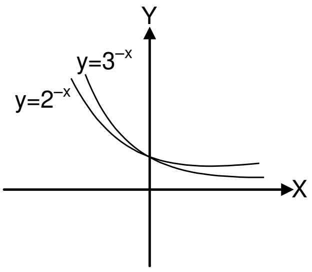
Case - II
For 0 < a < 1

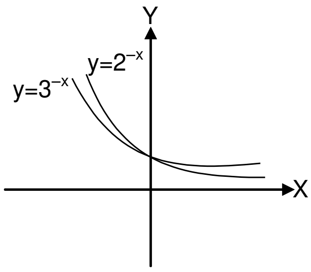


7*. Which of the following is correct :

(A)


(B)


(C)


(D)


8. Number of solutions of $3^x + x - 2 = 0$ is/are :

(A) 1 (B) 2 (C) 3 (D) 4

9. The number of positive solutions of $\log_{1/2} x = 7^x$ is/are :

(A) 0 (B) 1 (C) 2 (D) 3

Exercise-3

✍ **Marked questions are recommended for Revision.**

* **Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Indicate all correct alternatives, where base of the log is 2. **[JEE '89]**
The equation $x^{(3/4)(\log x)^2 + \log x - (5/4)} = \sqrt{2}$ has :
(A) at least one real solution
(B) exactly three real solutions
(C) exactly one irrational solution
(D) complex roots

2. The number $\log_2 7$ is :
(A) an integer
(B) a rational number
(C) an irrational number
(D) a prime number **[JEE '90]**

3.✍ Find all real numbers x which satisfy the equation
$2\log_2 \log_2 x + \log_{1/2} \log_2 (2\sqrt{2}\ x) = 1$. **[REE – 1999, 6]**

4.✍ Solve the equation $\log_{3/4} \log_8 (x^2 + 7) + \log_{1/2} \log_{1/4} (x^2 + 7)^{-1} = -2$. **[REE– 2000, 5]**

5. The number of solution(s) of $\log_4(x - 1) = \log_2(x - 3)$ is/are **[IIT-JEE-2002, Scr., (1, 0)/35]**
(A) 3 (B) 1 (C) 2 (D) 0

6. Let $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6}$ **[IIT-JEE 2007, Paper-2, (6, 0), 81]**

Column – I		**Column – II**	
(A)	If $-1 < x < 1$, then f(x) satisfies	(p)	$0 < f(x) < 1$
(B)	If $1 < x < 2$, then f(x) satisfies	(q)	$f(x) < 0$
(C)	If $3 < x < 5$, then f(x) satisfies	(r)	$f(x) > 0$
(D)	If $x > 5$, then f(x) satisfies	(s)	$f(x) < 1$

7.✍ Let (x_0, y_0) be the solution of the following equations
$(2x)^{\ell n 2} = (3y)^{\ell n 3}$
$3^{\ell n x} = 2^{\ell n y}$.
Then x_0 is **[IIT-JEE 2011, Paper-1, (3, –1), 80]**
(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

8.✍ The value of $6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}....}}}\right)$ is
[IIT-JEE 2012, Paper-1, (4, 0), 70]

9*. If $3^x = 4^{x-1}$, then x = **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**
(A) $\dfrac{2\log_3 2}{2\log_3 2 - 1}$ (B) $\dfrac{2}{2 - \log_2 3}$ (C) $\dfrac{1}{1 - \log_4 3}$ (D) $\dfrac{2\log_2 3}{2\log_2 3 - 1}$

10. The value of $((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is ___________.
[JEE(Advanced) 2018, Paper-1,(4, –2)/60]

PART - II : PREVIOUS YEARS PROBLEMS OF MAINS LEVEL

1. If $\log_p x = \alpha$ and $\log_q x = \beta$, then the value of $\log_{p/q} x$ is **[KCET-1997]**

(1) $\frac{\alpha-\beta}{\alpha\beta}$ (2) $\frac{\beta-\alpha}{\alpha\beta}$ (3) $\frac{\alpha\beta}{\alpha-\beta}$ (4) $\frac{\alpha\beta}{\beta-\alpha}$

2. If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in G.P. Then x is equal to **[KCET-1998]**

(1) $\log_a(\log_b a)$ (2) $\log_a(\log_e a) + \log_a \log_b b$

(3) $-\log_a(\log_a b)$ (4) none of these

3. If $\log_x 256 = 8/5$, then x is equal to **[KCET-2000]**

(1) 64 (2) 16 (3) 32 (4) 8

4. If log 2, $\log(2^x - 1)$ and $\log(2^x + 3)$ are in A.P., then x is equal to **[KCET-2000]**

(1) 5/2 (2) $\log_2 5$ (3) $\log_2 3$ (4) $\log_3 2$

5. The number $\log_2 7$ is **[DCE-2000]**

(1) an integer (2) a rational (3) an irrational (4) a prime number

6. The roots of the equation $\log_2(x^2 - 4x + 5) = (x - 2)$ are **[KCET-2001]**

(1) 4, 5 (2) 2, – 3 (3) 2, 3 (4) 3, 5

7. If x = 198 !, then value of the expression $\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + ... + \frac{1}{\log_{198} x}$ equals

[DCE-2005]

(1) –1 (2) 0 (3) 1 (4) 198

8. If $\log_{0.3}(x - 1) < \log_{0.09}(x - 1)$, then x lies in the intervel **[DCE-2006]**

(1) $(2, \infty)$ (2) (1, 2) (3) (–2, –1) (4) none of these

9. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then

[AIEEE-2009, (4, – 1), 144]

(1) A = C (2) B = C (3) $A \cap B = \phi$ (4) A = B

10. Let X = {1, 2, 3, 4, 5}. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is : **[AIEEE-2012, (4, – 1), 120]**

(1) 5^2 (2) 3^5 (3) 2^5 (4) 5^3

11. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to **[JEE(Main) 2014, (4, – 1), 120]**

(1) X (2) Y (3) N (4) Y – X

12. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is

[JEE(Main) 2016, (4, – 1), 120]

(1) – 4 (2) 6 (3) 5 (4) 3

13. In a class 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible 5 opted Chemistry course. Then the number of student who **did not** opt for any of the three courses is :

[JEE(Main) 2019, Online (10-01-19),P-1 (4, – 1), 120]

(1) 38 (2) 42 (3) 102 (4) 1

14. Let $X = \{n \in N: 1 \le n \le 50\}$. If $A = \{n \in X: n$ is a multiple of $2\}$; $B = \{n \in X: n$ is a multiple of $7\}$, then the number of elements in the smallest subset of X containing both A and B is__________.

[JEE(Main) 2020, Online (07-01-20),P-2 (4, 0), 120]

Answers

EXERCISE - 1

PART -I

Section (A) :

A-1. (i) Yes (ii) Yes (iii) Yes (iv) No

A-2. (i) {2,3,5,7} (ii) {3, 4, 5, 6, 7, 8, 9}

A-3. (i) $\{x : x = \frac{p}{q}, p \in I, q \in N\}$ (ii) $\{x : x = \lambda^2 + 1, \lambda \in N\}$

A-4. (i) Finite (ii) Finite and empty
(iii) Singleton & finite (iv) Infinite

A-5. $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$

Section (B) :

B-1. (i) {1, 2, 3, 4} (ii) {1, 2, 3} (iii) {4, 5, 6}

B-2. {3, 5, 9}

B-3. d = bc

B-4. (i) minimum $n(A \cap B) = 0$, maximum $n(A \cap B) = 3$
(ii) minimum $n(A \cup B) = 6$, maximum $n(A \cup B) = 9$

Section (C) :

C-1. 300 **C-2.** 25 **C-3.** 3300 **C-4.** (i) 45 (ii) 6

Section (D) :

D-1. (i) $(-3,-2) \cup (1,3)$ (ii) $\{-2\} \cup [1, 8)$
(iii) $(-\infty, -2)\cup(-2, -1/2)\cup(1, \infty)$ (iv) $[-\sqrt{2},-1)\cup(-1, \sqrt{2}]\cup[3, 4)$
(v) $(-\infty, -2] \cup (-1, 4)$

D-2. (i) $(-17/25, -3/8)$ (ii) $x \in (-6, -1) \cup (1, 4)$
(iii) $(-3,-2) \cup (-1,\infty)$ (iv) $x \in (-1,0) \cup (0,1)$

D-3. (i) $\left(-2, \frac{3-\sqrt{5}}{2}\right] \cup \left(1, \frac{3+\sqrt{5}}{2}\right]$ (ii) $(-\infty, -20) \cup (23, \infty)$
(iii) $\left(\frac{1}{2}, 3\right)$ (iv) $(-\infty, -1) \cup (5, \infty)$

D-4. (i) $x \in [-2, -1] \cup [1, 2]$ (ii) $x \in [-3, 3]$
(iii) $x\in(-\infty, -4]\cup [-2, -1) \cup [1,\infty)$

D-5. (i) $\left(-\infty, -\frac{5}{2}\right)$ (ii) $\left(\frac{25}{4}, \infty\right)$ (iii) $\left(-\frac{2}{5}, 0\right)$

D-6. 2

Section (E) :

E-1. (i) 1 (ii) –72 (iii) 2 (iv) 1 (v) $7 + \frac{1}{196}$ (vi) 0

E-2. (i) +ve (ii) – ve (iii) +ve (iv) +ve
(v) +ve (vi) – ve (vii) +ve (viii) – ve (ix) – ve

E-3. (i) b – 2a (ii) a + 3b (iii) $\frac{2b^2 + 3a^2}{ab}$ (iv) $\frac{4(2a+b)}{1-a+2b}$

E-4. (i) 1 (ii) 89 (iii) –1

Section (F) :

F-1. (i) 3 (ii) ± 2 (iii) 3 (iv) 16
(v) {1/3} (vi) {– 4} (vii) no root (viii) (2)

F-2. (i) 9 (ii) $\log_2 6$ (iii) 10 or $\frac{1}{100}$ (iv) $\{10^{-5}, 10^{3}\}$

Section (G) :

G-1. (i) $\left[-\frac{1}{2}, -\frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right]$ (ii) $(1, 2) \cup (3, 4)$

(iii) $\left(-\infty, \frac{1}{2}\right)$ (iv) $(-1, 0) \cup (1, 2)$

(v) $(-\infty, 2)$

G-2. 1

G-3 (i) $\left[\frac{1}{2}, 4\right]$ (ii) R (iii) $(0, \log_{\frac{3}{2}} 3)$

G-4. (i) $\left(\frac{3}{4}, 1\right) \cup (1, 3]$

(ii) $(-\infty, -1) \cup (1, \infty)$
(iii) $x \in (-2, -1) \cup (-1, 0) \cup (0, 1) \cup (2, \infty)$

PART - II

Section (A) :

A-1. (D) **A-2.** (A) **A-3.** (D) **A-4.** (A) **A-5.** (D) **A-6.** (B)

Section (B) :

B-1. (B) **B-2.** (A) **B-3.** (D) **B-4.** (B) **B-5.** (A) **B-6.** (B) **B-7.** (A)
B-8. (D)

Section (C) :

C-1. (B) **C-2.** (C) **C-3.** (C) **C-4.** (B)

Section (D) :

D-1. (B) **D-2.** (D) **D-3.** (D) **D-4.** (B) **D-5** (D) **D-6.** (D) **D-7.** (D)

Section (E) :

E-1. (A) **E-2.** (D) **E-3.** (B) **E-4.** (D) **E-5.** (D) **E-6.** (D) **E-7.** (A)
E-8. (A) **E-9.** (D)

Section (F) :

F-1. (D) **F-2.** (C) **F-3.** (B) **F-4.** (A) **F-5.** (D) **F-6.** (C) **F-7.** (B)

Section (G) :

G-1. (C) **G-2.** (A) **G-3.** (B) **G-4.** (A) **G-5.** (D) **G-6.** (A) **G-7.** (D)

PART - II

1. (A) → (q), (B) → (r), (C) → (s), (D) → (p)
2. (A) → r, (B) → s, (C) → p, (D) → p, (E) → q, (F) → q
3. (A) → r, (B) → p, (C) → s, (D) → q
4. (A) → r, (B) → s, (C) → p, (D) → q

EXERCISE - 2

PART - I

1.	(A)	**2.**	(B)	**3.**	(B)	**4.**	(A)	**5.**	(C)	**6.**	(D)	**7.**	(B)
8.	(B)	**9.**	(A)	**10.**	(B)	**11.**	(A)	**12.**	(A)	**13.**	(D)	**14.**	(A)
15.	(C)												

PART - II

1.	02.20	**2.**	12.28 or 12.29	**3.**	01.00	**4.**	22.00	**5.**	02.00	**6.**	01.00
7.	10.00	**8.**	03.82 or 03.83	**9.**	12.50	**10.**	03.00	**11.**	00.25	**12.**	01.00
13.	00.01	**14.**	01.00	**15.**	05.66 or 05.67						

PART - III

1.	(BD)	**2.**	(ABCD)	**3.**	(ABC)	**4.**	(ABCD)	**5.**	(ABCD)	**6.**	(AB)
7.	(CD)	**8.**	(ABCD)	**9.**	(BC)	**10.**	(ABD)				

PART - IV

1.	(D)	**2.**	(B)	**3.**	(C)	**4.**	(C)	**5.**	(A)	**6.**	(B)	**7.**	(BC)
8.	(A)	**9.**	(B)										

EXERCISE - 3

PART - I

1. (ABCD) **2.** (C) **3.** x = 8 **4.** x = 3 or – 3 **5.** (B)

6. (A) $\rightarrow$ (p), (r), (s) ; (B) $\rightarrow$ (q), (s) ; (C) $\rightarrow$ (q), (s) ; (D) $\rightarrow$ (p), (r), (s)

7. (C) **8.** (4) **9.** (ABC) **10.** (8)

PART - II

1.	(4)	**2.**	(1)	**3.**	(3)	**4.**	(2)	**5.**	(3)	**6.**	(3)	**7.**	(3)
8.	(1)	**9.**	(2)	**10.**	(2)	**11.**	(2)	**12.**	(4)	**13.**	(1)	**14.**	29

LOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	5 9 13 4 8 12	17 21 26 16 20 24	30 34 38 28 32 36
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 12 4 7 11	16 20 23 15 18 22	27 31 35 26 29 33
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 11 3 7 10	14 18 21 14 17 20	25 28 32 24 27 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	36 10 37 10	13 16 19 13 16 19	23 26 29 22 25 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	36 9 36 9	12 15 19 12 14 17	22 25 28 22 25 26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 9 3 6 8	11 14 17 11 14 17	20 23 26 19 22 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 6 8 3 5 8	11 14 16 10 13 16	19 22 24 18 21 23
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3 5 8 3 5 8	10 13 15 10 12 15	18 20 23 17 20 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 5 7 2 4 7	9 12 14 9 11 14	17 19 21 16 18 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7 2 4 6	9 11 13 8 11 13	16 18 20 15 17 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8

LOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	9766	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
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79	8976	9882	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
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81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
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93	9685	9689	9694	8699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
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98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 4 4

ANTILOGARITHM TABLE

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.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
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.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
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.34	2188	2193	2198	2203	2208	2213	2218	2223	2328	2234	1 1 2	2 3 3	4 4 5
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.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

ANTILOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1 1 2	3 4 4	5 6 7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1 2 2	3 4 5	5 6 7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1 2 2	3 4 5	5 6 7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1 2 2	3 4 5	6 6 7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1 2 2	3 4 5	6 6 7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1 2 2	3 4 5	6 6 7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1 2 3	3 4 5	6 7 8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1 2 3	3 4 5	6 7 8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1 2 3	4 4 5	6 7 8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1 2 3	4 5 5	6 7 8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1 2 3	4 5 6	6 7 8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4156	1 2 3	4 5 6	7 8 9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1 2 3	4 5 6	7 8 9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1 2 3	4 5 6	7 8 9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1 2 3	4 5 6	7 8 9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1 2 3	4 5 6	7 8 9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1 2 3	4 5 6	7 9 10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1 2 3	4 5 7	8 9 10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1 2 3	4 6 7	8 9 10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1 2 3	5 6 7	8 9 10
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.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1 2 4	5 6 7	8 10 11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1 2 4	5 6 7	9 10 11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1 3 4	5 6 8	9 10 11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1 3 4	5 6 8	9 10 12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1 3 4	5 7 8	9 10 12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1 3 4	5 7 8	9 11 12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1 3 4	5 7 8	10 11 12
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.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1 3 4	6 7 9	10 11 13
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.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2 3 5	6 8 9	11 12 14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2 3 5	6 8 9	11 13 14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2 3 5	6 8 10	11 13 15
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.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2 3 5	7 9 10	12 14 16
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.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2 4 6	8 9 11	13 15 17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2 4 6	8 10 12	14 15 17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2 4 6	8 10 12	14 16 18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2 4 6	8 10 12	14 16 18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2 4 6	8 10 12	15 17 19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2 4 6	8 11 13	15 17 19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2 4 7	9 11 13	15 17 20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2 4 7	9 11 13	16 18 20
.99	9772	9795	9817	9849	9863	9886	9908	9931	9954	9977	2 5 7	9 11 14	16 18 20

Exercise-1

✍ **Marked questions are recommended for Revision.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Representation of sets, Types of sets, subset and power set

A-1. State wether the following collections is a set or not ?
- (i) The collection of natural numbers between 2 and 20
- (ii) The collection of numbers which satisfy the equation $x^2 - 5x + 6 = 0$
- (iii) The collection of prime numbers between 1 and 100.
- (iv) The collection of all intelligent women in Jalandhar.

A-2. Write the following set in tabular form
- (i) A = {x : x is a positive prime < 10}
- (ii) $B = \{x : x = 3\lambda, x \in I, 1 \le \lambda \le 3\}$

A-3. Write the following set in builder form
- (i) set of all rational number
- (ii) {2, 5, 10, 17, 26, 37, ...}

A-4. Identifiy type of set in terms of empty/singleton/finite/infinite
- (i) {x : x is a real number and $x^2 - 1 = 0$}
- (ii) {x : x is a real number and $x^2 + 1 = 0$}
- (iii) {x : x is positive real number and $x^2 - 9 = 0$}
- (iv) {x : x is a real number and $x^2 + 2x + 2 \ge 0$}

A-5. Write power set of the set $A = \{\phi, \{\phi\}\}$.

Section (B) : Operations on sets, Law of Algebra of sets

B-1. Given the sets A = {1, 2, 3}, B = {3, 4}, C = {4, 5, 6}, then find the following
(i) $A \cup (B \cap C)$ (ii) $A - (B \cap C)$ (iii) $(B \cup C) - A$

B-2. Find the smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$

B-3. If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where $b, c \in N$, $b \ge 2$, $c \ge 2$ are relatively prime, then write 'd' in terms of b and c.

B-4.✍ Sets A and B have 3 and 6 elements respectively. What can be the minimum and maximum number of elements in
(i) $A \cap B$ (ii) $A \cup B$

Section (C) : Cardinal number Problems

C-1. Let n(U) = 700, n(A) = 200, n(B) = 300 and $n(A \cap B) = 100$, then find $n(A' \cap B')$

C-2.✍ In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. Find the number of newspaper.

C-3.✍ In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3 % buy B and C and 4% buy A and C. If 2% families buy all the three news papers, then find number of families which buy newspaper A only.

