7 Magnetism

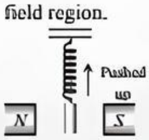
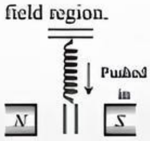
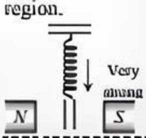
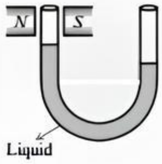
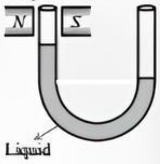
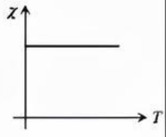
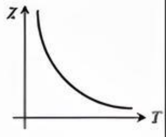
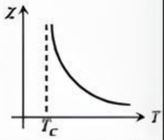
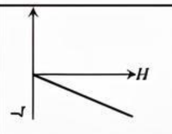
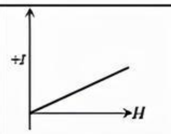
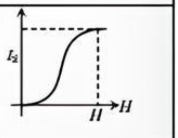
NITTY-GRITTY

1. Magnetism

Every molecule of a substance is a complete magnet in itself. However, in a magnetic substance the molecular magnets are randomly oriented to give net zero magnetic moment. On the basis of mutual interactions or behaviour of various materials in an external magnetic field, the materials are divided in three main categories.

A small number of crystalline substances exhibit strong magnetic effects called ferromagnetism. Some examples of ferromagnetic substances are iron, cobalt, nickel, gadolinium, and dysprosium. These substances contain permanent atomic magnetic moments that tend to align parallel to each other even in a weak external magnetic field. Paramagnetic substances have a small but positive magnetism resulting from the presence of atoms or ions that have permanent magnetic moments. These moments interact only weakly with one another and are randomly oriented in the absence of an external magnetic field. When an external magnetic field is applied to a diamagnetic substance, a weak magnetic moment is induced in the direction opposite the applied field, causing diamagnetic substances to be weakly repelled by a magnet.

Table 7.1: Comparative Study of Magnetic Materials

Property	Diamagnetic substances	Paramagnetic substances	Ferromagnetic substances
Cause of magnetism	Orbital motion of electrons	Spin motion of electrons	Formation of domains
Behaviour In a non-uniform magnetic field	These are repelled in an external magnetic field *i.e.*, have a tendency to move from high to low field region. 	These are feebly attracted in an external magnetic field *i.e.*, have a tendency to move from low to high field region. 	These are strongly attracted in an external magnetic field *i.e.*, they easily move from low to high field region. 
When the material in the form of liquid is filled in the U-tube and placed between pole pieces.	Liquid level in that limb gets depressed 	Liquid level in that limb rises up 	Liquid level in that limb rises up very much 
The gaseous material between pole pieces	Expands at right angles to the magnetic field.	Expands in the direction of magnetic field.	Rapidly expands in the direction of magnetic field.
Magnetic susceptibility χ and dependence on temperature	Low and negative $\lvert\chi\rvert \approx 1$ Does not depend on temperature (except *Bi* at low temperature) 	Low but positive $\chi \approx 1$ On cooling, these get converted to ferromagnetic materials at Curie temperature 	Positive and high $\chi \approx 10^2$ These get converted into paramagnetic materials at Curie temperature 
Relative permeability (μ_r)	$\mu_r < 1$	$\mu_r > 1$	$\mu_r >> 1$; $\mu_r = 10^2$
Intensity of magnetisation (I)	very low	low	very high.
I-H curves			
Magnetic moment(M)	Very low (≈ 0)	Very low	Very high
Examples	Cu, Ag, Au, Zn, Bi, Sb, NaCl, H_2O air and diamond etc.	Al, Mn, Pt, Na, $CuCl_2$, O_2 and crown glass	Fe, Co, Ni, Cd, Fe_3O_4 etc.

Earth's Magnetic Field

As per the most established theory, it is due to the rotation of the earth where by the various charged ions present in the molten state in the core of the earth rotate and constitute a current.

