

# CBSE SAMPLE PAPER - 03

## Class 11 - Mathematics

Time Allowed: 3 hours

Maximum Marks: 80

### General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

### Section A

1.  $162^\circ$  when measured in radians [1]  
a)  $\left(\frac{9\pi}{10}\right)^c$  b)  $\left(\frac{7\pi}{10}\right)^c$   
c)  $\left(\frac{4\pi}{3}\right)^c$  d)  $\left(\frac{5\pi}{4}\right)^c$
2. If the S.D. of the 1, 2, 3, 4, 5,.....10 is  $\sigma$ , then the S.D. of 11, 12, 13, 14,..... 20 is [1]  
a)  $\frac{\sigma}{10}$  b)  $10\sigma$   
c)  $\sigma + 10$  d)  $\sigma$
3. If three of the six vertices of a regular hexagon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is [1]  
a)  $\frac{1}{10}$  b)  $\frac{3}{20}$   
c)  $\frac{1}{5}$  d)  $\frac{3}{10}$
4.  $\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$  is equal to [1]  
a) 1 b) 0  
c) 2 d) -1
5. The angle between the lines  $2x - y + 3 = 0$  and  $x + 2y + 3 = 0$  is [1]  
a)  $90^\circ$  b)  $30^\circ$   
c)  $45^\circ$  d)  $60^\circ$
6. Which of the following is a null set? [1]

- a)  $C = \phi$  b)  $B = \{x : x + 3 = 3\}$
- c)  $D = \{0\}$  d)  $A = \{x : x > 1 \text{ and } x < 1\}$
7.  $z_1, z_2, z_3$  are complex numbers. Then the expression  $z_1 \operatorname{Im}(\bar{z}_2 z_3) + z_2 \operatorname{Im}(\bar{z}_3 z_1) + z_3 \operatorname{Im}(\bar{z}_1 z_2) =$  [1]
- a)  $\operatorname{Re}(z_1 z_2 \bar{z}_3 + z_2 z_3 \bar{z}_1 + z_1 z_2 \bar{z}_3)$  b)  $z_1 z_2 \bar{z}_3 + z_2 z_3 \bar{z}_1 + z_3 z_1 \bar{z}_2$
- c) 0 d)  $\operatorname{Im}(z_1 z_2 \bar{z}_3 + z_2 z_3 \bar{z}_1 + z_3 z_1 \bar{z}_2)$
8. If  $f(x) = \log\left(\frac{1+x}{1-x}\right)$  and  $g(x) = \frac{3x+x^3}{1+3x^2}$  Then  $f(g(x))$  is equal to [1]
- a)  $f(3x)$  b)  $-f(x)$
- c)  $[f(x)]^3$  d)  $3f(x)$
9. If  $\frac{|x-2|}{x-2} \geq 0$ , then [1]
- a)  $x \in (-\infty, 2)$  b)  $x \in (-\infty, 2]$
- c)  $x \in [2, \infty)$  d)  $x \in (2, \infty)$
10. The value of  $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$  is equal to [1]
- a) 2 b) 0
- c)  $\frac{1}{2}$  d) 1
11. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is [1]
- a) Atmost 20 b) None of these
- c) Atleast 30 d) Exactly 25
12. If five G.M.s are inserted between 486 and  $\frac{2}{3}$ , then fourth G.M. equals [1]
- a) 6 b) 12
- c) 4 d) -6
13. The total number of terms in the expansion of  $(x+k)^{100} + (x-k)^{100}$  after simplification is [1]
- a) 101 b) 50
- c) 202 d) 51
14. If a, b, c are real numbers such that  $a \leq b, c < 0$ , then [1]
- a)  $ac \leq bc$  b)  $ac > bc$
- c)  $ac \geq bc$  d) none of these
15. If  $A \subset B$ , then [1]
- a)  $A^c \subset B^c$  b)  $B^c \not\subset A^c$
- c)  $A^c = B^c$  d)  $B^c \subset A^c$
16. The maximum value of  $\sin(x + \frac{\pi}{6}) + \cos(x + \frac{\pi}{6})$  in the interval  $(0, \frac{\pi}{2})$  is attained at: [1]
- a)  $x = \frac{\pi}{12}$  b)  $x = \frac{\pi}{3}$
- c)  $x = \frac{\pi}{6}$  d)  $x = \frac{\pi}{2}$
17. If  $\omega$  is an imaginary cube root of unity, then  $(1 + \omega - \omega^2)^7$  is equal to [1]

a)  $-128\omega^2$

b)  $128\omega$

c)  $128\omega^2$

d)  $-128\omega$

18. The number of ways in which 15 identical objects can be distributed among 6 people, is [1]

a)  ${}^{20}C_5$

b)  ${}^{15}C_6$

c)  ${}^{21}C_6$

d)  ${}^{16}C_5$

19. **Assertion (A):** The coefficient of  $x^r$  in  $(1+x)^n$  is  ${}^nC_r$ . [1]

**Reason (R):** The number of dissimilar terms in the expansion of  $(1-3x+3x^2-x^3)^{20}$  is 61.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If  $A = \{1, 2, 3\}$ ,  $B = \{2, 4\}$ , then the number of relation from A to B is equal to 64. [1]

**Reason (R):** The total number of relation from set A to set B is equal to  $\{2^{n(A) \cdot n(B)}\}$ .

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

### Section B

21. Find the simplified form of [2]

$$f(x) = |x-2| + |2-x|, \text{ if } -3 \leq x \leq 3.$$

22. Evaluate:  $\lim_{x \rightarrow \sqrt{2}} \frac{x^4-4}{x^2+3\sqrt{2}x-8}$ . [2]

23. Find the equation of hyperbola having Foci  $(\pm 3\sqrt{5}, 0)$ , the latus rectum is of length 8. [2]

OR

Show that the equation  $x^2 + y^2 - 4x + 6y - 5 = 0$  represents a circle. Find its centre and radius.

24. Let  $X = \{1, 2, 3, 4, 5, 6\}$ . If n represent any member of X, express the set n is greater than 4 [2]

25. Using slopes show that the points A(6, -1), B(5, 0) and C(2, 3) are collinear. [2]

### Section C

26. The function f is defined by  $f(x) = \begin{cases} 1-x, & x < 0 \\ 1, & x = 0 \\ x+1, & x > 0 \end{cases}$  [3]

Draw the graph of f(x).

27. Three students are standing in a park with signboards "SAVE ENVIRONMENT", "DON'T LITTER", "KEEP YOUR PLACE CLEAN". Their positions are marked by the points A (0,7,10), B (-1,6,6) and C(-4,9,6). The three students are holding GREEN coloured ribbon together. Does the ribbons form sides of a right-angled triangle? Do you feel the need to promote? What message is given from this question to the society? [3]

OR

Find the points on the line  $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$  at a distance of 5 units from the point P(1,3,3).

