CIRCULAR MOTION

Contents

JEE (Advance) Syllabus

Uniform Circular Motion

JEE (Main) Syllabus

Uniform Circular Motion

Note: ✍ Marked Questions can be used for Revision.

CIRCULAR MOTION

CIRCULAR MOTION :

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant, then its motion is known as circular motion with respect to that fixed (or moving) point. The fixed point is called centre, and the distance of particle from it is called radius.

1. KINEMATICS OF CIRCULAR MOTION :

- **Variables of Motion :**

(a) Angular Position :

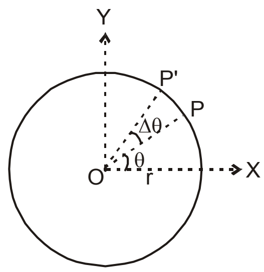
To decide the angular position of a point in space we need to specify (i) origin and (ii) reference line.

The angle made by the position vector w.r.t. origin, with the reference line is called angular position. Clearly angular position depends on the choice of the origin as well as the reference line.

Circular motion is a two dimensional motion or motion in a plane.

Suppose a particle P is moving in a circle of radius r and centre O.

The angular position of the particle P at a given instant may be described by the angle θ between OP and OX. This angle θ is called the **angular position** of the particle.



(b) Angular Displacement :

Definition: Angle through which the position vector of the moving particle rotates in a given time interval is called its angular displacement. Angular displacement depends on origin, but it does not depends on the reference line. As the particle moves on above circle its angular position θ changes. Suppose the point rotates through an angle $\Delta\theta$ in time Δt, then $\Delta\theta$ is angular displacement.

☞ **Important points :**

- Angular displacement is a dimensionless quantity. Its SI unit is radian, some other units are degree and revolution

  2π rad = 360° = 1 rev

- Infinitesimally small angular displacement is a vector quantity, but finite angular displacement is a scalar, because while the addition of the Infinitesimally small angular displacements is commutative, addition of finite angular displacement is not.

  $$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \quad \text{but} \quad \theta_1 + \theta_2 \neq \theta_2 + \theta_1$$

- Direction of small angular displacement is decided by right hand thumb rule. When the fingers are directed along the motion of the point then thumb will represents the direction of angular displacement.

(c) Angular Velocity ω

(i) Average Angular Velocity

$$\omega_{av} = \frac{\text{Angular displacement}}{\text{Total time taken}}$$

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are angular position of the particle at time t_1 and t_2. Since angular displacement is a scalar, average angular velocity is also a scalar.

(ii) Instantaneous Angular Velocity

It is the limit of average angular velocity as Δt approaches zero. i.e.

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t} = \frac{d\vec{\theta}}{dt}$$

Since infinitesimally small angular displacement $\overrightarrow{d\theta}$ is a vector quantity, instantaneous angular velocity $\vec{\omega}$ is also a vector, whose direction is given by right hand thumb rule.

☞ **Important points :**

- Angular velocity has dimension of $[T^{-1}]$ and SI unit rad/s.
- For a rigid body, as all points will rotate through same angle in same time, angular velocity is a characteristic of the body as a whole, e.g., angular velocity of all points of earth about earth's axis is $(2\pi/24)$ rad/hr.
- If a body makes 'n' rotations in 't' seconds then average angular velocity in radian per second will be

$$\omega_{av} = \frac{2\pi n}{t}$$

If T is the period and 'f' the frequency of uniform circular motion

$$\omega_{av} = \frac{2\pi}{T} = 2\pi f$$

SOLVED EXAMPLE

Example 1. If angular displacement of a particle is given by $\theta = a - bt + ct^2$, then find its angular velocity.

Solution : $\omega = \frac{d\theta}{dt} = -b + 2ct$

Example 2. Is the angular velocity of rotation of hour hand of a watch greater or smaller than the angular velocity of Earth's rotation about its own axis ?

Solution : Hour hand completes one rotation in 12 hours while Earth completes one rotation in 24 hours. So, angular velocity of hour hand is double the angular velocity of Earth. $\left(\omega = \frac{2\pi}{T}\right)$.

(d) Angular Acceleration α :

(i) Average Angular Acceleration :

Let ω_1 and ω_2 be the instantaneous angular speeds at times t_1 and t_2 respectively, then the average angular acceleration α_{av} is defined as

$$\vec{\alpha}_{av} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1} = \frac{\Delta\vec{\omega}}{\Delta t}$$

(ii) Instantaneous Angular Acceleration :

It is the limit of average angular acceleration as Δt approaches zero, i.e.,

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

since $\vec{\omega} = \frac{d\vec{\theta}}{dt}$, $\therefore\ \vec{\alpha} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}$, Also $\vec{\alpha} = \omega\frac{d\vec{\omega}}{d\theta}$

Important points :

- Both average and instantaneous angular acceleration are axial vectors with dimension $[T^{-2}]$ and unit rad/s^2.
- If $\alpha = 0$, circular motion is said to be uniform.

• Motion with constant angular velocity

$\theta = \omega t$, $\alpha = 0$

• Motion with constant angular acceleration

$\omega_0 \Rightarrow$ Initial angular velocity

$\omega \Rightarrow$ Final angular velocity

$\alpha \Rightarrow$ Constant angular acceleration

$\theta \Rightarrow$ Angular displacement

Circular motion with constant angular acceleration is analogous to one dimensional translational motion with constant acceleration. Hence even here equation of motion have same form.

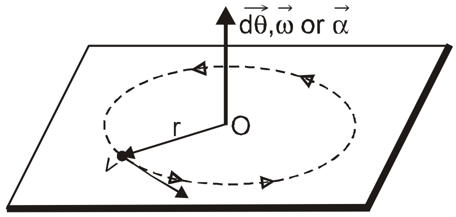
$$\omega = \omega_0 + \alpha t$$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\,\theta$

$\theta = \left(\frac{\omega + \omega_0}{2}\right) t$

$\theta_{n^{th}} = \omega_0 + \frac{\alpha}{2}(\theta_n - \theta_{n-1})$



2. RELATION BETWEEN SPEED AND ANGULAR VELOCITY :

$\vec{v} = \vec{\omega} \times \vec{r}$

Here, $\vec{v}$ is velocity of the particle, $\vec{\omega}$ is angular velocity about centre of circular motion and '$\vec{r}$' is position of particle w.r.t. center of circular motion.

Since $\vec{\omega} \perp \vec{r}$

$v = \omega\, r$ for circular motion.

SOLVED EXAMPLE

Example 3. A particle is moving with constant speed in a circular path. Find the ratio of average velocity to its instantaneous velocity when the particle describes an angle $\theta = \frac{\pi}{2}$

Solution : Time taken to describe angle θ, $t = \frac{\theta}{\omega} = \frac{\theta R}{v} = \frac{\pi R}{2v}$

Average velocity $= \frac{\text{Total displacement}}{\text{Total time}} = \frac{\sqrt{2}R}{\pi R/2v} = \frac{2\sqrt{2}}{\pi}v$

Instantaneous velocity = v

The ratio of average velocity to its instantaneous velocity $= \frac{2\sqrt{2}}{\pi}$ **Ans.**

Example 4. A fan is rotating with angular velocity 100 rev/sec. Then it is switched off. It takes 5 minutes to stop.

(a) Find the total number of revolution made before it stops. (Assume uniform angular retardation)

(b) Find the value of angular retardation

(c) Find the average angular velocity during this interval.

Solution : (a) $\theta = \left(\frac{\omega + \omega_0}{2}\right) t = \left(\frac{100 + 0}{2}\right) \times 5 \times 60 = 15000$ revolution.

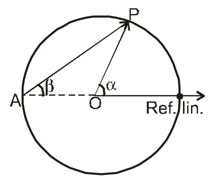
(b) $\omega = \omega_0 + \alpha t \quad \Rightarrow \quad 0 = 100 - \alpha\,(5 \times 60) \Rightarrow \alpha = \frac{1}{3}$ rev./sec^2

(c) $\omega_{av} = \frac{\text{Total Angle of Rotation}}{\text{Total time taken}} = \frac{15000}{50 \times 60} = 50$ rev./sec.

3. RELATIVE ANGULAR VELOCITY

Just as velocities are always relative, similarly angular velocity is also always relative. There is no such thing as absolute angular velocity. Angular velocity is defined with respect to origin, the point from which the position vector of the moving particle is drawn.

☞ Consider a particle P moving along a circular path shown in the figure given below



Here angular velocity of the particle P w.r.t. 'O' and 'A' will be different

Angular velocity of a particle P w.r.t. O, $\omega_{PO} = \frac{d\alpha}{dt}$

Angular velocity of a particle P w.r.t. A, $\omega_{PA} = \frac{d\beta}{dt}$

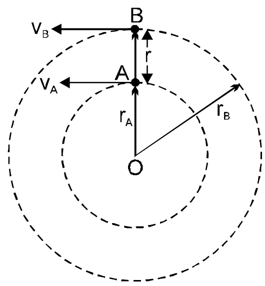
Definition :

Angular velocity of a particle 'A' with respect to the other moving particle 'B' is the rate at which position vector of 'A' with respect to 'B' rotates at that instant. (or it is simply, angular velocity of A with origin fixed at B). Angular velocity of A w.r.t. B, ω_{AB} is mathematically define as

$$\omega_{AB} = \frac{\text{Component of relative velocity of A w.r.t. B, perpendicular to line}}{\text{seperation between A and B}} = \frac{(V_{AB})_\perp}{r_{AB}}$$

☞ **Important points:**

- If two particles are moving on two different concentric circles with different velocities then angular velocity of B as observed by A will depend on their positions and velocities. Consider the case when A and B are closest to each other moving in same direction as shown in figure. In this situation



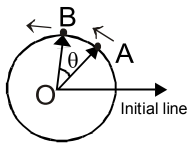
$$(V_{AB})_\perp = v_B - v_A$$

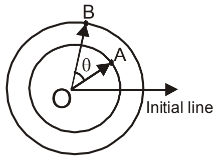
Separation between A and B is $\quad r_{BA} = r_B - r_A$

so, $\quad \omega_{AB} = \frac{(V_{AB})_\perp}{r_{AB}} = \frac{v_B - v_A}{r_B - r_A}$

- If two particles are moving on the same circle or different coplanar concentric circles in same direction with different uniform angular speed ω_A and ω_B respectively, the rate of change of angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is

$$\frac{d\theta}{dt} = \omega_B - \omega_A$$





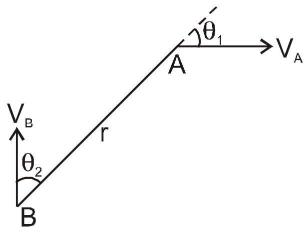
So the time taken by one to complete one revolution around O w.r.t. the other

$$T = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1T_2}{T_1 - T_2}$$

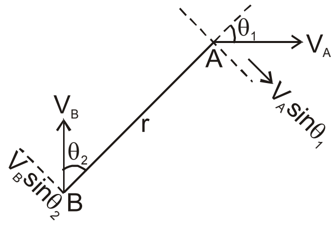
- $\omega_B - \omega_A$ is rate of change of angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. This is not angular velocity of B w.r.t. A. (Which is rate at which line AB rotates)

SOLVED EXAMPLE

Example 5. Find the angular velocity of A with respect to B in the figure given below:



Solution : Angular velocity of A with respect to B ;



$$\omega_{AB} = \frac{(V_{AB})_\perp}{r_{AB}}$$

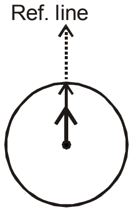
$\Rightarrow$ $(V_{AB})_\perp = V_A \sin\theta_1 + V_B \sin\theta_2$

$\Rightarrow$ $r_{AB} = r$

$$\omega_{AB} = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$$

Example 6. Find the time period of meeting of minute hand and second hand of a clock.

Solution :



$\omega_{min} = \frac{2\pi}{60}$ rad/min. , $\omega_{sec} = \frac{2\pi}{1}$ rad/min

$\theta_{sec} - \theta_{min} = 2\pi$ (for second and minute hand to meet again)

$(\omega_{sec} - \omega_{min})\, t = 2\pi$

$2\pi(1 - 1/60)\, t = 2\pi$ $\Rightarrow$ $t = \frac{60}{59}$ min.

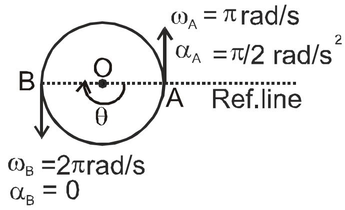
Example 7. Two particle A and B move on a circle. Initially Particle A and B are diagonally opposite to each other. Particle A move with angular velocity π rad/sec. , angular acceleration $\pi/2$ rad/sec^2 and particle B moves with constant angular velocity 2π rad/sec. Find the time after which both the particle A and B will collide.

Solution : Suppose angle between OA and OB = θ

then, rate of change of θ,

$$\dot{\theta} = \omega_B - \omega_A = 2\pi - \pi = \pi \text{ rad/sec}$$

$$\ddot{\theta} = \alpha_B - \alpha_A = -\frac{\pi}{2} \text{ rad/sec}^2$$



If angular displacement is $\Delta\theta$,

$$\Delta\theta = \dot{\theta}t + \frac{1}{2}\ddot{\theta}t^2$$

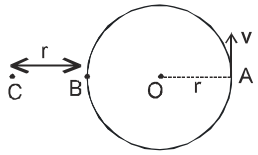
for A and B to collide angular displacement $\Delta\theta = \pi$

$$\Rightarrow \quad \pi = \pi t + \frac{1}{2}\left(\frac{-\pi}{2}\right)t^2$$

$$\Rightarrow \quad t^2 - 4t + 4 = 0$$

$\Rightarrow$ t = 2 sec. **Ans.**

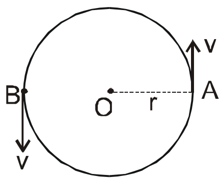
Example 8. A particle is moving with constant speed in a circle as shown, find the angular velocity of the particle A with respect to fixed point B and C if angular velocity with respect to O is ω.



Solution : Angular velocity of A with respect to O is ; $\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}} = \frac{v}{r} = \omega$

$$\therefore \quad \omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{v}{2r} = \frac{\omega}{2} \qquad \text{and} \qquad \omega_{AC} = \frac{(v_{AC})_\perp}{r_{AC}} = \frac{v}{3r} = \frac{\omega}{3}$$

Example 9. Particles A and B move with constant and equal speeds in a circle as shown, find the angular velocity of the particle A with respect to B, if angular velocity of particle A w.r.t. O is ω.



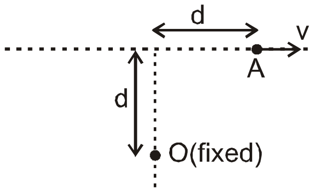
Solution : Angular velocity of A with respect to O is ; $\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}} = \frac{v}{r} = \omega$

Now, $\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}}$ $\Rightarrow$ $v_{AB} = 2v$,

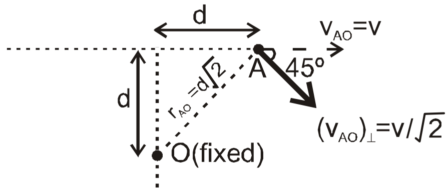
since v_{AB} is perpendicular to r_{AB},

$\therefore$ $(v_{AB})_\perp = v_{AB} = 2v$; $r_{AB} = 2r$ $\Rightarrow$ $\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{2v}{2r} = \omega$

Example 10. Find angular velocity of A with respect to O at the instant shown in the figure.



Solution : Angular velocity of A with respect to O is ;



$\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}}$

$v_{AO} = v$, $(v_{AO})_\perp = \frac{v}{\sqrt{2}}$

$r_{AO} = d\sqrt{2}$

$\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}} = \frac{v/\sqrt{2}}{d\sqrt{2}} = \frac{v}{2d}$

4. RADIAL AND TANGENTIAL ACCELERATION

There are two types of acceleration in circular motion ; Tangential acceleration and centripetal acceleration.

(a) Tangential acceleration :-

Component of acceleration directed along tangent of circle is called tangential acceleration. It is responsible for changing the speed of the particle. It is defined as,

$a_t = \frac{dv}{dt} = \frac{d|\vec{v}|}{dt}$ = Rate of change of speed.

$a_t = \alpha r$

☞ **IMPORTANT POINT**

(i) In vector form $\vec{a}_t = \vec{\alpha} \times \vec{r}$

(ii) If tangential acceleration is directed in direction of velocity then the speed of the particle increases.

(iii) If tangential acceleration is directed opposite to velocity then the speed of the particle decreases.

(b) Centripetal acceleration :-

It is responsible for change in direction of velocity. In circular motion, there is always a centripetal acceleration.

Centripetal acceleration is always variable because it changes in direction.

Centripetal acceleration is also called radial acceleration or normal acceleration.

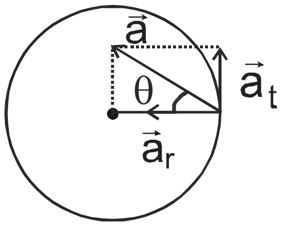
(c) Total acceleration :-

Total acceleration is vector sum of centripetal acceleration and tangential acceleration.

$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{a}_r + \vec{a}_t$$

$$a = \sqrt{a_t^2 + a_r^2}$$

$$\tan\theta = \frac{a_r}{a_t}$$



☞ **IMPORTANT POINT**

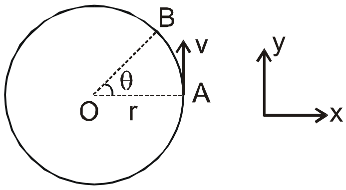
(i) Differentiation of speed gives tangential acceleration.

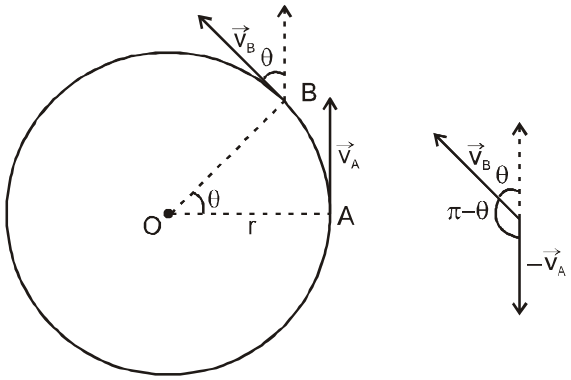
(ii) Differentiation of velocity ($\vec{v}$) gives total acceleration.

(iii) $\left|\frac{d\vec{v}}{dt}\right|$ & $\frac{d|\vec{v}|}{dt}$ are not same physical quantity. $\left|\frac{d\vec{v}}{dt}\right|$ is the magnitude of rate of change of velocity, i.e. magnitude of total acceleration and $\frac{d|\vec{v}|}{dt}$ is a rate of change of speed, i.e. tangential acceleration.

- **Calculation of centripetal acceleration :**

Consider a particle which moves in a circle with constant speed v as shown in figure.





