Circles

10

Objective Section (1 mark each)

Fill in the blanks

Q. 1. In given Fig. 2, the length PB = ... cm.

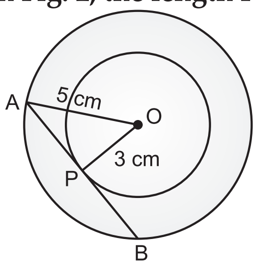


Fig. - 2

[CBSE OD, Set 1, 2020]

Ans. 4

Explanation :

We have, OP $\perp$ AB

$\therefore$ P is the mid-point of AB.

$\Rightarrow$ AP = PB

Now, in ΔOAP

$$OA^2 = OP^2 + AP^2$$

[Pythagoras theorem]

$\Rightarrow$ $(5)^2 = (3)^2 + AP^2$

$\Rightarrow$ $25 = 9 + AP^2$

$\Rightarrow$ $AP^2 = 16$

$\Rightarrow$ AP = 4

$\Rightarrow$ PB = 4 cm **Ans.**

Q. 2. In Fig 1, ΔABC is circumscribing a circle, the length of BC is .... cm.

[CBSE Delhi, Set 1, 2020]

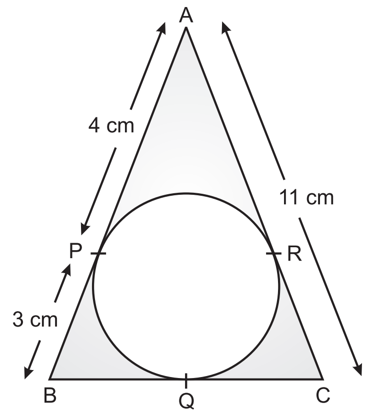


Fig. 1

Ans. 10

Explanation : We know that

The lengths of tangents drawn from an external point to a circle are equal.

$\therefore$ BP = BQ

AP = AR

BQ = 3 cm

AR = 4 cm

So, AC = AR + CR

11 = 4 + CR

CR = 7 cm

Now, CQ = CR

$\Rightarrow$ CQ = 7 cm

So, BC = BQ + CQ

= 3 + 7 = 10

BC = 10 cm. **Ans.**

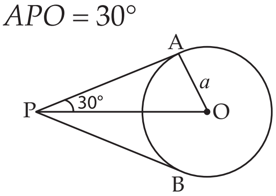
Very Short Answer Type Questions (1 mark each)

Q. 1. If the angle between two tangents drawn from an external point *P* to a circle of radius *a* and centre *O*, is 60°, then find the length of *OP*.

[CBSE OD, Term 2, Set 1, 2017]

Ans. Given, $\angle APB = 60°$

$\Rightarrow$ $\angle APO = 30°$



Also, $\angle OAP = 90°$ [$\because$ Tangent $\perp$ radius]

$\therefore$ In right angle ΔOAP,

$$\frac{OP}{OA} = \text{cosec } 30°$$

$$\frac{OP}{a} = 2 \Rightarrow OP = 2a.$$

Q. 2. In fig. 1, *PQ* is a tangent at a point *C* to a circle with centre *O*. If *AB* is a diameter and $\angle CAB$ = 30°, find $\angle PCA$.

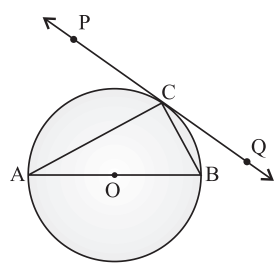


Figure 1

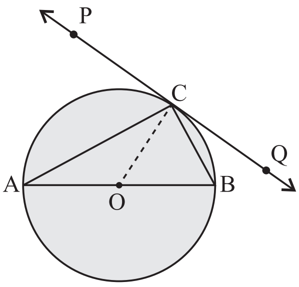
[CBSE OD, Term 2, Set 1, 2016]

Ans. Given, $\angle CAB = 30°$ and PQ is a tangent at a point C to a circle with centre O.

Since, AB is a diameter.

$\therefore \quad \angle ACB = 90°$

Join OC.



$$\angle CAO = \angle ACO = 30° \quad (\because OA = OC)$$

and, $\angle PCO = 90°$ (Tangent is perpendicular to the radius through the point of contact)

$$\therefore \quad \angle PCA = \angle PCO - \angle ACO$$
$$= 90° - 30°$$
$$= 60°$$

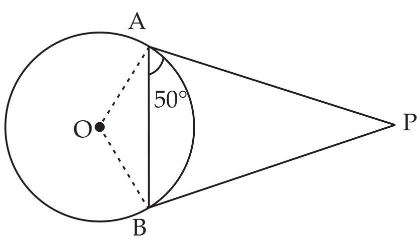
Q. 3. From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50°$, then find $\angle AOB$.
[CBSE Delhi, Term 2, Set 1, 2016]

Ans. Since, tangents from an external point are equal.

$\therefore \quad AP = BP$

Given, $\angle PAB = 50°$

$\therefore \quad \angle PBA = 50°$



In ΔAPB,

$$\angle APB = 180° - (50° + 50°) = 80°$$

$$\therefore \quad \angle AOB = 180° - 80° = 100$$

Q. 4. In figure 2, PQ is a chord of a circle with centre O and PT is a tangent. If $\angle QPT = 60°$, find $\angle PRQ$.

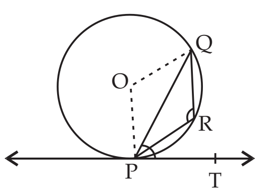


Figure 2

[CBSE OD, Term 2, Set 1, 2015]

Ans. Given, O is the centre of the given circle

$\therefore$ OQ and OP are the radius of circle.

$\because$ PT is a tangent

$\therefore \quad OP \perp PT$

So, $\quad \angle OPT = 90°$

$\therefore \quad \angle OPQ = 90° - \angle QPT$

$\Rightarrow \quad \angle OPQ = 90° - 60°$ [Given, $\angle QPT = 60°$]

$\Rightarrow \quad \angle OPQ = 30°$

$\therefore \quad \angle OQP = 30°$ [$\because \Delta OPQ$ is isosceles triangle]

Now, in ΔOPQ

$$\angle POQ + \angle OPQ + \angle OQP = 180°$$

$\Rightarrow \quad \angle POQ + 30° + 30° = 180°$

$\Rightarrow \quad \angle POQ = 120°$

reflex $\angle POQ = 360° - 120° = 240°$

$\therefore \quad \angle PRQ = \frac{1}{2}\text{reflex } \angle POQ$

[$\because$ The angle substended by an arc of a circle at the centre is double the angle substended by it at any point on the remaining part of the circle]

$\Rightarrow \quad \angle PRQ = \frac{1}{2} \times 240°$

Hence, $\angle PRQ = 120°$

Q. 5. In Fig. 1, PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$, Write the measure of $\angle OAB$.

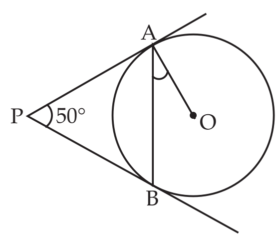


Figure 1

[CBSE Delhi, Term 2, Set 1, 2015]

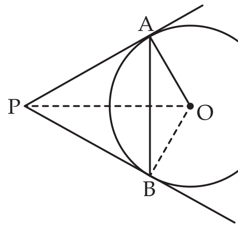
Ans. Since PA and PB are tangents to the circle with centre O then,

$$PA = PB$$

Join OP.

