Chapter - 1

Numbers

Question 1.
Fill in the blanks.
(i) The number of prime numbers between 11 and 60 is _____
(ii) The numbers 29 and _____ are twin primes.
(iii) 3753 is divisible by 9 and hence divisible by _____
(iv) The number of distinct prime factors of the smallest 4 digit number is_____
(v) The sum of distinct prime factors of 30 is _____

Solution:
(i) 12
(ii) 31
(iii) 3
(iv) 2
(v) 10

Question 2.
Say True or False.
(i) The sum of any number of odd numbers is always even.
(ii) Every natural number is either prime or composite.
(iii) If a number is divisible by 6, then it must be divisible by 3.
(iv) 16254 is divisible by 2, 3, 6, and 9.
(v) The number of distinct prime factors of 105 is 3.

Solution:
(i) False
(ii) False
(iii) True
(iv) True
(v) True

Question 3.
Write the smallest and the biggest two-digit prime number.

Solution:
Smallest two-digit prime number – 11
Biggest two-digit prime number – 97.

Question 4.
Write the smallest and the biggest three-digit composite number.

Solution:
Smallest three-digit composite number – 100
Biggest three-digit composite number – 999

Question 5.
The sum of any three odd natural number is odd. Justify this statement with an example.

Solution:
True.
$1 + 3 + 5 = 9$ (Odd)

Question 6.
The digits of the prime number 13 can be reversed to get another prime number 31. Find if any such pair exists up to 100.

Solution:
(17, 71), (37, 73) and (79, 97)

Question 7.
Your friend says that every odd number is prime. Give an example to prove him/her wrong.

Solution:
False. 15 is an odd number. But not prime.

Question 8.
Each of the composite numbers has at least three factors. Justify this statement with an example.

Solution:
Factors of 4 are 1, 2, 4

Question 9.
Find the dates of any month in a calendar which are divisible by both 2 and 3.

Solution:
Every month the dates 6, 12, 18, 24, and 30 (excluding February) are divisible by both 2 and 3.

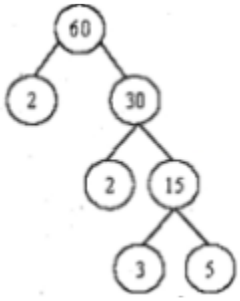
Question 10.
I am a two-digit prime number and the sum of my digits is 10.1 am also one of the factors of 57. Who am I?

Solution:
Two-digit prime numbers with a sum of digits 10 are 19, 37, 73.
of these factors of 57 is 19. the number is 19.

Question 11.
Find the prime factorisation of each number by factor tree method and division method.
(i) 60
(ii) 128
(iii) 144
(iv) 198
(v) 420
(vi) 999

Solution:

i)

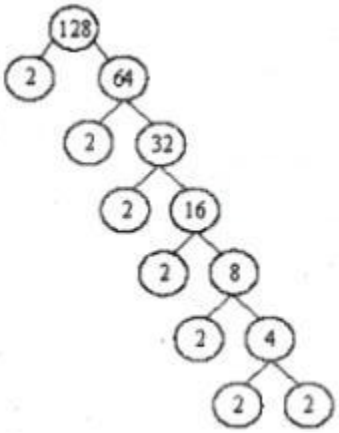


2	60
2	30
3	15
	5

$60 = 2 \times 2 \times 3 \times 5$

$60 = 2 \times 30 = 2 \times 2 \times 15$
$= 2 \times 2 \times 3 \times 5$

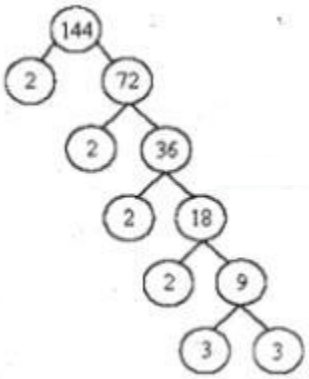
ii)



2	128
2	64
2	32
2	16
2	8
2	4
	2

$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$128 = 2 \times 64 = 2 \times 2 \times 32$
$= 2 \times 2 \times 2 \times 16$
$= 2 \times 2 \times 2 \times 2 \times 8$
$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 4$
$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

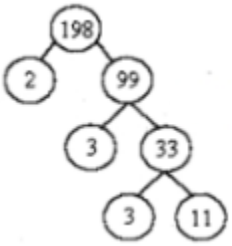
iii)