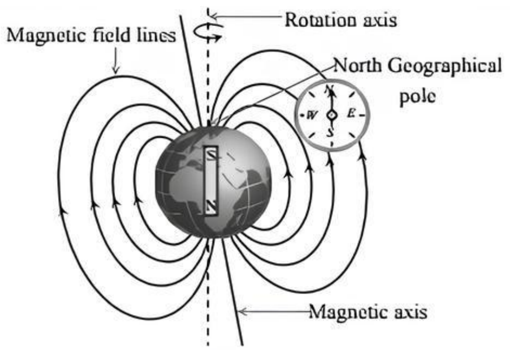


Figure: 7.1

At the poles and equator of earth the values of total intensity are 0.66 and 0.33 Oersted respectively. Magnetic axis and Geographical axis don't coincide but they make an angle of 17.5° with each other. The direction of earth's Horizontal magnetic field is from south to North. At poles Horizontal component H(B_H) = 0, while at equator vertical component V (B_V) = 0.

Magnetic Declination (*θ*): It is the angle between geographic and the magnetic meridian planes. Declination at a place is expressed at $\theta°E$ or $\theta°W$ depending upon whether the north pole of the compass needle lies to the east or to the west of the geographical axis.

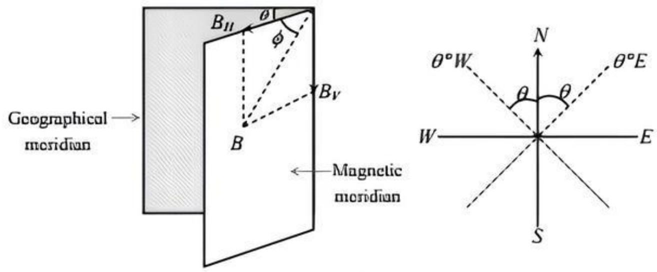


Figure: 7.2

Angle of inclination or Dip (*ϕ*): It is the angle between the direction of intensity of total magnetic field of earth and a horizontal line in the magnetic meridian.

Horizontal Component of Earth's Magnetic Field (B_H):
Earth's magnetic field is horizontal only at the magnetic equator. At any other place, the total intensity can be resolved into horizontal component (B_H) and vertical component (B_V).
Also $B_H = B\cos\phi$ and $B_V = B\sin\phi$

Therefore Earth's magnetic field is $B = \sqrt{B_{H^2} + B_{V^2}}$ and

$$\tan\phi = \frac{B_V}{B_H}$$

Isolated magnetic poles do not exist.

Magnetic dipole moment is a vector quantity; its direction is from south to north along the axis. Repulsion is the sure test to distinguish between a magnet and a piece of iron.

- Magnetic moment of bar-magnet $M = m.2l\ amp - m^2$

  where m = pole strength in amp-m, $2l$ = separation between poles.

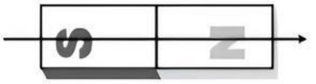


Figure: 7.3

- If a rectangular bar magnet is cut in n equal parts then time period of each part will be $\frac{1}{\sqrt{n}}$ times that of complete magnet (i.e. $T' = \frac{T}{\sqrt{n}}$) while for short magnet $T' = \frac{T}{n}$. If nothing is said then bar magnet is treated as short magnet.


Figure: 7.4

- Magnetic moment of current loop is a vector quantity. Its direction is perpendicular to the plane of the loop. Magnetic moment of a current loop, $M = NIA\ amp\text{-}m^2$. A dipole in a uniform magnetic field; Net force on dipole = 0
- Torque on dipole $\tau = MB\sin\theta$ $\tau = \vec{M} \times \vec{B}$ (vector form)
- Potential energy of dipole $U = MB\cos\theta = -\vec{M}.\vec{B}$ (vector form)
- Work done in rotating the dipole form equilibrium position ($\theta = 0°$) through an angle $\theta. W = MB(1-\cos\theta)$

Magnetic Field: Magnetic field produced by a short magnetic dipole at axial position $B = \frac{\mu_0}{4\pi}.\frac{2M}{r^3}$ (axial position)