Then $\quad \angle APO = \angle BPO = 25°$

Also, $PA \perp OA$.



In $\Delta\, APO$,

$$\angle APO + \angle POA + \angle OAP = 180°$$

$$\Rightarrow \quad 25° + \angle POA + 90° = 180°$$

$$\Rightarrow \quad \angle POA = 65°$$

Join OB, then

In ΔAOB

$$\angle OAB + \angle OBA + \angle AOB = 180°$$

$$\Rightarrow \quad 2\angle OAB + 2\angle POA = 180°$$

$$\begin{bmatrix} \because \angle OAB = \angle OBA \\ OA \,\&\, OB \text{ are radii} \end{bmatrix}$$

$$\Rightarrow \quad 2\angle OAB + 2 \times 65° = 180°$$

$$\Rightarrow \quad \angle OAB = 90° - 65°$$

$$\therefore \quad \angle OAB = 25°$$

Short Answer Type Questions-I (2 marks each)

Q. 1. In Fig. 3, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2 ∠OPQ. [CBSE Delhi, Set 1, 2020]

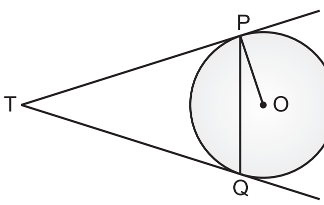


Fig. 3

Given : A circle with centre O, TP and TQ are two tangents from external point T.

To Prove : ∠PTQ = 2∠OPQ.

Proof : Let ∠PTQ = α

In ΔPTQ, TP = TQ

[∵ The lengths of the two tangents from an external point to a circle are equal]

$$\therefore \quad \angle TPQ = \angle TQP$$

[∵ Angles opposite to equal sides are equal]

So, ∠PTQ + ∠TPQ + ∠TQP = 180°

[Angle sum property of a triangle]

$$\Rightarrow \alpha + \angle TPQ + \angle TPQ = 180°$$

$$\Rightarrow \quad 2\angle TPQ = 180° - \alpha$$

$$\Rightarrow \quad \angle TPQ = \frac{1}{2}(180° - \alpha)$$

$$\Rightarrow \quad \angle TPQ = 90° - \frac{\alpha}{2}$$

Now, OP ⊥ TP

{∵ The tangent to a circle is perpendicular to the radius through the point of contact}

$$\therefore \quad \angle OPT = 90°$$

$$\Rightarrow \quad \angle OPQ + \angle TPQ = 90°$$

$$\Rightarrow \quad 90° - \frac{\alpha}{2} + \angle OPQ = 90°$$

$$\Rightarrow \quad \angle OPQ = 90° - 90° + \frac{\alpha}{2}$$

$$\Rightarrow \quad \angle OPQ = \frac{\alpha}{2}$$

$$\Rightarrow \quad \angle OPQ = \frac{\angle PTQ}{2}$$

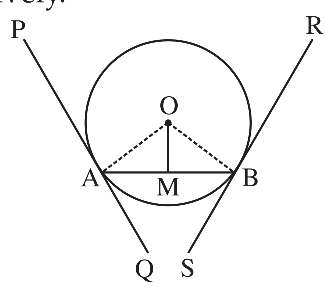
$$\text{or} \quad \angle PTQ = 2\angle OPQ$$

Hence Proved.

Q. 2. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

[CBSE OD, Term 2, Set 1, 2017]

Ans. Given, a circle of radius OA and centred at O with chord AB and tangents PQ & RS are drawn from point A and B respectively.



Draw $OM \perp AB$, and join OA and OB.

In $\Delta\, OAM$ and $\Delta\, OMB$,

$OA = OB$ (Radii)

$OM = OM$ (Common)

$\angle\, OMA = \angle\, OMB$ (Each 90°)

$\therefore \quad \Delta\, OMA \cong \Delta\, OMB$

(By R.H.S. Congurency)

$\therefore \quad \angle\, OAM = \angle\, OBM$ (C.P.C.T.)

Also, $\angle\, OAP = \angle\, OBR = 90°$ (Line joining point of contact of tangent to centre is perpendicular on it)

On addition,

$$\angle OAM + \angle OAP = \angle OBM + \angle OBR$$

$\Rightarrow$ $\angle PAB = \angle RBA$

$\Rightarrow$ $\angle 180° - \angle PAB = \angle 180° - \angle RBA$

$\Rightarrow$ $\angle QAB = \angle SBA$

Hence Proved.

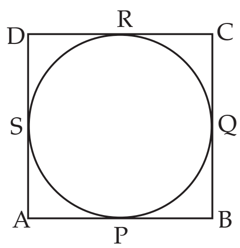
Q. 3. A circle touches all the four sides of a quadrilateral *ABCD*. Prove that

AB* + *CD* = *BC* + *DA

[CBSE OD, Term 2, Set 1, 2017]
[CBSE OD, Term 2, Set 1, 2016]

Ans.

Given, a quad. *ABCD* and a circle touches its all four sides at *P*, *Q*, *R*, and *S* respectively.



To prove: $AB + CD = BC + DA$

Now, L.H.S. $= AB + CD$

$= AP + PB + CR + RD$

$= AS + BQ + CQ + DS$

(Tangents from same external point are always equal)

$= (AS + SD) + (BQ + QC)$

$= AD + BC$

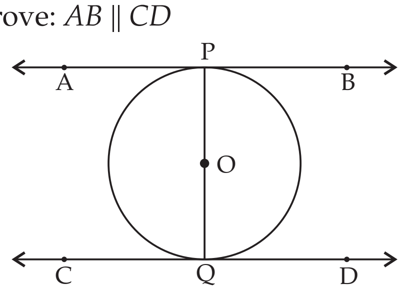
$=$ R.H.S. **Hence Proved.**

Q. 4. Prove that tangents drawn at the ends of a diameter of a circle are parallel to each other.

[CBSE Delhi, Term 2, Set 1, 2017]

Ans. Given, *PQ* is a diameter of a circle with centre *O*. The lines *AB* and *CD* are tangents at *P* and *Q* respectively.