28. Find  $(x+1)^6 + (x-1)^6$ . Hence or otherwise evaluate  $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$  [3]

OR

Using binomial theorem, expand:  $(\sqrt{x} + \sqrt{y})^8$

29. Find the multiplicative inverse of the complex numbers  $= \sqrt{5} + 3i$  [3]



OR

Find the square root of  $-i$

30. A man wants to cut three lengths from a single piece of board of length 91cm. The second length is to be 3cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5cm longer than the second? [3]
31. Find:  $r$ , if  ${}^4P_r = 6 {}^5P_{r-1}$  [3]

**Section D**

32. One urn contains two black balls (labelled B1 and B2) and one white ball. A second urn contains one black ball and two white balls (labelled W1 and W2). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball. [5]
- Write the sample space showing all possible outcomes
  - What is the probability that two black balls are chosen?
  - What is the probability that two balls of opposite colour are chosen?

33. Solve:  $\lim_{x \rightarrow 1} \frac{x^4 - 3x^3 + 2}{x^3 - 5x^2 + 3x + 1}$  [5]

OR

Find the derivative of  $x \sin x$  from first principle.

34. If  $\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha) = \frac{-3}{2}$ , then prove that  $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$ . [5]

OR

Prove that:  $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$ .

35. The mean and variance of five observations are 6 and 4 respectively. If three of these are 5, 7 and 9, find the other two observations. [5]

**Section E**

36. **Read the text carefully and answer the questions:** [4]
- Arun is running in a racecourse note that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m.



- Path traced by Arun represents which type of curve. Find the length of major axis?
- Find the equation of the curve traced by Arun?
- Find the eccentricity of path traced by Arun?

OR

Find the length of latus rectum for the path traced by Arun.

37. **Read the text carefully and answer the questions:** [4]
- On the roof of Monesh's house, a water tank of capacity 9000 litres is installed. A water pump fills the tank, the pump uses water from the municipality water supply, In the beginning, the water flow of the pump remains 100

litres/hour for the first hour.

The water flow from the pump is 1.25th after each 1 hour.

Once Monesh's mother was not at home and told him to switch off the pump when the tank is almost full.

He calculated that after how many hours should he stop the pump so water does not get overflow in the next one hour.



- (i) After how many hours Monesh should stop the pump so that in the next hour the tank does not get overflow?
- |             |             |
|-------------|-------------|
| a) 13 hours | b) 16 hours |
| c) 14 hours | d) 15 hours |
- (ii) After 10 hours how much water was filled in the tank?
- |                   |                |
|-------------------|----------------|
| a) 3325.29 Liters | b) 3200 Liters |
| c) 3300 Liters    | d) 3000 Liters |
- (iii) In 7th hour how much water was filled in the tank?
- |                  |                  |
|------------------|------------------|
| a) 375.25 Liters | b) 450 Liters    |
| c) 400 Liters    | d) 381.47 Liters |

**OR**

What was the water flow in 5th hour?

- |                  |                     |
|------------------|---------------------|
| a) 250 Liters/hr | b) 300 Liters/hr    |
| c) 400 Liters/hr | d) 244.14 Liters/hr |

38. Read the text carefully and answer the questions:

[4]

In a company, 100 employees offered to do a work. In out of them, 10 employees offered ground floor only, 15 employees offered first floor only, 10 employees offered second floor only, 30 employees offered second floor and ground floor to work, 25 employees offered first and second floor, 15 employees offered ground and first floor, 60 employees offered second floor.



- (i) Find the number of employees who offered all three floors.

(ii) Find the number of employees who offered ground and first floor but not second floor.

**Solution**

**CBSE SAMPLE PAPER - 03**

**Class 11 - Mathematics**

**Section A**

1. (a)  $\left(\frac{9\pi}{10}\right)^c$

**Explanation:**  $180^\circ = \pi^c \Rightarrow 1^\circ = \left(\frac{\pi}{180}\right)^c \Rightarrow 162^\circ = \left(\frac{\pi}{180} \times 162\right)^c = \left(\frac{9\pi}{10}\right)^c$

2. (d)  $\sigma$

**Explanation:** S.D =  $\sqrt{\frac{n^2-1}{12}}$

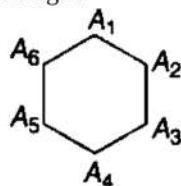
Where n is the number of observations

In case of 1, 2, 3, .....,10 as well as in case of 11, 12, 13,....., 20

S.D =  $\sqrt{\frac{(10)^2-1}{12}} = \sqrt{\frac{99}{12}} = 2.87$

3. (a)  $\frac{1}{10}$

**Explanation:** Since, there is a regular hexagon, then the number of ways of choosing three vertices is  ${}^6C_3$ . And, there is only two ways i.e. choosing vertices of a regular hexagon alternate, here  $A_1, A_3, A_5$  or  $A_2, A_4, A_6$  will result in an equilateral triangle.



$\therefore$  Required probability =  $\frac{2}{{}^6C_3} = \frac{2}{\frac{6!}{3!3!}} = \frac{2 \times 3 \times 2 \times 3 \times 2}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{1}{10}$

4. (a) 1

**Explanation:** Given,  $\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$

=  $\lim_{x \rightarrow 0} \frac{\sin x [\sqrt{x+1}+\sqrt{1-x}]}{(\sqrt{x+1}-\sqrt{1-x})(\sqrt{x+1}+\sqrt{1-x})}$

=  $\lim_{x \rightarrow 0} \frac{\sin x [\sqrt{x+1}+\sqrt{1-x}]}{x+1-1+x}$

=  $\lim_{x \rightarrow 0} \frac{\sin x [\sqrt{x+1}+\sqrt{1-x}]}{2x} = \frac{1}{2} \cdot \lim_{x \rightarrow 0} \frac{\sin x}{x} [\sqrt{x+1} + \sqrt{1-x}]$

Taking limits, we get

=  $\frac{1}{2} \times 1 \times [\sqrt{0+1} + \sqrt{1-0}] = \frac{1}{2} \times 1 \times 2 = 1$

5. (a)  $90^\circ$

**Explanation:** Let  $m_1$  and  $m_2$  be the slope of the lines  $2x - y + 3 = 0$  and  $x + 2y + 3 = 0$ , respectively.

Let  $\theta$  be the angle between them.