2	144
2	72
2	36
2	18
3	9
	3

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$144 = 2 \times 72 = 2 \times 2 \times 36$
$= 2 \times 2 \times 2 \times 18 = 2 \times 2 \times 2 \times 2 \times 9$
$= 2 \times 2 \times 2 \times 2 \times 3 \times 3$

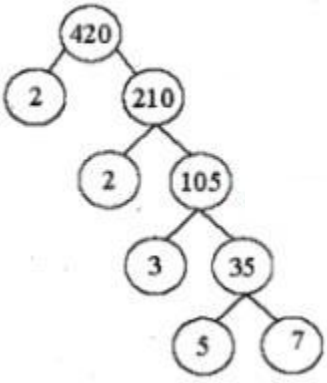
iv)



2	198
3	99
3	33
	11

$198 = 2 \times 3 \times 3 \times 11$
$198 = 2 \times 99 = 2 \times 3 \times 33 = 2 \times 3 \times 3 \times 11$

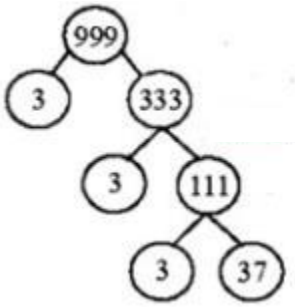
v)

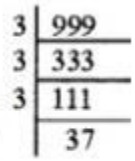


2	420
2	210
3	105
5	35
	7

$420 = 2 \times 210$
$= 2 \times 2 \times 105$
$= 2 \times 2 \times 3 \times 35 = 2 \times 2 \times 3 \times 5 \times 7$

vi)

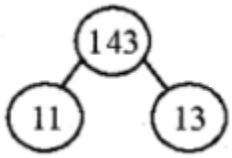


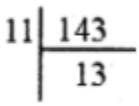


$999 = 3 \times 333 = 3 \times 3 \times 111$
$= 3 \times 3 \times 3 \times 37$

Question 12.
If there are 143 math books to be arranged in equal numbers in all the stacks, then find the number of books in each stack and also the number of stacks.

Solution:





$143 = 11 \times 13$ (11, 13) or (13, 11)
$143 = 11 \times 13$

Question 13.
The difference between two successive odd numbers is
(a) 1
(b) 2
(c) 3
(d) 0

Solution:
(b) 2

Question 14.
The only even prime number is
(a) 4

(b) 6
(c) 2
(d) 0

Solution:
(c) 2

Question 15.
Which of the following numbers is not a prime?
(a) 53
(b) 92
(c) 97
(d) 71

Solution:
(b) 92

Question 16.
The sum of the factors of 27 is
(a) 28
(b) 37
(c) 40
(d) 31

Solution:
(c) 40

Question 17.
The factors of the number are 1, 2, 4, 5, 8, 10, 16, 20, 40, and 80. What is the number?
(a) 80
(b) 100
(c) 128
(d) 160

Solution:
(a) 80

Question 18.
The prime factorisation of 60 is $2 \times 2 \times 3 \times 5$. Any other number which has the same prime factorisation as 60 is
(a) 30
(b) 120

(c) 90
(d) Impossible

Solution:
(d) Impossible

Question 19.
If the number 6354*97 is divisible by 9 then the value of * is
(a) 2
(b) 4
(c) 6
(d) 7

Solution:
(a) 2

Question 20.
The number 87846 is divisible by
(a) 2 only
(b) 3 only
(c) 11 only
(d) all of these

Solution:
(d) all of these

Ex 1.2

Question 1.
Fill in the blanks
(i) The HCF of 45 and 75 is …….
(ii) The HCF of two successive even numbers is …….
(iii) If the LCM of 3 and 9 is 9, then their HCF is …….
(iv) The LCM of 26, 39 and 52 is ……..
(v) The least number that should be added to 57 so that the sum is exactly divisible by 2, 3, 4 and 5 is ……

Solution:
(i) 15
(ii) 2
(iii) 3

(iv) 156
(v) 3

The Least common multiple (LCM) of 15 and 20 is 60.

