2. MOTION IN A STRAIGHT LINE

1. INTRODUCTION

Mechanics is a branch of Physics which deals with motion of bodies and the cause behind it. Motion of a body that includes position and time can be determined with respect to other bodies. Branch of physics that deals with the motion of particles and rigid bodies irrespective of the forces responsible for their motion is known as Kinematics. When the size of a body is too small such that its motion can be described by a point mass moving along straight line, motion is known as rectilinear motion or one-dimensional motion.

1.1 Motion

A body is said to be in motion when it changes its position with respect to the observer while it is said to be at rest when there is no change in its position with respect to the observer. For instance, two passengers travelling in a moving train are at rest with respect to each other but in motion for a ground observer.

1.2 Particle

Physically, a particle is considered as analogues to a point. A body with a definite size is considered as a particle when all of its parts have same displacement, velocity and acceleration. The motion of any such body can be studied by the motion of any point on that body.

1.3 Basic Definitions

1.3.1 Position and Position Vectors

Position of any point can be represented by its coordinates with respect to an origin in Cartesian system. For example, point A can be represented by (x_A, y_A, z_A).

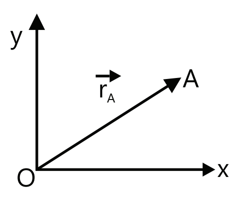


Figure 2.1

$$\vec{r}_A = \overline{OA} = x_A\hat{i} + y_A\hat{j} + z_A\hat{k}$$

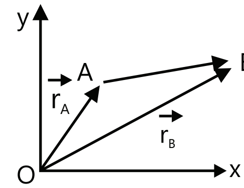


Figure 2.2

Given the co-ordinates of two points A & B, the position vector of B w.r.t. A can be determined as follows:

$$\overrightarrow{AB} = \vec{r}_B - \vec{r}_A \Rightarrow \overrightarrow{AB} = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$$

Illustration 1: Find the torque (τ) exerted by force $\hat{i} + \hat{j} + 2\hat{k}$ at point P (2, 3, 4) w.r.t origin. Given, $\vec{\tau} = \vec{r} x \vec{F}$

$\vec{r}$: is the position vector of point at which force is acting w.r.t to the given point (in this case origin) **(JEE MAIN)**

Sol: The force $\vec{F}$ is given in Cartesian coordinates. Express the position vector of point P in Cartesian coordinates and find the cross product $\vec{\tau} = \vec{r} x \vec{F}$

Here $\vec{r} = \overrightarrow{op} = \vec{P}$ [$\vec{P}$: Position vector of point P]

$\Rightarrow \vec{r} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

and $\vec{F} = \hat{i} + \hat{j} + 2\hat{k}$

$\therefore \vec{\tau} = \vec{r} \times \vec{F}$

$= \hat{i} + \hat{j} + 2\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & 1 & 2 \end{vmatrix} = \hat{i}(2) - \hat{j}(0) + \hat{k}(-1) \qquad = \vec{\tau} = 2\hat{i} - \hat{k}$$

1.3.2. Distance and Displacement

A particle follows either a curve or a straight line when moving in the space. This curve or line is known as its trajectory. Distance is the length of the path or trajectory covered by the particle and displacement is the difference between the vectors of the first and the last position on this path.

Distance and displacement are illustrated in the Fig. 2.3 where AB (curve length) is the distance and vector $\Delta\vec{r}_{AB}$ is the displacement $\left(\vec{r}_B - \vec{r}_A\right)$. Following points should be considered about both the quantities:

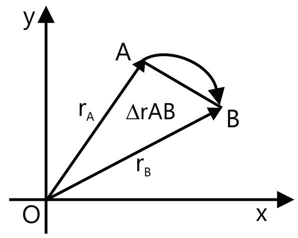


Figure 2.3

(a) Distance is a scalar quantity while displacement is vector.

(b) Displacement is always less than or equal to distance in magnitude.

(c) Displacement can be zero but distance cannot be zero for a moving body.

CONCEPTS

For a particle moving in a straight line:

- the distance travelled is always equal to the displacement when there is no change in direction,

  i.e. distance travelled = |displacement|

- Else, distance travelled is always greater than displacement,

  i.e. distance travelled ≥ |displacement|

Vaibhav Gupta (JEE 2009 AIR 54)

Illustration 2: Find the distance and displacement of a particle travelling from one point to another, say from pt. A to B, in a given path. **(JEE MAIN)**

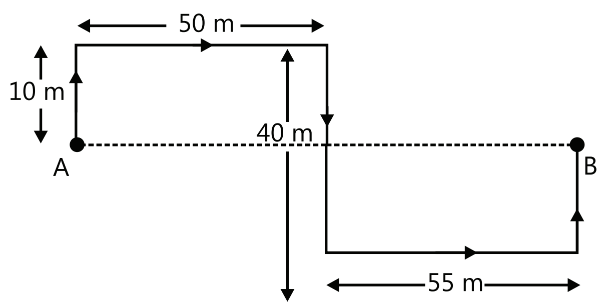


Figure 2.4

Sol: Distance is length of the path travelled. Displacement is the vector from initial point to final point.

Total distance travelled = 10+50+40+55+ (40 - 10) = 185 m

Total Displacement = 50 + 55 =105 m

Deduction of displacement is from A (initial position) to B (final position)

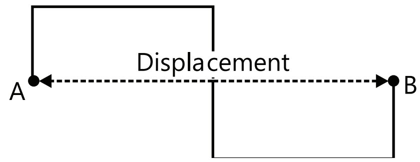


Figure 2.5

Illustration 3: If a particle travels a distance of 5 m in straight line and returns back to the initial point, then find

(i) Total distance travelled (ii) Total displacement **(JEE ADVANCED)**

Sol: Distance is length of the path travelled. Displacement is the vector from initial point to final point.

Total distance travelled = 5 + 5 = 10 m (since, initial and final position of the particle are same).

Displacement = 0 m (5-5=0, since the directions are opposite and cancel with each other)

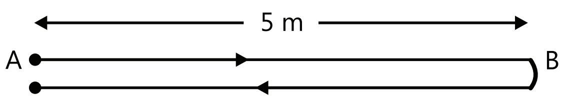


Figure 2.6

1.3.3 Average Speed and Average Velocity

The total distance travelled by a particle divided by the total time taken is known as its average speed.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

While the average velocity is defined as $v_{av} = \frac{\text{displacement}}{\text{time elapsed}}$ or $v = \frac{r_B - r_A}{t_B - t_A} = \frac{\Delta r_{AB}}{\Delta t}$

Both average speed and average velocity are expressed in ms^{-1} or kmh^{-1}, the former is a scalar quantity while the latter is a vector.