$\therefore$ change in velocity between the point A and B is ;

$$\Delta \vec{v} = \vec{v}_B - \vec{v}_A$$

Magnitude of change in velocity.

$$|\Delta\vec{v}| = |\vec{v}_B - \vec{v}_A| = \sqrt{v_B^2 + v_A^2 + 2v_A v_B \cos(\pi - \theta)}$$

($v_A = v_B = v$, since speed is same)

$$\therefore |\Delta \vec{v}| = 2v \sin\frac{\theta}{2}$$

Distance travelled by particle between A and B = $r\theta$

Hence time taken, $\Delta t = \dfrac{r\theta}{v}$

Net acceleration , $|\vec{a}_{net}| = \left|\dfrac{\overrightarrow{\Delta v}}{\Delta t}\right| = \dfrac{2v\sin\theta/2}{r\theta/v} = \dfrac{v^2}{r}\dfrac{2\sin\theta/2}{\theta}$

If $\Delta t \to 0$, then θ is small, $\sin(\theta/2) = \theta/2$

$$\lim_{\Delta t\to 0}\left|\frac{\Delta\vec{v}}{\Delta t}\right| = \left|\frac{d\vec{v}}{dt}\right| = \frac{v^2}{r}$$

i.e. net acceleration is $\dfrac{v^2}{r}$ but speed is constant so that tangential acceleration, $a_t = \dfrac{dv}{dt} = 0$.

$$\therefore a_{net} = a_r = \frac{v^2}{r}$$

** Through we have derived the formula of centripetal acceleration under condition of constant speed, the same formula is applicable even when speed is variable.

☞ **IMPORTANT POINT**

In vector form $\vec{a}_c = \vec{\omega} \times \vec{v}$

SOLVED EXAMPLE

Example 11. The speed of a particle traveling in a circle of radius 20 cm increases uniformly from 6.0 m/s to 8.0 m/s in 4.0 s, find the angular acceleration.

Solution : Since speed increases uniformly, average tangential acceleration is equal to instantaneous tangential acceleration

$\therefore$ The instantaneous tangential acceleration is given by

$$a_t = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$= \frac{8.0 - 6.0}{4.0} \text{ m/s}^2 = 0.5 \text{ m/s}^2.$$

The angular acceleration is $\alpha = a_t / r$

$$= \frac{0.5 \text{ m/s}^2}{20 \text{ cm}} = 2.5 \text{ rad/s}^2.$$

Example 12. A particle is moving in a circle of radius 10 cm with uniform speed completing the circle in 4s, find the magnitude of its acceleration.

Solution : The distance covered in completing the circle is $2\pi r = 2\pi \times 10$ cm. The linear speed is

$$v = 2\pi r/t = \frac{2\pi \times 10 \text{ cm}}{4\text{s}} = 5\pi \text{ cm/s}.$$

The acceleration is $a = \dfrac{v^2}{r} = \dfrac{(5\pi \text{ cm/s})^2}{10 \text{ cm}} = 2.5\pi^2$ cm/s^2.

Example 13. A particle moves in a circle of radius 2.0 cm at a speed given by v = 4t, where v is in cm/s and t is in seconds.

(a) Find the tangential acceleration at t = 1s.

(b) Find total acceleration at t = 1s.

Solution : (a) Tangential acceleration

$$a_t = \frac{dv}{dt}$$

or $\quad a_t = \dfrac{d}{dt}(4t) = 4 \text{ cm/s}^2$

$$a_C = \frac{v^2}{R} = \frac{(4)^2}{2} = 8 \text{ cm/s}^2$$

$\Rightarrow \quad a = \sqrt{a_t^2 + a_C^2} = \sqrt{(4)^2 + (8)^2} = 4\sqrt{5}$ cm/s^2

Example 14. A particle begins to move with a tangential acceleration of constant magnitude 0.6 m/s^2 in a circular path. If it slips when its total acceleration becomes 1 m/s^2, Find the angle through which it would have turned before it starts to slip.

Solution : $a_{Net} = \sqrt{a_t^2 + a_c^2} \quad \Rightarrow \quad \omega^2 = \omega_0^2 + 2\alpha\theta$

$\because \omega_0 = 0 \quad$ so $\quad \omega^2 = 2\alpha\theta$

$\omega^2 R = 2(\alpha R\theta)$

$a_c = \omega^2 R = 2a_t\theta$

$1 = \sqrt{0.36 + (1.2 \times \theta)^2} \quad \Rightarrow \quad 1 - 0.36 = (1.2\,\theta)^2$

$\Rightarrow \quad \dfrac{0.8}{1.2} = \theta \quad \Rightarrow \quad \theta = \dfrac{2}{3}$ radian **Ans.**

5. DYNAMICS OF CIRCULAR MOTION :

If there is no force acting on a body it will move in a straight line (with constant speed). Hence if a body is moving in a circular path or any curved path, there must be some force acting on the body.

If speed of body is constant, the net force acting on the body is along the inside normal to the path of the body and it is called centripetal force.

$$\text{Centripetal force } (F_c) = ma_c = \frac{mv^2}{r} = m\,\omega^2 r$$

However if speed of the body varies then, in addition to above centripetal force which acts along inside normal, there is also a force acting along the tangent of the path of the body which is called tangential force.

$$\text{Tangential force } (F_t) = Ma_t = M\frac{dv}{dt} = M\,\alpha\, r$$; where α is the angular acceleration

☞ **IMPORTANT POINT**

Remember $\frac{mv^2}{r}$ is not a force itself. It is just the value of the net force acting along the inside normal which is responsible for circular motion. This force may be friction, normal, tension, spring force, gravitational force or a combination of them.

So to solve any problem in uniform circular motion we identify all the forces acting along the normal (towards center), calculate their resultant and equate it to $\frac{mv^2}{r}$.

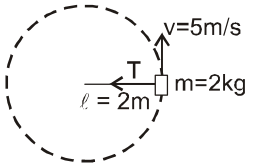
If circular motion is non uniform then in addition to above step we also identify all the forces acting along the tangent to the circular path, calculate their resultant and equate it to $\frac{mdv}{dt}$ or $\frac{md|\vec{v}|}{dt}$.

6. CIRCULAR MOTION IN HORIZONTAL PLANE :

SOLVED EXAMPLE

Example 15. A block of mass 2kg is tied to a string of length 2m, the other end of which is fixed. The block is moved on a smooth horizontal table with constant speed 5 m/s. Find the tension in the string.

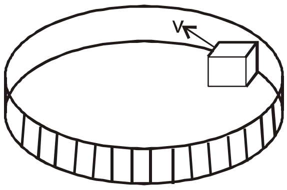
Solution :



here centripetal force is provided by tension.

$$T = \frac{mv^2}{r} = \frac{2\times 5^2}{2} = 25\text{ N}$$

Example 16. A block of mass m moves with speed v against a smooth, fixed vertical circular groove of radius r kept on smooth horizontal surface.



Find :

(i) normal reaction of the floor on the block.

(ii) normal reaction of the vertical wall on the block.

Solution : Here centripetal force is provided by normal reaction of vertical wall.

(i) normal reaction of floor $N_F = mg$

(ii) normal reaction of vertical wall $N_W = \dfrac{mv^2}{r}$.

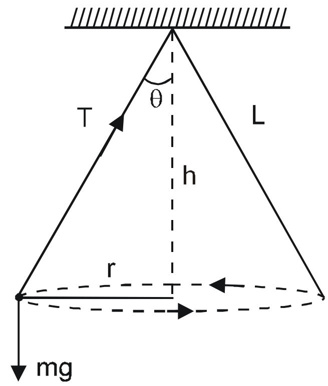
Example 17. A block of mass m is kept on the edge of a horizontal turn table of radius R, which is rotating with constant angular velocity ω (along with the block) about its axis. If coefficient of friction is μ, find the friction force between block and table

Solution : Here centripetal force is provided by friction force.

Friction force = centripetal force = $m\omega^2R$

Example 18. Consider a conical pendulum having bob of mass m is suspended from a ceiling through a string of length L. The bob moves in a horizontal circle of radius r. Find (a) the angular speed of the bob and (b) the tension in the string.

Solution : The situation is shown in figure. The angle θ made by the string with the vertical is given by



$$\sin\theta = r / L,\ \cos\theta = h/L = \frac{\sqrt{L^2 - r^2}}{L} \qquad \text{...(i)}$$

The forces on the particle are

(a) the tension T along the string and

(b) the weight mg vertically downward.

The particle is moving in a circle with a constant speed v. Thus , the radial acceleration towards the centre has magnitude v^2 / r. Resolving the forces along the radial direction and applying Newton's second law,

$$T\sin\theta = m(v^2 / r) \qquad \text{....(ii)}$$

As there is no acceleration in vertical direction, we have from Newton's law,

$$T\cos\theta = mg \qquad \text{.....(iii)}$$

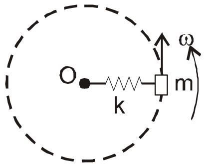
Dividing (ii) by (iii),

$$\tan\theta = \frac{v^2}{rg} \quad \text{or,} \quad v = \sqrt{rg\tan\theta}$$

$$\Rightarrow \quad \omega = \frac{v}{r} = \sqrt{\frac{g\tan\theta}{r}} = \sqrt{\frac{g}{h}} = \sqrt{\frac{g}{L\cos\theta}} = \sqrt{\frac{g}{(L^2 - r^2)^{\frac{1}{2}}}} \quad \textbf{Ans.}$$

And from (iii), $$T = \frac{mg}{\cos\theta} = \frac{mgL}{(L^2 - r^2)^{\frac{1}{2}}} \quad \textbf{Ans.}$$

Example 19. A block of mass m is tied to a spring of spring constant k , natural length ℓ, and the other end of spring is fixed at O. If the block moves in a circular path on a smooth horizontal surface with constant angular velocity ω , find tension in the spring.



Solution : Assume extension in the spring is x

Here centripetal force is provided by spring force.

Centripetal force, $kx = m\omega^2(\ell + x)$

$$\Rightarrow \quad x = \frac{m\omega^2\ell}{k - m\omega^2}$$

therefore,

$$\text{Tension} = kx = \frac{km\omega^2\ell}{k - m\omega^2} \quad \textbf{Ans.}$$

Example 20. A string breaks under a load of 50 kg. A mass of 1 kg is attached to one end of the string 10 m long and is rotated in horizontal circle. Calculate the greatest number of revolutions that the mass can make in one second without breaking the string.

Solution : $\omega = 2\pi n$,

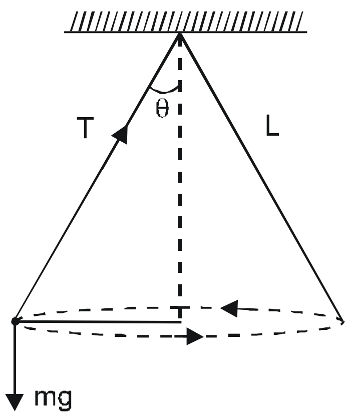
$T_{max} = 500$ N, $\quad r = L\sin\theta$

$T\sin\theta = m\omega^2 r$

$\Rightarrow \quad T = m\omega^2 L$

$\Rightarrow \quad T_{max} = m\,\omega^2_{max} L$

$\Rightarrow \quad T_{max} = m(2\pi n_{max})^2 L$

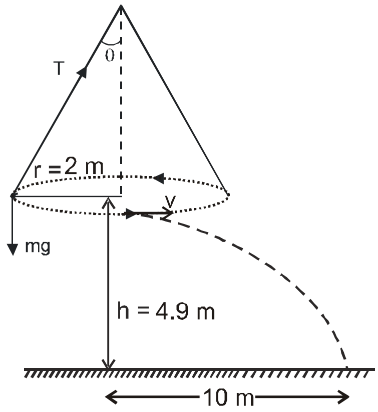


$$n_{max} = \frac{1}{2\pi}\sqrt{\frac{T_{max}}{mL}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{500}{1\times 10}} = \frac{\sqrt{50}}{2\pi} \text{ revolution per second.} \quad \textbf{Ans.}$$

Example 21. A boy whirls a stone in a horizontal circle of radius 2 m and at height 4.9 m above level ground. The string breaks, and the stone files off horizontally and strikes the ground at a point which is 10 m away from the point on the ground directly below the point where the string had broken. What is the magnitude of the centripetal acceleration of the stone while in circular motion? (g = 9.8 m/s^2)

Solution : $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2\times 4.9}{9.8}} = 1\text{ s}$



$v = \dfrac{10}{t} = 10\text{ m/s}$

$a = \dfrac{v^2}{R} = 50\text{ m/s}^2$

Example 22. A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its smooth surface and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.

Solution : Let ω be the angular speed of rotation of the bowl. Two force are acting on the ball.

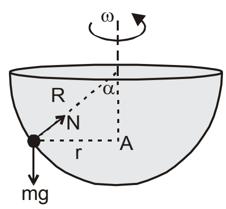
1. Normal reaction N 2. weight mg

The ball is rotating in a circle of radius r (= R sin α) with centre at A at an angular speed ω. Thus,

$N \sin\alpha = mr\omega^2 = mR\omega^2 \sin\alpha$

$N = mR\omega^2$ ....(i)

and $N\cos\alpha = mg$ ...(ii)



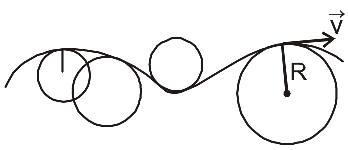
Dividing Eqs. (i) by (ii),

we get $\dfrac{1}{\cos\alpha} = \dfrac{\omega^2 R}{g}$

$\therefore \quad \boldsymbol{\omega = \sqrt{\dfrac{g}{R\cos\alpha}}}$

7. RADIUS OF CURVATURE

Any curved path can be assumed to be made of infinite circular arcs. Radius of curvature at a point is the radius of the circular arc at a particular point which fits the curve at that point.



If R is radius of the circular arc at a given point P, where velocity is $\vec{v}$, then centripetal force at that point is,

$$F_c = \frac{mv^2}{R} \quad \Rightarrow \quad R = \frac{mv^2}{F_C}$$

Now centripetal force F_c is simply the component of force perpendicular to velocity (let us say $F_\perp$).

$$\therefore \quad R = \frac{mv^2}{F_\perp} \quad \Rightarrow \quad R = \frac{v^2}{a_\perp}$$

Where, $a_\perp$ = Component of acceleration perpendicular to velocity.

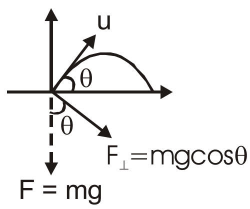
☞ If a particle moves in a trajectory given by $y = f(x)$ then radius of curvature at any point (x, y) of the trajectory is given by

$$\Rightarrow R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

SOLVED EXAMPLE

Example 23. A particle of mass m is projected with speed u at an angle θ with the horizontal. Find the radius of curvature of the path traced out by the particle at the point of projection and also at the highest point of trajectory.

Solution : at point of projection



$$R = \frac{mv^2}{F_\perp} = \frac{mu^2}{mg\cos\theta}$$

$$R = \frac{u^2}{g\cos\theta}$$ **Ans.**

at highest point

$$a_\perp = g \ , \ v = u\cos\theta \ : \quad R = \frac{v^2}{a_\perp} = \frac{u^2\cos^2\theta}{g}$$ **Ans.**

Example 24. A particle moves along the plane trajectory y(x) with constant speed v. Find the radius of curvature of the trajectory at the point x = 0 if the trajectory has the form of a parabola $y = ax^2$ where 'a' is a positive constant.

Solution : If the equation of the trajectory of a particle is given we can find the radius of trajectory of the instantaneous circle by using the formula

$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

As ; $y = ax^2$ $\Rightarrow$ $\frac{dy}{dx} = 2ax = 0$ (at x = 0) and $\frac{d^2y}{dx^2} = 2a$

Now radius of trajectory is given by

$$R = \frac{[1+0]^{3/2}}{2a} = \frac{1}{2a}$$

Aliter : This problem can also be solved by using the formula : $R = \frac{v^2}{a_\perp}$. $y = ax^2$,

differentiate with respect to time $\frac{dy}{dt} = 2ax\frac{dx}{dt}$ ....(1)

at x = 0, $v_y = \frac{dy}{dt} = 0$ hence $v_x = v$

since v_x is constant, $a_x = 0$

Now, differentiate (1) with respect to time $\frac{d^2y}{dt^2} = 2ax\frac{d^2x}{dt^2} + 2a\left(\frac{dx}{dt}\right)^2$

at x = 0, $v_x = v$

$\therefore$ net acceleration, $a = a_y = 2av^2$ (since $a_x = 0$)

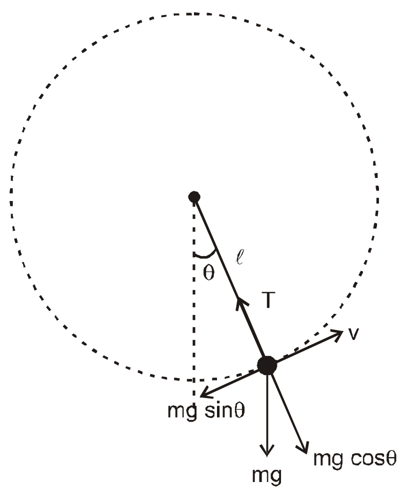
this acceleration is perpendicular to velocity (v_x)

Hence it is equal to centripetal acceleration

$$R = \frac{v^2}{a_\perp} = \frac{v^2}{2av^2} = \frac{1}{2a}$$ **Ans.**

8. MOTION IN A VERTICAL CIRCLE :

Let us consider the motion of a point mass tied to a string of length ℓ and whirled in a vertical circle. If at any time the body is at angular position θ, as shown in the figure, the forces acting on it are tension T in the string along the radius towards the center and the weight of the body mg acting vertically down wards.



Applying Newton's law along radial direction

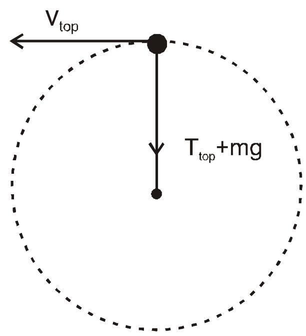
$$T - mg\cos\theta = m.a_c = \frac{mv^2}{\ell}$$

or $$T = \frac{mv^2}{\ell} + mg\cos\theta \qquad .....(1)$$

The point mass will complete the circle only and only if tension is never zero (except momentarily, if at all) if tension becomes zero at any point, string will go slack and subsequently, the only force acting on the body is gravity. Hence its subsequent motion will be similar to that of a projectile.

From equation ...(1), it is evident that tension decreases with increase in θ because cos θ is a decreasing function and v decreases with height. Hence tension is minimum at the top most point. i.e. $T_{min} = T_{topmost}$.

$T > 0$ at all points. $\Rightarrow$ $T_{min} > 0$.



However if tension is momentarily zero at highest point the body would still be able to complete the circle.

Hence condition for completing the circle (or looping the loop) is

$T_{min} \geq 0$ or $T_{top} \geq 0$.

$$T_{top} + mg = \frac{mv_{top}^2}{\ell} \quad ....(2)$$

Equation...(2) could also be obtained by putting $\theta = \pi$ in equation ..(1).