To Prove: $AB \parallel CD$



Proof: *AB* is a tangent to the circle at *P* and *OP* is the radius through the point of contact.

$\therefore$ $\angle OPA = 90°$

Similarly, *CD* is a tangent to circle at *Q* and *OQ* is radius through the point of contact.

$\therefore$ $\angle OQD = 90°$

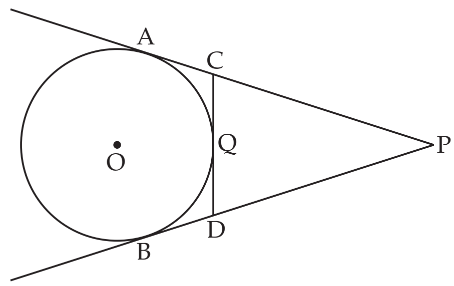
$\Rightarrow$ $\angle OPA = \angle OQD$

But both form a pair of alternate angles

$\therefore$ $AB \parallel CD$ **Hence Proved.**

Q. 5. In the given figure, *PA* and *PB* are tangents to the circle from an external point *P*. *CD* is another tangent touching the circle at *Q*. If *PA* = 12 cm, *QC* = *QD* = 3 cm, then find *PC* + *PD*.

[CBSE Delhi, Term 2, Set 1, 2017]



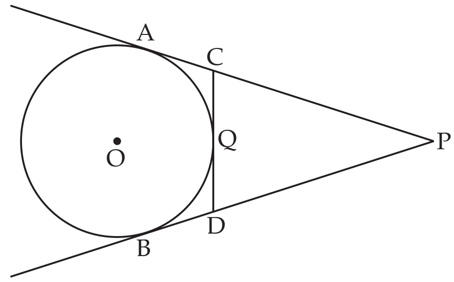
Ans. Given, $PA = PB = 12$ cm

[Tangent from external point]

$AC = CQ = 3$ cm

$BD = QD = 3$ cm

[Tangent from external point]

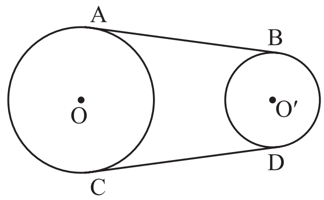


So, $PC + PD = (PA - AC) + (PB - BD)$

$= (12 - 3) + (12 - 3)$

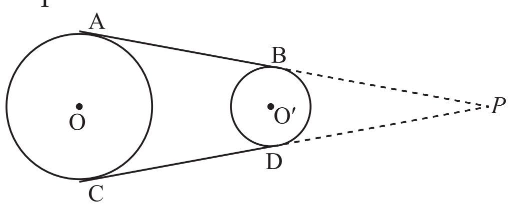
$= 9 + 9 = 18$ cm

Q. 6. In the figure, *AB* and *CD* are common tangents to two circles of unequal radii. Prove that *AB* = *CD*.



[CBSE Delhi, Term 2, Set 3, 2017]

Ans. Construction: Extend *AB* and *CD* to meet at a point *P*.



Now, *PA* and *PC* are tangents of circle with centre *O*.

So, $PA = PC$ ...(i)

PB and *PD* are tangent on circle with centre *O*'

So, $PB = PD$ ...(ii)

On subtracting equation (ii) from equation (i),

$$PA - PB = PC - PD$$

$$AB = CD$$ **Hence Proved.**

Q. 7. In Fig. 3, from an external point *P*, two tangents *PT* and *PS* are drawn to a circle with centre *O* and radius *r*. If *OP* = 2*r*, show that ∠*OTS* = ∠*OST* = 30°.

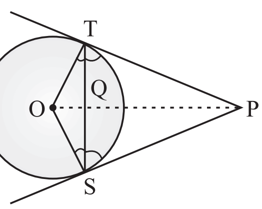


Figure 3

[CBSE OD, Term 2, Set 1, 2016]

Ans. We have, $OP = 2r$

$\angle OTP = 90°$ [∵ Tangent ⊥ radius]

Let $\angle TOP = \theta$

In Δ *OTP*, $\cos\theta = \dfrac{OT}{OP} = \dfrac{r}{2r} = \dfrac{1}{2} = \cos 60°$

$\Rightarrow \theta = 60°$

Hence, $\angle TOS = 2\theta = 2 \times 60° = 120°$

In Δ *TOS*,

$\angle TOS + \angle OTS + \angle OST = 180°$

$\Rightarrow 120° + 2\angle OTS = 180°$ (∵ $\angle OTS = \angle OST$)

$\Rightarrow 2\angle OTS = 180° - 120°$

$\Rightarrow \angle OTS = 30°$

Hence, $\angle OTS = \angle OST = 30°$

Hence Proved.

Q. 8. In Fig. 2, a circle is inscribed in a Δ *ABC*, such that it touches the sides *AB*, *BC* and *CA* at points *D*, *E* and *F* respectively. If the lengths of sides *AB*, *BC* and *CA* are 12 cm, 8 cm and 10 cm respectively, find the lengths of *AD*, *BE* and *CF*.

[CBSE Delhi, Term 2, Set 1, 2016]

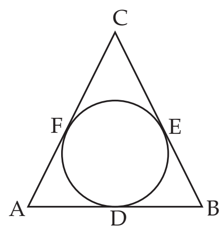


Fig. 2

Ans. Given, *AB* = 12 cm; *BC* = 8 cm and *CA* = 10 cm

Let $AD = AF = x$

$\Rightarrow DB = BE = 12 - x$

and, $CF = CE = 10 - x$

Now, $BC = BE + EC$

$\Rightarrow 8 = 12 - x + 10 - x$

$\Rightarrow 8 = 22 - 2x$

$\Rightarrow 2x = 14$

$\Rightarrow x = 7$ cm

$\Rightarrow AD = 7$ cm, $BE = 5$ cm and $CF = 3$ cm

Q. 9. In Fig. 3, *AP* and *BP* are tangents to a circle with centre *O*, such that *AP* = 5 cm and ∠ *APB* = 60°. Find the length of chord *AB*.