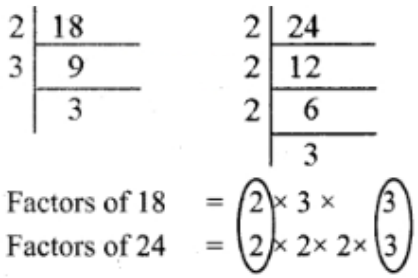
Question 2.
Say True or False
(i) The numbers 57 and 69 are co-primes.
(ii) The HCF of 17 and 18 is 1.
(iii) The LCM of two successive numbers is the product of the numbers.
(iv) The LCM of two co-primes is the sum of the numbers.
(v) The HCF of two numbers is always a factor of their LCM.

Solution:
(i) False
(ii) True
(iii) True
(iv) False
(v) True

So, now the prime factorization of 84 with the upside-down division method is 2 × 2 × 3 × 7.

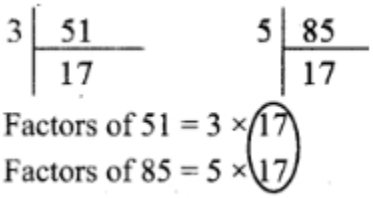
Question 3.
Find the HCF of each set of numbers using the prime factorisation method.
(i) 18, 24
(ii) 51, 85
(iii) 61, 76
(iv) 84, 120
(v) 27, 45, 81
(vi) 45, 55, 95

Solution:
(i) 18, 24



HCF $= 2 \times 3 = 6$

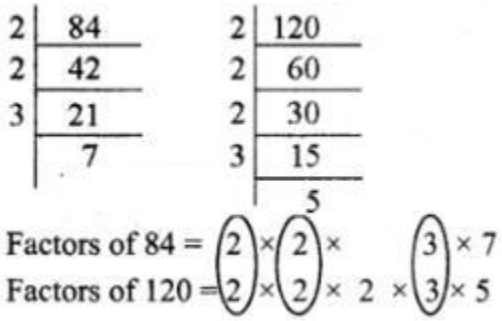
(ii) 51, 85



HCF $= 17$

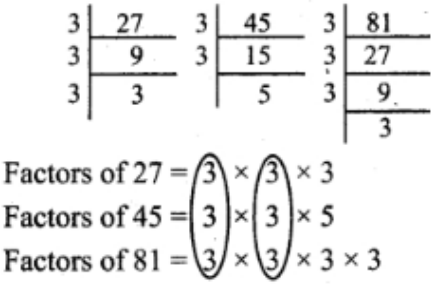
(iii) 61, 76
$61 = 1 \times 61$
For 61, 76, the common factor is 1
HCF $= 1$

(iv) 84, 120



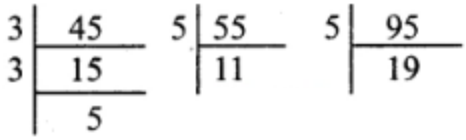
HCF $= 2 \times 2 \times 3 = 12$

(v) 27, 45, 81



HCF $= 3 \times 3 = 9$

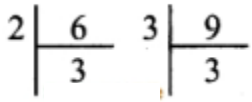
(vi) 45, 55, 95



Factors of $55 = 5 \times 11$
Factors of $45 = 3 \times 3 \times 5$
Factors of $95 = 5 \times 19$
HCF $= 5$

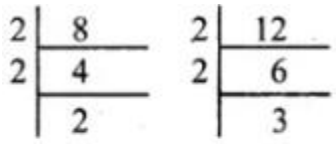
Question 4.
Find the LCM of each set of numbers using the prime factorisation method.
(i) 6, 9
(ii) 8, 12
(iii) 10, 15
(iv) 14, 42
(v) 30, 40, 60
(vi) 15, 25, 75

Solution:
(i) 6, 9



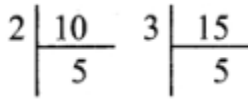
Factors of $6 = 2 \times 3$
Factors of $9 = 3 \times 3$
LCM of 6 and $9 = 3 \times 2 \times 3 = 18$

(ii) 8, 12



Factors of 8 $= 2 \times 2 \times 2$
Factors of 12 $= 2 \times 2 \times 3$
LCM of 8 and $12 = 2 \times 2 \times 2 \times 3 = 24$

(iii) 10, 15

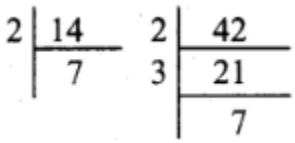


Factors of $10 = 2 \times 5$

Factors of $15 = 3 \times 5$

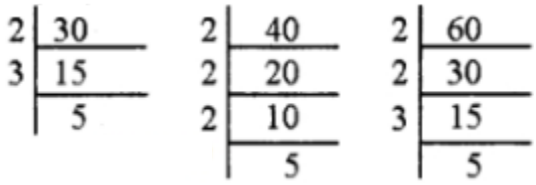
LCM of 10 and $15 = 5 \times 2 \times 3 = 30$