For looping the loop, $T_{top} \geq 0$.

$$\Rightarrow \quad \frac{mv_{top}^2}{\ell} \geq mg \qquad \Rightarrow \quad v_{top} \geq \sqrt{g\ell} \quad ....(3)$$

Condition for looping the loop is $v_{top} \geq \sqrt{g\ell}$.

If speed at the lowest point is u, then from conservation of mechanical energy between lowest point and top most point.

$$\frac{1}{2}mu^2 = \frac{1}{2}m v_{top}^2 + mg \cdot 2\ell$$

using equation ..(3) for v_{top} we get $u \geq \sqrt{5g\ell}$

i.e., for looping the loop, velocity at lowest point must be $\geq \sqrt{5g\ell}$.

☞ If velocity at lowest point is just enough for looping the loop, value of various quantities. (True for a point mass attached to a string or a mass moving on a smooth vertical circular track.)

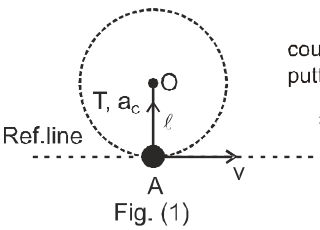


Fig. (1)

P.E. = 0

$v = \sqrt{5g\ell}$

$T - mg = \frac{mv^2}{\ell}$

could also be obtained by putting θ=0 in equation ..(1)

$\Rightarrow T = 6mg$

$a_c = 5g$

$a_t = 0$

$a_{net} = 5g$

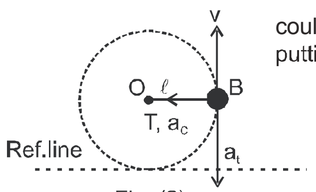


Fig. (2)

P.E.=mgℓ

By energy conservation, $v = \sqrt{3g\ell}$

$T = mv^2/\ell$

could also be obtained by putting θ =90° in equation ..(1)

T=3mg

$a_c = 3g$

$a_t = g$

$a_{net} = g\sqrt{10}$

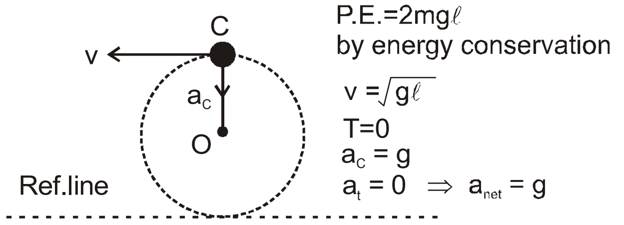


Fig. (3)

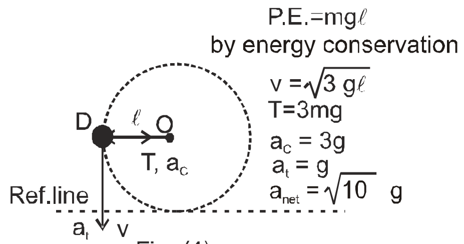


Fig. (4)

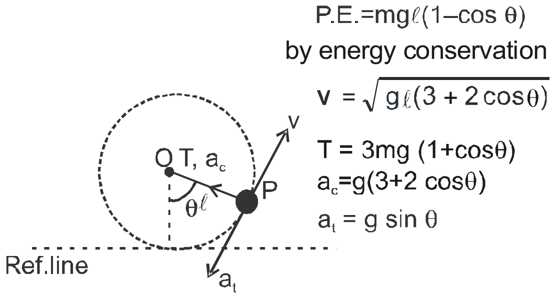


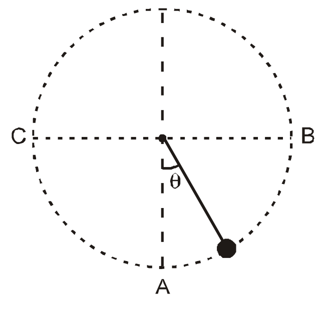
Fig. (5)

		A	B,D	C	P(generalpoint)
1	Velocity	$\sqrt{5g\ell}$	$\sqrt{3g\ell}$	$\sqrt{g\ell}$	$\sqrt{g\ell(3+2\cos\theta)}$
2	Tension	6mg	3mg	0	$3mg(1+\cos\theta)$
3	PotentialEnergy	0	$mg\ell$	$2mg\ell$	$mg\ell(1-\cos\theta)$
4	Radial acceleration	5g	3g	g	$g(3+2\cos\theta)$
5	Tangential acceleration	0	g	0	$g\sin\theta$

Note :- From above table we can see , $T_{bottom} - T_{top} = T_C - T_A = 6$ mg , this difference in tension remain same even if V > $\sqrt{5g\ell}$

SOLVED EXAMPLE

Example 25. Find minimum speed at A so that the ball can reach at point B as shown in figure. Also discuss the motion of particle when T = 0, v = 0 simultaneously at θ = 90º.



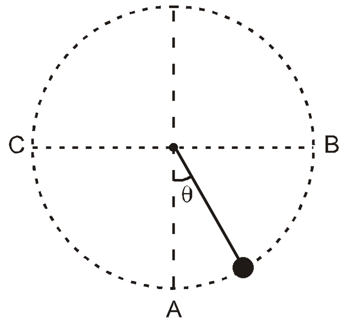
Solution : From energy conservation

$$\frac{1}{2}mv_A^2 + 0 = 0 + mg\ell \qquad \text{(for minimum speed } v_B = 0)$$

$$v_{min} = \sqrt{2g\ell}$$

at the position B, v = 0 and T = 0 (putting v_B = 0 or θ = 90º , in equation ....(1))
ball will return back, motion is oscillatory

CONDITION FOR OSCILLATION OR LEAVING THE CIRCLE :



In case of non uniform circular motion in a vertical plane if velocity of body at lowest point is lesser than $\sqrt{5g\ell}$, the particle will not complete the circle in vertical plane. In this case, the motion of the point mass which depend on 'whether tension becomes zero before speed becomes zero or vice versa.

Case I (Speed becomes zero before tension)

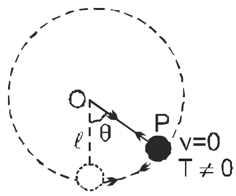
In this case the ball never rises above the level of the center O i.e. the body is confined to move within C and B, (| θ | < 90°) for this the speed at A,

v < $\sqrt{2g\ell}$ (as proved in above example)

In this case tension cannot be zero, since a component of gravity acts radially outwards.

Hence the string will not go slack, and the ball will reverse back as soon as its speed becomes zero.

Its motion will be oscillatory motion.



(A)

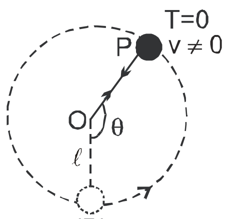
For Oscillation

$0 < v_L < \sqrt{2g\ell}$

$0 < \theta < 90°$

Case II (Tension becomes zero before speed)

In this case the ball rises above the level of center O i.e. it goes beyond point B (θ > 90°) for this v > $\sqrt{2g\ell}$ (as proved in above example)



(B)

For Leaving the circular path after which motion converts into projectile motion.

$\sqrt{2g\ell} < v_L < \sqrt{5g\ell}$

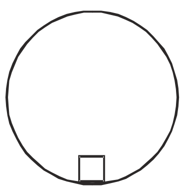
$90° < \theta < 180°$

In this case a component of gravity will always act towards center, hence centripetal acceleration or speed will remain nonzero. Hence tension becomes zero first.

As soon as, Tension becomes zero at any point, string will go slack and subsequently, the only force acting on the body is gravity. Hence its subsequent motion will be similar to that of a projectile. In this case motion is a combination of circular and projectile motion.

• CONDITION FOR LOOPING THE LOOP IN SOME OTHER CASES

Case 1 : A mass moving on a smooth vertical circular track.



(B)

Mass moving along a smooth vertical circular loop.

condition for just looping the loop, normal at highest point = 0

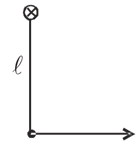
By calculation similar to article (motion in vertical circle)

Minimum horizontal velocity at lowest point = $\sqrt{5g\ell}$

Case 2 : A particle attached to a light rod rotated in vertical circle.

Condition for just looping the loop, velocity v = 0 at highest point (even if tension is zero, rod won't slack, and a compressive force can appear in the rod).

By energy conservation,

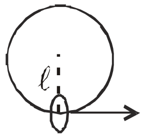


velocity at lowest point = $\sqrt{4g\ell}$

$V_{min} = \sqrt{4g\ell}$ (for completing the circle)

Case 3 : A bead attached to a ring and rotated.

Condition for just looping the loop, velocity v = 0 at highest point (even if normal is zero, the bead will not lose contact with the track, normal can act radially outward).



By energy conservation,

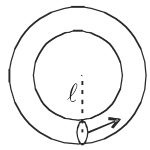
velocity at lowest point = $\sqrt{4g\ell}$

$V_{min} = \sqrt{4g\ell}$ (for completing the circle)

Case 4 : A block rotated between smooth surfaces of a pipe.

Condition for just looping the loop, velocity v = 0 at highest point (even if normal is zero, the bead will not lose contact with the track, normal can act radially outward).

By energy conservation,

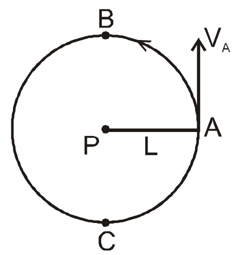


velocity at lowest point = $\sqrt{4g\ell}$

$V_{min} = \sqrt{4g\ell}$ (for completing the circle)

SOLVED EXAMPLE

Example 26. If a particle of mass M is tied to a light inextensible string fixed at point P and particle is projected at A with velocity $V_A = \sqrt{4gL}$ as shown. Find :



(i) velocity at points B and C

(ii) tension in the string at B and C

Assume particle is projected in the vertical plane.

Solution : $V_B = \sqrt{2gL}$ (from energy conservation)

$V_C = \sqrt{6gL}$

$T_B + Mg = \frac{Mv_B^2}{L}$

$T_B = Mg$

$T_C - Mg = \frac{Mv_C^2}{L}$

$T_C = 7Mg$ (where M $\Rightarrow$ Mass of the particle)

Example 27. Two point mass m are connected the light rod of length ℓ and it is free to rotate in vertical plane as shown. Calculate the minimum horizontal velocity is given to mass so that it completes the circular motion in vertical lane.

Solution : Here tension in the rod at the top most point of circle can be zero or negative for completing the loop. So velocity at the top most point is zero.

From energy conservation

$$\frac{1}{2}mv^2 + \frac{1}{2}m\frac{v^2}{4} = mg(2\ell) + mg(4\ell) + 0$$

$$\Rightarrow \qquad v = \sqrt{\frac{48g\ell}{5}} \quad \textbf{Ans.}$$

Example 28. You may have seen in a circus a motorcyclist driving in vertical loops inside a 'death well' (a hollow spherical chamber with holes, so that the cyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m?

Solution : When the motorcyclist is at the highest point of the death-well, the normal reaction R on the motorcyclist by the ceiling of the chamber acts downwards. His weight mg also act downwards.

$$F_{net} = ma_c \qquad \Rightarrow \qquad \therefore \qquad R + mg = \frac{mv^2}{r}$$

Here v is the speed of the motorcyclist and m is the mass of the motorcyclist (including the mass of the motor cycle). Because of the balancing of the forces, the motorcyclist does not fall down.

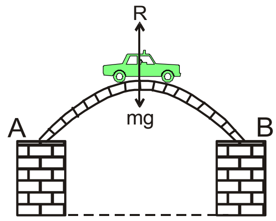
The minimum speed required to perform a vertical loop is given by equation (1) when R = 0.

$$\therefore \qquad mg = \frac{mv_{min}^2}{r} \text{ or } v^2_{min} = gr \quad \text{or} \quad v_{min} = \sqrt{gr} = \sqrt{9.8 \times 25} \text{ m s}^{-1} = 15.65 \text{ ms}^{-1}.$$

So, the minimum speed, at the top, required to perform a vertical loop is 15.65 m s^{-1}.

Example 29. Prove that a motor car moving over a convex bridge is lighter than the same car resting on the same bridge.

Solution : The motion of the motor car over a convex bridge AB is the motion along the segment of a circle AB (Figure) ;



The centripetal force is provided by the difference of weight mg of the car and the normal reaction R of the bridge.

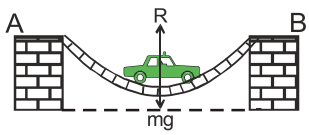
$\therefore \quad mg - R = \dfrac{mv^2}{r} \quad$ or $\quad R = mg - \dfrac{mv^2}{r}$

Clearly R < mg, i.e., the weight of the moving car is less

than the weight of the stationary car.

Example 30. Prove that a motor car moving over a concave bridge is heavier than the same car resting on the same bridge.

Solution : The motion of the motor car over a concave bridge AB is the motion along the segment of a circle AB (Figure) ;



The centripetal force is provided by the difference of normal reaction R of the bridge and weight mg of the car.

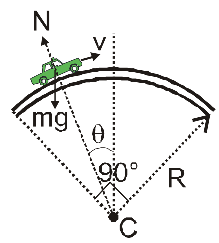
$$\therefore \quad R - mg = \frac{mv^2}{r}$$

$$\text{or} \quad R = mg + \frac{mv^2}{r}$$

Clearly R > mg, i.e., the weight of the moving car is greater than the weight of the stationary car.

Example 31. A car is moving with uniform speed over a circular bridge of radius R which subtends an angle. of 90° at its centre. Find the minimum possible speed so that the car can cross the bridge without losing the contact any where.

Solution : Let the car losses the contact at angle θ with the vertical



$$mg\cos\theta - N = \frac{mv^2}{R}$$

$$N = mg\cos\theta - \frac{mv^2}{R} \quad .... (1)$$

for losing the contact N = 0 ,

$\Rightarrow$ v = $\sqrt{Rg\cos\theta}$ (from (1))

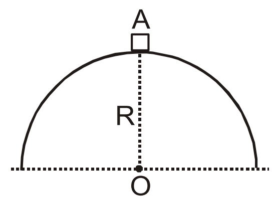
for minimum speed, cosθ should be minimum so that θ should be maximum.

$$\theta_{max} = 45° \Rightarrow \cos 45° = \frac{1}{\sqrt{2}}$$

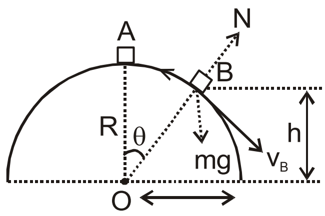
$$v_{min} = \left(\frac{Rg}{\sqrt{2}}\right)^{1/2}$$ **Ans.**

So that if car cannot lose the contact at initial or final point, car cannot be lose the contact anywhere.

Example 32. A block of mass m is released from the top of a frictionless fixed hemisphere as shown. Find (i) the angle with the vertical where it breaks off. (ii) the velocity at the instant when it breaks off. (iii) the height where it breaks off.



Solution : At B ; N = 0



$$\therefore \; mg\cos\theta = \frac{mv_B^2}{R}$$

$$\therefore \; v_B = \sqrt{gR\cos\theta} \quad .... (1)$$

Now by equation of energy between A and B we have ;

$$0 + mgR = \frac{1}{2}mv_B^2 + mgh$$

put v_B from (1)

and h = R cos θ

$$\therefore \; v_B = \sqrt{\frac{2}{3}gR} \text{ and } h = \frac{2R}{3} \text{ from the bottom and } \cos\theta = \frac{2}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

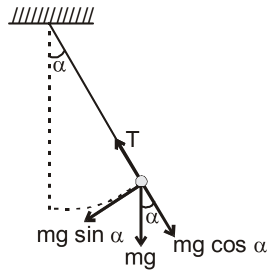
Example 33. Consider a simple pendulum having a bob of mass m suspended by string of length L fixed at its upper end. The bob is oscillating in a vertical circle. It is found that the speed of the bob is v when the string makes an angle α with the vertical. Find (i) tension in the string and (ii) magnitude of net force on the bob at the instant.

Solution : (i) The forces acting on the bob are :

(a) the tension T

(b) the weight mg

As the bob moves in a circle of radius L with centre at O.



A centripetal force of magnitude $\frac{mv^2}{L}$ is required towards

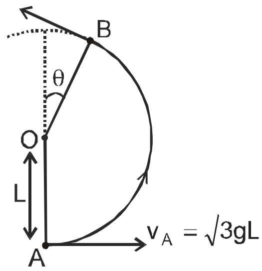
O. This force will be provided by the resultant of T and mg cos α. Thus,

or $T - mg\cos\alpha = \frac{mv^2}{L}$ $T = m\left(g\cos\alpha + \frac{v^2}{L}\right)$

(ii) $a_{net} = \sqrt{a_t^2 + a_r^2} = \sqrt{(g\sin\alpha)^2 + \left(\frac{v^2}{\ell}\right)^2}$

$|\vec{F}_{net}| = ma_{net} = m\sqrt{g^2\sin^2\alpha + \frac{v^4}{L^2}}$ **Ans.**

Example 34. A particle is projected with velocity $\sqrt{3gL}$ at point A (lowest point of the circle) in the vertical plane. Find the maximum height above horizontal level of point A if the string slacks at the point B as shown.



Solution : As tension at B ; T = 0

$\therefore$ mgcos $\theta = \frac{mv_B^2}{L}$

$\therefore$ $v_B = \sqrt{gL\cos\theta}$ .... (1)

Now by equation of energy between A and B.

$0 + \frac{1}{2}m\,3gL = \frac{1}{2}mv_B^2 + mgL(1+\cos\theta)$

put v_B

$\therefore$ $\cos\theta = \frac{1}{3}$

$\therefore$ height attend by particle after the point B where the string slacks is ;

$h' = \frac{v_B^2\sin^2\theta}{2g} = \frac{gL\cos\theta(1-\cos^2\theta)}{2g} = \frac{4L}{27}$

$\therefore$ Maximum height about point A is given by ;

$H_{max} = L + L\cos\theta + h' = L + \frac{L}{3} + \frac{4L}{27} = \frac{40L}{27}$

9. CIRCULAR TURNING ON ROADS :

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways.

1. **By friction only**
2. **By banking of roads only.**
3. **By friction and banking of roads both.**

In real life the necessary centripetal force is provided by friction and banking of roads both. Now let us write equations of motion in each of the three cases separately and see what are the constant in each case.

● By Friction Only

Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction f acting towards center

Thus, $$f = \frac{mv^2}{r}$$

Further, limiting value of f is μN

or $$f_L = \mu N = \mu mg \quad (N = mg)$$

Therefore, for a safe turn without sliding $\frac{mv^2}{r} \le f_L$

or $$\frac{mv^2}{r} \le \mu mg \quad \text{or} \quad \mu \ge \frac{v^2}{rg} \quad \text{or} \quad v \le \sqrt{\mu rg}$$

Here, two situations may arise. If μ and r are known to us, the speed of the vehicle should not exceed $\sqrt{\mu rg}$ and if v and r are known to us, the coefficient of friction should be greater than $\frac{v^2}{rg}$.

