3 Pair of Linear Equations in Two Variables

Topics Covered

1. Pair of Linear Equations in Two Variables and Finding its Solution by Graphical Method
2. Solving a Pair of Linear Equations by Algebraic Methods
3. Solving a System of Equations Reducible to Simultaneous Linear Equations
4. Solving Word Problems on Pair of Linear Equations in Two Variables

Introduction

An equation which can be expressed in the form $ax + by + c = 0$, where a, b and c are real numbers, and $a \neq 0$, $b \neq 0$ is called **a linear equation in two variables** x and y. A **solution** of a linear equation in two variables is a pair of values of x and y, which satisfy the equation. The graph of a linear equation in two variables is a **straight line.**

Every linear equation in two variables has **infinitely many solutions** and each one of them is represented by a point on the graph of the equation.

1. Pair of Linear Equations in Two Variables and Finding its Solution by Graphical Method

Pair of Linear Equations in Two Variables

Two linear equations in the same two variables are called **a pair of linear equations in two variables.** General form of a pair of linear equations is

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers such that $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$.

Some examples of a pair of linear equations in two variables are:

(*i*) $x + 2y = 17$, $3x - 5y = 0$

(*ii*) $2a + 3b - 2 = 0$, $a + b - 2 = 0$

The **solution of a given pair of linear equations** in x and y is a pair of values of x and y, which satisfy each of the equations. A pair of linear equations in two variables is said to be consistent, if it has at least one solution. A pair of linear equations in two variables is said to be inconsistent, if it has no solution.

Graphical Method of Solution of a Pair of Linear Equations

In order to find a solution of a pair of linear equations in two variables by graphical method, we draw the graph of each of the given linear equations. When we represent a pair of linear equations graphically as two lines, we can observe that the lines

(*i*) may intersect. (*ii*) may be parallel. (*iii*) may coincide.

The number of solutions for a system of two linear equations in two variables is given by one of the following observations.

What this Means Graphically	Number of Solutions	Type of Equations
The two lines intersect at a single point.	Unique common solution.	Consistent
The two lines are parallel.	No common solution.	Inconsistent
The two lines are coincident.	Infinitely many solutions.	Dependent Consistent

Conditions for Consistency of a Pair of Linear Equations

Again, consider a pair of two linear equations

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

where a_1, b_1, c_1 and a_2, b_2, c_2 denote the coefficients of the equations given above in the general form.

By comparing the values of $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, we can observe that if the lines represented by the above stated equations are intersecting or parallel or coincident.

For the system of two linear equations $a_1x + b_1y + c_1 = 0;\ a_2x + b_2y + c_2 = 0$			
S.No.	**Compare the ratios**	**Graphical representations**	**Algebraic interpretation**
1.	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	• Exactly one solution (Unique) • Consistent system of equations • Equations are independent
2.	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	• Infinitely many solutions • System is consistent • Equations are dependent
3.	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	• No solution • System is inconsistent

Example 1. Given: 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

The pair of linear equations representing the above situation and their solution, respectively are

(*a*) $x + y = 10, x - y = -4; (3, 7)$
(*b*) $x - y = 10, x + y = 4; (3, 5)$
(*c*) $2x + y = 10, 2x - y = 4; (2, 5)$
(*d*) $2x - y = 10, 2x + y = 4$

Solution. Let the number of boys and girls who took part in Mathematics quiz be x and y respectively.

Then, $x + y = 10$ ...(*i*)

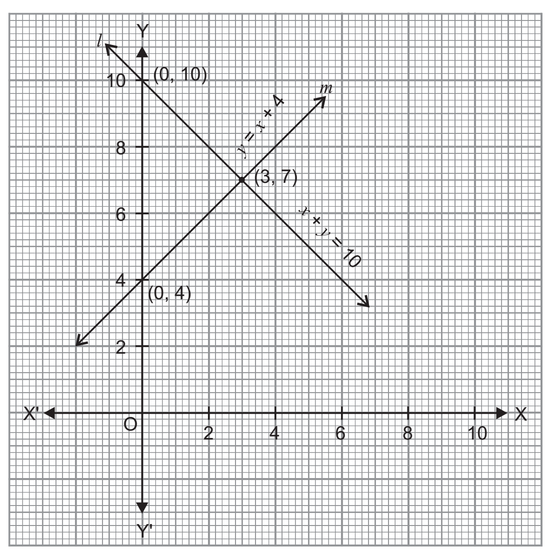
and $y = x + 4$ ...(*ii*)

Now, we draw the graph for equations (*i*) and (*ii*).

We find two solutions of each of the equations which are given in table:

Table for $x + y = 10$		
x	0	3
$y = 10 - x$	10	7

Table for $y = x + 4$		
x	0	3
$y = x + 4$	4	7



From graph, we observe that the two lines representing two equations intersect each other at point (3, 7). So, the solution of the given pair of linear equations is $x = 3$ and $y = 7$.

Thus, 3 boys and 7 girls took part in Mathematics quiz.

Hence, option (*a*) is the correct ansswer.

Example 2. Given: 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46.

The pair of linear equations representing the above situation and the cost of one pencil and that of one pen, respectively are

(*a*) $5x - 7y = 50$, $7x - 5y = 46$; (3, 5)
(*b*) $5x + 7y = 50$, $7x + 5y = 46$; (3, 5)
(*c*) $3x - 8y = 48$, $3x + 8y = 48$; (5, 3)
(*d*) None of these

Solution. Let cost of one pencil be ₹ x and cost of one pen be ₹ y.

Then, $5x + 7y = 50$ ...(*i*)

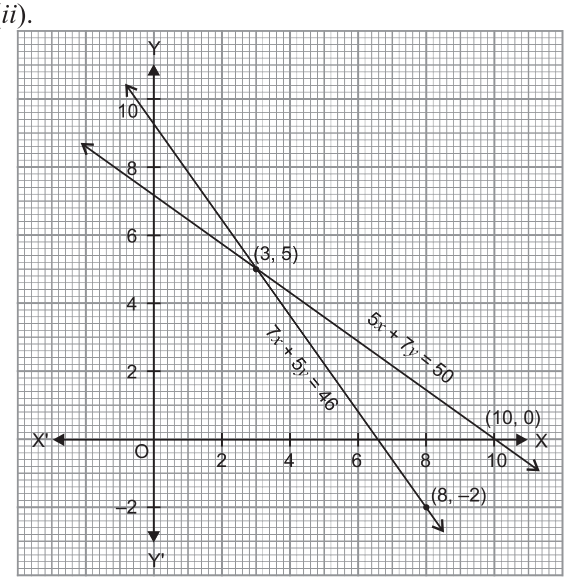
and $7x + 5y = 46$ ...(*ii*)

Now, we draw the graph of equations (*i*) and (*ii*).

We will now complete the tables as follows:

Table for $5x + 7y = 50$		
x	10	3
$y = \dfrac{50 - 5x}{7}$	0	5

Table for $7x + 5y = 46$		
x	8	3
$y = \dfrac{46 - 7x}{5}$	–2	5



Plotting the values of x and y from these tables on the graph, we observe that two lines intersect each other at point (3, 5). So, the solution of the pair of linear equations is $x = 3$ and $y = 5$.

Thus, the cost of 1 pencil is ₹ 3 and that of 1 pen is ₹ 5.

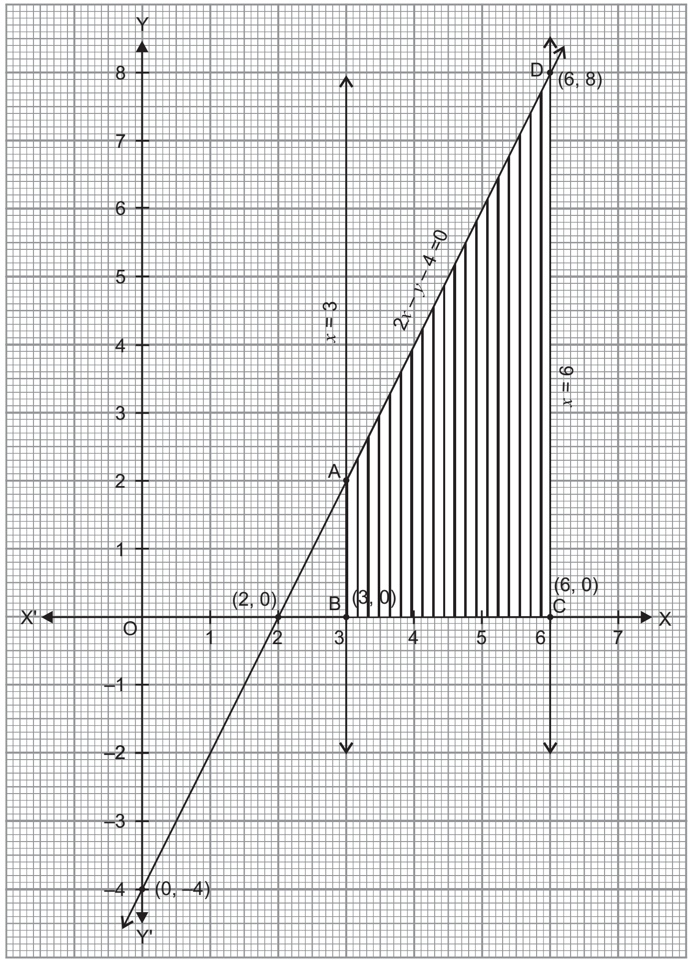
Hence, option (*b*) is the correct answer.

Example 3. The area of the quadrilateral formed by the lines $x = 3$, $x = 6$, $2x - y - 4 = 0$ and x-axis is

(*a*) 8 sq. units
(*b*) 12 sq. units
(*c*) 15 sq. units
(*d*) None of these

Solution. First we find three solutions of the equation $2x - y - 4 = 0$ (see table) and then plot these solutions on the graph.

Table for $2x - y - 4 = 0 \Rightarrow y = 2x - 4$			
x	0	2	6
y	– 4	0	8



Drawing the graph of $x = 3$, $x = 6$, we get the lines parallel to y-axis and passing through the points (3, 0) and (6, 0) respectively.

Area of the trapezium ABCD

$= \frac{1}{2}$ (sum of parallel sides) × height

$= \frac{1}{2}(2 + 8) \times 3$

$= 5 \times 3 = 15$ sq. units

Hence, option (c) is the correct answer.

Example 4. If the pair of equations $x + y = 5$ and $2x + 2y = 10$ is consistent, the two solutions obtained graphically are

(a) (0, 4), (4, 0)
(b) (7, –2), (2, 7)
(c) (0, 5), (5, 0)
(d) None of these **[Imp.]**

Solution. The given equations can be written

$$x + y - 5 = 0 \quad ...(i)$$

and $$2x + 2y - 10 = 0 \quad ...(ii)$$

Here, $a_1 = 1, b_1 = 1, c_1 = -5$

and $a_2 = 2, b_2 = 2, c_2 = -10$

Now, $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the linear equations represented by (i) and (ii) are coincident. The system of equations has infinitely many solutions. The system is consistent and the equations are dependent.

Now, we draw the graph of equations. First we find two solutions of each equation (see tables).

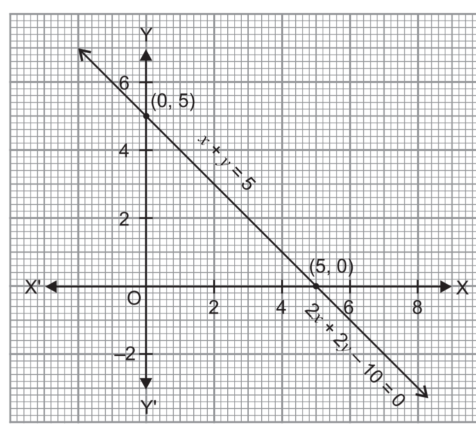


Table for $x + y - 5 = 0$		
x	0	5
$y = 5 - x$	5	0

Table for $2x + 2y - 10 = 0 \Rightarrow x + y - 5 = 0$		
x	0	5
$y = 5 - x$	5	0

The graph of the given system of equations is shown in the figure.

There are infinitely many solutions.

Hence, option (c) is the correct answer.

Example 5. The nature of the system of equations $2x - 2y - 2 = 0$ and $4x - 4y - 5 = 0$ is

(a) Unique (b) Consistent (c) Inconsistent (d) None of these

Solution. We have

$$2x - 2y - 2 = 0 \quad \text{...}(i)$$

and

$$4x - 4y - 5 = 0 \quad \text{...}(ii)$$

Here, $a_1 = 2, b_1 = -2, c_1 = -2$ and $a_2 = 4, b_2 = -4, c_2 = -5$

Now, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the lines are parallel, *i.e.*, they have no point in common. So, the given system of equations has no solution, *i.e.*, it is inconsistent.

Hence, option (c) is the correct answer.

Example 6. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. The dimensions of the garden are (use graphical method)

(a) Length = 20 m, width = 16 m (b) Length = 16 m, width = 24 m

(c) Length = 18 m, width = 12 m (d) Length = 24 m, width = 15 m

Solution. Let the length and width of the garden be x m and y m respectively.