(iv) 14, 42



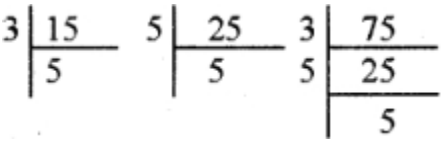
Factors of $14 = 2 \times 7$
Factors of $42 = 2 \times 3 \times 7$
LCM of 14 and $42 = 7 \times 2 \times 3 = 42$

(v) 30, 40, 60



Factors of $30 = 2 \times 3 \times 5$
Factors of $40 = 2 \times 2 \times 2 \times 5$
Factors of $60 = 2 \times 2 \times 3 \times 5$
LCM of 30, 40, $60 = 2 \times 5 \times 3 \times 2 \times 2 = 120$

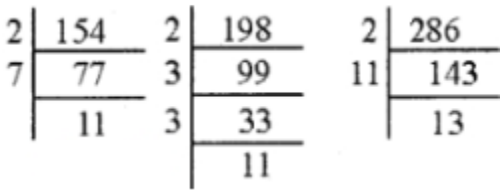
(vi) 15, 25, 75



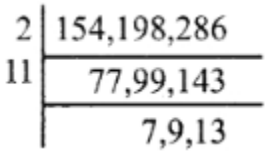
Factors of $15 = 3 \times 5$
Factors of $25 = 5 \times 5$
Factors of $75 = 3 \times 5 \times 5$
LCM of 15, 25, $75 = 5 \times 3 \times 5 = 75$

Question 5.
Find the HCF and the LCM of the numbers 154, 198, 286

Solution:
HCF



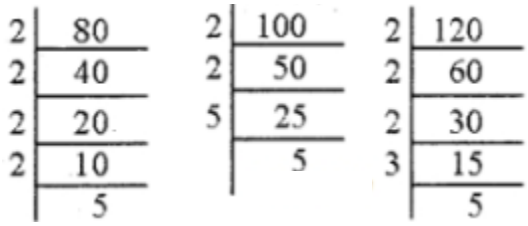
Factors of $154 = 2 \times 7 \times 11$
Factors of $198 = 2 \times 3 \times 3 \times 11$
Factors of $286 = 2 \times 13 \times 11$
HCF of 154, 198, 286 $= 11 \times 2 = 22$
LCM



LCM $= 2 \times 11 \times 7 \times 9 \times 13$
$= 22 \times 63 \times 13$
$= 18018$

Question 6.
What is the greatest possible volume of a vessel that can be used to measure exactly the volume of milk in cans (in full capacity) of 80 litres, 100 litres and 120 litres?

Solution:
This is an HCF related problem. So, we need to find the HCF of 80, 100, 120



$80 = 2 \times 2 \times 2 \times 2 \times 5$
$100 = 2 \times 2 \times 5 \times 5$
$120 = 2 \times 2 \times 2 \times 3 \times 5$
HCF of 80, 100 and $120 = 2 \times 2 \times 5 = 20$
Volume of the vessel = 20 litres

Question 7.
The traffic lights at three different road junctions change after every 40 seconds, 60 seconds, and 72 seconds respectively. If they changed simultaneously together at 8 a.m at the junctions, at what time will they simultaneously change together again?

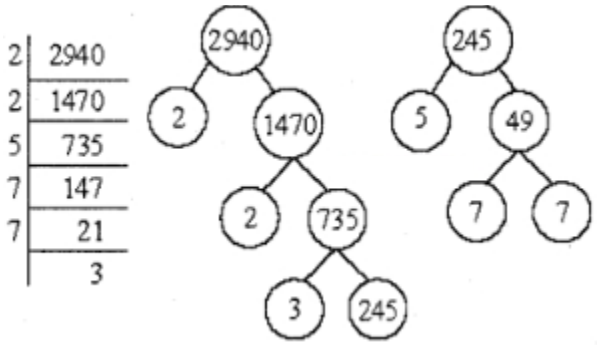
Solution:
This is an LCM related problem. So, we need to find the LCM of 40, 60, 72

2	40, 60, 72
2	20, 30, 36
3	10, 15, 18
5	10, 5, 6
2	2, 1, 6
	1, 1, 3

LCM of 40, 60 and 72
$= 2 \times 2 \times 3 \times 5 \times 2 \times 1 \times 1 \times 3$
$= 360$ seconds
$= 6$ minutes
The traffic lights will change simultane¬ously again at 8 : 06 am.