C-4. In a survery, it was found that 21 persons liked product A, 26 liked product B and 29 liked product C. If 14 persons liked products A and B, 12 liked products C and A, 13 persons liked products B and C and 8 liked all the three products then

(i) Find the number of persons who liked atleast one product

(ii) The number of persons who like the products A and B but not C

Section (D) : Rational Inequalities

D-1. Solve the following rational in equalities

(i) $\frac{(x-1)(x+2)}{(x-3)(x+3)} < 0$

(ii) $\frac{(1-x)^3(x+2)^4}{(x+9)^2(x-8)} \geq 0$

(iii) $\frac{x^2+4x+4}{2x^2-x-1} > 0$

(iv) $\frac{(2-x^2)(x-3)^3}{(x+1)(x^2-3x-4)} \geq 0$

(v) $\frac{(x+2)(x^2-2x+1)}{4+3x-x^2} \geq 0$

D-2. Solve the following Inequalities

(i) $\frac{7x-5}{8x+3} > 4$

(ii) $\frac{14x}{x+1} < \frac{9x-30}{x-4}$

(iii) $\frac{(x-1)(x-2)(x-3)}{(x+1)(x+2)(x+3)} \leq 1$

(iv) $\frac{x^2+2}{x^2-1} < -2$

D-3. Solve the following rational in equalities

(i) $\frac{(x^2-3x+1)^3}{(x-1)(x+2)} \leq 0$

(ii) $\frac{2x^2-3x-459}{x^2+1} > 1$

(iii) $\frac{x^2-5x+12}{x^2-4x+5} > 3$

(iv) $\frac{x^4+x^2+1}{x^2-4x-5} > 0$

D-4. Solve the following rational in equalities

(i) $x^4 - 5x^2 + 4 \leq 0$

(ii) $x^4 - 2x^2 - 63 \leq 0$

(iii) $(x^2+3x+1)(x^2+3x-3) \geq 5$

D-5. If $1 < \frac{x-1}{x+2} < 7$ then find the range of

(i) x (ii) x^2 (iii) $\frac{1}{x}$

D-6. Find the number of positive integral value of x satisfying the inequality $\frac{(3^x-5^x)(x-2)}{(x^2+5x+2)} \geq 0$

Section (E) : Logarithmic Properties

E-1. Find the value of

(i) $\log_{10}5.\log_{10}20 + (\log_{10}2)^2$

(ii) $5^{\log_{\sqrt{5}} 2} + 9^{\log_3 7} - 8^{\log_2 5}$

(iii) $\sqrt[3]{5^{\frac{1}{\log_7 5}} + \frac{1}{(-\log_{10} 0.1)}}$

(iv) $\log_{0.75} \log_2 \sqrt{\sqrt{\frac{1}{0.125}}}$

(v) $\left(\frac{1}{49}\right)^{1+\log_7 2} + 5^{-\log_{1/5} 7}$

(vi) $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

E-2. Which of the following numbers are positive/negative

(i) $\log_{\sqrt{3}} \sqrt{2}$ (ii) $\log_{1/7}(2)$ (iii) $\log_{1/3}(1/5)$

(iv) $\log_3(4)$ (v) $\log_7(2.11)$ (vi) $\log_3(\sqrt{7}-2)$

(vii) $\log_4\left(\dfrac{\sqrt{2}+1}{\sqrt{2}-1}\right)$ (viii) $\log_3\left(\dfrac{2\cdot\sqrt[3]{3}}{3}\right)$ (ix) $\log_{10}(\log_{10} 9)$

E-3. Let $\log_{10}2 = a$ and $\log_{10}3 = b$ then determine the following logarithms in terms of a and b.

(i) $\log_{10}\left(\sin^2\dfrac{\pi}{3}\right)$ (ii) $\log_{100}4 + 2\log_{100}27$

(iii) $\log_2 9 + \log_3 8$ (iv) $\log_{\sqrt{45}} 144$

E-4. (i) Let n = 75600, then find the value of $\dfrac{4}{\log_2 n}+\dfrac{3}{\log_3 n}+\dfrac{2}{\log_5 n}+\dfrac{1}{\log_7 n}$

(ii) If $\log_2(\log_3(\log_4(x)))=0$ and $\log_3(\log_4(\log_2(y)))=0$ and $\log_4(\log_2(\log_3(z)))=0$ then find the sum of x, y and z is

(iii) Suppose n be an integer greater than 1. let $a_n = \dfrac{1}{\log_n 2002}$. Suppose $b = a_2 + a_3 + a_4 + a_5$ and $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then find the value of (b – c)

E-5. Show that the number $\log_2 7$ is an irrational number.

E-6. If $\dfrac{\log a}{b-c}=\dfrac{\log b}{c-a}=\dfrac{\log c}{a-b}$, show that $a^a.\ b^b.\ c^c = 1$.

Section (F) : Logarithmic Equation

F-1. Solve the following equations :

(i) $\log_x(4x-3) = 2$ (ii) $\log_2(\log_3(x^2-1)) = 0$

(iii) $4^{\log_2 x} - 2x - 3 = 0$ (iv) $\log_4(\log_2 x) + \log_2(\log_4 x) = 2$.

(v) $\log_3\left(\log_9 x + \dfrac{1}{2} + 9^x\right) = 2x$. (vi) $2\log_4(4-x) = 4 - \log_2(-2-x)$.

(vii) $x^{(\log_{\sqrt{x}} 2x)} = 4$ (viii) $x^{0.5\log_{\sqrt{x}}(x^2-x)} = 3^{\log_9 4}$.

F-2. (i) Find the product of roots of the equation
$(\log_3 x)^2 - 2(\log_3 x) - 5 = 0$

(ii) Find sum of roots of the equation $4^x - 7.2^x + 6 = 0$

(iii) Solve for x : $x^{\log_{10} x+2} = 10^{\log_{10} x+2}$

(iv) Solve for x : $x^{\frac{\log_{10} x+5}{3}} = 10^{5+\log_{10} x}$

Section (G) : Logarithmic inequalities

G-1. Solve the following inequalities

(i) $\log_{\frac{5}{8}}\left(2x^2 - x - \dfrac{3}{8}\right) \geq 1$ (ii) $\log_{\frac{1}{2}}(x^2-5x+6) > -1$

(iii) $\log_7 \dfrac{2x-6}{2x-1} > 0$ (iv) $\log_{1/4}(2-x) > \log_{1/4}\left(\dfrac{2}{x+1}\right)$

(v) $\log_{1/3}(2^{x+2} - 4^x) \geq -2$

G-2. Find the number of integers satisfying $\log_{1/5}\frac{4x+6}{x} \geq 0$

G-3 Solve the inequalities

(i) $(\log_5 x)^2 + \log_5 x - 2 \leq 0$ (ii) $15^x - 25.3^x - 9.5^x + 225 \geq 0$

(iii) $8.\left(\frac{3^{x-2}}{3^x - 2^x}\right) > 1 + \left(\frac{2}{3}\right)^x$

G-4. Solve the following inequalities :

(i) $\log_x(4x - 3) \geq 2$ (ii) $\log_{(3x^2+1)} 2 < \frac{1}{2}$

(iii) $\log_{x^2}(2 + x) < 1$

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Representation of sets, Types of sets, subset and power set

A-1. The set of intelligent students in a class is-
(A) a null set (B) a singleton set
(C) a finite set (D) not a well defined collection

A-2. The set A = $\{x : x \in R, x^2 = 16 \text{ and } 2x = 6\}$ is
(A) Null set (B) Singleton set
(C) Infinite set (D) not a well defined collection

A-3. If A = $\{x : -3 < x < 3, x \in Z\}$ then the number of subsets of A is –
(A) 120 (B) 30 (C) 31 (D) 32

A-4. Which of the following are true ?
(A) $[3, 7] \subset (2, 10)$ (B) $(0, \infty) \subset (4, \infty)$ (C) $(5, 7] \subset [5, 7)$ (D) $[2, 7] \subset (2.9, 8)$

A-5. The number of subsets of the power set of set A = {7, 10, 11} is
(A) 32 (B) 16 (C) 64 (D) 256

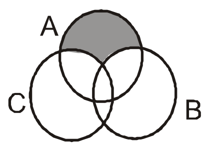
A-6. Which of the folowing sets is an infinite set ?
(A) Set of divisors of 24
(B) Set of all real number which lie between 1 and 2
(C) Set of all humman beings living in India.
(D) Set of all three digit natural numbers

Section (B) : Operations on sets, Law of Algebra of sets

B-1. Let A = $\{x : x \in R, -1 < x < 1\}$, B = $\{x : x \in R, x \leq 0 \text{ or } x \geq 2\}$ and $A \cup B = R - D$, then the set D is
(A) $\{x : 1 < x \leq 2\}$ (B) $\{x : 1 \leq x < 2\}$ (C) $\{x : 1 \leq x \leq 2\}$ (D) $\{x : 1 < x < 2\}$

B-2. If A = {2, 3, 4, 8, 10}, B = {3, 4, 5, 10, 12}, C = {4, 5, 6, 12, 14} then $(A \cap B) \cup (A \cap C)$ is equal to
(A) {3, 4, 10} (B) {2, 8, 10} (C) {4, 5, 6} (D) {3, 5, 14}

B-3. The shaded region in the given figure is



(A) $A \cap (B \cup C)$ (B) $A \cup (B \cap C)$ (C) $A \cap (B - C)$ (D) $A - (B \cup C)$

B-4. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 5}, B = {6, 7}, then $A \cap B'$ is
(A) B' (B) A (C) A' (D) B

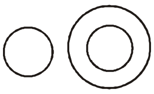
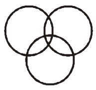
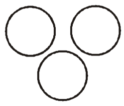
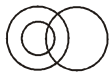
B-5. If A = $\{x : x = 4n + 1, n \le 5, n \in N\}$ and B = $\{3n : n \le 8, n \in N\}$, then A – (A – B) is :
(A) {9, 21} (B) {9, 12} (C) {6, 12} (D) {6, 21}

B-6. $A \cup B = A \cap B$ iff :
(A) $A \subset B$ (B) A = B (C) $A \supset B$ (D) $A \not\subset B$

B-7. Consider the following statements :
1. $N \cup (B \cap Z) = (N \cup B) \cap Z$ for any subset B of R, where N is the set of positive integers, Z is the set of integers, R is the set of real numbers.
2. Let A = $\{n \in N : 1 \le n \le 24$, n is a multiple of 3$\}$. There exists no subset B of N such that the number of elemets in A is equal to the number of elements in B.

Which of the above statements is/are correct ?
(A) 1 only (B) 2 only (C) Both 1 and 2 (D) Neither 1 nor 2

B-8. Which of the following venn-diagrams best represents the sets of females, mothers and doctors ?

(A)  (B)  (C)  (D) 

Section (C) : Cardinal number Problems

C-1. Let A and B be two sets. Then
(A) $n(A \cup B) \le n(A \cap B)$ (B) $n(A \cap B) \le n(A \cup B)$
(C) $n(A \cap B) = n(A \cup B)$ (D) can't be say

C-2. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is
(A) 80 percent (B) 40 percent (C) 60 percent (D) 70 percent

C-3. A class has 175 students. The following data shows the number of students obtaining one or more subjects : Mathematics 100, Physics 70, Chemistry 40, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics & Physics & Chemistry 18. How many students have offered Mathematics alone ?
(A) 35 (B) 48 (C) 60 (D) 22

C-4. 31 candidates appeared for an examination, 15 candidates passed in English, 15 candidates passed in Hindi, 20 candidates passed in Sanskrit. 3 candidates passed only in English. 4. candidates passed only in Hindi, 7 candidates passed only in Sanskrit. 2 candidates passed in all the three subjects How many candidates passed only in two subjects ?
(A) 17 (B) 15 (C) 22 (D) 14

Section (D) : Rational Inequalities

D-1. The complete solution set of the inequality $\dfrac{x^4 - 3x^3 + 2x^2}{x^2 - x - 30} \ge 0$ is:
(A) $(-\infty, -5) \cup (1, 2) \cup (6, \infty) \cup \{0\}$ (B) $(-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$
(C) $(-\infty, -5] \cup [1, 2] \cup [6, \infty) \cup \{0\}$ (D) $(-\infty, -5] \cup [1, 2] \cup [6, \infty)$

D-2. Number of positive integral values of x satisfying the inequality
$$\frac{(x-4)^{2017}.\ (x+8)^{2016}\ (x+1)}{x^{2016}(x-2)^3\ .\ (x+3)^5\ .\ (x-6)\ (x+9)^{2018}} \le 0 \text{ is}$$
(A) 0 (B) 1 (C) 2 (D) 3

D-3. The number of prime numbers satisfying the inequality $\dfrac{x^2 - 1}{2x + 5} < 3$ is
(A) 1 (B) 2 (C) 3 (D) 4

D-4. The complete solution of $\frac{x^2-1}{x+3} \geq 0$ & $x^2 - 5x + 2 \leq 0$ is :

(A) $x \in \left[\frac{5-\sqrt{17}}{2}, \frac{5+\sqrt{17}}{2}\right]$ (B) $x \in \left[1, \frac{5+\sqrt{17}}{2}\right]$

(C) $x \in (-3, -1]$ (D) $x \in (-3, -1] \cup [1, \infty)$

D-5 The solution of the inequality $2x - 1 \leq x^2 + 3 \leq x - 1$ is

(A) $x \in R$ (B) $[-2, 2]$ (C) $(-2, 2)$ (D) $x \in \phi$

D-6. The number of the integral solutions of $x^2 + 9 < (x + 3)^2 < 8x + 25$ is :

(A) 1 (B) 3 (C) 4 (D) 5

D-7. Number of non-negative integral values of x satisfying the inequality $\frac{2}{x^2-x+1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \geq 0$ is

(A) 0 (B) 1 (C) 2 (D) 3

Section (E) : Logarithmic Properties

E-1. If $a^4 \cdot b^5 = 1$ then the value of $\log_a(a^5b^4)$ equals

(A) 9/5 (B) 4 (C) 5 (D) 8/5

E-2. $\frac{1}{1+\log_b a+\log_b c} + \frac{1}{1+\log_c a+\log_c b} + \frac{1}{1+\log_a b+\log_a c}$ has the value equal to

(A) abc (B) $\frac{1}{abc}$ (C) 0 (D) 1

E-3. $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to :

(A) 1/2 (B) 1 (C) 2 (D) 4

E-4. $(\log_2 10) . (\log_2 80) - (\log_2 5) . (\log_2 160)$ is equal to :

(A) $\log_2 5$ (B) $\log_2 20$ (C) $\log_2 10$ (D) $\log_2 16$

E-5. The ratio $\frac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to :

(A) $a^2 - a - 1$ (B) $a^2 + a - 1$ (C) $a^2 - a + 1$ (D) $a^2 + a + 1$

E-6. If $\log_a(ab) = x$, then $\log_b(ab)$ is equal to

(A) $\frac{1}{x}$ (B) $\frac{x}{1+x}$ (C) $\frac{x}{1-x}$ (D) $\frac{x}{x-1}$

E-7. $10^{\log_p(\log_q(\log_r x))} = 1$ and $\log_q(\log_r(\log_p x)) = 0$ then 'p' equals

(A) $r^{q/r}$ (B) rq (C) 1 (D) $r^{r/q}$

E-8. Which one of the following is the smallest?

(A) $\log_{10}\pi$ (B) $\sqrt{\log_{10}\pi^2}$ (C) $\left(\frac{1}{\log_{10}\pi}\right)^3$ (D) $\left(\frac{1}{\log_{10}\sqrt{\pi}}\right)$

E-9. $\log_{10}(\log_2 3) + \log_{10}(\log_3 4) + \log_{10}(\log_4 5) + \ldots\ldots + \log_{10}(\log_{1023} 1024)$ simplifies to

(A) a composite (B) a prime number

(C) rational which is not an integer (D) an integer

Section (F) : Logarithmic Equation

F-1. The sum of all the solutions to the equation $2\log_{10} x - \log_{10}(2x - 75) = 2$

(A) 30 (B) 350 (C) 75 (D) 200

F-2. If the solution of the equation $\log_x(125x) \cdot \log_{25}^2 x = 1$ are α and β ($\alpha < \beta$). Then the value of $1/\alpha\beta$ is :

(A) 5 (B) 25 (C) 125 (D) 625

F-3. The positive integral solution of the equation $\log_x \sqrt{5} + \log_x 5x = \frac{9}{4} + \log_x^2 \sqrt{5}$ is :

(A) Composite number (B) Prime number
(C) Even number (D) Divisible by 3

F-4. The expression $\log_p \underbrace{\log_p \sqrt[p]{\sqrt[p]{\sqrt[p]{\cdots\cdots\sqrt[p]{p}}}}}_{\text{n radical sign}}$, where $p \geq 2$, $p \in N$; $n \in N$ when simplified is

(A) independent of p (B) independent of p and of n
(C) dependent on both p and n (D) positive

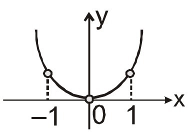
F-5. If $\log_x \log_{18}\left(\sqrt{2} + \sqrt{8}\right) = \frac{1}{3}$. Then the value of 1000 x is equal to

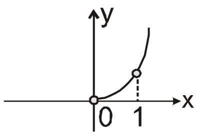
(A) 8 (B) 1/8 (C) 1/125 (D) 125

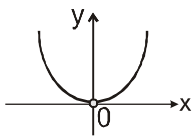
F-6. Number of real solutions of the equation $\sqrt{\log_{10}(-x)} = \log_{10}\sqrt{x^2}$ is :

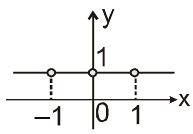
(A) zero (B) exactly 1 (C) exactly 2 (D) 4

F-7. The correct graph of $y = x^{\log_x x^2}$ is

(A) 

(B) 

(C) 

(D) 

Section (G) : Logarithmic inequalities

G-1. The solution set of the inequality $\log_{\sin\left(\frac{\pi}{3}\right)}(x^2 - 3x + 2) \geq 2$ is

(A) $\left(\frac{1}{2}, 2\right)$ (B) $\left(1, \frac{5}{2}\right)$ (C) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$ (D) $(1, 2)$

G-2. If $\log_{0.3}(x - 1) < \log_{0.09}(x - 1)$, then x lies in the interval

(A) $(2, \infty)$ (B) $(1, 2)$ (C) $(-2, -1)$ (D) $\left(1, \frac{3}{2}\right)$

G-3. Solution set of the inequality $2 - \log_2(x^2 + 3x) \geq 0$ is :

(A) $[-4, 1]$ (B) $[-4, -3) \cup (0, 1]$
(C) $(-\infty, -3) \cup (1, \infty)$ (D) $(-\infty, -4) \cup [1, \infty)$

G-4. If $\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1$, then 'x' lies in the interval

(A) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3)$ (B) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 2)$
(C) $(\sqrt{5}, 3\sqrt{5})$ (D) ϕ

G-5. The set of all solutions of the inequality $(1/2)^{x^2 - 2x} < 1/4$ contains the set

(A) $(-\infty, 0)$ (B) $(-\infty, 1)$ (C) $(1, \infty)$ (D) $(3, \infty)$

G-6. The number of positive integers not satisfying the inequality $\log_2(4^x - 2.2^x + 17) > 5$.

