(MATHS) # AREAS RELATED TO CIRCLES

DPP – 04
CLASS – 10th
TOPIC – FORMULA BASED QUESTIONS

Q.1 AB is a chord of a circle with centre O and radius 4 cm. AB is of length 4 cm and divides the circle into two segments. Find the area of the minor segment.

Q.2 A chord PQ of length 12 cm subtends an angle of 120° at the centre of a circle. Find the area of the minor segment cut off by the chord PQ.

Q.3 A chord of circle of radius 14cm makes a right angle at the centre. Find the areas of minor and major segments of circle.

Q.4 A chord 10 cm long is drawn in a circle whose radius is $5\sqrt{2}$ cm. Find the area of both segments.

Q.5 A chord AB of circle, of radius 14cm makes an angle of 60° at the centre. Find the area of minor segment of circle.

Q.6 Find the area of minor segment of a circle of radius 14 cm, when the angle of the corresponding sector is 60°.

Q.7 A chord of a circle of radius 20 cm sub tends an angle of 90° at the centre. Find the area of the corresponding major segment of the circle.

(Use π = 3.14)

Q.8 AB is the diameter of a circle, centre O. C is a point on the circumference such that $\angle COB = \theta$. The area of the minor segment cutoff by AC is equal to twice the area of sector BOC.Prove that

$$\sin\frac{\theta}{2}.\ \text{COS}\frac{\theta}{2} = \pi\left(\frac{1}{2}-\frac{\theta}{120^\circ}\right)$$

Q.9 A chord of a circle subtends an angle θ at the centre of circle. one the area of the minor segment cut off by the chord is one eighth of the area of circle . prove that 8

$$\frac{\theta}{2}.\ \text{COS}\frac{\theta}{2} + \pi = \frac{\pi\theta}{45}$$

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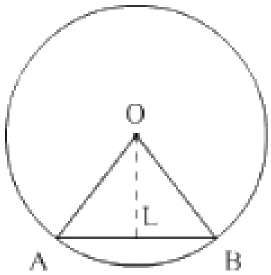
TOPIC – FORMULA BASED QUESTIONS

Sol.1

We know that the area of minor segment of angle θ in a circle of radius r is,

$$A = \left\{ \frac{\pi\theta}{360^\circ} - \sin\frac{\theta}{2}\cos\frac{\theta}{2} \right\} r^2$$

It is given that the chord AB divides the circle in two segment.



We have OA=4 cm and AB=4 cm. so,

$AL = \frac{AB}{2} cm$

$= \frac{4}{2} cm$

= 2 cm

Let $\angle AOB = 2\theta$. Then,

$\angle AOL = \angle BOL$

$= \theta$

In$\triangle OLA$,We have

$\sin\theta = \frac{AL}{OA}$

$= \frac{2}{4}$

$= \frac{1}{2}$

$\theta = \sin^{-1}\frac{1}{2}$

$= 30^\circ$

Hence, $\angle AOB = 60^\circ$

Now using the value of r and θ, we will find the area of minor segment

$$A = \left\{ \frac{\pi \times 60^\circ}{360^\circ} - \sin\frac{60^\circ}{2}\cos\frac{60^\circ}{2} \right\} \times 4 \times 4$$

$$= \left\{ \frac{\pi}{6} - \sin 30^{\circ} \cos 30^{\circ} \right\} \times 16$$

$$= \left\{ \frac{16 \times \pi}{6} - \frac{1}{2} \times \frac{\sqrt{3}}{2} \times 16 \right\}$$

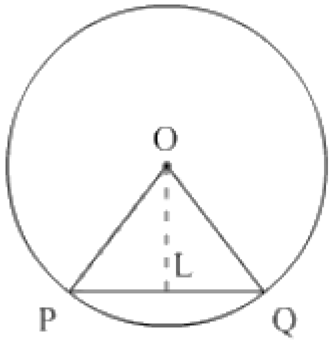
$$= \left\{ \frac{8\pi}{3} - 4\sqrt{3} \right\} cm^2$$

Sol.2

We know that the area of minor segment of angle θ in a circle of radius r is,

$$A = \left\{ \frac{\pi\theta}{360}^{\circ} - \sin\frac{\theta}{2} \cos\frac{\theta}{2} \right\} r^2$$

It is given that the chord PQ divides the circle in two segments.