CONCEPTS

For a moving body:

- Average velocity can be zero but average speed cannot be zero.
- The magnitude of average velocity is always less than or equal to the average speed because,

  $|\text{displacement}| \le \text{distance}$

  $$\left|\frac{\text{displacement}}{\text{time elapsed}}\right| \le \frac{\text{distance}}{\text{time elapsed}}$$

  therefore,
- Average speed does not mean the magnitude of the average velocity vector.

Vaibhav Krishnan (JEE 2009 AIR 22)

Illustration 4: If a train moves from station A to B with a constant speed v =40 km/h and returns back to the initial point A with a constant speed V_2=30 km/h, then calculate the average speed and average velocity. **(JEE MAIN)**

Sol: Average speed is distance covered divided by time taken. Distance is length of the path travelled. Average velocity is displacement divided by time taken. Displacement is the vector from initial point to final point.

Let the distance AB = s, Time taken by train from A to B, $t_1 = \frac{s}{v_1}$

Time taken by train From B to A, $t_2 = \frac{s}{v_2}$; Average speed $= \frac{\text{Total distance}}{\text{Total time taken}} = \frac{s+s}{t_1+t_2} = \frac{s+s}{\frac{s}{v_1}+\frac{s}{v_2}}$

$V_{avg} = \frac{2v_1v_2}{v_1+v_2} = \frac{2x40x30}{40+30} = \frac{240}{7} = 34.3\,km/h$; Average velocity $= \frac{\text{Net displacement}}{\text{Total time}} = \frac{0}{t_1+t_2} = 0$

Illustration 5: Consider a train moving from station A to B with a constant speed of 40 km/h for half the time and with constant speed of 30 km/h for the next half time of that journey. Calculate the average speed of the whole journey. **(JEE MAIN)**

Sol: Average speed is distance covered divided by time taken. Here we need to assume total time of journey as T. The speed in each half of the time T is constant. The distance covered in each half of the time can be easily written in terms of T. Average speed is distance covered divided by total time taken.

Let AB = s and T = Total time of journey.

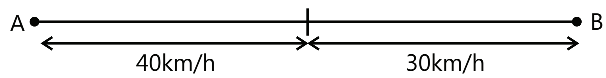


Figure 2.7

$\therefore$ Distance travelled in first half time $\frac{T}{2}$ is, $s_1 = v_1\frac{T}{2}$

Distance travelled in second half time $\frac{T}{2}$ is, $s_2 = v_2\frac{T}{2}$; Average speed $= \frac{\text{Total distance}}{\text{Total time}}$

$V_{avg} = \frac{v_1(T/2)+v_2(T/2)}{T}$; $V_{avg} = \frac{v_1+v_2}{2}$; $V_{avg} = \frac{40+30}{2} = 35\,km/h$

Illustration 6: A particle travels half of the journey with speed 2 m/s. For second half of the journey, the particle travels with a speed of 3 m/s for half of remaining time, and for the other half it travel with a speed of 6 m/s. Find its average speed. **(JEE ADVANCED)**

Sol: Average speed is distance covered divided by total time taken. Here we need to assume total distance of journey. The speed in each part of the journey is constant. The time taken to cover each part of the journey can be calculated in terms of distance covered and speed in that part.

Let total distance = 4 s

Let the time taken in covering last half of journey = 2t

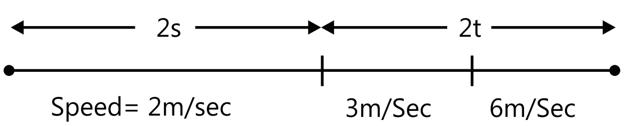


Figure 2.8

For the first half of the journey

speed (v_1) = 2 m/s and distance (d_1) = 2s

So, time taken $(t_1) = \frac{\text{distance}}{\text{speed}} = \frac{2s}{2} = s$

For the first part of second half of the journey

v_2 = 3 m/s ; t_2 = t s; $\therefore d_2 = v_2 x t_2 = 3t$

For the second part of second half of the journey

V_3 = 6 m/s ; T_3 = t s ; $d_3 = v_3 x t_3 = 6t$ m

We know $d_2 + d_3 = \frac{\text{Total distance}}{2} = \frac{4s}{2} = 2s$; $\Rightarrow$ 3t + 6t = 2s; $\Rightarrow t = \frac{2s}{9}$

Therefore, total time taken for journey = $t_1 + t_2 + t_3 = s + t + t = s + \frac{2s}{9} + \frac{2s}{9} = \frac{13s}{9}$

$\therefore V_{avg} = \frac{\text{Total distance}}{\text{Time interval}} = \frac{4s}{\frac{13s}{9}} = \frac{36}{13}$ m/s

Illustration 7: Consider the following:

Men running in a straight line with a speed of 15 m/s one behind the other at equal intervals of 20 m. Cyclists are also riding along the straight line with a speed of 25 m/s at equal intervals of 30 m.

An observer moving in the opposite direction with a speed of v such that comes across men and cyclists at the same time. Find the velocity v. **(JEE ADVANCED)**

Sol: The relative velocity between observer and men is (15 + v). The relative velocity between observer and cyclists is (25 + v). In the reference frame of observer, the time elapsed between the passing of two men is 20/(15 + v) and the time elapsed between the passing of two cyclists is 30/(25 + v). Both these timings should be equal so that the men and cyclists pass the observer together.