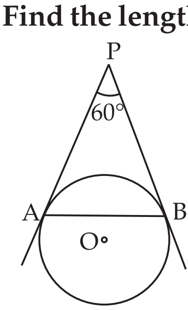


Fig. 3

[CBSE Delhi, Term 2, Set 1, 2016]

Ans. Given, *AP* and *BP* are tangents to a circle with centre *O*.

$\therefore \quad AP = BP$

Now, $\angle APB = 60°$ (Given)

$\therefore \quad \angle PAB = \angle PBA = 60°$

$(\because AP = BP)$

Thus, ΔAPB is an equilateral triangle.

Hence, the length of chord AB is equal to the length of AP *i.e.* 5 cm.

Q. 10. In figure 3, two tangents *RQ* and *RP* are drawn from an external point *R* to the circle with centre *O*. If $\angle PRQ = 120°$, then prove that, *OR* = *PR* + *RQ*.

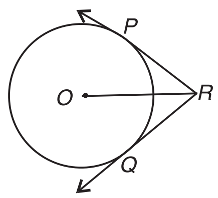


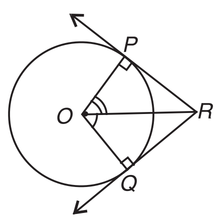
Figure 3

[CBSE OD, Term 2, Set 1, 2015]

Ans. O is the centre of the circle and $\angle PRQ = 120°$

Construction: Join OP, OQ

To prove: $OP = PR + RQ$



Proof: We know, tangents drawn from an external point are equal.

$\therefore \quad PR = RQ$

Also, tangents are equally inclined to the line joining the centre of circle and the external point.

$$\therefore \quad \angle ORP = \angle ORQ = \frac{1}{2}\angle PRQ$$

$$= \frac{1}{2} \times 120° = 60°$$

And $\angle OPR = 90°$ $[\because$ Tangent $\perp$ Radius$]$

Now, in Δ OPR

$$\cos 60° = \frac{PR}{OR}$$

$$\left[\because \ \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}\right]$$

$$\Rightarrow \quad \frac{1}{2} = \frac{PR}{OR}$$

$\Rightarrow \quad OR = 2PR$

$\Rightarrow \quad OR = PR + PR$

$\Rightarrow \quad OR = PR + RQ \quad [\because PR = RQ]$

Hence, $OR = PR + RQ$.

Hence Proved.

Q. 11. In figure 4, a triangle *ABC* is drawn to circumscribe a circle of radius 3 cm, such that the segments *BD* and *DC* are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm^2, then find the lengths of sides AB and AC.

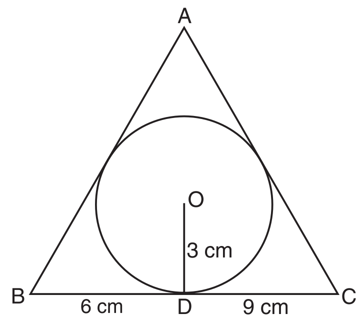
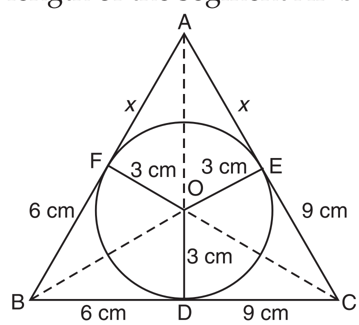


Figure 4

[CBSE OD, Term 2, Set 1, 2015]

Ans. Given, in ΔABC, circle touch the triangle at points D, F and E respectively and let the length of the segment AF be x.



So, $BF = BD = 6$ cm

[Tangents from point B]

$CE = CD = 9$ cm

[Tangent from point C]

and $AE = AF = x$ cm

[Tangents from point A]

$$\text{Now, Area of } \Delta OBC = \frac{1}{2} \times BC \times OD$$

$$= \frac{1}{2} \times (6 + 9) \times 3$$

$$= \frac{45}{2} \text{ cm}^2$$

$$\text{Area of } \Delta OCA = \frac{1}{2} \times AC \times OE$$

$$= \frac{1}{2} \times (9 + x) \times 3$$

$$= \frac{3}{2} (9 + x) \text{ cm}^2$$

$$\text{Area of } \Delta BOA = \frac{1}{2} \times AB \times OF$$

$$= \frac{1}{2} \times (6 + x) \times 3$$

$$= \frac{3}{2} (6 + x) \text{ cm}^2$$

Area of $\Delta ABC = 54 \text{ cm}^2$ [Given]

$\because$ Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔBOA

$$54 = \frac{45}{2} + \frac{3}{2}(9+x) + \frac{3}{2}(6+x)$$

$\Rightarrow$ $54 \times 2 = 45 + 27 + 3x + 18 + 3x$

$\Rightarrow$ $108 - 45 - 27 - 18 = 6x$

$\Rightarrow$ $6x = 18$

$\Rightarrow$ $x = 3$

So, $AB = AF + FB = x + 6$

$= 3 + 6 = 9$ cm

and $AC = AE + EC = x + 9$

$= 3 + 9 = 12$ cm

Hence, lengths of AB and AC are 9 cm and 12 cm respectively.

Q. 12. In Fig. 2, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58°$, find $\angle ATQ$.

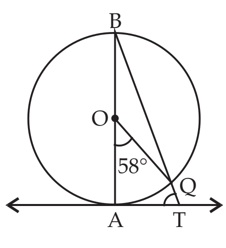


Figure 2

[CBSE Delhi, Term 2, Set 1, 2015]

Ans. Given, AB is a diameter of a circle with centre O and AT is a tangent, then

$BA \perp AT$

Also $\angle ABQ = \frac{1}{2}\angle AOQ$

($\because$ Angle subtended on the arc is half of the angle subtended at centre)

$\Rightarrow$ $\angle ABQ = \frac{1}{2} \times 58° = 29°$

Now, $\angle ATQ = 180° - (\angle ABQ + \angle BAT)$

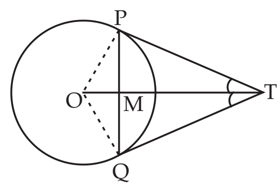
$= 180° - (29° + 90°)$

$\therefore$ $\angle ATQ = 61°$

Q. 13. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ.

[CBSE Delhi, Term 2, Set 1, 2015]

Ans. Given, TP and TQ are the tangents drawn on a circle with centre O.



To prove: OT is the right bisector of PQ.

