# BINOMIAL THEOREM

**1.** **Statement of Binomial theorem :** If a, b $\in$ R and n $\in$ N, then

$(a + b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + ... + {}^nC_r\, a^{n-r} b^r + ... + {}^nC_n\, a^0 b^n$

$$= \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$$

**2.** **Properties of Binomial Theorem :**

(i) **General term :** $T_{r+1} = {}^nC_r\, a^{n-r} b^r$

(ii) **Middle term (s) :**

(a) If n is even, there is only one middle term,

which is $\left(\frac{n+2}{2}\right)$th term.

(b) If n is odd, there are two middle terms,

which are $\left(\frac{n+1}{2}\right)$th and $\left(\frac{n+1}{2}+1\right)$th terms.

## 3. Multinomial Theorem :

$(x_1 + x_2 + x_3 + \ldots\ldots\ldots\ldots x_k)^n$

$$= \sum_{r_1+r_2+\ldots+r_k=n} \frac{n!}{r_1!\, r_2! \ldots r_k!}\; x_1^{r_1} \,.\, x_2^{r_2} \ldots x_k^{r_k}$$

Here total number of terms in the expansion = ${}^{n+k-1}C_{k-1}$

## 4. Application of Binomial Theorem :

**If $(\sqrt{A}+B)^n$ = I + f where I and n are positive integers, n being odd and**

**0 < f < 1 then (I + f) f = $k^n$ where A – $B^2$ = k > 0 and $\sqrt{A}$ – B < 1.**

**If n is an even integer, then (I + f) (1 – f) = $k^n$**

## 5. Properties of Binomial Coefficients :

(i) ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots\ldots + {}^nC_n = 2^n$

(ii) ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \ldots\ldots\ldots\ldots + (-1)^n\, {}^nC_n = 0$

(iii) ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots. = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots. = 2^{n-1}$

(iv) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ (v) $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$

## 6. Binomial Theorem For Negative Integer Or Fractional Indices

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots. +$$

$$\frac{n(n-1)(n-2)\ldots\ldots.(n-r+1)}{r!}x^r + \ldots.,|\,x\,| < 1.$$

$$T_{r+1} = \frac{n(n-1)(n-2)\ldots\ldots\ldots(n-r+1)}{r!}x^r$$