Let us assume that a man, a cyclist and the observer are in line. Now after t time, the observer again meets with the next man and cyclist

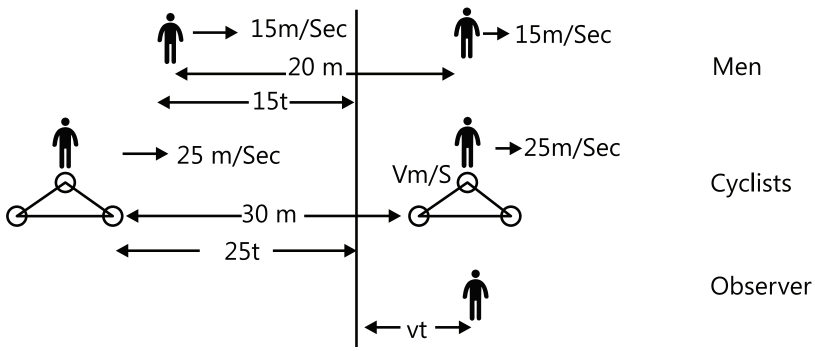


Figure 2.9

Then the distance travelled by the observer = vt

Distance travelled by the man = 15t = 20 –vt ... (i)

Distance travelled by the cyclist = 25t = 30 – vt ... (ii)

Simplifying (i) and (ii) we get

15t + vt = 20 ... (iii)

25t + vt = 30 ... (iv)

Dividing (iii) and (iv) we get $\frac{15t+vt}{25t+vt}=\frac{20}{30}$; $\Rightarrow$ $\frac{15+v}{25+v}=\frac{2}{3}$; $\Rightarrow$ 45 + 3v = 50 + 2v; $\Rightarrow$ v = 5 m/sec

Illustration 8: 2 Cars A and B simultaneously start with speed 20 m/sec and 30 m/sec, respectively. Both cars have constant but different acceleration. On completing the race simultaneously, if the final velocity of A is 90 m/sec, then find the final velocity of B (i.e. v_B) **(JEE ADVANCED)**

Sol: Both cars travel equal distance in equal time. Initial velocities and accelerations of both the cars are different. Problem can be best solved by equating the average velocities.

Since both cars travel equal distance at equal intervals of time, both cars have equal average speeds, i.e. average speed of car A = average speed of car B and for

for constant acceleration, avg velocity $=\frac{u+v}{2}$; $\therefore$ $\frac{20+90}{2}=\frac{30+v_B}{2}$; $\Rightarrow$ V_B = 80 m/s

1.3.4. INSTANTANEOUS SPEED AND INSTANTANEOUS VELOCITY

The average speed/velocity of a moving body with

infinitesimally small time interval (i.e. $\Delta t \rightarrow 0$) is known as instantaneous speed/velocity. Therefore

Instantaneous velocity $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$.

(a) As the time interval tends to zero (i.e. $\Delta t \to 0$), the displacement vector $\Delta \vec{r}$ is along the direction of motion of the particle i.e. tangential to the path of the particle at that instant. Thus, the instantaneous velocity direction is always tangential to the path of the the particle.

(b) Instantaneous speed and the magnitude of instantaneous velocity are always the same.

i.e. Instantaneous Speed= | Instantaneous Velocity |

A particle moving on a straight line, say along the x-axis, has an instantaneous velocity as follows

$$v(t) - \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} - \lim_{\Delta t \to 0} v_{av}$$

A particle is said to move in a uniform velocity when the velocity of a particle remains constant with respect to time. It is said to be accelerated when velocity changes with respect to time.

Illustration 9: The distance travelled by a particle in time t is given by $s(t) = (2.5\ m/s^2)t^2$.

Find (a) the average speed of the particle during time 0 to 5 s?

(b) The instantaneous speed at t = 5.0 sec **(JEE MAIN)**

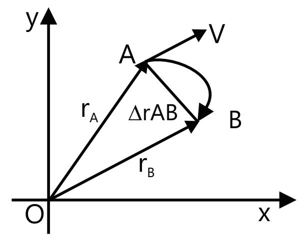


Figure 2.10

Sol: Average speed is distance covered divided by total time taken. Instantaneous speed is the rate of change of distance at a particular instant.

(a) The distance travelled during time 0 to 5.0 sec

= s (5 sec) = 2.5 x $(5)^2$ = 62.5 m

$\therefore$ Average speed $= \frac{\text{distance travelled}}{\text{time interval}}$; $\Rightarrow V_{av} = \frac{62.5m}{5.0sec - 0.0sec} = 12.5 m/s$

(b) $s(t) = 2.5t^2$; $\therefore \frac{ds(t)}{dt} = (2.5)x(2t) = 5t$; $\therefore$ instantaneous speed $(v) = \frac{ds}{dt}\Big|_{t=5.0} = 5(5.0) = 25$ m/s

1.3.5. Average acceleration and Instantaneous Acceleration

Rate of change of velocity is defined as acceleration. Velocity changes with change in magnitude or direction or both.

Suppose the velocity of a particle at time t_1 is $\overrightarrow{v_1}$ and at time t_2 it is $\overrightarrow{v_2}$. The change produced in time interval t_1 to t_2 is $\vec{v}_2 - \vec{v}_1$. Average acceleration $\overrightarrow{a_{av}}$ is defined as change in velocity with respect to change in time, i.e.

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_1 - t_2}$$

The average acceleration with infinitesimally small time interval Δt is known as instantaneous acceleration, i.e.

$$\vec{a} = \lim \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

(a) Acceleration is a vector quantity and its SI unit is ms^{-2}.

(b) The average acceleration vector and the change in velocity vector are in the same direction.

(c) The direction of velocity vector and the direction of acceleration vector are independent of each other.

(d) Acceleration is perpendicular to the velocity vector only when there is change in direction of velocity with time, with its magnitude being constant.

If a body moves with uniform acceleration along a straight line, then the average acceleration and instantaneous acceleration will always be the same.

CONCEPTS

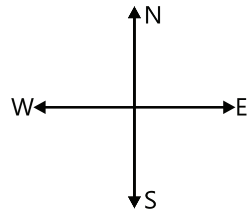
Retardation:

- Negative acceleration does not imply retardation.
- Retardation refers to decrease in speed and not velocity.

Nivvedan JEE 2009 AIR 113

Illustration 10: Consider a particle moving with a speed of 5 m/s towards east. After 10 sec velocity of particle is 5 m/s towards north. Find the average acceleration and its direction.

(JEE MAIN)



Sol: Average acceleration is change in velocity divided by total time taken.