SOLVED EXAMPLE

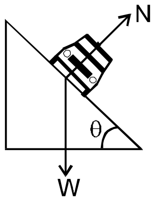
Example 35. A bend in a level road has a radius of 100 m. Calculate the maximum speed which a car turning this bend may have without skidding. Given : $\mu = 0.8$.

Solution : $V_{max} = \sqrt{\mu rg} = \sqrt{0.8 \times 100 \times 10} = \sqrt{800} = 28$ m/s

- **By Banking of Roads Only**

Friction is not always reliable at circular turns if high speeds and sharp turns are involved to avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.

Applying Newton's second law along the radius and the first law in the vertical direction.



$$N \sin\theta = \frac{mv^2}{r}$$

or $$N \cos\theta = mg$$

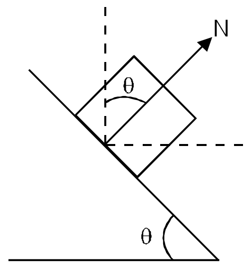
from these two equations, we get

$$\tan\theta = \frac{v^2}{rg} \quad \text{or} \quad v = \sqrt{rg\tan\theta}$$

SOLVED EXAMPLE

Example 36. What should be the angle of banking of a circular track of radius 600 m which is designed for cars at an average speed of 180 km/hr ?

Solution : Let the angle of banking be θ. The forces on the car are (figure)



(a) weight of the car Mg downward and

(b) normal force N.

For proper banking , static frictional force is not needed.

For vertical direction the acceleration is zero. So,

$$N\cos\theta = Mg \quad .....(i)$$

For horizontal direction , the acceleration is v^2 / r towards the centre , so that

$$N\sin\theta = Mv^2 / r \quad .....(ii)$$

From (i) and (ii), $\tan\theta = v^2 / rg$

Putting the values , $\tan\theta = \dfrac{180\,(km/h)^2}{(600\,m)(10\,m/s^2)} = 0.4167$

$\Rightarrow \quad \theta = 22.6°$.

- **By Friction and Banking of Road Both**

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction.

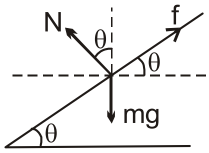
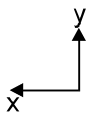


Figure (i)



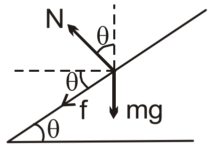


Figure (ii)

The direction of second force, i.e., normal reaction N is also fixed (perpendicular to road) while the direction of the third force i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_L = \mu N$). So the magnitude of normal reaction N and directions plus magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the center. Of these m and r are also constant. Therefore, magnitude of N and directions plus magnitude of friction mainly depends on the speed of the vehicle v. Thus, situation varies from problem to problem. Even though we can see that :

(i) Friction f will be outwards if the vehicle is at rest v = 0. Because in that case the component of weight mg $\sin\theta$ is balanced by f.

(ii) Friction f will be inwards if

$$v > \sqrt{rg\tan\theta}$$

(iii) Friction f will be outwards if

$$v < \sqrt{rg\tan\theta} \text{ and}$$

(iv) Friction f will be zero if

$$v = \sqrt{rg\tan\theta}$$

(v) For maximum safe speed (figure (ii))

$$N\sin\theta + f\cos\theta = \frac{mv^2}{r} \qquad \text{........(i)}$$

$$N\cos\theta - f\sin\theta = mg \qquad \text{.......(ii)}$$

As maximum value of friction

$$f = \mu N$$

$$\therefore \quad \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{rg} \qquad \therefore \quad v_{max} = \sqrt{\frac{rg(\tan\theta + \mu)}{(1 - \mu\tan\theta)}}$$

Similarly ; $$v_{min} = \sqrt{\frac{rg(\tan\theta - \mu)}{(1 + \mu\tan\theta)}}$$

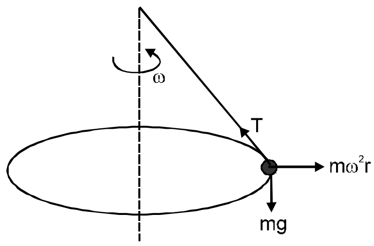
Note :
- The expression $\tan\theta = \frac{v^2}{rg}$ also gives the angle of banking for an aircraft, i.e., the angle through which it should tilt while negotiating a curve, to avoid deviation from the circular path.
- The expression $\tan\theta = \frac{v^2}{rg}$ also gives the angle at which a cyclist should lean inward, when rounding a corner. In this case, θ is the angle which the cyclist must make with the vertical which will be discussed in chapter rotation.

10. CENTRIFUGAL FORCE :

When a body is rotating in a circular path and the centripetal force vanishes, the body would leave the circular path. To an observer A who is not sharing the motion along the circular path, the body appears to fly off tangentially at the point of release. To another observer B, who is sharing the motion along the circular path (i.e., the observer B is also rotating with the body which is released, it appears to B, as if it has been thrown off along the radius away from the centre by some force. This inertial force is called centrifugal force.)

Its magnitude is equal to that of the centripetal force. $= \frac{mv^2}{r} = m\omega^2 r$

Direction of centrifugal force, it is always directed radially outward.

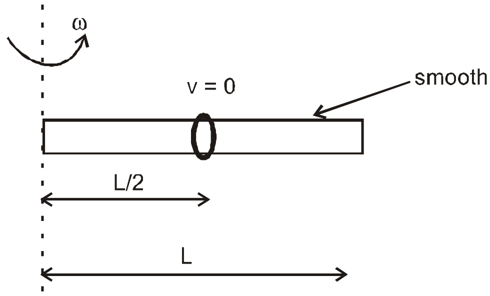


Centrifugal force is a fictitious force which has to be applied as a concept only in a rotating frame of reference to apply Newton's law of motion in that frame. FBD of ball w.r.t. non inertial frame rotating with the ball.

Suppose we are working from a frame of reference that is rotating at a constant, angular velocity ω with respect to an inertial frame. If we analyses the dynamics of a particle of mass m kept at a distance r from the axis of rotation, we have to assume that a force $mr\omega^2$ react radially outward on the particle. Only then we can apply Newton's laws of motion in the rotating frame. This radially outward pseudo force is called the centrifugal force.

SOLVED EXAMPLE

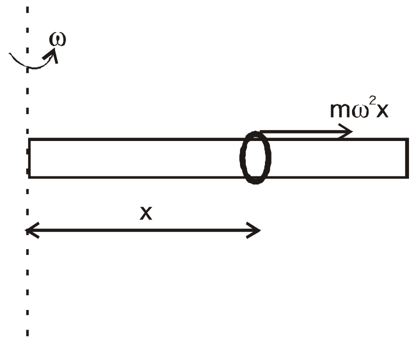
Example 37. A ring which can slide along the rod are kept at mid point of a smooth rod of length L. The rod is rotated with constant angular velocity ω about vertical axis passing through its one end. Ring is released from mid point. Find the velocity of the ring when it just leave the rod.



Solution : Centrifugal force

$m\omega^2 x = ma$

$\omega^2 x = \frac{vdv}{dx}$



$\int_{L/2}^{L} \omega^2 x \, dx = \int_0^v v \, dv$ (integrate both side.)

$\omega^2 \left(\frac{x^2}{2}\right)_{L/2}^{L} = \left(\frac{v^2}{2}\right)_0^v$

$\omega^2 \left(\frac{L^2}{2} - \frac{L^2}{8}\right) = \frac{v^2}{2}$

$v = \frac{\sqrt{3}}{2} \omega L.$

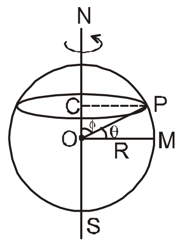
Velocity at time of leaving the rod

$v' = \sqrt{(\omega L)^2 + \left(\frac{\sqrt{3}}{2}\omega L\right)^2} = \frac{\sqrt{7}}{2}\omega L$ **Ans.**

11. EFFECT OF EARTHS ROTATION ON APPARENT WEIGHT :

The earth rotates about its axis at an angular speed of one revolution per 24 hours. The line joining the north and the south poles is the axis of rotation.

Every point on the earth moves in a circle. A point at equator moves in a circle of radius equal to the radius of the earth and the centre of the circle is same as the centre of the earth. For any other point on the earth, the circle of rotation is smaller than this. Consider a place P on the earth (figure).



Draw a perpendicular PC from P to the axis SN. The place P rotates in a circle with the centre at C. The radius of this circle is CP. The angle between the line OM and the radius OP through P is called the latitude of the place P. We have

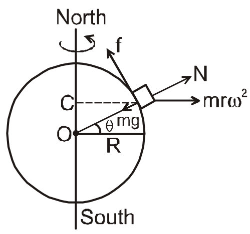
$CP = OP \cos\theta$

or, $r = R \cos\theta$

where R is the radius of the earth and ϕ is colatitude angle.

If we work from the frame of reference of the earth, we shall have to assume the existence of pseudo force. In particular, a centrifugal force mw^2r has to be assumed on any particle of mass m placed at P.

If we consider a block of mass m at point P then this block is at rest with respect to earth. If resolve the forces along and perpendicular the centre of earth then



$$N + mr\omega^2 \cos\theta = mg$$

$$\Rightarrow \quad N = mg - mr\omega^2 \cos\theta$$

$$\Rightarrow \quad N = mg - mR\omega^2 \cos^2\theta$$

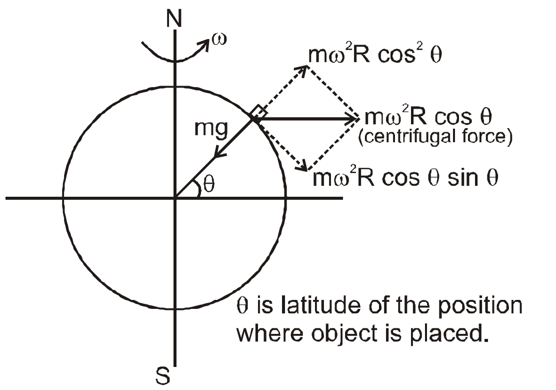


Figure (1) Earth's gravity & centrifugal force due rotation of Earth.

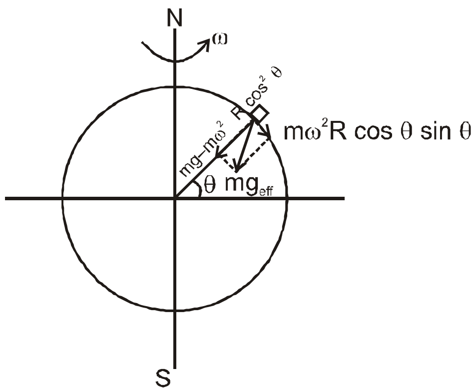


Figure (2) Resultant of Earth's gravity & centrifugal force is shown.

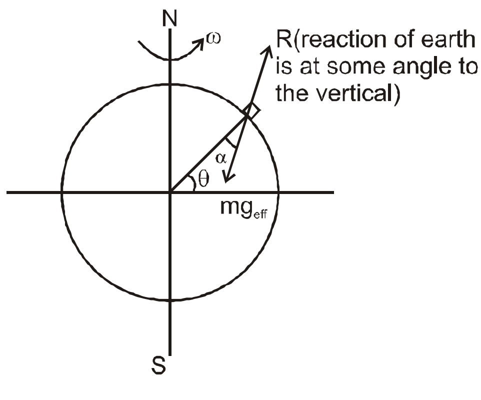


Figure (3)
The reaction on an object kept at rest w.r.t. Earth is called a apparent weight
$W_{app.} = R = mg_{eff} \approx m(g - \omega^2 R\cos^2\theta)$

Note: At equator ($\theta = 0$) $W_{app.}$ is minimum and at pole ($\theta = \pi/2$) $W_{app.}$ is maximum.
This apparent weight is not along normal but at some angle α w.r.t. it. At all point except poles and equator ($\alpha = 0$ at poles and equator)

SOLVED EXAMPLE

Example 38. A body weighs 98N on a spring balance at the north pole. What will be the reading on the same scale if it is shifted to the equator? Use $g = GM/R^2 = 9.8$ m/s^2 and $R_{earth} = 6400$ km.

Solution : At poles , the apparent weight is same as the true weight.

Thus, $98N = mg = m(9.8 \text{ m/s}^2)$

At the equator , the apparent weight is

$$mg' = mg - m\omega^2 R$$

The radius of the earth is 6400 km and the angular speed is

$$\omega = \frac{2\pi \text{ rad}}{24 \times 60 \times 60 s} = 7.27 \times 10^{-6} \text{ rad/s}$$

$$mg' = 98N - (10 \text{ kg})(7.27 \times 10^{-5} \text{ s}^{-1})^2 (6400 \text{ km})$$

$$= 97.66N$$ **Ans.**

MISCELLANEOUS SOLVED EXAMPLE

Problem 1. A fan rotating with $\omega = 100$ rad/s, is switched off. After 2n rotation its angular velocity becomes 50 rad/s. Find the angular velocity of the fan after n rotations.

Solution : $\omega^2 = \omega_0^2 + 2\alpha\,\theta$

$50^2 = (100)^2 + 2\alpha\,(2\pi \,.\, 2n)$ ...(1)

If angular velocity after n rotation is ω_n

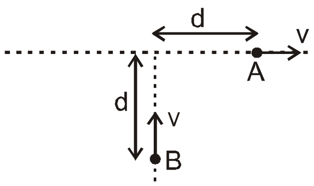
$\omega_n^2 = (100)^2 + 2\alpha\,(2\pi \,.\, n)$ ....(2)

from equation (1) and (2)

$$\frac{50^2 - 100^2}{\omega_n^2 - 100^2} = \frac{2\alpha(2\pi.2n)}{2\alpha 2\pi n} = 2 \quad \Rightarrow \quad \omega_n^2 = \frac{50^2 + 100^2}{2}$$

$$\omega = 25\sqrt{10} \text{ rad/s}$$ **Ans.**

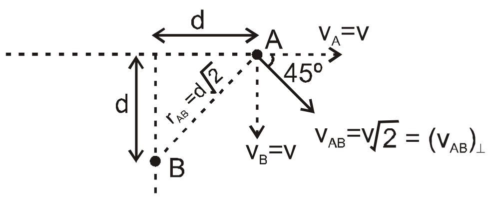
Problem 2. Find angular velocity of A with respect to B at the instant shown in the figure.



Solution : Angular velocity of A with respect to B is ;

$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}}$$

$v_{AB} = \sqrt{2}\,v = (v_{AB})_\perp \quad \Rightarrow \quad r_{AB} = \sqrt{2}\,d$



$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{v\sqrt{2}}{d\sqrt{2}} = \frac{v}{d}$$

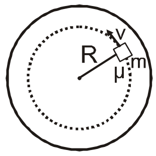
Problem 3. A particle is moving with a constant angular acceleration of 4 rad./sec^2 in a circular path. At time t = 0 particle was at rest. Find the time at which the magnitudes of centripetal acceleration and tangential acceleration are equal.

Solution : $a_t = \alpha R \Rightarrow v = 0 + \alpha Rt$

$$a_c = \frac{v^2}{R} = \frac{\alpha^2 R^2 t^2}{R}$$

$$\because |a_t| = |a_c| \Rightarrow \alpha R = \frac{\alpha^2 R^2 t^2}{R} \Rightarrow t^2 = \frac{1}{\alpha} = \frac{1}{4} \Rightarrow t = \frac{1}{2} \text{ sec.}$$ **Ans.**

Problem 4. The coefficient of friction between block and table is μ. Find the tension in the string if the block moves on the horizontal table with speed v in circle of radius R.



Solution : The magnitude of centripetal force is $\frac{mv^2}{R}$.

(i) If limiting friction is greater than or equal to $\frac{mv^2}{R}$, then static friction alone provides centripetal force, so tension is equal to zero.

T = 0 **Ans.**

(ii) If limiting friction is less than $\frac{mv^2}{R}$, then friction as well as tension both combine to provide the necessary centripetal force.

$$T + f_r = \frac{mv^2}{R}$$

In this case friction is equal to limiting friction, $f_r = \mu mg$

$$\therefore \text{Tension} = T = \frac{mv^2}{R} - \mu mg$$ **Ans.**

Problem 5. A block of mass m is kept on rough horizontal turn table at a distance r from centre of table. Coefficient of friction between turn table and block is μ . Now turn table starts rotating with uniform angular acceleration α.

(i) Find the time after which slipping occurs between block and turn table.

(ii) Find angle made by friction force with velocity at the point of slipping.

Solution : (i) $a_t = \alpha r$

speed after t time

$$\frac{dv}{dt} = \alpha r \Rightarrow v = 0 + \alpha rt$$

Centripetal acceleration

$$a_c = \frac{v^2}{r} = \alpha^2 r t^2$$

Net acceleration $a_{net} = \sqrt{a_t^2 + a_c^2} = \sqrt{\alpha^2 r^2 + \alpha^4 r^2 t^4}$

block just start slipping

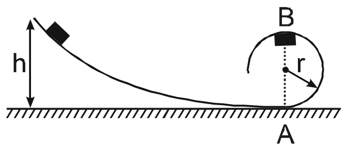
$\mu mg = ma_{net} = m\sqrt{\alpha^2 r^2 + \alpha^4 r^2 t^4}$

$$t = \left(\frac{\mu^2 g^2 - \alpha^2 r^2}{\alpha^4 r^2}\right)^{1/4} \Rightarrow t = \left[\left(\frac{\mu g}{\alpha^2 r}\right)^2 - \left(\frac{1}{\alpha}\right)^2\right]^{1/4}$$ **Ans.**

(ii) $\tan\theta = \frac{a_c}{a_t} \Rightarrow \tan\theta = \frac{\alpha^2 r t^2}{\alpha r}$

$\Rightarrow \theta = \tan^{-1}(\alpha t^2)$ **Ans.**

Problem 6. A block is released from the top of a smooth vertical track, which ends in a circle of radius r as shown.



(i) Find the minimum value of h so that the block completes the circle.

(ii) If h = 3r, find normal reaction when the block is at the points A and B.

(iii) If h = 2r , find the velocity of the block when it loses the contact with the track.

Solution : (i) For completing the circle, velocity at lowest point of circle (say A) is $\sqrt{5gr}$

from energy conservation $mgh = \frac{1}{2}m\left(\sqrt{5gr}\right)^2 \Rightarrow h = \frac{5r}{2}$ **Ans.**

(ii) h = 3r

From energy conservation velocity at point A and B are

$mg.3r = \frac{1}{2}mv_A^2 \Rightarrow v_A = \sqrt{6gr}$

$mg.3r = mg2r + \frac{1}{2}mv_B^2 \Rightarrow v_B = \sqrt{2gr}$

Therefore normal reaction at A and B is -

$N_A - mg = \frac{mv_A^2}{r} \Rightarrow N_A = 7mg$

$N_B + mg = \frac{mv_B^2}{r} \Rightarrow N_B = mg$

(iii) $h = 2r$

It loses contact with the track when normal reaction is zero

$$\frac{mv^2}{r} = mg\cos\theta \text{ .... (1)}$$

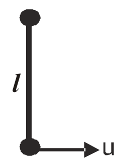
from energy conservation

$$mgh = mgr\,(1 + \cos\theta\,) + \frac{1}{2}mv^2 \text{ .... (2)}$$

from (1) and (2)

$$v = \sqrt{\frac{2g(h-r)}{3}} = \sqrt{\frac{2gr}{3}} \quad \textbf{Ans.}$$

Problem 7. A point mass m connected to one end of inextensible string of length ℓ and other end of string is fixed at peg. String is free to rotate in vertical plane. Find the minimum velocity give to the mass in horizontal direction so that it hits the peg in its subsequent motion.