Proof: In ΔTPM and ΔTQM

$TP = TQ$ (Tangents drawn from external point are equal)

$TM = TM$ (Common)

$\angle PTM = \angle QTM$

(TP and TQ are equally inclined to OT)

$\therefore$ $\Delta TPM \cong \Delta TQM$ (By SAS congruence)

$\therefore$ $PM = MQ$

and $\angle PMT = \angle QMT$ (By C.P.C.T)

since, PMQ is a straight line, then

$\therefore$ $\angle PMT + \angle QMT = 180°$

$\Rightarrow$ $\angle PMT = \angle QMT = 90°$

$\therefore$ OT is the right bisector of PQ.

Hence Proved.

Short Answer Type Questions-II (3 marks each)

Q. 1. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that

$$\mathbf{AQ = \frac{1}{2}(BC + CA + AB)}$$

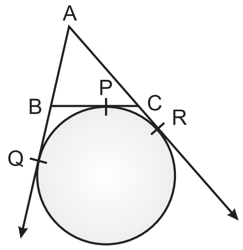
[CBSE OD, Set-I, 2020]

Ans. We know that the lengths of tangents drawn from an external point to a circle are equal.

$\therefore$ AQ = AR ...(i) [Tangents from A]

BP = BQ ...(ii) [Tangents from B]

CP = CR ...(iii) [Tangents from C]



We have to show that

$$AQ = \frac{1}{2}(BC + CA + AB)$$

Taking L.H.S.

$$AQ = \frac{1}{2} \times 2AQ$$

$$\Rightarrow \quad AQ = \frac{1}{2} \times (AQ + AQ)$$

$$\Rightarrow \quad AQ = \frac{1}{2} \times (AQ + AR)$$

[From equation (i)]

$$\Rightarrow \quad AQ = \frac{1}{2} \times (AB + BQ + AC + CR)$$

$$\Rightarrow \quad AQ = \frac{1}{2} \times (AB + BP + AC + CP)$$

[Using equations (ii) and (iii)]

$$\Rightarrow \quad AQ = \frac{1}{2} \times \{AB + (BP + CP) + AC\}$$

$$\Rightarrow \quad AQ = \frac{1}{2} \times (AB + BC + AC)$$

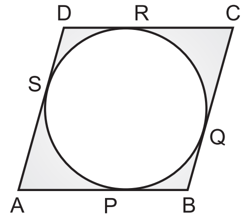
Hence Proved.

Q. 2. Prove that the parallelogram circumscribing a circle is a rhombus.

[CBSE Delhi, Set 2, 2020]

Ans. Given : A parallelogram ABCD circumscribing a circle.

To prove : ABCD is a rhombus.



Proof : We know that, the length of two tangents from an external points to a circle are equal.

$\therefore$ for point A, AP = AS ...(i)

for point B, BP = BQ ...(ii)

for point C, CR = CQ ...(iii)

and for point D, DR = DS ...(iv)

On adding equations (i), (ii), (iii) and (iv), we get

$\Rightarrow$ AP + BP + CR + DR = AS + BQ + CQ + DS

$\Rightarrow$ (AP + BP) + (CR + DR)

= (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

$\Rightarrow$ AB + AB = AD + AD

{$\because$ opposite sides of a parallelogram are equal

i.e., AB = CD, AD = BC}

$\Rightarrow$ 2AB = 2AD

$\Rightarrow$ AB = AD

$\therefore$ ABCD is a parallelogram with adjacent sides AB = AD.

Hence, it is a rhombus. **Hence Proved.**

Q. 3. In given Fig. 5, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.

[CBSE Delhi, Set 3, 2020]

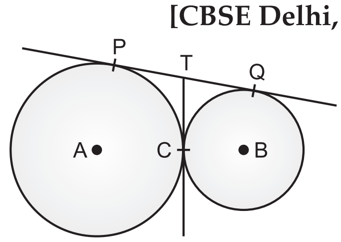


Fig. 5

Ans. Given : Two circles with centres A and B. Also, PT, TQ and TC are the tangents.

To Prove : CT bisects PQ *i.e.*, PT = QT

Proof : We know that,

The length of two tangents drawn to a circle from an external point are equal.

$\therefore$ For point T and circle with centre A,

$$TP = TC \quad \text{...(i)}$$

For point T and circle with centre B,

$$TQ = TC \quad \text{...(ii)}$$

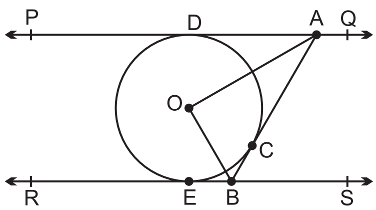
From (i) and (ii),

$$TP = TQ$$

$\therefore$ T is mid-point of PQ

Hence, point T bisects PQ **Hence Proved.**

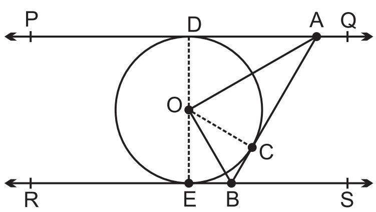
Q. 4. In Figure 3, PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting PQ at A and RS at B. Prove that $\angle$AOB = 90°.



[CBSE OD, Set 1, 2019]

Ans. Given, $PQ \parallel RS$

To prove: $\angle AOB = 90°$



Construction: Join OC, OD and OE.

In ΔODA and ΔOCA

$OD = OC$ (radii of circle)

$OA = OA$ (common)

$AD = AC$

(tangents drawn from same point)

By SSS congruency

$$\Delta ODA \cong \Delta OCA$$

Then, $\angle DOA = \angle AOC$ ...(i)

Similiarly, in ΔEOB and ΔBOC, we have

$$\Delta EOB \cong \Delta BOC$$

$$\angle EOB = \angle BOC \quad \text{...(ii)}$$

EOD is a diameter of circle, therefore it is a straight line.

Hence,

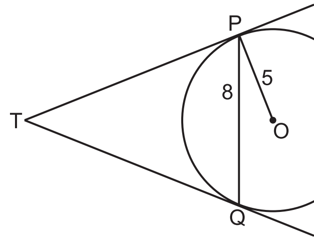
$$\angle DOA + \angle AOC + \angle EOB + \angle BOC = 180°$$

$$2(\angle AOC) + 2(\angle BOC) = 180°$$

$$\angle AOC + \angle BOC = 90°$$

$\angle AOB = 90°$. **Hence Proved.**

Q. 5. In Fig. *PQ* is a chord of length 8 cm of a circle of radius 5 cm and centre *O*. The tangents at *P* and *Q* intersect at point *T*. Find the length of *TP*.