$\vec{v}_i = 5\hat{i}$; $\vec{v}_f = 5\hat{j}$ $\therefore$ $\overline{v_f} - \overline{f_i} = 5\hat{j} - 5\hat{i}$

Times interval = 10 sec

We know that , Average acceleration $= \dfrac{\vec{v}_f - \vec{v}_i}{\text{Time interval}}$

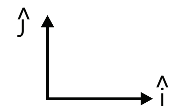
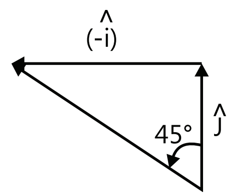


Figure 2.11

$\Rightarrow$ $acc^n_{avg} = \dfrac{5\hat{j} - 5i}{10} = \dfrac{1}{2}(\hat{j} - \hat{i})m/sec^2$

$\therefore$ $|acc^n_{avg}| = \left|\dfrac{1}{2}(\hat{j} - \hat{i})\right| = \dfrac{1}{\sqrt{2}}$ m / sec^2



Unit vector along that direction is $= \dfrac{1}{\sqrt{2}}(\hat{j} - \hat{i})$ [45° due west of north]

Figure 2.12

CONCEPTS

Motion of bodies in three dimensional space:

If a body has coordinates (x, y, z) in space, its position vector r at time is given by $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

The velocity vector v is given by $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

where v_x ,v_y v_z are magnitudes of components of the velocity along the x-, y-, and z-axes, respectively, at time t. $v_x = \dfrac{dx}{dt}, v_y = \dfrac{dy}{dt}, v_z = \dfrac{dz}{dt}$

The acceleration a is given by

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a = \hat{k}$

where a_x,a_y and a_z are components of acceleration along x, y, and z directions respectively at time t.

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2. UNIFORMLY ACCELERATED MOTION FOR 1-D MOTION

2.1 Equations of Motion

Consider that the acceleration of a particle 'a' is constant.

First Equation: Acceleration is defined as the rate of change of velocity

$$a=\frac{dv}{dt}\Rightarrow dv=a.dt \qquad \text{... (i)}$$

The velocity at time 0 is u and at time t is v. Thus, at t = 0, v= u and

At t = t, v = v.

Integrating equation (i) for these limits: V = v to u = v and t = 0 to t = t

$$\int_u^v dv=\int_o^t adt\,;\ \Rightarrow\left[v\right]_u^v=a\left[t\right]_0^t\Rightarrow v-u=a\left(t-0\right)\ ;\ \Rightarrow v-u=at\,;\ \Rightarrow \boxed{v-u=at}$$

Second Equation: Velocity is defined as the rate of change of displacement.

$$v=\frac{ds}{dt}\Rightarrow ds=vdt\,;\ \Rightarrow ds=(u+at)dt \qquad [\therefore v=u+at] \qquad \text{... (ii)}$$

Suppose the position of the particle is '0' at time '0' and 's' at time 't'. Hence,

at t = 0, s = 0 and

at t = t, s = s

Integrating equation (ii) for these limits: $\int_o^s ds=\int_o^t (u+at)$

$$\Rightarrow\left[s\right]_0^s=\int_0^t udt+\int_0^t at.dt \quad \Rightarrow \left[s-0\right]=u\left[t\right]_0^t+\int_0^t tdt \qquad \Rightarrow s=ut+\frac{1}{2}at^2$$

Third Equation: From the definition of acceleration

$$a=\frac{dv}{dt}\Rightarrow a\frac{dv}{dx}.\frac{dx}{dt}; \quad \Rightarrow a=\frac{dv}{dt}.v\ ; \quad \Rightarrow v.dv=a.dx \qquad \text{... (iii)}$$

If the velocity at position 0 is u and at position s is v,

at s=0, v=u and

at s=s, v=v

Integrating equation (iii) for these limits: V=u to v = v and x = 0 to x = s

$$\int_u^v vdv=\int_o^s adx\,;\ \Rightarrow\left[\frac{v^2}{2}\right]=a[x]_o^s \quad \Rightarrow \frac{v^2}{2}-\frac{u^2}{2}=a[s-0]$$

$$\frac{v^2-u^2}{2}=as\,;\ \Rightarrow \boxed{v^2-u^2=2as}$$

Displacement In n[th] Second

Displacements S_n of a particle in n seconds and S_{n-1} in (n-1) seconds are given as: $s_n=un+\frac{1}{2}an^2$; $s_n=u(n-1)+\frac{1}{2}a(n-1)^2$

Displacement in n[th] second: $s_n-s_{n-1}=un+\frac{1}{2}an^2-u(n-1)-\frac{1}{2}a(n-1)^2$; $x_n=u+\frac{a}{2}(2n-1)$

Illustration 11: Consider a particle moving in straight line with constant acceleration "a" traveling 50 m in 5th second and 100 m in 10th second. Find

(1) Initial velocity (u)

(2) Acceleration (a)

(3) Displacement till 7 s

(4) Velocity after 7 s

(5) Displacement between t = 6 s and t = 8 s **(JEE MAIN)**

Sol: We know the formula for displacement in nth second. For 5th second and 10th second we get two equations and two variables u and a. So we solve the equations to get the values of u and a.

We know that displacement in n^{th} second (x_{nth})

$$x_{nth} = u + \frac{a}{2}(2n-1) \quad \text{... (i)}$$

Given $x_{5th} = 50\,;\ \Rightarrow u + \frac{a}{2}[2x5-1] = 50$

$$\Rightarrow u + \frac{9}{2}a\ = 50 \quad \text{... (ii)}$$

and X_{10th} = 100 ; $\Rightarrow u\frac{a}{2}[2x10-1] = 100$

$$\Rightarrow u + \frac{19}{2}\ a\ = 100 \quad \text{... (iii)}$$

Subtracting (ii) from (i), we get $\left(\frac{19}{2} - \frac{9}{2}\right)a = 100 - 50\ \ \Rightarrow a = 10\,m/sec^2$

Substituting the value of a in (i) we get, $u + \frac{9}{2}(10) = 50\ \Rightarrow\ u = 50 - 45 = 5m/s$

We know $x = ut + \frac{1}{2}at^2$

At t= 7 sec $x = (5)(7) + \frac{1}{2}(10)(7)^2\ = 35 + 245 = 280$ m

We know v = u + at

∴ At t = 7sec $\Rightarrow v = (5) + (10)(7)$ = 75 m/sec

$$X_{6\,sec\,to\,8\,sec} = x_8 - x_6\ = \left(ut + \frac{1}{2}at^2\right)_{at\quad t=8\,sec} - \left(ut + \frac{1}{2}at^2\right)_{at\quad t=6\,sec} = u(8-6) + \frac{1}{2}a\left[8^2 - 6^2\right]\ = 5(2) + \frac{1}{2}x10(28) = 150m$$

Illustration 12: Consider a particle moving in a straight line with constant acceleration, has a velocity (v_p) = 7 m/s and V_Q = 17 m/s, when it crosses the point P and Q respectively. Find the speed of the particle at mid-point of PQ. **(JEE ADVANCED)**

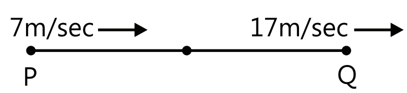


Figure 2.13

Sol: Initial and final velocity are known for constant acceleration and a particular displacement. The final velocity for half the displacement is to be calculated. This problem can be easily solved by using the third equation of motion with constant acceleration.