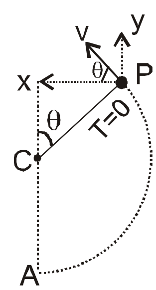
Solution : Tension in string is zero at point P in its subsequent motion, after this point its motion is projectile.

Velocity at point P, T = 0 $\Rightarrow$ $mg\cos\theta = \frac{mv^2}{\ell}$ $\Rightarrow$ $v = \sqrt{g\ell\cos\theta}$

Assume its projectile motion start at point P and it passes through point C. So that equation of trajectory satisfy the co-ordinate of C ($\ell\sin\theta$, $-\ell\cos\theta$)

Equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$



$$-\ell\cos\theta = \ell\sin\theta\tan\theta - \frac{g(\ell\sin\theta)^2}{2(g\ell\cos\theta)\cos^2\theta}$$

$$\Rightarrow -\cos\theta = \frac{\sin^2\theta}{\cos\theta} - \frac{1}{2}\frac{\sin^2\theta}{\cos^3\theta}$$

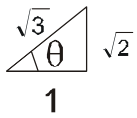
$$\Rightarrow -2\cos^4\theta = 2\sin^2\theta\cos^2\theta - \sin^2\theta$$

$\Rightarrow \sin^2\theta = 2\sin^2\theta \cos^2\theta + 2\cos^4\theta$

$\Rightarrow \sin^2\theta = 2\cos^2\theta\,(\sin^2\theta + \cos^2\theta)$

$\Rightarrow \tan^2\theta = 2$

$\Rightarrow \tan\theta = \sqrt{2}$ $\Rightarrow$



$\Rightarrow \cos\theta = \dfrac{1}{\sqrt{3}}$, $\sin\theta = \sqrt{\dfrac{2}{3}}$

From energy conservation between point P and A.

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\ell\,(1+\cos\theta)$$

$\Rightarrow u^2 = v^2 + 2g\ell\,(1 + \cos\theta\,)$

$\Rightarrow u^2 = 2g\ell + 3g\ell\cos\theta$

$\Rightarrow u^2 = 2g\ell + 3g\ell\dfrac{1}{\sqrt{3}}$ $\Rightarrow u = \left[\left(2+\sqrt{3}\right)g\ell\right]^{1/2}$ **Ans.**

Problem 8. A simple pendulum of length ℓ and mass m free to oscillate in vertical plane. A nail is located at a distance 'd = ℓ – a' vertically below the point of suspension of a simple pendulum. The pendulum bob is released from the position where the string makes an angle of 90° from vertical. Discuss the motion of the bob if (a) ℓ = 2a, (b) ℓ = 2.5 a .

Solution : (a) ℓ = 2a, Velocity at lowest point from energy conservation

$$0 + mg2a = \frac{1}{2}mv^2$$

$$v = \sqrt{4ga}$$

Here radius of circle is 'a' about nail and velocity at lowest point is not sufficient to complete the loop. Therefore motion of bob is combination of circular and projectile motion. Because velocity at lowest point is lie between $\sqrt{3ga}$ and $\sqrt{5ga}$.

(b) ℓ = 2.5 a, Velocity at lowest point from energy conservation

$$0 + mg(2.5a) = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{5ga}$$

here radius of circle is 'a' about nail and velocity at lowest point is just sufficient to complete the loop so that here looping the loop about nail.

Exercise # 1

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Kinematics of circular motion

A 1. Find the ratio of angular speeds of minute hand and hour hand of a watch and also find the angular speed of the second's hand in a watch.

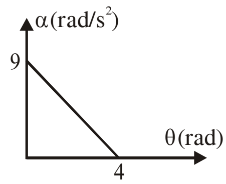
A 2. A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 seconds, it rotates through an angle θ_1. In the next 2 seconds, it rotates through an additional angle θ_2. find the ratio of θ_2/θ_1 .

A 3. A disk rotates about its central axis starting from rest and accelerates with constant angular acceleration. At one instant it is rotating at 12 rad/s and after 80 radian of more angular displacement, its angular speed becomes 28 rad/s. How much time (seconds) does the disk takes to complete the mentioned angular displacement of 80 radians.

A 4. The length of second's hand in a watch is 1 cm. Find the magnitude of change in velocity of its tip in 15 seconds. Also find out the magnitude of average acceleration during this interval.

A 5. If the equation for the angular displacement of a particle moving on a circular path is given by $(\theta) = 2t^3 + 0.5$, where θ is in radians and t in seconds, then find the angular velocity of the particle after 2 seconds from its start.

A 6. A particle starts moving in a non-uniform circular motion, has angular acceleration as shown in figure. The angular velocity at the end of 4 radian is given by ω rad/s then find the value of ω.



Section (B) : Radial and Tangential acceleration

B 1. A car is moving with speed 30 m/sec on a circular path of radius 500 m. Its speed is increasing at the rate of 2 m/sec^2. What is the acceleration of the car at that moment?

B 2. A particle moves in a circle of radius 1.0 cm at a speed given by v= 2t where v is in cm/s and t in seconds.

(a) Find the radial acceleration of the particle at t = 1s.

(b) Find the tangential acceleration at t = 1s

(c) Find the magnitude of the acceleration at t = 1s.

B 3. A particle is travelling in a circular path of radius 4m. At a certain instant the particle is moving at 20m/s and its acceleration is at an angle of 37° from the direction to the centre of the circle as seen from the particle

(i) At what rate is the speed of the particle increasing?

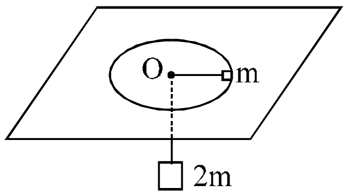
(ii) What is the magnitude of the acceleration?

Section (C) : Circular Motion in Horizontal plane

C 1. A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev/min in a horizontal plane. What is the tension in the string ? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N ?

C 2. A mosquito is sitting on an L.P. record of a gramophone disc rotating on a turn table at $33\frac{1}{3}$ revolution per minute. The distance of the mosquito from the centre of the disc is 10 cm. Show that the friction coefficient between the record and the mosquito is greater than $\pi^2/81$. Take g = 10 m/s^2.

C 3. A mass m rotating freely in a horizontal circle of radius 1 m on a frictionless smooth table supports a stationary mass 2m, attached to the other end of the string passing through smooth hole O in table, hanging vertically. Find the angular velocity of rotation.



C 4. A mass is kept on a horizontal frictionless surface. It is attached to a string and rotates about a fixed centre at an angular velocity ω_0. If the length of the string and angular velocity are doubled, find the tension in the string which was initially T_0 .

C 5. A ceiling fan has a diameter (of the circle through the outer edges of the three blades) of 120 cm and rpm 1500 at full speed. Consider a particle of mass 1g sticking at the outer end of a blade. What is the net force on it, when the fan runs at full speed ? Who exerts this force on the particle ? How much force does the particle exert on the blade in the plane of motion ?

Section (D) : Radius of curvature

D 1. A particle is projected at an angle θ with horizontal. What is the radius of curvature of the parabola traced out by the projectile at a point where the particle velocity makes an angle $\theta/2$ with the horizontal?

Section (E) : Circular motion in vertical plane

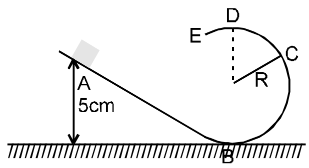
E 1. A bucket tied at the end of a 1.6 m long rod is whirled in a vertical circle with constant speed. What should be the minimum speed at highest point so that the water from the bucket does not spill, when the bucket is at the highest position (Take g = 10 m/sec^2)

E 2. A weightless thread can support tension upto 30 N. A stone of mass 0.5 kg is tied to it and is revolved in a circular path of radius 2 m in a vertical plane. If g = 10 m/s^2, find the maximum angular velocity of the stone.

E 3. A small body of mass m hangs at one end of a string of length a, the other end of which is fixed. It is given a horizontal velocity u at its lowest position so that the string would just becomes slack, when it makes an angle of 60° with the upward drawn vertical line. Find the tension in the string at point of projection.

E 4. A simple pendulum oscillates in a vertical plane. When it passes through the mean position, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum angular displacement of the pendulum of the string with respect to the downward vertical. **[Orissa JEE 2002]**

E 5. A frictionless track ABCDE ends in a circular loop of radius R. A body slides down the track from point A which is at height h = 5 cm. Find the maximum value of R for a body to complete the loop successfully.



E 6. A body attached to a string of length ℓ describes a vertical circle such that it is just able to cross the highest point. Find the minimum velocity at the bottom of the circle.

Section (F) : Motion of a vehicle, Centrifugal force and rotation of earth

F 1. A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The coefficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn?

F 2. Find the maximum speed at which a car can turn round a curve of 30 m radius on a level road if the coefficient of friction between the tyres and the road is 0.4 [g = 10 m/s^2] **[REE 1986]**

F 3. When the road is dry and coefficient of friction is μ, the maximum speed of a car in a circular path is 10 ms^{-1}. If the road becomes wet and coefficient of friction become $\frac{\mu}{2}$, what is the maximum speed permitted?

F 4. A train has to negotiate a curve of radius 400 m. By how much height should the outer rail be raised with respect to inner rail for a speed of 48 km/hr ? The distance between the rails is 1 m :

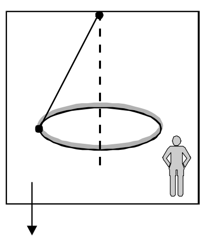
F 5. A road surrounds a circular playing field having radius of 10 m. If a vehical goes around it at an average speed of 18 km/hr, find proper angle of banking for the road. If the road is horizontal (no banking), what should be the minimum friction coefficient so that a scooter going at 18 km/hr does not skid.

F 6. A circular road of radius 1000 m has banking angle 45°. Find the maximum safe speed of a car having mass 2000 kg, if the coefficient of friction between tyre and road is 0.5.

F 7. A turn of radius 20 m is banked for the vehicles going at a speed of 36 km/h. If the coefficient of static friction between the road and the tyre is 0.4, what are the possible speeds of a vehicle so that it neither slips down nor skids up ?

F 8. In the figure shown a lift goes downwards with a constant retardation. An observer in the lift observers a conical pendulum in the lift, revolving in a horizontal circle with time period 2 seconds. The distance between the centre of the circle and the point of suspension is 2.0 m. Find the retardation of the lift in m/s^2.

Use π^2 = 10 and g = 10 m/s^2



PART - II : OBJECTIVE QUESTIONS

*** Marked Questions may have more than one correct option.**

Section (A) : Kinematics of circular motion

A 1. Two racing cars of masses m_1 and m_2 are moving in circles of radii r and 2r respectively and their angular speeds are equal. The ratio of the time taken by cars to complete one revolution is :

(A) $m_1 : m_2$ (B) 1 : 2 (C) 1 : 1 (D) $m_1 : 2m_2$

A 2. A mass is revolving in a circle which lies in a plane of paper. The direction of angular acceleration can be:-

(A) perpendicular to radius and velocity

(B) towards the radius

(C) tangential

(D) at right angle to angular velocity

A 3. A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ m with tangential acceleration of constant magnitude. If the speed of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is:

(A) $160\,\pi$ m/s^2 (B) $40\,\pi$ m/s^2 (C) 40 m/s^2 (D) $640\,\pi$ m/s^2

A 4. During the circular motion with constant speed :

(A) Both velocity and acceleration are both constant

(B) velocity is constant but the acceleration changes

(C) acceleration is constant but the velocity changes

(D) velocity and acceleration both change

A 5. Which of the following statements is false for a particle moving in a circle with a constant angular speed ?

(A) The velocity vector is tangent to the circle

(B) The acceleration vector is tangent to the circle

(C) the acceleration vector points to the centre of the circle

(D) The velocity and acceleration vectors are perpendicular to each other

Section (B) : Radial and Tangential acceleration

B 1. Two particles P and Q are located at distances r_P and r_Q respectively from the axis of a rotating disc such that $r_P > r_Q$:

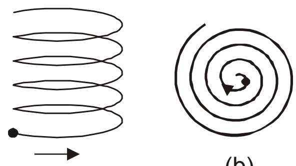
(A) Both P and Q have the same acceleration (B) Both P and Q do not have any acceleration

(C) P has greater acceleration than Q (D) Q has greater acceleration than P

B 2. Let a_r and a_t represent radial and tangential acceleration. The motion of a particle may be circular if :

(A) $a_r = 0$, $a_t = 0$ (B) $a_r = 0$, $a_t \neq 0$ (C) $a_r \neq 0$, $a_t = 0$ (D) none of these

B 3. A particle is going with constant speed along a uniform helical and spiral path separately as shown in figure (in case (a), verticle acceleration of particle is negligible)



(a) (b)

(A) The velocity of the particle is constant in both cases

(B) The magnitude of acceleration of the particle is constant in both cases

(C) The magnitude of accleration is constant in (a) and decreasing in (b)

(D) The magnitude of accleration is decreasing continuously in both the cases

B 4. A stone is thrown horizontally with the velocity 15m/s from some height. Determine the tangential and normal accelerations of the stone in 1 second after it begins to move.

(A) $a_t = \frac{20}{\sqrt{13}}$, $a_n = \frac{30}{\sqrt{13}}$ (B) $a_t = \frac{30}{\sqrt{13}}$, $a_n = \frac{20}{\sqrt{13}}$

(C) $a_t = \frac{20}{\sqrt{13}}$, $a_n = \frac{20}{\sqrt{13}}$ (D) $a_t = \frac{20}{\sqrt{13}}$, $a_n = \frac{10}{\sqrt{13}}$

Section (C) : Circular Motion in Horizontal plane

C 1. A coin placed on a rotating turntable just slips if it is placed at a distance of 16 cm from the centre. If the angular velocity of the turntable is doubled, it will just slip at a distance of

(A) 1 cm (B) 2 cm (C) 4 cm (D) 8 cm

C 2. A stone of mass of 16 kg is attached to a string 144 m long and is whirled in a horizontal smooth surface. The maximum tension the string can withstand is 16 N. The maximum speed of revolution of the stone without breaking it, will be :

(A) 20 ms^{-1} (B) 16 ms^{-1} (C) 14 ms^{-1} (D) 12 ms^{-1}

C 3. On horizontal smooth surface a mass of 2 kg is whirled in a horizontal circle by means of a string at an initial angular speed of 5 revolutions per minute. Keeping the radius constant the tension in the string is doubled. The new angular speed is nearly:

(A) 14 rpm (B) 10 rpm (C) 2.25 rpm (D) 7 rpm

C 4. A particle is kept fixed on a uniformly rotating turn-table As seen from the ground , the particle goes in a circle, its speed is 10 cm/s and acceleration is 10 cm/s^2.The particle is now shifted to a new position to make the radius half of the original value.The new values of the speed and acceleration will be

(A) 20 cm/s, 20 cm/s^2 (B) 5 cm/s, 5 cm/s^2 (C) 40 cm/s, 10 cm/s^2 (D) 40 cm/s,40 cm/s^2

C 5. A rod of length L is hinged at one end and it is rotated with a constant angular velocity in a horizontal plane . Let T_1 and T_2 be the tensions at the points L/4 and 3L/4 away from the hinged end.

(A) $T_1 > T_2$

(B) $T_2 > T_1$

(C) $T_1 = T_2$

(D) The relation between T_1 and T_2 depends on whether the rod rotates clockwise or anticlockwise

C 6. A particle moving along a circular path due to a centripetal force having constant magnitude is an example of motion with :

(A) constant speed and velocity (B) variable speed and velocity

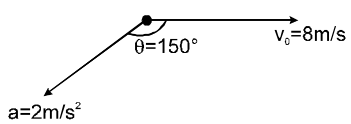
(C) variable speed and constant velocity (D) constant speed and variable velocity.

Section (D) : Radius of curvature

D 1. A particle of mass m is moving with constant velocity $\vec{v}$ on smooth horizontal surface. A constant force $\vec{F}$ starts acting on particle perpendicular to velocity v. Radius of curvature after force F start acting is :

(A) $\frac{mv^2}{F}$ (B) $\frac{mv^2}{F\cos\theta}$ (C) $\frac{mv^2}{F\sin\theta}$ (D) none of these

D 2. The figure shows the velocity and acceleration of a point like body at the initial moment of its motion. The acceleration vector of the body remains constant. The minimum radius of curvature of trajectory of the body is



(A) 2 meter (B) 4 meter (C) 8 meter (D) 16 meter.

Section (E) : Circular motion in vertical plane

E 1. A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angles 30° and 60° from downward vertical are T_1 and T_2 respectively. Then **[Orissa JEE 2002]**

(A) $T_1 = T_2$

(B) $T_2 > T_1$

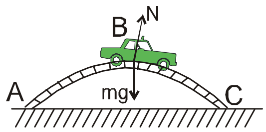
(C) $T_1 > T_2$

(D) Tension in the string always remains the same

E 2. A stone tied to a string is rotated in a vertical plane. If mass of the stone is m, the length of the string is r and the linear speed of the stone is v when the stone is at its lowest point, then the tension in the string at the lowest point will be :(Acceleration due to gravity is g)

(A) $\frac{mv^2}{r}+mg$ (B) $\frac{mv^2}{r}-mg$ (C) $\frac{mv^2}{r}$ (D) mg

E 3. A car is going on an overbridge of radius R, maintaining a constant speed. As the car is descending on the overbridge from point B to C, the normal force on it :



(A) increase (B) decreases

(C) remains constant (D) first increases then decreases.

E 4. In a circus, stuntman rides a motorbike in a circular track of radius R in the vertical plane. The minimum speed at highest point of track will be :

(A) $\sqrt{2gR}$ (B) 2gR (C) $\sqrt{3gR}$ (D) $\sqrt{gR}$

E 5. A bucket is whirled in a vertical circle with a string attached to it.The water in bucket does not fall down even when the bucket is inverted at the top of its path. In this position choose most appropiate option if v is the speed at the top.

(A) mg = $\frac{mv^2}{r}$ (B) mg is greater than $\frac{mv^2}{r}$

(C) mg is not greater than $\frac{mv^2}{r}$ (D) mg is not less than $\frac{mv^2}{r}$

Section (F) : Motion of a vehicle, Centrifugal force and rotation of earth

F 1. A car moving on a horizontal road may be thrown out of the road in taking a turn :

(A) By the gravitational force

(B) Due to lack of sufficient centripetal force

(C) Due to friction between road and the tyre

(D) Due to reaction of earth

F 2. A train A runs from east to west and another train B of the same mass runs from west to east at the same speed with respect to earth along the equator. Normal force by the track on train A is N_1 and that on train B is N_2:

(A) $N_1 > N_2$

(B) $N_1 < N_2$

(C) $N_1 = N_2$

(D) the information is insufficient to find the relation between N_1 and N_2.