[CBSE Delhi, Set 1, 2019]

Ans. Join OT, let it intersect PQ at the point R

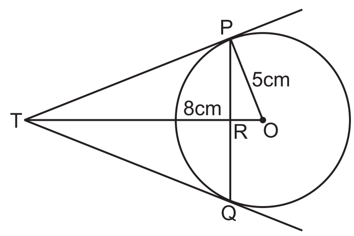
Now, ΔTPQ is an isosceles triangle and TO is the angle bisector of $\angle PTQ$. So, $OT \perp PQ$ and therefore, OT bisects PQ

$\therefore \quad PR = RQ = 4$ cm

Also, $OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2}$

$= \sqrt{25 - 16}$

$= \sqrt{9} = 3$ cm



Now, $\angle TPR + \angle RPO = 90° = \angle TPR + \angle PTR$

$[\because$ In ΔTRP, $\angle TRP = 90°]$

$\Rightarrow \quad \angle RPO = \angle PTR$

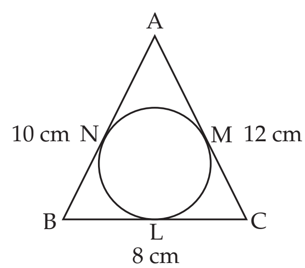
So, $\Delta TRP \sim \Delta PRO$ (By AA rule)

$$\because \quad \frac{TP}{PO} = \frac{RP}{RO}$$

$$\frac{TP}{5} = \frac{4}{3}, \; TP = \frac{20}{3} \text{cm}$$

Hence, the length of $TP = \dfrac{20}{3}$ cm

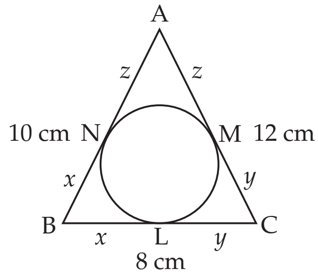
Q. 6. In Fig. a circle is inscribed in a ΔABC having sides $BC = 8$ cm, $AB = 10$ cm and $AC = 12$ cm. Find the lengths *BL*, *CM* and *AN*.



[CBSE Delhi, Set 2, 2019]

Ans. A circle is inscribed in a ΔABC

$AB = 10$ cm, $BC = 8$ cm and $AC = 12$ cm



Let $BN = BL = x$

$CL = CM = y$

$AN = AM = z$

(Tangents drawn from an exterior points are equal in length.)

Perimeter of $\Delta = AB + BC + CA = 10 + 8 + 12$

$\Rightarrow \quad x + z + x + y + y + z = 30$

$2(x + y + z) = 30$

$x + y + z = 15 \quad ...(i)$

Also, AB = 10 cm

$\Rightarrow \quad x + z = 10 \quad ...(ii)$

and AC = 12

$\Rightarrow \quad y + z = 12 \quad ...(iii)$

and BC = 8 cm

$x + y = 8 \quad ...(iv)$

From equations (i) and (ii),

$y = 5$ cm

From equations (i) and (iii)

$x = 3$ cm

From equations (i) and (iv)

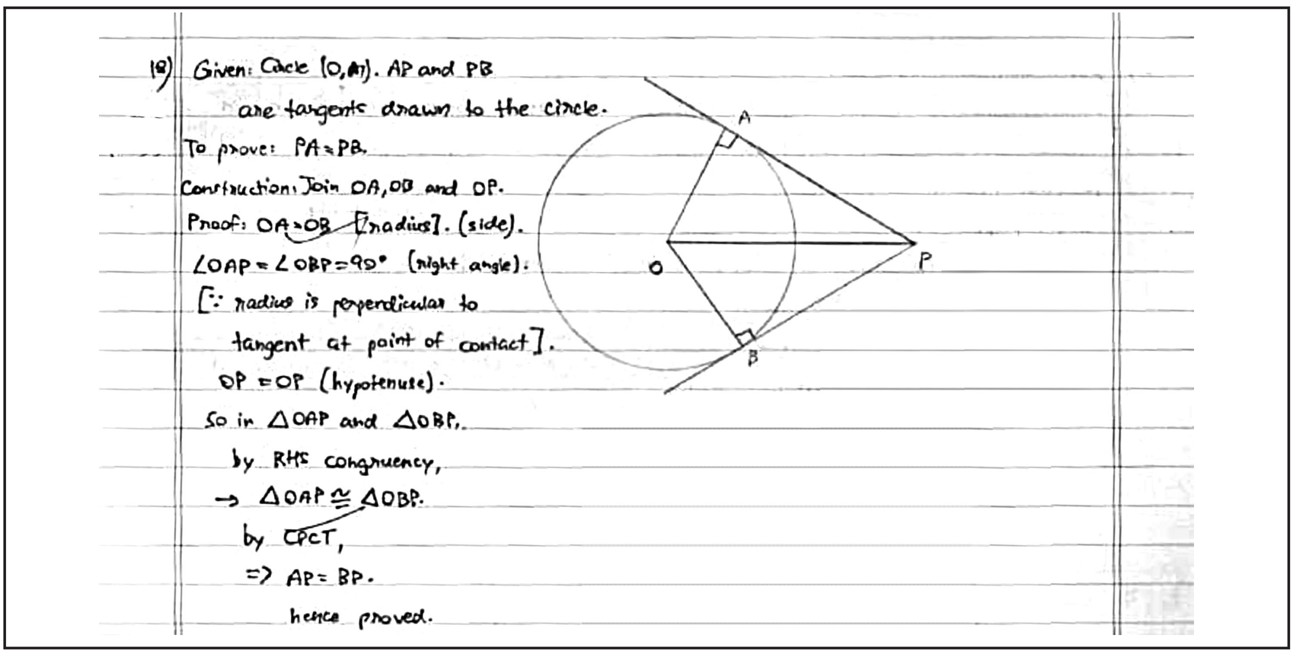
$z = 7$ cm

So, BL = 3 cm, CM = 5 cm, AN = 7 cm.

Q. 7. Prove that the lengths of tangents drawn from an external point of a circle are equal.
[CBSE, 2018]
[CBSE OD, Term 2, Set 1, 2017]
[CBSE OD, Term 2, Set 1, 2016]
[CBSE Delhi, Term 2, Set 1, 2016]
[CBSE OD, Term 2, Set 1, 2015]

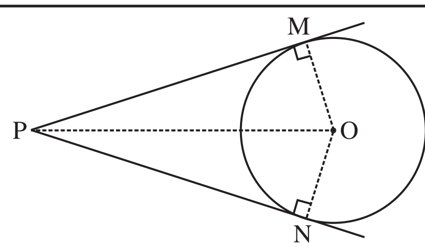
Ans.