(A) 2 (B) 3 (C) 4 (D) 1

G-7. The set of all the solutions of the inequality $\log_{1-x}(x - 2) \geq -1$ is

(A) $(-\infty, 0)$ (B) $(2, \infty)$ (C) $(-\infty, 1)$ (D) ϕ

PART - III : MATCH THE COLUMN

1. Match the set P in column one with its super set Q in column II

Column – I (set P)		**Column– II** (set Q)	
(A)	$[3^{2n} - 8n - 1 : n \in N\}$	(p)	$\{49(n-1) : n \in N\}$
(B)	$\{2^{3n} - 1 : n \in N\}$	(q)	$\{64(n-1) : n \in N\}$
(C)	$\{3^{2n} - 1 : n \in N\}$	(r)	$\{7n : n \in N\}$
(D)	$\{2^{3n} - 7n - 1 : n \in N\}$	(s)	$\{8n : n \in N\}$

2.✍

Column-I		**Column-II**	
(A)	If $a = 3\left(\sqrt{8+2\sqrt{7}} - \sqrt{8-2\sqrt{7}}\right)$, $b = \sqrt{(42)(30)+36}$ then the value of $\log_a b$ is equal to	(p)	-1
(B)	If $a = \sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}$, $b = \sqrt{11+6\sqrt{2}} - \sqrt{11-6\sqrt{2}}$, then the value of $\log_a b$ is equal to	(q)	1
(C)	If $a = \sqrt{3+2\sqrt{2}}$, $b = \sqrt{3-2\sqrt{2}}$ then the value of $\log_a b$ is equal to	(r)	2
(D)	If $a = \sqrt{7+\sqrt{7^2-1}}$, $b = \sqrt{7-\sqrt{7^2-1}}$, then the value of $\log_a b$ is equal to	(s)	$\frac{3}{2}$
(E)	The number of zeroes at the end of the product of first 20 prime numbers, is	(t)	None
(F)	The number of solutions of $2^{2x} - 3^{2y} = 55$, in which x and y are integers, is		

3.

Column-I		**Column-II**	
(A)	When the repeating decimal 0.363636............. is written as a rational fraction in the simplest form, the sum of the numerator and denominator is	(p)	4
(B)	Given positive integer p, q and r with $p = 3^q \cdot 2^r$ and $100 < p < 1000$. The difference between maximum and minimum values of (q + r), is	(q)	0
(C)	If $\log_8 a + \log_8 b = (\log_8 a)(\log_8 b)$ and $\log_a b = 3$, then the value of 'a' is	(r)	15
(D)✍	If $P = 3^{\sqrt{\log_3 2}} - 2^{\sqrt{\log_2 3}}$ then value of P is	(s)	16

4.

Column-I		**Column-II**	
(A)	Anti logarithm of $(0.\bar{6})$ to the base 27 has the value equal to	(p)	5
(B)	Characteristic of the logarithm of 2008 to the base 2 is		
(C)	The value of b satisfying the equation, $\log_e 2 \cdot \log_b 625 = \log_{10} 16 \cdot \log_e 10$ is	(q)	7
(D)	Number of naughts after decimal before a significant figure comes in the number $\left(\frac{5}{6}\right)^{100}$, is (Given $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$)	(r)	9
		(s)	10

Exercise-2

✍ **Marked questions are recommended for Revision.**

*** Marked Questions may have more than one correct option.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1.✍ Let A_1, A_2 and A_3 be subsets of a set X. Which one of the following is correct ?

(A) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containing elements of each of A_1, A_2 and A_3

(B) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containing either A_1 or $A_2 \cup A_3$ but not both

(C) The smallest subset of X containing $A_1 \cup A_2$ and A_3 equals the smallest subset of X containing both A_1 and $A_2 \cup A_3$ only if $A_2 = A_3$

(D) None of these

2. Let A, B, C be distinct subsets of a universal set U. For a subset X of U, let X ' denote the complement of X in U.

Consider the following sets :

1. $(((A \cap B) \cup C)' \cap B')' = B \cap C$
2. $(A' \cap B') \cap (A \cup B \cup C') = (A \cup (B \cup C))'$

Which of the above statements is/are correct ?

(A) 1 only (B) 2 only (C) Both 1 and 2 (D) Neither 1 nor 2

3.✍ In an examination of a certain class, at least 70% of the students failed in Physics, at least 72% failed in Chemistry, at least 80% failed in Mathematics and at least 85% failed in English. How many at least must have failed in all the four subjects ?

(A) 9% (B) 7% (C) 15%

(D) Cannot be determined due to insufficient data

4. Let X and Y be two sets.

Statement-1 $X \cap (Y \cup X)' = \phi$

Statement-2 If $X \cup Y$ has m elements and $X \cap Y$ has n elements then symmetric difference $X \Delta Y$ has m – n elements.

(A) Both the statements are true. (B) Statement-I is true, but Statement-II is false.

(C) Statement-I is false, but Statement-II is true. (D) Both the statements are false.

5. If $\dfrac{6x^2 - 5x - 3}{x^2 - 2x + 6} \le 4$, then the least and the highest values of $4x^2$ are:

(A) 36 & 81 (B) 9 & 81 (C) 0 & 81 (D) 9 & 36

6. Sum of all the real solutions of the inequality $\dfrac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x^2-9)} \le 0$ is

(A) 5 (B) 4 (C) 8 (D) 0

7. If $\log_a b = 2$; $\log_b c = 2$ and $\log_3 c = 3 + \log_3 a$ then (a + b + c) equals

(A) 90 (B) 93 (C) 102 (D) 243

8. Let $x = \left(\log_{1/3} 5\right)\left(\log_{125} 343\right)\left(\log_{49} 729\right)$ and y = $25^{3\log_{289} 11 \, \log_{28} \sqrt{17} \, \log_{1331} 784}$, then value of $\dfrac{y}{x}$ is

(A) $\dfrac{5}{3}$ (B) $-\dfrac{5}{3}$ (C) $-\dfrac{4}{5}$ (D) $\dfrac{3}{7}$

9. The expression: $\dfrac{\left(\dfrac{x^2+3x+2}{x+2}\right)+3x-\dfrac{x(x^3+1)}{(x+1)(x^2-x+1)}\log_2 8}{(x-1)(\log_2 3)(\log_3 4)(\log_4 5)(\log_5 2)}$ reduces to

(A) $\dfrac{x+1}{x-1}$ (B) $\dfrac{x^2+3x+2}{(\log_2 5)x-1}$ (C) $\dfrac{3x}{x-1}$ (D) x

10. If a, b, c are positive real numbers such that $a^{\log_3 7} = 27$; $b^{\log_7 11} = 49$ and $c^{\log_{11} 25} = \sqrt{11}$. The value of $\left(a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}\right)$ equals

(A) 489 (B) 469 (C) 464 (D) 400

11. Consider the statement : $x(\alpha - x) < y(\alpha - y)$ for all x, y with $0 < x < y < 1$. The statement is true

(A) if and only if $\alpha \geq 2$
(B) if and only if $\alpha > 2$
(C) if and only if $\alpha < -1$
(D) for no values of α

12. The set of values of x satisfying simultaneously the inequalities $\dfrac{\sqrt{(x-8)\ (2-x)}}{\log_{0.3}\left(\dfrac{10}{7}\ (\log_2 5-1)\right)} \geq 0$ and $2^{x-3} - 31 > 0$ is :

(A) a unit set
(B) an empty set
(C) an infinite set
(D) a set consisting of exactly two elements.

13. The solution set of the inequality $\dfrac{(3^x - 4^x)\cdot \ell n(x+2)}{x^2-3x-4} \leq 0$ is

(A) $(-\infty, 0] \cup (4, \infty)$
(B) $(-2, 0] \cup (4, \infty)$
(C) $(-1, 0] \cup (4, \infty)$
(D) $(-2, -1) \cup (-1, 0] \cup (4, \infty)$

14. If $\sqrt{\log_4\{\log_3\{\log_2(x^2-2x+a)\}\}}$ is defined $\forall x \in R$, then the set of values of 'a' is

(A) $[9, \infty)$ (B) $[10, \infty)$ (C) $[15, \infty)$ (D) $[2, \infty)$

15. If $\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$ then value of x is equal to

(A) $\dfrac{3}{4}$ (B) $-\dfrac{3}{4}$ (C) $-\dfrac{1}{4}$ (D) $\dfrac{3}{2}$

PART-II: NUMERICAL VALUE QUESTIONS

INSTRUCTION :

- The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto two digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.

1. Let U be set with number of elements in it is 2009 and A, B are subsets of U with n (A ∪ B) = 280. If $n(A' \cap B') = x_1^3 + x_2^3 = y_1^3 + y_2^3$ for some positive integers $x_1 < y_1 < y_2 < x_2$, then find value of $\dfrac{x_2 + y_2}{x_1 + y_1}$

2. Let U be set with number of elements in it is 2009. A is a subset of U with n (A) = 1681 and out of these 1681 elements, exactly 1075 elements belong to a subset B of U. If n (A – B) = $m^2 + p_1 p_2 p_3$ for some positive integer m and distinct primes $p_1 < p_2 < p_3$ then for least m find $\dfrac{p_1 p_3}{p_2}$

3. Let A = {(x, y) : x ∈ R, y ∈ R, $x^3 + y^3 = 1$], B = {(x, y) : x ∈ R, y ∈ R, x – y = 1} and C = {(x, y) : x ∈ R, y ∈ R, x + y = 1}. If A ∩ B contains 'p' elements and A ∩ C contains 'q' elements then find (q – p).

4. In a class of 42 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. Then find number of students who have taken exactly one subject.

5. If c(a – b) = a(b – c) then find the value of $\frac{\log(a+c)+\log(a-2b+c)}{\log(a-c)}$ (Assume all terms are defined)

6. If $\log_b a.\ \log_c a + \log_a b.\ \log_c b + \log_a c.\ \log_b c = 3$ (where a, b, c are different positive real numbers ≠ 1), then find the value of a b c.

7. If $4^A + 9^B = 10^C$, where $A = \log_{16}4$, $B = \log_3 9$ & $C = \log_x 83$, then find x.

8. Let a, b, c, d are positive integers such that $\log_a b = \frac{3}{2}$ and $\log_c d = \frac{5}{4}$. If (a – c) = 9, find the value of $\frac{b+d}{a+c}$.

9. Find the positive number, x , which satisfies the equation $\log_{10}(2x^2 - 21x + 50) = 2$

10. Find the value of x satisfying the equation $\log_{\frac{1}{2}}(x-1) + \log_{\frac{1}{2}}(x+1) - \log_{\frac{1}{\sqrt{2}}}(7-x) = 1$

11. Find sum of roots of equation $\log_{10}^2 x + \log_{10}x^2 = \log_{10}^2 2 - 1$

12. If the product of all solutions of the equation $\frac{(2009)x}{2010} = (2009)^{\log_x(2010)}$ can be expressed in the lowest form as $\frac{m}{n}$ then the value of (m – n) is

13. If the complete solution set of the inequality $(\log_{10} x)^2 \geq \log_{10} x + 2$ is (0, a] ∪ [100, ∞) then find the value of a.

14. The complete solution set of the inequality $\frac{1}{\log_4 \frac{x+1}{x+2}} < \frac{1}{\log_4(x+3)}$, is (–a, ∞), then determine 'a'.

15. If complete solution set of inequality $\log_{1/2}(x+5)^2 > \log_{1/2}(3x-1)^2$ is (–∞,p) ∪ (q,r) ∪ (s,∞) then find $\frac{p^2+q^2+r^2}{s^2}$

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. Let a > 2, a ∈ N be a constant. If there are just 18 positive integers satisfying the inequality $(x-a)(x-2a)(x-a^2) < 0$ then which of the option(s) is/are correct?
(A) 'a' is composite (B) 'a' is odd
(C) 'a' is greater than 8 (D) 'a' lies in the interval (3, 11)

2. Let N = $\frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$. Then N is :
(A) a natural number (B) a prime number (C) a rational number (D) an integer

3.✍ Values of x satisfying the equation $\log_5^2 x + \log_{5x}\left(\frac{5}{x}\right) = 1$ are

(A) 1 (B) 5 (C) $\frac{1}{25}$ (D) 3

4. The equation $\log_{x^2} 16 + \log_{2x} 64 = 3$ has :

(A) one irrational solution (B) no prime solution
(C) two real solutions (D) one integral solution

5.✍ The equation $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$ has

(A) exactly three real solution (B) at least one real solution
(C) exactly one irrational solution (D) complex roots.

6. The solution set of the system of equations $\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_{27}(x + y) = \frac{2}{3}$ is :

(A) {6, 3} (B) {3, 6} (C) {6, 12} (D) {12, 6}

7. Consider the quadratic equation, $(\log_{10}8)x^2 - (\log_{10}5)x = 2(\log_2 10)^{-1} - x$. Which of the following quantities are irrational.

(A) sum of the roots (B) product of the roots
(C) sum of the coefficients (D) discriminant

8.✍ If $\log_a x = b$ for permissible values of a and x then identify the statement(s) which can be correct?

(A) If a and b are two irrational numbers then x can be rational.
(B) If a rational and b irrational then x can be rational.
(C) If a irrational and b rational then x can be rational.
(D) If a rational and b rational then x can be rational.

9. Which of the following statements are true

(A) $\log_2 3 < \log_{12} 10$ (B) $\log_6 5 < \log_7 8$
(C) $\log_3 26 < \log_2 9$ (D) $\log_{16} 15 > \log_{10} 11 > \log_7 6$

10. If $\frac{1}{2} \le \log_{0.1} x \le 2$, then

(A) maximum value of x is $\frac{1}{\sqrt{10}}$ (B) x lies between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$
(C) minimum value of x is $\frac{1}{10}$ (D) minimum value of x is $\frac{1}{100}$

PART - IV : COMPREHENSION

Comprehension # 1 (1 to 3)

In a group of 1000 people, there are 750 people, who can speak Hindi and 400 people, who can speak Bengali.

1. Number of people who can speak Hindi only is

(A) 300 (B) 400 (C) 500 (D) 600

2. Number of people who can speak Bengali only is

(A) 150 (B) 250 (C) 50 (D) 100

3. Number of people who can speak both Hindi and Bengali is

(A) 50 (B) 100 (C) 150 (D) 200

Comprehension # 2 (4 to 6)

Let A denotes the sum of the roots of the equation $\frac{1}{5-4\log_4 x} + \frac{4}{1+\log_4 x} = 3$.

B denotes the value of the product of m and n, if $2^m = 3$ and $3^n = 4$.

C denotes the sum of the integral roots of the equation $\log_{3x}\left(\frac{3}{x}\right) + (\log_3 x)^2 = 1$.

4. The value of A + B equals

(A) 10 (B) 6 (C) 8 (D) 4

5. The value of B + C equals

(A) 6 (B) 2 (C) 4 (D) 8

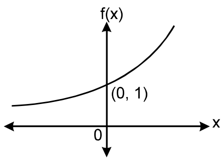
6. The value of A + C ÷ B equals

(A) 5 (B) 8 (C) 7 (D) 4

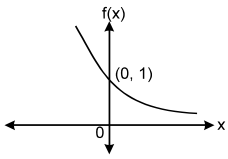
Comprehension # 3 (Q.7- to Q.9)

A function $f(x) = a^x (a > 0, a \neq 1, x \in R)$ is called an exponential function. Graph of exponential function can be as follows :

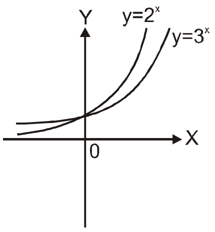
Case - I
For a > 1

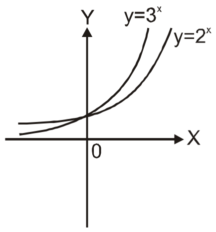


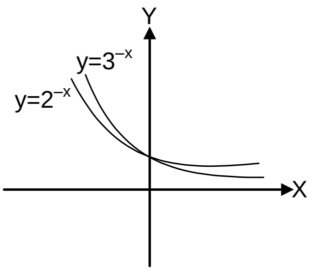
Case - II
For 0 < a < 1

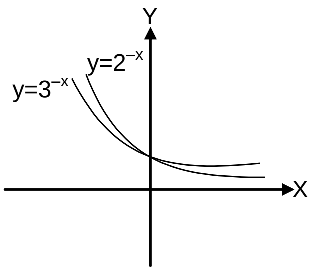


7*. Which of the following is correct :

(A)


(B)


(C)


(D)


8. Number of solutions of $3^x + x - 2 = 0$ is/are :

(A) 1 (B) 2 (C) 3 (D) 4

9. The number of positive solutions of $\log_{1/2} x = 7^x$ is/are :

(A) 0 (B) 1 (C) 2 (D) 3

Exercise-3

✍ **Marked questions are recommended for Revision.**

*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Indicate all correct alternatives, where base of the log is 2. **[JEE '89]**

The equation $x^{(3/4)(\log x)^2 + \log x - (5/4)} = \sqrt{2}$ has :

(A) at least one real solution (B) exactly three real solutions

(C) exactly one irrational solution (D) complex roots

2. The number $\log_2 7$ is :

(A) an integer (B) a rational number

(C) an irrational number (D) a prime number **[JEE '90]**

3.✍ Find all real numbers x which satisfy the equation

$2\log_2\log_2 x + \log_{1/2}\log_2(2\sqrt{2}\,x) = 1.$ **[REE – 1999, 6]**

4.✍ Solve the equation $\log_{3/4}\log_8(x^2+7) + \log_{1/2}\log_{1/4}(x^2+7)^{-1} = -2.$ **[REE– 2000, 5]**

5. The number of solution(s) of $\log_4(x-1) = \log_2(x-3)$ is/are **[IIT-JEE-2002, Scr., (1, 0)/35]**

(A) 3 (B) 1 (C) 2 (D) 0

6. Let $f(x) = \dfrac{x^2-6x+5}{x^2-5x+6}$ **[IIT-JEE 2007, Paper-2, (6, 0), 81]**

Column – I		**Column – II**	
(A)	If $-1 < x < 1$, then f(x) satisfies	(p)	$0 < f(x) < 1$
(B)	If $1 < x < 2$, then f(x) satisfies	(q)	$f(x) < 0$
(C)	If $3 < x < 5$, then f(x) satisfies	(r)	$f(x) > 0$
(D)	If $x > 5$, then f(x) satisfies	(s)	$f(x) < 1$

7.✍ Let (x_0, y_0) be the solution of the following equations

$(2x)^{\ell n2} = (3y)^{\ell n3}$

$3^{\ell nx} = 2^{\ell ny}$.

