

Bar Magnet and Earth's Magnetism

1. Magnetic Charge

Based on similarities between electrostatic & magnetic phenomenon, the behaviour of a current loop can be understood using the following hypothetical model :

There are two types of magnetic charges, positive magnetic charge called north pole and negative magnetic charge called south pole. A magnetic charge with pole strength m placed in a magnetic field experiences a force

$$\vec{F} = m\vec{B}$$

A magnetic charge m produces a magnetic field

$$B = \frac{\mu_0}{4\pi} \frac{m}{r^2}$$

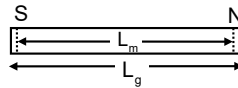
at a distance r from it. The field is radially outward if magnetic charge is positive & is inward if it is negative.

2. Bar Magnet & Pole Strength

2.1 Bar Magnet

It is most commonly used form of an artificial magnet. It consists of two equal and opposite magnetic poles, separated by a small distance.

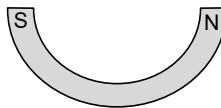
The shortest distance between the two poles is called effective length or magnetic length (L_m), whereas the actual length of the magnet is known as geometrical length (L_g). Geometrical length is always greater than its magnetic length as poles do not lie exactly at ends as shown.



For a bar magnet $L_m \approx 5/6 L_g$

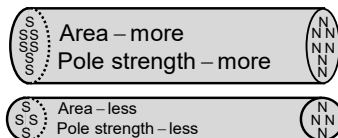
- For a semi-circular magnet of radius 'R'

$$L_g = \pi R \quad ; \quad L_m = 2R$$



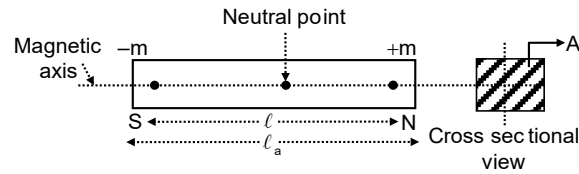
2.2 Pole Strength (m)

The strength of a magnetic pole to attract magnetic materials towards itself is known as pole strength.



- Pole strength of the magnet depends on the nature of material of magnet and area of cross section. It doesn't depend upon its length.
- It is a scalar quantity.
- Pole strength of N and S pole of a magnet is conventionally represented by $+m$ and $-m$ respectively.
- Its SI units are ampere metre and newton/tesla.
- Its dimensional formula is $[LA]$.

3. Magnetic Moment of Bar Magnet



The magnetic moment of a bar magnet is defined as a vector quantity having magnitude equal to the product of pole strength (m) with effective length (ℓ) and directed along the axis of the magnet from south pole to north pole.

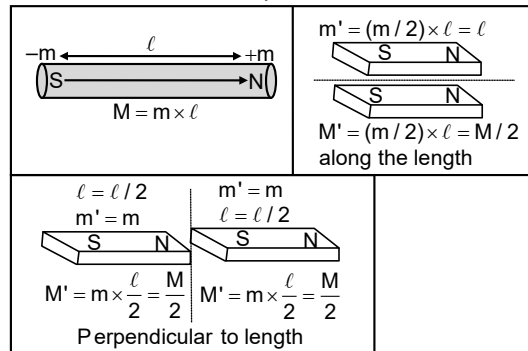
$$\vec{M} = m \vec{\ell}$$

It is an axial vector
S.I. unit: A.m^2

3.1 Cutting of Bar magnet :

- If a magnet is cut into two equal parts along the length then pole strength is reduced to half and length remains unchanged. New magnetic dipole moment $M' = m'(\ell) = \frac{m}{2} \times \ell = \frac{M}{2}$.

The new magnetic dipole moment of each part becomes half of original value.



- If a magnet is cut into two equal parts transverse to the length then pole strength remains unchanged and length is reduced to half. New magnetic dipole moment $M' = m \left(\frac{\ell}{2}\right) = \frac{M}{2}$.

The new magnetic dipole moment of each part becomes half of original value.

- Effective length or magnetic length :-** It is distance between two poles along the axis of a bar magnet. As pole are not exactly at the ends, the effective length (ℓ) is less than the geometrical length (ℓ_0) of the bar magnet. $\ell \approx 0.83 \ell_0$
- Inverse square law (Coulomb law) :** The magnetic force between two isolated magnetic poles of strength m_1 and m_2 lying at a distance 'r' is directly proportional to the product of pole strength and inversely proportional to the square of distance between their centres. The magnetic force between the poles can be attractive or repulsive according to the nature of the poles.

$$\left. \begin{array}{l} F_m \propto m_1 m_2 \\ F_m \propto \frac{1}{r^2} \end{array} \right\} F_m = k \frac{m_1 m_2}{r^2} \text{ where } k \begin{cases} \frac{\mu_0}{4\pi} \text{ (S.I.)} \\ 1 \text{ (C.G.S.)} \end{cases}$$

- Inverse square law of Coulomb in magnetism is applicable only for two long bar magnets because isolated poles cannot exist.

Example 1:

The force between two magnetic poles in air is 9.604 mN. If one pole is 10 times stronger than the other, calculate the pole strength of each if distance between two poles is 0.1 m?

Solution:

$$\text{Force between poles } F = \frac{\mu_0}{4\pi} \frac{m_1 m_2}{r^2}$$

$$\text{or } 9.604 \times 10^{-3} = \frac{10^{-7} \times m \times 10m}{0.1 \times 0.1}$$

$$\text{or } m^2 = 96.04 \text{ N}^2 \text{T}^{-2} \Rightarrow m = 9.8 \text{ N/T}$$

So strength of other pole is $9.8 \times 10 = 98 \text{ N/T}$

Example 2:

A steel wire of length L has a magnetic moment M. It is then bent into a semicircular arc. What is the new magnetic moment ?

Solution:

If m is the pole strength then

$$M = m.L \Rightarrow m = \frac{M}{L}$$

If it is bent into a semicircular arc then

$$L = \pi r \Rightarrow r = \frac{L}{\pi}$$

So new magnetic moment

$$M' = m \times 2r = \frac{M}{L} \times 2 \times \frac{L}{\pi} = \frac{2M}{\pi}$$

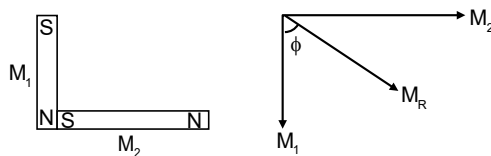
Example 3:

Two identical bar magnets each of length L and pole strength m are placed at right angles to each other with the north pole of one touching the south pole of other. Evaluate the magnetic moment of the system.

Solution:

$$M_1 = M_2 = mL$$

$$\therefore M_R = \sqrt{M_1^2 + M_2^2 + 2M_1 M_2 \cos \frac{\pi}{2}} = \sqrt{2} \text{ mL}$$



$$\text{and } \tan \phi = \frac{M \sin 90^\circ}{M + M \cos 90^\circ} = 1$$

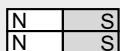
$$\text{i.e., } \phi = \tan^{-1} 1 = 45^\circ$$

Concept Builder-1

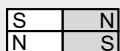


1. A bar magnet of magnetic moment 3.0 A-m^2 is placed in a uniform magnetic induction field of $2 \times 10^{-5} \text{ T}$. If each pole of the magnet experiences a force of $6 \times 10^{-4} \text{ N}$, the length of the magnet is
- (1) 0.5 m (2) 0.3 m (3) 0.2 m (4) 0.1 m

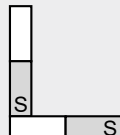
2. Two identical bar magnets are placed together in three different ways as shown below.