Here,

$m_1 = 2$  and  $m_2 = -\frac{1}{2}$

$\therefore m_1 m_2 = -1$

Therefore, the angle between the given lines is  $90^\circ$ , as it satisfy the condition of product of slopes of two lines is -1.

6. (a)  $C = \phi$

**Explanation:**  $\phi$  is denoted as null set.

7. (c) 0

**Explanation:**  $\text{Im}(\bar{z}_2 z_3) = \frac{\bar{z}_2 z_3 - z_2 \bar{z}_3}{2i}$

$\Rightarrow z_1 \text{Im}(\bar{z}_2 z_3) = \frac{1}{2i} (z_3 z_1 \bar{z}_2 - z_1 z_2 \bar{z}_3) \dots(i)$

Similarly

$z_2 \text{Im}(\bar{z}_3 z_1) = \frac{1}{2i} (z_1 z_2 \bar{z}_3 - z_2 z_3 \bar{z}_1) \dots(ii)$



$$\text{and } z_3 \operatorname{Im}(\bar{z}_1 z_2) = \frac{1}{2i} (z_2 z_3 \bar{z}_1 - z_3 z_1 \bar{z}_2) \dots \text{(iii)}$$

Adding (i), (ii) and (iii) we get,

$$\text{Required expression} = 0$$

8. (d)  $3f(x)$

$$\text{Explanation: } f(g(x)) = \log\left(\frac{1+g(x)}{1-g(x)}\right)$$

$$= \log\left(\frac{1+\frac{3x+x^2}{1+3x^2}}{1-\frac{3x+x^2}{1+3x^2}}\right)$$

$$= \log\left(\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}\right)$$

$$= \log\left(\frac{1+x}{1-x}\right)^3 = 3\log\left(\frac{1+x}{1-x}\right)$$

$$f(g(x)) = 3f(x)$$

9. (d)  $x \in (2, \infty)$

**Explanation:** Since  $\frac{|x-2|}{x-2} \geq 0$ , for  $|x-2| \geq 0$ , and  $x-2 \neq 0$  solution set  $(2, \infty)$

10. (b) 0

**Explanation:** We have,  $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$

$$= 2 \cos\left(\frac{50^\circ+70^\circ}{2}\right) \sin\left(\frac{50^\circ-70^\circ}{2}\right) + \sin 10^\circ$$

$$= 2 \cos 60^\circ (-\sin 10^\circ) + \sin 10^\circ$$

$$= -2 \times \frac{1}{2} \sin 10^\circ + \sin 10^\circ$$

$$= -\sin 10^\circ + \sin 10^\circ$$

$$= 0$$

11. (d) Exactly 25

**Explanation:** Let  $n$  be the number of newspapers which are read by the students.

$$\text{Then, } 60n = (300) \times 5$$

$$\Rightarrow n = 25$$

12. (a) 6

**Explanation:** Let  $G_1, G_2, G_3, G_4, G_5$  be the G.M.'s inserted between 486 and  $\frac{2}{3}$ .

Then, total terms are 7.

$$\text{Now, } T_n = ar^{n-1} \Rightarrow \frac{2}{3} = 486 (r)^6 \Rightarrow r = \frac{1}{3}$$

$\Rightarrow$  4<sup>th</sup> G.M. is,

$$T_5 = ar^4 = 486 \left(\frac{1}{3}\right)^4 = 6$$

13. (d) 51

**Explanation:** When  $n$  is even, the number of terms in  $\{(x+y)^n + (x-y)^n\}$  is  $\left(\frac{n}{2} + 1\right)$

$\therefore (x+k)^{100} + (x-k)^{100}$  has  $\left(\frac{100}{2} + 1\right) = 51$  terms. This is the required answer.

14. (c)  $ac \geq bc$

**Explanation:** The sign of the inequality is to be reversed ( $\leq$  to  $\geq$  or  $\geq$  to  $\leq$ ) if both sides of an inequality are multiplied by the same negative real number.

15. (d)  $B^c \subset A^c$

**Explanation:** Let  $A \subset B$

To prove  $B^c \subset A^c$ , it is enough to show that  $x \in B^c \Rightarrow x \in A^c$

Let  $x \in B^c$

$\Rightarrow x \notin B$

$\Rightarrow x \notin A$  since  $A \subset B$

$\Rightarrow x \in A^c$

Hence  $B^c \subset A^c$

16. (a)  $x = \frac{\pi}{12}$

**Explanation:** Let  $f(x) = \sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{6}\right)$ , then



$$\begin{aligned}
 f(x) &= \sqrt{2} \left[ \cos\left(x + \frac{\pi}{6}\right) \frac{1}{\sqrt{2}} + \sin\left(x + \frac{\pi}{6}\right) \frac{1}{\sqrt{2}} \right] \\
 &= \sqrt{2} \left[ \cos\left(x + \frac{\pi}{6}\right) \cos \frac{\pi}{4} + \sin\left(x + \frac{\pi}{6}\right) \sin \frac{\pi}{4} \right] \\
 &= \sqrt{2} \cos\left(x + \frac{\pi}{6} - \frac{\pi}{4}\right) \dots [\because \cos(A - B) = \cos A \cos B + \sin A \sin B] \\
 &= \sqrt{2} \cos\left(x - \frac{\pi}{12}\right)
 \end{aligned}$$

Since,  $-1 \leq \cos\left(x - \frac{\pi}{12}\right) \leq 1$

$\therefore f(x)$  is maximum, if  $x - \frac{\pi}{12} = 0 \Rightarrow x = \frac{\pi}{12}$

17. (a)  $-128\omega^2$

**Explanation:**  $(1 + \omega - \omega^2)^7 = (-\omega - \omega^2)^7$  [ $\because 1 + \omega + \omega^2 = 0$ ]  
 $= (-2\omega^2)^7 = (-2)^7 \omega^{14} = -128\omega^2$

18. (a)  ${}^{20}C_5$

**Explanation:** Distributing 15 identical objects among 6 people is similar to finding non negative integer solutions to  $x_1 + x_2 + \dots + x_6 = 15$  and the number of non negative integer solutions is  ${}^{15+6-1}C_{6-1}$

$\Rightarrow$  The number of ways of distributing  $n$  identical objects among  $r$  persons is  ${}^{n+r-1}C_{r-1}$

$\Rightarrow$  Required number of ways =  ${}^{15+6-1}C_{6-1} = {}^{20}C_5$

19. (b) Both A and R are true but R is not the correct explanation of A.