Then x_0 is **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

8.✍ The value of $6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots}}}\right)$ is

[IIT-JEE 2012, Paper-1, (4, 0), 70]

9*. If $3^x = 4^{x-1}$, then x = **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) $\dfrac{2\log_3 2}{2\log_3 2 - 1}$ (B) $\dfrac{2}{2-\log_2 3}$ (C) $\dfrac{1}{1-\log_4 3}$ (D) $\dfrac{2\log_2 3}{2\log_2 3 - 1}$

10. The value of $((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is ___________.

[JEE(Advanced) 2018, Paper-1,(4, –2)/60]

PART - II : PREVIOUS YEARS PROBLEMS OF MAINS LEVEL

1. If $\log_p x = \alpha$ and $\log_q x = \beta$, then the value of $\log_{p/q} x$ is **[KCET-1997]**

(1) $\frac{\alpha-\beta}{\alpha\beta}$ (2) $\frac{\beta-\alpha}{\alpha\beta}$ (3) $\frac{\alpha\beta}{\alpha-\beta}$ (4) $\frac{\alpha\beta}{\beta-\alpha}$

2. If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in G.P. Then x is equal to **[KCET-1998]**

(1) $\log_a (\log_b a)$ (2) $\log_a(\log_e a) + \log_a \log_b b$

(3) $-\log_a(\log_a b)$ (4) none of these

3. If $\log_x 256 = 8/5$, then x is equal to **[KCET-2000]**

(1) 64 (2) 16 (3) 32 (4) 8

4. If log 2, $\log(2^x - 1)$ and $\log(2^x + 3)$ are in A.P., then x is equal to **[KCET-2000]**

(1) 5/2 (2) $\log_2 5$ (3) $\log_2 3$ (4) $\log_3 2$

5. The number $\log_2 7$ is **[DCE-2000]**

(1) an integer (2) a rational (3) an irrational (4) a prime number

6. The roots of the equation $\log_2(x^2 - 4x + 5) = (x - 2)$ are **[KCET-2001]**

(1) 4, 5 (2) 2, – 3 (3) 2, 3 (4) 3, 5

7. If x = 198 !, then value of the expression $\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + ... + \frac{1}{\log_{198} x}$ equals

[DCE-2005]

(1) –1 (2) 0 (3) 1 (4) 198

8. If $\log_{0.3}(x - 1) < \log_{0.09}(x - 1)$, then x lies in the intervel **[DCE-2006]**

(1) $(2, \infty)$ (2) (1, 2) (3) (–2, –1) (4) none of these

9. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then

[AIEEE-2009, (4, – 1), 144]

(1) A = C (2) B = C (3) $A \cap B = \phi$ (4) A = B

10. Let X = {1, 2, 3, 4, 5}. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is : **[AIEEE-2012, (4, – 1), 120]**

(1) 5^2 (2) 3^5 (3) 2^5 (4) 5^3

11. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to **[JEE(Main) 2014, (4, – 1), 120]**

(1) X (2) Y (3) N (4) Y – X

12. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is

[JEE(Main) 2016, (4, – 1), 120]

(1) – 4 (2) 6 (3) 5 (4) 3

13. In a class 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible 5 opted Chemistry course. Then the number of student who **did not** opt for any of the three courses is :

[JEE(Main) 2019, Online (10-01-19),P-1 (4, – 1), 120]

(1) 38 (2) 42 (3) 102 (4) 1

14. Let $X = \{n \in N: 1 \le n \le 50\}$. If $A = \{n \in X: n$ is a multiple of $2\}$; $B = \{n \in X: n$ is a multiple of $7\}$, then the number of elements in the smallest subset of X containing both A and B is__________.

[JEE(Main) 2020, Online (07-01-20),P-2 (4, 0), 120]

Answers

EXERCISE - 1

PART -I

Section (A) :

A-1. (i) Yes (ii) Yes (iii) Yes (iv) No

A-2. (i) {2,3,5,7} (ii) {3, 4, 5, 6, 7, 8, 9}

A-3. (i) $\{x : x = \frac{p}{q}, p \in I, q \in N\}$ (ii) $\{x : x = \lambda^2 + 1, \lambda \in N\}$

A-4. (i) Finite (ii) Finite and empty
(iii) Singleton & finite (iv) Infinite

A-5. $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$

Section (B) :

B-1. (i) {1, 2, 3, 4} (ii) {1, 2, 3} (iii) {4, 5, 6}

B-2. {3, 5, 9}

B-3. d = bc

B-4. (i) minimum $n(A \cap B) = 0$, maximum $n(A \cap B) = 3$
(ii) minimum $n(A \cup B) = 6$, maximum $n(A \cup B) = 9$

Section (C) :

C-1. 300 **C-2.** 25 **C-3.** 3300 **C-4.** (i) 45 (ii) 6

Section (D) :

D-1. (i) $(-3,-2) \cup (1,3)$ (ii) $\{-2\} \cup [1, 8)$
(iii) $(-\infty, -2)\cup(-2, -1/2)\cup(1, \infty)$ (iv) $[-\sqrt{2},-1)\cup(-1, \sqrt{2}]\cup[3, 4)$
(v) $(-\infty, -2] \cup (-1, 4)$

D-2. (i) $(-17/25, -3/8)$ (ii) $x \in (-6, -1) \cup (1, 4)$
(iii) $(-3,-2) \cup (-1,\infty)$ (iv) $x \in (-1,0) \cup (0,1)$

D-3. (i) $\left(-2, \frac{3-\sqrt{5}}{2}\right] \cup \left(1, \frac{3+\sqrt{5}}{2}\right]$ (ii) $(-\infty, -20) \cup (23, \infty)$
(iii) $\left(\frac{1}{2}, 3\right)$ (iv) $(-\infty, -1) \cup (5, \infty)$

D-4. (i) $x \in [-2, -1] \cup [1, 2]$ (ii) $x \in [-3, 3]$
(iii) $x \in (-\infty, -4] \cup [-2, -1) \cup [1,\infty)$

D-5. (i) $\left(-\infty, -\frac{5}{2}\right)$ (ii) $\left(\frac{25}{4}, \infty\right)$ (iii) $\left(-\frac{2}{5}, 0\right)$

D-6. 2

Section (E) :

E-1. (i) 1 (ii) –72 (iii) 2 (iv) 1 (v) $7 + \frac{1}{196}$ (vi) 0

E-2. (i) +ve (ii) – ve (iii) +ve (iv) +ve
(v) +ve (vi) – ve (vii) +ve (viii) – ve (ix) – ve

E-3. (i) b – 2a (ii) a + 3b (iii) $\frac{2b^2 + 3a^2}{ab}$ (iv) $\frac{4(2a+b)}{1-a+2b}$

E-4. (i) 1 (ii) 89 (iii) –1

Section (F) :

F-1. (i) 3 (ii) ± 2 (iii) 3 (iv) 16
(v) {1/3} (vi) $\{-4\}$ (vii) no root (viii) (2)

F-2. (i) 9 (ii) $\log_2 6$ (iii) 10 or $\frac{1}{100}$ (iv) $\{10^{-5}, 10^3\}$

Section (G) :

G-1. (i) $\left[-\frac{1}{2}, -\frac{1}{4}\right) \cup \left(\frac{3}{4}, 1\right]$ (ii) $(1, 2) \cup (3, 4)$

(iii) $\left(-\infty, \frac{1}{2}\right)$ (iv) $(-1, 0) \cup (1, 2)$

(v) $(-\infty, 2)$

G-2. 1

G-3 (i) $\left[\frac{1}{2}, 4\right]$ (ii) R (iii) $(0, \log_{\frac{3}{2}} 3)$

G-4. (i) $\left(\frac{3}{4}, 1\right) \cup (1, 3]$

(ii) $(-\infty, -1) \cup (1, \infty)$

(iii) $x \in (-2, -1) \cup (-1, 0) \cup (0, 1) \cup (2, \infty)$

PART - II

Section (A) :

A-1. (D) **A-2.** (A) **A-3.** (D) **A-4.** (A) **A-5.** (D) **A-6.** (B)

Section (B) :

B-1. (B) **B-2.** (A) **B-3.** (D) **B-4.** (B) **B-5.** (A) **B-6.** (B) **B-7.** (A)
B-8. (D)

Section (C) :

C-1. (B) **C-2.** (C) **C-3.** (C) **C-4.** (B)

Section (D) :

D-1. (B) **D-2.** (D) **D-3.** (D) **D-4.** (B) **D-5** (D) **D-6.** (D) **D-7.** (D)

Section (E) :

E-1. (A) **E-2.** (D) **E-3.** (B) **E-4.** (D) **E-5.** (D) **E-6.** (D) **E-7.** (A)
E-8. (A) **E-9.** (D)

Section (F) :

F-1. (D) **F-2.** (C) **F-3.** (B) **F-4.** (A) **F-5.** (D) **F-6.** (C) **F-7.** (B)

Section (G) :

G-1. (C) **G-2.** (A) **G-3.** (B) **G-4.** (A) **G-5.** (D) **G-6.** (A) **G-7.** (D)

PART - II

1. (A) $\rightarrow$ (q), (B) $\rightarrow$ (r), (C) $\rightarrow$ (s), (D) $\rightarrow$ (p)

2. (A) $\rightarrow$ r, (B) $\rightarrow$ s, (C) $\rightarrow$ p, (D) $\rightarrow$ p, (E) $\rightarrow$ q, (F) $\rightarrow$ q

3. (A) $\rightarrow$ r, (B) $\rightarrow$ p, (C) $\rightarrow$ s, (D) $\rightarrow$ q

4. (A) $\rightarrow$ r, (B) $\rightarrow$ s, (C) $\rightarrow$ p, (D) $\rightarrow$ q

EXERCISE - 2

PART - I

1.	(A)	**2.**	(B)	**3.**	(B)	**4.**	(A)	**5.**	(C)	**6.**	(D)	**7.**	(B)
8.	(B)	**9.**	(A)	**10.**	(B)	**11.**	(A)	**12.**	(A)	**13.**	(D)	**14.**	(A)
15.	(C)												

PART - II

1.	02.20	**2.**	12.28 or 12.29	**3.**	01.00	**4.**	22.00	**5.**	02.00	**6.**	01.00
7.	10.00	**8.**	03.82 or 03.83	**9.**	12.50	**10.**	03.00	**11.**	00.25	**12.**	01.00
13.	00.01	**14.**	01.00	**15.**	05.66 or 05.67						

PART - III

1.	(BD)	**2.**	(ABCD)	**3.**	(ABC)	**4.**	(ABCD)	**5.**	(ABCD)	**6.**	(AB)
7.	(CD)	**8.**	(ABCD)	**9.**	(BC)	**10.**	(ABD)				

PART - IV

1.	(D)	**2.**	(B)	**3.**	(C)	**4.**	(C)	**5.**	(A)	**6.**	(B)	**7.**	(BC)
8.	(A)	**9.**	(B)										

EXERCISE - 3

PART - I

1.	(ABCD)	**2.**	(C)	**3.**	x = 8	**4.**	x = 3 or – 3	**5.**	(B)

6. (A) → (p), (r), (s) ; (B) → (q), (s) ; (C) → (q), (s) ; (D) → (p), (r), (s)

7.	(C)	**8.**	(4)	**9.**	(ABC)	**10.**	(8)

PART - II

1.	(4)	**2.**	(1)	**3.**	(3)	**4.**	(2)	**5.**	(3)	**6.**	(3)	**7.**	(3)
8.	(1)	**9.**	(2)	**10.**	(2)	**11.**	(2)	**12.**	(4)	**13.**	(1)	**14.**	29

LOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	5 9 13 4 8 12	17 21 26 16 20 24	30 34 38 28 32 36
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 12 4 7 11	16 20 23 15 18 22	27 31 35 26 29 33
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 11 3 7 10	14 18 21 14 17 20	25 28 32 24 27 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	36 10 37 10	13 16 19 13 16 19	23 26 29 22 25 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	36 9 36 9	12 15 19 12 14 17	22 25 28 22 25 26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 9 3 6 8	11 14 17 11 14 17	20 23 26 19 22 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 6 8 3 5 8	11 14 16 10 13 16	19 22 24 18 21 23
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3 5 8 3 5 8	10 13 15 10 12 15	18 20 23 17 20 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 5 7 2 4 7	9 12 14 9 11 14	17 19 21 16 18 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7 2 4 6	9 11 13 8 11 13	16 18 20 15 17 19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8

LOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	9766	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	9882	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	8699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 4 4

ANTILOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1	1 1 1	2 2 2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 2	2 2 2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1	1 2 2	2 3 3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1	1 2 2	2 3 3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1	1 2 2	2 3 3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1	1 2 2	3 3 3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1	2 2 2	3 3 3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1	2 2 2	3 3 3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1841	1845	0 1 1	2 2 3	3 4 4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2328	2234	1 1 2	2 3 3	4 4 5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1 1 2	2 3 3	4 4 5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
.38	2399	2404	2410	2415	2421	2432	2427	2432	2443	2449	1 1 2	2 3 3	4 4 5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

ANTILOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1 1 2	3 4 4	5 6 7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1 2 2	3 4 5	5 6 7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1 2 2	3 4 5	5 6 7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1 2 2	3 4 5	6 6 7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1 2 2	3 4 5	6 6 7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1 2 2	3 4 5	6 6 7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1 2 3	3 4 5	6 7 8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1 2 3	3 4 5	6 7 8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1 2 3	4 4 5	6 7 8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1 2 3	4 5 5	6 7 8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1 2 3	4 5 6	6 7 8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4156	1 2 3	4 5 6	7 8 9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1 2 3	4 5 6	7 8 9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1 2 3	4 5 6	7 8 9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1 2 3	4 5 6	7 8 9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1 2 3	4 5 6	7 8 9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1 2 3	4 5 6	7 9 10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1 2 3	4 5 7	8 9 10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1 2 3	4 6 7	8 9 10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1 2 3	5 6 7	8 9 10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1 2 4	5 6 7	8 9 11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1 2 4	5 6 7	8 10 11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1 2 4	5 6 7	9 10 11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1 3 4	5 6 8	9 10 11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1 3 4	5 6 8	9 10 12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1 3 4	5 7 8	9 10 12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1 3 4	5 7 8	9 11 12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1 3 4	5 7 8	10 11 12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1 3 4	6 7 8	10 11 13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1 3 4	6 7 9	10 11 13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1 3 4	6 7 9	10 12 13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2 3 5	6 8 9	11 12 14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2 3 5	6 8 9	11 12 14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2 3 5	6 8 9	11 13 14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2 3 5	6 8 10	11 13 15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2 3 5	7 8 10	12 13 15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2 3 5	7 8 10	12 13 15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2 3 5	7 9 10	12 14 16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2 4 5	7 9 11	12 14 16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2 4 5	7 9 11	13 14 16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2 4 6	7 9 11	13 15 17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2 4 6	8 9 11	13 15 17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2 4 6	8 10 12	14 15 17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2 4 6	8 10 12	14 16 18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2 4 6	8 10 12	14 16 18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2 4 6	8 10 12	15 17 19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2 4 6	8 11 13	15 17 19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2 4 7	9 11 13	15 17 20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2 4 7	9 11 13	16 18 20
.99	9772	9795	9817	9849	9863	9886	9908	9931	9954	9977	2 5 7	9 11 14	16 18 20

SOLUTIONS OF FUNDAMENTALS OF MATHEMATICS-I

EXERCISE # 1

PART–1

Section (A)

A-1. Collection of all intelligent women in Jalandhar is not a set as it is not a well defined collection. It is not possible to decide logically which woman is to be included in the collection and which is not to be included.

A-2. 2, 3, 5 and 7 are the only positive primes less than 10.

A-3. Obvious

A-4. (i) $x^2 - 1 = 0 \quad x = \pm 1$
(ii) $x^2 + 1 = 0 \quad x = \pm i \quad x \in \phi$
(iii) $x^2 - 9 = 0 \quad x = \pm 3$
(iv) $x^2 - x - 2 = 0, \quad x = 2, -1$

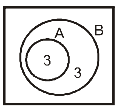
A-5. $P(A) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\} = \{\phi, \{\phi\}, \{\{\phi\}\}, A\}$

Section (B)

B-1. $A = \{1, 2, 3\}$
$B = \{3, 4\}$
$C = \{4, 5, 6\}$
$B \cap C = \{4\}$
$B \cup C = \{3, 4, 5, 6\}$
$A \cup (B \cap C) = \{1, 2, 3, 4\}$
$A - (B \cap C) = \{1, 2, 3\}$
$(B \cup C) - A = \{4, 5, 6\}$

B-2. Obvious

B-3. $bN \cap. cN$
(+ve integral multiple of b) $\cap$ (+ve integral multiple of c)
since b & c are relatively primes $= b\,c\,N$
$\therefore \quad d = bc$

B-4. 

for minimum $n(A \cup B)$, the set A is subset of B and for maximum $n(A \cup B)$, the sets A and B are disjoint set.
also $\quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$\Rightarrow \quad$ minimum $n(A \cap B) = 0$, maximum $n(A \cap B) = 3$
$\Rightarrow \quad$ minimum value of $n(A \cup B) = 3 + 6 - 3 = 6$ or maximum value of $n(A \cup B) = 3 + 6 - 0 = 9$

Section (C)

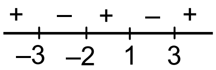
C-1. $n(A^c \cap B^c) = n[(\{A \cup B)^c] = n(U) - n(A \cup B) = n(U) - [n(A) + n(B) - n(A \cap B)]$
$= 700 - [200 + 300 - 100] = 300.$

C-2. Let number of newspapers is x. As every newspaper is read by 60 students
Since, every students reads 5 newspapers
$\therefore \quad 60x = 300(5) \quad \Rightarrow \quad x = 25.$

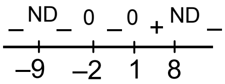
C-3. n (A) = 40% of 10,000 = 4,000
n(B) = 20% of 10,000 = 2,000
n(C) = 10% of 10,000 = 1,000
$n(A \cap B)$ = 5% of 10,000 = 500
$n(B \cap C)$ = 3% of 10,000 = 300
$n(C \cap A)$ = 4% of 10,000 = 400
$n(A \cap B \cap C)$ = 2% of 10,000 = 200
$n(A \cap B^c \cap C^c) = n[A \cap (B \cup C)^c]$
$= n(A) - n[A \cap (B \cup C)] = n(A) - n[(A \cap B) \cup (A \cap C)]$
$= n(A) - [n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)]$
$= 4000 - [500 + 400 - 200] = 4000 - 700 = 3300.$

C-4. n(A) = 21, n(B) = 26, n(C) = 29
$n(A \cap B) = 14$, $n(A \cap C) = 12$, $m(B \cap C) = 13$, $n(A \cap B \cap C) = 8$
$n(C \cap A' \cap B') = n(C \cap \overline{A \cup B}) = n(C) - n((C \cap A) \cup (C \cap B))$
$n(C) - [n(C \cap A) + n(C \cap B) - n(A \cap B \cap C)]$
$29 - [12 + 13 - 8] = 12$
$n(A \cap B \cap C') = n(A \cap B) - n(A \cap B \cap C) = 14 - 8 = 6$

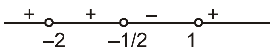
Section (D)

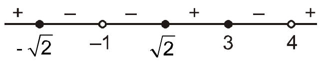
D-1. (i)


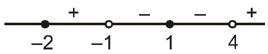
$\Rightarrow x \in (-3,-2) \cup (1,3)$

(ii)


$x \in \{-2\} \cup [1,8)$

(iii) $\dfrac{(x+2)^2}{(2x+1)(x-1)} > 0$


(iv) $\dfrac{(x-\sqrt{2})(x+\sqrt{2})(x-3)^3}{(x+1)^2(x-4)} \le 0$


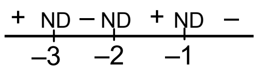
(v) $\dfrac{(x+2)(x-1)^2}{(x+1)(x-4)} \le 0 \quad \Rightarrow$


D-2. (i) $\dfrac{7x-5}{8x+3} - 4 > 0$

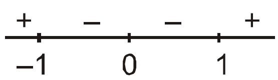
(ii) $\dfrac{14x}{x+1} - \left(\dfrac{9x-30}{x-4}\right) < 0 \quad \Rightarrow \quad \dfrac{14x^2 - 56x - 9x^2 - 9x + 30x + 30}{(x+1)(x-4)} < 0$