(1)



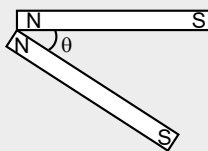
(2)



(3)

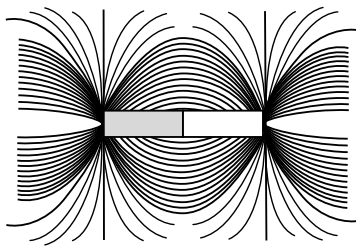
Calculate the magnetic moment of the combination in each case if that of each magnet is M .

3. Find the magnetic dipole moment of the combination of the identical bar magnets each of dipole moment p_0 .

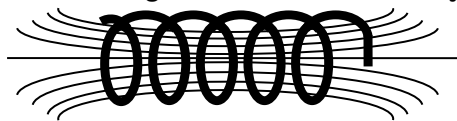


4. Magnetic Field and Field Lines

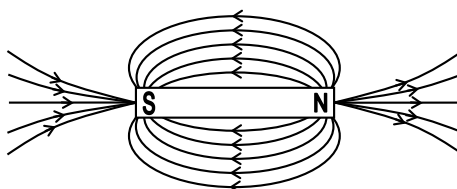
Space around a magnetic pole of a magnet or a current carrying wire, within which its effect can be experienced is defined as magnetic field. The pattern of iron filings permits us to plot the magnetic field lines.



The magnetic field lines for a bar magnet and a current carrying solenoid are shown below.



Field Lines of a Solenoid



Field lines of a Bar Magnet

The properties of magnetic field lines are

- (i) The magnetic field lines of a magnet (or a solenoid) form continuous closed loops. This is unlike the electric dipole where these field lines begin from a positive charge and end on the negative charge or escape to infinity.
- (ii) Inside the magnet, field lines from south to north while outside the magnet, they go from north to south.
- (iii) The tangent to the field line at an given point represents the direction of the magnetic field at that point.
- (iv) The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field .
- (v) The magnetic field lines do not intersect, for if they did, the direction of the magnetic field would not be unique at the point of intersection.
- (vi) Magnetic field lines have tendency to contract longitudinally indicating attraction between unlike magnetic poles. The lines also have tendency to dilate laterally, indicating repulsion between like magnetic poles as shown below.



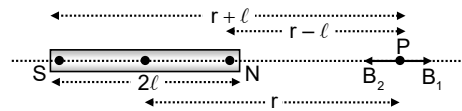
5. Strength of Magnetic Field

It is defined as the force experienced by a unit north pole placed at the given point in the magnetic field.

Hence, magnetic field due to an imaginary magnetic pole with-pole strength m is

$$B = \frac{F}{m} = \frac{\mu_0}{4\pi} \frac{m}{r^2}$$

6. Magnetic Field due to Bar Magnet



(i) At Axial position

Magnetic field at point 'P' due to north pole

$$B_1 = \frac{\mu_0}{4\pi} \frac{m}{(r - \ell)^2} \quad (\text{away from north pole})$$

Magnetic field at point 'P' due to south pole

$$B_2 = \frac{\mu_0}{4\pi} \frac{m}{(r + \ell)^2} \quad (\text{towards north pole})$$

Net magnetic field at point 'P'

$$B_{\text{axis}} = B_1 - B_2, \quad (B_1 > B_2)$$

$$= \frac{\mu_0}{4\pi} \frac{2Mr}{(r^2 - \ell^2)^2} = \frac{\mu_0 m}{4\pi} \left[\frac{5r\ell}{(r^2 - \ell^2)} \right]$$

$$B_{\text{axis}} = \frac{\mu_0}{4\pi} \frac{2Mr}{(r^2 - \ell^2)^2}, \quad \text{where } M = m (2\ell)$$

If magnet is short $r \gg \ell$, then

$$B_{\text{axis}} \approx \frac{\mu_0}{4\pi} \frac{2M}{r^3}$$

(ii) At equatorial position

Magnetic field at point 'P' due to north pole :-

$$B_1 = \frac{\mu_0}{4\pi} \frac{m}{(\sqrt{r^2 + \ell^2})^2} \dots (1) \text{ (along NP line)}$$

Magnetic field at point 'P' due to south pole

$$B_2 = \frac{\mu_0}{4\pi} \frac{m}{(\sqrt{r^2 + \ell^2})^2} \dots (2) \text{ (along PS line)}$$

From equation (1) & (2)

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^2 + \ell^2} = B \text{ (Let)}$$

Net magnetic field at point 'P'

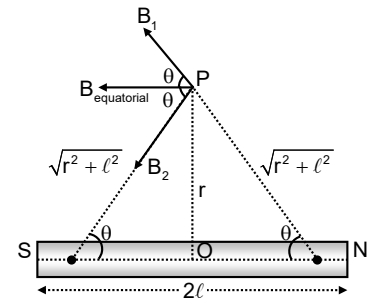
$$B_{eq} = 2 B \cos\theta = 2 \cdot \frac{\mu_0}{4\pi} \frac{m}{(r^2 + \ell^2)} \cos\theta, \text{ [where } \cos\theta = \frac{\ell}{\sqrt{r^2 + \ell^2}} \text{]}$$

$$= 2 \cdot \frac{\mu_0}{4\pi} \frac{m}{(r^2 + \ell^2)} \frac{\ell}{\sqrt{r^2 + \ell^2}}$$

$$B_{eq} = \frac{\mu_0}{4\pi} \frac{M}{(r^2 + \ell^2)^{3/2}} \quad \text{where } M = m (2\ell)$$

If magnet is short $r \gg \ell$, then

$$B_{eq} \simeq \frac{\mu_0}{4\pi} \frac{M}{r^3}$$



7. Gauss Law in Magnetism

Net magnetic flux through any closed surface is always zero.

$$\oint \vec{B} \cdot d\vec{S} = 0$$

It implies that monopole doesn't exist.

A consequence of the fact that magnetic monopoles do not exist is that the magnetic field lines are continuous and form closed loops. In contrast, the electrostatic lines of force begin on a positive charge and terminate on the negative charge.

Example 4:

Calculate the magnetic induction at a point 1\AA away from a proton, measured along its axis of spin. The magnetic moment of the proton is $1.4 \times 10^{-26} \text{ A-m}^2$.

Solution:

On the axis of a magnetic dipole magnetic induction is given by,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}.$$

Substituting the values, we get

$$B = \frac{(10^{-7})(2)(1.4 \times 10^{-26})}{(10^{-10})^3} = 2.8 \times 10^{-3} \text{ T} = 2.8 \text{ mT}$$

Example 5:

What is the magnitude of the equatorial and axial fields due to a bar magnet of length 5.0 cm at a distance of 50 cm from its mid point? The magnetic moment of the bar magnet is 0.40 A m^2 .

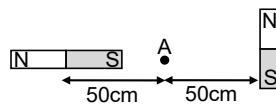
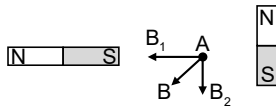
Solution:

$$B_{\text{eq}} = \frac{\mu_0}{4\pi} \frac{M}{r^3} = \frac{10^{-7} \times 0.4}{(0.5)^3} = 3.2 \times 10^{-7} \text{ T}$$

$$B_{\text{axial}} = \frac{\mu_0}{4\pi} \frac{2M}{r^3} = \frac{10^{-7} \times 2 \times 0.4}{(0.5)^3} = 6.4 \times 10^{-7} \text{ T}$$

Example 6:

In the figure below, calculate the magnetic field at point A. Dipole moment of each magnet shown is 1 A m^2 .