We have ∠POQ=120° and $PQ = 12cm$. So,

$$PL = \frac{PQ}{2} cm$$

$$= \frac{12}{2} cm$$

$$= 6cm$$

Since $\angle POQ = 120^{\circ}$

$$\angle POL = \angle QOL$$

$$= 60^{\circ}$$

In ΔOPQ, We have

$$\sin\theta = \frac{PL}{OA}$$

$$\sin 60^{\circ} = \frac{6}{OA}$$

$$\frac{\sqrt{3}}{2} = \frac{6}{OA}$$

$$OA = \frac{12}{\sqrt{3}}$$

Thus the radius of circle is $OA = 4\sqrt{3}cm$

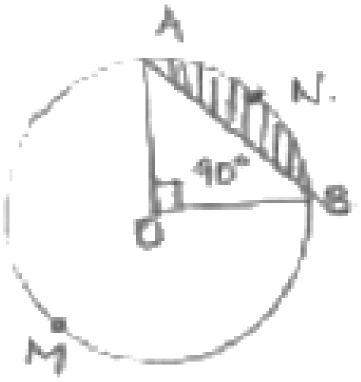
Now using the value of radius r and angle θ we will find the area of minor segment

$$A = \left\{ \frac{120^{\circ}\pi}{360^{\circ}} - \sin\frac{120^{\circ}}{2} \cos\frac{120^{\circ}}{2} \right\} \left(4\sqrt{3}\right)^2$$

$$= \left\{ \frac{\pi}{3} - \frac{\sqrt{3}}{2} \times \frac{1}{2} \right\} \times 48$$

$$= 4\left\{ 4\pi - 3\sqrt{3} \right\} cm^2$$

Sol.3



Radius (r) = 14cm

$\theta = 90°$

= OA = OB

Area of minor segment (ANB)

$= (area\ of\ ANB\ sector) - (area\ of\ \Delta AOB)$

$= \frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} \times OA \times OB$

$= \frac{90}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{2} \times 14 \times 14$

$= 154 - 98 = 56 cm^2$

Area of major segment (other than shaded)

= area of circle – area of segment ANB

$= \pi r^2 - 56$

$= \frac{22}{7} \times 14 \times 14 - 56$

= 616 – 56

= 560 cm^2.

Sol.4

Given radius = r = $5\sqrt{2}$ cm = OA = OB

Length of chord AB = 10cm

In ΔOAB, OA = OB =$5\sqrt{2}\ cm$ AB = 10cm

$$OA^2 + OB^2 = \left(5\sqrt{2}\right)^2 + \left(5\sqrt{2}\right)^2 = 50 + 50 = 100 = (AB)^2$$

Pythagoras theorem is satisfied OAB is right triangle

θ = angle subtended by chord = ∠AOB = 90°

Area of segment (minor) = shaded region

= area of sector - area of △OAB

$$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times OA \times OB$$

$$= \frac{90}{360} \times \frac{22}{7}\left(5\sqrt{2}\right)^2 - \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}$$

$$= \frac{275}{7} - 25 - \frac{100}{7} cm^2$$

Area of major segment = (area of circle) – (area of minor segment)

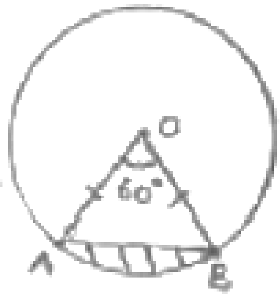
$$= \pi r^2 2 - \frac{100}{7}$$

$$= \frac{22}{7} \times \left(5\sqrt{2}\right)^2 - \frac{100}{7}$$

$$= \frac{1100}{7} - \frac{100}{7}$$

$$= \frac{1000}{7} cm^2$$

Sol.5



Given radius (r) = 14cm = OA = OB

θ = angle at centre = 60°

In ΔAOB, ∠A = ∠B [angles opposite to equal sides OA and OB] = x

By angle sum property ∠A + ∠B + ∠O = 180°

x + x + 60° = 180° ⇒ 2x = 120° ⇒ x = 60°

All angles are 60°, OAB is equilateral OA = OB = AB

Area of segment = area of sector – area Δle OAB

$$= \frac{\theta}{360^\circ} \times \pi r^2 - \frac{\sqrt{3}}{4} \times (-AB)^2$$

$$= \frac{60}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{\sqrt{3}}{4} \times 14 \times 14$$

$$= \frac{308}{3} - 49\sqrt{3}$$

$$= \frac{308 - 147\sqrt{3}}{3} cm^2$$

Sol.6

Area of the minor segment of the circle $= \sqrt{2}\frac{\theta}{360} \times \pi r^2 - r^2 \sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$= \frac{60}{360} \times \pi(14)^2 - (14)^2 \sin\frac{60}{2}\cos\frac{60}{2}$$

$$= (14)^2\left[\frac{1}{6}\pi - \frac{1}{2} \times \frac{\sqrt{3}}{2}\right]$$

$$= \left(\frac{308}{3} - 49\sqrt{3}\right) cm^2$$

Sol.7

We know area of minor segment of the circle is $A=\left\{\frac{\pi\theta}{360}-\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\}r^2$

$\Rightarrow A=\left\{\frac{\pi\times 90^\circ}{360}-\sin\frac{90}{2}\cos\frac{90}{2}\right\}(20)^2$

$\Rightarrow A=\left(\frac{\pi}{4}-\frac{1}{2}\right)(400)$

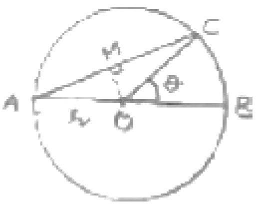
Area of the major segment = Area of the circle − area of the minor segment