At equatorial position, $B = \frac{\mu_0}{4\pi}.\frac{M}{r^3}$

At any general point (r, θ) relative to centre of dipole $B = \frac{\mu_0}{4\pi}.\frac{M}{r^3}\sqrt{1+3\cos\theta}$

Force between to short magnetic dipoles (magnets) at separation r (magnetic moments M_1 and M_2)

When they are co-axial, $B=\frac{\mu_0}{4\pi}.\frac{6M_1M_2}{r^4}$

When they are broadside on position, $B=\frac{\mu_0}{4\pi}.\frac{3M_1M_2}{r^4}$

- Intensity of magnetization $I=\frac{M}{V}$ amp/meter; where V = volume.
- Magnetic susceptibility $\chi_m=\frac{1}{H}$; where H = magnetizing field in A/m^2
- Absolute permeability, $\mu=\frac{B}{H}$ Weber/Amp-meter
- Relative permeability, $\mu_r=\frac{\mu}{\mu_0}=1+\chi_m$
- Curie law of paramagnetic substances, $\chi_m\propto\frac{1}{T}$
- Deflection magnetometer Tan A position (arms along $E-W$ and magnet parallel to arms) $\frac{\mu_0}{4\pi}.\frac{3Md}{(d^2-l^2)^2}=H\tan\theta$

  Tab B position (arms long $N-S$ and magnet perpendicular to arms) $\frac{\mu_0}{4\pi}.\frac{M}{(d^2+l^2)^{3/2}}=H\tan\theta$

Vibration Magnetometer: If a small magnet is placed in magnetic meridian and vibrates in horizontal plane, the time period is $T=2\pi\sqrt{\frac{I}{MH}}$ Where I = moment of inertia of magnet about axis of rotation $I=\frac{M_0(l^2+b^2)}{12}$ (Where M_0 = mass of magnet) If breadth of magnet is negligible $I=\frac{M_0l^2}{12}$

- If a magnet is placed parallel to magnetic meridian and oscillates in vertical plane $T=2\pi\sqrt{\frac{I}{MB_2}}$
- If a magnet is placed perpendicular to magnetic meridian and oscillates in a vertical plane $T=2\pi\sqrt{\frac{I}{MV}}$
- Comparison of magnetic moments; Sum and difference method $\frac{M_1}{M_2}=\frac{T_1^2+T_2^2}{T_1^2-T_2^2}$

  Magnetic moment of a current loop = NiA $amp\times m^2$ where A = area of loop, N = number of loops
- Torque on a current loop in a magnetic filed $\tau=MB\sin\theta$

Where θ= angle between $\vec{M}$ and $\vec{B}$; In vector from $\tau=\vec{M}\times\vec{B}$. In moving coil galvanometer, the pole pieces of a magnet are strong and cylindrical to make the field radial (sin θ = 1).

- Deflection of moving coil galvanometer is $\theta=\frac{NBA}{C}=i$

$\Rightarrow\theta\propto i$

Where C = torsional rigidity of suspension wire.

- Sensitivity of galvanometer: $\frac{\theta}{i}=\frac{NBA}{C}$

Galvanometer: A normal galvanometer measures current. But a B.G measures charge due to impulse in the coil (sudden flow of charges for a short interval of time. A ballistic galvanometer measures the charge and its deflection is proportional to charge i.e. $\theta\propto q$.

Tangent galvanometer

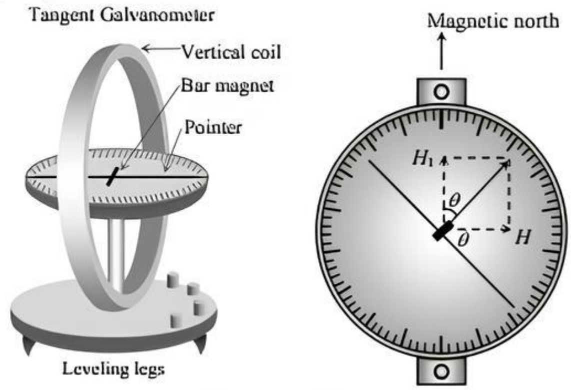


Figure: 7.5

When the plane of vertical circular coil is in magnetic meridian, then $i=K\tan\theta$

Where $K=\frac{2rH}{\mu_0N}$ = reduction factor

r = radius of coil,

H = horizontal component of earth's magnetic field.