**Solution:**

Point A is on the axial line of first magnet and the field produced by the first magnet is

$$B_1 = \frac{\mu_0}{4\pi} \frac{2M}{r^3} = 16 \times 10^{-7} \text{ T}$$

Point A is on the equatorial line of second magnet and the field produced by the second magnet is

$$B_2 = \frac{\mu_0}{4\pi} \frac{M}{r^3} = \frac{10^{-7} \times 1}{(0.5)^3} = 8 \times 10^{-7} \text{ T}$$

Therefore net magnetic field at A is

$$B = \sqrt{B_1^2 + B_2^2} = 8\sqrt{5} \times 10^{-7} \text{ T}$$

8. Dipole in a Uniform Magnetic Field**8.1 Torque**

A small compass needle of known magnetic moment \vec{M} and moment of inertia I is placed in a uniform magnetic field \vec{B} at an angle θ as shown.

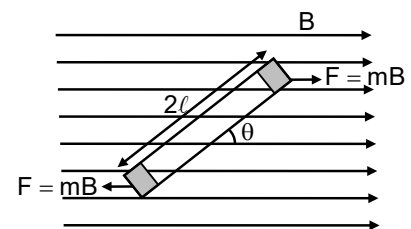
Both of its poles experience equal and opposite forces due to which net force on it is always zero. Hence the magnetic dipole is always in translational equilibrium in a uniform magnetic field. But torque on it is non-zero. It can be calculated as

$$\tau = \vec{r} \times \vec{F} = (2\ell \sin \theta) mB$$

$$= (m \times 2\ell \times B \sin \theta)$$

$$\tau = MB \sin \theta$$

In vector form $\tau = \vec{M} \times \vec{B}$



Special cases

(1) When $\theta = 0^\circ$, $\tau = 0$

The dipole is in rotational equilibrium also.

(2) When $\theta = 180^\circ$, $\vec{\tau} = 0$ again

(3) When $\theta = 90^\circ$, $|\vec{\tau}| = \tau_{\max} = MB$

8.2 Work Done in rotating a dipole in external field

Torque on a dipole in magnetic field is given by

$$\tau = MB \sin \theta$$

If dipole is rotated by a small angle $d\theta$, then work is done against torque which is given by

$$dW = \tau d\theta = MB \sin \theta d\theta$$

Total work done is given by

$$W = \int dW = \int_{\theta_1}^{\theta_2} MB \sin \theta d\theta$$

$$W = MB (\cos \theta_1 - \cos \theta_2)$$

This work done is stored in the form of potential energy of system.

8.3 Potential Energy of magnetic dipole in external field

Change in potential energy = work done

$$\Delta U = W = MB (\cos \theta_1 - \cos \theta_2)$$

As we are interested only in the change in the potential energy of the dipole, let us define its potential energy to be zero when it is perpendicular to the magnetic field.

$$U_{\text{initial}} = 0, \text{ when } \theta_1 = 90^\circ$$

$$\text{and } U_{\text{final}} = U, \text{ when } \theta_2 = \theta$$

$$U = -MB \cos \theta = -\vec{M} \cdot \vec{B}$$

Special cases

(1) When $\theta = 0^\circ$, $U = U_{\min} = -MB$

The dipole is in total equilibrium and its potential energy is minimum, so its equilibrium is stable.

(2) When $\theta = 180^\circ$, $\vec{\tau} = 0$, $U = U_{\max} = MB$

(3) When $\theta = 90^\circ$, $|\vec{\tau}| = \tau_{\max} = MB$, $U = 0$

Example 8:

An magnetic dipole is placed at an angle of 30° with an magnetic field of intensity 10^4 T. It experiences a torque equal to 5 Nm. Calculate the pole strength of the dipole, If dipole length is 1 cm.

Solution:

$$\begin{aligned} \text{As } \tau &= MB \sin \theta, M = \frac{\tau}{B \sin \theta} \\ &= \frac{5}{[(10^4)(0.5)]} = 10^{-3} \text{ Am}^2 \end{aligned}$$

$$\text{Thus, } m = \frac{M}{d} = 10^{-1} \text{ Am} = 0.1 \text{ Am}$$

Example 9:

A bar magnet of magnetic moment 1.5 JT^{-1} lies aligned with the direction of a uniform magnetic field of 0.22 T .

- (a) What is the amount of work required to turn the magnet so as to align its magnetic moment-
 (i) Normal to the field direction?
 (ii) Opposite to the field direction?
 (b) What is the torque on the magnet in case (i) and (ii)?

Solution:

Here, $M = 1.5 \text{ JT}^{-1}$, $B = 0.22 \text{ T}$.

- (a) P.E. with magnetic moment aligned to field = $-MB$
 P.E. with magnetic moment normal to field = 0
 P.E. with magnetic moment antiparallel to field = $+MB$
 (i) Work done = increase in P.E. = $0 - (-MB)$
 $= MB = 1.5 \times 0.22 = 0.33 \text{ J}$.
 (ii) Work done = increase in P.E. = $MB - (-MB)$
 $= 2MB = 2 \times 1.5 \times 0.22 = 0.66 \text{ J}$.
 (b) We have $\tau = MB \sin \theta$
 (i) $\tau = MB \sin \theta = 1.5 \times 0.22 \times 1 = 0.33 \text{ J}$.
 $(\theta = 90^\circ \Rightarrow \sin \theta = 1)$
 This torque will tend to align M with B .
 (ii) $\tau = MB \sin \theta = 1.5 \times 0.22 \times 0 = 0$
 $(\theta = 180^\circ \Rightarrow \sin \theta = 0)$

Example 10:

A short bar magnet of magnetic moment 0.32 J/T is placed in uniform field of 0.15 T . If the bar is free to rotate in plane of field then which orientation would correspond to its

- (i) stable and
 (ii) unstable equilibrium? What is potential energy of magnet in each case?

Solution:

- (i) If M is parallel to B then $\theta = 0^\circ$. So potential energy $U = U_{\min} = -MB$
 $U_{\min} = -MB = -0.32 \times 0.15 \text{ J} = -4.8 \times 10^{-2} \text{ J}$
 (stable equilibrium)
 (ii) If M is antiparallel to B then $\theta = \pi$
 So potential energy
 $U = U_{\max} = +MB = +0.32 \times 0.15 = 4.8 \times 10^{-2} \text{ J}$
 (unstable equilibrium)

9. Oscillations of Bar Magnet in a Uniform Magnetic Field

Let a bar magnet of dipole moment M and moment of inertial I is slightly disturbed by a small angle θ from its stable equilibrium position in a uniform magnetic field B . Restoring torque acting on it is

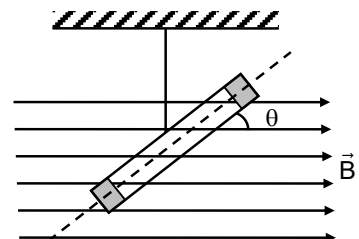
$$\tau = MB \sin \theta$$

for small angle $\sin \theta \approx \theta$
 $\therefore \tau = I\alpha = MB\theta$

$$\text{or } \alpha = \frac{MB}{I} \theta \quad \text{or } \ddot{\alpha} = - \frac{MB}{I} \theta$$

Which is the standard equation of angular SHM. Hence

$$\omega = \sqrt{\frac{MB}{I}} \quad \text{or } T = 2\pi \sqrt{\frac{I}{MB}}$$



Example 11:

A magnetic needle has magnetic moment $8 \times 10^{-2} \text{ Am}^2$ and moment of inertia $= 7.5 \times 10^{-6} \text{ kg-m}^2$. It performs 10 complete oscillations in 8s when placed in a uniform magnetic field. What is the magnitude of the magnetic field ?