**Explanation:** Assertion is true

Reason!

$$\begin{aligned}
 &(1 - 3x + 3x^2 - x^3)^{20} \\
 &= [(1 - x)^3]^{20} \\
 &= (1 - x)^{60}
 \end{aligned}$$

$\therefore$  No. of dissimilar terms in the expansion of  $(1 - 3x + 3x^2 - x^3)^{20}$  is 61

20. (a) Both A and R are true and R is the correct explanation of A.

**Explanation:** We know by the property of relation, the total number of relation from set A to set B is  $2^{n(A) \cdot n(B)}$ .

$2^{3 \times 2} = 64$

### Section B

21. Given,  $|x - 2| = \begin{cases} x - 2, & x \geq 2 \\ -(x - 2), & x < 2 \end{cases}$  and  $|x + 2| = \begin{cases} (x + 2), & x \geq -2 \\ -(x + 2), & x < -2 \end{cases}$

$$\Rightarrow f(x) = \begin{cases} (x - 2) + (2 + x), & 2 \leq x \leq 3 \\ -(x - 2) + (x + 2), & -2 \leq x < 2 \\ -(x - 2) - (x + 2), & -3 \leq x < -2 \end{cases}$$

$$= \begin{cases} x - 2 + 2 + x, & 2 \leq x \leq 3 \\ -x + 2 + x + 2, & -2 \leq x < 2 \\ -x + 2 - x - 2, & -3 \leq x < -2 \end{cases}$$

$$= \begin{cases} 2x, & 2 \leq x \leq 3 \\ 4, & -2 \leq x < 2 \\ -2x, & -3 \leq x < -2 \end{cases}$$

22.  $\lim_{x \rightarrow \sqrt{2}} \frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8}$

$$= \lim_{x \rightarrow \sqrt{2}} \frac{(x^2 - 2)(x^2 + 2)}{x^2 + 4\sqrt{2}x - \sqrt{2}x - 8}$$

$$= \lim_{x \rightarrow \sqrt{2}} \frac{(x - \sqrt{2})(x + \sqrt{2})(x^2 + 2)}{(x - \sqrt{2})(x + 4\sqrt{2})}$$

$$= \lim_{x \rightarrow \sqrt{2}} \frac{(x + \sqrt{2})(x^2 + 2)}{(x + 4\sqrt{2})}$$

$$= \frac{(\sqrt{2} + \sqrt{2})[(\sqrt{2})^2 + 2]}{\sqrt{2} + 4\sqrt{2}}$$

$$= \frac{2\sqrt{2} \cdot (4)}{5\sqrt{2}} = \frac{8}{5}$$

23. Here foci are  $(\pm 3\sqrt{5}, 0)$  which lie on x-axis.

So the equation of hyperbola in standard form is  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\therefore$  foci  $(\pm c, 0)$  is  $(\pm 3\sqrt{5}, 0)$

$$\Rightarrow c = 3\sqrt{5}$$

Length of latus rectum  $\frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a$

We know that  $c^2 = a^2 + b^2$

$$\therefore (3\sqrt{5})^2 = a^2 + 4a$$

$$\Rightarrow a^2 + 4a - 45 = 0$$

$$\Rightarrow (a + 9)(a - 5) = 0$$

$$\Rightarrow a = 5 (\because a = -9 \text{ is not possible})$$

Also  $a = 5$

$$\Rightarrow b^2 = 4 \times 5 = 20$$

Thus required equation of hyperbola is

$$\frac{x^2}{25} - \frac{y^2}{20} = 1$$

OR

The general equation of a conic is as follows

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  where  $a, b, c, f, g, h$  are constants ... (i)

For a circle,  $a = b$  and  $h = 0$ .

The equation becomes:  $x^2 + y^2 + 2gx + 2fy + c = 0$

Given,  $x^2 + y^2 - 4x + 6y - 5 = 0$

Comparing with (i),  $a = b$  and  $h = 0$ .  $2g = -4 \Rightarrow g = -2$ ,  $2f = 6 \Rightarrow f = 3$  and  $c = -5$ .

Here  $a = b$  and  $h = 0$  so given equation is equation of circle

Centre  $(-g, -f) = (-(-2), -3)$

$$= (2, -3)$$

$$\text{Radius} = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{(-2)^2 + 3^2 - (-5)}$$

$$= \sqrt{4 + 9 + 5} = \sqrt{18} = 3\sqrt{2}$$

24. Suppose  $C = \{x \mid x \in X, x > 4\}$  where  $X = \{1, 2, 3, 4, 5, 6\}$

Therefore,  $C = \{5, 6\}$

25. For three points to be collinear, the slope of all pairs must be equal, that is the slope of AB = slope of BC = slope of CA

Here, it is given  $A(6, -1)$ ,  $B(5, 0)$  and  $C(2, 3)$

Therefore, slope =  $\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$

$$\text{Slope of AB} = \left(\frac{0 - (-1)}{5 - 6}\right) = \frac{1}{-1} = -1$$

$$\text{And the slope of BC} = \left(\frac{3 - 0}{2 - 5}\right) = \frac{3}{-3} = -1$$

$$\text{Slope of CA} = \left(\frac{3 - (-1)}{2 - 6}\right) = \frac{4}{-4} = -1$$

Thus slopes of AB, BC and CA are equal, thus Points A, B, C are collinear.

### Section C

26. Here it is given that,  $f(x) = 1 - x$ ,  $x < 0$ , this gives

$$f(-4) = 1 - (-4) = 5;$$

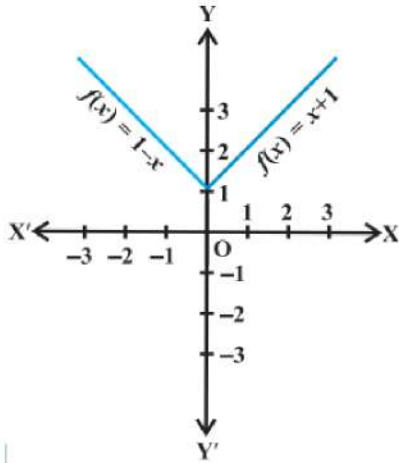
$$f(-3) = 1 - (-3) = 4,$$

$$f(-2) = 1 - (-2) = 3$$

$$f(-1) = 1 - (-1) = 2; \text{ etc, and } f(1) = 2, f(2) = 3, f(3) = 4$$

$$f(4) = 5 \text{ and so on for } f(x) = x + 1, x > 0.$$

Thus, the graph of the given function  $f(x)$  is as shown in figure given below.