Topper's Answers



19) Given: Circle (O, r). AP and PB are tangents drawn to the circle.
To prove: PA = PB.
Construction: Join OA, OB and OP.
Proof: OA = OB [radius]. (side).
∠OAP = ∠OBP = 90° (right angle).
[∵ radius is perpendicular to tangent at point of contact].
OP = OP (hypotenuse).
So in ΔOAP and ΔOBP,
by RHS congruency,
→ ΔOAP ≅ ΔOBP.
by CPCT,
⇒ AP = BP.
hence proved.

Given: A circle with centre O on which two tangents PM and PN are drawn from an external point P.



To prove: $PM = PN$

Construction: Join OM, ON, and OP.

Proof: Since tangent and radius are perpendicular at point of contact,

$\therefore \quad \angle OMP = \angle ONP = 90°$

In ΔPOM and ΔPON,

$OM = ON$ (Radii)

$\angle OMP = \angle ONP$ (Each 90°)

$PO = OP$ (Common)

$\therefore \quad \Delta OMP \cong \Delta ONP$ (RHS cong.)

$\therefore \quad PM = PN$ (C.P.C.T)

Hence Proved.

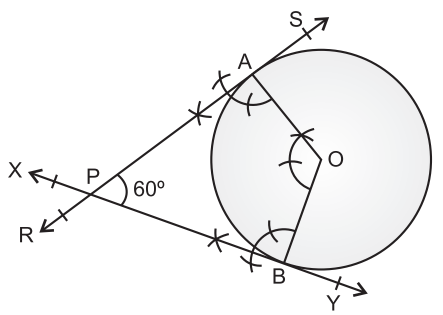
Long Answer Type Questions (4 marks each)

Q. 1. Draw two tangents to a circle or radius 4 cm, which are inclinded to each other at an angle of 60°. [CBSE OD, Set 2, 2020]

Ans. Steps of construction :

(i) Draw a circle with O as centre and radius = 4 cm.

(ii) Take a point A on the circumference of the circle and join OA.



(iii) Construct ∠AOB = 120° such that point B is on circumference of the circle.

(iv) Draw RS perpendicular to OA and XY perpendicular to OB.

Let XY and RS intersect each other at P.

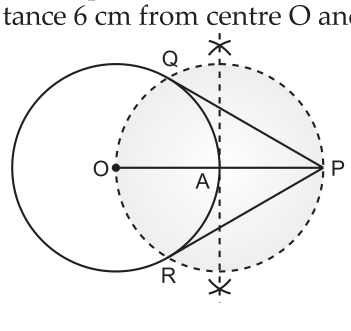
Hence, AP and BP are the tangents inclined at 60°.

Q. 2. Draw a circle of radius 3.5 cm. From a point P, 6 cm from its centre, draw two tangents to the circle. [CBSE OD, Set 3, 2020]

Ans. Steps of construction :

(i) Draw a circle with centre O and radius 3.5 cm.

(ii) Take a point P outside the circle at a distance 6 cm from centre O and join OP.



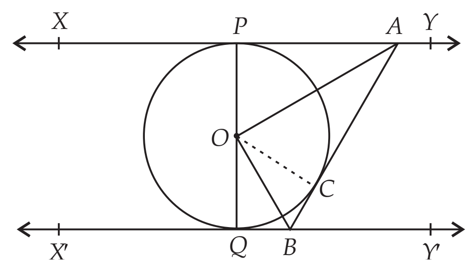
(iii) Draw perpendicular bisector of OP to get its mid-point A.

(iv) Taking A as centre and OA (or AP) as radius, draw a circle which intersects the circle of radius 3.5 cm at points Q and R.

(v) Join PQ and PR.

Thus, PQ and PR are the required tangents.

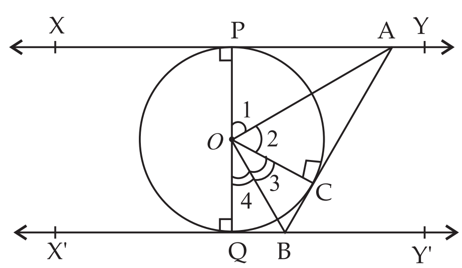
Q. 3. In the given figure, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C, is intersecting XY at A and $X'Y'$ at B. Prove that $\angle AOB = 90°$.



[CBSE OD, Term 2, Set 1, 2017]

Ans. Given, XY & $X'Y'$ are parallel.

Tangent AB is another tangent which touches the circle at C.



To prove: $\angle AOB = 90°$

Const.: Join OC.

Proof: In ΔOPA and ΔOCA,

$OP = OC$ (Radii)

$\angle OPA = \angle OCA$ (Radius ⊥ Tangent)

$OA = OA$ (Common)

$\therefore \quad \Delta OPA \cong \Delta OCA$

$\therefore \quad \angle 1 = \angle 2$ (CPCT) ...(i)

Similarly, $\quad \Delta OQB \cong \Delta OCB$

$\therefore \quad \angle 3 = \angle 4$ ...(ii)

Also, POQ is a diameter of circle

$\therefore \quad \angle POQ = 180°$

(Straight angle)

$\therefore \quad \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

From eq. (i) and (ii),

$$\angle 2 + \angle 2 + \angle 3 + \angle 3 = 180°$$

$$2(\angle 2 + \angle 3) = 180°$$

$$\angle 2 + \angle 3 = 90°$$

Hence, $\quad \angle AOB = 90°$

Hence Proved.

Q. 4. In Fig. 7, two equal circles, with centres *O* and *O'*, touch each other at *X*. *OO'* produced meets the circle with centre *O'* at *A*. *AC* is tangent to the circle with centre *O*, at the point *C*. *O'D* is perpendicular *AC*. Find the value of $\frac{DO'}{CO}$.

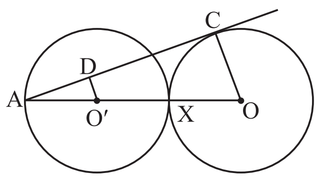


Figure 7

[CBSE OD, Term 2, Set 1, 2016]

Ans. Given, AC is tangent to the circle with centre O and $O'D$ is perpendicular to AC.

then, $\quad \angle ACO = 90°$

Also, $\quad \angle ADO' = 90°$

$\quad \angle CAO = \angle DAO'$

($\because$ Common angle)

$\therefore \quad \Delta AO'D \sim \Delta AOC$

$$\Rightarrow \quad \frac{AO'}{AO} = \frac{DO'}{CO}$$

$$\therefore \quad \frac{AO'}{3AO'} = \frac{DO'}{CO}$$

$$\left(\begin{array}{ll} \because & AX = 2AO' \\ \text{and} & OX = AO' \end{array}\right)$$

$$\Rightarrow \quad \frac{DO'}{CO} = \frac{1}{3}$$

Q. 5. Prove that tangent drawn at any point of a circle is perpendicular to the radius through the point of contact.

[CBSE OD, Term 2, Set 2, 2016]

[CBSE Delhi, Term 2, Set 3, 2016]

[CBSE OD, Set 2, 2015]

[CBSE Delhi, Term 2, Set 1, 2015]

Ans. Given, a tangent AB at point P of the circle with centre O.