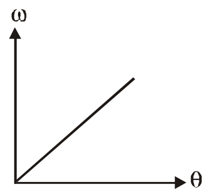
F 3. If the apparent weight of the bodies at the equator is to be zero, then the earth should rotate with angular velocity

(A) $\sqrt{\frac{g}{R}}$ rad/sec (B) $\sqrt{\frac{2g}{R}}$ rad/sec (C) $\sqrt{\frac{g}{2R}}$ rad/sec (D) $\sqrt{\frac{3g}{2R}}$ rad/sec

PART - III : MATCH THE COLUMN

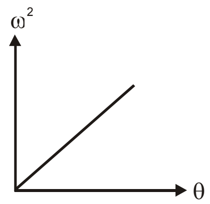
1. Each situation in column I gives graph of a particle moving in circular path. The variables ω, θ and t represent angular speed (at any time t) , angular displacement (in time t) and time respectively. Column II gives certain resulting interpretation. Match the graphs in column I with statements in column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

Column-I **Column-II**

(A) 

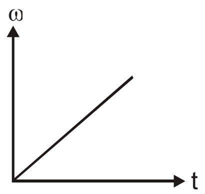
ω - θ graph

(p) Angular acceleration of particle is uniform

(B) 

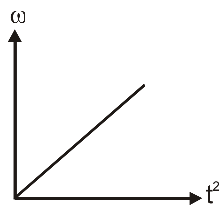
ω^2 - θ graph

(q) Angular acceleration of particle is non-uniform

(C) 

ω - t graph

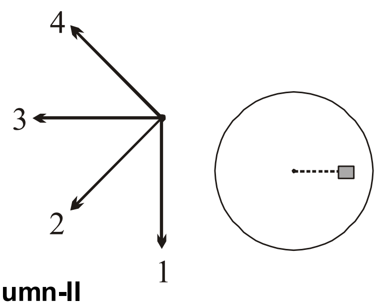
(r) Angular acceleration of particle is directly proportional to t.

(D) 

ω - t^2 graph

(s) Angular acceleration of particle is directly proportional to θ.

2. A block is placed on a horizontal table which can rotate about its axis. The block is placed at a certain distance from centre as shown in figure. Table rotates such that particle does not slide. Select possible direction of net acceleration of block at the instant shown in figure.

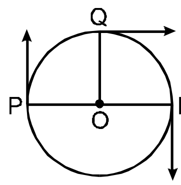


	Column-I		**Column-II**
(A)	When rotation is clockwise with constant ω	(P)	1
(B)	When rotation is clock wise with decreasing ω	(Q)	2
(C)	When rotation is clockwise with increasing ω	(R)	3
(D)	Just after clockwise rotation begins from rest	(S)	4

Exercise # 2

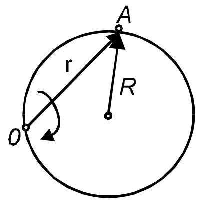
PART - I : ONLY ONE OPTION CORRECT TYPE

1. Three point particles P, Q, R move in a circle of radius 'r' with different but constant speeds. They start moving at t = 0 from their initial positions as shown in the figure. The angular velocities (in rad/sec) of P, Q and R are 5π, 2π & 3π respectively, in the same sense. The time at which they all meet is:



(A) 2/3 sec (B) 1/6 sec (C) 1/2 sec (D) 3/2 sec

2. A particle A moves along a circle of radius R = 50 cm so that its radius vector r relative to the fixed point O (Fig.) rotates with the constant angular velocity ω = 0.40 rad/s. Then modulus v of the velocity of the particle,and the modulus a of its total acceleration will be



(A) v = 0.4 m/s, a = 0.4 m/s^2

(B) v = 0.32 m/s, a = 0.32 m/s^2

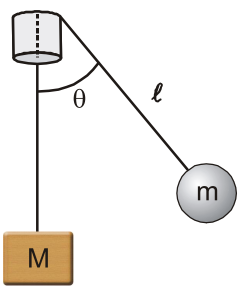
(C) v = 0.32 m/s, a = 0.4 m/s^2

(D) v = 0.4 m/s, a = 0.32 m/s^2

3. A boy whirls a stone in a horizontal circle 1.8 m above the ground by means of a string with radius 1.2 m. while whirling the stone string was horizontal, it breaks and stone flies off horizontally, striking the ground 9.1 m away. The centripetal acceleration during the circular motion was nearly: (use g = 9.8 m/s^2)

(A) 94 m/s^2 (B) 141 m/s^2 (C) 188 m/s^2 (D) 282 m/s^2

4. A large mass M hangs stationary at the end of a light string that passes through a smooth fixed ring to a small mass m that moves around in a horizontal circular path. If ℓ is the length of the string from m to the top end of the tube and θ is angle between this part and vertical part of the string as shown in the figure, then time taken by m to complete one circle is equal to

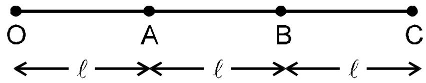


(A) $2\pi\sqrt{\frac{\ell}{g\sin\theta}}$ (B) $2\pi\sqrt{\frac{\ell}{g\cos\theta}}$ (C) $2\pi\sqrt{\frac{m\ell}{gM\sin\theta}}$ (D) $2\pi\sqrt{\frac{\ell m}{gM}}$

5. A stone of mass 1 kg tied to a light inextensible string of length L = $\frac{10}{3}$ m, whirling in a circular path in a vertical plane. The ratio of maximum tension in the string to the minimum tension in the string is 4, If g is taken to be 10 m/s^2, the speed of the stone at the highest point of the circle is :

(A) 10 m/s (B) $5\sqrt{2}$ m/s (C) $10\sqrt{3}$ m/s (D) 20 m/s

6. Three identical particles are joined together by a thread as shown in figure. All the three particles are moving on a smooth horizontal plane about point O. If the speed of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is : (Assume that the string remains straight)



(A) 3 : 5 : 7 (B) 3 : 4 : 5 (C) 7 : 11 : 6 (D) 3 : 5 : 6

7. A Toy cart attached to the end of an unstretched string of length a, when revolved moves on a smooth horizontal table in a circle of radius 2a with a time period T. Now the toy cart is speeded up until it moves in a circle of radius 3a with a period T'. If Hook's law holds then (Assume no friction) :

(A) $T' = \sqrt{\frac{3}{2}}\,T$ (B) $T' = \left(\frac{\sqrt{3}}{2}\right)T$ (C) $T' = \left(\frac{3}{2}\right)T$ (D) T' = T

8. A particle is projected horizontally from the top of a tower with a velocity v_0. If v be its velocity at any instant, then the radius of curvature of the path of the particle at that instant is directly proportional to:

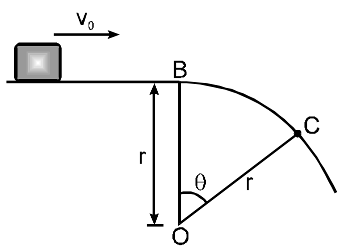
(A) v^3 (B) v^2 (C) v (D) 1/v

9. A small sphere of mass m suspended by a thread is first taken aside so that the thread forms the right angle with the vertical and then released. The total acceleration of the sphere and the thread tension as a function of θ, the angle of deflection of the thread from the vertical will be

(A) $g\sqrt{1+3\cos^2\theta}$, T = 3mg cos θ (B) g cos θ, T = 3 mg cos θ.

(C) $g\sqrt{1+3\sin^2\theta}$, T = 5mg cos θ (D) g sin θ , T = 5 mg cos θ.

10. A small frictionless block slides with velocity $0.5\sqrt{gr}$ on the horizontal surface as shown in the Figure. The block leaves the surface at point C. The angle θ in the Figure is :



(A) $\cos^{-1}$ (4/9) (B) $\cos^{-1}$(3/4) (C) $\cos^{-1}$(1/2) (D) none of the above

11. The kinetic energy k of a particle moving along a circle of radius R depends on the distance covered s as $k = as^2$ where a is a positive constant. The total force acting on the particle is :

(A) $2a\frac{s^2}{R}$ (B) $2as\left(1+\frac{s^2}{R^2}\right)^{1/2}$ (C) $2as$ (D) $2a\frac{R^2}{s}$

12. A particle moves with deceleration along the circle of radius R so that at any moment of time its tangential and normal accelerations are equal in magnitude. At the initial moment t = 0 the speed of the particle equals v_0, then the speed of the particle as a function of the distance covered s will be

(A) $v = v_0 e^{-s/R}$ (B) $v = v_0 e^{s/R}$ (C) $v = v_0 e^{-R/s}$ (D) $v = v_0 e^{R/s}$

13. A particle moves along an arc of a circle of radius R. Its velocity depends on the distance covered s as $v = a\sqrt{s}$, where a is a constant then the angle α between the vector of the total acceleration and the vector of velocity as a function of s will be

(A) $\tan\alpha = \frac{R}{2s}$ (B) $\tan\alpha = 2s / R$ (C) $\tan\alpha = \frac{2R}{s}$ (D) $\tan\alpha = \frac{s}{2R}$

14. A particle of mass m begins to slide down a fixed smooth sphere from the top with negligible intial velocity. What is its tangential acceleration when it breaks off the sphere ?(Acceleration due to gravity is g)

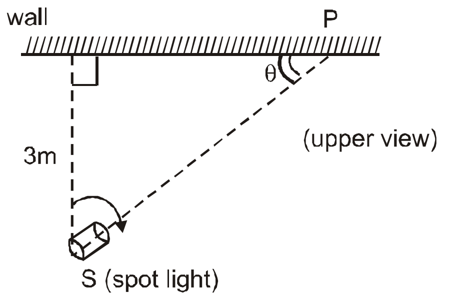
(A) $\frac{2g}{3}$ (B) $\frac{\sqrt{5}g}{3}$ (C) g (D) $\frac{g}{3}$

15. A sphere of mass m is suspended by a thread of length 'ℓ' is oscillating in a vertical plane, the angular amplitude being θ_0. What is the tension in the thread when it makes an angle θ with the vertical during oscillations ? If the thread can support a maximum tension of 2mg, then what can be the maximum angular amplitude of oscillation of the sphere without breaking the rope? **[IIT 1978]**

(A) $3\ mg\cos\theta - 2mg\cos\theta_0$, $\theta_0 = 60^0$ (B) $3\ mg\cos\theta + 2mg\cos\theta_0$,, $\theta_0 = 60^0$

(C) $2mg\cos\theta - 3mg\cos\theta_0$, $\theta_0 = 30^0$ (D) $2mg\cos\theta + 3mg\cos\theta_0$,, $\theta_0 = 30^0$

16. A spot light S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light P moves along the wall at a distance 3 m. What is the velocity of the spot P when $\theta = 45^0$? **[IIT 1987]**



(A) 0.6 m/s (B) 0.5 m/s (C) 0.4 m/s (D) 0.3 m/s

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. A particle moves clockwise in a circle of radius 1 m with centre at (x, y) = (1m, 0). It starts at rest at the origin at time t = 0. Its speed increases at the constant rate of $\left(\frac{\pi}{2}\right)$ m/s^2 . If the net acceleration at t = 2 sec is $\frac{\pi}{M}\sqrt{(1+N\pi^2)}$ then what is the value of M + N ?

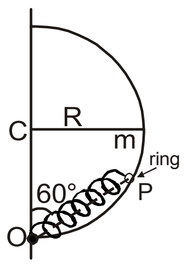
2. Two particles A and B move anticlockwise with the same speed v in a circle of radius R and are diametrically opposite to each other. At t = 0, A is imparted a tangential acceleration of constant magnitude $a_t = \frac{72v^2}{25\pi R}$. If the time in which A collides with B is $\frac{5\pi R}{N_1 v}$, the angle traced by A during this time is $\frac{11\pi}{N_2}$, its angular velocity is $\frac{17v}{N_3 R}$ and radial acceleration at the time of collision is $\frac{289v^2}{5RN_4}$. Then calculate the value of $N_1 + N_2 - N_3 - N_4$.

3. A block of mass 'm' = 1 kg moves on a horizontal circle against the wall of a cylindrical room of radius R = $2\sqrt{2}$ m. The floor of the room on which the block moves is smooth but the friction coefficient between the wall and the block is μ = 1. The block is given an initial speed v_0. If speed at any instant is v = 4m/s then calculate resultant accleration of block in m/s^2 at that instant.

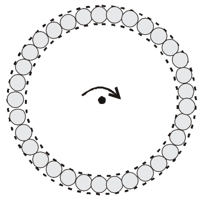
4. A car goes on a horizontal circular road of radius R = $\sqrt{27}$ m, the speed increasing at a constant rate $\frac{dv}{dt}$ = a = 1 m/s^2. The friction coefficient between the road and the tyre is μ = 0.2. Find the time at which the car will skid.

5. A nail is located at a certain distance vertically below the point of suspension of a simple pendulum. The pendulum bob is released from the position where the string makes an angle of 60° from the vertical. Calculate the value of x if distance of the nail from the point of suspension is $\frac{x}{5}$ such that the bob will just perform revolution with the nail as centre. Assume the length of pendulum to be 2m.

6. A smooth semicircular wire-track of radius R is fixed in a vertical plane shown in fig. One end of a massless spring of natural length (3R/4) is attached to the lower point O of the wire track. A small ring of mass m, which can slide on the track, is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle of 60° with the vertical. The spring constant K = mg/R. Consider the instant when the ring is released, if the tangential acceleration of the ring is $\frac{x\sqrt{3}g}{8}$ and the normal reaction is $\frac{ymg}{8}$. Then calculate the value of x – y.



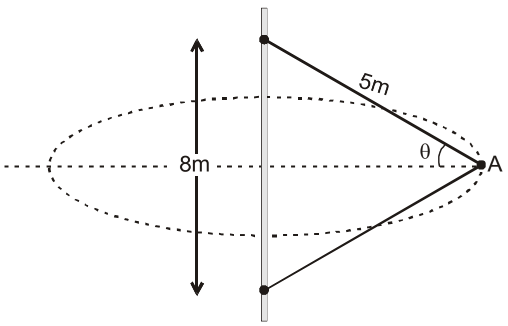
7. A uniform metallic chain in a form of circular loop of mass m = 3kg with a length ℓ = 1m rotates at the rate of n = 3 revolutions per second. Find the tension T (in N) in the chain.



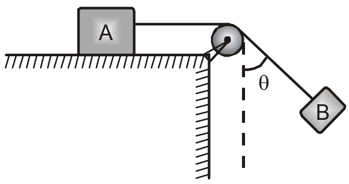
8. A particle of mass m is suspended by string of length ℓ from a fixed rigid support. A sufficient horizontal velocity $v_0 = \sqrt{3g\ell}$ is imparted to it suddenly. Calculate the angle made by the string with the vertical when the acceleration of the particle is inclined to the string by 45º.

9. A particle moves along the plane trajectory y (x) with velocity v whose modulus is constant. Find the curvature radius of the trajectory at the point x = 0 if the trajectory has the form of a parabola $y = \frac{1}{10}x^2$.

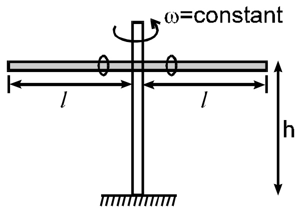
10. A 4 kg block is attached to a vertical rod by means of two strings of equal length. When the system rotates about the axis of the rod, the strings are extended as shown in figure. If tension in upper and lower chords are 200 N and 30x N respectively and angular velocity and particle is $\sqrt{\frac{y}{2}}$, then calculate the value of $\frac{y}{x}$.



11. Two particles A and B each of mass m are connected by a massless string. A is placed on the rough table. The string passes over a small, smooth peg. B is left from a position making an $\angle\theta$ with the vertical. If the minimum coefficient of friction between A and the table is $\mu_{min} = P - Q\cos\theta$ so that A does not slip during the motion of mass B. Then calculate the value of P + Q.

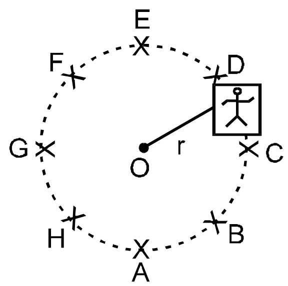


12. Two identical rings which can slide along the rod are kept near the mid point of a smooth rod of length $2\ell(\ell = 1m)$ The rod is rotated with constant angular velocity $\omega = 3$ rad/s about vertical axis passing through its centre. The rod is at height ' h' = 5m from the ground. Find the distance between the points on the ground where the rings will fall after leaving the rods.



PART - III : ONE OR MORE THAN ONE CORRECT OPTIONS

1. A machine, in an amusement park, consists of a cage at the end of one arm, hinged at O. The cage revolves along a vertical circle of radius r (ABCDEFGH) about its hinge O, at constant linear speed $v = \sqrt{gr}$. The cage is so attached that the man of weight 'w' standing on a weighing machine, inside the cage, is always vertical. Then which of the following is correct:



(A) the reading of his weight on the machine is the same at all positions

(B) the weight reading at A is greater than the weight reading at E by 2 w.

(C) the weight reading at G = w

(D) the ratio of the weight reading at E to that at A = 0

(E) the ratio of the weight reading at A to that at C = 2.

2. A car of mass M is travelling on a horizontal circular path of radius r. At an instant its speed is v and tangential acceleration is 'a' :

(A) The acceleration of the car is towards the centre of the path

(B) The magnitude of the frictional force on the car is greater than $\frac{mv^2}{r}$

(C) The friction coefficient between the ground and the car is not less than a/g.

(D) The friction coefficient between the ground and the car is $\mu = \tan^{-1}\frac{v^2}{rg}$

3. A stone is projected from level ground at t = 0 sec such that its horizontal and vertical components of initial velocity are 10 m/s and 20 m/s respectively. Then the instant of time at which magnitude of tangential and magnitude of normal components of acceleration of stone are same is : (neglect air resistance) [g = 10 m/s^2].

(A) $\frac{1}{2}$ sec (B) 1 sec (C) 3 sec (D) 4 sec.

4. Let $\vec{v}(t)$ be the velocity of a particle at time t. Then :

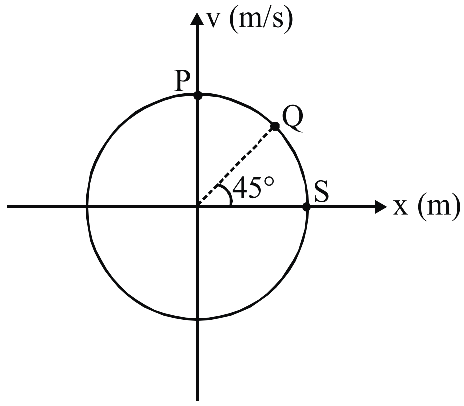
(A) $|d\vec{v}(t)/dt|$ and $d|\vec{v}(t)|/dt$ are always equal

(B) $|d\vec{v}(t)/dt|$ and $d|\vec{v}(t)|/dt$ may be equal

(C) $d|\vec{v}(t)|/dt$ can be zero while $|d\vec{v}(t)/dt|$ is not zero

(D) $d|\vec{v}(t)|/dt \neq 0$ implies $|d\vec{v}(t)/dt| \neq 0$

5. A particle is in motion on the x-axis. The variation of its velocity with position is as shown. The graph is circle and its equation is $x^2 + v^2 = 1$, where x is in m and v in m/s. The **CORRECT** statement(s) is/are :-



(A) When x is positive, acceleration is negative.