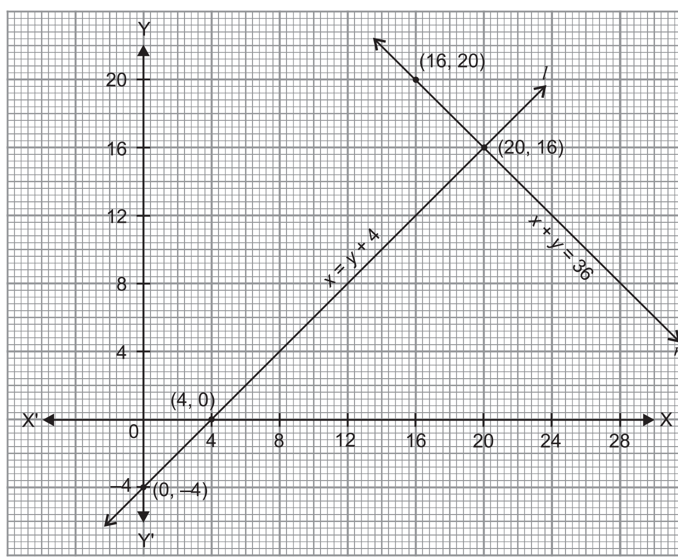
Then, $x = y + 4$ ...(i)

Also $x + y = 36$ ...(ii)

Now, we find two solutions of each equation and then draw their graphs.

Table for $x = y + 4$		
x	0	4
$y = x - 4$	–4	0

Table for $x + y = 36$		
x	16	20
$y = 36 - x$	20	16



From the graph, we observe that the lines intersect each other at the point (20, 16).

Therefore, $x = 20$ and $y = 16$ are the required solutions.

Thus, the length of the garden is 20 m and the width of the garden is 16 m.

Hence, option (a) is the correct answer.

Example 7. The solution of the following system of equations when solved graphically is:

$$2x - 3y - 6 = 0$$
$$2x + y + 10 = 0$$

(*a*) (3, 4) (*b*) (–3, –4) (*c*) (4, 5) (*d*) (–4, –5)

Solution. We have, $2x - 3y - 6 = 0$ ...(*i*)

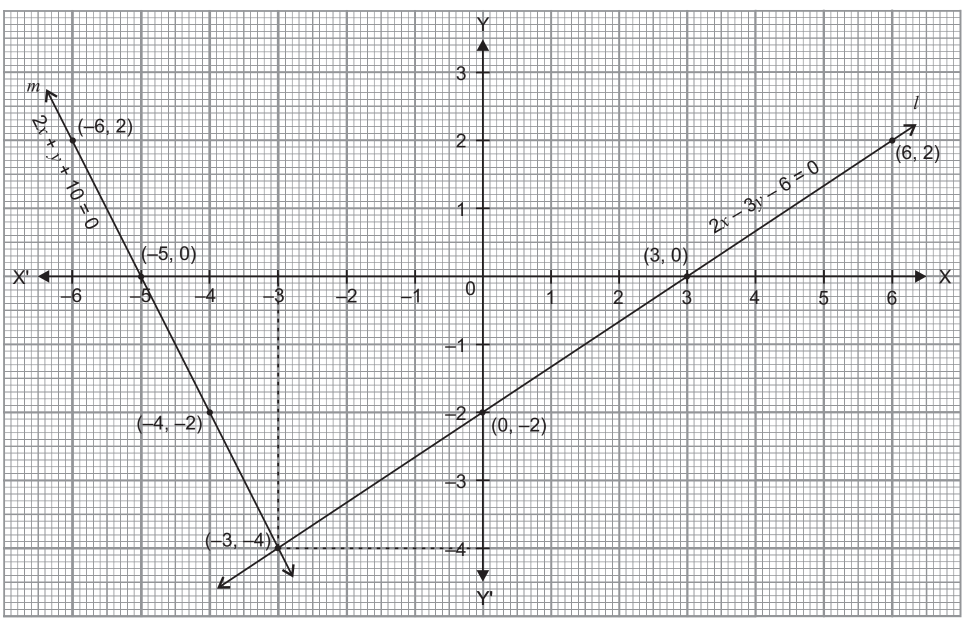
$2x + y + 10 = 0$ ...(*ii*)

Let us find the table of values for each equation.

Table for $2x - 3y - 6 = 0 \Rightarrow y = \frac{2x-6}{3}$			
x	3	6	0
y	0	2	–2

Table for $2x + y + 10 = 0 \Rightarrow y = -10 - 2x$			
x	–5	–6	–4
y	0	2	–2

We plot these values to get the following graph:



We observe that the point (–3, –4) is an intersecting point of graphs of the equations (*i*) and (*ii*).

Thus, the required solution is $x = -3$, $y = -4$.

Hence, option (*b*) is the correct answer.

Example 8. The coordinates of vertices of a triangle formed by the equations $x - y + 1 = 0$, $3x + 2y - 12 = 0$ and x-axis are

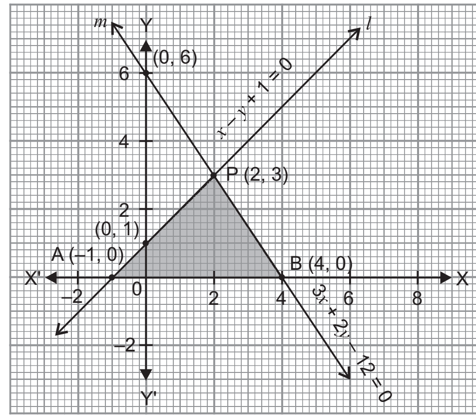
(*a*) (1, 3), (–2, 3), (3, 0) (*b*) (3, 4), (–3, 5), (2, 3)

(*c*) (2, 5), (–3, 0), (5, 0) (*d*) (2, 3), (–1, 0), (4,0)

Solution. We find the tables of values to draw the graph of the equations,

Table for $x - y + 1 = 0$		
x	0	–1
$y = x + 1$	1	0

Table for $3x + 2y - 12 = 0$		
x	2	0
$y = \dfrac{12 - 3x}{2}$	3	6



From the graph we observe that the two lines intersect each other at the point P(2, 3). So, $x = 2$ and $y = 3$ is the solution of the given system of equations.

The triangle formed by these lines and the x-axis is ΔPAB whose vertices are P(2, 3), A(–1, 0) and B(4, 0).

Hence, option (*d*) is the correct answer.

Example 9. The coordinates of the vertices of a triangle formed by the equations of sides are: $y = x$; $y = 2x$; $x + y = 6$ are

(*a*) (0, 0), (3, 3), (2, 4)
(*b*) (0, 1), (5, 5), (2, 5)
(*c*) (4, 4), (3, 0), (1, 6)
(*d*) None of these

Solution. We find the table of values for drawing the graph of equations.

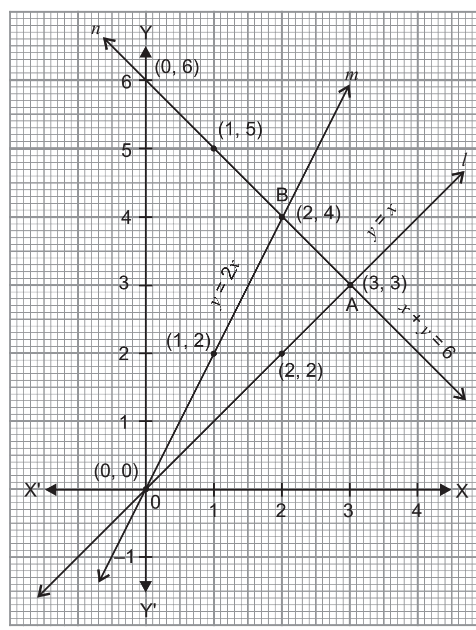
Table of values for $y = x$		
x	0	2
y	0	2

Table of values for $y = 2x$		
x	0	1
y	0	2

Now, we find the table of values for

$$x + y = 6$$
$$\Rightarrow \qquad y = 6 - x$$

Table for $x + y = 6$		
x	0	1
y	6	5



From the graph, we observe that a triangle is formed, namely OAB and the vertices of triangle OAB are O(0, 0), A(3, 3) and B (2, 4).

Hence, option (*a*) is the correct answer.

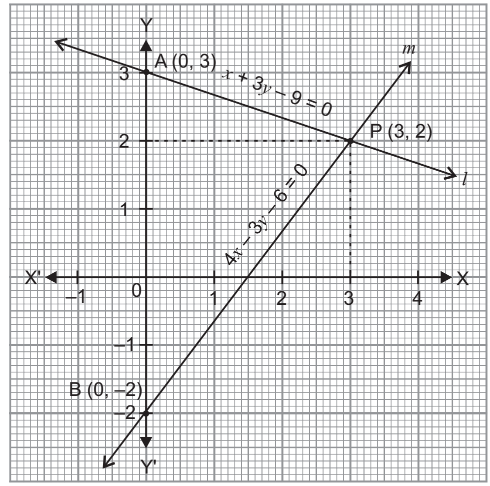
Example 10. The vertices of the triangle formed by the graphs of the equations $4x - 3y - 6 = 0$, $x + 3y - 9$ and y-axis are

(*a*) (1, 3), (2, 4), (–3, 5)
(*b*) (0, 3), (3, 2), (0, –2)
(*c*) (0, 4), (3, 3), (2, 6)
(*d*) None of these

Solution. We first find the table of solutions of each of the equations, (see table).

Table for $4x - 3y - 6 = 0$		
x	3	0
$y = \dfrac{4x-6}{3}$	2	–2

Table for $x + 3y - 9 = 0$		
x	0	3
$y = \dfrac{9-x}{3}$	3	2



Now, we plot the points of both tables in the graph which is shown alongside.

We observe that there is a point (3, 2) common to both the lines l and m, so the solution of linear equations is $x = 3$ and $y = 2$, *i.e.*, the given system of equations is consistent. The vertices of the triangle formed by lines l, m and the y-axis are A(0, 3), P(3, 2) and B(0, –2).

Hence, option (*b*) is the correct answer.

Example 11. The nature of graphs of equations $x + 4y = 3$, $2x + 8y = 6$ and the number of their solutions are

(*a*) Consistent, one
(*b*) Consistent, two
(*c*) Dependent, many
(*d*) Inconsistent, no solution

Solution. We have $x + 4y = 3$ ...(*i*)

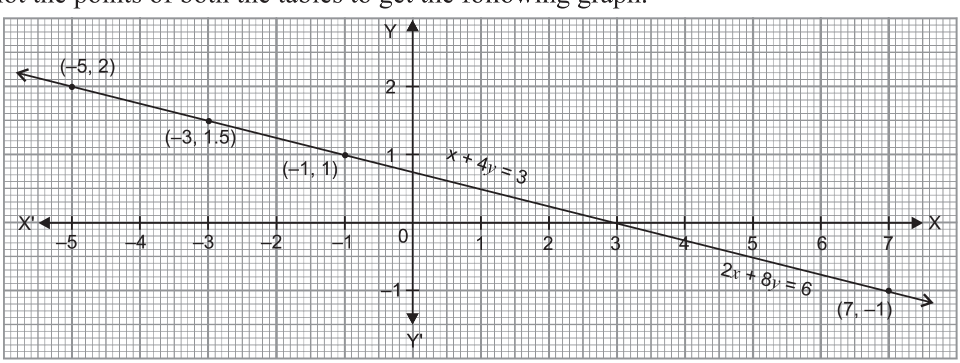
and $2x + 8y = 6$ ...(*ii*)

To draw the graph for given equations, we find the tables of values which are shown below:

Table for $x + 4y = 3$			
x	7	–1	–5
$y = \dfrac{3-x}{4}$	–1	1	2

Table for $2x + 8y = 6 \Rightarrow x + 4y = 3$			
x	–1	–5	–3
$y = \dfrac{3-x}{4}$	1	2	1.5

We plot the points of both the tables to get the following graph.



We observe that graphs of given two equations coincide. So, the given equations are pair of dependent equations and they have infinitely many solutions.

Hence, option (*c*) is the correct answer.

Example 12. The value of k so that the following system of equations has no solution is

$$3x - y - 5 = 0, \; 6x - 2y + k = 0$$ **[Imp.]**

(*a*) 10 (*b*) -10

(*c*) Both 10 and -10 (*d*) All real values of k except -10

Solution. We have $3x - y - 5 = 0$ ...(*i*)

and $6x - 2y + k = 0$ ...(*ii*)

Here, $a_1 = 3, a_2 = 6, b_1 = -1, b_2 = -2, c_1 = -5, c_2 = k$

For no solution, $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\therefore \quad \frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{k} \Rightarrow \frac{1}{2} \neq \frac{-5}{k} \Rightarrow k \neq -10$$

$\therefore$ For all real values of k except -10, the system of equations has no solution.

Hence, option (*d*) is the correct answer.

Example 13. If the pair of equations $x \sin\theta + y\cos\theta = 1$ and $x + y = \sqrt{2}$ has infinitely many solutions, then the value of θ is

(*a*) 30° (*b*) 45° (*c*) 60° (*d*) 90°

Solution. We have $x\sin\theta + y\cos\theta = 1$ ...(*i*)

and $x + y = \sqrt{2}$ ...(*ii*)

Here, $a_1 = \sin\theta, a_2 = 1, b_1 = \cos\theta, b_2 = 1, c_1 = -1, c_2 = -\sqrt{2}$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \sin\theta = \cos\theta = \frac{-1}{-\sqrt{2}} \Rightarrow \theta = 45°$$

Hence, option (*b*) is the correct answer.

Example 14. The number of solutions of the following pair of linear equations is

$$x + 2y - 8 = 0$$
$$2x + 4y = 16$$

(*a*) No solutions (*b*) One solution

(*c*) Two solutions (*d*) Infinitely many solutions

Solution. We have $x + 2y - 8 = 0$

and $2x + 4y = 16$

Here, $a_1 = 1, a_2 = 2, b_1 = 2, b_2 = 4, c_1 = -8, c_2 = -16$

Now, $$\frac{a_1}{a_2} = \frac{1}{2}, \; \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}, \; \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

Since $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$$

$\therefore$ The pair of linear equations is consistent and has infinitely many solutions.

Hence, option (*d*) is the correct answer.