To prove: $OP \perp AB$.

Construction: Join OQ, where Q is a point (other than P) on AB.

Proof: Since Q is a point on the tangent AB (other than P),

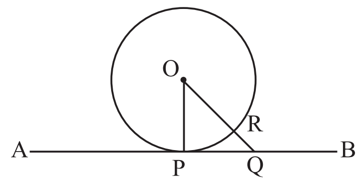
$\therefore$ Q lies outside the circle.

Let OQ intersect the circle at R.

$\Rightarrow \quad OR < OQ$

But, $\quad OP = OR$

(Radii of the circle)



$\therefore \quad OP < OQ.$

Thus, OP is the shortest distance than any other line segment joining O to any point of AB.

But, we know that the shortest distance between a point and a line is the perpendicular distance.

$\therefore \quad OP \perp AB$ **Hence Proved.**

Q. 6. In Fig. 8, *O* is the centre of a circle of radius 5 cm. *T* is a point such that *OT* = 13 cm and *OT* intersects circle at *E*. If *AB* is a tangent to the circle at *E*, find the length of *AB*, where *TP* and *TQ* are two tangents to the circle.

[CBSE Delhi, Term 2, Set 1, 2016]

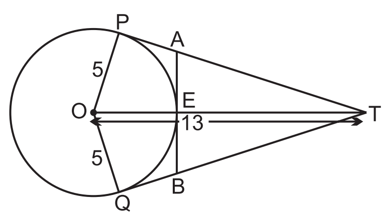


Fig. 8

Ans. Given, a circle with centre of radius 5 cm

and $\quad OT = 13$ cm

Since, PT is a tangent at P and OP is a radius through P

$\therefore \quad OP \perp PT$

In Δ OPT,

$$(PT)^2 = (OT)^2 - (OP)^2$$

$\Rightarrow \quad PT = \sqrt{(13)^2 - (5)^2}$

$\Rightarrow \quad PT = \sqrt{(169 - 25} = \sqrt{144}$

$\Rightarrow \quad PT = 12$ cm

And, $\quad TE = OT - OE = (13 - 5)$ cm $= 8$ cm

Now, $\quad PA = AE$

Let $\quad PA = AE = x$

Then, in Δ *AET*,

$$(AT)^2 = (AE)^2 + (ET)^2$$

$\Rightarrow \quad (12 - x)^2 = (x)^2 + (8)^2$

$\Rightarrow \quad 144 + x^2 - 24x = x^2 + 64$

$\Rightarrow \quad 24x = 80$

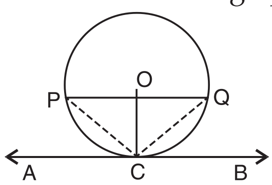
$\Rightarrow \quad AE = x = 3.33$ cm

$\therefore \quad AB = 2AE = 2 \times 3.33$

$= 6.66$ cm

Q. 7. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.
[CBSE OD, Term 2, Set 1, 2015]

Ans. Given: *C* is the mid-point of the minor arc *PQ* and *O* is the centre of the circle and *AB* is tangent to the circle through point *C*.



Construction: Join *PC* and *QC*.

To prove: $\quad PQ \parallel AB$

Proof: It is given that C is the mid-point of the arc *PQ*.

So, $\quad$ Minor arc *PC* = Minor arc *QC*

$\Rightarrow \quad PC = QC$

Hence ΔPQC is an isosceles triangle.

Thus the perpendicular bisector of the side *PQ* of ΔPQC passes through vertex *C*.

But we know that the perpendicular bisector of a chord passes through centre of the circle.

So, the perpendicular bisector of *PQ* passes through the center *O* of the circle.

Thus, the perpendicular bisector of *PQ* passes through the points *O* and *C*.

$\Rightarrow \quad PQ \perp OC \quad$ ...(i)

AB is a tangent to the circle through the point *C* on the circle

$\therefore \quad AB \perp OC \quad$ ...(ii)

From equations (i) and (ii), the chord *PQ* and tangent *AB* of the circle are perpendicular to the same line *OC*.

Hence, $\quad AB \parallel PQ$

Hence Proved.

Q. 8. In Fig. 7, tangents *PQ* and *PR* are drawn from an external point *P* to a circle with centre *O*, such that $\angle RPQ = 30°$. *A* chord RS is drawn parallel to the tangent *PQ*. Find $\angle RQS$.

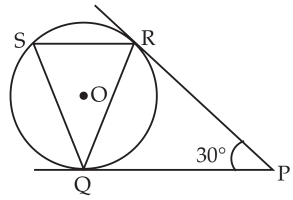


Figure 7

[CBSE Delhi, Term 2, Set 1, 2015]

Ans. We have, $\quad PR = PQ$

and $\quad \angle PRQ = \angle PQR$

In ΔPQR,

$\angle PRQ + \angle PQR + \angle RPQ = 180°$

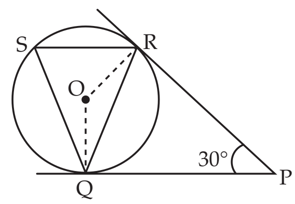
$\Rightarrow \quad 2\angle PRQ + 30° = 180°$

$\Rightarrow \quad \angle PRQ = \dfrac{180° - 30°}{2} = 75°$

$\because SR \parallel QP$ and *QR* is a transversal

$\therefore \quad \angle SRQ = \angle PQR = 75°$

Join *OR*, *OQ*.



$\therefore \quad \angle ORQ = \angle RQO = 90° - 75° = 15°$

$\therefore \quad \angle QOR = (180° - 2 \times 15°)$

$= 180° - 30° = 150°$

$\angle QSR = \dfrac{1}{2}\angle QOR$

$= 75°$

(Angle subtended on arc is half the angle subtended on centre)

$\therefore$ In ΔSQR

$\angle RQS = 180° - (\angle SRQ + \angle RSQ)$

$= 180° - (75° + 75°)$

$\therefore \quad \angle RQS = 30°$