Question 8.
The LCM of two numbers is 210 and their HCF is 14. How many such pairs are possible.

Solution:
Product of two numbers = LCM × HCF
$x \times y = 210 \times 14$
$x \times y = 2940$
(12, 245), (20, 147)
Two pairs are possible.



Question 9.
The LCM of two numbers is 6 times their HCF. If the HCF is 12 and one of the numbers is 36, then find the other number.

Solution:
$36x = 6 \times 12 \times 12$

$\Rightarrow x = \frac{6 \times 12 \times 12}{36} = 24$

Objective Type Questions

Question 10.
Which of the following pairs is co-prime?
(a) 51, 63
(b) 52, 91
(c) 71, 81
(d) 81, 99

Solution:
(c) 71, 81

Question 11.
The greatest four-digit number which is exactly divisible by 8, 9, and 12 is
(a) 9999
(b) 9996
(c) 9696
(d) 9936

Solution:
(d) 9936

Question 12.
The HCF of two numbers is 2 and their LCM is 154. If the difference between the numbers is 8, then the sum is
(a) 26
(b) 36
(c) 46
(d) 56

Solution:
(b) 36

Question 13.
Which of the following cannot be the HCF of two numbers whose LCM is 120?
(a) 60
(b) 40
(c) 80
(d) 30

Solution:
(c) 80

Ex 1.3

Miscellaneous Practice Problems

Question 1.
Every even number greater than 2 can be expressed as the sum of two prime numbers. Verify this statement for every even number upto 16.

Solution:
Even numbers greater then 2 upto 16 are 4, 6, 8, 10, 12, 14 and 16
$4 = 2 + 2$
$6 = 3 + 3$
$8 = 3 + 5$
$10 = 3 + 7$ (or) $5 + 5$
$12 = 5 + 7$
$14 = 7 + 7$ (or) $3 + 11$
$16 = 5 + 11$ (or) $3 + 13$

Question 2.
Is 173 a prime? Why?

Solution:
Yes, 173 is a prime. Because it has only 1 and itself as factors.

Question 3.
For which of the numbers, from $n = 2$ to 8.
Is $2n - 1$ a prime?

Solution:
$n = 2 \Rightarrow 2n - 1 = 2 \times 2 - 1$
$= 4 - 1$
$= 3$ (prime)
$n = 3 \Rightarrow 2n - 1 = 2 \times 3 - 1$

$= 6 - 1$
$= 5$ (prime)
$n = 4 \Rightarrow 2n - 1 = 2 \times 4 - 1$
$= 8 - 1$
$= 7$ (prime)
$n = 5 \Rightarrow 2n - 1 = 2 \times 5 - 1$
$= 10 - 1$
$= 9$ (Not prime)
$n = 6 \Rightarrow 2n - 1 = 2 \times 6 - 1$
$= 12 - 1$
$= 11$ (prime)
$n = 7 \Rightarrow 2n - 1 = 2 \times 7 - 1$
$= 14 - 1$
$= 13$
$n = 8 \Rightarrow 2n - 1 = 2 \times 8 - 1$
$= 16 - 1$
$= 15$ (Not prime)

Question 4.
Explain your answer with the reason for the following statements.
(a) A number is divisible by 9 if it is divisible by 3.
(b) A number is divisible by 6 if it is divisible by 12.

Solution:
(i) False, 42 is divisible by 3 but it is not divisible by 9
(ii) True, 36 is divisible by 12. Also divisible by 6.

Question 5.
Find A as required
(i) The greatest 2 digit number 9 A is divisible by 2.
(ii) The least number 567A is divisible by 3.
(iii) The greatest 3 digit number 9A6 is divisible by 6.
(iv) The number A08 is divisible by 4 and 9.
(v) The number 225A85 is divisible by 11.

Solution:
(i) 98 A = 8
(ii) 5670 A = 0
(iii) 996 A = 9
(iv) 108 A = 1
(v) 225885 A = 8

Question 6.
Numbers divisible by 4 and 6 are divisible by 24. Verify this statement and support your answer with an example.