Let the mid-point be R

Then PR=RQ = s (say)

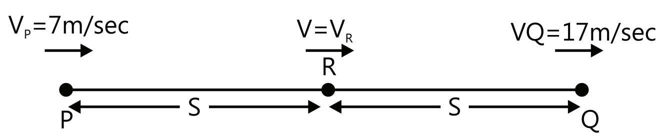


Figure 2.14

From PR From RQ

$u = v_p = 7$ m/s $u = V_R$

$V = V_R$ V=17 m/s

x=s x=s

Using formula $v^2 = u^2 + 2ax$

We get: $V_R^2 = 7^2 + 2as$ ... (i)

And $17^2 = V_R^2 + 2as$ ... (ii)

Subtracting (i) from (ii) we get $17^2 - V_R^2 = V_R^2 - 7^2 \Rightarrow 2V_R^2 = 17^2 + 7^2$

$\Rightarrow 2V_R^2 = 338 \therefore V_R = 13m/sec$

Illustration 13: A body moving with uniform acceleration covers 24 m in the 4th second and 36 m in the 6th second. Calculate the acceleration and initial velocity. **(JEE MAIN)**

Sol: We know the formula for displacement in nth second. For 4th second and 6th second we get two equations and two variables u and a. So we solve the equations to get the values of u and a.

$$S_n = u + \frac{a}{2}(2n-1)$$

$$\therefore \quad 24 = u + \frac{a}{2}(2x4-1) \quad \text{... (i)}$$

$$36 = u + \frac{a}{2}(6x2-1) \quad \text{... (ii)}$$

From equation (i) and (ii) we get $12 = 2a \Rightarrow a = \frac{12}{2} = 6m/s^2$

From equation (i) $24 = u + \frac{6}{2}(2x4-1) \Rightarrow u = 3\ ms^{-1}$

Observation: Motion is independent of the mass of the body and hence no equation of motion considers mass.

CONCEPTS

- For uniformly accelerated motion i.e. constant acceleration:

  Average velocity $\{V_{avg}\} = (v + u)/2$

  Proof: V_{avg} =Displacement (s)/time interval (t) = s/t

  = (ut + ½ at^2) / t

  = u + ½ at

  = (2u + at)/2

  = [u + (u+at)]/2

  = (u + v)/2

- If initial vector of a particle is $\vec{r}_o$, then position vector at time t can be written as

  $$\vec{r} = \vec{r}_o + \vec{s} = \vec{r}_o + \left(\vec{u}t + \frac{1}{2}\vec{a}t^2\right)$$

- Difference between distance (d) and displacement (s)

  From equations of motion

  $$s = ut + \frac{1}{2}at^2 \text{ and } v^2 = u^2 + 2as$$

s is the displacement and not the distance of the particle. The values are different when u and a are of opposite sign or $u\uparrow\downarrow a$.

Case 1: When velocity u is either zero or parallel to a, then motion is simply accelerated and in this case distance is equal to displacement. So, we can write, $d = s = ut + \frac{1}{2}at^2$.

Case 2: When u is not parallel to a, the motion is first retarded and then accelerated in opposite direction. Hence distance is either greater than or equal to displacement ($d \geq |s|$).

Nitin Chandrol (JEE 2012 AIR 134)

Illustration 14: Consider an object moving with an initial velocity of 10 m/s and acceleration of 2 m/s^2. Find distance travelled from t = 0 to 6 s. **(JEE MAIN)**

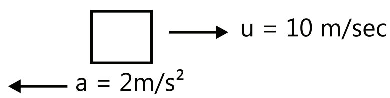


Figure 2.15

Sol: Distance covered is equal to displacement if the object moves in a straight line and there is no change in direction of motion. If direction of motion changes, distance should be calculated separately for different parts of the path.

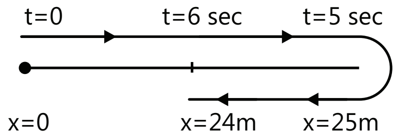


Figure 2.16

$\because V = u + at$; $\Rightarrow 0 = 10 + (-2)t$; $\Rightarrow t = 5\,sec$; $x_5 = ut + \frac{1}{2}at^2$

$= (10)(5) + \frac{1}{2}(-2)(5)^2$; $= 25m$ and $x_6 = (10)(6) + \frac{1}{2}(-2)(6)^2 = 24m$

$\therefore$ Total distance travelled = 25 + (25-24) = 26 m.

Illustration 15: Consider a body moving with velocity 9 m/s. It is subjected to acceleration of -2 m/s^2. Calculate the distance travelled by the body in fifth second. **(JEE ADVANCED)**

Sol: Distance covered is equal to displacement if the object moves in a straight line and there is no change in direction of motion. If direction of motion changes, distance should be calculated separately for different parts of the path.

Advice: Distance travelled in 5th sec need to be calculated and not the displacement.

Hence displacement formula cannot be used directly to calculate the distance in nth second.

According to equations of motion,

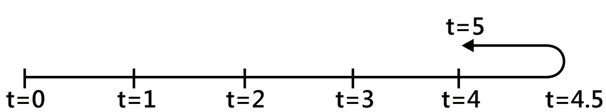


Figure 2.17

$S_n = u + \frac{a}{2}(2t-1) = 9 - \frac{2}{2}(2x5-1) = 9 - 9 = 0$

The value obtained is for displacement and not distance.