A tangent galvanometer is most accurate when its deflection is 45°.

Conversion of Galvanometer: With increase of range of ammeter, its resistance decreases. With the increases of range of voltmeter, its resistance increases. Out of voltmeter, ammeter and galvanometer, the resistance of voltmeter is largest and that of ammeter is smallest.

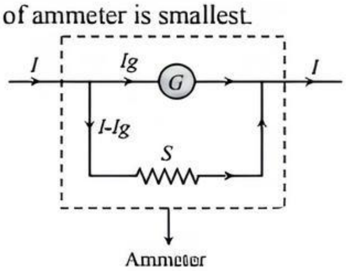


Figure: 7.6

- Working equation of conversion of galvanometer into ammeter. $i_g = \frac{S}{S+G} i$
- Shunt resistance $S = \frac{i_g}{i - i_g} . G$
- The resistance of ammeter so formed.

$$R_A = \frac{SG}{S+G} \Rightarrow R_A < G$$

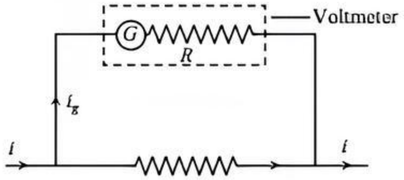


Figure: 7.7

- Working equation of conversion of galvanometer into voltmeter. $i_g = \frac{V}{R+G} i$
- Series resistance $R = \frac{V}{i_g} - G$
- Resistance of voltmeter so formed is $R_V = R + G$.

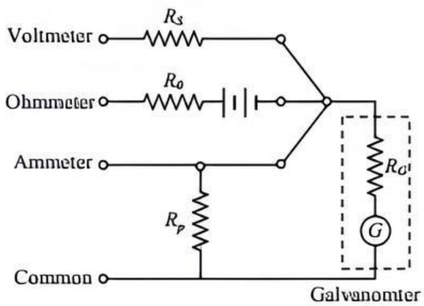


Figure: 7.8

- The voltmeter is a high resistance device so that it does not draw appreciable current from the circuit. A series resistor limits the current.
- The ohmmeter has a voltage source to drive a small current through the external resistance to be measured. It contains a calibration resistor.
- The ammeter has a parallel resistor of very small value to shunt most of the current away from the sensitive current measuring element. It must carry the total current of the circuit to be measured without appreciable voltage drop

Hysteresis Curve: The complete cycle of magnetisation and demagnetisation is represented by $BCDEFGB$. This curve is known as hysteresis curve. Hysteresis energy loss = Area bound by the hysteresis loop = $VAnt$ *Joule*; Where, V = Volume of ferromagnetic sample, A= Area of B–H loop P, n= Frequency of alternating magnetic field and t = Time

- **Retentivity:** When H is reduced, I reduces but is not zero when $H = 0$. The remainder value OC of magnetisation when $H = 0$ is called the residual magnetism or retentivity.
- **Coercivity or coercive force:** When magnetic field H is reversed, the magnetisation decreases and for a particular value of H, denoted by H_c, it becomes zero *i.e.*, $H_c = OD$ when $I = 0$. This value of H is called the corecivity.