Solution:
False 12 is divisible by both 4 and 6. But not divisible by 24

Question 7.
The sum of any two successive odd numbers is always divisible by 4. Justify this statement with an example.

Solution:
True $3 + 5 = 8$ is divisible by 4.

Question 8.
Find the length of the longest rope that can be used to measure exactly the ropes of length 1 m 20cm, 3m 60 cm and 4 m.

Solution:
1 m 20 cm = 120 cm
3 m 60 cm = 360 cm
4 m = 400 cm
This is a HCF related problem. So, we need to find the HCF of 120,360 and 400.

2	120
2	60
2	30
3	15
	5

2	360
2	180
2	90
3	45
3	15
	5

2	400
2	200
2	100
2	50
5	25
	5

$120 = 2 \times 2 \times 2 \times 3 \times 5$
$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$
$400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5$
$HCF = 2 \times 2 \times 2 \times 5 = 40$
The length of the longest rope = 40 cm

Challenge Problems

Question 9.
The sum of three prime numbers is 80. The difference between the two of them is 4. Find the numbers.

Solution:
Given the sum of three prime numbers is 80
The numbers will be one or two-digit prime numbers. Also one of them is 2
Sum of the remaining 2 numbers = 78 [∵ one number must be even]
Also their difference = 4 (given) [Otherwise sum of three odd numbers is odd]
The numbers will be 37 and 41
The required numbers are 2, 37, 41

Question 10.
Find the sum of all the prime numbers between 10 and 20 and check whether that sum is divisible by all the single-digit numbers.

Solution:
Prime numbers between 10 and 20 are 11, 13, 17 and 19
Sum = 11 + 13 + 17 + 19 = 60
60 is divisible by 1, 2, 3, 4, 5 and 6.

Question 11.
Find the smallest number which is exactly divisible by all the numbers from 1 to 9.

Solution:
2520

Question 12.
The product of any three consecutive numbers is always divisible by 6. Justify this statement with an example.

Solution:
Yes. Because one of every two consecutive integers is even and so the product of three consecutive integers is even and divisible by 2.
Also one of every 3 consecutive integers is divisible by 3.
Product of any three consecutive integers is divisible by 6.
Example: 5 × 6 × 7

Question 13.
Malarvizhi, Karthiga, and Anjali are friends and natives of the same village. They work in different places. Malarvizhi comes to her home once in 5 days. Similarly, Karthiga and Anjali come to their homes once in 6 days and 10 days respectively, Assuming that they met each other on the 1st of October, when will all the three meet again?

Solution:
This is an LCM related problem. So, we need to find the LCM of 5, 6, and 10.

$$\begin{array}{r|l} 5 & 5, 6, 10 \\ \hline 2 & 1, 6, 2 \\ \hline & 1, 3, 1 \end{array}$$

LCM $= 5 \times 2 \times 1 \times 3 \times 1$
$= 30$
All the three will meet again once in 30 days.

Question 14.
In an apartment consisting of 108 floors, two lifts A & B starting from the ground floor, stop at every 3rd and 5th floors respectively. On which floors, will both of them stop together?

Solution:
LCM of 3 and $5 = 3 \times 5 = 15$

The lifts stop together at floors 15, 30, 45, 60, 75, 90, and 105.

Question 15.
The product of 2 two-digit numbers is 300 and their HCF is 5. What are the numbers?

Solution:
$15 \times 20 = 300$
HCF of 15 and 20 is 5
The numbers are 15 and 20

Question 16.
Find whether the number 564872 is divisible by 88. (use of the test of divisibility rule for 8 and 11 will help)

Solution:
564872 Divisibility by 8
564872 It is divisible by 8
Divisibility by 11
$5 + 4 + 7 = 16$
$6 + 8 + 2 = 16$
$16 - 16 = 0$
It is divisible by both 8 and 11 and hence divisible by 88.

Question 17.
Wilson, Mathan, and Guna can complete one round of a circular track in 10, 15, and

20 minutes respectively. If they start together from at the starting point of 7 am, at what time will they meet together again at the same starting point?

Solution:
This is an LCM related problem. So, we need to find the LCM of 10, 15, and 20.

5	10, 15, 20
2	2, 3, 4
	1, 3, 2

LCM $= 5 \times 2 \times 1 \times 3 \times 2 = 60$ min
They will meet together again after 60 minutes.
ie. 7.am + 60 minutes = 8 am.