Hence distance S in 5th sec. can be calculated as S = 2

$(S_{4.5} - S_4) = 2\left[\left\{9x4.5 - \frac{1}{2}x2x(4.5)^2\right\} - \left\{9x4 - \frac{1}{2}x2(4)^2\right\}\right]$

= 2 [4.5-4.25] = 2 x 0.25 = 0.5 m

3. MOTION OF BODY UNDER GRAVITY (FREE FALL)

The force of attraction that the earth exerts on all the bodies is called force of gravity and the acceleration induced by gravity is called acceleration due to gravity, represented by g. All bodies irrespective of their size, weight or composition fall with the same acceleration near the surface of earth in the absence of air. Motion of a body falling towards the earth from a small altitude (h<<R) is known as free fall (R: Radius of Earth).

3.1 Body Projected Vertically Upwards

(a) Equation of motion: Considering point of projection as origin and direction of motion (i.e. vertically up) as positive

$a = -g$ [as acceleration is downwards]

If a body is projected with velocity u and after time *t*

it reaches to a height *h* then

$v = u - gt$ ... (i)

$h = ut - ½ gt^2$ ... (ii)

$v^2 = u^2 - 2gh$ ... (iii)

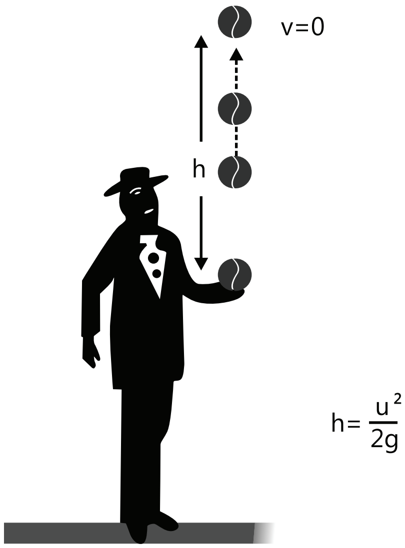


Figure 2.18

(b) For maximum height (H): v =0

Using equation (ii) we get $0^2 = u^2 - 2gH \Rightarrow H = \frac{u^2}{2g}$ (c) Time taken to reach maximum height (t): v=0

Using equation (ii) we get: 0 = u-gt; T = u/g.

(c) Time of flight (T) is the time during which the object travels. In this case, it is the time between the the maximum height and the ground.

Thus, h = 0. Using equation (iii): $0 = uT - \frac{1}{2}gt^2$; $0 = T\,(u - \frac{1}{2}gt)$

$\Rightarrow$ either T=0 or T=2u/g = 2 x. Time taken to reach maximum height (t)

(d) The following graphs show the displacement, velocity and acceleration with respect to time (for maximum height) when body is thrown upwards:

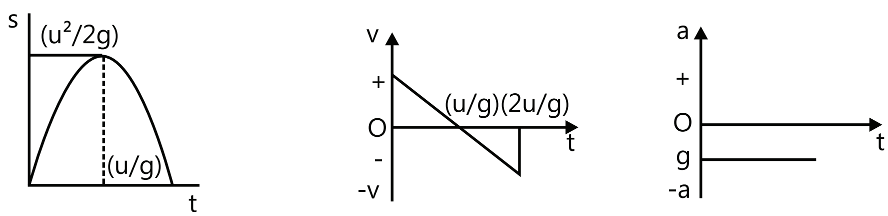


Figure 2.19

Observation:

(a) Time taken by the body to travel up is equal to the time taken by the body to fall down. Time of descent (t_2) = time of ascent (t_1) = u/g

(b) The speed with which a body is projected up is equal to the speed with which it comes down. The magnitude of velocity at any point is same whether the body is moving up or down.

Illustration 16: Consider a ball being thrown upwards with an initial speed of u. Find out u if the ball is at a height of 80 m and the interval between two times is 6 s. ($g=10\ m/s^2$) **(JEE MAIN)**

Sol: Body thrown vertically upwards reaches the maximum height, stops momentarily and then starts falling vertically downwards. So for any point at height less than the maximum height, the body will reach the point twice during its travel, first time while ascending and the second time while descending.

u = u m/s, a=g= $-10\ m/s^2$ and s=80 m

Substituting the value, $s = ut + \frac{1}{2}at^2$, we have $80 = ut - st^2$ or $st^2 - ut + 80 = 0$

Or, $t = \frac{u + \sqrt{u^2 - 1600}}{10}$ and $\frac{u - \sqrt{u^2 - 1600}}{10}$

Given that $\frac{u + \sqrt{u^2 - 1600}}{10} - \frac{u - \sqrt{u^2 - 1600}}{10} = 6$

$\frac{\sqrt{u^2 - 1600}}{5} = 6$ or $\sqrt{u^2 - 1600} = 30$

Or $u^2 - 1600 = 900$; $\therefore$ $u^2 = 2500$; Or $u = \pm 50$ m/s

Ignoring the negative sign, u =50 m/s.

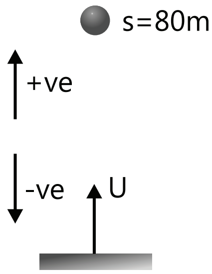


Figure 2.20

3.2 Body Dropped From Some Height (Initial Velocity Zero)

(a) Equations of motion: Let the initial position be the origin and direction of motion (i.e. downward direction) to be positive, then

u = 0 [As body starts from rest]

a = g [As acceleration is in the direction of motion]

Therefore, equations of motion are:

$v = u + gt$ ...(i)

$v = ut + \frac{1}{2} gt^2$ ...(ii)

$v^2 = u^2 + 2gh$ ...(iii)

(b) Velocity (v) of the particle just before hitting the ground: h = H

Therefore, using equation (iii): $V^2 = 0 + 2gH$; $v = \sqrt{2gH}$ (c) Time (t) taken by the object to reach the ground: h = H

Therefore, using equation (ii): $H = (0)T + \frac{1}{2}gT^2$; $T = \sqrt{\frac{2H}{g}}$