$\Rightarrow \quad \dfrac{5x^2 - 35x + 30}{(x+1)(x+4)} < 0 \quad \Rightarrow \quad \dfrac{(x-6)(x-1)}{(x+1)(x+4)} < 0$

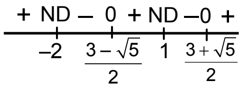
$\Rightarrow$ **$x \in (-6, -1) \cup (1, 4)$**

(iii) $\dfrac{-12x^2 - 12}{(x+1)(x+2)(x+3)} \le 0$


$x \in (-3,-2) \cup (-1,\infty)$

(iv) $\dfrac{x^2 + 2 + 2x^2 - 2}{x^2 - 1} < 0 \quad \Rightarrow \quad \dfrac{3x^2}{(x-1)(x+1)} < 0$


$\Rightarrow \quad x \in (-1,0) \cup (0,1)$

D-3. (i) 

$x \in \left(-2, \frac{3-\sqrt{5}}{2}\right] \cup \left(1, \frac{3+\sqrt{5}}{2}\right]$

(ii) $2x^2 - 3x - 459 > x^2 + 1$

$\Rightarrow \quad x^2 - 3x - 460 > 0 \quad \Rightarrow \quad x \in (-\infty, -20) \cup (23, \infty)$

(iii) $x^2 - 5x + 12 > 3(x^2 - 4x + 5)$ (since $x^2 - 4x + 5 = (x-2)^2 + 1 > 0$)

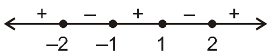
$\Rightarrow \quad 2x^2 - 7x + 3 < 0 \quad \Rightarrow \quad 2x^2 - 6x - x + 3 < 0$

$\Rightarrow \quad 2x(x-3) - 1(x-3) < 0 \quad \Rightarrow \quad x \in \left(\frac{1}{2}, 3\right)$

(iv) $x^4 + x^2 + 1$ is always positive सदैव धनात्मक

$\Rightarrow \quad \frac{1}{(x-5)(x+1)} > 0$

D-4. (i) $(x^2 - 1)(x^2 - 4) \le 0 \quad \Rightarrow \quad (x-1)(x+1)(x+2)(x-2) \le 0$



$x \in [-2, -1] \cup [1, 2]$

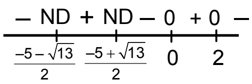
(ii) $(x^2 - 9)(x^2 + 7) \le 0 \quad \Rightarrow \quad (x-3)(x+3) \le 0$

$x \in [-3, 3]$

D-5. $\frac{x-1-7x-14}{x+2} < 0$ and $\frac{x-1-x-2}{x+2} > 0$

$x \in (-\infty, -\frac{5}{2}) \cup (-2, \infty)$ and $x \in (-\infty, -2) \Rightarrow x \in (-\infty, -\frac{5}{2})$

(i) $x \in (-\infty, -\frac{5}{2})$ (ii) $x^2 \in (\frac{25}{4}, \infty)$ (iii) $\frac{1}{x} \in \left(-\frac{2}{5}, 0\right)$

D-6. 

Number of positive integer satisfying the inequality equal to 2 (which are 1 and 2)

Section (E)

E-1. (i) $\log_{10}(\log_{10}5 + 2\log_{10}2) + (\log_{10}2)^2 = (\log_{10}5 + \log_{10}2)^2 = (\log_{10}10)^2 = 1$

(ii) $2^{\log_{\sqrt{5}} 5} + 7^{\log_3 9} + -5^{\log_2 8} = 2^2 + 7^2 - 5^3 = 53 - 125 = -72$

(iii) $\left(5^{\log_5 7} + \frac{1}{\left(\log_{10}\left(\frac{1}{0.1}\right)\right)}\right)^{1/3} = (7+1)^{1/3} = 2$

(iv) $\log_{3/4} \log_2\left(\left(8\right)^{1/2}\right)^{1/2} = \log_{3/4} \log_2 (2)^{3/4} = 1$

(v) $\left(\frac{1}{49}\right)^{1+\log_7^2} = (7^{-2})^{\log_7 7 + \log_7 2} = (7^{-2})^{\log_7^{14}} = 7^{\log_7 (14)^{-2}} = \frac{1}{196}$ & $5^{-\log_{1/5} 7} = 5^{\log_5 7} = 7$

$\therefore \; 7 + \frac{1}{196}$

(vi) $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3} = 7^{\log_3 5} + 3^{\log_5 7} - 7^{\log_3 5} - 3^{\log_5 7}$

{using property $a^{\log_c b} = b^{\log_c a}$ }= 0

E-2. (i) $\sqrt{2} > 1$ & $\sqrt{3} > 1$, hence positive

(ii) Number & base on oppsite sides of unity, hence negative

(iii) Number & base on same sides of unity, hence positive

(iv) Number & base on same sides of unity, hence positive

(v) Number & base on same sides of unity, hence positive

(vi) Number & base on oppsite sides of unity, hence negative

(vii) Number & base on same sides of unity, hence positive

(viii) Number & base on oppsite sides of unity, hence negative

(ix) Number & base on oppsite sides of unity, hence negative

E-3. (i) $\log_{10}\left(\frac{3}{4}\right) = \log_{10}3 - 2\log_{10}2 = b - 2a$

(ii) $\log_{10}2 + 3\log_{10}3 = a + 3b$

(iii) $\frac{2\log_{10}3}{\log_{10}2} + \frac{3\log_{10}2}{\log_{10}3} = \frac{2b}{a} + \frac{3a}{b} = \frac{2b^2 + a^2}{ab}$

(iv) $2\left(\frac{4\log_{10}2 + 2\log_{10}3}{2\log_{10}3 + \log_{10}5}\right) = \frac{4(2a + b)}{1 - a + 2b}$

E-4. (i) n = 75600

Now $4\log_n 2 + 3\log_n 3 + 2\log_n 5 + \log_n 7 = \log_n(2^4.3^3.5^2.7^1) = \log_n(75600) = 1$

(ii) $x = 4^{3^2} = 4^3 = 64$

$y = (2^4)^1 = 16$, $z = 3^{2^1} = 9$

sum = 64 + 16 + 9 = 89

(iii) $a_n = \frac{1}{\log_n 2002} = \log_{2002} n$

$b = a_2 + a_3 + a_4 + a_5 = \log_{2002} 2 + \log_{2002} 3 + \log_{2002} 4 + \log_{2002} 5 = \log_{2002} (2.3.4.5) = \log_{2002} 120$

$c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14} = \log_{2002}10 + \log_{2002}11 + \log_{2002}12 + \log_{2002}13 + \log_{2002}14$

$= \log_{2002}(10.11.12.13.14) = \log_{2002}240240$

Now $b - c = \log_{2002}120 - \log_{2002}240240 = \log_{2002}10 +$ =

E-5. Let $\log_2 7 = \frac{p}{q}$, where p & q are coprime numbers.

$\Rightarrow$ $7 = 2^{p/q}$ $\Rightarrow$ $7^q = 2^p$

$\because$ 7^q is an odd number

while 2^p is an even number

$\therefore$ this is not possible & $\log_2 7$ is an irrational number

E-6. $\frac{\log a}{b - c} = \frac{\log b}{c - a} = \frac{\log c}{a - b} = k$ (let)

$\log_a = k(b - c)$ $\Rightarrow$ $a \log a = ka\ (b - c)$

$\log_b = k(c - a)$ $\Rightarrow$ $b \log b = kb\ (c - a)$

$\log_c = k(a - b)$ $\Rightarrow$ $c \log c = kc\ (a - b)$

$\therefore$ $\log a^a + \log b^b + \log c^c = k\ (ab - ac + bc - ab + ca - bc)$

$\Rightarrow$ $\log a^a b^b c^c = 0$ $\Rightarrow$ $a^a b^b c^c = 1$

Section (F)

F-1. (i) $\log_x(4x - 3) = 2 \Rightarrow 4x - 3 = x^2 \Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow$ $x^2 - 4x + 3 = 0$ $\Rightarrow$ $(x - 1)(x - 3) = 0$ $\Rightarrow$ $x = 1, x = 3$

But $4x - 3 > 0$ $\Rightarrow$ $x > \frac{3}{4}$ and $x > 0, x \neq 1$

Ans. x = 3

(ii) $\log_2\log_3(x^2 - 1) = 0$ $\Rightarrow$ $\log_3(x^2 - 1) = 2^0 = 1$ $\Rightarrow$ $(x^2 - 1) = 3^1$

$\Rightarrow$ $x^2 - 1 = 3$ $\Rightarrow$ $x^2 = 4$ $\Rightarrow$ $x = \pm 2$

both are satisfied

(iii) $x^2 - 2x - 3 = 0 \Rightarrow (x-3)(x+1) = 0 \quad x = 3, x = -1$

but $x > 0$

Ans. $x = 3$

(iv) $\log_4(\log_2 x) + \log_2(\log_4 x) = 2 \Rightarrow \frac{1}{2}\log_2(\log_2 x) + \log_2[\frac{1}{2}\log_2 x] = 2$

$\frac{1}{2}\log_2(\log_2 x) + \log_2(\log_2 x) = 2 \Rightarrow \frac{3}{2}\log_2(\log_2 x) = 3$

$\log_2(\log_2 x) = 2 \Rightarrow \log_2 x = 4 \Rightarrow x = 2^4 = 16$

(v) $\log_3\left(\log_9 x + \frac{1}{2} + 9^x\right) = 2x \Rightarrow \log_9 x + \frac{1}{2} + 9^x = 3^{2x} \Rightarrow \log_9 x + \frac{1}{2} + 9^x = 9^x$

$\Rightarrow \log_9 x = -\frac{1}{2} \Rightarrow x = 9^{-1/2} \Rightarrow x = \frac{1}{3}$

(vi) $2\log_4(4-x) = 4 - \log_2(-2-x)$

(i) $4 - x > 0 \Rightarrow x < 4$

(ii) $-2 - x > 0 \Rightarrow x < -2$

(iii) $\log_2(4-x) = 4 - \log_2(-2-x) \Rightarrow \log_2(4-x)(-2-x) = 4$

$\Rightarrow (4-x)(-2-x) = 16 \Rightarrow -8 - 2x + x^2 = 16$

$\Rightarrow x^2 - 2x - 24 = 0 \Rightarrow (x-6)(x+4) = 0$

$x = 6$ (not possible) , $x = -4$.

(viii) $x^{\frac{1}{2}\log_{\sqrt{x}}(x^2-x)} = 3\log_{3^2}(2^2) = 2$

$\Rightarrow (\sqrt{x})^{\log_{\sqrt{x}}}(x^2 - x) = 2$

$\Rightarrow x^2 - x = 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = 2, x = -1$ but $x > 0$ and $x^2 - x = 0 \Rightarrow x(x-1) > 0$

$x \in (-\infty, 0) \cup (1, \infty)$

F-2. (i) $t^2 - 2t - 5 = 0$

sum of roots = 2

$\log_3 x_1 + \log_3 x_2 = 2 \Rightarrow \log_3(x_1 x_2) = 2 \Rightarrow x_1 x_2 = 3^2 = 9$

(ii) $(2^x)^2 - 7(2^x) + 6 = 0$

$\Rightarrow t^2 - 7t + 6 = 0$

$t = 1, t = 6$

Roots $2^x = 1, 2^x = 6$

$\Rightarrow x = 1, x = \log_2 6$

Product of roots = $(1)(\log_2 6) = \log_2 6$

(iii) $x^{\log_{10} x + 2} = 10^{\log_{10} x + 2}$

$x = 10$ or $\log_{10} x + 2 = 0 \Rightarrow x = 10^{-2} = \frac{1}{100}$

(iv) $x^{\frac{\log x + 5}{3}} = 10^{5 + \log x}$; $\left(\frac{\log x + 5}{3}\right)\log x = 5 + \log x$

$\log^2 x + 2\log x - 15 = 0$; $(\log x + 5)(\log x - 3) = 0$

$\log x = -5, \log x = 3$; $x = 10^{-5}$, $x = 10^3$.

Section (G) :

G-1. (i) $\log_{\frac{5}{8}}\left(2x^2 - x - \frac{3}{8}\right) \ge 1 \Rightarrow 2x^2 - x - \frac{3}{8} \le \frac{5}{8} \Rightarrow 16x^2 - 8x - 8 \le 0$

$\Rightarrow 2x^2 - x - 1 \le 0 \Rightarrow (2x+1)(x-1) \le 0$

$x \in \left[-\frac{1}{2}, 1\right]$ ......(i)

also $2x^2 - x - \frac{3}{8} > 0 \Rightarrow 16x^2 - 8x - 3 > 0$

$\Rightarrow \quad 16x^2 - 12x + 4x - 3 > 0 \Rightarrow \quad (4x - 3)(4x + 1) > 0$

$\Rightarrow \quad x \in \left(-\infty, \frac{-1}{4}\right) \cup \left(\frac{3}{4}, 1\right]$ ....(ii)

(i) $\cap$ (ii) $\Rightarrow \quad x \in \left[\frac{-1}{2}, -\frac{-1}{4}\right) \cup \left(\frac{3}{4}, 1\right]$

(ii) $x^2 - 5x + 6 > 0 \Rightarrow \quad (x - 3)(x - 2) > 0$

$\Rightarrow \quad x \in (-\infty, 2) \cup (3, \infty)$ ......(i)

and $\quad x^2 - 5x + 6 < \left(\frac{1}{2}\right)^{-1}$

$\Rightarrow \quad x^2 - 5x + 4 < 0 \Rightarrow \quad (x - 1)(x - 4) < 0 \quad \Rightarrow \quad x \in (1, 4)$ .....(ii)

(i) $\cap$ (ii) $\Rightarrow \quad x \in (1, 2) \cup (3, 4)$

(iii) $\log_7\left(\frac{2x-6}{2x-1}\right) > 0 \quad \Rightarrow \frac{2x-6}{2x-1} > 7^0 \quad \Rightarrow \frac{2x-6}{2x-1} - 1 > 0$

$\Rightarrow \frac{-5}{2x-1} > 0 \quad \Rightarrow \frac{1}{2x-1} < 0 \quad \Rightarrow x \in \left(-\infty, \frac{1}{2}\right)$

and $\quad \frac{2x-6}{2x-1} > 0 \quad \Rightarrow$ + (1/2) − (3) +

Ans. $\left(-\infty, \frac{1}{2}\right)$

(iv) $2 - x < \frac{2}{x+1} \quad \Rightarrow \quad \frac{2}{x+1} + x - 2 > 0 \quad \Rightarrow \frac{2 + x^2 - x - 2}{x+1} > 0$

$\Rightarrow \quad \frac{x(x-1)}{x+1} > 0 \quad \Rightarrow$ − (−1) + (0) − (1) +

and $\quad 2 - x > 0 \quad \Rightarrow \quad x < 2 \quad$ and $\quad \frac{2}{x+1} > 0 \quad \Rightarrow \quad x > -1$

Ans. $(-1, 0) \cup (1, 2)$

(v) $2^2.2^x - 4^x \le \left(\frac{1}{3}\right)^{-2} \quad \Rightarrow \quad 4.2^x - (2^x)^2 \le 9$

Let $\quad 2^x = t \quad \Rightarrow \quad t^2 - 4t + 9 \ge 0$ always true

$2^{x+2} - 4^x > 0 \quad \Rightarrow \quad 2^x.(4 - 2^x) > 0$

$4 - 2^x > 0 \quad \Rightarrow \quad 2^x < 2^x$

$\Rightarrow \quad x < 2 \quad \Rightarrow \quad x \in (-\infty, 2)$

(vi) $\log_x(4x - 3) \ge 2$

Case-I $\quad 0 < x < 1 \quad$ and $4x - 3 > 0 \Rightarrow \quad x > \frac{3}{4}$

then $\quad 4x - 3 \le x^2 \quad \Rightarrow \quad x^2 - 4x + 3 \ge 0 \Rightarrow \quad (x - 3)(x - 1) \ge 0$

$\Rightarrow \quad x \in \left(\frac{3}{4}, 1\right)$

Case -II $\quad x > 1, 4x - 3 > 0 \quad \Rightarrow \quad x > \frac{3}{4}$

then $4x - 3 \ge x^2 \quad \Rightarrow \quad x^2 - 4x + 3 \le 0$

$\Rightarrow \quad (x - 3)(x - 1) \le 0 \Rightarrow \quad x \in (1,3]$

Ans. $\quad x \in \left(\frac{3}{4}, 1\right) \cup (1, 3]$

G-2. $\log_{\frac{1}{5}} \frac{4x+6}{x} \geq 0$

$\frac{4x+6}{x} > 0 \quad \Rightarrow \quad x \in \left(-\infty, -\frac{3}{2}\right) \cup (0, \infty)$ ....(i)

& $\frac{4x+6}{x} \leq 1 \quad \Rightarrow \quad \frac{x+2}{x} \leq 0 \quad \Rightarrow \quad x \in (-\infty, -2] \cup (0, \infty)$ ....(ii)

$(i) \cap (ii) \Rightarrow x \in \left[-2, \frac{-3}{2}\right)$

G-3 (i) $-2 \leq \log_{.5} x \leq 1$

$x \in [(0.5)^1, (0.5)^{-2}] \quad \Rightarrow \quad x \in \left[\frac{1}{2}, 4\right]$

(ii) $15^x - 25\ 3^x - 9.5^x + 225 > 0 \qquad (5^x - 25)(3^x - 9) \geq 0 \qquad \Rightarrow x \in R$

(iii) $8\left(\frac{3^{x-2}}{3^x - 2^x}\right) > 1 + \left(\frac{2}{3}\right)^x \quad \Rightarrow \quad \frac{8}{9}\left(\frac{\left(\frac{3}{2}\right)^x}{\left(\frac{3}{2}\right)^x - 1}\right) > 1 + \left(\frac{2}{3}\right)^x$

Let $\left(\frac{3}{2}\right)^x = t$ then $\frac{8t}{9(t-1)} > \frac{t+1}{t} \Rightarrow \frac{8t^2 - 9(t^2-1)}{9t(t-1)} > 0$

$\Rightarrow \quad \frac{(t-3)(t+3)}{t(t-1)} < 0 \Rightarrow t \in (-3,0) \cup (1, 3)$

$\Rightarrow \quad \left(\frac{3}{2}\right)^x \in (1,3) \Rightarrow x \in (0, \log_{\frac{3}{2}} 3)$

G-4. (i) $\log_x(4x - 3) \geq 2$

Case -I : When $x > 1 \quad \Rightarrow \quad 4x - 3 \geq x^2 \quad \Rightarrow \quad x^2 - 4x + 3 \leq 0$

$\Rightarrow \quad (x-1)(x-3) \leq 0$

$x \in (1, 3]$ ....(i)

Case-II : When $0 < x < 1 \quad \Rightarrow \quad 4x - 3 \leq x^2$

and $4x - 3 > 0 \quad \Rightarrow \quad (x-1)(x-3) \geq 0$

$x > 3/4 \qquad x \in \left(\frac{3}{4}, 1\right)$ ......(ii) Ans. $(i) \cup (ii)$

(ii) $\log_{3x^2+1} 2 < \frac{1}{2}$

$3x^2 + 1 > 1 \quad \Rightarrow \quad x^2 > 0 \quad \Rightarrow \quad x \in R - \{0\}$

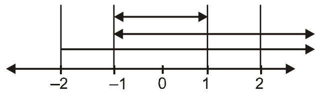
$2 < (3x^2 + 1)^{1/2} \quad \Rightarrow \quad 3x^2 + 1 > 4 \quad \Rightarrow \quad (x-1)(x+1) > 0$

$\Rightarrow \quad x \in (-\infty, -1) \cup (1, \infty)$

(iii) $\log_{x^2} (x + 2) < 1 \qquad x + 2 > 0 \qquad x > -2$

Case -I : when $0 < x^2 < 1 \qquad x \in (-1, 0) \cup (0, 1)$

then $x + 2 > x^2 \Rightarrow \quad x^2 - x - 2 < 0$

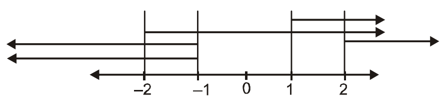


$x \in (-1, 1) - \{0\}$

Case -II : $x^2 > 1 \qquad |x| > 1$

$x + 2 < x^2 \quad \Rightarrow \quad x^2 - x - 2 > 0$

$x \in (-2, -1) \cup (2, \infty)$



Hence, $\quad x \in (-2, -1) \cup (-1, 0) \cup (0, 1) \cup (2, \infty)$

PART - II

Section (A) :

A-1. Since, intelligency is not defined for students in a class so set of intelligent students in a class is not well defined collection.