27. Let  $A(0, 7, 10)$ ,  $B(-1, 6, 6)$  and  $C(-4, 9, 6)$  be the vertices of a triangle. Then

$$\text{Side } AB = \sqrt{(0+1)^2 + (7-6)^2 + (10-6)^2} \quad [\because \text{distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}]$$

$$\Rightarrow AB = \sqrt{1^2 + 1^2 + 4^2}$$

$$= \sqrt{1 + 1 + 16} = \sqrt{18} = 3\sqrt{2}$$

$$\text{Side } BC = \sqrt{(-1+4)^2 + (6-9)^2 + (6-6)^2}$$

$$= \sqrt{3^2 + 3^2 + 0}$$

$$\Rightarrow BC = \sqrt{9 + 9 + 0} = \sqrt{18} = 3\sqrt{2}$$

$$\text{and Side } CA = \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2}$$

$$= \sqrt{4^2 + 2^2 + 4^2}$$

$$\Rightarrow CA = \sqrt{16 + 4 + 16} = \sqrt{36} = 6$$

$$\text{Now, } AB^2 + BC^2 = (3\sqrt{2})^2 + (3\sqrt{2})^2$$

Hence,  $\triangle ABC$  is the right-angled triangle at B.

Yes, this question gives us a message to protect our environment and help us to follow these signboards in our daily lives to make ourselves healthy.

OR

Given equation of line is:

$$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda \text{ (say)}$$

$$\Rightarrow \frac{x+2}{3} = \lambda, \frac{y+1}{2} = \lambda, \frac{z-3}{2} = \lambda$$

$$\Rightarrow x = 3\lambda - 2, y = 2\lambda - 1, z = 2\lambda + 3$$

So, we have a point on the line is

$$Q(3\lambda - 2, 2\lambda - 1, 2\lambda + 3) \dots (i)$$

Now, given that distance between two points

$P(1, 3, 3)$  and  $Q(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$  is

5 Units, i.e.  $PQ=5$

$$\Rightarrow \sqrt{[(3\lambda - 2 - 1)^2 + (2\lambda - 1 - 3)^2 + (2\lambda + 3 - 3)^2]} = 5$$

$$\left[ \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \right]$$

$$\Rightarrow \sqrt{(3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2} = 5$$

Therefore, on squaring both sides, we get,

$$(3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2 = 25$$

$$\Rightarrow 9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 16 - 16\lambda + 4\lambda^2 = 25$$

$$\Rightarrow 17\lambda^2 - 34\lambda = 0 \Rightarrow 17\lambda(\lambda - 2) = 0$$

$$\Rightarrow \text{Either } 17\lambda = 0 \text{ or } \lambda - 2 = 0$$

$$\therefore \lambda = 0 \text{ or } 2$$

On putting  $\lambda = 0$  and  $\lambda = 2$  in Eq. (i). we get the required points as  $(-2, -1, 3)$  or  $(4, 3, 7)$ .

$$28. (x+1)^6 + (x-1)^6 = [{}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6]$$

$$+ [{}^6C_0x^6 + {}^6C_1x^5(-1) + {}^6C_2x^4(-1)^2 + {}^6C_3x^3(-1)^3 + {}^6C_4x^2(-1)^4 + {}^6C_5x(-1)^5 + {}^6C_6(-1)^6]$$

$$= [x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1] + [x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1]$$

$$= 2x^6 + 30x^4 + 30x^2 + 2$$

$$= 2(x^6 + 15x^4 + 15x^2 + 1)$$

Putting  $x = \sqrt{2}$

$$(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6 = 2[(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1]$$

$$= 2[8 + 15 \times 4 + 15 \times 2 + 1]$$

$$= 2[8 + 60 + 30 + 1]$$

$$= 2 \times 99 = 198$$

OR

We hand to find value of  $(\sqrt{x} + \sqrt{y})^8$

Formula used:  ${}^n C_r = \frac{n!}{(n-r)!(r)!}$

$$(a + b)^n = {}^n C_0 a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_{n-1} a b^{n-1} + {}^n C_n b^n$$

We have,  $(\sqrt{x} + \sqrt{y})^8$

We can write  $\sqrt{x}$  as  $x^{\frac{1}{2}}$  and  $\sqrt{y}$  as  $y^{\frac{1}{2}}$

Now, we have to solve for  $(x^{\frac{1}{2}} + y^{\frac{1}{2}})^8$

$$= \left[ {}^8 C_0 \left(x^{\frac{1}{2}}\right)^{8-0} \right] + \left[ {}^8 C_1 \left(x^{\frac{1}{2}}\right)^{8-1} \left(y^{\frac{1}{2}}\right)^1 \right] + \left[ {}^8 C_2 \left(x^{\frac{1}{2}}\right)^{8-2} \left(y^{\frac{1}{2}}\right)^2 \right] + \left[ {}^8 C_3 \left(x^{\frac{1}{2}}\right)^{8-3} \left(y^{\frac{1}{2}}\right)^3 \right]$$

$$+ \left[ {}^8 C_4 \left(x^{\frac{1}{2}}\right)^{8-4} \left(y^{\frac{1}{2}}\right)^4 \right] + \left[ {}^8 C_5 \left(x^{\frac{1}{2}}\right)^{8-5} \left(y^{\frac{1}{2}}\right)^5 \right] + \left[ {}^8 C_6 \left(x^{\frac{1}{2}}\right)^{8-6} \left(y^{\frac{1}{2}}\right)^6 \right]$$

$$+ \left[ {}^8 C_7 \left(x^{\frac{1}{2}}\right)^{8-7} \left(y^{\frac{1}{2}}\right)^7 \right] + \left[ {}^8 C_8 \left(y^{\frac{1}{2}}\right)^8 \right]$$

$$= \left[ \frac{8!}{0!(8-0)!} \left(x^{\frac{5}{2}}\right) \right] + \left[ \frac{8!}{1!(8-1)!} \left(x^{\frac{3}{2}}\right) \left(y^{\frac{1}{2}}\right) \right] + \left[ \frac{8!}{2!(8-2)!} \left(x^{\frac{6}{2}}\right) \left(y^{\frac{2}{2}}\right) \right]$$

$$+ \left[ \frac{8!}{3!(8-3)!} \left(x^{\frac{5}{2}}\right) \left(y^{\frac{3}{2}}\right) \right] + \left[ \frac{8!}{4!(8-4)!} \left(x^{\frac{4}{2}}\right) \left(y^{\frac{4}{2}}\right) \right]$$

$$+ \left[ \frac{8!}{5!(8-5)!} \left(x^{\frac{3}{2}}\right) \left(y^{\frac{5}{2}}\right) \right] + \left[ \frac{8!}{6!(8-6)!} \left(x^{\frac{2}{2}}\right) \left(y^{\frac{6}{2}}\right) \right] + \left[ \frac{8!}{7!(8-7)!} \left(x^{\frac{1}{2}}\right) \left(y^{\frac{7}{2}}\right) \right] + \left[ \frac{8!}{8!(8-8)!} \left(y^{\frac{8}{2}}\right) \right]$$

$$= [1(x^4)] + [8(x^{\frac{7}{2}})(y^{\frac{1}{2}})] + [28(x^3)(y)] + [56(x^{\frac{5}{2}})(\frac{2}{y^2})]$$

$$+ [70(x^2)(y^2)] + [56(x^{\frac{3}{2}})(y^{\frac{5}{2}})] + [28(x^2)(y^3)] + [8(x^{\frac{1}{2}})(y^{\frac{7}{2}})] + [1(y^4)]$$