Example 15. The lines represented by the equations $5x - 4y + 8 = 0$, $7x + 6y - 9$ will

(*a*) intersect at a point (*b*) be parallel (*c*) be coincident (*d*) None of these

Solution. We have

$$5x - 4y + 8 = 0 \qquad ...(i)$$

and
$$7x + 6y - 9 = 0 \qquad ...(ii)$$

Here, $a_1 = 5, b_1 = -4, c_1 = 8$ and $a_2 = 7, b_2 = 6, c_2 = -9$

Since, $\frac{a_1}{a_2} = \frac{5}{7}$ and $\frac{b_1}{b_2} = \frac{-4}{6} = -\frac{2}{3}$. So, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Thus, the lines representing the given equations are intersecting. The system of equations is consistent and has a unique solution. Equations are independent.

Hence, option (*a*) is the correct answer.

Example 16. The lines represented by the equations $9x + 3y + 12 = 0$ and $18x + 6y + 24$ will

(*a*) intersect at a point (*b*) be parallel (*c*) be coincident (*d*) None of these

Solution. We have

$$9x + 3y + 12 = 0 \qquad ...(i)$$

and
$$18x + 6y + 24 = 0 \qquad ...(ii)$$

Here, $a_1 = 9, b_1 = 3, c_1 = 12$ and $a_2 = 18, b_2 = 6, c_2 = 24$

Since, $\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$ So, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Thus, the lines representing the given equations are coincident. The system of equations is consistent and the equations are dependent. There are infinitely many solutions.

Hence, option (*c*) is the correct answer.

Example 17. The lines represented by the equations $6x - 3y + 10 = 0$ and $2x - y + 9 = 0$ will

(*a*) intersect at a point (*b*) be parallel (*c*) be coincident (*d*) None of these

Solution. We have

$$6x - 3y + 10 = 0 \qquad ...(i)$$

and
$$2x - y + 9 = 0 \qquad ...(ii)$$

Here, $a_1 = 6, b_1 = -3, c_1 = 10$ and $a_2 = 2, b_2 = -1, c_2 = 9$

Since, $\frac{a_1}{a_2} = \frac{6}{2} = 3, \frac{b_1}{b_2} = \frac{-3}{-1} = 3$ and $\frac{c_1}{c_2} = \frac{10}{9}$. So, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Thus, the lines representing the given equations are parallel. The system of equations is inconsistent and has no solution because the lines never intersect.

Hence, option (*b*) is the correct answer.

Example 18. On comparing $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$, the graphical representation of equations $3x + 2y = 5, 2x - 3y = 7$ will be

(*a*) Intersecting (*b*) Coincident (*c*) Parallel (*d*) None of these

Solution. We have

$$3x + 2y - 5 = 0 \qquad ...(i)$$

and
$$2x - 3y - 7 = 0 \qquad ...(ii)$$

Here, $a_1 = 3, b_1 = 2, c_1 = -5$ and $a_2 = 2, b_2 = -3, c_2 = -7$

Now, $\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2}{-3} = -\frac{2}{3} \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Thus, the lines representing the given equations are intersecting. The system of equations is consistent and has a unique solution. Equations are independent.

Hence, option (*a*) is the correct answer.

Example 19. On comparing $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$, the graphical representation of equations $2x - 3y = 8$ and $4x - 6y - 9 = 0$ will

(*a*) Intersecting lines (*b*) Coincident lines (*c*) Parallel lines (*d*) None of these

Solution. We have

$$2x - 3y - 8 = 0 \quad \text{...}(i)$$

and

$$4x - 6y - 9 = 0 \quad \text{...}(ii)$$

Here, $a_1 = 2, b_1 = -3, c_1 = -8$ and $a_2 = 4, b_2 = -6, c_2 = -9$

Now, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9} \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Thus, the lines representing the given equations are parallel. The system of equations is inconsistent and has no solution.

Hence, option (*c*) is the correct answer.

Exercise 3.1

A. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

1. The pair of linear equations $\frac{3x}{2} + \frac{5y}{3} = 7$ and $9x + 10y = 14$ is
   (*a*) consistent (*b*) inconsistent
   (*c*) consistent with one solution (*d*) consistent with many solutions
2. The value of k for which the system of linear equations $x + 2y = 3, 5x + ky + 7 = 0$ is inconsistent is
   (*a*) $-\frac{14}{3}$ (*b*) $\frac{2}{5}$ (*c*) 5 (*d*) 10
3. If a pair of linear equations is consistent, then the line represented by them are
   (*a*) parallel (*b*) intersecting or coincident
   (*c*) always coincident (*d*) always intersecting
4. The value of k for which the system of equations $kx + 4y = k - 4, 16x + ky = k$ have infinite number of solutions is
   (*a*) $k = 2$ (*b*) $k = 4$ (*c*) $k = 6$ (*d*) $k = 8$
5. The value of k for which the system of linear equations $3x + y = 1, (2k - 1)x + (k - 1)y = 2k + 1$ have no solution is
   (*a*) $k = 2$ (*b*) $k = 4$ (*c*) $k = 6$ (*d*) $k = 8$
6. If the equations $kx - 2y = 3$ and $3x + y = 5$ represent two intersecting lines at unique point, then the value of k is
   (*a*) Only 4 (*b*) Only 5
   (*c*) Only 6 (*d*) Any number other than –6
7. The value of k for which the given system has unique solution
   $$2x + 3y - 5 = 0, \quad kx - 6y - 8 = 0 \text{ is}$$
   (*a*) $k = 2$ (*b*) $k \neq 4$ (*c*) $k = 4$ (*d*) $k \neq 4$

8. For which values of k, the pair of equations $kx + 3y = k - 3$ and $12x + ky = k$ have no solution? **[Imp.]**

(a) $k = 2$ (b) $k = 6$ (c) -6 (d) $k = -2$

9. For what value of k, the following system of equations have infinite solutions: **[Imp.]**

$$2x - 3y = 7, \ (k+2)x - (2k+1)y = 3(2k-1)?$$

(a) $k = 2$ (b) $k = 3$ (c) $k = 4$ (d) $k = 8$

10. The value of p if the lines represented by the equations $3x - y - 5 = 0$ and $6x - 2y - p = 0$ are parallel is **[Imp.]**

(a) only 8 (b) only 10

(c) only 15 (d) All values of 'p' except 10

11. The value of m for which the pair of linear equations $2x + 3y - 7 = 0$ and $(m - 1)x + (m + 1)y = (3m - 1)$ has infinitely many solutions is

(a) 5 (b) 8 (c) -5 (d) 8

12. For what value of k, the pair of linear equations $3x + y = 3$ and $6x + ky = 8$ does not have a solution?

(a) 2 (b) -2 (c) 4 (d) -4

13. For what values of p, the pair of equations $4x + py + 8 = 0$ and $2x + 2y + 2 = 0$ have unique solution?

(a) $p = 4$ (b) $p \neq 4$ (c) $p = 7$ (d) $p \neq 7$

14. What type of straight lines will be represented by the system of equations $2x + 3y = 5$ and $4x + 6y = 7$?

(a) Intersecting (b) Parallel (c) Conincident (d) None of these

15. The values of a and b for which the following pair of linear equations have infinitely many solutions:

$$2x + 3y = 7, \ (a+b)x + (2a-b)y = 21, \text{ respectively are}$$

(a) $a = 5, b = 1$ (b) $a = 2, b = 3$ (c) $a = 4, b = 7$ (d) None of these

16. For what value of p, the following pair of linear equations have infinitely many solutions?

$$(p-3)x + 3y = p, \ px + py = 12$$

(a) 4 (b) 6 (c) 9 (d) 11

17. The value of k for which the following pair of linear equations have infinitely many solutions:

$$2x + 3y = 7, \ (k-1)x + (k+2)y = 3k \text{ is}$$

(a) 2 (b) 4 (c) 7 (d) 9

18. The value(s) of k for which the pair of linear equations $kx + y = k^2$ and $x + ky = 1$ have infinitely many solutions is

(a) 1 (b) 2 (c) 3 (d) 4

19. The value of m and n so that the pair of linear equations $(2m - 1)x + 3y = 5$; $3x + (n - 1)y = 2$ has infinite number of solutions respectively are

(a) $\frac{15}{4}, \frac{13}{8}$ (b) $\frac{17}{4}, \frac{11}{5}$ (c) $\frac{11}{8}, \frac{11}{9}$ (d) None of these

20. For what values of p and q, the following pair of linear equations have infinitely many solutions?

$$4x + 5y = 2; (2p + 7q)x + (p + 8q)y = 2q - p + 1$$

(a) $p = 1, q = 3$ (b) $p = 3, q = 4$ (c) $p = -2, q = 5$ (d) $p = -1, q = 2$

21. The vertices of the triangle formed by the lines, $5x - y = 5$, $x + 2y = 1$ and $6x + y = 17$ are

(a) (1, 0), (3, –1), (2, 5) (b) (2, 3), (5, 6), (3, –1)

(c) (1, 2), (2, 5), (3, 6) (d) None of these

B. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement reason (R). Choose the correct choice as:

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(*c*) Assertion (A) is true but reason (R) is false.

(*d*) Assertion (A) is false but reason (R) is true.

1. **Assertion (A):** Pair of linear equations $x + y = 14$, $x - y = 4$ is consistent.

   Reason (R): By comparing $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$ if we get $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then given system of equations is consistent.

2. **Assertion (A):** For $k = 6$, the system of linear equations $x + 2y + 3 = 0$ and $3x + ky + 6 = 0$ is inconsistent.

   Reason (R): The system of linear equations $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ is inconsistent if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Case Study Based Questions

I. The state governments revise fares from time to time based on various factors such as inflation, fuel price, demand from various quarters, etc. The government notifies different fares for different types of vehicles like Auto Rickshaws, Taxis, Radio Cab, etc.

The auto charges in a city comprise of a fixed charge together with the charge for the distance covered. Study the following situations:

Situation-I: In city A, for a journey of 10 km, the charge paid is ₹ 75 and for a journey of 15 km, the charge paid is ₹ 110.

Situation-II: In city B, for a journey of 8 km, the charge paid is ₹ 91 and for a journey of 14 km, the charge paid is ₹ 145.

Refer Situation I

1. If the fixed charges of auto rickshaw be ₹ x and the running charges be ₹ y km/hr, the pair of linear equations representing the situation is

   (*a*) $x + 10y = 110$, $x + 15y = 75$ (*b*) $x + 10y = 75$, $x + 15y = 110$

   (*c*) $10x + y = 110$, $15x + y = 75$ (*d*) $10x + y = 75$, $15x + y = 110$

2. What will a person have to pay for travelling a distance of 25 km?

   (*a*) ₹ 160 (*b*) ₹ 280 (*c*) ₹ 180 (*d*) ₹ 260

3. A person travels a distance of 50 km. The amount he has to pay is

   (*a*) ₹ 155 (*b*) ₹ 255 (*c*) ₹ 355 (*d*) ₹ 455

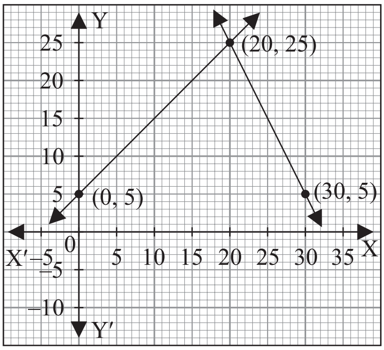
Refer Situation II

4. What will a person have to pay for travelling a distance of 30 km?

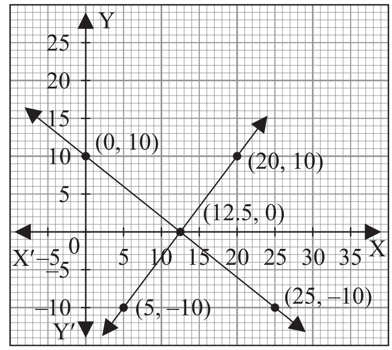
   (*a*) ₹ 185 (*b*) ₹ 289 (*c*) ₹ 275 (*d*) ₹ 305

5. The graphs of lines representing the conditions are

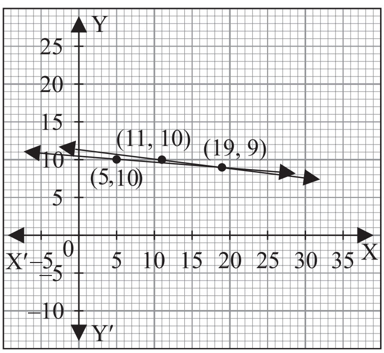
(*a*)



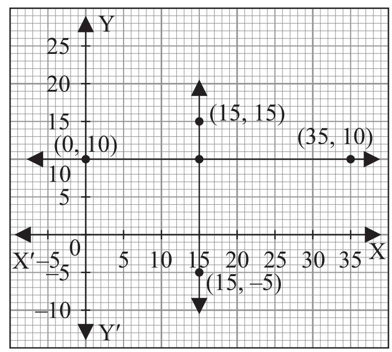
(*b*)



(*c*)



(*d*)



Answers and Hints

A. Multiple Choice Questions (MCQs)

1. (*b*) inconsistent
2. (*d*) 10
3. (*b*) intersecting or coincident
4. (*d*) $k = 8$
5. (*a*) $k = 2$
6. (*d*) any number other than –6
7. (*d*) $k \neq -4$
8. (*c*) –6

   For no solution,

   $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{k}{12} = \frac{3}{k} \neq \frac{-k+3}{-k}$$

   Now, $\frac{k}{12} = \frac{3}{k} \Rightarrow k^2 = 36 \Rightarrow k = -6$

9. (*c*) $k = 4$
10. (*c*) All real values of ‘*p*’ except ‘10’.
11. (*a*) 5

    For infinitely many solutions the condition is

    $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{m-1} = \frac{3}{m+1} = \frac{7}{3m-1}$$

    Now, $2(m+1) = 3(m-1) \Rightarrow m = 5$

    and $3(3m-1) = 7(m+1) \Rightarrow m = 5$

    Hence, for $m = 5$, the system has infinitely many solutions.