$=\pi(20)^2-(400)\left[\frac{\pi}{2}-\frac{1}{2}\right]$

$=(400)\left[\pi-\frac{\pi}{2}+\frac{1}{2}\right]$

$=1142cm^2$

Sol.8



Given AB is diameter of circle with centre O

∠COB = θ

Area of sector BOC = $\frac{\theta}{360^\circ}\times\pi r^2$

Area of segment cut off, by AC = (area of sector) – (area of ΔAOC)

∠AOC = 180 – θ [∠AOC and ∠BOC form linear pair]

Area of sector = $\frac{180-\theta}{360^\circ}\times\pi r^2=\frac{\pi r^2}{2}-\frac{(\pi\theta r)^2}{360^\circ}$

In ΔAOC, drop a perpendicular AM, this bisects ∠AOC and side AC.

Now, In ΔAMO, sin∠AOM = $\frac{AM}{DA}\Rightarrow\sin\left(\frac{180-\theta}{2}\right)=\frac{AM}{R}$

$\Rightarrow$ AM = R sin(90 – $\frac{\theta}{2}$) = $R.\cos\frac{\theta}{2}$

cos ∠ADM = $\frac{OM}{OA}\Rightarrow\cos\left(90-\frac{\theta}{2}\right)=\frac{OM}{Y}\Rightarrow OM=R.\sin\frac{\theta}{2}$

Area of segment = $\frac{\pi r^2}{2}-\frac{\pi\theta r^2}{360^\circ}-\frac{1}{2}(AC\times OM)$ $[AC=2\ AM]$

$=\frac{\pi r^2}{2}-\frac{\pi\theta r^2}{360^\circ}-\frac{1}{2}\times\left(2R\cos\frac{\theta}{2}R\sin\frac{\theta}{2}\right)$

$$= r^2\left[\frac{\pi}{2} - \frac{\pi\theta}{360^\circ} - \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right]$$

Area of segment by AC = 2 (Area of sector BDC)

$$r^2\left[\frac{\pi}{2} - \frac{\pi\theta}{360^\circ} - \cos\frac{\theta}{2}.\ \sin\frac{\theta}{2}\right] = 2r^2\left[\frac{\pi\theta}{360^\circ}\right]$$

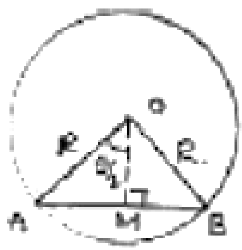
$$\cos\frac{\theta}{2}.\ \sin\frac{\theta}{2} = \frac{\pi}{2} - \frac{\pi\theta}{360} - \frac{2\pi\theta}{360^\circ}$$

$$= \frac{\pi}{2} - \frac{\pi\theta}{360^\circ}[1+2]$$

$$= \frac{\pi}{2} - \frac{\pi\theta}{360^\circ} = \pi\left(\frac{1}{2} - \frac{\theta}{120^\circ}\right)$$

$$\cos\frac{\theta}{2}.\sin\frac{\theta}{2} = \pi\left(\frac{1}{2} - \frac{\theta}{120^\circ}\right)$$

Sol.9



Let radius of circle = r

Area of circle = πr^2

AB is a chord, OA, OB are joined drop OM ⊥ AB. This OM bisects AB as well as ∠AOB.

∠AOM = ∠MOB = $\frac{1}{2}(0) = \frac{\theta}{2}$ AB = 2AM

In ΔAOM, ∠AMO = 90°

$\text{Sin}\frac{\theta}{2} = \frac{AM}{AD} \Rightarrow AM = R.\sin\frac{\theta}{2}$ AB = 2R sin$\frac{\theta}{2}$

$\text{Cos}\frac{\theta}{2} = \frac{OM}{AD} \Rightarrow OM = R\cos\frac{\theta}{2}$

Area of segment cut off by AB = (area of sector) – (area of triangles)

$$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times AB \times OM$$

$$= r^2\left[\frac{\pi\theta}{360^\circ} - \frac{1}{2}.\,2\text{rsin}\frac{\theta}{2}.\,R\ \cos\frac{\theta}{2}\right]$$

$$= R^2\left[\frac{\pi\theta}{360^\circ} - \sin\frac{\theta}{2}.\cos\frac{\theta}{2}\right]$$

Area of segment $=\frac{1}{2}(area\ of\ circle)$

$$r^2\left[\frac{\pi\theta}{360} - \sin\frac{\theta}{2}.\ \ \cos\frac{\theta}{2}\right] = \frac{1}{8}\pi r^2$$

$$\frac{8\pi\theta}{360^\circ} - 8\sin\frac{\theta}{2}.\cos\frac{\theta}{2} = \pi$$

$$8\sin\frac{\theta}{2}.\cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45}$$