A-2. $x^2 = 16 \Rightarrow x = \pm 4$
$2x = 6 \quad x = 3$
No common value of x

A-3. $A = \{-2, -1, 0, 1, 2\}$
No. of subsets $= 2^n = 2^5 = 32$

A-4. Obvious

A-5. $P(A) = \{\phi, \{7\}, \{10\}, \{11\}, \{7, 10\}, \{7, 11\}, \{10, 11\}, \{7, 10, 11\}\}$
Number of subsets $= 2^n = 2^8 = 256$

A-6. Between any two real numbers there lie infinitely many real numbers.

Section (B) :

B-1. $A = [x : x \in R, -1 < x < 1]$
$B = [x : x \in R : x \leq 0 \text{ or } x \geq 2]$
$\therefore \quad A \cup B = R - D$, where $D = [x : x \in R, 1 \leq x < 2]$

B-2. $A \cap B = \{3, 4, 10\}$
$A \cap C = \{4\}$
$(A \cap B) \cup (A \cap C) = \{3, 4, 10\}$

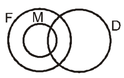
B-3. Obviously $A - (B \cup C)$

B-4. $B' = U - B = \{1, 2, 3, 4, 5, 8, 9, 10\}$
$A \cap B' = \{1, 2, 5\} = A$

B-5. $A = \{5, 9, 13, 17, 21\}$ and $B = \{3, 6, 9, 12, 15, 18, 21, 24\}$
$A - B = \{5, 13, 17\}$
$A - (A - B) = \{9, 21\}$

B-6. Let $A \cup B = A \cap B$
Now, $x \in A \Rightarrow x \in A \cup B \quad (\because A \subset A \cup B)$
$\Rightarrow x \in A \cap B \quad (\because A \cup B = A \cap B)$
$\Rightarrow x \in B$
Similarly, $x \in B$ implies $x \in A \quad \therefore A = B$
Conversly, let A = B
$\therefore A \cup B = A \cup A = A = A \cap A = A \cap B$
$\therefore A \cup B = A \cap B$

B-7. 1. $(N \cup B) \cap Z = (N \cap Z) \cup (B \cap Z) = N \cup (B \cap Z)$
2. $A = \{3, 6, 9, 12, 15, 18, 21, 24\}$

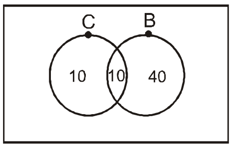
B-8. 

$M \equiv$ Mother; $F \equiv$ Female; D = Doctor

Section (C) :

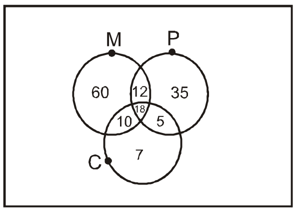
C-1. (i) $A \cup B \geq A \cup B$ (ii) $A \cap B \leq A \cup B$ (iii) $A \cap B = A \cup B$ not always

C-2.



P = 10 + 10 + 40 = 60 %

C-3.



Number of students offered maths alone = 60

n (M) = 100

n(P) = 70

n (C) = 40

n (M $\cap$ P) = 30

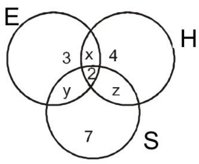
n (M $\cap$ P) = 28

n (P $\cap$ C) = 23

n (M $\cap$ P $\cap$ C) = 18

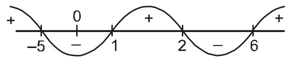
C-4. x+ y = 10; x + z = 9; y + z = 11 $\Rightarrow$ x + y + z = 15

x = 4, y = 6, z = 5



Section (D) :

D-1. $\frac{x^2(x^2-3x+2)}{x^2-x-30} \geq 0 \quad \Rightarrow \quad \frac{x^2(x-1)(x-2)}{(x+5)(x-6)} \geq 0$



$x \neq -5, 6$

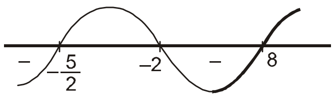
$x \in (-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$

D-2. x $\in$ (–∞, –9) $\cup$ (–9, –3) $\cup$ [–1, 0) $\cup$ (0, 2) [4, 6)

so +ve integral solution

D-3. $\frac{x^2-1}{2x+5} - 3 < 0 \quad \Rightarrow \frac{x^2-1-6x-15}{2x+5} < 0 \quad \Rightarrow \frac{x^2-6x-16}{2x+5} < 0 \quad \Rightarrow \frac{x^2-8x+2x-16}{\left(x+\frac{5}{2}\right)} < 0$

$\Rightarrow \frac{x(x-8)+2(x-8)}{x+\frac{5}{2}} < 0 \Rightarrow \frac{(x-8)(x-2)}{x+\frac{5}{2}} < 0$



x $\in$ (–∞, –5/2) $\cup$ (–2, 8)

D-4. $\frac{x^2-1}{x+3} \geq 0$ $\qquad$ $x^2 - 5x + 2 \leq 0$.

$\Rightarrow \quad x \in (-3, -1] \cup [1, \infty) \qquad \Rightarrow \quad x \in \left[\frac{5-\sqrt{17}}{2}, \frac{5+\sqrt{17}}{2}\right]$

so the common solution is $x \in \left[1, \ \frac{5+\sqrt{17}}{2}\right]$

D-5. $2x - 1 \le x^2 + 3 \le x - 1$

$x^2 + 3 \le x - 1 \quad \Rightarrow \quad x^2 - x + 4 \le 0$ which is not true for $x \in R$

D-6. $x^2 + 9 < (x + 3)^2 < 8x + 25$

$\Rightarrow \quad (x + 3)^2 > x^2 + 9 \Rightarrow \quad x > 0$ ....(i)

and $(x + 3)^2 < 8x + 25$

$\Rightarrow \quad x^2 - 2x - 16 < 0$

$\Rightarrow \quad x \in (1 - \sqrt{17}, 1 + \sqrt{17})$ .....(ii)

(i) $\cap$ (ii) $\Rightarrow x \in (0, 1 + \sqrt{17})$

Noumber of integers = 5

D-7. $\frac{2}{x^2 - x + 1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \ge 0$

$\frac{2x + 2 - x^2 + x - 1}{x^3 + 1} - \frac{(2x-1)}{x^2+1} \ge 0 = \frac{3x + 1 - x^2 - 2x + 1}{x^3+1} \ge 0 = \frac{-x^2 + x + 2}{x^3 + 1} \ge 0 \quad = \frac{x^2 - x - 2}{x^3+1} \le 0$

$= \frac{(x+1)(x-2)}{(x+1)(x^2 - x + 1)} \le 0 \quad \Rightarrow \frac{(x-2)}{(x^2 - x + 1)} \le 0$ $(-\infty, -1) \cup (-1, 2]$ – + 2

required value of x, {0, 1, 2}

Section (E) :

E-1. $a^4 b^5 = 1 \Rightarrow$ log w.r.t. $a \to 4 + 5\log_a b = \log_a 1 \Rightarrow \log_a b = -\frac{4}{5}$

Now $\log_a(a^5b^4) = 5 + 4\log_a b = 5 + 4\left(-\frac{4}{5}\right) = \frac{25-16}{5} = \frac{9}{5}$

E-2. $\frac{1}{1 + \log_b a + \log_b c} + \frac{1}{1 + \log_c a + \log_c b} + \frac{1}{1 + \log_a b + \log_a c} = \frac{1}{\log_b abc} + \frac{1}{\log_c abc} + \frac{1}{\log_a abc}$

$= \log_{abc} b + \log_{abc} c + \log_{abc} a = \log_{abc} abc = 1$

E-3. $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc} = \log_{abc}\sqrt{bc} + \log_{abc}\sqrt{ca} + \log_{abc}\sqrt{ab} = \log_{abc} abc = 1$

E-4. Obvious

E-5. $y = \frac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1} \quad \Rightarrow \quad 2^{\log_{2^{1/4}} a} = 2^{4\log_2 a} = a^4$

$3^{\log_{27}(a^2+1)^3} = 3^{\log_3(a^2+1)} = a^2 + 1 \quad \Rightarrow \quad 7^{4\log_{49} a} = 7^{2\log_7 a} = a^2$

$\therefore \quad y = \frac{a^4 - (a^2 + 1 + 2a)}{a^2 - a - 1} = \frac{a^4 - (a+1)^2}{a^2 - a - 1} = a^2 + a + 1$

E-6. $\log_a(ab) = x \quad \Rightarrow \quad 1 + \log_a b = x \quad \Rightarrow \quad \log_a b = x - 1 \quad \Rightarrow \quad \log_b a = \frac{1}{x-1}$

Now $\log_b(ab) = 1 + \log_b a = 1 + \frac{1}{x-1} = \frac{x - 1 + 1}{x-1} = \frac{x}{x-1}$

E-7. $\log_p(\log_q(\log_r x)) = 0 \quad \Rightarrow \quad \log_q(\log_r x) = b \quad \Rightarrow \quad \log_r x = q \quad \Rightarrow \quad x = r^q$ ......(i)

and $\log_q(\log_r(\log_p x)) = 0$

$\Rightarrow \quad \log_r(\log_p x) = 1 \quad \Rightarrow \quad \log_p x = r \quad \Rightarrow \quad x = p^r$ ......(ii)

from (i) and (ii) $p^r = r^q$

$\Rightarrow \quad p = r^{q/r}$

E-8. $\log_{10}\pi$ is quantity lie between 0 to 1.

E-9. $\log_{10}(\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \ldots\ldots \log_{1023}{}^{1024}) = \log_{10}(\log_2 1024) = \log_{10}(\log_2 2^{10}) = \log_{10}(10) = 1$

Section (F) :

F-1. $2\log_{10}x - \log_{10}(2x - 75) = 2 \Rightarrow \frac{x^2}{2x-75} = 10^2 = 100$

$\Rightarrow x^2 - 200x + 7500 = 0 \Rightarrow x = 50, x = 150$

sum = 200

F-2. Take $\log_5 x = t$

on solving we get x = 1/625 & x = 5.

$\log_5 x = t$

x = 1/625 & x = 5.

F-3. Take $\log_x 5 = t$ on solving we get $x = \sqrt[3]{5}$ and 5.

$\log_x 5 = t$

F-4. $\log_p \log_p (p)^{\frac{1}{p^n}} = \log_p \left(\frac{1}{p}\right)^n = -\log_p p^n = -n$ independent of p.

F-5. $\log_x \log_{18} \left(\sqrt{2} + 2\sqrt{2}\right) = \frac{1}{3} \Rightarrow \log_x \log_{18}\left(\sqrt{18}\right) = \frac{1}{3} \Rightarrow \log_x \frac{1}{2} = \frac{1}{3}$

$\Rightarrow x^{1/3} = \frac{1}{2} \Rightarrow x = \frac{1}{8} \Rightarrow 1000\,x = 125.$

F-6. $\sqrt{\log_{10}(-x)} = \log_{10}|x| \Rightarrow -x > 0 \Rightarrow x < 0$

$\therefore |x| = -x \Rightarrow \sqrt{\log_{10}(-x)} = \log_{10}(-x)$

$\log_{10}(-x)\left(\log_{10}(-x) - 1\right) = 0$

$\log_{10}(-x) = 0$ $\qquad$ $\log_{10}(-x) = 1$

$\Rightarrow -x = 1$ $\qquad$ $-x = 10$

$x = -1$ $\qquad$ $x = -10.$

F-7. Clearly Domain is $x > 0$ and $x \neq 1$

Section (G) :

G-1. $\log_{\sin\frac{\pi}{3}}(x^2 - 3x + 2) \geq 2 \Rightarrow x^2 - 3x + 2 \leq \left(\frac{\sqrt{3}}{2}\right)^2 \Rightarrow 4x^2 - 12x + 8 \leq 3$

$\Rightarrow 4x^2 - 12x + 5 \leq 0 \Rightarrow (2x - 5)(2x - 1) \leq 0 \Rightarrow x \in \left[\frac{1}{2}, \frac{5}{2}\right]$

But domain $x^2 - 3x + 2 > 0 \Rightarrow (x - 1)(x - 2) > 0 \Rightarrow x \in (-\infty, 1) \cup (2, \infty)$

Hence $x \in \left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$

G-2. $\log_{0.3}(x - 1) < \log_{0.09}(x - 1)$; $\log_{0.3}(x - 1) < \frac{\log_{0.3}(x-1)}{2}$

$\Rightarrow \log_{0.3}(x - 1) < 0 \Rightarrow x - 1 > 1 \Rightarrow x > 2$

G-3. $2 - \log_2(x^2 + 3x) \geq 0 \Rightarrow \log_2(x^2 + 3x) \leq 2$

$x^2 + 3x > 0 \Rightarrow x \in (-\infty, -3) \cup (0, \infty)$ .....(i)

and $x^2 + 3x \leq 4$

$\Rightarrow (x - 1)(x + 4) \leq 0 \Rightarrow x \in [-4, 1]$ .....(ii)

(i) $\cap$ (ii) $\Rightarrow x \in [-4, -3) \cup (0, 1]$

G-4. $\log_{0.5}\log_5(x^2 - 4) > \log_{0.5} 1$; $\qquad \log_{0.5}\log_5(x^2 - 4) > 0$

$\Rightarrow x^2 - 4 > 0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$ ....(i)

$\log_5(x^2 - 4) > 0 \Rightarrow x^2 - 5 > 0$

$\Rightarrow x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$ ....(ii)

$\log_5(x^2 - 4) < 1$

$\Rightarrow x^2 - 9 < 0 \Rightarrow x \in (-3, 3)$ ....(iii)

(i) $\cap$ (ii) $\cap$ (iii) $\Rightarrow x \in (-3, \sqrt{5}) \cup (\sqrt{5}, 3)$

G-5. $\left(\frac{1}{2}\right)^{x^2-2x} < \left(\frac{1}{2}\right)^{2} \Rightarrow x^2 - 2x > 2 \Rightarrow x^2 - 2x - 2 > 0 \Rightarrow x \in (-\infty, 2-\sqrt{3}) \cup (2+\sqrt{3}, \infty)$

G-6. $\log_2 (4^x - 2.2^x + 17) > 5$

$4^x - 2.2^x + 17 > 0$

$(2^x)^2 - 2.2^x + 17 > 0 \quad \Rightarrow \quad \forall\, x \in R \quad$ and $\quad 4^x - 2.2^x + 17 > 32$

$\Rightarrow \quad (2^x)^2 - 2.2^x - 15 > 0 \quad \Rightarrow \quad (2^x + 3)(2^x - 5) > 0 \quad \Rightarrow \quad 2^x < -3$ or $2^x > 5$

$\Rightarrow \quad x \in \phi$ or $x > \log_2 5 \quad \Rightarrow \quad x \in (\log_2 5, \infty)$

G-7. $\log_{1-x}(x-2) \geq -1$

$x > 2$ ....(1)

(i) When $0 < 1 - x < 1 \Rightarrow 0 < x < 1$. So no common range comes out.

(ii) When $1 - x > 1 \Rightarrow x < 0$ but $x > 2$

here, also no common range comes out. , hence no solution. Finally, no solution

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PART - III

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1. (A) The set $\{3^{2n} - 8n - 1 : n \in N\}$ contains 0 and every element of this set is a multiple of 64.

(B) $2^{3n} - 1$ is always divisible by 7.

(C) $3^{2n} - 1$ is always divisible by 8.

(D) $2^{2n} - 7n - 1$ is always divisible by 49 and $2^{3n} - 7n - 1 = 0$ for n = 1.

2. (A) $a = 3 \quad ((\sqrt{7}+1) - (\sqrt{7}-1))$

$a = 3(2) = 6$

$b = \sqrt{(42)(30)+36} = 6\sqrt{7\times5+1} = 6 \times 6 = 36$

$\log_a b = \log_6 36 = 2$

(B) $a = (\sqrt{3}+1) - (\sqrt{3}-1) = 2$

$b = (3+\sqrt{2}) - (3-\sqrt{2}) = 2\sqrt{2}$

$\log_a b = \log_2(2\sqrt{2}) = \log_2(2^{3/2}) = 3/2$

(C) $a\sqrt{3+2\sqrt{2}} = (\sqrt{2}+1)$ = , $b = \sqrt{3-2\sqrt{2}} = (\sqrt{2}-1) = \frac{1}{\sqrt{2}+1}$

$\log_a b = \log_{\sqrt{2}+1}(\sqrt{2}+1)^{-1} = -1$

(D) $a = \sqrt{7+\sqrt{7^2-1}} = \sqrt{7+\sqrt{48}} = \sqrt{7+4\sqrt{3}} = (2+\sqrt{3})$

$b = 2-\sqrt{3} = \frac{1}{2+\sqrt{3}} = (2+\sqrt{3})^{-1}$ Now $\log_a b = \log_{2+\sqrt{3}}(2+\sqrt{3})^{-1}$ $\log_{2+\sqrt{3}} = -1$

(E) First 20 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, ...
zero are formed by multiple of 2 and 5 that is one times 2 and one times 5 there fore one zero at end of product of first 20 prime numbers.

(F) $2^{2x} - 3^{2y} = 55$, x, y $\in$ I $\Rightarrow 4^x - 9^y = 55$

only x = 3, y = 1 satisfy

There fore number of solution is one set (x, y) g.e. (3, 1)

3. (A) x = 0.363636......

100x = 36.363636.......