12. (*a*) 2

    $$\frac{3}{6} = \frac{1}{k} \neq \frac{3}{8}$$

    $$\frac{3}{6} = \frac{1}{k}$$

    $$k = 2$$

13. (*b*) $p \neq 4$

    $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ is the condition for the given pair of equations to have a unique solution.

    $$\Rightarrow \quad \frac{4}{2} \neq \frac{p}{2}$$

    $$\Rightarrow \quad p \neq 4$$

Therefore, for all real values of p except 4, the given pair of equations will have a unique solution.

14. (*b*) Parallel

$$\text{Here, } \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{5}{7}$$

$$\frac{1}{2} = \frac{1}{2} \neq \frac{5}{7}$$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ is the condition for which the given system of equations will represent parallel lines.

So, the given system of linear equations will represent a pair of parallel lines.

15. (*a*) $a = 5, b = 1$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a+b} = \frac{3}{2a-b} = \frac{7}{21}$$

$$\text{Now } \frac{2}{a+b} = \frac{1}{3} \Rightarrow a+b=6 \quad ...(i)$$

$$\text{Also } \frac{3}{2a-b} = \frac{1}{3} \Rightarrow 2a-b=9 \quad ...(ii)$$

Solving (*i*) and (*ii*), we get, $a = 5$, $b = 1$.

16. (*b*) 6

For infinitely may solutions,

$$\frac{p-3}{p} = \frac{3}{p} = \frac{p}{12} \quad ...(i)$$

$$\text{Now } \frac{p-3}{p} = \frac{3}{p}$$

$$\Rightarrow \quad p-3=3 \Rightarrow p=6$$

$$\text{and } \quad \frac{3}{p} = \frac{p}{12}$$

$$\Rightarrow \quad p^2 = 36 \Rightarrow p = \pm 6$$

$\because$ For $p = 6$, (*i*) is true. So, for $p = 6$, system has infinitely many solutions.

17. (*c*) 7

For infinitely many solutions the condition is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{k-1} = \frac{3}{k+2} = \frac{7}{3k}$$

Now, $2k + 4 = 3k - 3 \Rightarrow k = 7$

and $9k = 7k + 14 \Rightarrow k = 7$

Hence, the value of k is 7.

18. (*a*) 1

For pair of equations $kx + y = k^2$ and $x + ky = 1$

$$\frac{a_1}{a_2} = \frac{k}{1}, \frac{b_1}{b_2} = \frac{1}{k}, \frac{c_1}{c_2} = \frac{k^2}{1}$$

For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\therefore \quad \frac{k}{1} = \frac{1}{k}$$

$$\Rightarrow \quad k^2 = 1 \Rightarrow k = 1, -1 \quad ...(i)$$

$$\text{and} \quad \frac{k}{1} = \frac{k^2}{1}$$

$$\Rightarrow \quad k = 1 \quad ...(ii)$$

From (*i*) and (*ii*), $k = 1$

19. (*b*) $\frac{17}{4}, \frac{11}{5}$

For infinite number of solutions,

$$\frac{2m-1}{3} = \frac{3}{n-1} = \frac{5}{2}$$

$$\text{Now } \frac{2m-1}{3} = \frac{5}{2} \text{ and } \frac{3}{n-1} = \frac{5}{2}$$

$$\Rightarrow \quad 4m - 2 = 15$$

$$\text{and} \quad 6 = 5n - 5$$

$$\Rightarrow \quad 4m = 17$$

$$\text{and} \quad 5n = 11$$

$$\Rightarrow \quad m = \frac{17}{4} \text{ and } n = \frac{11}{5}$$

20. (*d*) $p = -1, q = 2$

For infinitely many solutions,

$$\frac{4}{2p+7q} = \frac{5}{p+8q} = \frac{2}{2q-p+1}$$

$$\text{Now, } \frac{4}{2p+7q} = \frac{5}{p+8q}$$

$$\text{and } \frac{5}{p+8q} = \frac{2}{2q-p+1}$$

$$\Rightarrow \quad 4p + 32q = 10p + 35q$$

and $10q - 5p + 5 = 2p + 16q$

$\Rightarrow \quad 6p + 3q = 0$

and $-5p - 2p + 10q - 16q + 5 = 0$

$\Rightarrow \quad 2p + q = 0 \quad \ldots(i)$

and $\quad 7p + 6q = 5 \quad \ldots(ii)$

On solving the equations (*i*) and (*ii*), we get

$p = -1$

and $\quad q = 2$

Hence, for $p = -1$ and $q = 2$ the given system has infinitely many solutions.

21. (*a*) (1, 0), (3, –1), (2, 5)

B. Assertion-Reason Type Questions

1. (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

2. (*c*) Assertion (A) is true but reason (R) is false.

Case Study Based Questions

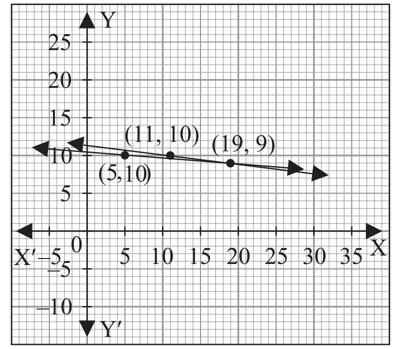
I. 1. (*a*) $x + 10y = 110$, $x + 15y = 75$

2. (*c*) ₹ 180

3. (*c*) ₹ 355

4. (*b*) ₹ 289

5. (*c*)



2. Solving a Pair of Linear Equations by Algebraic Methods

The solution of a pair of linear equations can be obtained by using any one of the following algebraic methods:

(*i*) Substitution Method (*ii*) Elimination Method

A. To find the solution of a pair of linear equations using **Substitution Method**, we follow these steps:

Step 1: From one equation, find the value of one variable (say y) in terms of other variable, *i.e.* x.

Step 2: Substitute the value of variable obtained in step 1 in the other equation to get an equation in one variable.

Step 3: Solve the equation obtained in step 2 to get the value of one variable.

Step 4: Substitute the value of variable so obtained in any given equation to find the value of other variable.

B. To find the solution of a pair of linear equations using **Elimination Method**, we follow these steps:

Step 1: Multiply the given equations to make the coefficient of one of the unknown variables (either x or y), numerically equal.

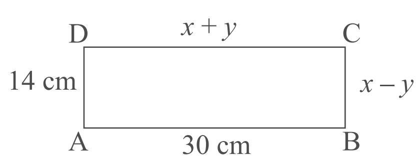
Step 2: Add the equations obtained after multiplication, if the numerically equal coefficients are opposite in signs or else subtract them.

Step 3: Solve the resulting linear equation in one unknown variable to find its value.

Step 4: Substitute this value in any one of the equations and find the value of the other variable.

The values of the two variables constitute the solution of the given pair of linear equations.

Example 1. In figure, ABCD is a rectangle. The values of x and y, respectively are



(*a*) $x = 12, y = 16$ (*b*) $x = 16, y = 10$ (*c*) $x = 22, y = 8$ (*d*) $x = 15, y = 18$

Solution. $$AB = DC \text{ and } BC = AD$$

$\Rightarrow$ $$x + y = 30 \quad ...(i)$$

and $$x - y = 14 \quad ...(ii)$$

We have $x + y = 30 \Rightarrow x = 30 - y$. Substituting $x = 30 - y$ in (ii), we get

$$x - y = 14$$

$$\Rightarrow \quad 30 - y - y = 14$$

$$\Rightarrow \quad 30 - 14 = 2y$$

$$\Rightarrow \quad y = \frac{16}{2} = 8$$

$$\therefore \quad x = 14 + 8 = 22$$

Thus, $$x = 22 \text{ and } y = 8$$

Hence, option (c) is correct answer.

Example 2. The solution of the system of equations $x + y = 5,\ x - y = 2$ using substitution method is:

(a) $x = \frac{7}{2}, y = \frac{3}{2}$ (b) $x = \frac{3}{5}, y = \frac{1}{2}$ (c) $x = \frac{3}{5}, y = \frac{1}{4}$ (d) $x = \frac{2}{5}, y = \frac{5}{2}$

Solution. We have $$x + y = 5 \quad \Rightarrow \quad y = 5 - x$$

Substituting $y = 5 - x$ in $x - y = 2$, we get

$$x - (5 - x) = 2$$

$$\Rightarrow \quad 2x - 5 = 2 \quad \Rightarrow \quad 2x = 7 \quad \Rightarrow \quad x = \frac{7}{2}$$

$$\therefore \quad y = 5 - \frac{7}{2} = \frac{3}{2}$$

Thus, $$x = \frac{7}{2} \quad \text{and} \quad y = \frac{3}{2}$$

Hence, option (a) is correct answer.

Example 3. The solution of given system of equations:

$$x + y = a + b,\ ax - by = a^2 - b^2 \text{ is}$$

(a) $x = 2a, y = b$ (b) $x = a, y = 2b$ (c) $x = a, y = b$ (d) $x = \frac{1}{a}, y = \frac{1}{b}$

Solution. We have

$$x + y = a + b \quad ...(i)$$

and $$ax - by = a^2 - b^2 \quad ...(ii)$$

From (i), we get $$x = a + b - y \quad ...(iii)$$

Substituting this value of x in (ii), we get

$$a(a + b - y) - by = a^2 - b^2$$

$$a^2 + ab - ay - by = a^2 - b^2 \quad \Rightarrow \quad -(a + b)y = a^2 - b^2 - a^2 - ab$$

$$\Rightarrow \quad -(a + b)y = -b^2 - ab \quad \Rightarrow \quad -(a + b)y = -b(b + a)$$

$$\Rightarrow \quad y = \frac{-b(a + b)}{-(a + b)} = b$$

Substituting this value of y in (iii), we get $x = a + b - b = a$

Thus, the solution of the given system is $x = a, y = b$.

Hence, option (c) is correct answer.

Example 4. When $3x + 2y = \frac{11}{3}$ and $-7x + 5y = \frac{31}{3}$ are solved by elimination method, we get

(*a*) $x = \frac{5}{19}, y = \frac{111}{37}$ (*b*) $x = \frac{9}{85}, y = \frac{160}{27}$ (*c*) $x = \frac{-4}{71}, y = \frac{5}{28}$ (*d*) $x = \frac{-7}{87}, y = \frac{170}{87}$

Solution. We have

$$3x + 2y = \frac{11}{3} \Rightarrow 9x + 6y = 11 \quad ...(i)$$

and

$$-7x + 5y = \frac{31}{3} \Rightarrow -21x + 15y = 31 \quad ...(ii)$$

Multiplying equation (*i*) by 7 and equation (*ii*) by 3, we get

$$63x + 42y = 77 \quad ...(iii)$$

$$-63x + 45y = 93 \quad ...(iv)$$

Adding (*iii*) and (*iv*), we get

$$87y = 170 \Rightarrow y = \frac{170}{87}$$

Substituting this value of y in equation (*iii*), we get

$$63x + 42 \times \frac{170}{87} = 77 \Rightarrow 63x + \frac{2380}{29} = 77$$

$$\Rightarrow \quad 63x = 77 - \frac{2380}{29} = \frac{2233 - 2380}{29} = -\frac{147}{29}$$

$$\Rightarrow \quad x = -\frac{147}{29 \times 63} = -\frac{7}{87}$$

Thus, the required solution is $x = -\frac{7}{87}$ and $y = \frac{170}{87}$.

Hence, option (*d*) is correct answer.

Example 5. Solving $3x - 5y - 4 = 0$ and $9x = 2y + 7$ by the elimination method, we get the values of x and y as

(*a*) $x = \frac{9}{13}, y = \frac{-5}{13}$ (*b*) $x = \frac{11}{24}, y = \frac{15}{23}$ (*c*) $x = \frac{17}{25}, y = \frac{16}{9}$ (*d*) None of these

Solution. We have

$$3x - 5y - 4 = 0 \quad ...(i)$$

and

$$9x - 2y - 7 = 0 \quad ...(ii)$$

Multiplying (*i*) by 3, we get

$$9x - 15y - 12 = 0 \quad ...(iii)$$

Subtracting (*iii*) from (*ii*), we get

$$13y + 5 = 0 \Rightarrow y = -\frac{5}{13}$$

Substituting this value of y in (*i*), we get

$$3x - 5\left(-\frac{5}{13}\right) - 4 = 0 \Rightarrow 3x + \frac{25}{13} - 4 = 0$$

$$\Rightarrow \quad 3x - \frac{27}{13} = 0 \Rightarrow x = \frac{9}{13}$$

Thus, the solution is $x = \frac{9}{13}, y = -\frac{5}{13}$.

Hence, option (*a*) is correct answer.

Exercise 3.2

A. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

1. Solution of the simultaneous linear equations: $\frac{2x}{y} - \frac{y}{2} = -\frac{1}{6}$ and $\frac{x}{2} + \frac{2y}{3} = 3$ is

   (*a*) $x = 2, y = -3$ (*b*) $x = -2, y = 3$ (*c*) $x = 2, y = 3$ (*d*) $x = -2, y = -3$

2. The value of x satisfying both the equations $4x - 5 = y$ and $2x - y = 3$, when $y = -1$ is

   (*a*) 1 (*b*) –1 (*c*) 2 (*d*) –2

3. Which of the following is not a solution of the pair of equations $3x - 2y = 4$ and $6x - 4y = 8$?

   (*a*) $x = 2, y = 1$ (*b*) $x = 4, y = 4$ (*c*) $x = 6, y = 7$ (*d*) $x = 5, y = 3$

Answer the questions (Q 4 to Q 13) using best suitable algebric method.