29. M.I. of  $= \sqrt{5} + 3i$

$$= \frac{1}{\sqrt{5} + 3i} = \frac{1}{\sqrt{5} + 3i} \times \frac{\sqrt{5} - 3i}{\sqrt{5} - 3i}$$

$$= \frac{\sqrt{5} - 3i}{(\sqrt{5})^2 - (3i)^2}$$

$$= \frac{\sqrt{5} - 3i}{5 - 9i^2} = \frac{\sqrt{5} - 3i}{5 + 9} = \frac{1}{14}(\sqrt{5} - 3i)$$

OR

Let  $x + yi = \sqrt{-i}$

Squaring both sides, we get

$$(x + yi)^2 = -i$$

$$x^2 - y^2 + 2xyi = -i$$

Equating the real and imaginary parts

$$x^2 - y^2 = 0 \dots (i)$$

$$\text{and } 2xy = -1$$

$$\therefore xy = -\frac{1}{2}$$

Now using the identity

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$$

$$= (0)^2 + 4\left(-\frac{1}{2}\right)^2$$

$$= 1$$

$$\therefore x^2 + y^2 = 1 \dots (ii) \text{ [Neglecting (-) sign as } x^2 + y^2 > 0]$$



Solving (i) and (ii), we get

$$x^2 = \frac{1}{2} \text{ and } y^2 = \frac{1}{2}$$

$$\therefore x = \pm \frac{1}{\sqrt{2}} \text{ and } y = \pm \frac{1}{\sqrt{2}}$$

Since the sign of  $xy$  is  $(-)$  then if

$$x = \frac{1}{\sqrt{2}}, y = -\frac{1}{\sqrt{2}}$$

$$\text{If } x = -\frac{1}{\sqrt{2}}, y = \frac{1}{\sqrt{2}}$$

$$\therefore \sqrt{-i} = \pm \left( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right)$$

30. Let the length of the shortest board be  $x$  cm

Then length of the second board =  $(x + 3)$  cm

length of the third board =  $2x$  cm

$$\text{Now } x + (x + 3) + 2x \leq 91 \text{ and } 2x \geq (x + 3) + 5$$

$$\Rightarrow 4x + 3 \leq 91 \text{ and } 2x - (x + 3) \geq 5$$

$$\Rightarrow 4x \leq 91 - 3 \text{ and } 2x - x - 3 \geq 5$$

$$\Rightarrow 4x \leq 88 \text{ and } x \geq 5 + 3$$

$$\Rightarrow x \leq 22 \text{ and } x \geq 8$$

Thus minimum length of shortest board is 8 cm and maximum length is 22 cm.

31. We have,  $5 \cdot {}^4P_r = 6 \cdot ({}^5P_{r-1})$

$$\Rightarrow 5 \cdot \frac{4!}{(4-r)!} = 6 \times \frac{5!}{[5-(r-1)]!}$$

$$\Rightarrow \frac{5 \cdot 4!}{(4-r)!} = \frac{6 \times 5 \times 4!}{(6-r)!}$$

$$\Rightarrow \frac{1}{(4-r)!} = \frac{6}{(6-r)(5-r)(4-r)!}$$

$$\Rightarrow (6-r)(5-r) = 6$$

$$\Rightarrow 30 - 11r + r^2 = 6$$

$$\Rightarrow r^2 - 11r + 24 = 0$$

$$\Rightarrow (r-3)(r-8) = 0$$

$$\Rightarrow r = 3, 8$$

But  $r \neq 8$ , because in  ${}^4P_r$ ,  $r$  cannot be greater than 4.

Hence,  $r = 3$

#### Section D

32. Given that one urn contains two black balls and one white ball and second urn contains one black ball and two white balls as expressed in the figure below



It is also given that one of the two urns is chosen, then a ball is randomly chosen from the urn, then second ball is chosen at random from the same urn without replacing the first ball and this condition can also be treated as taking out two balls at a time from one of the two urns. So,

i. Sample Space  $S = \{B_1B_2, B_1W, B_2W, B_2B_1, WB_1, WB_2, W_1W_2, W_1B, W_2B, W_2W_1, BW_1, BW_2\}$

Total number of sample space = 12

ii. If two black balls are chosen

Total outcomes = 12

Favourable outcomes are  $B_1B_2, B_2B_1$

$\therefore$  Total favourable outcomes = 2

We know that,

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$\therefore \text{Required probability} = \frac{2}{12} = \frac{1}{6}$$

iii. If two balls of opposite colours are chosen i.e. one black and one white

Favourable outcomes are  $B_1W, B_2W, WB_1, WB_2, W_1B, W_2B, BW_1, BW_2$

$\therefore$  Total favourable outcomes = 8 and Total outcomes = 12

We know that,

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$\therefore \text{Required probability} = \frac{8}{12} = \frac{2}{3}$$

33. Dividing  $x^4 - 3x^3 + 2$  by  $x^3 - 5x^2 + 3x + 1$

$$\begin{array}{r} x^3 - 5x^2 + 3x + 1 \overline{) x^4 - 3x^3 + 2} \\ \underline{\pm x^4 \mp 5x^3 \pm 3x^2 \pm x} \\ 2x^3 - 3x^2 - x + 2 \\ \underline{\pm 2x^3 \mp 10x^2 \pm 6x \pm 2} \\ 7x^2 - 7x \end{array}$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 1} \frac{x^4 - 3x^3 + 2}{x^3 - 5x^2 + 3x + 1} &= \lim_{x \rightarrow 1} (x + 2) + \lim_{x \rightarrow 1} \frac{7x^2 - 7x}{x^3 - 5x^2 + 3x + 1} \\ &= \lim_{x \rightarrow 1} x + 2 + \lim_{x \rightarrow 1} \frac{7x(x-1)}{x^3 - 5x^2 + 3x + 1} \\ &= \lim_{x \rightarrow 1} x + 2 + \lim_{x \rightarrow 1} \frac{7x(x-1)}{(x-1)(x^2 - 4x - 1)} \\ &= \lim_{x \rightarrow 1} x + 2 + \lim_{x \rightarrow 1} \frac{7x}{(x^2 - 4x - 1)} \\ &= 1 + 2 + \frac{7}{(1 - 4 - 1)} \\ &= 3 - \frac{7}{4} \\ &= \frac{12 - 7}{4} \\ &= \frac{5}{4} \end{aligned}$$