  Magnetic hard substance (steel) → High coercvity

  Magnetic soft substance (soft iron) → Low coercivity

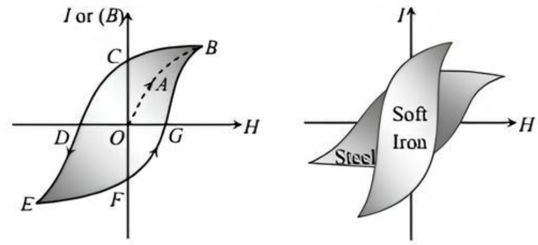


Figure: 7.9

2. Magnetic Effect of Current

A magnetic field is the magnetic effect of electric currents and magnetic materials. The magnetic field at any given point is specified by both a direction and a magnitude (or strength); as such it is a vector field

- Magnetic flux $\phi_m = BA\cos\theta = \vec{B}.\vec{A}$ weber

  Where B = magnetic field in Tesla, A = area of loop and θ angle between magnetic field and normal to loop.
- Magnetic force on a current carrying wire $F_m = Bil\sin\theta$

  Where θ = angle between current element $i\vec{\delta l}$ and magnetic field $\vec{B}$

  Maximum force, $F_m = Bil$ when $\theta = 90°$.
- When a current carrying wire is placed parallel to direction of magnetic field, the force on the conductor is zero.

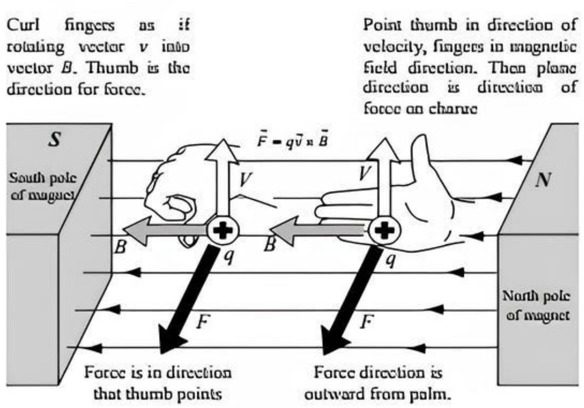


Figure: 7.10

- Magnetic Lorentz force on moving charge particle $F_m = qvB\sin\theta$. Where $\theta =$ angle between velocity $\vec{v}$ and magnetic field $\vec{B}$. $\vec{F}_m = v\times\vec{B}$ (vector form)
- Lorentz force is perpendicular to both $\vec{v}$ and $\vec{B}$. When a charged particle moves along the direction of magnetic field, the magnetic force on it is zero.
- Magnetic force between charges moving with velocity v_1 and v_2 is weaker than electric force $\frac{F_m}{F_e} = \frac{v_1 v_2}{c^2}$
- When charge q enters perpendicular to magnetic field. The path is circular having radius r given by $r = \frac{mv}{qB} = \frac{\sqrt{2mE_k}}{qB}$

  Where $E_k =$ kinetic energy of particle.

  Time period $T = \frac{2\pi m}{qB}$ Frequency $f = \frac{1}{T} = \frac{qB}{2\pi m}$
- A charged particle entering perpendicular to magnetic field can suffer 180°C deflection if length of magnetic field ≥ radius of path.
- When charge enters at angle $\theta = 0°$ or 90°, the path is helix having radius, $r = \frac{mv\sin\theta}{qB}$

  Pitch $p = v\cos\theta . T = v\cos\theta \frac{2\pi m}{qB}$

Biot-Savart Law: Biot-Savart's law is used to determine the magnetic field at any point due to a current carrying conductor. This law is although for infinitesimally small conductor yet it can be used for long conductors. The Magnetic field for current-element *idl* in different form:

Table: 7.2 Biot-Savarts Law in Vector Form and in Terms of Current Density

Vector form	Biot-Savarts Law in terms of current density	Biot-Savarts Law in terms of charge and its velocity
Vectorially, $d\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{I(d\vec{l}\times\hat{r})}{r^2}$ $= \frac{\mu_0}{4\pi}\cdot\frac{I(d\vec{l}\times\vec{r})}{r^3}$ Direction of $d\vec{B}$ is perpendicular to both $d\vec{l}$ and $\hat{r}$. This is given by right hand screw rule. *Weber/m*^{2} or Tesla	In terms of current density $d\vec{B} = \frac{\mu_0}{4\pi}\frac{\vec{J}\times\vec{r}}{r^3}dV$ Where $j = \frac{i}{A} = \frac{idl}{Adl} = \frac{idl}{dV}$ current density at any point of the element, dV = volume of element	In terms of charge and its velocity, $d\vec{B} = \frac{\mu_0}{4\pi}q\frac{(\vec{v}\times\vec{r})}{r^3}$ $\because id\vec{l} = \frac{q}{dt}d\vec{l}$ $= q\frac{d\vec{l}}{dt}q\vec{v}$