4. If $2x + 5y - 1 = 0$, $2x + 3y - 3 = 0$, then

   (*a*) $x = 1, y = -3$ (*b*) $x = 3, y = -1$ (*c*) $x = 2, y = 5$ (*d*) $x = 5, y = -3$

5. If $x + 2y - 3 = 0$, $3x - 2y + 7 = 0$, then

   (*a*) $x = -1, y = 2$ (*b*) $x = 1, y = 2$ (*c*) $x = 2, y = 3$ (*d*) $x = -2, y = -3$

6. If $2x = 5y + 4$, $3x - 2y + 16 = 0$, then

   (*a*) $x = 2, y = -2$ (*b*) $x = 3, y = -3$ (*c*) $x = 4, y = 5$ (*d*) $x = -8, y = -4$

7. If $6(ax + by) = 3a + 2b$; $6(bx - ay) = 3b - 2a$, then

   (*a*) $x = \frac{1}{2}, y = \frac{1}{2}$ (*b*) $x = -\frac{1}{2}, y = -\frac{1}{2}$ (*c*) $x = \frac{1}{2}, y = \frac{1}{3}$ (*d*) $x = -\frac{1}{2}, y = -\frac{1}{3}$

8. If $\frac{4}{x} + 3y = 8$; $\frac{6}{x} - 4y = -5$, then

   (*a*) $x = 2, y = 2$ (*b*) $x = 1, y = -1$ (*c*) $x = 2, y = -2$ (*d*) $x = 3, y = -3$

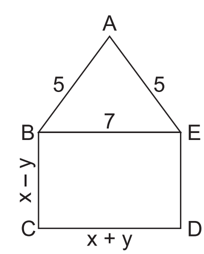
9. If $2x + 3y = 11$ and $2x - 4y = -24$, then the value of 'm' for which $y = mx + 3$ is

   (*a*) 0 (*b*) 1 (*c*) –1 (*d*) – 2

10. If $2x + 3y = 11$ and $x - 2y = -12$, then the value of 'm' for which $y = mx + 3$ is

    (*a*) 1 (*b*) –1 (*c*) 2 (*d*) – 2

11. In the figure, ABCDE is a pentagon with BE ∵ CD and BC ∵ DE. BC is perpendicular to CD. AB = 5 cm, AE = 5 cm, BE = 7 cm, BC = $x - y$ and CD = $x + y$. If the perimeter of ABCDE is 27 cm, the value of x and y, given $x, y \neq 0$, respectively are

    (*a*) $x = 6, y = 1$ (*b*) $x = 3, y = 4$

    (*c*) $x = 2, y = 5$ (*d*) $x = 0, y = 2$

12. The value of x and y for the following system of equations, respectively are

$$\frac{21}{x} + \frac{47}{y} = 110$$

$$\frac{47}{x} + \frac{21}{y} = 162 \quad \Rightarrow \quad x, y \neq 0$$

    (*a*) $x = \frac{1}{2}, y = -1$ (*b*) $x = \frac{1}{3}, y = 1$ (*c*) $x = \frac{1}{4}, y = 2$ (*d*) $x = \frac{1}{3}, y = \frac{1}{4}$

13. If $2x + y = 23$ and $4x - y = 19$, then the value of $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$, respectively are

(*a*) $31, \frac{-5}{7}$ (*b*) $28, \frac{3}{11}$ (*c*) $24, \frac{5}{8}$ (*d*) $10, \frac{17}{21}$

Answer the questions (Q 14 to Q 16) by elimination method.

14. If $\frac{x}{2} + y = 0.8$; $\frac{7}{x + \frac{y}{2}} = 10$, then

(*a*) $x = 2, y = 0.5$ (*b*) $x = 0.4, y = 0.6$ (*c*) $x = 0.3, y = 3$ (*d*) $x = 0.5, y = 0.8$

15. If $7(y + 3) - 2(x + 2) = 14$, $4(y - 2) + 3(x - 3) = 2$, then

(*a*) $x = 1, y = 4$ (*b*) $x = 3, y = 5$ (*c*) $x = 5, y = 1$ (*d*) None of these

16. If $\frac{4}{x} + 5y = 7$; $\frac{3}{x} + 4y = 5$, then

(*a*) $x = \frac{1}{3}, y = -1$ (*b*) $x = 8, y = 3$ (*c*) $x = 4, y = 7$ (*d*) $x = 5, y = 9$

Answer the questions (Q 17 and Q 18) by the method of substitution.

17. If $3 - (x - 5) = y + 2$, $2(x + y) = 4 - 3y$, then

(*a*) $x = \frac{13}{4}, y = \frac{9}{10}$ (*b*) $x = \frac{7}{16}, y = \frac{5}{8}$ (*c*) $x = \frac{4}{9}, y = \frac{9}{12}$ (*d*) $x = \frac{26}{3}, y = \frac{-8}{3}$

18. If $2x + y = 35$ and $3x + 2y = 65$, the value of $\frac{x}{y}$ is

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{1}{5}$

Answers and Hints

A. Multiple Choice Questions (MCQs)

1. (*c*) $x = 2, y = 3$

2. (*a*) 1

3. (*d*) $x = 5, y = 3$

4. (*b*) $x = 3, y = -1$

5. (*a*) $x = -1$ and $y = 2$.

$$x + 2y = 3 \quad ...(i)$$
$$3x - 2y = -7 \quad ...(ii)$$

On adding the equations (*i*) and (*ii*), we get

$$4x = -4$$
$$\Rightarrow \quad x = -1$$

Putting $x = -1$ in eq. (*i*), we get

$$-1 + 2y = 3$$
$$\Rightarrow \quad 2y = 4 \quad \Rightarrow \quad \boxed{y = 2}$$

Hence, solution of the system is $x = -1$ and $y = 2$.

6. (*d*) $x = -8, y = -4$

7. (*c*) $x = \frac{1}{2}, y = \frac{1}{3}$

$$6ax + 6by = 3a + 2b \quad ...(i)$$
$$6bx - 6ay = -2a + 3b \quad ...(ii)$$

Multiplying eq. (*i*) by '*a*' and eq. (*ii*) by '*b*' and then adding, we get

$$6a^2x + 6aby = 3a^2 + 2ab \quad ...(iii)$$
$$6b^2x - 6aby = -2ab + 3b^2 \quad ...(iv)$$
$$6(a^2 + b^2)x = 3(a^2 + b^2)$$
$$\Rightarrow \quad x = \frac{1}{2}$$

Putting $x = \frac{1}{2}$ in eq. (i), we get

$$6a \times \frac{1}{2} + 6by = 3a + 2b \Rightarrow 6by = 2b \Rightarrow y = \frac{1}{3}$$

Hence, solution of the system is $x = \frac{1}{2}$ and $y = \frac{1}{3}$

8. (a) $x = 2,\ y = 2$

9. (c) –1

$$2x + 3y = 11 \quad ...(i)$$
$$2x - 4y = -24 \quad ...(ii)$$

From (i), $\quad x = \left[\frac{11 - 3y}{2}\right] \quad ...(iii)$

Substituting this value of x in (ii), we get

$$2\left[\frac{11-3y}{2}\right] - 4y = -24$$

$\Rightarrow \quad 11 - 7y = -24 \Rightarrow y = 5 \quad (1)$

Substituting $y = 5$ in (iii), we get

$$x = \frac{11 - 3(5)}{2}$$

$\Rightarrow \quad x = -2$

$\therefore \quad x = -2$ and $y = 5$

Now, $\quad y = mx + 3$

$\Rightarrow \quad 5 = m(-2) + 3$

Thus, $\quad m = -1$

10. (b) –1

11. (a) $x = 6, y = 1$

$x + y = 7$ and $2(x - y) + x + y + 5 + 5 = 27$

$\therefore \quad x + y = 7$ and $3x - y = 17$

Solving, we get, $x = 6$ and $y = 1$

12. (b) $x = \frac{1}{3}, y = 1$

Let $\quad \frac{1}{x} = a$ and $\frac{1}{y} = b$

$\Rightarrow 21a + 47b = 110$ and $47a + 21b = 162$

Adding and subtracting the two equations, we get

$$a + b = 4 \text{ and } a - b = 2$$

Solving the above two equations, we get $a = 3$ and $b = 1$

$\therefore \quad x = \frac{1}{3}$ and $y = 1$

13. (a) $31, -\frac{5}{7}$

Given equation are

$$2x + y = 23 \quad ...(i)$$

and $\quad 4x - y = 19 \quad ...(ii)$

On adding both equations, we get

$$6x = 42 \quad \Rightarrow x = 7$$

Putting the value of x in (i), we get

$$2 \times 7 + y = 23 \Rightarrow 14 + y = 23$$

$\Rightarrow \quad y = 23 - 14 \Rightarrow y = 9$

We have $\quad 5y - 2x = 5 \times 9 - 2 \times 7 = 45 - 14 = 31$

and $\quad \frac{y}{x} - 2 = \frac{9}{7} - 2 = \frac{9 - 14}{7} = -\frac{5}{7}$

Hence the value of $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$ are 31 and $-\frac{5}{7}$ respectively.

14. (b) $x = 0.4,\ y = 0.6$

$$\frac{x}{2} + y = 0.8$$

$\Rightarrow \quad x + 2y = \frac{16}{10}$

$\Rightarrow \quad 10x + 20y = 16 \quad ...(i)$

Also $\quad \frac{7}{x + \frac{y}{2}} = 10$

$\Rightarrow \quad 7 = 10x + 5y$

$\Rightarrow \quad 10x + 5y = 7 \quad ...(ii)$

Solving (i) and (ii) by elimination method,

$$15y = 9$$

$\Rightarrow \quad y = \frac{9}{15} = \frac{3}{5} = 0.6$

From (ii), $\quad 10x + 3 = 7$

$\Rightarrow \quad x = \frac{4}{10} = 0.4$

$\therefore \quad x = 0.4$ and $y = 0.6$

15. (c) $x = 5, y = 1$

$$7(y + 3) - 2(x + 2) = 14$$

$\Rightarrow \quad -2x + 7y + 3 = 0 \quad ...(i)$

and $4(y - 2) + 3(x - 3) = 2$

$\Rightarrow \quad 3x + 4y - 19 = 0 \quad ...(ii)$

Solving (i) and (ii) by elemination method,

$$x = 5, y = 1$$

16. (a) $x = \frac{1}{3}$, $y = -1$

17. (d) $x = \frac{26}{3}$, $y = \frac{-8}{3}$

$$3 - (x - 5) = y + 2$$

$$\Rightarrow \quad y + 2 - 8 + x = 0$$

$$\Rightarrow \quad x + y - 6 = 0 \quad ...(i)$$

$$2(x + y) = 4 - 3y$$

$$\Rightarrow \quad 2x + 2y - 4 + 3y = 0$$

$$\Rightarrow \quad 2x + 5y - 4 = 0 \quad ...(ii)$$

Solving by elimination method,

$$x = \frac{26}{3} \text{ and } y = \frac{-8}{3}$$

18. (d) $\frac{1}{5}$

$$2x + y = 35 \quad ...(i)$$

$$3x + 2y = 65 \quad ...(ii)$$

Using elimination method,

$$6x + 3y = 105$$

$$6x + 4y = 130$$

$$- \quad - \quad -$$

$$-y = -25$$

$$\Rightarrow \quad y = 25$$

From (i), $\quad 2x + 25 = 35$

$$\Rightarrow \quad x = \frac{35 - 25}{2} = 5$$

So $\quad x = 5$ and $y = 25$

Thus the value of $\frac{x}{y} = \frac{1}{5}$

3. Solving a System of Equations Reducible to Simultaneous Linear Equations

Now, we shall learn the method to find the solutions of such pairs of equations which are not linear but can be reduced to linear form by making some suitable substitutions. Consider following examples.

Example 1. When $\frac{1}{2x} + \frac{1}{3y} = 2$ and $\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$ are solved by reducing them to a pair of linear equations, we get

(a) $x = \frac{1}{2}, y = \frac{1}{7}$ (b) $x = \frac{1}{4}, y = \frac{1}{5}$ (c) $x = \frac{1}{5}, y = \frac{1}{4}$ (d) $x = \frac{1}{5}, y = \frac{1}{7}$

Solution. We have $\frac{1}{2x} + \frac{1}{3y} = 2$ ...(i) and $\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$ ...(ii)

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$, then the given equations (i) and (ii) become

$$\frac{u}{2} + \frac{v}{3} = 2 \quad \Rightarrow \quad 3u + 2v = 12 \quad ...(iii)$$

and

$$\frac{u}{3} + \frac{v}{2} = \frac{13}{6} \quad \Rightarrow \quad 2u + 3v = 13 \quad ...(iv)$$

Solving (iii) and (iv), we get

$$x = \frac{1}{2}, y = \frac{1}{3}$$

Hence, option (a) is the correct answer.

Example 2. The value of x and y for the following system of equations:

$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \quad \text{and} \quad \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1, \text{ respectively are}$$

(a) $x = 4, y = 9$ (b) $x = 5, y = 10$ (c) $x = 8, y = -9$ (d) None of these

Solution. We have $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$ ...(*i*) and $\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$ ...(*ii*)

Let $\frac{1}{\sqrt{x}}=u$ and $\frac{1}{\sqrt{y}}=v$, then equations (*i*) and (*ii*) become

$$2u+3v=2 \quad ...(iii)$$

and $$4u-9v=-1 \quad ...(iv)$$

Multiplying (*iii*) by 2, we get

$$4u+6v=4 \quad ...(v)$$

Subtracting (*iv*) from (*v*), we get

$$15v=5 \Rightarrow v=\frac{5}{15}=\frac{1}{3}$$

From (*iii*), we get $$2u+3\left(\frac{1}{3}\right)=2 \Rightarrow 2u=1 \Rightarrow u=\frac{1}{2}$$

Now, $$u=\frac{1}{2} \Rightarrow \frac{1}{\sqrt{x}}=\frac{1}{2} \Rightarrow \sqrt{x}=2 \Rightarrow x=4$$

and $$v=\frac{1}{3} \Rightarrow \frac{1}{\sqrt{y}}=\frac{1}{3} \Rightarrow \sqrt{y}=3 \Rightarrow y=9$$

Thus, the solution is $x=4$, $y=9$.