(B) When x is negative, acceleration is positive.

(C) At Q, acceleration has magnitude $\frac{1}{\sqrt{2}}$ m/s^2

(D) At S, acceleration is infinite.

6. For a curved track of radius R, banked at angle θ (Take $v_0 = \sqrt{Rg\tan\theta}$)

(A) a vehicle moving with a speed v_0 is able to negotiate the curve without calling friction into play at all

(B) a vehicle moving with any speed $v > v_0$ is always able to negotiate the curve, with friction called into play

(C) a vehicle moving with any speed $v < v_0$ must have the force of friction into play

(D) the minimum value of the angle of banking for a vehicle parked on the banked road can stay there without slipping, is given by $\theta = \tan^{-1}\mu_0$ (μ_0 = coefficient of static friction)

7. A car runs around a curve of radius 10 m at a constant speed of 10 ms^{-1}. Consider the time interval for which car covers a curve of 120° arc :-

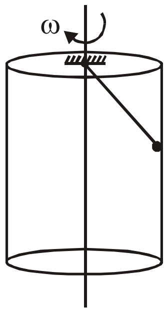
(A) Resultant change in velocity of car is $10\sqrt{3}$ ms^{-1}

(B) Instantaneous acceleration of car is 10 ms^{-2}

(C) Average acceleration of car is $\frac{5}{24}$ ms^{-2}

(D) Instantaneous and average acceleration are same for the given period of motion.

8. In the shown figure inside a fixed hollow cylinder with vertical axis a pendulum is rotating in a conical path with its axis same as that of the cylinder with uniform angular velocity. Radius of cylinder is 30 cm, length of string is 50 cm and mass of bob is 400 gm. The bob makes contact with the inner frictionless wall of the cylinder while moving :-



(A) The minimum value of angular velocity of the bob so that it does not leave contact is 5 rad/s

(B) Tension in the string is 5N for all values of angular velocity

(C) For angular velocity of 10 rad/s the bob pushes the cylinder with a force of 9N

(D) For angular velocity of 10 rad/s, tension in the string is 20N

9. An ant travels along a long rod with a constant velocity $\vec{u}$ relative to the rod starting from the origin. The rod is kept initially along the positive x-axis. At t = 0, the rod also starts rotating with an angular velocity ω (anticlockwise) in x-y plane about origin. Then

(A) the position of the ant at any time t is $\vec{r} = ut[\cos\omega t\hat{i} + \sin\omega t\hat{j}]$

(B) the speed of the ant at any time t is $u\sqrt{1+\omega^2 t^2}$

(C) the magnitude of the tangential acceleration of the ant at any time t is $\frac{\omega^2 tu}{\sqrt{1+\omega^2 t^2}}$

(D) the speed of the ant at any time t is $\sqrt{1+2\omega^2 t^2 u}$

PART - IV : COMPREHENSIONS

Comprehension # 1

A particle undergoes uniform circular motion. The velocity and angular velocity of the particle at an instant of time is $\vec{v} = 3\hat{i} + 4\hat{j}$ m/s and $\vec{\omega} = x\hat{i} + 6\hat{j}$ rad/sec.

1. The value of x in rad/s is

(A) 8 (B) – 8 (C) 6 (D) can't be calculated

2. The radius of circle in metres is

(A) 1/2 m (B) 1 m (C) 2 m (D) can't be calculated

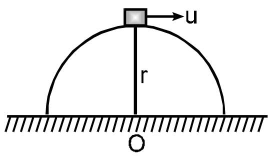
3. The acceleration of particle at the given instant is

(A) $-50\hat{k}$ (B) $-42\hat{k}$ (C) $2\hat{i} + 3\hat{j}$ (D) can't be calculated

Comprehension # 2

A small block of mass m is projected horizontally from the top of the smooth and fixed hemisphere of radius r with speed u as shown. For values of $u \geq u_0$, ($u_0 = \sqrt{gr}$) it does not slide on the hemisphere.

[i.e. leaves the surface at the top itself]



4. For $u = 2\,u_0$, it lands at point P on ground. Find OP.

(A) $\sqrt{2}$ r (B) 2 r (C) 4r (D) $2\sqrt{2}$ r

5. For $u = u_0/3$, find the height from the ground at which it leaves the hemisphere.

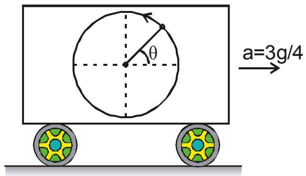
(A) $\frac{19r}{9}$ (B) $\frac{19\,r}{27}$ (C) $\frac{10r}{9}$ (D) $\frac{10r}{27}$

6. Find its net acceleration at the instant it leaves the hemisphere.

(A) g/4 (B) g/2 (C) g (D) g/3

Comprehension # 3

A bus is moving with a constant acceleration a = 3g/4 towards right. In the bus, a ball is tied with a rope of length ℓ and is rotated in vertical circle as shown.



7. At what value of angle θ, tension in the rope will be minimum

(A) θ = 37° (B) θ = 53° (C) θ = 30° (D) θ = 90°

8. At above mentioned position, find the minimum possible speed V_{min} during whole path to complete the circular motion :

(A) $\sqrt{5g\ell}$ (B) $\frac{5}{2}\sqrt{g\ell}$ (C) $\frac{\sqrt{5g\ell}}{2}$ (D) $\sqrt{g\ell}$

9. For above value of V_{min} find maximum tension in the string during circular motion.

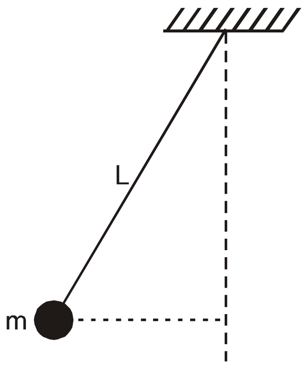
(A) 6 mg (B) $\frac{117}{20}$mg (C) $\frac{15}{2}$mg (D) $\frac{17}{2}$mg

Exercise # 3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

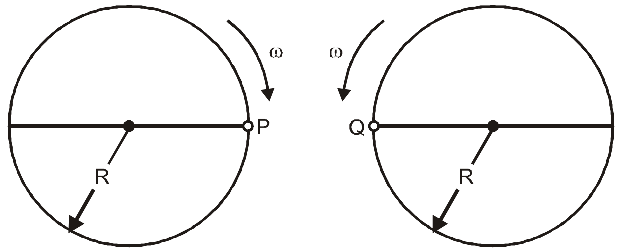
*** Marked Questions may have more than one correct option.**

1. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is : **[JEE 2011, 3/160, –1]**

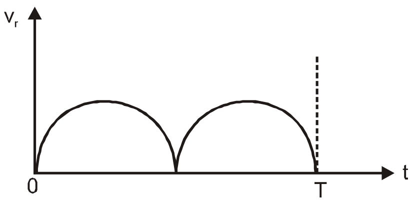


(A) 9 (B) 18 (C) 27 (D) 36

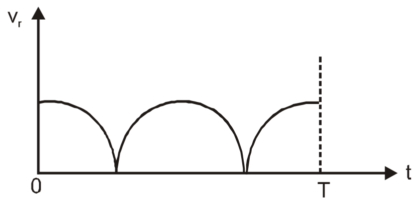
2. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The disc are in the same horizontal plane.At time t = 0 , the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. as function of times best represented by **[IIT-JEE-2012, Paper-2; 3/66, –1]**



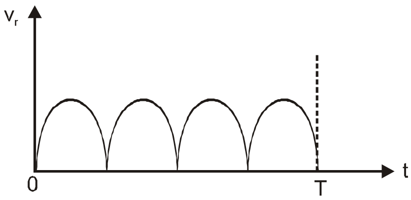
(A)



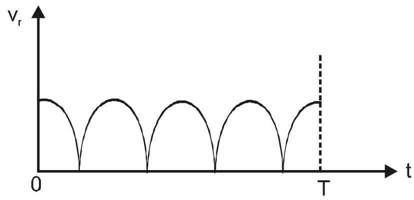
(B)



(C)

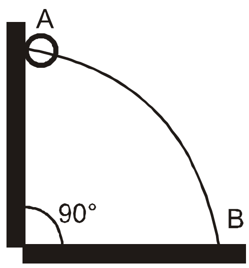


(D)



3. A wire, which passes through the hole is a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is

[JEE (Advanced)-2014, 3/60, –1]



(A) always radially outwards

(B) always radially inwards

(C) radially outwards initially and radially inwards later

(D) radially inwards initially and radially outwards later.

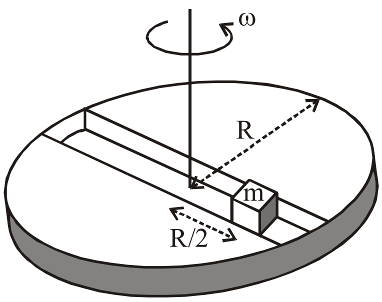
Paragraph for Question No. 4 and 5

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{\upsilon}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega},$$

where $\vec{\upsilon}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the centre of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $(\vec{\omega} = \omega\hat{k})$. A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ at t = 0 and is constrained to move only along the slot. **[IIT-JEE Advanced-2016]**



4. The distance r of the block at time t is :

(A) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$

(B) $\frac{R}{2}\cos 2\omega t$

(C) $\frac{R}{2}\cos \omega t$

(D) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$

5. The net reaction of the disc on the block is :

(A) $-m\omega^2 R\cos\omega t\hat{j} - mg\hat{k}$

(B) $m\omega^2 R\sin\omega t\hat{j} - mg\hat{k}$

(C) $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}$

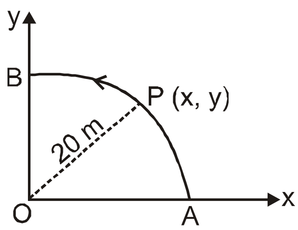
(D) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} + mg\hat{k}$

6.* The potential energy of a particle of mass m at a distance r from a fixed point O is given by V (r) = $kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true ? **[JEE Advanced 2018]**

(A) $v = \sqrt{\frac{k}{2m}}R$ (B) $v = \sqrt{\frac{k}{m}}R$ (C) $L = \sqrt{mk}R^2$ (D) $L = \sqrt{\frac{mk}{2}}R^2$

PART - II : JEE(MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length s = t^3 + 5, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when t = 2 s is nearly. **[AIEEE - 2010, 4/144]**



(A) 13 m/s² (B) 12 m/s² (C) 7.2 m/s² (D) 14 m/s²

2. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P (R, θ) on the circle of radius R is (Here θ is measured from the x-axis) **[AIEEE - 2010, 4/144]**

(A) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$ (B) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$

(C) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$ (D) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

3. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is :

[AIEEE 2012 ; 4/120, –1]

(A) $m_1 r_1 : m_2 r_2$ (B) $m_1 : m_2$ (C) $r_1 : r_2$ (D) 1 : 1

4. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute(rpm) to ensure proper mixing is close to : (Take the radius of the drum to be 1.25 m and its axle to be horizontal): **[JEE Main (Online) 2016]**

(A) 8.0 (B) 0.4 (C) 1.3 (D) 27.0

5. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then, **[JEE Main 2018; 4/120, –1]**

(A) $T \propto R^{\frac{n}{2}+1}$ (B) $T \propto R^{(n+1)/2}$ (C) $T \propto R^{n/2}$ (D) $T \propto R^{3/2}$ for any n

6. A body is projected at t = 0 with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1s is R. Neglecting air resistance and taking acceleration due to gravity g = 10 ms^{-2}, the value of R is : **[JEE Main 2019, Jan.; 4/120, –1]**

(A) 2.5 m (B) 10.3 m (C) 2.8 m (D) 5.1 m

7. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change is velocity of the particle, when it moves through an angle of 60° around the centre of the circle? **[JEE Main 2019, Jan.; 4/120, –1]**

(A) zero (B) 10 m/s (C) $10\sqrt{3}$ m/s (D) $10\sqrt{2}$ m/s

Answers

Exercise # 1

PART - I

Section (A) :

A 1. 12 : 1, $\frac{\pi}{30}$ rad/sec.

A 2. 3 : 1 **A 3.** 4

A 4. $\frac{\pi\sqrt{2}}{30}$ cm/sec, $\frac{\pi\sqrt{2}}{30\times15}$ cm/s^2

A 5. 24 rad/sec **A 6.** 6

Section (B) :

B 1. $\left(\frac{\sqrt{181}}{5}\text{m/sec}^2\right)$

B 2. (a) 4.0 cm/s^2, (b) 2.0 cm/s^2, (c) $\sqrt{20}$ cm/s^2

B 3. (i) 75m/s^2, (ii) 125m/s^2

Section (C) :

C 1. T = 6.6 N, v_{max} = 34.64 ms^{-1}

C 2. $\mu \geq \frac{\pi^2}{81}$ **C 3.** $\sqrt{2g}$ rad/s

C 4. $8T_0$

C 5. $\frac{15\pi^2}{10}$ = 14.8N, $\frac{15\pi^2}{10}$ = 14.8 N.

Section (D) :

D 1. $\frac{u^2\cos^2\theta}{g\cos^3(\theta/2)}$

Section (E) :

E 1. 4 m/sec **E 2.** 5 rad/s

E 3. $\frac{9}{2}$ mg **E 4.** 90°

E 5. 2 cm **E 6.** $\sqrt{5g\ell}$

Section (F) :

F 1. Yes, a_c = 8, μg = 1 **F 2.** $\sqrt{120}$ m/s

F 3. $5\sqrt{2}$ ms^{-1} **F 4.** $\frac{2}{45}$m

F 5. tan^{-1}(1/4) , 1/4 **F 6.** $100\sqrt{3}$ m/s

F 7. Between $\sqrt{\frac{50}{3}}\times\frac{18}{5}$ = 14.7 km/h and 54 km/hr

F 8. 10 m/s^2

PART - II :

Section (A) :

A 1. (C) **A 2.** (A) **A 3.** (C)

A 4. (D) **A 5.** (B)

Section (B) :

B 1. (C) **B 2.** (C) **B 3.** (C)

B 4. (A)

Section (C) :

C 1. (C) **C 2.** (D) **C 3.** (D)

C 4. (B) **C 5.** (A) **C 6.** (D)

Section (D) :

D 1. (A) **D 2.** (C)

Section (E) :

E 1. (C) **E 2.** (A) **E 3.** (B)

E 4. (D) **E 5.** (C)

Section (F) :

F 1. (B) **F 2.** (A) **F 3.** (A)

PART - III :

1. (A) $\rightarrow$ q,s (B) $\rightarrow$ p (C) $\rightarrow$ p (D) $\rightarrow$ q,r

2. (A) $\rightarrow$ R; (B) $\rightarrow$ S; (C) $\rightarrow$ Q; (D) $\rightarrow$ P

Exercise # 2

PART - I

1.	(D)	**2.**	(D)	**3.**	(C)
4.	(D)	**5.**	(A)	**6.**	(D)
7.	(B)	**8.**	(A)	**9.**	(A)
10.	(B)	**11.**	(B)	**12.**	(A)
13.	(B)	**14.**	(B)	**15.**	(A)
16.	(A)				

PART - II

1.	6	**2.**	2	**3.**	8
4.	3	**5.**	8	**6.**	2
7.	27	**8.**	90	**9.**	5
10.	7	**11.**	5	**12.**	10

PART - III

1.	(B,C,D,E)	**2.**	(B, C)
3.	(B, C)	**4.**	(B, C, D)
5.	(A, B, C)	**6.**	(A, C)
7.	(A, B)	**8.**	(A,B,C)
9.	(A,B,C)		

PART - IV

1.	(B)	**2.**	(A)	**3.**	(A)
4.	(D)	**5.**	(B)	**6.**	(C)
7.	(B)	**8.**	(C)	**9.**	(C)

Exercise # 3

PART - I

1.	(D)	**2.**	(A)	**3.**	(D)
4.	(D)	**5.**	(C)	**6.**	(B, C)

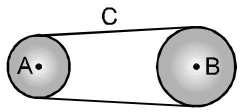
PART - II

1.	(D)	**2.**	(C)	**3.**	(C)
4.	(D)	**5.**	(B)	**6.**	(C)
7.	(B)				

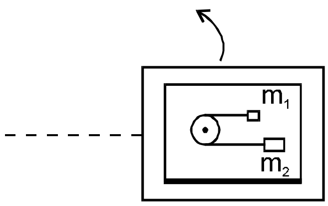
RANKER PROBLEMS

SUBJECTIVE QUESTIONS

1. Wheel A of radius r_A = 10cm is coupled by a belt C to another wheel of radius r_B = 25 cm as in the figure. The belt does not slip. At time t = 0 wheel A increases it's angular speed from rest at a uniform rate of $\pi/2$ rad/sec$^{2.}$ Find the time in which wheel B attains a speed of 100 rpm (wheel are fixed).



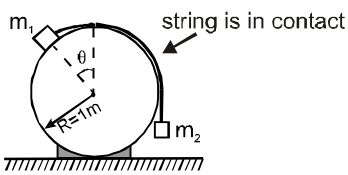
2. A table with smooth horizontal surface is placed in a cabin which moves in a circle of a large radius R (figure). A smooth pulley of small radius is fastened to the table. Two masses m and 2m placed on the table are connected through a string over the pulley. Initially the masses are held by a person with the string along the outward radius and then the system is released from rest (with respect to the cabin). Find the magnitude of the initial acceleration of the masses as seen from the cabin and the tension in the string.



3. A person stands on a spring balance at the equator. (a) By what percentage is the balance reading less than his true weight ? (b) If the speed of earth's rotation is increased by such an amount that the balance reading is half the true weight, what will be the length of the day in this case ?

4. A block of mass m is kept on a horizontal ruler. The friction coefficient between the ruler and the block is μ. The ruler is fixed at one end and the block is at a distance L from the fixed end. The ruler is rotated about the fixed end in the horizontal plane through the fixed end. (A) What can the maximum angular speed before which the block does not slip ? (B) If the angular speed of the ruler is uniformly increased from zero at a constant angular acceleration α, at what angular speed will the block slip?

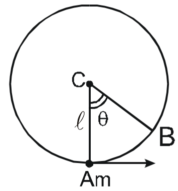
5. A mass m_1 lies on fixed, smooth cylinder. An ideal cord attached to m_1 passes over the cylinder and is connected to mass m_2 as shown in the figure.