Magnetic field due to a straight current carrying conductor of finite length $B = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2)$

For infinite length θ_1 and θ_2 both are 90°

Therefore, $\sin\theta_1 = 1$ and $\sin\theta_2 = 1 \Rightarrow B = \frac{\mu_0 I}{4\pi a}$

Magnetic field due to a current carrying circular loop of radius

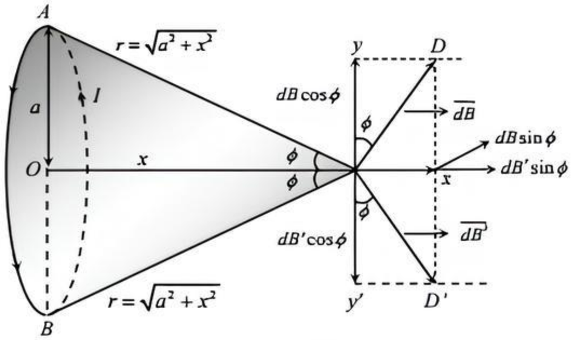


Figure: 7.11

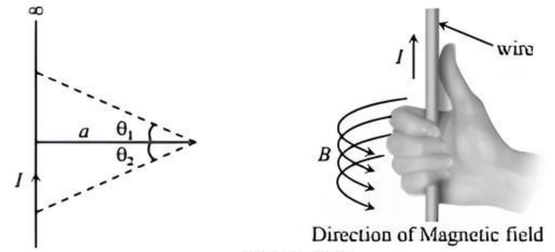


Figure: 7.12

- At its centre $B = \frac{\mu_0 NI}{2a}$ where $N =$ number of turns.
- At its axis distance x from centre $B = \frac{\mu_0 NIa^2}{2(a^2+x^2)^{3/2}}$

Helmholtz Coils: A pair of Helmholtz coil is to calculate magnetic field intensity B produced by each ring. If a current (I) is allowed to flow through a wire of length l, and the wire is bent into an arc of radius r, then the magnetic field intensity at center of the arc is $B = \frac{\mu_0 Il}{4\pi a^2}$

Where $\mu_0 =$ permeability of free space $(8.854\times10^{-12} F/m)$

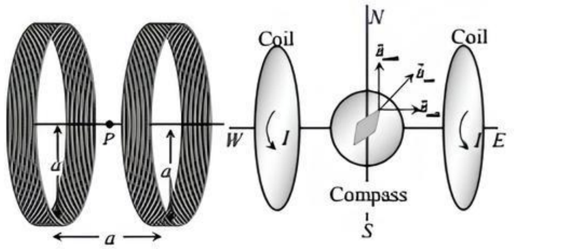


Figure: 7.13

- For a circular coil of n turns $B = \frac{\mu_0 In}{2a}$ or $B = \frac{2\pi nI}{a\times10^7}$
- The magnetic field at any point on axis at a distance x from center of coil is $B = \frac{2\pi nIa^2}{10^7(x^2+a^2)^{3/2}}$

Magnetic Field due to Current Carrying Circular Arc $B=\frac{\mu_0}{4\pi}\cdot\frac{\theta i}{r}$

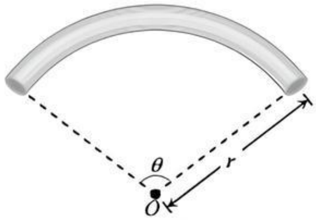
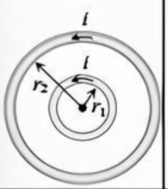
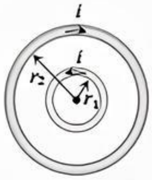
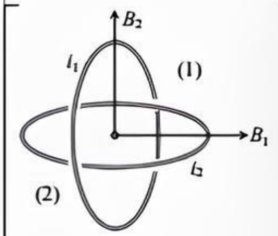