Hence, option (*a*) is the correct answer

Example 3. Given the following system of equations:

$$\frac{10}{x+y}+\frac{2}{x-y}=4 \text{ and } \frac{15}{x+y}-\frac{5}{x-y}=-2$$

Then which of the following is true?

(*a*) $x=2, y=5$ (*b*) $x=1, y=2$ (*c*) $x=3, y=2$ (*d*) None of these

Solution. We have $\frac{10}{x+y}+\frac{2}{x-y}=4$ ...(*i*) and $\frac{15}{x+y}-\frac{5}{x-y}=-2$ ...(*ii*)

Let $\frac{1}{x+y}=u$ and $\frac{1}{x-y}=v$, then equations (*i*) and (*ii*) become

$$10u+2v=4 \quad ...(iii)$$

and $$15u-5v=-2 \quad ...(iv)$$

Solving (*iii*) and (*iv*), we get $x=3$, $y=2$.

Hence, option (*c*) is the correct option

Example 4. If $\frac{5}{x-1}+\frac{1}{y-2}=2$ and $\frac{6}{x-1}-\frac{3}{y-2}=1$

(*a*) $x=1, y=2$ (*b*) $x=2, y=3$ (*c*) $x=4, y=5$ (*d*) $x=5, y=7$

Solution. We have $\frac{5}{x-1}+\frac{1}{y-2}=2$ ...(*i*) and $\frac{6}{x-1}-\frac{3}{y-2}=1$ ...(*ii*)

Let $\frac{1}{x-1}=u$ and $\frac{1}{y-2}=v$, the equations (*i*) and (*ii*) become

$$5u+v=2 \quad ...(iii)$$

and $$6u-3v=1 \quad ...(iv)$$

Solving (*iii*) and (*iv*), we get $x=4$, $y=5$

Hence, option (*c*) is the correct answer.

Exercise 3.3

A. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

1. If $\frac{2}{x}+\frac{3}{y}=13$ and $\frac{5}{x}-\frac{4}{y}=-2$, then $x+y$ equals

(*a*) $\frac{1}{6}$ (*b*) $\frac{-1}{6}$ (*c*) $\frac{5}{6}$ (*d*) $\frac{-5}{6}$

2. If $\frac{2}{x}+2y=15$ and $\frac{2}{x}-4y=3$, then the values of x and y, respectively are

(*a*) $\frac{2}{11}, 2$ (*b*) $3, \frac{1}{3}$ (*c*) $4, \frac{1}{4}$ (*d*) $\frac{1}{4}, 4$

3. If $\frac{1}{x}=u$ and $\frac{1}{y}=v$, then $\frac{2}{x}+\frac{3}{y}=13$ becomes

(*a*) $3u+2v=13$ (*b*) $2u+3v=13$ (*c*) $\frac{2}{u}+\frac{3}{v}=13$ (*d*) $\frac{2}{v}+\frac{3}{u}=13$

4. Equation $\frac{5}{x-1}+\frac{1}{y-2}=2$ is reduced in linear equation as $5p+q=2$. Then the values of p and q respectively are

(*a*) $\frac{1}{x-1}, \frac{1}{y-2}$ (*b*) $\frac{1}{y-1}, \frac{1}{x-2}$ (*c*) $x-1, y-2$ (*d*) $x+1, y+2$

5. If $\frac{2}{x}+\frac{3}{y}=13$ and $\frac{5}{x}-\frac{4}{y}=-2$, then the values of x and y respectively are

(*a*) $\frac{1}{2}, \frac{1}{3}$ (*b*) $\frac{1}{3}, \frac{1}{4}$ (*c*) $\frac{1}{2}, \frac{1}{5}$ (*d*) $2, 3$

6. $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$ and $\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$, where $3x+y\neq 0$, $3x-y\neq 0$, then

(*a*) 1, 2 (*b*) 1, 1 (*c*) 1, 3 (*d*) 1, 4

7. If $\frac{3a}{x}-\frac{2b}{y}+5=0$ and $\frac{a}{x}+\frac{3b}{y}-2=0$, then

(*a*) $x=\frac{1}{a}$, $y=\frac{1}{b}$ (*b*) $x=\frac{1}{b}$, $y=\frac{1}{a}$ (*c*) $x-a, y=b$ (*d*) $x=a, y=-b$

8. If $\frac{1}{2x}-\frac{1}{y}=-1$ and $\frac{1}{x}+\frac{1}{2y}=8$, $(x\neq 0, y\neq 0)$, then

(*a*) $x=\frac{1}{4}, y=\frac{1}{2}$ (*b*) $x=\frac{1}{3}, y=\frac{1}{5}$ (*c*) $x=\frac{1}{6}, y=\frac{1}{8}$ (*d*) $x=\frac{1}{6}, y=\frac{1}{4}$

9. When the following pair of equations is solved by reducing them to a pair of linear equations, we get

$$\frac{1}{x} - \frac{4}{y} = 2 \text{ and } \frac{1}{x} + \frac{3}{y} = 9$$

(*a*) $x = \frac{1}{6}, y = 1$ (*b*) $x = 1, y = \frac{1}{5}$ (*c*) $x = \frac{1}{3}, y = 5$ (*d*) $x = \frac{1}{4}, y = \frac{1}{3}$

10. If $\frac{44}{x+y} + \frac{30}{x-y} = 10$ and $\frac{55}{x+y} + \frac{40}{x-y} = 13$, then

(*a*) $x = 7, y = 7$ (*b*) $x = 2, y = 3$ (*c*) $x = 5, y = 2$ (*d*) $x = 8, y = 3$

11. If $\frac{57}{x+y} + \frac{6}{x-y} = 5$ and $\frac{38}{x+y} + \frac{21}{x-y} = 9$, then

(*a*) $x = 5, y = 3$ (*b*) $x = 11, y = 8$ (*c*) $x = 9, y = -9$ (*d*) $x = 3, y = 4$

12. If $\frac{7x - 2y}{xy} = 5$ and $\frac{8x + 7y}{xy} = 15$, then

(*a*) $x = 1, y = 1$ (*b*) $x = 2, y = 2$ (*c*) $x = 3, y = 4$ (*d*) $x = 2, y = 1$

Answers and Hints

A. Multiple Choice Questions (MCQs)

1. (*c*) $\frac{5}{6}$

2. (*a*) $\frac{2}{11}, 2$

3. (*b*) $2u + 3v = 13$

4. (*a*) $\frac{1}{x-1}, \frac{1}{y-2}$

5. (*a*) $\frac{1}{2}, \frac{1}{3}$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$. Then the given equation become

$$2p + 3q = 13 \quad \text{...}(i)$$

and

$$5p - 4q = -2 \quad \text{...}(ii)$$

Multiplying equation (*i*) by 5 and equation (*ii*) by 2, we get

$$10p + 15q = 65 \quad \text{...}(iii)$$

$$10p - 8q = -4$$

$$- \quad + \quad -$$

$$23q = 69 \qquad q = 3$$

From (*i*), $2p + 9 = 13 \qquad p = 2$

But $p = \frac{1}{x} \qquad \frac{1}{x} = 2 \qquad x = \frac{1}{2}$

and $q = \frac{1}{y} \qquad \frac{1}{y} = 3 \qquad y = \frac{1}{3}$

6. (*b*) 1, 1

$$\frac{1}{(3x+y)} + \frac{1}{(3x-y)} = \frac{3}{4} \quad \text{...}(i)$$

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8} \quad \text{...}(ii)$$

Let $\frac{1}{(3x+y)} = p$ and $\frac{1}{(3x-y)} = q$

$\therefore$ (*i*) and (*ii*) can be expressed as

$$p + q = \frac{3}{4} \quad \text{...}(iii)$$

$$\frac{p}{2} - \frac{q}{2} = -\frac{1}{8} \quad \text{...}(iv)$$

Multiplying equation (*iii*) by $\frac{1}{2}$ and adding it to (*iv*), we get

$$\frac{p}{2} + \frac{q}{2} = \frac{3}{8}$$

$$\frac{p}{2} - \frac{q}{2} = -\frac{1}{8}$$

$$\left(\frac{p}{2} + \frac{p}{2}\right) = \left(\frac{3}{8} - \frac{1}{8}\right) \Rightarrow p = \frac{2}{8} = \frac{1}{4}$$

From (*iii*),

$$\frac{1}{4}+q = \frac{3}{4}$$

$$\Rightarrow \quad q = \frac{3}{4}-\frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

But $p = \dfrac{1}{3x+y} \Rightarrow \dfrac{1}{3x+y} = \dfrac{1}{4}$

$$\Rightarrow \quad 3x+y = 4 \qquad ...(v)$$

and $q = \dfrac{1}{3x-y} \Rightarrow \dfrac{1}{3x-y} = \dfrac{1}{2}$

$$\Rightarrow \quad 3x-y = 2 \qquad ...(vi)$$

Adding (*v*) and (*vi*)

$$\begin{array}{r} 3x+y=4 \\ 3x-y=2 \\ \hline 6x = 6 \end{array} \Rightarrow x = \frac{6}{6} = 1$$

Subtracting (*vi*) from (*v*)

$$\begin{array}{r} 3x+y=4 \\ 3x-y=2 \\ (-)\ \ (+)\ \ (-) \\ \hline 2y = 2 \end{array} \Rightarrow y = \frac{2}{2} = 1$$

Thus, the required solution is: $\begin{cases} x = 1 \\ y = 1 \end{cases}$.

7. (*c*) $x = -a, y = b$

8. (*d*) $x = \frac{1}{6}, y = \frac{1}{4}$

Given equations are

$$\frac{1}{2x}-\frac{1}{y} = -1 \qquad ...(i)$$

$$\frac{1}{x}+\frac{1}{2y} = 8 \qquad ...(ii)$$

Multiplying eqn. (*i*) by $\frac{1}{2}$ and then adding the result to (*ii*), we get

$$\begin{array}{l} \dfrac{1}{4x} - \dfrac{1}{2y} = -\dfrac{1}{2} \\ \dfrac{1}{x} + \dfrac{1}{2y} = 8 \\ \hline \dfrac{1}{4x} + \dfrac{1}{x} = 8 - \dfrac{1}{2} \end{array}$$

$$\Rightarrow \quad \frac{1+4}{4x} = \frac{16-1}{2}$$

$$\Rightarrow \quad \frac{5}{4x} = \frac{15}{2}$$

$$\Rightarrow \quad x = \frac{1}{6}$$

Now, $\dfrac{1}{x}+\dfrac{1}{2y} = 8 \qquad$ [From (*ii*)]

$$\Rightarrow \quad \frac{1}{\left(\frac{1}{6}\right)}+\frac{1}{2y} = 8 \qquad \left[\because x = \frac{1}{6}\right]$$

$$\Rightarrow \quad 6+\frac{1}{2y} = 8 \Rightarrow \frac{1}{2y} = 2$$

$$\Rightarrow \quad y = \frac{1}{4}$$

So, $x = \frac{1}{6}$ and $y = \frac{1}{4}$

9. (*a*) $x = \frac{1}{6}, y = 1$

Given system is

$$\frac{1}{x}-\frac{4}{y} = 2 \qquad ...(i)$$

$$\frac{1}{x}+\frac{3}{y} = 9 \qquad ...(ii)$$

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$ then the system becomes,

$$a-4b = 2 \qquad ...(iii)$$

$$a+3b = 9 \qquad ...(iv)$$

On multiplying equation (*iii*) by 3 and equation (*iv*) by 4 and then adding, we get

$$\begin{array}{r} 3a-12b = 6 \\ 4a+12b = 36 \\ \hline 7a = 42 \end{array} \Rightarrow a = 6$$

Putting $a = 6$ in equation (*iii*), we get

$$6-4b = 2 \Rightarrow 4b = 4 \Rightarrow b = 1$$

Thus $\quad a = 6$ and $b = 1$

$$\Rightarrow \quad \frac{1}{x} = 6 \text{ and } \frac{1}{y} = 1$$

$$\Rightarrow \quad x = \frac{1}{6} \text{ and } y = 1$$

10. (*d*) $x = 8, y = 3$

11. (*b*) $x = 11, y = 8$

12. (*a*) $x = 1, y = 1$

4. Solving Word Problems on Pair of Linear Equations in Two Variables

To solve the word problems, we should consider the following points:

(*i*) Reduce the given situation in terms of x and y as mathematical statements.

(*ii*) Solve the linear equations thus formed using any algebraic method.

Example 1. Read the following problem.

Ritu can row downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

The pair of linear equations representing the above situation is

(*a*) $x + y = 10, x + y = 2$ (*b*) $x + y = 10, x - y = 2$

(*c*) $x - y = 10, x + y = 2$ (*d*) $x - y = 10, x - y = 2$

Solution. Let her speed of rowing in still water be x km/h.

and Speed of the current = y km/h.