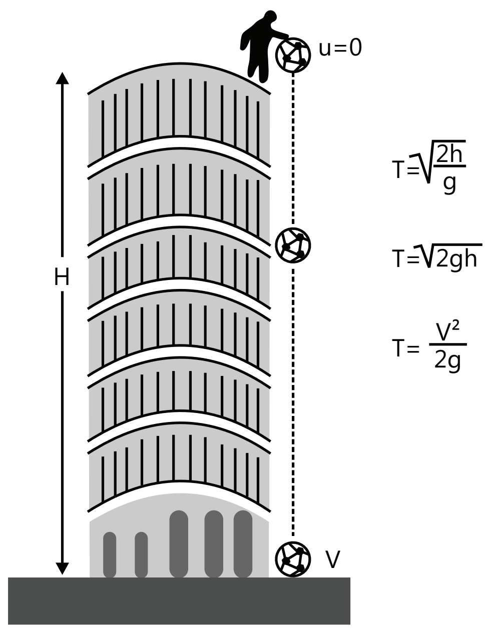


Figure 2.21

(c) The following graphs show the distance, velocity and acceleration with respect to time (for free fall):

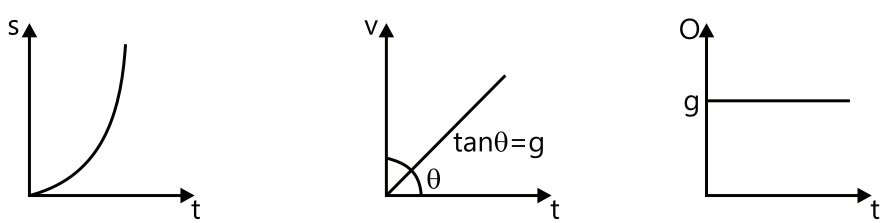


Figure 2.22

(d) The distance covered in the nth sec, $h_n = \frac{1}{2}g(2n - 1)$

Hence the ratio of the distance covered in 1st, 2nd, 3rd sec, etc. is 1:3:5 i.e. only odd integers.

These results obtained are the corollary of the Galileo's Theorem:

For a uniform accelerating body, the distance travelled is always odd ratio, i.e. 1:3:5:7, for regular time interval.

(e) As $h = (1/2)\, gt^2$, i.e. $h \propto t^2$, distance covered in time t, 2t, 3t, etc., will be in the ratio of $1^2:2^2:3^2$, i.e. square of integers.

3.3 Body Thrown Vertically from a Height

There are two possibilities when an object is thrown.

First, when an object is thrown in upward direction, velocity is upwards whereas acceleration acts downwards, i.e. they are in opposite directions. Hence initially the object undergoes retardation and rises through a certain height and then it undergoes free fall from that height.

Second, when an object is thrown from a certain height, both the velocity and acceleration are in the downward direction, i.e. velocity and acceleration are in the same direction. In this case, the object undergoes acceleration. It hits the ground with a speed greater than the speed if it had gone through free fall.

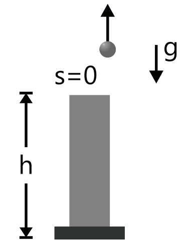


Figure 2.23

Equations of motion are used in both the cases.

Assume a sign convention for the direction.

Then, note the values of displacement, velocity and acceleration with appropriate signs.

Finally, use the appropriate equations of motion.

Illustration 17: A ball is thrown upwards from 40 m high tower with a velocity of 10 m/s. Calculate the time when it strikes the ground. ($g = 10\ m/s^2$) **(JEE MAIN)**

Sol: In the second equation of motion with constant acceleration, value of all the quantities need to be substituted with proper sign. If the displacement and acceleration are in the opposite direction of initial velocity (taken as positive) then substitute there values with negative sign.

$u = +10\ m/s$, $a = -10\ m/s^2$

$s = -40$ m (at the point where the ball strikes the ground)

Substituting in $S = ut + \frac{1}{2}at^2$, we have $-40 = 10t - 5t^2$

or $5t^2 - 10t - 40 = 0$ or $t^2 - 2t - 8 = 0$

Solving this, we get t = 4 s and -2 s. Considering the positive value, t=4 s.

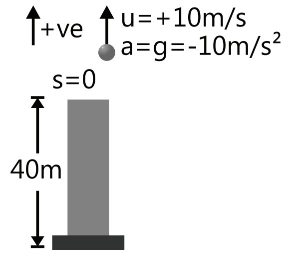


Figure 2.24

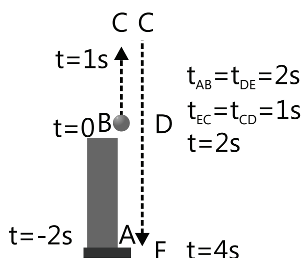


Figure 2.25

Note: The significance of t=-2 s can be understood by Fig. 2.25:

CONCEPTS

We have studied the formula of maximum height (H) and time taken (T) to reach the point where the velocity of an object becomes zero under gravity.

The retardation of the object is 'g'. $H = u^2/2g$ and $T = u/g$.

This can be used for an object having an initial velocity 'u' and retardation 'a'.

Thus, distance at which velocity of the particle becomes zero H= $u^2/2a$;

Distance after which the particle changes its direction;

Time taken to reach this distance = u/a

Time taken to reach its initial position = 2u/a

Anand K (JEE 2011 AIR 47)

4. NON-UNIFORMLY ACCELERATED MOTION

Equations of motion cannot be considered for particles travelling with constant or uniform acceleration. While deriving equations of motion, we considered acceleration to be constant. So, for solving non-uniformly accelerated motion, we will follow the two basic equations: (i) $\vec{v}=\frac{d\vec{s}}{dt}$ or sometimes $\vec{v}=\frac{d\vec{r}}{dt}$ (ii) $\vec{a}=\frac{d\vec{v}}{dt}$

Vector quantity is not required for one-dimensional motion. Therefore the above equations can be re-written as: $(i)\ v=\frac{ds}{dt}\quad (ii)\ a=\frac{dv}{dt}=v\frac{dv}{ds}$

CONCEPTS

$a=v\frac{dv}{ds}$ is useful when acceleration displacement is known and velocity displacement is required.

Yashwanth Sandupatla (JEE 2012, AIR 821)

Illustration 18: For a particle moving along x-axis, displacement time equation is $x = 20 + t^3 - 12t$.

(a) Find the position and velocity of the particle at time t = 0

(b) Find out whether the motion is uniformly accelerated or not.

(c) Find out the position of particle when velocity is zero. **(JEE MAIN)**

Sol: Displacement is given as a function of time. Differentiating the equation of displacement w.r.t time we get the velocity as a function of time. Differentiating the equation of velocity w.r.t time we get the acceleration as a function of time.