Solution:

The time period of oscillation is

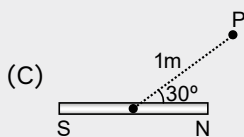
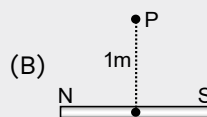
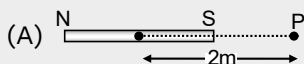
$$T = \frac{8}{10} = 0.8 \text{ s}$$

As $T = 2\pi\sqrt{\frac{I}{MB}}$, therefore

$$B = \frac{4\pi^2 I}{MT^2} = \frac{4 \times (3.14)^2 \times 7.5 \times 10^{-6}}{8 \times 10^{-2} \times (0.8)^2} = 0.006 \text{ T}$$

Concept Builder-2

Q.1 What is the Intensity of magnetic field at following points due to the bar magnet ($M = 1 \text{ A-m}^2$)



(Assume length of bar magnet to be very small)

Q.2 A short bar magnet has a magnetic moment of 0.48 JT^{-1} . Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of magnet on (a) the axis (b) the equatorial lines (normal bisector) of the magnet.

Q.3 A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.25 T experiences a torque of magnitude equal to $4.5 \times 10^{-2} \text{ J}$. What is the magnitude of magnetic moment of the magnet ?

Q.4 A magnetic dipole of length of 10 cm having pole strength $\pm 2 \times 10^{-3} \text{ Am}$, placed at 53° with respect to a uniform magnetic field experiences a torque of 8 Nm. Calculate
(a) magnitude of magnetic field
(b) The potential energy of the dipole

Q.5 A bar magnet of pole strength $\pm 6 \times 10^{-2} \text{ A-m}$ is placed in a magnetic field of strength $4 \times 10^{-2} \text{ T}$, making an angle 60° with the field. Assuming length of the magnet to be 10 mm. Find out
(A) Torque on the magnet
(B) The orientation it will tend to rotate. Stable or unstable equilibrium.
(C) Change in P.E. when it attains
(i) Stable equilibrium
(ii) Unstable equilibrium

10. Earth's Magnetism

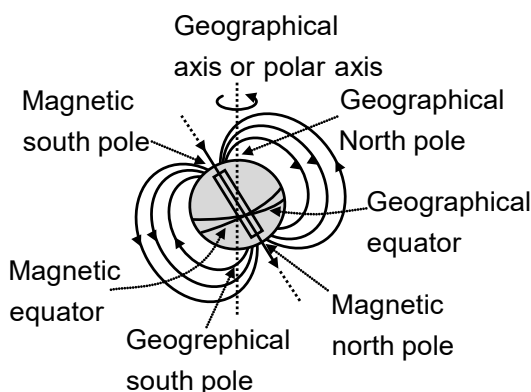
10.1 Introduction of basic elements

The branch of physics which deals with the study of earth's magnetic field is called geomagnetism.

Important definitions

- (a) **Geographic axis** : It is a straight line passing through the geographical poles of the earth. It is also called axis of rotation or polar axis of the earth.
- (b) **Geographic Meridian (GM)** : It is a vertical plane at any place which passing through geographical axis of the earth.
- (c) **Geographic equator** : It is a great circle on the surface of the earth, in a plane perpendicular to the geographic axis. All the points on the geographic equator are at equal distance from the geographic poles.

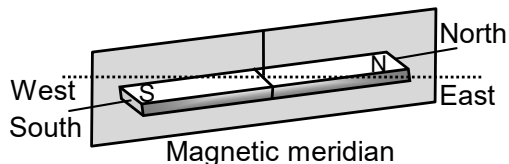
A great plane which passes through geographic equator and perpendicular to the geographic axis called **geographic equatorial plane**. This plane cuts the earth in two equal parts, a part has geographic north called **northern hemisphere** (NHS) and another part has geographic south called **southern hemisphere** (SHS).



- (d) **Magnetic axis** : It is a straight line passing through magnetic poles of the earth. It is inclined to the geographic axis at nearly 17° .

(e) **Magnetic Meridian (MM)** :

- (i) It is a vertical plane at any place which passing through magnetic axis of the earth.



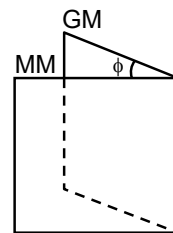
- (ii) It is a vertical plane at any place which passes through axis of free suspended bar magnet or magnetic needle.
 - (iii) It is a vertical plane at any place which contains all the magnetic field lines of earth of that place.
- (f) **Magnetic Equator** : It is a great circle on the surface of the earth, in a plane perpendicular to the magnetic axis. All the points on the magnetic equator are at equal distance from the magnetic poles.

10.2 Components of Earth's Magnetic Field

Main elements of earth's magnetic field

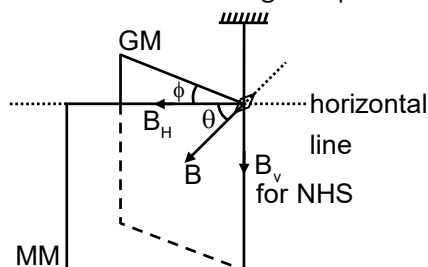
Angle of Declination (ϕ)

At a given place, the acute angle between geographic meridian and the magnetic meridian is called angle of declination, i.e. at a given place it is the angle between the geographical north south direction and the direction indicated by a magnetic compass needle in its equilibrium.



Angle of Dip (θ) :

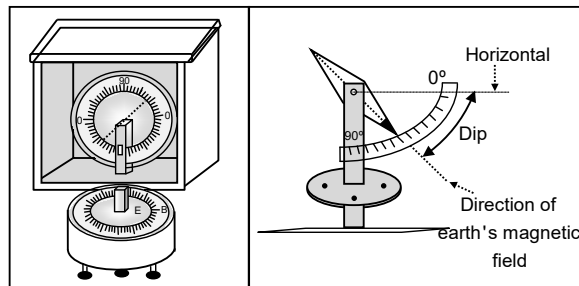
- (i) It is an angle which the direction of resultant magnetic field of the earth subtends with the horizontal line in magnetic meridian at the given place.



- (ii) It is an angle which the axis of freely suspended magnetic needle (up or down) subtends with the horizontal line in magnetic meridian at a given place.

In northern hemisphere, north pole of freely suspended magnetic needle will dip downwards i.e. towards the earth surface. In southern hemisphere, south pole of freely suspended magnetic needle will dip downwards i.e. towards the earth surface.

Dip circle : Angle of dip at a place is measured by the instrument called 'Dip-circle' in which a magnetic needle is free to rotate in vertical plane, about its horizontal axis. The ends of the needle move over a vertical scale graduated in degree.