Figure: 7.14

Table: 7.3 Concentric Circular Loops ($N = 1$)

Coplanar		Non coplanar
Current in same direction	Current in opposite direction	Plane of both coils are perpendicular to each other
$B_1=\frac{\mu_0}{4\pi}2\pi i\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$	$B_2=\frac{\mu_0}{4\pi}2\pi i\left[\frac{1}{r_1}+\frac{1}{r_2}\right]$	$B=\sqrt{B_1^2+B_2^2}\ \frac{\mu_0}{2r}=\sqrt{i_1^2+i_2^2}$
		
$\frac{B_1}{B_2}=\left(\frac{r_2+r_1}{r_2-r_1}\right)$		

Magnetic Field due to a Solenoid: When solenoid having n number of turns/metre and carrying current i.

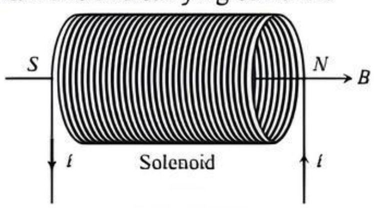


Figure: 7.15

- For finite length of solenoid $B=\frac{\mu_0 ni}{2}(\cos\alpha-\cos\beta)$
- For infinite length $B=\mu_0 ni$

Magnetic Field due to Current in Toroid: $B=\mu_0 ni$

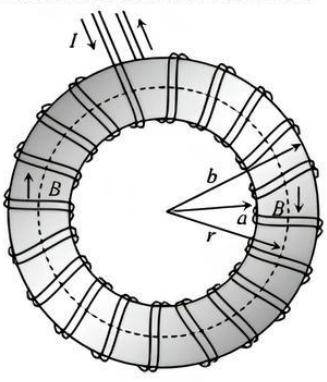


Figure: 7.16

Force between the Parallel Current Carrying Wires

When the wires are at separation r per unit length

$$f=\frac{F}{l}=\frac{\mu_0 i_1 i_2}{2\pi r}N/m$$

Parallel currents attract while anti-parallel currents repel.

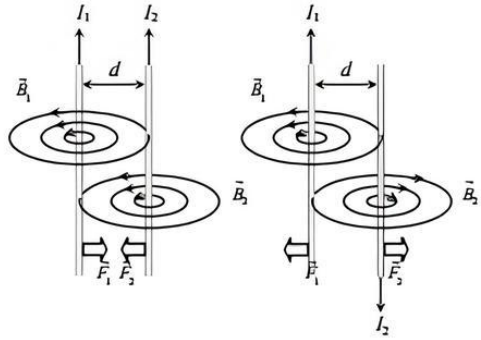


Figure: 7.17

Ampere's Circuital Law: The line integration of magnetic field B around the circular path in vacuum is equal to μ_0 times the total current I threading the closed path.

$$\oint \vec{B}.d\vec{l}=\mu_0 i$$

where i = current enclosed by path.

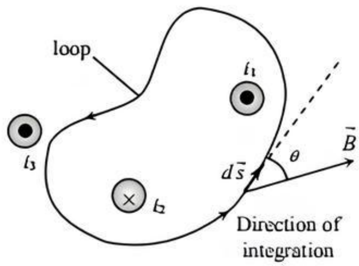


Figure: 7.18

Magnetic Field through Cylinder: Magnetic field due to solid current carrying cylindrical conductor at distance r from axis is

- Inside $B=\frac{\mu_0 i}{2\pi R^2}(r<R)$
- Outside $B=\frac{\mu_0 i}{2\pi R}(r>R)$

Note: Magnetic filed within a hollow current carrying conductor is zero

Cyclotron: Cyclotron is used to accelerate charged particles (+) or (–) by means of magnetic field. Frequency $f=\frac{qB}{2\pi m}$ and Energy $E=\frac{q^2B^2R^2}{2m}$; where R is the radius of Dee.