$\therefore$ Upstream speed = $(x - y)$ km/h

and Downstream speed = $(x + y)$ km/h

Since $$\frac{\text{Distance}}{\text{Speed}} = \text{Time}$$

$\therefore$ $$\frac{20}{x+y} = 2 \quad \Rightarrow \quad x + y = 10 \qquad ...(i)$$

and $$\frac{4}{x-y} = 2 \quad \Rightarrow \quad x - y = 2 \qquad ...(ii)$$

Hence, option (*b*) is the correct answer.

Example 2. A part of monthly hostel charges in a college is fixed and the remaining depends on the number of days one has taken food in the mess. When a student 'A' takes food for 22 days, he has to pay ₹ 1380 as hostel charges; whereas a student 'B', who takes food for 28 days, pays ₹ 1680 as hostel charges. The fixed charges and the cost of food per day respectively are

(*a*) ₹280, ₹50 (*b*) ₹240, ₹60 (*c*) ₹290, ₹100 (*d*) ₹170, ₹210

Solution. Let the fixed hostel charges be ₹ x and the cost of food per day be ₹ y.

According to the question, we get

$$x + 22y = 1380 \qquad ...(i)$$

and $$x + 28y = 1680 \qquad ...(ii)$$

Subtracting (*i*) from (*ii*), we get

$$6y = 300 \quad \Rightarrow \quad y = 300 \div 6 = 50$$

Putting $y = 50$ in (*i*), we get

$$x + 22(50) = 1380 \quad \Rightarrow \quad x + 1100 = 1380 \Rightarrow x = 280$$

$\therefore$ Fixed hostel charges = ₹ 280 and cost of the food per day = ₹ 50.

Hence, option (*a*) is the correct answer.

Example 3. Atul sold a television set and a mobile phone for ₹ 10500, thereby making a profit of 10% on the television set and 25% on the mobile phone. If he had taken a profit of 25% on the television set and 10% on the mobile phone, he would have got ₹ 10650. The cost of each item is

(*a*) TV : ₹7000, Mobile : ₹5000 (*b*) TV : ₹6000, Mobile : ₹4000

(*c*) TV : ₹4000, Mobile : ₹5000 (*d*) TV : ₹5000, Mobile : ₹4000

Solution. Let the cost price of a television set be ₹ x and the cost price of the mobile phone be ₹ y.

The selling price of the television set when it is sold at a profit of 10%

$$= ₹\left(x + \frac{10}{100}x\right) = ₹\frac{110}{100}x$$

The S.P. of the mobile phone when it is sold at a profit of 25%.

$$= ₹\left(y + \frac{25}{100}y\right) = ₹\frac{125}{100}y$$

So, $$\frac{110x}{100} + \frac{125y}{100} = 10500 \quad ...(i)$$

Similarly, $$\frac{125x}{100} + \frac{110y}{100} = 10650 \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$110x + 125y = 1050000 \quad ...(iii)$$

and $$125x + 110y = 1065000 \quad ...(iv)$$

On adding equations (*iii*) and (*iv*), we get

$$235x + 235y = 2115000$$

$$\Rightarrow \quad x + y = 9000$$

$$\Rightarrow \quad x = 9000 - y$$

Putting this value of x in (*i*), we get

$$\frac{110\,(9000 - y)}{100} + \frac{125\,y}{100} = 10500$$

$$\Rightarrow \quad 990000 - 110y + 125y = 1050000$$

$$\Rightarrow \quad 15y = 60000$$

$$\therefore \quad y = 4000$$

and $$x = 9000 - 4000$$

$$= 5000$$

So, the cost price of the television set is ₹ 5000 and cost price of the mobile phone is ₹ 4000.

Hence, option (*d*) is the correct answer.

Example 4. Base of an isosceles triangle is $\frac{2}{3}$ times its congruent sides. Perimeter of the triangle is 32 cm. The length of each side of that triangle is

(*a*) 8 cm, 8 cm, 6 cm (*b*) 12 cm, 12 cm, 10 cm

(*c*) 10 cm, 10 cm, 12 cm (*d*) None of these

Solution. Let each of the congruent sides of an isosceles triangle be x cm long and base be y cm long.

According to the first condition,

$$y = \frac{2}{3}x \Rightarrow 2x - 3y = 0 \qquad ...(i)$$

According to the second condition,

$$x + x + y = 32 \quad \Rightarrow \quad 2x + y = 32 \qquad ...(ii)$$

Subtracting (i) from (ii), we get

$$4y = 32 \quad \Rightarrow \quad y = 32 \div 4 = 8$$

Putting $y = 8$ in equation (i), we get

$$2x - 3(8) = 0$$

$\Rightarrow$ $$2x = 24 \quad \Rightarrow \quad x = 24 \div 2 = 12$$

$\therefore$ Each of the congruent sides is 12 cm long and base 8 cm long.

Hence, option (b) is the correct answer.

Example 5. Places A and B are 80 km apart from each other on a highway. A car starts from A and another from B at the same time. If they move in same direction they meet in 8 hrs and if they move in opposite directions they meet in 1 hr 20 minutes.

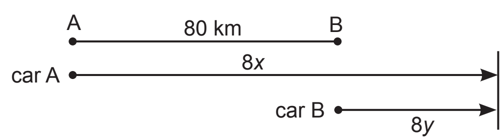
The speeds of cars started from places A and B respectively are

(a) 40 km/hr, 55 km/hr (b) 20 km/hr, 30 km/hr

(c) 35 km/hr, 25 km/hr (d) 30 km/hr, 28 km/hr

Solution. Let the speed of car starts from A or car A be x km/hr and the speed of car starts from B or car B be y km/hr

Case I:



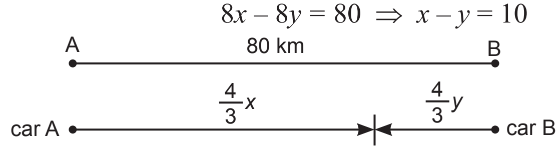
After 8 hours,

distance covered by car A = $8x$ $\left[\text{as speed} = \dfrac{\text{distance}}{\text{time}}\right]$

and distance covered by car B = $8y$

So, $$8x - 8y = 80 \Rightarrow x - y = 10 \qquad ...(i)$$

Case II:



After 1 hr 20 minutes, i.e. $\frac{4}{3}$ hrs, distance covered by car A = $\frac{4}{3}x$

and distance covered by car B = $\frac{4}{3}y$

So, $$\frac{4}{3}x + \frac{4}{3}y = 80 \quad \Rightarrow x + y = 60 \qquad ...(ii)$$

On solving equations (i) and (ii), we get $x = 35$ and $y = 25$

Thus, speed of car A = 35 km/hr and speed of car B = 25 km/hr.

Hence, option (c) is the correct answer.

Example 6. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. The speed of the stream and that of the boat in still water, respectively are

(*a*) 10 km/hr, 12 km/hr (*b*) 8 km/hr, 3 km/hr

(*c*) 5 km/hr, 4 km/hr (*d*) 4 km/hr, 6 km/hr

Solution. Let the speed of the boat in still water be x km/h and the speed of the stream be y km/h.

$\therefore$ Speed of the boat going upstream = $(x - y)$ km/h

and speed of the boat going downstream = $(x + y)$ km/h

We know that Time = $\dfrac{\text{Distance}}{\text{Speed}}$

$\therefore$ Time taken in going 30 km upstream = $\dfrac{30}{x-y}$ h

Time taken in going 44 km downstream = $\dfrac{44}{x+y}$ h

$$\therefore \quad \frac{30}{x-y} + \frac{44}{x+y} = 10 \quad \text{...}(i) \text{ (From given condition)}$$

Again, time taken in going 40 km upstream = $\dfrac{40}{x-y}$ h

and time taken in going 55 km downstream = $\dfrac{55}{x+y}$ h.

$$\therefore \quad \frac{40}{x-y} + \frac{55}{x+y} = 13 \quad \text{...}(ii) \text{ (From given condition)}$$

Now, putting $\dfrac{1}{x-y} = a$ and $\dfrac{1}{x+y} = b$, we get

$$30a + 44b = 10 \quad \text{...}(iii)$$
$$40a + 55b = 13 \quad \text{...}(iv)$$

Multiplying equation (*iii*) by 4 and equation (*iv*) by 3 and subtracting, we get

$$120a + 176b = 40$$
$$120a + 165b = 39$$
$$- \quad - \quad -$$

$$11b = 1 \Rightarrow b = \frac{1}{11}$$

$\therefore$ From (*iii*), $30a + 44 \times \dfrac{1}{11} = 10 \Rightarrow 30a + 4 = 10 \Rightarrow 30a = 10 - 4 = 6$

$$\Rightarrow \quad a = \frac{6}{30} = \frac{1}{5}$$

$$\because \quad a = \frac{1}{5} \Rightarrow \frac{1}{x-y} = \frac{1}{5} \Rightarrow x - y = 5 \quad \text{...}(v)$$

$$\text{and} \quad b = \frac{1}{11} \Rightarrow \frac{1}{x+y} = \frac{1}{11} \Rightarrow x + y = 11 \quad \text{...}(vi)$$

On solving (*v*) and (*vi*), we get $x = 8$, $y = 3$

Thus, speed of the boat in still water = 8 km/h and speed of the stream = 3 km/h

Hence, option (*b*) is the correct answer.

Exercise 3.4

A. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

1. Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively

   (*a*) 35 and 15 (*b*) 35 and 20 (*c*) 15 and 35 (*d*) 25 and 25

2. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, (in years) of the son and the father are, respectively

   (*a*) 4 and 24 (*b*) 5 and 30 (*c*) 6 and 36 (*d*) 3 and 24

3. A purse contains 25 paise and 10 paise coins. The total amount in the purse is ₹ 8.25. If the number of 25 paise coins is one-third of the number of 10 paise coins in the purse, then the total number of coins in the purse is

   (*a*) 60 (*b*) 40 (*c*) 80 (*d*) 72

4. A man has some hens and cows. If the number of heads be 48 and the number of feet equals 140, the number of hens will be

   (*a*) 18 (*b*) 26 (*c*) 32 (*d*) 40

5. If 3 chairs and 1 table costs ₹ 1500 and 6 chairs and 1 table costs ₹ 2400, the pair of linear equations to represent this situation is

   (*a*) $6x + y = 1500,\ 3x + y = 2400$ (*b*) $\frac{x}{3} + y = 1500,\ \frac{x}{6} + y = 2400$

   (*c*) $3x + y = 1500,\ 6x + y = 2400$ (*d*) None of these

6. A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator. The fraction is

   (*a*) $\frac{2}{5}$ (*b*) $\frac{5}{18}$ (*c*) $\frac{4}{13}$ (*d*) $\frac{7}{15}$

7. The difference between two numbers is 26 and the larger number exceeds thrice of the smaller number by 4. The numbers are

   (*a*) 39, 13 (*b*) 12, 38 (*c*) 37, 11 (*d*) None of these

8. Meena went to a bank to withdraw ₹ 2,000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. How many notes of ₹ 50 and ₹ 100 she received?

   (*a*) ₹50 : 10, ₹100 : 15 (*b*) ₹50 : 12, ₹100 : 10 (*c*) ₹50 : 15, ₹100 : 10 (*d*) None of these

9. A motor boat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. The speed of the stream is

   (*a*) 60 km/hr (*b*) 6 km/min (*c*) 6 km/s (*d*) 6 km/hr

10. The two consecutive odd positive integers, sum of whose squares is 290 are

    (*a*) 5, 13 (*b*) 11, 13 (*c*) 13, 17 (*d*) None of these

11. Jamila sold a table and a chair for ₹ 1050, thereby making a profit of 10% on the table and 25% on the chair. If she had taken a profit of 25% on the table and 10% on the chair, she would have got ₹ 1065. Then the cost price of each is

    (*a*) Table : ₹500, chair : ₹400 (*b*) Table : ₹400, chair : ₹500

    (*c*) Table : ₹700, chair : ₹600 (*d*) None of these

12. A part of monthly Hostel charge is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 20 days, she has to pay ₹ 3000 as hostel charges whereas, Mansi who takes food for 25 days pays ₹ 3500 as hostel charges. The fixed charges and the cost of food per day, respectively are

(*a*) ₹400, ₹300 (*b*) ₹300, ₹200 (*c*) ₹1000, ₹100 (*d*) ₹500, ₹100

13. Children were fallen in for drill. If each row contained 4 children less, 10 more rows would have been made. But if 5 more children were fallen in each row, the number of rows would have reduced by 5. The number of children in the school is

(*a*) 150 (*b*) 160 (*c*) 170 (*d*) 120

14. A two-digit number is obtained by either multiplying the sum of digits by 8 and then subtracting 5 or by multiplying the difference of digits by 16 and adding 3. The number is

(*a*) 23 (*b*) 34 (*c*) 83 (*d*) 119

15. The sum of a two-digit number and the number obtained by interchanging the digits is 132. If the two digits differ by 2, the number is

(*a*) 45 (*b*) 75 (*c*) 85 (*d*) 115

16. Two years ago, a father was five times as old as his son. Two years later, his age will be 8 more than three times the age of the son. The present age of father and son, respectively are

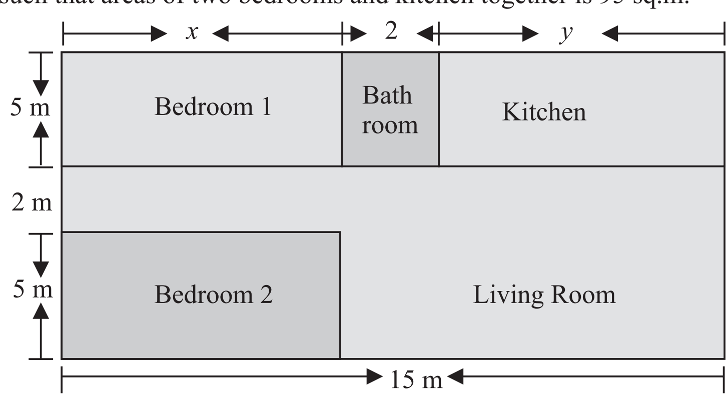
(*a*) 40 years, 12 years (*b*) 30 years, 6 years (*c*) 32 years, 8 years (*d*) 42 years, 10 years

17. A boat travels for 7 hours. If it travels 4 hours downstream and 3 hours upstream, then it covers the distance of 116 km. But if it travels 3 hours downstream and 4 hours upstream, it covers the distance of 108 km. The speed of the boat is

(*a*) 16 km/h (*b*) 22 km/h (*c*) 18 km/h (*d*) None of these

Case Study Based Questions

I. Amit is planning to buy a house and the layout is given below. The design and the measurement has been made such that areas of two bedrooms and kitchen together is 95 sq.m.