Horizontal component of earth magnetic field (B_H)

Horizontal component of earth magnetic field at a given place is the component of resultant magnetic field of the earth along the horizontal line in magnetic meridian.

$$B_H = B \cos \theta \quad \text{and} \quad B_V = B \sin \theta \quad \dots(1)$$

$$\text{so that } \tan \theta = \frac{B_V}{B_H} \quad \text{and} \quad B = \sqrt{B_H^2 + B_V^2} \quad \dots(2)$$

At magnetic poles $\theta = 90^\circ$

$B_H = 0$ and only B_V exist

At magnetic equator $\theta = 0^\circ$

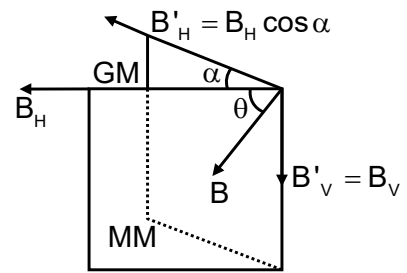
$B_V = 0$ and only B_H exist

ϕ decides the plane in which magnetic field lies at any place, (ϕ) and (θ) decides the direction of magnetic field and (θ) and (B_H) decides the magnitude of the field.

Apparent Angle of Dip (θ')

When the plane of vertical scale of dip circle is in the magnetic meridian, the needle rests in the direction of earth's magnetic field. The angle made by the needle with the horizontal is called true dip or actual dip. If the plane of vertical scale of dip circle is not kept in magnetic meridian, then the needle will not indicate the correct direction of earth magnetic field.

In this situation the angle made by the needle with the horizontal is called the apparent angle of dip. Suppose the dip circle is set at an angle α to the magnetic meridian. Effective horizontal component in this plane will be $B_H \cos \alpha$ and no effect on vertical component B_V



Apparent angle of dip $\tan \theta' = \frac{B'_V}{B'_H}$

$$\tan \theta' = \frac{B_V}{B_H \cos \alpha} \Rightarrow \tan \theta' = \frac{\tan \theta}{\cos \alpha}$$

- For a vertical plane other than magnetic meridian

$$\alpha > 0 \quad \cos \alpha < 1 \quad \tan \theta' > \tan \theta$$

$$\theta' > \theta$$

so apparent angle of dip is always more than actual angle of dip at any place.

- For a vertical plane perpendicular to magnetic meridian $\alpha = 90^\circ$

$$\tan \theta' = \frac{\tan \theta}{\cos 90^\circ} = \infty$$

$\theta' = 90^\circ$, so in a plane perpendicular to magnetic meridian dip needle becomes just vertical.

Example 12:

At a certain place, the horizontal component of earth's magnetic field is times of the vertical component. What is the angle of dip at that place?

Solution:

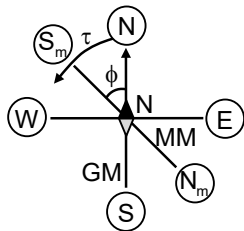
$$B_H = \sqrt{3} B_V$$

$$\tan \theta = \frac{B_V}{B_H} = \frac{B_V}{\sqrt{3} B_V} = \frac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow \theta = 30^\circ$$

Example 13:

A compass needle of magnetic moment is 60 A-m^2 pointing towards geographical north at a certain place where the horizontal component of earth's magnetic field is $40 \mu\text{T}$, experiences a torque $1.2 \times 10^{-3} \text{ N-m}$. What is the declination of that place.

Solution:



$$\tau = MB \sin \phi \Rightarrow \sin \phi = \frac{\tau}{MB} = \frac{1.2 \times 10^{-3}}{24 \times 10^{-4}} = \frac{1}{2}$$

$$\Rightarrow \phi = 30^\circ$$

Concept Builder-3



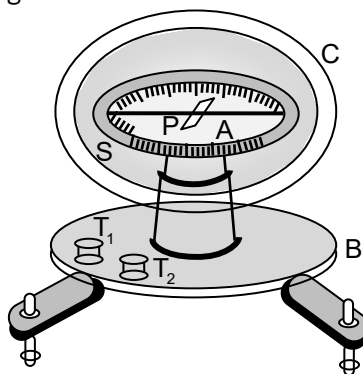
- Q.1** In the magnetic meridian of a certain place, the horizontal component of the earth's magnetic field is 0.26 G and the dip angle is 60° . Find
 (a) vertical component of the earth's magnetic field.
 (b) the net magnetic field at this place.
- Q.2** If the dip circle is set at 45° to the magnetic meridian, then the apparent dip is 30° . Calculate the true dip.
- Q.3** A dip circle shows an apparent dip of 60° at a place where the true dip is 45° . If the dip circle is rotated through 90° what apparent dip will it show ? ($\tan 51^\circ = 1.22$)
- Q.4** A magnetic needle suspended in a vertical plane at 30° from the magnetic meridian makes an angle of 45° with the horizontal. Find the true angle of dip. ($\tan 41^\circ = 0.866$)

11. Application of Geo Magnetism

11.1 Tangent Galvanometer

It is an instrument which can detect/measures electric currents. It is also called moving magnet galvanometer.

Principle :- It is based on 'tangent law'



Construction

- It consists of a circular coil of a large number of turns of insulated copper wire wound over a vertical circular frame.
- A small magnetic compass needle is pivoted at the centre of vertical circular coil. This needle can rotate freely in a horizontal plane.

Tangent law :- If a current is passed through the vertical coil, then magnetic field produced at its centre is perpendicular to the horizontal component of earth's magnetic field since coil is in magnetic meridian. So in the effect of two crossed fields ($B_H \perp B_0$) compass needle comes in equilibrium according to tangent law.

Torque on needle due to (B_0) = $\tau_1 = MB_0 \sin(90 - \theta)$

Torque on needle due to (B_H) = $\tau_2 = MB_H \sin \theta$

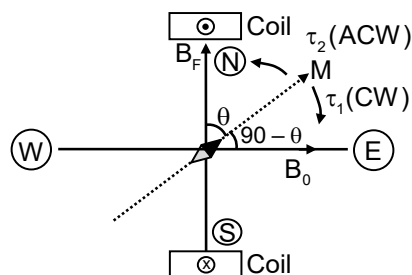
At equilibrium condition of needle net torque on it is zero

$$MB_0 \sin(90 - \theta) = MB_H \sin \theta$$

$$B_0 \cos \theta = B_H \sin \theta$$

$$B_0 = B_H \frac{\sin \theta}{\cos \theta}; B_0 = B_H \tan \theta$$

where $B_H = B \cos \theta$, $\theta \rightarrow$ angle of dip



$$\therefore \frac{\mu_0 N I}{2R} = B_H \tan \theta \quad \text{so} \quad I = \left(\frac{2B_H R}{\mu_0 N} \right) \tan \theta$$

$I = K \tan \theta$, so for this galvanometer $I \tan \theta$

The electric current is proportional to the tangent of the angle of deflection

Reduction factor : is a constant for the given galvanometer at given place.

The reduction factor of a tangent galvanometer is numerically equal to the current required to produce a deflection of 45° in it.

$$\theta = 45^\circ \quad I = K \tan(45^\circ) ; I = K$$

SI unit of 'K' ampere

- **Sensitivity** : A tangent galvanometer is both sensitive and accurate if the change in its deflection is large for a given fractional change in current.

$$I = K \tan \theta \quad \text{or} \quad dI = K \sec^2 \theta d\theta$$

$$\frac{dI}{I} = \frac{d\theta}{I \sin \theta \cos \theta} = \frac{2d\theta}{\sin 2\theta} \quad \text{or} \quad d\theta = \frac{\sin 2\theta}{2} \frac{dI}{I}$$

$$d\theta = (d\theta)_{\max} \quad \text{if} \quad \sin 2\theta = 1 = \sin \frac{\pi}{2}$$

$$\text{So} \quad \theta = \frac{\pi}{4}$$

The tangent galvanometer has maximum sensitivity when $\theta = 45^\circ$.