99x = 36

$\Rightarrow \quad x = \frac{36}{99} = \frac{4}{11}$ sum of numerator and denominator is 4 + 11 = 15

(C) $\frac{1}{\log_a 8} + \frac{1}{\log_b 8} = \frac{1}{\log_a 8 . \log_b 8} \quad \Rightarrow \quad \log_b 8 + \log_a 8 = 1$ (given $\log_a b = 3$)

$\Rightarrow \quad 4\log_b 8 = 1 \qquad \log_b 8 = 3\log_8 a$ ....(1)

$\Rightarrow \quad \log_b 8 = 1/4 \qquad \log_a 8 = 3\log_b 8$

$8 = b^{1/4} \Rightarrow b = 8^4$

$\log_8(8^4) = 3\log_8 a \quad \Rightarrow \quad \log_8 a = \frac{4}{3} \quad \Rightarrow a = (8^{4/3}) = (2^3) = 2^4 = 16$

4. (A) $\text{Antilog}_{27}(0.\bar{6}) = x$ $\Rightarrow$ $0.\bar{6} = \log_{27}x = \frac{2}{3}$ $\Rightarrow x = (27)^{2/3} = (3^3)^{2/3} = 9$

(B) Since $2^{10} < 2008 < 2^{11}$ $\Rightarrow$ $\log_2(2^{10}) < \log_2 2008 < \log_2(2^{11}) \Rightarrow 10 < \log_2 2008 < 11$

(C) $\log_e 2.\log_b 625 = \log_{10}16.\log_e 10$ $\Rightarrow$ $\log_e 2.\log_b 265 = \log_e 16$ $\Rightarrow$ $\log_b 625 = \log_2 16 = 4$

$\Rightarrow$ $625 = b^4$

(D) $x = \left(\frac{5}{6}\right)^{100} = \left(\frac{10}{2\times 6}\right)^{100} = \left(\frac{10}{2^2.3}\right)^{100}$

$\log_{10}x = 10(1 - 2\log 2 - \log 3) = 100(1 - 2(0.3010) - 0.4771) = 100(-0.0791) = 7.91$

EXERCISE–2

PART - I

1. $A_1 \cup A_2 \cup A_3$ is the smallest element containing subset of all we set A_1, A_2 and A_3

2. 1. $((A \cap B) \cup C)' \cap B')'$

$= (A \cap B) \cup C) \cup B$

$= (A \cap B) \cup B \cup C$

$= B \cup C \neq B \cap C$

2. $(A' \cap B') \cap (A \cup B \cup C')$

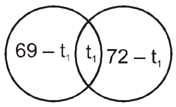
$= (A \cup B)' \cap ((A \cup B) \cup C')$

$= \phi \cup ((A \cup B)' \cap C')$

$= ((A \cup B) \cup C)'$

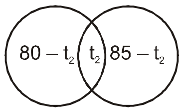
$= (A \cup (B \cup C))'$

3.

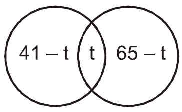


$70 + 72 - t_1 = 100$

$t_1 = 42\%$ $\Rightarrow$ min. in $P \cap C = 42\%$



$t_2 = 85\% - 20\% = 65\%$ $\Rightarrow$ min. $M \cap E = 65\%$



$t = 42 - 35 = 7\%$

min. in $((P \cap C) \cap (M \cap E)) = 7\%$

4. $X \cap (Y \cup X)' = X \cap (Y' \cap X') = X \cap X' \cap Y' = \phi$

$\Rightarrow$ Statement – 1 true.

$X \Delta Y = (X \sim Y) \cup (Y \sim X) = (X \cup Y) \sim (X \cap Y)$ $\Rightarrow$ number of element in $X \Delta Y = m - n$.

$\Rightarrow$ Statement-2 is true but does explain statement-1

5. $\frac{6x^2 - 5x - 3}{x^2 - 2x + 6} - 4 \le 0$ $\Rightarrow$ $\frac{2x^2 + 3x - 27}{x^2 - 2x + 6} \le 0$

denominator $x^2 - 2x + 6 > 0$ $\forall$ $x \in R (\because D < 0)$

then $2x^2 + 3x - 27 \le 0$ $\Rightarrow$ $(2x + 9)(x - 3) \le 0$

$-\frac{9}{2} \le x \le 3$ $\Rightarrow$ $0 \le x^2 \le \frac{81}{4}$

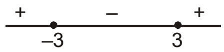
$(4x^2)_{max} = 4\left(-\frac{9}{2}\right)^2 = 81$ $\Rightarrow$ $(4x^2)_{min} = 4(0) = 0$

6. $x^2 - 16 \ge 0$

$\therefore$ $(x - 4)(x + 4) \ge 0$

$\therefore \quad x \in (-\infty, -4] \cup [4, \infty)$ ....(1)

Now $\dfrac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x-3)(x+3)} \le 0$



$x \in (-3, 3)$ ....(2)

By (1) and (2) $\quad x \in \{-4, 4\}$

7. $b = a^2, c = b^2, \dfrac{c}{a} = 3^3 \quad \Rightarrow \quad c = 27a$

$\Rightarrow \quad b^2 = 27a$

$\Rightarrow \quad a^4 = 27a$

$\Rightarrow \quad a = 3, a > 0$

$c = 81, b = 9$

$\therefore \quad a + b + c = 3 + 9 + 81 = 93$

8. $x = \left(-\log_3 5\right)\left(\log_{5^3}(7^3)\right)\left(\log_{7^2}(3^6)\right)$

$x = -\dfrac{3}{3}.\dfrac{6}{2}\log_3 5 \,.\, \log_5 7 \,.\, \log_7 3$ and और $y = 25^{\left(3\log_{17^2} 11\right)\left(\log_{28} 17^{1/2}\right)\left(\log_{(11)^3}(28)^2\right)}$

$y = 25^{\frac{3}{2}.\frac{1}{2}.\frac{2}{3}(\log_{17} 11).(\log_{28} 17)(\log_{11} 28)}$

$y = 5$

$\therefore x^2 + y^2 = (-3)^2 + 5^2 = 34$

9. $\dfrac{(x+1)+3x-x\log_2 8}{(x-1)(1)} = \dfrac{(x+2)+3x-3x}{(x-1)} = \dfrac{x+1}{x-1}$

10. $a^{(\log_3 7)^2} = \left(a^{\log_3 7}\right)^{\log_3 7} = 27^{\log_3 7} = 27^{\log_3 7} = 7^3 = 343$

$b^{(\log_7 11)^2} = \left(b^{\log_7 11}\right)^{\log_7 11} = 49^{\log_7 11} = 11^{\log_7 49} = 121$

$c^{(\log_{11} 25)^2} = \left(c^{\log_{11} 25}\right)^{\log_{11} 25} = \left(\sqrt{11}\right)^{\log_{11} 25} = 25^{\log_{11}\sqrt{11}} = 5$

hence the sum is 343 + 121 + 5 = 469

11. For $0 < x < y < 1$

$f(x) = x(\alpha - x) = -x^2 + x \; f(x) < f(1)$

so f(x) should be increasing in (0, 1) and one root is 0 so vertex should be ≥ 1

$\Rightarrow \quad \dfrac{-\alpha}{-2} \ge 1, \alpha \ge 2$

12. $\dfrac{\sqrt{(x-8)(2-x)}}{\log_{0.3}\left(\dfrac{10}{7}(\log_2 5 - 1)\right)} \ge 0$

For $\sqrt{(x-8)(2-x)}$ to be defined

(i) $(x-8)(2-x) \ge 0$

$(x-2)(x-8) \le 0 \quad \Rightarrow \quad 2 \le x \le 8$

Now Let say $y = \log_{0.3}\dfrac{10}{7}(\log_2 5 - \log_2 2) = \log_{0.3}\dfrac{10}{7}(\log_2 5/2)$

Let $y < 0$ (assume) then $\log_{0.3}\dfrac{10}{7}(\log_2 5/2) < 0$

$\Rightarrow \quad \dfrac{10}{7}\log_2 5/2 > 1 \quad \Rightarrow \quad \log_2 5/2 > \dfrac{7}{10} \quad \Rightarrow \quad \dfrac{5}{2} > 2^{(7/10)}$ which is true

So $y < 0$

so denominator is – ve and numerator is +ve, so inequality is not satisfied,

thus $\sqrt{(x-8)(2-x)} = 0$

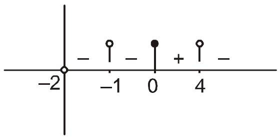
$x = 2, 8$ .....(i)

Now $2^{x-3} > 31$

$\Rightarrow$ $(x - 3) > \log_2 31 \Rightarrow x > 3 + \log_2 2^{4.9}$ (approx)

$\Rightarrow$ $x > 7.9$ $\Rightarrow x = 8$

13✍. $\dfrac{4^x\left(\left(\frac{3}{4}\right)^x - 1\right)\ell n\,(x+2)}{(x-4)\,(x+1)} \le 0$



14. $\sqrt{\log_4\{\log_3\{\log_2(x^2-2x+a)\}\}}$

for defined $\log_4\log_3\log_2(x^2 - 2x + a) \ge 0$

$\Rightarrow$ $\log_3\log_2(x^2 - 2x + a) \ge 1 \Rightarrow$ $\log_2(x^2 - 2x + a) \ge 3$

$\Rightarrow$ $x^2 - 2x + a \ge 8$ $\Rightarrow$ $x^2 - 2x + (a - 8) \ge 0$

$\Rightarrow$ $D \le 0$

$4 - 4(a - 8) \le 0$ $\Rightarrow$ $1 - a + 8 \le 0$

$\Rightarrow$ $a \ge 9$

15. domain $x > -\dfrac{3}{2}$

$\log_{2x+3}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$

$\log_{(2x+3)}(2x + 3)(3x + 7) = 4 - \log_{3x+7}(2x + 3)^2$

$1 + \log_{2x+3}(3x + 7) = 4 - 2\log_{3x+7}(2x + 3)$

$\log_{2x+3}(3x + 7) = y$

$y + \dfrac{2}{y} - 3 = 0$ $\Rightarrow$ $y = 1$ or $y = 2$ $\Rightarrow$ $x = -2, , \dfrac{-1}{4} - 4$

$x \ne -2, -4$ so $x = -\dfrac{1}{4}$

PART - II

1.✍ $n(A \cup B) = 280$

Now $n(A' \cap B') = n(A \cup B)' = 2009 - n(A \cup B) = 2009 - 280 = 1729 = 12^3 + 1^3 = 10^3 + 9^3$

2.✍ $n(A - B) = 1681 - 1075 = 606 = 4 + 2 \times 301 = 4 + 2 \times 7 \times 43 = (2)\ 2 + 2 \times 7 \times 43$

3. For $A \cap B$

$x^3 + (x-1)^3 = 1$ $\Rightarrow$ $x^3 + x^3 - 3x^2 + 3x - 1 = 1$ $\Rightarrow$ $2x^3 - 3x^2 + 3x - 2 = 0 \Rightarrow (x-1)(2x^2 - x + 2) = 0$

$\Rightarrow$ $x = 1$ $\Rightarrow$ $y = 0$ $\Rightarrow$ $(x, y) = (1, 0)$

For $A \cap C$

$x^3 + (1 - x)^3 = 1 \Rightarrow x^3 + 1 - 3x + 3x^2 - x^3 = 1$ $\Rightarrow$ $x^2 - x = 0 \Rightarrow$ $x = 0, 1 \Rightarrow (x, y) = (0, 1)\ (1, 0)$

4. $n(M) = 23,\ n(P) = 24,\ n(C) = 19$

$n(M \cap P) = 12,\ n(M \cap C) = 9,\ n(P \cap C) = 7$

$n(M \cap P \cap C) = 4$

$n\,(M \cap P' \cap C') = n[M \cap (P \cup C)'] = n(M) - n(M \cap (P \cup C)) = n(M) - n[(M \cap P) \cup (M \cap C)]$

$= n(M) - n(M \cap P) - n(M \cap C) + n(M \cap P \cap C) = 23 - 12 - 9 + 4 = 27 - 21 = 6$

$n(P \cap M' \cap C) = n[P \cap (M \cup C)']$

$= n(P) - n[P \cap (M \cup C) = n(P) - n\,[P \cap M) \cup (P \cap C)] = n(P) - n(P \cap M) - n(P \cap C) + n(P \cap M \cap C)$

$= 24 - 12 - 7 + 4 = 9$

$n(C \cap M' \cap P') = n(C) - n(C \cap P) - n(C \cap M) + n(C \cap P \cap M)$

$= 19 - 7 - 9 + 4 = 23 - 16 = 7$

5. $c(a-b) = a(b-c) \Rightarrow ac - bc = ab - ac \Rightarrow 2ac = ab + bc$

$\Rightarrow \frac{2ac}{b} = a + c \Rightarrow \frac{2ac}{a+c} = b$

Now $\frac{\log(a+c)+\log(a+c-2b)}{\log(a-c)} = \frac{\log(a+c)+\log\left(a+c-\frac{4ac}{a+c}\right)}{\log(a-c)}$

$= \frac{\log(a+c)+2\log(a-c)-\log(a+c)}{\log(a-c)} = 2$

6. $\log_b a \,.\, \log_c a + \log_a b \,.\, \log_c b + \log_a c \,.\, \log_b c = 3$

$\Rightarrow \frac{(\log a)^2}{\log b . \log c} + \frac{(\log b)^2}{\log a . \log c} + \frac{(\log c)^2}{\log a . \log b} = 3$

$\Rightarrow (\log a)^3 + (\log b)^3 + (\log c)^3 = 3 \log a . \log b . \log c$

$\Rightarrow \log a + \log b + \log c = 0$ ($\because$ a, b, c are distinct)

$\Rightarrow \log abc = 0 \Rightarrow abc = 1$

7. $A = \log_{16} 4 = \frac{1}{2}$

$B = \log_3 9 = 2$

$\therefore 4^{1/2} + 9^2 = 10^{\log_x 83} \Rightarrow 83 = 83^{\log_x 10} \Rightarrow x = 10$

8. $b^2 = a^3 = k \Rightarrow b = k^{1/2}, a = k^{1/3}$ and $d^4 = c^5 = \lambda \Rightarrow c = \lambda^{1/5}, d = \lambda^{1/4}$

Now $a - c = 9 \Rightarrow k^{1/3} - \lambda^{1/5} = 9$

Let $k^{1/3} = 25, \lambda^{1/5} = 16 \Rightarrow k = 25^3 = 5^6$ and $\lambda = 16^5 = 2^{20}$

$\Rightarrow b = (5^6)^{1/2} = 125, d = (2^{20})^{1/4} = 32$

Now $a - c = 9 \Rightarrow k^{1/3} - \lambda^{1/5} = 9$

let $k^{1/3} = 25, \lambda^{1/5} = 16$

$\Rightarrow b = (5^6)^{1/2} = 125, d = (2^{20})^{1/4} = 32$

Now $b - d = 125 - 32 = 93$

$\frac{b+d}{a+c} = \frac{157}{41}$ = **03.829**

9. $\log_{10}(2x^2 - 21x + 50) = 2$

(i) $2x^2 - 21x + 50 = 100$

$\Rightarrow 2x^2 - 21x - 50 = 0 \Rightarrow x = -2, \frac{25}{2}$

10. Domain $x - 1 > 0$ and $x + 1 > 0$ and $y - x > 0$

$x > 1 \quad x > -1 \quad x < 7$

$\Rightarrow x \in (1, 7)$ ....(i)

$-\log_2(x-1) - \log_2(x+1) = 1 + \log_{2^{-\frac{1}{2}}}(7-x)$

$-\log_2(x^2-1) + \log_2(7-x)^2 = 1$; $\log_2 \frac{(7-x)^2}{x^2-1} = 1 \Rightarrow \frac{(7-x)^2}{x^2-1} = 2$

$\Rightarrow x^2 + 14x - 51 = 0$; $(x+17)(x-3) = 0$

$x = 3$ or $x = -17$ (rejected) ; $x = 3$

11. $\log^2_{10} x + \log_{10} x^2 = \log^2_{10} 2 - 1$; $\log^2_{10} x + 2\log_{10} x + 1 = \log^2_{10} 2$

$\Rightarrow (\log_{10} x + 1)^2 = \log_{10}^2 2$

$\log_{10} x + 1 = \pm \log_{10} 2$; $x = \frac{1}{20}$ and $\frac{1}{5}$

-

12. $\frac{2009}{2010}x = (2009)^{\log_x(2010)}$

$\Rightarrow$ Taking log both sides w.r.t. x

$\log_x 2009 + 1 - \log_x 2010 = \log_x 2010.\log_x 2009 \quad \Rightarrow \frac{\ell n2009}{\ell nx} + 1 - \frac{\ell n2010}{\ell nx} = \frac{\ell n2009\ell n2010}{(\ell nx)^2}$

$\Rightarrow \quad \ell n\left(\frac{2009}{2010}\right).\ell n x + (\ell nx)^2 = \ell n2009\ \ell n2010$

Let $\ell nx = t$, $t^2 + \ell n\left(\frac{2009}{2010}\right)x - \ell n2009\ \ell n2010 = 0$

sum of roots $= -\ell n\left(\frac{2009}{2010}\right) \quad \Rightarrow \quad \ell nx_1 + \ell nx_2 = -\ell n\left(\frac{2009}{2010}\right) = \ell n\left(\frac{2010}{2009}\right)$

$\Rightarrow \quad x_1x_2 = \frac{2010}{2009}$

m = 2010, n = 2009 $\quad \Rightarrow \quad$ m – n = 1

13. $(\log x)^2 - \log x - 2 \geq 0$

$x > 0$ ....(i)

$(\log x - 2)(\log x + 1) \geq 0$

$\Rightarrow \quad \log x \leq -1 \quad$ or $\quad \log x \geq 2$

$\Rightarrow \quad x \leq \frac{1}{10}$ or $\quad x \geq 100$ ....(ii)

(i) $\cap$ (ii) $\quad \Rightarrow \quad x \in \left(0, \frac{1}{10}\right] \cup [100, \infty)$.