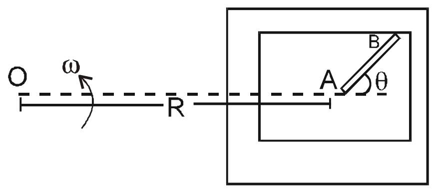
(a) Find the value of θ (shown in diagram) for which the system is in equilibrium

(b) Given m_1 = 5 kg, m_2 = 4kg . The system is released from rest when θ = 30º. Find the magnitude of acceleration of mass m_1 just after the system is released.

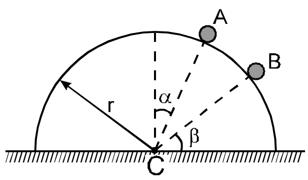
6. A particles of mass m is attached at one end of a light, inextensible string of length ℓ whose other end is fixed at the point C. At the lowest point the particle is given minimum velocity to complete the circular path in the vertical plane. As it moves in the circular path the tension in the string changes with θ. θ is defined in the figure. As θ varies from '0' to '2π' (i.e. the particle completes one revolution) plot the variation of tension 'T' against 'θ'.



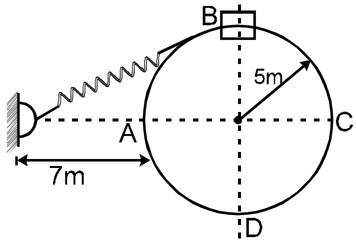
7. A table with smooth horizontal surface is fixed in a cabin that rotates with a uniform angular velocity ω in a circular path of radius R (figure). A smooth groove AB of length L(< < R) is made on the surface of the table .The groove makes an angle θ with the radius OA of the circle in which the cabin rotates. A small particle is kept at the point A in the groove and is released to move along AB. Find the time taken by the particle to reach the point B.



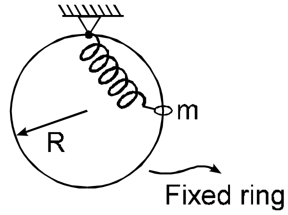
8. A particle initially at rest starts moving from point A on the surface of a fixed smooth hemisphere of radius r as shown. The particle looses its contact with hemisphere at point B. C is centre of the hemisphere. The equation relating α and β is



9. A collar 'B' of mass 2 kg is constrained to move along a horizontal smooth and fixed circular track of radius 5 m. The spring lying in the plane of the circular track and having spring constant 200 N/m is undeformed when the collar is at 'A'. If the collar starts from rest at 'B', what will be the normal reaction exerted by the track on the collar when it passes through 'A' ? [no gravitational force]



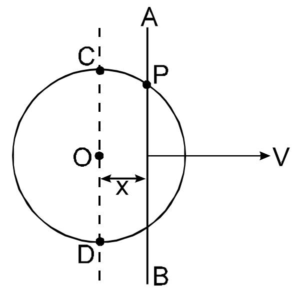
10. A ring of radius R is placed such that it lies in a vertical plane. The ring is fixed. A bead of mass m is constrained to move along the ring without any friction. One end of the spring is connected with the mass m and other end is rigidly fixed with the topmost point of the ring. Initially the spring is in un-extended position and the bead is at a vertical distance R from the lowermost point of the ring. The bead is now released from rest.



(a) What should be the value of spring constant K such that the bead is just able to reach bottom of the ring.

(b) The tangential and centripetal accelerations of the bead at initial and bottommost position for the same value of spring constant K.

11. A rod AB is moving on a fixed circle of radius R with constant velocity 'v' as shown in figure. P is the point of intersection of the rod and the circle. At an instant the rod is at a distance $x = \frac{3R}{5}$ from centre of the circle. The velocity of the rod is perpendicular to the rod and the rod is always parallel to the diameter CD.

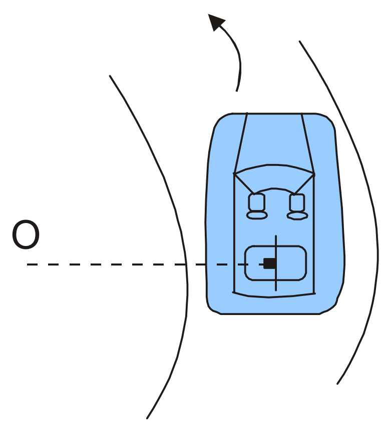


(a) Find the speed of point of intersection P.

(b) Find the angular speed of point of intersection P with respect to centre of the circle.

12. A chain of mass m forming a circle of radius R is slipped on a smooth round cone with half-angle θ. Find the tension of the chain if it rotates with a constant angular velocity ω about a vertical axis coinciding with the symmetry axis of the cone.

13. A car moving at a speed of 36 km/hr is taking a turn on a circular road of radius 50 m. A small wooden plate is kept on the seat with its plane perpendicular to the radius of the circular road (figure). A small block of mass 100g is kept on the seat which rests against the plate. The friction coefficient between the block and the plate is $\mu = \frac{1}{\sqrt{3}} = 0.58$.



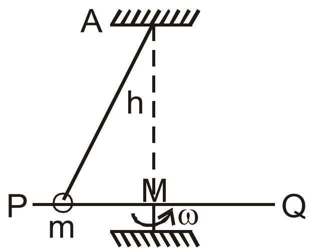
(a) Find the normal contact force exerted by the plate on the block.

(b) The plate is slowly turned so that the angle between the normal to the plate and the radius of the road slowly increases. Find the angle at which the block will just start sliding on the plate.

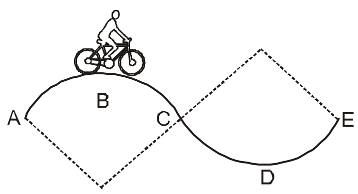
14. A hemispherical bowl of radius r = 0.1m is rotating about its axis (which is vertical) with an angular velocity ω. A particle of mass 10^{-2}kg on the frictionless inner surface of the bowl is also rotating with the same ω. the particle is at a height h from the bottom of the bowl. (a) Obtain the relation between h and ω. What is the minimum value of ω needed in order to have a nonzero value of h. (b) It is desired to measure 'g' using this setup by measuring h accurately. Assuming that r and ω are known precisely and that the least count in the measurement of h is 10^{-4} m. What is minimum error Δg in the measured value of g.[g = 9.8m/s^2]

15. A block is placed inside a horizontal hollow cylinder. The cylinder is rotating with constant angular speed one revolution per second about its axis. The angular position of the block at which it begins to slide is 30° below the horizontal level passing through the center. Find the radius of the cylinder if the coefficient of friction is 0.6. What should be the minimum constant angular speed of the cylinder so that the block reach the highest point of the cylinder?

16. A smooth rod PQ is rotated in a horizontal plane about its mid point M which is h = 0.1 m vertically below a fixed point A at a constant angular velocity 14 rad/s. A light elastic string of natural length 0.1 m requiring 1.47 N/cm has one end fixed at A and its other end attached to a ring of mass m = 0.3 kg which is free to slide along the rod. When the ring is stationary relative to rod, then find inclination of string with vertical, tension in string, force exerted by ring on the rod. (g = 9.8 m/s^2)



17. A track consists of two circular parts ABC and CDE of equal radius 100 m and joined smoothly as shown in fig. Each part subtends a right angle at its centre. A cycle weighing 100 kg together with the rider travels at a constant speed of 18 km/h on the track. (a) Find the normal contact force by the road on the cycle when it is at B and D. (b) Find the force of friction exerted by the track on the tyres when the cycle is at B, C and D. (c) Find the normal force between the road and the cycle just, before and just after the cycle crosses C. (d) What should be the minimum friction coefficient between the road and the tyre, which will ensure that the cyclist can move with constant speed ? Take g = 10m/s^2.



Answers

1. 50/3 sec.

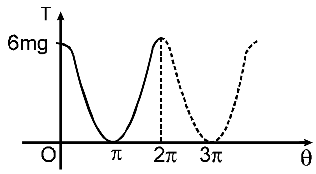
2. $\frac{\omega^2 R}{3}$, $\frac{4}{3} m\omega^2 R$

3. (a) $\frac{\omega^2 R}{g} \times 100 = 0.65\%$, (b) $2\pi\sqrt{\frac{2R}{g}}$ = 2.0 hour]

4. (A) $\sqrt{\mu g / L}$, (B) $\left[\left(\frac{\mu g}{L}\right)^2 - \alpha^2\right]^{\frac{1}{4}}$

5. (a) $\sin\theta = \frac{m_2}{m_1}$ (b) $a = \frac{15}{9}$ m/s^2

6.



7. $\sqrt{\frac{2L}{\omega^2 R \cos\theta}}$

8. 3 sin β = 2 cos α

9. 1440 N

10. (a) $K = \frac{mg}{R(3-2\sqrt{2})}$ (b) at intial instant $a_t = g$, $a_c = 0$; at bottommost position $a_t = 0$ $a_c = 0$

11. (a) $V_P = \frac{5}{4} V$ (b) $\omega = \frac{V_P}{R} = \frac{5V}{4R}$

12. $T = (\cot\theta + \omega^2 R / g)\, mg / 2\pi$

13. (a) 0.2N, (b) 30º

14. (a) $7\sqrt{2}$ rad/s (b) -9.8 x 10^{-3} m/s^2

15. 0.24m, 8.9 rad/sec

16. cos θ = 3/5 , T = 9.8 N , N = $\frac{147}{50}$ = 2.94 N

17. (a) 975N, 1025 N ; (b) 0,707N, 0 ; (c) 682N , 732 N ; (d) 1.037

SELF ASSESSMENT PAPER

JEE (ADVANCED) PAPER

SECTION-1 : ONE OPTION CORRECT TYPE (Maximum Marks - 12)

1. When the angular velocity of a uniformly rotating body has increased to thrice of initial value the resultant of forces applied to it increases by 60 N. Find the accelerations of the body in the two cases. The mass of the body, m = 3kg:

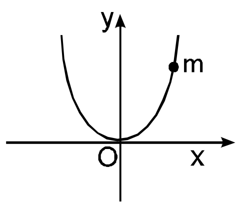
(A) 2.5 ms^{-2}, 7.5 ms^{-2} (B) 7.5 ms^{-2}, 22.5 ms^{-2} (C) 5 ms^{-2}, 45 ms^{-2} (D) 2.5 ms^{-2}, 22.5 ms^{-2}

2. A heavy particle hanging vertically from a point by a light inextensible string of length l is started so as to make a complete revolution in a vertical plane. The sum of the magnitude of tension at the ends of any diameter :

(A) first increase then decreases (B) is constant

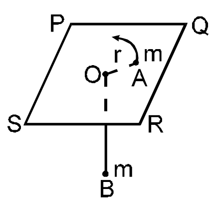
(C) first decrease then increases (D) decreases continuously

3. A bead of mass m is located on a parabolic wire with its axis vertical and vertex at the origin as shown in figure and whose equation is $x^2 = 4ay$. The wire frame is fixed and the bead can slide on it without friction. The bead is released from the point y = 4a on the wire frame from rest. The tangential acceleration of the bead when it reaches the position given by y = a is :



(A) $\frac{g}{2}$ (B) $\frac{\sqrt{3}g}{2}$ (C) $\frac{g}{\sqrt{2}}$ (D) $\frac{g}{\sqrt{5}}$

4. In the figure PQRS is a frictionless horizontal plane on which a particle A of mass m moves in a circle of radius r with an angular velocity ω such that $\omega^2 r = g/3$. Another particle of mass m is tied to A through an inextensible massless string. O is the hole through which string passes down to B. B can move only vertically. The tension in the string at this instant will be:



(A) mg/3 (B) 2 mg/3 (C) mg/6 (D) none

SECTION-2 : ONE OR MORE THAN ONE CORRECT TYPE (Maximum Marks - 32)

5. A curved section of a road is banked for a speed v. If there is no friction between road and tyres of the car, then:

(A) car is more likely to slip at speeds higher than v than speeds lower than v

(B) car cannot remain in static equilibrium on the curved section

(C) car will not slip when moving with speed v

(D) none of the above

6. A heavy particle is tied to the end A of a string of length 1.6 m. Its other end O is fixed. It revolves as a conical pendulum with the string making 60° with the vertical. Then

(A) its period of revolution is $\frac{4\pi}{7}$ sec.

(B) the tension in the string is double the weight of the particle

(C) the velocity of the particle = $2.8\sqrt{3}$ m/s

(D) the centripetal acceleration of the particle is $9.8\sqrt{3}$ m/s^2.

7. The position vector of a particle in a circular motion about the origin sweeps out equal area in equal time. Its

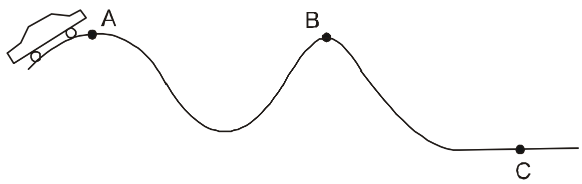
(A) velocity remains constant

(B) speed remains constant

(C) acceleration remains constant

(D) tangential acceleration remains constant

8. A car is moving with constant speed on a road as shown in figure. The normal reaction by the road on the car is N_A, N_B and N_C when it is at the points A, B and C respectively.



(A) $N_A = N_B$

(B) $N_A > N_B$

(C) $N_A < N_B$

(D) $N_C > N_A$

9. Two particles move on a circular path (one just inside and the other just outside) with angular velocities ω and 5ω starting from the same point. Then

(A) they cross each other at regular intervals of time $\frac{2\pi}{4\omega}$ when their angular velocities are oppositely directed.

(B) they cross each other at points on the path subtending an angle of 60° at the centre if their angular velocities are oppositely directed.

(C) they cross at intervals of time $\frac{\pi}{3\omega}$ if their angular velocities are oppositely directed.

(D) they cross each other at points on the path subtending 90° at the centre if their angular velocities are in the same sense.

10. A ball tied to the end of a string swings in a vertical circle under the influence of gravity

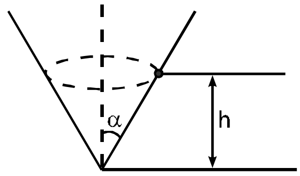
(A) when the string makes an angle 90º with the vertical, the tangential acceleration is zero & radial acceleration is somewhere between maximum and minimum

(B) when the string makes an angle 90º with the vertical, the tangential acceleration is maximum & radial acceleration is somewhere between maximum and minimum

(C) at no place in the circular motion, tangential acceleration is equal to radial acceleration

(D) throughout the path whenever radial acceleration has its extreme value, the tangential acceleration is zero.

11. A particle is describing circular motion in a horizontal plane in contact with the smooth inside surface of a fixed right circular cone with its axis vertical and vertex down. The height of the plane of motion above the vertex is h and the semivertical angle of the cone is α. The period of revolution of the particle:



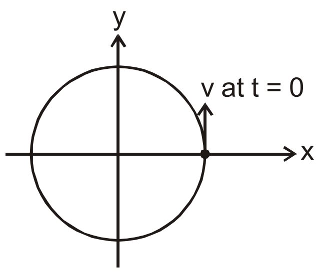
(A) increases as h increases

(B) decreases as h increases

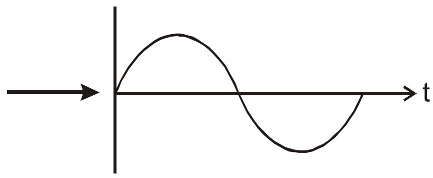
(C) increases as α increases

(D) decreases as α increases

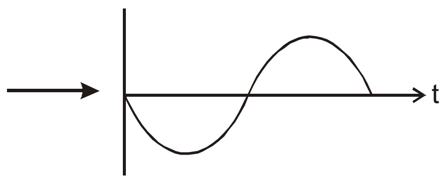
12. A particle is moving in a uniform circular motion on a horizontal surface. Particle position and velocity at time t = 0 are shown in the figure in the coordinate system. Which of the indicated variable on the vertical axis is/are correctly matched by the graph(s) shown alongside for particle's motion ?



(A) x component of velocity

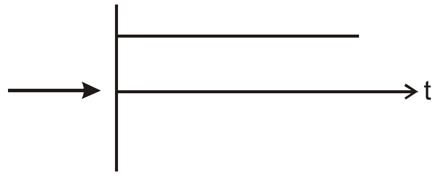


(B) y component of force keeping particle moving

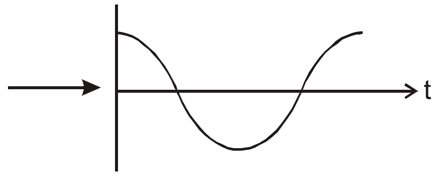


in a circle

(C) Angular velocity of the particle



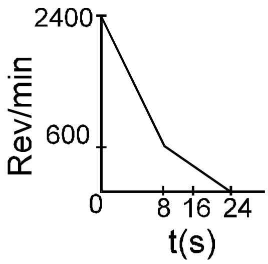
(D) x coordinate of the particle



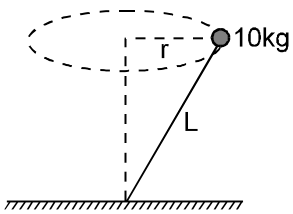
SECTION-3 : NUMERICAL VALUE TYPE (Maximum Marks - 18)

13. The velocity and acceleration vectors of a particle undergoing circular motion are $\vec{v} = 2\hat{i}$ m/s and $\vec{a} = 2\hat{i} + 4\hat{j}$ m/s^2 respectively at an instant of time. The radius of the circle is

14. A table fan rotating at a speed of 2400 rpm is switched off and the resulting variation of the rpm with time is shown in the figure. The total number of revolutions of the fan before it come to rest is :

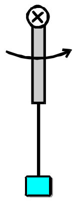


15. A 10kg ball attached at the end of a rigid rod of length 1m rotates at constant speed in a horizontal circle of radius 0.5m and period 1.57 s as shown in the figure. What will be the force exerted by the rod on the ball?

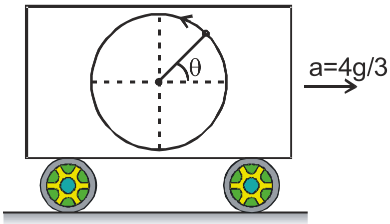


16. A stone is thrown horizontally with a velocity of 10m/sec. If the radius of curvature of it's trajectory at the end of 3 sec after motion began is $100\sqrt{n}$ m. Find is n?

17. One end of a light rod of length 1 m is attached with a string of length 1m. Other end of the rod is attached at point O such that rod can move in a vertical circle. Other end of the string is attached with a block of mass 2kg. The minimum velocity that must be given to the block in horizontal direction so that it can complete the vertical circle is ($g = 10\ m/s^2$).



18. A bus is moving with a constant acceleration $a = 4g/3$ towards right. In the bus, a ball is tied with a rope and is rotated in vertical circle as shown. The tension in the rope will be minimum, when the ropemakes an angle θ (in degree). Find θ.



Answers

1.	(D)	**2.**	(B)	**3.**	(C)	**4.**	(B)	**5.**	(B, C)
6.	(A, B, C, D)	**7.**	(B, D)	**8.**	(B, D)	**9.**	(B, C, D)	**10.**	(B, D)
11.	(A, C)	**12.**	(B, C, D)	**13.**	01.00	**14.**	280.00	**15.**	128.00
16.	10.00	**17.**	10.00	**18.**	37.00				