Example 18:

A cell of an emf of 2V and internal resistance of 0.5Ω is sending current through a tangent galvanometer of resistance 4.5Ω . If another external resistance of 95Ω is introduced, the deflection of galvanometer is 45° . Calculate the reduction factor of galvanometer.

Solution:

$$I = \frac{E}{r + R + R'} = \frac{2}{0.5 + 4.5 + 95} = \frac{2}{100} \text{ ampere}$$

From $I = K \tan \theta$ reduction factor

$$K = \frac{I}{\tan \theta} = \frac{2}{100 \times \tan 45^\circ} = 0.02 \text{ ampere}$$

Example 19:

Two tangent galvanometers A and B have their number of turns in the ratio 1 : 3 and diameters in the ratio 1 : 2

- Which galvanometer has greater reduction factor
- Which galvanometer shown greater deflection, when both are connected in series to a d.c. source.

Solution:

$$(a) \quad \frac{N_A}{N_B} = \frac{1}{3}, \quad \frac{R_A}{R_B} = \frac{1}{2}; \text{ Reduction factor}$$

$$K = \frac{K_A}{K_B} = \frac{R_A}{R_B} \times \frac{N_B}{N_A} = \frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$$

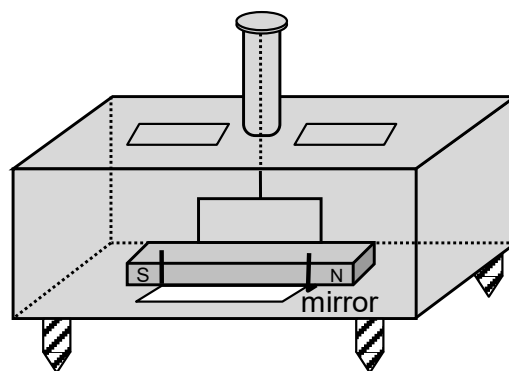
$$\Rightarrow K_A > K_B$$

- From $I = K \tan \theta$, (I = same in series combination) $\therefore K_A > K_B \Rightarrow \tan \theta_A < \tan \theta_B \Rightarrow \theta_A < \theta_B$

11.2 Vibration Magnetometer

It is an instrument used to compare the horizontal components of magnetic field of earth at two different places, to compare magnetic fields and magnetic moments of two bar magnets. It is also called oscillation magnetometer.

Principle : This device works on the principle, that whenever a freely suspended bar magnet lies in horizontal component of earth magnetic field (B_H), it is slightly disturbed from its equilibrium position. Then, it will experience a torque and executes angular S.H.M. Rotation is possible only in horizontal plane.



Angular S.H.M of magnetic dipole :- When a dipole is suspended in a uniform magnetic field, it will align itself parallel to field. Now if it is given a small angular displacement θ about its equilibrium position. The restoring torque acts on it :

$$\tau = - M B_H \sin \theta$$

$$\Rightarrow I \alpha = - M B_H \sin \theta = - M B_H \theta$$

($\because \sin \theta \approx \theta$)

$$\Rightarrow \alpha = \frac{M B_H}{I} (-\theta) \Rightarrow \alpha = \omega^2 (-\theta)$$

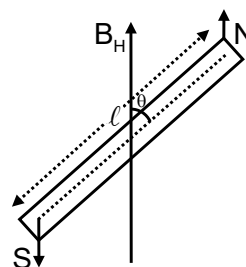
$$\Rightarrow \omega^2 = \frac{M B_H}{I}$$

The time period of angular S.H.M.

$$\Rightarrow T = 2\pi \sqrt{\frac{I}{M B_H}}$$

M = magnetic moment of bar magnet

I = moment of inertia of bar magnet about its geometric axis



- Comparison of magnetic moments of magnets of the same size**

Let the two magnets of same size have moment of inertia I and magnetic moments M_1 and M_2 . Suspend the two given magnets turn by turn in the metal stirrup of the vibration magnetometer and note the time period in each case.

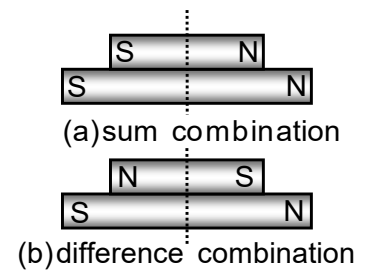
$$\text{Then } T_1 = 2\pi \sqrt{\frac{I}{M_1 B}} \text{ and } T_2 = 2\pi \sqrt{\frac{I}{M_2 B}}$$

$$\text{Dividing, } \frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}} \text{ or } \frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}$$

Since T_1 and T_2 are known therefore the ratio $\frac{M_2}{M_1}$ can be determined.

- **Comparison of magnetic moments of magnets of different sizes**

Let the two magnets have moments of inertia I_1 and I_2 and magnetic moments M_1 and M_2 respectively. Place the two given magnets one upon the other as shown in Fig. (a). This combination is called 'sum combination'. It has moment of inertia $(I_1 + I_2)$ and magnetic moment $(M_1 + M_2)$. Put this combination in the magnetometer and set it into oscillations. The time period T_1 is determined.



$$T_1 = 2\pi \sqrt{\frac{I_1 + I_2}{(M_1 + M_2)B}} \quad \text{.....(1)}$$

Now, the two magnets are placed as shown in Fig. (b). This combination is called 'difference combination'. It has moment of inertia $(I_1 + I_2)$ and magnetic moment $(M_1 - M_2)$. This combination is put in the magnetometer and its time period T_2 is determined.

$$T_2 = 2\pi \sqrt{\frac{I_1 + I_2}{(M_1 - M_2)B}} \quad \text{.....(2)}$$

$$\text{Dividing, } \frac{T_1}{T_2} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}} \quad [\text{from equation (1) and (2)}]$$

knowing T_1 and T_2 , we can determine $\frac{M_1}{M_2}$.

- **Comparison of earth's magnetic field at two different places**

Let the vibrating magnet have moment of inertia I and magnetic moment M . Let it be vibrated in places where earth's magnetic field is and

$$\text{Then, } T_1 = 2\pi \sqrt{\frac{I}{MB_{H_1}}} \quad \text{and} \quad T_2 = 2\pi \sqrt{\frac{I}{MB_{H_2}}}$$

T_1 and T_2 are determined by placing magnetometer at two different places, turn by turn.

$$\text{Dividing, } \frac{T_1}{T_2} = \sqrt{\frac{B_{H_2}}{B_{H_1}}} \quad \text{or} \quad \frac{T_1^2}{T_2^2} = \frac{B_{H_2}}{B_{H_1}}$$

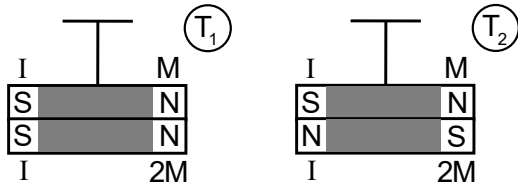
$$= \frac{B_2 \cos \theta_2}{B_1 \cos \theta_1} \Rightarrow \frac{B_1}{B_2} = \frac{T_2^2 \cos \theta_2}{T_1^2 \cos \theta_1}$$

Knowing T_1 , T_2 and θ_1 , θ_2 the ratio $\frac{B_1}{B_2}$ can be determined.