14. $\frac{x+1}{x+2} \quad \Rightarrow \quad x \in (-\infty, -2) \cup (-1, \infty)$

and $x + 3 > 0 \quad \Rightarrow \quad x > -3$

If $\quad \log_4\left(\frac{x+1}{x+2}\right) > \log_4(x + 3)$

$\Rightarrow \quad \frac{x+1}{x+2} > x - 3 \quad \Rightarrow \quad \frac{x+1-(x+3)(x+2)}{(x+1)} > 0$

$\Rightarrow \quad \frac{x+1-x^2-5x-6}{x+1} > 0 \quad \Rightarrow \quad \frac{x^2+4x+5}{x+1} < 0$

$\Rightarrow \quad x > -1$

ans. $(-1, \infty) \quad \Rightarrow \quad -a = -1 \quad \Rightarrow \quad a = -1$

If $\log_4\left(\frac{x+1}{x+2}\right) < \log_4(x + 3)$

$\frac{x+1}{x+2} < x + 3$ then solution is $(-3, -1)$

15. $\log_{1/2}(x + 5)^2 > \log_{1/2}(3x - 1)^2$

$(x + 5)^2 > 0 \quad \Rightarrow \quad x \in R - \{-5\}$ ....(i)

$(3x - 1)^2 > 0 \quad \Rightarrow \quad x \in R - \left\{\frac{1}{3}\right\}$ ....(ii)

$(x + 5)^2 < (3x - 1)^2$

$\Rightarrow \quad 8x^2 - 16x - 24 > 0$

$\Rightarrow \quad x^2 - 2x - 3 > 0$

$\Rightarrow \quad (x - 3)(x + 1) > 0$

$\Rightarrow \quad x \in (-\infty, -1) \cup (3, \infty)$ ....(iii)

(i) $\cap$ (ii) $\cap$ (iii) gives

$(-\infty, -5) \cup (-5, -1) \cup (3, \infty)$

p = –5, q = –5 , r = –1, s = 3

PART - III

1.

so $a - 1 + a^2 - 2a - 1 = 18$

$a = 5, -4 \qquad \therefore \qquad a = 5$

2. N = $\dfrac{\log_3 135}{\log_{15} 3} - \dfrac{\log_3 5}{\log_{405} 3} = \left(\log_3 27 + \log_3 5\right) - \left(\log_3 15\right)\ \log_3 5 . \log_3 405$

$= \left(3 + \log_3 5\right)\left(1 + \log_3 5\right) - \log_3 5\ \log_3(81 \times 5) = (3 + \log_3 5)\ (1 + \log_3 5) - \log_3 5(4 + \log_3 5) = 3$

3. $(\log_5 x)^2 + \log_{5x} \dfrac{5}{x} = 1$

$\Rightarrow \quad (\log_5 x)^2 + \log_{5x} 5 - \log_{5x} x = 1$

$\Rightarrow \quad (\log_5 x)^2 + \dfrac{\log_5 5}{\log_5 5 + \log_5 x} - \dfrac{\log_5 x}{\log_5 5 + \log_5 x} = 1$

$\Rightarrow \quad (\log_5 x)^2 + \dfrac{1}{1 + \log_5 x} - \dfrac{\log_5 x}{1 + \log_5 x} = 1$

Let $\log_5 x = t$

$\therefore \quad t^2 + \dfrac{1}{1+t} - \dfrac{t}{1+t} = 1 \quad \Rightarrow \quad \dfrac{t^2(1+t) + 1 - t}{1+t} = 1 \quad \Rightarrow \quad t^3 + t^2 + 1 - t = 1 + t$

$t^3 + t^2 - 2t = 0 \quad ; \quad t(t^2 + t - 2) = 0 \quad ; \quad t(t - 1)\ (t + 2) = 0$

$t = 0, 1, -2$

$\therefore \quad \log_5 x = 0, 1, -2$

$\therefore \quad x = 1, 5, \dfrac{1}{25}$

4. $\log_{x^2} 16 + \log_{2x} 64 = 3 \quad \Rightarrow \quad 4\ \log_{x^2} 2\ + 6 \log_{2x} 2 = 3$

$\Rightarrow \quad \dfrac{4}{\log_2 x^2} + \dfrac{6}{\log_2 2x} = 3 \quad \Rightarrow \quad \dfrac{2}{\log_2 x} + \dfrac{6}{1 + \log_2 x} = 3$ but $\log_2 x = t$

$\therefore \quad \dfrac{2}{t} + \dfrac{6}{1+t} = 3 \quad \Rightarrow \quad 2 + 2t + 6t = 3t + 3t^2$

$\Rightarrow \quad 3t^2 - 5t - 2 = 0 \quad \Rightarrow \quad 3t^2 - 5t - 2 = 0 \quad \Rightarrow \quad (3t + 1)(t - 2) = 0$

$\Rightarrow \quad t = -\dfrac{1}{3}, t = 2 \quad \Rightarrow \quad \log_2 x = -\dfrac{1}{3} \qquad \log_2 x = 2$

$\Rightarrow \quad x = 2^{-1/3} \qquad x = 4 = \dfrac{1}{2^{1/3}}.$

5. $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3} \quad \Rightarrow \quad \left(\log_3 x\right)^3 - \dfrac{9}{2}\ \log_3 x + 5 = \log_x 3\ \sqrt{3}$

$\Rightarrow \quad \left(\log_3 x\right)^2 - \dfrac{9}{2}\log_3 x + 5 = \dfrac{3}{2}\log_x 3$

Let $\log_3 x = t \quad \Rightarrow \quad t^2 - \dfrac{9}{2}t + 5 = \dfrac{3}{2t} \quad \Rightarrow \quad 2t^3 - 9t^2 + 10t - 3 = 0$

t = 1 satisfies it $\qquad 2t^3 - 9t^2 + 10t - 3 = 2t^2(t - 1) - 7t\ (t - 1) + 3(t - 1)$

$= (t - 1)\left(2t^2 - 7t + 3\right) = (t - 1)\ (2t - 1)\ (t - 3) \quad \Rightarrow \quad t = 1 \quad t = \dfrac{1}{2} \quad t = 3$

$\Rightarrow \quad \log_3 x = 1 \qquad \log_3 x = \dfrac{1}{2} \qquad \log_3 x = 3$

$\Rightarrow \quad x = 3 \qquad x = 3^{1/2} \qquad x = 27.$

6. $\log_3 x + \log_3 y = 2 + \log_3 2$, $\log_{27}(x + y) = \frac{2}{3}$

$\Rightarrow$ $\log_3 xy = \log_3 9 + \log_3 2$,

$\Rightarrow$ $x + y = (27)^{2/3}$

$\Rightarrow$ $xy = 18$,

$\Rightarrow$ $x + y = 9$

$\Rightarrow$ $x = 6$ or $x = 3$ $y = 3$, $y = 6$ as $x > 0$, $y > 0$.

7. $(\log_{10}8)x^2 - (\log_{10}5)x + x - 2\log_{10}2 = 0$

(A) sum of roots = $\frac{-(1-\log_{10}5)}{\log_{10}8} = \frac{-\log_{10}\left(\frac{10}{5}\right)}{\log_{10}8} = \frac{-\log_{10}2}{3\log_{10}2} = \frac{-1}{3}$ rational

(B) Product of roots = $\frac{-2\log_{10}2}{\log_{10}8} = \frac{-2}{3}$

(C) sum of cofficient = $\log_{10}8 - \log_{10}5 + 1 - \log_{10}4 = \log_{10}\left(\frac{8\times 10}{5\times 4}\right)$

$= \log_{10}4$ = irrational

(D) Dirscriminant = $(\log_{10}2)^2 - 4\log_{10}8(-2\log_{10}2) = (\log_{10}2)^2 + 24(\log_{10}2)^2$

$= 25(\log_{10}2)^2 = (5\log_{10}2)^2$ irrational.

8. $x = a^b$

(D) If a is rational & b is rational then x may be rational

e.g. $= 2^2$

(C) $(\sqrt{2})^4$

(B) $(2)^{\log_2 3}$

(A) $(\sqrt{2})^{\log_2 9}$

9. $\log_2 3 > 1$, $\log_{12} 10 < 1$ $\Rightarrow$ $\log_2 3 > \log_{12} 10$

$\log_6 5 < 1$, $\log_7 8 > 1$ $\Rightarrow$ $\log_6 5 < \log_7 8$

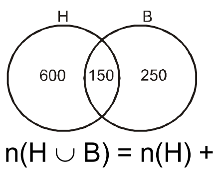
$\log_3 26 < 3$, $\log_2 9 > 3$ $\Rightarrow$ $\log_3 26 < \log_2 9$

$\log_{16} 15 < 1$, $\log_{10} 11 > 1$ $\Rightarrow$ $\log_{16} 15 < \log_{10} 11$

10. $\frac{1}{2} \le \log_{1/10} x \le 2 \Rightarrow$ $\frac{1}{100} \le x \le \frac{1}{\sqrt{10}}$

PART - IV

Sol. **(Q 1 to 3)**



$n(H \cup B) = n(H) + n(B) - n(H \cap B)$

$1000 = 750 + 400 - n(H \cap B) = 150$

Now n(only hindi) = $n(H) - n(H \cap B) = 750 - 150 = 600$

n(only bengali) = $n(B) - n(H \cap B)$

$400 - 150 = 250$

Sol. (Q 4 to 6) Let $\log_4 x = t$ $\Rightarrow$ $1 + t + 4(5 - 4t) = 3(5 - 4t)(1 + t)$

$21 - 15t = 15 + 3t - 12t^2$

$\Rightarrow$ $12t^2 - 18t + 6 = 0$ $\Rightarrow$ $2t^2 - 3t + 1 = 0$

$t = 1, t = 1/2$ $\Rightarrow$ $x = 4'$ or

$x = 4^{1/2} = 2$

A = sum of roots = 4 + 2 = 6

for B: $2^m = 3$, $3^n = 4$ $\Rightarrow$ $m = \log_2 3$, $\Rightarrow$ $n = \log_3 4$

$\Rightarrow$ Now $mn = \log_2 3.(\log_3 2^2) = 2$ B = 2

for C: $\frac{1-\log_3 x}{1+\log_3 x} + (\log_3 x)^2 = 1$ $\Rightarrow$ $1 - t + t^2(1 + t) = 1 + t$

$\Rightarrow$ $t^2 + t^3 - t = t$

$t^3 + t^2 - 2t = 0$ $\Rightarrow$ $t(t^2 + t - 2) = 0$ $\Rightarrow$ $t = 0, t = -2, t = 1$

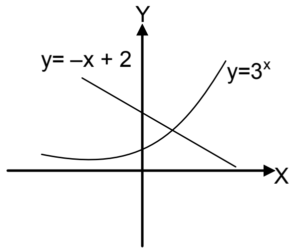
$x = 3^t$ $\Rightarrow$ $x = 3° = 1$, $x = 3' = 3$ $x = 3^{-2} = 1/9$

sum of integral root = C = 1 + 3 = 4

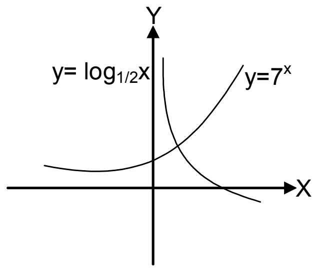
(4) A + B = 6 + 2 = 8 (5) B + C = 2 + 4 = 6

(6) A + C ÷ B = 6 + 4 ÷ 2 = 6 + 2 = 8

8.



9.



EXERCISE # 3

PART - I

1. Obvious

2. Obvious

3. $2 \log_2 \log_2 x + \log_{1/2} \log_2 \left(2\sqrt{2}\,x\right) = 1$ $\Rightarrow$ $\log_2 (\log_2 x)^2 - \log_2 \log_2 \left(2\sqrt{2}\,x\right) = 1$

$\Rightarrow$ $\log_2 \frac{(\log_2 x)^2}{\log_2\left(2\sqrt{2}\,x\right)} = 1$ $\Rightarrow$ $\frac{(\log_2 x)^2}{\frac{3}{2}+\log_2 x} = 2$

let $\log_2 x = y$

$\therefore$ $y^2 - 2y - 3 = 0$ $\Rightarrow$ $(y - 3)(y + 1) = 0$

$\therefore$ $y = 3, -1$ $\Rightarrow$ $\log_2 x = 3, -1,$

but $\log_2 x > 0$

$\therefore$ $\log_2 x = -1$ is not possible $\Rightarrow$ $x = 8$

4. $\log_{3/4} \log_8 (x^2 + 7) + \log_{1/2} \log_{1/4} (x^2 + 7)^{-1} = -2$

$\Rightarrow$ $\log_{3/4} \frac{1}{3} \log_2 (x^2 + 7) - \log_2 \frac{\log_2(x^2+7)}{2} = -2$

let $\log_2 (x^2 + 7) = t$

$\Rightarrow \quad \log_{3/4}\frac{t}{3} - \log_2\frac{t}{2} + 2 = 0 \quad \Rightarrow \quad \log_{3/4}\frac{t}{3} + 1 - \left(\log_2\frac{t}{2} - 1\right) = 0$

$\Rightarrow \quad \log_{3/4}\frac{t}{4} = \log_2\frac{t}{4} \quad \Rightarrow \quad \frac{t}{4} = 1 \Rightarrow \quad t = 4$

$\therefore \quad \log_2 (x^2 + 7) = 4$

this gives $x = \pm 3$

5. $\frac{1}{2}\log_2(x - 1) = \log_2 (x - 3) \Rightarrow \sqrt{x-1} = x - 3$

$(x - 1) = x^2 - 6x + 9 \quad \Rightarrow \quad x^2 - 7x + 10 = 0$

$(x - 5)(x - 2) = 0$ but $x \neq 2$

$\therefore \quad x = 5$

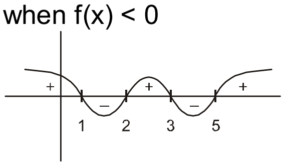
6. Given $f(x) = \frac{x^2 - 6x + 5}{x^2 - 5x + 6}$

(i) when $0 < f(x) < 1$ then $0 < \frac{x^2 - 6x + 5}{(x^2 - 5x + 6)} < 1$

So $\frac{(x-5)(x-1)}{(x-2)(x-3)} > 0$ and $\frac{x^2 - 6x + 5}{x^2 - 5x + 6} - 1 < 0 \quad \Rightarrow \quad \frac{(x+1)}{(x-2)(x-3)} > 0$

$\Rightarrow \quad x \in (-1, 1) \cup (5, 0)$

(ii) when $f(x) < 0$

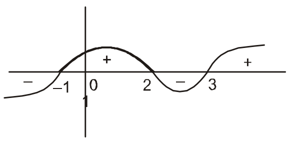


$\frac{(x-1)(x-5)}{(x-2)(x-3)} < 0$

$\Rightarrow \quad x \in (1, 2) \cup (3, 5)$

(iii) $f(x) > 0, x \in (-\infty, 1) \cup (2, 3) \cup (5, \infty)$

(iv) $f(x) < 1 \Rightarrow \frac{x+1}{(x-2)(x-3)} > 0$



$x \in (-1, 2) \cup (3, \infty)$

(A) $-1 < x < 1$, f(x) satisfies p, q, s

(B) $1 < x < 2$, f(x) satisfies q, s

(C) $3 < x < 5$, f(x) satisfies q, s

(D) $x > 5$, f(x) satisfies p, r, s

7. $(2x)^{\ell n2} = (3y)^{\ell n3}$

$\Rightarrow \ell n2\ \ell n(2x) = \ell n3\ \ell n(3y) = \ell n3\ (\ell n3 + \ell ny)$ .... (1)

also $3^{\ell nx} = 2^{\ell ny} \quad \Rightarrow \ell nx\ \ell n3 = \ell ny\ \ell n2$ .... (2)

by (1) $\Rightarrow \ell n2\ \ell n(2x) = \ell n3\ (\ell n3 + \ell ny) \quad \Rightarrow \quad \ell n\, 2 \,.\, \ell n\,(2x) = \ell n3 \left\{\ell n3 + \frac{\ell nx \ell n3}{\ell n2}\right\}$

$\Rightarrow \quad \ell n^2 2\ \ell n2x = \ell n^2 3\ (\ell n2 + \ell nx) \quad \Rightarrow \left(\ell n^2 2 - \ell n^2 3\right)(\ell n2x) = 0 \quad \Rightarrow \ell n2x = 0 \Rightarrow x = \frac{1}{2}$

8. Let $\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\ldots\ldots}} = t \quad \Rightarrow \sqrt{4 - \frac{1}{3\sqrt{2}}t} = t \quad \Rightarrow 4 - \frac{1}{3\sqrt{2}}\ t = t^2 \Rightarrow$

$t^2 + \frac{1}{3\sqrt{2}} t - 4 = 0 \Rightarrow 3\sqrt{2}\, t^2 + t - 12\sqrt{2} = 0 \quad \Rightarrow \quad t = \frac{-1 \pm \sqrt{1 + 4 \times 3\sqrt{2} \times 12\sqrt{2}}}{2 \times 3\sqrt{2}} = \frac{-1 \pm 17}{2 \times 3\sqrt{2}}$

$t = \frac{16}{6\sqrt{2}}, \frac{-18}{6\sqrt{2}} \Rightarrow \quad t = \frac{8}{3\sqrt{2}}, \frac{-3}{\sqrt{2}}$ and $\frac{-3}{\sqrt{2}}$ is rejected

so $6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}} \times \frac{8}{3\sqrt{2}}\right) = 6 + \log_{3/2}\left(\frac{4}{9}\right) = 6 + \log_{3/2}\left(\left(\frac{2}{3}\right)^2\right) = 6 - 2 = 4$

9*. $3^x = 4^{x-1} \quad \Rightarrow \quad x = (x-1)\log_3 4 \quad \Rightarrow \quad x(1 - 2\log_3 2) = -2\log_3 2$

$x = \frac{2\log_3 2}{2\log_3 2 - 1}$ **Ans. (A)**

Again $x \log_2 3 = (x-1) \cdot 2 \Rightarrow x(\log_2 3 - 2) = -2 \Rightarrow x = \frac{2}{2 - \log_2 3}$ **Ans. (B)**

$x = \frac{1}{1 - \frac{1}{2}\log_2 3} = \frac{1}{1 - \log_4 3}$ **Ans.(C)**

10. $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\log_7 4}$

$(\log_2 9)^{2\log_{(\log_2 9)} 2}.(2) = 4.2 = 8$

PART - II

1. **Obvious** **2.** **Obvious** **3.** **Obvious** **4.** **Obvious**
5. **Obvious** **6.** **Obvious** **7.** **Obvious** **8.** **Obvious**

9. We have, $A \cup B = A \cup C \Rightarrow (A \cup B) \cap C = (A \cup C) \cap C$

$\Rightarrow (A \cap C) \cup (B \cap C) = C$ [$\because (A \cup C) \cap C = C$]

$\Rightarrow (A \cap B) \cup (B \cap C) = C$ ...(i) [$\because A \cap C = A \cap B$]

Again, $A \cup B = A \cup C$

$\Rightarrow (A \cup B) \cap B = (A \cup C) \cap B \Rightarrow B = (A \cap B) \cup (C \cap B)$

$\Rightarrow (A \cap B) \cup (C \cap B) = B \Rightarrow (A \cap B) \cup (B \cap C) = B$ ...(ii)

From (i) and (ii), we get B = C

10. Every element has 3 options. Either set Y or set Z or none
so number of ordered pairs = 3^5

11. $X = \{0, 9, ... 4^n - 3n - 1\}$
$Y = \{0, 9, ..., 9(n-1)\}$
Now $4^n - 3n - 1 = (3+1)^n - 3n - 1$
$= 3^n + n.3^{n-1} + ... + {}^nC_2 .9.$
is a multiple of 9.
Also Y consists of all multiples of '9' from 0, 9,.......
Hence all values of X are subset of values of Y.
Thus $X \cup Y = Y$

12. $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$

$x^2 - 5x + 5 = 1$, $x^2 - 5x + 4 = 0$, $x = 1, x = 4$

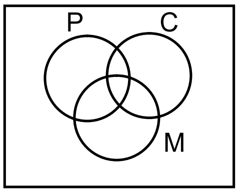
$x^2 + 4x - 60 = 0$, $x = -10, x = 6$

$x^2 - 5x + 5 = -1$, $x^2 - 5x + 6 = 0$, $x = 2, 3$

at $x=2$, $x^2 + 4x - 60 = -48$
$\therefore$ $x=2$ is valid
at $x=3$, $x^2 + 4x - 60 = -39$ (odd)
$\therefore$ $x = 3$ is invalid
$x = 1, 2, 4, 6, -10$

13. $n(P) = \left[\frac{140}{3}\right] = 46$

$n(C) = \left[\frac{140}{5}\right] = 28$



$n(M) = \left[\frac{140}{2}\right] = 70$

$n(P \cup C \cup M) = n(P) + n(C) + n(M) - n(P \cap C) - n(C \cap M) - n(M \cap P) + n(P \cap M \cap C)$

$= 46 + 28 + 70 - \left[\frac{140}{15}\right] - \left[\frac{140}{10}\right] - \left[\frac{140}{6}\right] + \left[\frac{140}{30}\right]$

$= 144 - 9 - 14 - 23 + 4 = 102$

so required number of student = 140 – 102 = 38

14. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 25 + 7 - 3$

$= 29$