OR

We have,  $f(x) = x \sin x$

By using first principle of derivative,

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h) \sin(x+h) - x \sin x}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)[\sin x \cos h + \cos x \sin h] - x \sin x}{h} \quad [\because \sin(x+y) = \sin x \cos y + \cos x \sin y] \\ &= \lim_{h \rightarrow 0} \frac{x \sin x \cos h + x \cos x \sin h + h \sin x \cos h + h \cos x \sin h - x \sin x}{h} \\ &= \lim_{h \rightarrow 0} \frac{x \sin x (\cos h - 1) + x \cos x \sin h + h(\sin x \cos h + \cos x \sin h)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x \sin x (\cos h - 1)}{h} + \lim_{h \rightarrow 0} x \cos x \cdot \frac{\sin h}{h} + \lim_{h \rightarrow 0} \frac{h(\sin x \cos h + \cos x \sin h)}{h} \\ &= x \sin x \lim_{h \rightarrow 0} \left[ \frac{-(1 - \cos h)}{h} \right] + x \cos x + \sin x \\ &= -2x \sin x \cdot \lim_{\frac{h}{2} \rightarrow 0} \frac{\sin^2 \frac{h}{2}}{h \times \frac{h}{4}} \times \frac{h}{4} + x \cos x + \sin x \\ &= -x \cdot \sin x \cdot \frac{2}{4} \lim_{\frac{h}{2} \rightarrow 0} \left( \frac{\sin \frac{h}{2}}{\frac{h}{2}} \right)^2 \times h + x \cos x + \sin x \\ &= -x \sin x \cdot \frac{1}{2} (1) \times 0 + x \cos x + \sin x \quad [\because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1] \\ &= x \cos x + \sin x \end{aligned}$$

34. Given,

$$\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha) = \frac{-3}{2}$$

$$\Rightarrow 2[\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha)] = -3$$

$$\Rightarrow 2[\cos \alpha \cos \beta + \sin \alpha \sin \beta + \cos \beta \cos \gamma + \sin \beta \sin \gamma + \cos \gamma \cos \alpha + \sin \gamma \sin \alpha] = -3 \quad [\because$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B]$$

$$\Rightarrow [2 \cos \alpha \cos \beta + 2 \cos \beta \cos \gamma + 2 \cos \gamma \cos \alpha] + [2 \sin \alpha \sin \beta + 2 \sin \beta \sin \gamma + 2 \sin \gamma \sin \alpha] + 3 = 0$$

$$\Rightarrow [2 \cos \alpha \cos \beta + 2 \cos \beta \cos \gamma + 2 \cos \gamma \cos \alpha] + [2 \sin \alpha \sin \beta + 2 \sin \beta \sin \gamma + 2 \sin \gamma \sin \alpha] + (\cos^2 \alpha + \sin^2 \alpha)$$

$$+ (\cos^2 \beta + \sin^2 \beta) + (\cos^2 \gamma + \sin^2 \gamma) = 0$$

$$[\because \cos^2 x + \sin^2 x = 1]$$

$$\Rightarrow [\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + 2 \cos \alpha \cos \beta + 2 \cos \beta \cos \gamma + 2 \cos \gamma \cos \alpha] + [\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma + 2 \sin \alpha \sin \beta + 2 \sin \beta \sin \gamma + 2 \sin \gamma \sin \alpha] = 0$$

$$\Rightarrow (\cos \alpha + \cos \beta + \cos \gamma)^2 + (\sin \alpha + \sin \beta + \sin \gamma)^2 = 0$$

We know that, sum of two positive terms will be zero, if both are equal to zero.

∴  $\cos\alpha + \cos\beta + \cos\gamma = 0$   
 and  $\sin\alpha + \sin\beta + \sin\gamma = 0$   
 Hence proved.

OR

We have,

$$\begin{aligned} \text{LHS} &= \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ \\ \Rightarrow \text{LHS} &= \cos 60^\circ (\cos 20^\circ \cos 40^\circ) \cos 80^\circ \\ \Rightarrow \text{LHS} &= \frac{1}{2} \times \frac{1}{2} (2 \cos 20^\circ \cos 40^\circ) \cos 80^\circ \left[ \because \cos \frac{\pi}{3} = \frac{1}{2} \right] \\ \Rightarrow \text{LHS} &= \frac{1}{4} \{ [\cos (40^\circ + 20^\circ) + \cos (40^\circ - 20^\circ)] \cos 80^\circ \} \left[ \because 2 \cos A \cos B = \cos (A + B) + \cos (A - B) \right] \\ \Rightarrow \text{LHS} &= \frac{1}{4} \{ (\cos 60^\circ + \cos 20^\circ) \cos 80^\circ \} \\ \Rightarrow \text{LHS} &= \frac{1}{4} \left\{ \left( \frac{1}{2} + \cos 20^\circ \right) \cos 80^\circ \right\} \\ \Rightarrow \text{LHS} &= \frac{1}{4} \left\{ \frac{1}{2} \cos 80^\circ + \cos 80^\circ \cos 20^\circ \right\} \\ \Rightarrow \text{LHS} &= \frac{1}{8} \{ \cos 80^\circ + 2 \cos 80^\circ \cos 20^\circ \} \\ \Rightarrow \text{LHS} &= \frac{1}{8} [\cos 80^\circ + \{ \cos (80^\circ + 20^\circ) + \cos (80^\circ - 20^\circ) \}] \\ \Rightarrow \text{LHS} &= \frac{1}{8} \{ \cos 80^\circ + \cos 100^\circ + \cos 60^\circ \} \\ \Rightarrow \text{LHS} &= \frac{1}{8} \{ \cos 80^\circ + \cos (180^\circ - 80^\circ) + \cos 60^\circ \} \\ \Rightarrow \text{LHS} &= \frac{1}{8} \{ \cos 80^\circ - \cos 80^\circ + \cos 60^\circ \} \left[ \because \cos (180^\circ - x) = -\cos x \right] \\ \Rightarrow \text{LHS} &= \frac{1}{8} \times \frac{1}{2} = \frac{1}{16} = \text{RHS} \end{aligned}$$