Based on the above information, answer the following questions:

1. The pair of linear equations in two variables from above situation is

(*a*) $x + y = 13, 2x + y = 19$ (*b*) $2x + y = 13, x + y = 19$

(*c*) $x + 2y = 13, x - y = 19$ (*d*) None of these

2. The length of the outer boundary of the layout is

(*a*) 35 m (*b*) 54 m (*c*) 42 m (*d*) 60 m

3. The area of each bedroom and kitchen in the layout respectively is

(*a*) 25 m, 35 m (*b*) 15 m, 25 m (*c*) 30 m, 35 m (*d*) 25 m, 30 m

4. The area of living room in the layout is

(*a*) 55 sq. m (*b*) 65 sq. m (*c*) 75 sq. m (*d*) 85 sq. m

5. The cost of laying tiles in kitchen at the rate of ₹ 50 per sq m is

(*a*) ₹ 1260 (*b*) ₹ 1750 (*c*) ₹ 1590 (*d*) ₹ 1810

II. Ankit and his friends went to a shop to purchase some daily use items. He purchased five copies and one book from the shop which cost him ₹ 500. His friends purchased the same copies and same books from other shop. If his friends purchased 10 copies and 3 books for ₹ 1300, then using variables 'x' and 'y' for the cost of one copy and one book respectively, answer the following questions:

1. The algebraic representation of the above situation is given by the equations as

(*a*) $5x + y = 500,\ 10x + 3y = 1300$ (*b*) $x + 5y = 500,\ 3x + 10y = 1300$

(*c*) $5x - y = 500,\ 10x - 3y = 1300$ (*d*) $x - 5y = 500,\ 3x - 10y = 1300$

2. The above situations represent a pair of linear equations. The pair of linear equations show a/an

(*a*) unique solution (*b*) infinitely many solutions

(*c*) no solution (*d*) None of the above

3. The above situations are representing a pair of linear equations which can be shown by drawing two lines in a plane. The following possibilities can happen. The two lines are

(*a*) intersecting at one point (*b*) parallel to each other

(*c*) coincident lines (*d*) perpendicular to each other

4. The pair of linear equations shown by above situation are

(*a*) consistent (*b*) inconsistent

(*c*) dependent (*d*) Both (*a*) and (*c*)

5. Using above situations, the cost of one copy and one book separately is

(*a*) ₹ 40, ₹ 300 (*b*) ₹ 60, ₹ 200

(*c*) ₹ 20, ₹ 400 (*d*) ₹ 60, ₹ 300

Answers and Hints

A. Multiple Choice Questions (MCQs)

1. (*d*) 25 and 25
2. (*c*) 6 and 36
3. (*a*) 60
4. (*b*) 26
5. (*c*) $3x + y = 1500$, $6x + y = 2400$

Let the cost of 1 chair = ₹ x

And the cost of 1 table = ₹ y

$$3x + y = 1500$$
$$6x + y = 2400$$

6. (*d*) $\frac{7}{15}$

Let the numerator be x and denominator be y

Then, fraction $= \frac{x}{y}$

ATQ, $\frac{x-2}{y} = \frac{1}{3}$

$\Rightarrow$ $3x - 6 = y \Rightarrow 3x - y = 6$ ...(*i*)

and $\frac{x}{y-1} = \frac{1}{2} \Rightarrow 2x = y - 1$

$\Rightarrow$ $2x - y = -1$ ...(*ii*)

Subtracting (*ii*) from (*i*), we have

$$\begin{array}{r} 3x - y = 6 \\ 2x - y = -1 \\ - \quad + \quad + \\ \hline x = +7 \\ \hline \end{array}$$

From equation (*i*)

$3 \times 7 - y = 6 \Rightarrow y = 21 - 6 = 15$

Thus required fraction $= \frac{7}{15}$.

7. (*c*) 37, 11

Let the larger number be x and the smaller number be y.

Then, $x - y = 26$ ...(*i*)

and $x - 3y = 4$ ...(*ii*)

$$\begin{array}{r} - \quad + \qquad - \\ \hline 2y = 22 \end{array} \Rightarrow y = 11$$

From (*i*), $x = 26 + 11 = 37$

So, the larger number is 37 and the smaller number is 11.

8. (*a*) ₹ 50 : 10, ₹ 100 : 15

Let Meena has received x no. of ₹ 50 notes and y no. of ₹ 100 notes.

So, $50x + 100y = 2000$ ... (*i*)

$x + y = 25$ ... (*ii*)

Multiply (*ii*) by 50, we get

$$\begin{array}{r} 50x + 100y = 2000 \\ 50x + 50y = 1250 \\ - \quad - \qquad - \\ \hline 50y = 750 \end{array}$$

$\Rightarrow$ $y = 15$

Putting $y = 15$ in equation (*ii*), we get

$x + 15 = 25$

$\Rightarrow$ $x = 10$

Meena has received 10 pieces of ₹ 50 notes and 15 pieces of ₹ 100 notes.

9. (*d*) 6 km/hr

Given: Speed of boat = 18 km/hr,

Distance = 24 km

Let x be the speed of stream.

Let t_1 and t_2 be the time for upstream and downstream.

As we know that,

$\Rightarrow$ time $= \frac{\text{distance}}{\text{speed}}$

For upstream, Speed $= (18 - x)$ km/hr,

Time $= t_1$

Therefore, $t_1 = \frac{24}{18 - x}$

For downstream, Speed $= (18 + x)$ km/hr,

Time $= t_2$

Therefore, $t_2 = \frac{24}{18 + x}$

Now according to the question:

$$t_1 = t_2 + 1$$

$$\frac{24}{18 - x} = \frac{24}{18 + x} + 1$$

$\Rightarrow$ $48x = (18 - x)(18 + x)$

$\Rightarrow$ $48x = 324 + 18x - 18x - x^2$

$\Rightarrow$ $x^2 + 48x - 324 = 0$

$\Rightarrow$ $x^2 + 54x - 6x - 324 = 0$

$\Rightarrow x(x + 54) - 6(x + 54) = 0$

$$\Rightarrow \quad (x+54)(x-6) = 0$$
$$\Rightarrow \quad x = -54 \text{ or } x = 6$$

Since speed cannot be negative.

$$\Rightarrow \quad x = -54 \text{ will be rejected}$$
$$\therefore \quad x = 6$$

Thus, the speed of stream is 6 km/hr.

10. (*b*) 11, 13

Let one of the odd positive integer be x then the other odd positive integer is $x + 2$

ATQ, $\quad x^2 + (x+2)^2 = 290$

$$\Rightarrow \quad x^2 + x^2 + 4x + 4 = 290$$
$$\Rightarrow \quad 2x^2 + 4x - 286 = 0$$
$$\Rightarrow \quad 2(x^2 + 2x - 143) = 0$$
$$\Rightarrow \quad x^2 + 2x - 143 = 0$$
$$\Rightarrow \quad x^2 + 13x - 11x - 143 = 0$$
$$\Rightarrow \quad x(x+13) - 11(x+13) = 0$$
$$\Rightarrow \quad (x-11)(x+13) = 0$$
$$\Rightarrow \quad (x-11) = 0, (x+13) = 0$$

Therefore, $\quad x = 11 \text{ or } -13$

According to question, x is a positive odd integer.

Hence, we take positive value of x

So, $x = 11$ and $(x + 2) = 11 + 2 = 13$

Therefore, the odd positive integers are 11 and 13.

11. (*a*) Table : ₹ 500, Chair : ₹ 400

Let C.P of table be ₹x and C.P of chair be ₹y.

Profit on table is 10%.

$$\therefore \quad \text{S.P of table} = x + 10\% \text{ of } x$$
$$= x + \frac{10x}{100} = \frac{110x}{100}$$

Profit on chair = 25%

$$\therefore \quad \text{S.P of chair} = y + 25\% \text{ of } y$$
$$= y + \frac{25y}{100} = \frac{125}{100}y$$

As per question

$$\frac{110x}{100} + \frac{125y}{100} = 1050 \quad \ldots(i)$$

Again if profit on table and chair are 25% and 10% respectively, then

$$\text{the S.P of table} = \frac{125x}{100}$$

and $\quad \text{S.P of chair} = \dfrac{110y}{100}$

Then, $\quad \dfrac{125x}{100} + \dfrac{110y}{100} = 1065 \quad \ldots(ii)$

Solving (*i*) and (*ii*), we get

$$x = 500$$

and $\quad y = 400$

Thus, CP of table = ₹ 500 and CP of chair = ₹ 400

12. (*c*) ₹1000, ₹100

Let fixed hostel charges be ₹ x

Charge per day is ₹ y

Charge paid by Swati = ₹ 3000

$\therefore$ Ist condition is $x + 20y$ = ₹ 3000 $\quad \ldots(i)$

Charge paid by Mansi = ₹ 3500

$\therefore$ 2nd condition is $x + 25y$ = ₹ 3500 $\quad \ldots(ii)$

Subtracting (*ii*) from (*i*)

$$x + 20y = 3000 \quad \ldots(i)$$
$$x + 25y = 3500 \quad \ldots(ii)$$
$$- \quad - \quad\quad -$$
$$-5y = -500$$

$\Rightarrow \quad y$ = ₹ 100

Put value of $y = 100$ in equation (*i*), we get

$$\Rightarrow \quad x + 20(100) = 3000$$

$x = 3000 - 2000$ = ₹ 1000

Hence, fixed charges is ₹ 1000 and charges per day is ₹ 100.

13. (*a*) 150 **14.** (*c*) 83

15. (*b*) 75 **16.** (*d*) 42 years, 10 years

17. (*a*) 16 km/hr

Case Study Based Questions

I. **1.** (*a*) $x + y = 13, 2x + y = 19$

2. (*b*) 54 m

3. (*c*) 30 m, 35 m

4. (*c*) 75 sq m

5. (*b*) ₹ 1750

II. **1.** (*a*) $5x + y = 500, \ 10x + 3y = 1300$

2. (*a*) unique solution

3. (*a*) intersecting at one point

4. (*d*) both (*a*) and (*c*)

5. (*a*) ₹ 40, ₹ 300

Experts' Opinion

Questions based on following types are very important for exams. So, students are advised to revise them thoroughly.

1. To find the value of unknown for which the given pair of linear equations has unique solution, infinitely many solutions or no solutions.
2. To find out whether the lines representing the given pairs of linear equations intersect at a point, are parallel or coincident by comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$.
3. To check whether the given pairs of linear equations are consistent/inconsistent and obtain the solution graphically if consistent.
4. To draw the graphs of given equations and determine the coordinates of the vertices of the triangle formed by the lines and the x-axis.
5. To solve the given pair of linear equations by the substitution method.
6. To solve the given pair of linear equations by the elimination method and substitution method.
7. To solve the given pair of equations by reducing them to a pair of linear equations.
8. To formulate the given problem as a pair of linear equations and hence find its solution.

IMPORTANT FORMULAE

For the system of two linear equations $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$		
Compare the ratios	**Graphical representations**	**Algebraic interpretation**
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (Unique)
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

QUICK REVISION NOTES

- Two linear equations in the same two variables are called a pair of linear equations in two variables. The general form of a pair of linear equations is

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers, such that $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$.

- A pair of linear equations in two variables can be represented and solved by the

  (*i*) Graphical Method (*ii*) Algebraic Method

  Graphical Method: The graph of a pair of linear equations in two variables is represented by two lines.

  (*i*) If the lines intersect at a point, then the point gives the unique solution of the two equations. The pair of equations is consistent.

  (*ii*) If the lines are parallel, the pair of lines of equations has no solution. The pair of equations are inconsistent.

  (*iii*) If the lines coincide, then there are infinitely many solutions. The pair of equations is consistent.

  Algebraic Method: The following are the algebraic methods:

  (*i*) Substitution Method (*ii*) Elimination Method

- If a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then the following situations can arise:

  (*i*) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ Equations are consistent.

  (*ii*) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ Equations are inconsistent.

  (*iii*) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$ Equations are dependent and are consistent.

- There are many situations which can be mathematically represented by two equations that are not linear to start with. But, when we alter them, they are reduced to a pair of linear equations.

COMMON ERRORS

Errors	Corrections
(*i*) Drawing axes as line segments.	(*i*) Do not forget to mark arrow heads for axes (lines).
(*ii*) Incorrectly transposing the terms.	(*ii*) Be careful about sign of terms while transposing.
(*iii*) Writing incorrectly that if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, there are infinitely many solutions.	(*iii*) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations is inconsistent and so has no solution.
(*iv*) Making mistake in the problem of upstream and downstream. For example, let the speed of the boat in still water be x km/h and speed of the stream be y km/h. Then, the speed of the boat downstream $= (x - y)$ km/h and the speed of the boat upstream $= (x + y)$ km/h	(*iv*) The speed of the boat downstream $= (x + y)$ km/h and the speed of the boat upstream $= (x - y)$ km/h