Example 20:

Magnetic moments of two identical magnets are M and $2M$ respectively. Both are combined in such a way that their similar poles are same side. The time period in this case is ' T_1 '. If polarity of one of the magnets is reversed, its period becomes ' T_2 '. Then find out ratio of their time periods respectively.

Solution:



$$\begin{aligned}
 M_{\text{system}} &= 2M + M = 3M & M_{\text{system}} &= 2M - M = M \\
 I_{\text{system}} &= 2I & I_{\text{system}} &= 2I \\
 T &= 2\pi \sqrt{\frac{I}{MB_H}} & (I_{\text{system}} \rightarrow \text{same}, B_H \rightarrow \text{same}) \\
 \boxed{T \propto \frac{1}{\sqrt{M}}} & & ; \frac{T_1}{T_2} &= \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{M}{3M}} = \frac{1}{\sqrt{3}}
 \end{aligned}$$

Example 21:

A magnet is suspended in such a way when it oscillates in the horizontal plane. It makes 20 oscillations per minute at a place where dip angle is 30° and 15 oscillations per min at a place where dip angle is 60° . Find the ratio of the total earth's magnetic field at the two places.

Solution:

$$\begin{aligned}
 f &= \frac{1}{2\pi} \sqrt{\frac{MB_H}{I}} \Rightarrow f^2 = \frac{1}{4\pi^2} \cdot \frac{MB \cos \theta}{I} \\
 I \text{ and } M \text{ are same in given cases} \\
 \frac{B_1}{B_2} &= \frac{f_1^2}{f_2^2} \times \frac{\cos \theta_2}{\cos \theta_1} = \frac{20 \times 20}{15 \times 15} = \frac{\cos 60^\circ}{\cos 30^\circ} = \frac{16}{9\sqrt{3}}
 \end{aligned}$$

Example 22:

A vibration magnetometer consists of two identical bar magnets placed one over the other such that they are perpendicular and bisect each other. The time period of oscillation in a horizontal magnetic field is $2^{5/4}$ sec. One of the magnets is removed and if the other magnet oscillates in the same field, calculate the time period.

Solution:

Magnetic moment is a vector quantity. If the magnetic moments of the two magnets are M each then, the net magnetic moment when the magnets are placed perpendicular to each other, is

$$M_{\text{eff.}} = \sqrt{M^2 + M^2} = M\sqrt{2} \text{ and the moment of inertia is } 2I. \text{ So, } T = \sqrt{\frac{2I}{M\sqrt{2}B}}$$

$$\text{When one of the magnets is withdrawn, the time period is } T' = 2\pi \sqrt{\frac{I}{MB}}$$

$$\therefore \frac{T'}{T} = \sqrt{\frac{1}{2^{1/2}}} \text{ or } T' = \frac{T}{2^{1/4}} = 2^{5/4 - 1/4} = 2 \text{ sec}$$

11.3 Neutral Points

It is a point where net magnetic field is zero.

At this point, magnetic field of bar magnet or current carrying coil or current carrying wire is just neutralised by magnetic field of earth. (B_H)

A compass needle placed at this neutral point can set itself in any direction.

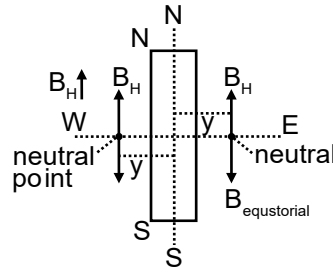
Location of Neutral Points

(a) When N–pole of magnet directed towards North :- Two neutral points symmetrically located on equatorial line of magnet. Let distance of each neutral point from centre of magnet is 'y' then

$$B_{eq} = B_H$$

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{M}{(y^2 + \ell^2)^{3/2}}$$

$$\frac{\mu_0}{4\pi} \cdot \frac{M}{y^3} = B_H \text{ (If } y \gg \ell \text{)}$$



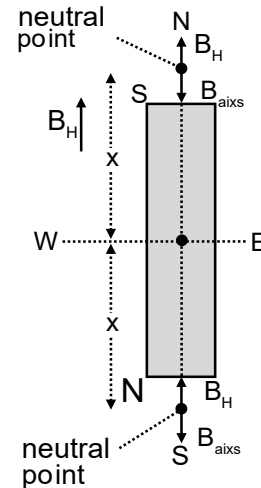
(b) When S–pole of magnet directed towards North

Two neutral points symmetrically located on the axial line of magnet. Let distance of each neutral point from centre of the magnet is x, then

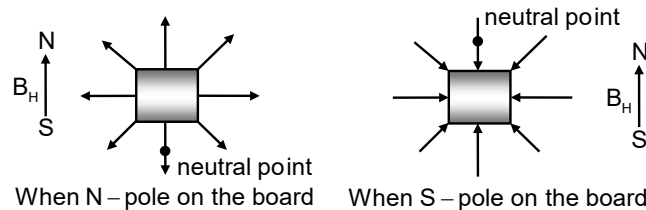
$$B_{axis} = B_H \Rightarrow B_H = \frac{\mu_0}{4\pi} \frac{2Mx}{(x^2 - \ell^2)^2}$$

$$\frac{\mu_0}{4\pi} \frac{2M}{x^3} = B_H \text{ (If } x \gg \ell \text{)}$$

$$\frac{\mu_0}{4\pi} \frac{2M}{x^3} = B_H \text{ (If } x \gg \ell \text{)}$$



(c) If magnet is held vertically on the board, then only one neutral point is obtained on the horizontal board.



Example 23:

The magnetic field at a point x on the axis of a small bar magnet is equal to the field at a point y on the equator of the same magnet. Find the ratio of the distances of x and y from the centre of the magnet.

Solution:

$$B_{axis} = B_{equatorial} = \frac{\mu_0}{4\pi} \frac{2M}{x^3} = \frac{\mu_0}{4\pi} \frac{M}{y^3}$$

$$\Rightarrow \frac{2}{x^3} = \frac{1}{y^3} \Rightarrow \frac{x^3}{y^3} = \frac{2}{1} \Rightarrow \frac{x}{y} = 2^{1/3}$$

Concept Builder-4



- Q.1** The tangent galvanometer coil has 35 turns and radius 11 cm. The pointer shift by 45° when a current of 2 A flows through it. What is the horizontal component of magnetic field at that place ?
 (1) 4×10^{-4} T (2) 2×10^{-4} T (3) 10^{-5} T (4) 9×10^{-5} T
- Q.2** Time period of thin rectangular bar magnet of vibration magnetometer is 'T'. If it is broken into two equal parts then find out time period of each part at the same place.
 (a) Along its length
 (b) Perpendicular to its length

ANSWER KEY FOR CONCEPT BUILDERS

CONCEPT BUILDER-1

1. (4)
2. In Case I, $M_{eq} = M + M = 2M$
 In Case II, $M_{eq} = M - M = 0$
 In Case III, $M_{eq} = \sqrt{M^2 + M^2} = \sqrt{2} M$
3. $2p_0 \cos \frac{\theta}{2}$

CONCEPT BUILDER-2

1. (a) $B = 0.25 \times 10^{-7}$ T
 (b) $B = 10^{-7}$ T
 (c) $B_p = \frac{\sqrt{13}}{2} \times 10^{-7}$ T
2. (a) 0.96×10^{-4} T along S-N direction
 (b) 0.48 G along N-S direction
3. 0.36 JT^{-1}
4. (a) 5×10^4 T (b) -6 J