35. Let the other two observations be  $x$  and  $y$

Therefore, our observations are 5, 7, 9,  $x$  and  $y$

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Total number of observations}}$$

$$6 = \frac{5+7+9+x+y}{5} \Rightarrow 6 \times 5 = 21 + x + y$$

$$\Rightarrow 30 - 21 = x + y \text{ or } x + y = 9 \dots\dots\dots(i)$$

Now prepare the following table we have,

$x_i$	$x_i - \bar{x} = x_i - 6$	$(x_i - \bar{x})^2$
5	5 - 6 = -1	(-1) <sup>2</sup> = 1
7	7 - 6 = 1	(1) <sup>2</sup> = 1
9	9 - 6 = 3	(3) <sup>2</sup> =
$x$	$x - 6$	$(x - 6)^2$
$y$	$y - 6$	$(y - 6)^2$
		$\sum (x_i - \bar{x})^2 = 11 + (x - 6)^2 + (y - 6)^2$

$$\text{So, Variance, } \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$4 = \frac{11 + (x - 6)^2 + (y - 6)^2}{5}$$

$$\Rightarrow 20 = 11 + (x^2 + 36 - 12x) + (y^2 + 36 - 12y)$$

$$\Rightarrow 20 - 11 = x^2 + 36 - 12x + y^2 + 36 - 12y$$

$$\Rightarrow x^2 + y^2 + 72 - 12(9) - 9 = 0 \text{ from (i)}$$

$$\Rightarrow x^2 + y^2 + 63 - 108 = 0$$

$$\Rightarrow x^2 + y^2 = 45 \dots\dots\dots(ii)$$

From eq.(i)

Now,  $x + y = 9$

$$(x + y)^2 = (9)^2$$

$$\Rightarrow x^2 + y^2 + 2xy = 81$$

$$\Rightarrow 45 + 2xy = 81 \text{ [from (ii)]}$$

$$\Rightarrow 2xy = 81 - 45 \Rightarrow 2xy = 36$$

$$\Rightarrow xy = 18 \Rightarrow x = \frac{18}{y} \dots\dots\dots(iii)$$



After Putting the value of x in eq. (i) we get

$$\Rightarrow \frac{18}{y} + y = 9 \Rightarrow \frac{18+y^2}{y} = 9 \Rightarrow y^2 + 18 = 9y$$

$$\Rightarrow y^2 - 9y + 18 = 0 \Rightarrow y^2 - 6y - 3y + 18 = 0$$

$$\Rightarrow y(y - 6) - 3(y - 6) = 0 \Rightarrow (y - 3)(y - 6) = 0$$

$$\Rightarrow y - 3 = 0 \text{ and } y - 6 = 0 \Rightarrow y = 3 \text{ and } y = 6$$

For  $y = 3$

$$x = \frac{18}{y} = \frac{18}{3} = 6$$

Hence,  $x = 6, y = 3$  are the remaining two observation

For  $y = 6$

$$x = \frac{18}{y} = \frac{18}{6} = 3$$

Hence,  $x = 3, y = 6$  are the remaining two observation

Therefore, remaining two observations are 3 and 6

### Section E

#### 36. Read the text carefully and answer the questions:

Arun is running in a racecourse note that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m.



- (i) An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.  
Hence path traced by Arun is ellipse.

Sum of the distances of the point moving point to the foci is equal to length of major axis = 10m

- (ii) Given  $2a = 10$  &  $2c = 8$

$$\Rightarrow a = 5 \text{ & } c = 4$$

$$c^2 = a^2 + b^2$$

$$\Rightarrow 16 = 25 + b^2$$

$$\Rightarrow b^2 = 25 - 16 = 9$$

$$\text{Equation of ellipse } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{Required equation is } \frac{x^2}{25} + \frac{y^2}{9} = 1$$

- (iii) equation is of given curve is  $\frac{x^2}{25} + \frac{y^2}{9} = 1$

$$a = 5, b = 3 \text{ and given } 2c = 8 \text{ hence } c = 4$$

$$\text{Eccentricity} = \frac{c}{a} = \frac{4}{5}$$

OR

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$\text{Hence } a = 5 \text{ and } b = 3$$

$$\text{Length of latus rectum of ellipse is given by } \frac{2b^2}{a} = \frac{2 \times 9}{5} = \frac{18}{5}$$

#### 37. Read the text carefully and answer the questions:

On the roof of Monesh's house, a water tank of capacity 9000 litres is installed. A water pump fills the tank, the pump uses water from the municipality water supply, In the beginning, the water flow of the pump remains 100 litres/hour for the first hour.

The water flow from the pump is 1.25th after each 1 hour.

Once Monesh's mother was not at home and told him to switch off the pump when the tank is almost full.



He calculated that after how many hours should he stop the pump so water does not get overflow in the next one hour.



- (i) (c) 14 hours

**Explanation:** 14 hours

- (ii) (a) 3325.29 Liters

**Explanation:** 3325.29 Liters

- (iii) (d) 381.47 Liters

**Explanation:** 381.47 Liters

OR

- (d) 244.14 Liters/hr

**Explanation:** 244.14 Liters/hr

**38. Read the text carefully and answer the questions:**

In a company, 100 employees offered to do a work. In out of them, 10 employees offered ground floor only, 15 employees offered first floor only, 10 employees offered second floor only, 30 employees offered second floor and ground floor to work, 25 employees offered first and second floor, 15 employees offered ground and first floor, 60 employees offered second floor.



- (i) Let 'x' the number of employees who offered all three floors.

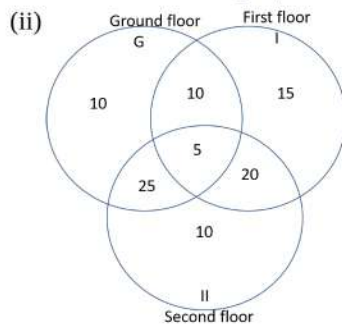
$$\text{only } n(\text{II}) = n(\text{II}) - n(\text{II} \cap \text{G}) - n(\text{II} \cap \text{I}) + n(\text{G} \cap \text{I} \cap \text{II})$$

$$10 = 60 - 30 - 25 + x$$

$$\Rightarrow x = 10 - 5 = 5$$

$$\Rightarrow n(\text{I} \cap \text{II} \cap \text{G}) = 5$$

The number of employees who offered all three floors = 5



$$\text{only } n(\text{G} \cap \text{I}) = n(\text{G} \cap \text{I}) - n(\text{G} \cap \text{I} \cap \text{II})$$

$$\Rightarrow n(\text{G} \cap \text{I}) = 15 - 5 = 10$$

The number of employees who offered ground and first floor but not second floor = 10