5. (A) $12\sqrt{3} \times 10^{-6}$ N-m
 (B) The torque on the magnet will always try to bring the magnet into stable equilibrium due to minimum potential energy in this orientation.
 (C) (i) $\Delta U = U_{\text{stable}} - U_i = -12 \times 10^{-6}$ J
 (ii) $\Delta U = U_{\text{unstable}} - U_i = 36 \times 10^{-6}$ J

CONCEPT BUILDER-3

1. (a) 0.45 G (b) 0.52 G
2. $\theta = \tan^{-1} \left(\frac{1}{\sqrt{6}} \right)$
3. $\theta_2 = 51^\circ$ 4. $\approx 41^\circ$

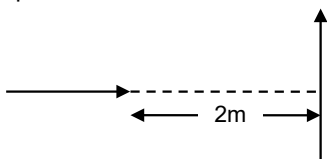
CONCEPT BUILDER-3

1. (1)
2. (a) $T' = T$ (b) $T' = \frac{T}{2}$

Exercise - I

Bar Magnet and Earth's Magnetism

1. Tangent galvanometer measures :
 (1) capacitance
 (2) current
 (3) resistance
 (4) potential difference

2. Two identical magnetic dipoles of magnetic moments 1.0 A-m^2 each, placed at a separation of 2 m with their axes perpendicular to each other. The resultant magnetic field at a point midway between the dipole is:

 (1) $5 \times 10^{-7} \text{ T}$ (2) $\sqrt{5} \times 10^{-7} \text{ T}$
 (3) 10^{-7} T (4) $2 \times 10^{-7} \text{ T}$

3. A bar magnet has a magnetic moment 2.5 JT^{-1} and is placed in a magnetic field of 0.2 T . Work done in turning the magnet from parallel to antiparallel position relative to the field direction.
 (1) 0.5 J (2) 1 J
 (3) 2.0 J (4) Zero


4. Magnetic field lines produced by a bar magnet, cuts each other :
 (1) At neutral points
 (2) Near the poles of the magnets
 (3) At equatorial axis
 (4) Never intersects to each other

5. At any place horizontal and vertical component of earth magnetic field are equal then angle of dip at that place :
 (1) 0° (2) 45°
 (3) 90° (4) 180°

6. Value of earth's magnetic field at any point is $7 \times 10^{-5} \text{ wb/m}^2$. This field is neutralised by field which is produced at the centre of a current carrying loop of radius 5 cm. The current in the loop (approx) :
 (1) 0.56 A (2) 5.6 A
 (3) 0.28 A (4) 28 A

7. At a certain place, the horizontal component B_0 and the vertical component B_v of the earth's magnetic field are equal in magnitude. The total field at that place will be :
 (1) B_0 (2) B_0^2
 (3) $2B_0$ (4) $\sqrt{2} B_0$

8. For a magnetic needle placed in a uniform magnetic field, which of following are correct :-
 (a) $F \neq 0, \tau \neq 0$ (b) $F \neq 0, \tau = 0$
 (c) $F = 0, \tau \neq 0$ (d) $F = 0, \tau = 0$
 (1) a, b (2) a, c
 (3) c, d (4) b, d

9. A bar magnet of length 3 cm has points A and B along its axis at distances of 24 cm and 48 cm from centre on the opposite sides. Ratio of magnetic fields at these points will be :

 (1) 8 (2) $\frac{1}{2\sqrt{2}}$
 (3) 3 (4) 4

10. Magnetic field intensity of a short magnet at a distance 1m. on axial line is 1 oersted. At a distance 2 m. on the same line the intensity in oersted is :-
 (1) 0.75 (2) 0.125
 (3) 0.25 (4) 0.5

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	2	2	4	2	2	4	3	1	2

Exercise - II

1. A magnetic needle of magnetic moment $6.7 \times 10^{-2} \text{ Am}^2$ and moment of inertia $7.5 \times 10^{-6} \text{ kg m}^2$ is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is-
 (1) 8.89 s (2) 6.98 s
 (3) 8.76 s (4) 6.65 s
2. The length of a magnet is large compared to its width and breadth. The time period of its oscillations in a vibration magnetometer is 2 sec. The magnet is cut along its length into three equal parts and these parts are then placed on each other with their like poles together. The time period of this combination will be
 (1) 2 sec (2) $2/3$ sec
 (3) 4 sec (4) $1/3$ sec
3. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T_1 , the ratio T_1/T is :
 (1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{2}$
 (3) 2 (4) $\frac{1}{4}$
4. The magnetic lines of force inside a bar magnet:
 (1) are from north – pole to south – pole of the magnet
 (2) do not exist
 (3) depend upon the area of cross-section of the bar magnet
 (4) are from south-pole to north-pole of the magnet
5. A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The coercivity of the bar magnet is :
 (1) 1200 A/m (2) 2600 A/m
 (3) 520 A/m (4) 285 A/m
6. A magnetic dipole in a constant magnetic field has :
 (1) maximum potential energy when the torque is maximum.
 (2) zero potential energy when the torque is maximum
 (3) zero potential energy when the torque is minimum
 (4) minimum potential energy when the torque is maximum
7. A short bar magnet is placed in the magnetic meridian of the earth with north pole pointing north. Neutral points are found at a distance of 30 cm from the magnet on the East-West line, drawn through the middle point of the magnet. The magnetic moment of the magnet in Am^2 is close to:
 (Given in SI units and Horizontal component of earth's magnetic field $= 3.6 \times 10^{-5} \text{ Tesla}$).
 (1) 9.7 (2) 4.9
 (3) 19.4 (4) 14.6

ANSWER KEY

Que.	1	2	3	4	5	6	7
Ans.	4	2	2	4	2	2	1

Exercise – III (Previous Years Question)

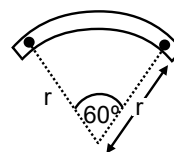
- [AIPMT-2009]**

- [AIPMT-2010]**

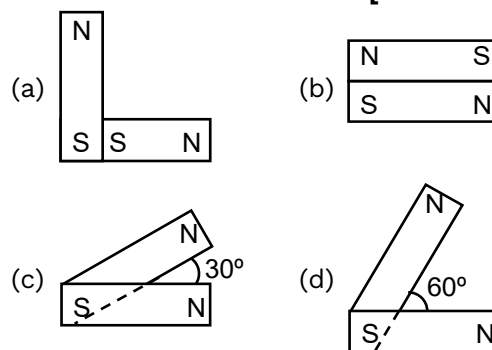
- [AIPMT (Mains) 2011]**

- [AIPMT (Mains) 2012]**

- [NEET- 2013]**



- [NEET - 2014]**



- [NEET- 2016]**

- (1) $\frac{\sqrt{3}W}{2}$ (2) $\frac{2W}{\sqrt{3}}$
(3) $\frac{W}{\sqrt{3}}$ (4) $\sqrt{3}W$

9. Magnetic field of earth is 0.3 gauss. A magnet oscillating with rate of 5 oscillation/min. How much the magnetic field of earth is increased, so the number of oscillations become 10 per min:

[NEET-2017]

- (1) 0.3G (2) 0.6G
(3) 0.9G (4) 0.12G

10. A thin diamagnetic rod is placed vertically between the poles of an electromagnet. When the current in the electromagnet is switched on, then the diamagnetic rod is

pushed up, out of the horizontal magnetic field. Hence the rod gains gravitational potential energy. The work required to do this comes from

[NEET-2018]

- (1) the current source
(2) the magnetic field
(3) the lattice structure of the material of the rod
(4) the induced electric field due to the charging magnetic field

ANSWER KEY										
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	4	2	4	4	1	1	4	3	1