(a) $x = 20 + t^3 - 12t$ ... (i)

At t = 0, x = 20 + 0 – 0 = 20 m

By differentiating Equation (i) w.r.t. time i.e. $v=\frac{dx}{dt}=3t^2-12$ ... (ii)

Velocity of particle can be obtained at time t.

At t = 0, v = 0 -12 =- 12 m/s

(b) Differentiating equation (ii) w.r.t. time, we get the acceleration $a=\frac{dv}{dt}=6t$

As acceleration is a function of time, the motion is non-uniformly accelerated.

(c) Substituting v=0 in equation (ii) $0=3t^2-12$

From the above equation, t = 2 sec. Substituting it in equation (i) we have $x=20 + (2)^3 - 12(2)$ or x=4 m

Illustration 19: Acceleration of an object moving in straight line is $a=v^2$ and initial velocity of that object is u m/sec Find. (i) v(x) i.e. velocity as a function of displacement (ii) v(t) i.e. velocity as a function of time **(JEE ADVANCED)**

Sol: Acceleration is the differentiation of velocity with respect to time. We can use the following transformation:

$$a=\frac{dv}{dt}=\frac{dv}{dx}\frac{dx}{dt}=v\frac{dv}{dx}$$

(i) $a=v\frac{dv}{dx}=v^2;\ \Rightarrow\frac{dv}{v}=dx$ Integration $\Rightarrow\int_u^v\frac{dv}{v}=\int_o^x dx\ \Rightarrow \ell nv\Big|_u^v=x\Big|_o^x;\ \ell nv-\ell nu=x;\ \Rightarrow\ell n\frac{v}{u}=x;\ \therefore\ v=u\,e^x$

(ii) $a=\frac{dv}{dt}=v^2;\ \frac{dv}{v^2}=dt$ Integration $\Rightarrow\int_u^v\frac{dv}{v^2}=\int_o^t dt;\ \Rightarrow-\frac{1}{v}\Big|_u^v=t\Big|_o^t\Rightarrow-\left[\frac{1}{v}-\frac{1}{u}\right]=t;\ \Rightarrow\frac{1}{v}=-t+\frac{1}{u}=\frac{1-ut}{u};\ \therefore\ v=\frac{u}{1-ut}$

5. ANALYSIS OF MOTION THROUGH GRAPHS

Let us see some basics points of graphs before analyzing the motion of an object through graphs.

Basic Graphs: (a) A linear relationship between x and y represents a straight line.

E.g., y=4x-2, y=5x + 3, 3x = y-2

(b) A proportionality relationship between x and y (i.e. $x\propto y$ or $y=kx$) represents a straight line passing through origin.

(c) Inverse proportionality $\left(x\propto\frac{1}{y}\right)$ or xy=k represents a rectangular hyperbola.

Shape of a rectangular hyperbola is given in the graph:

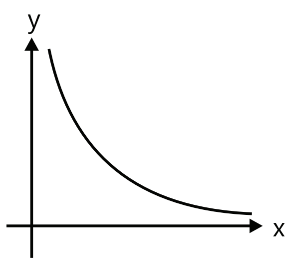


Figure 2.26

(d) A quadratic equation in x and y represents a parabola in x-y graph. E.g., $y=3x^2+2$, $y^2=4x$, $x^2=y-2$

Analysis of Graphs: (a) If $z=\frac{dy}{dx}$, the value of z at any point on x-y graph can be obtained by the slope of the graph at that point.

(b) If z = y(dx) or x(dy), the value of z between x_1 and x_2 or y_1 and y_2 is obtained by the area of graph between x_1 and x_2 or y_1 and y_2.

5.1 Displacement–Time Graph

With displacement of a body plotted on y-axis against time on x-axis, displacement–time curve is obtained.

(a) The slope of the tangent at any point of time gives the instantaneous velocity at any given instant.

(b) At a uniform motion the displacement–time graph is a straight line.

(i) If the graph obtained is parallel to time axis, the velocity is zero.

(ii) If the graph is an oblique line, the velocity is constant (OC and EF in Fig. 2.28).

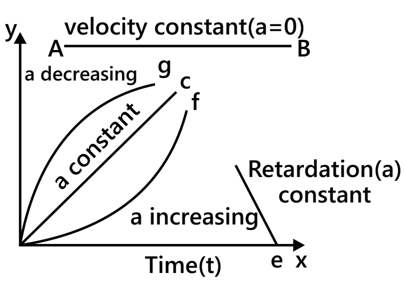


Figure 2.27

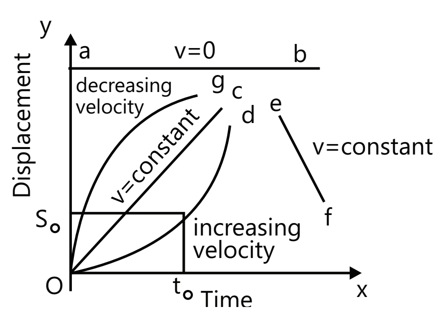


Figure 2.28

Illustration 20: Displacement–time graph of a particle moving in a straight line is shown in the Fig. 2.30. State whether the motion is accelerated or not. Describe the motion of the particle in detail. Given s_0=20 m and t_0=4 s. **(JEE MAIN)**

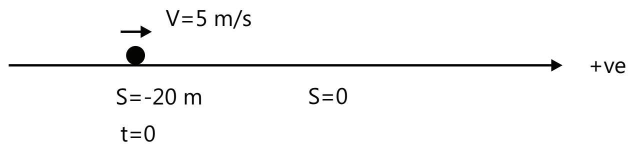


Figure 2.29

Sol: The velocity of the particle at any instant is the slope of the displacement time graph at that instant. If the slope is constant, velocity is constant.

Slope s–t is a straight line. Hence, velocity of particle is constant. At time t = 0, displacement of the particle from its mean position is $-s_0$ i.e. -20 m. Velocity of particle,

$$V = \text{slope} = \frac{s_0}{t_0} = \frac{20}{4} = 5\text{m/s}$$

At t = 0 particle is at –20 m and has a constant velocity of 5 m/s. At t_0 = 4 sec, particle will pass through its mean position.

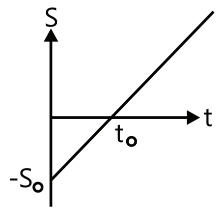


Figure 2.30

5.2 Velocity–Time Graph

Displacement, velocity and acceleration, specifying the entire motion, can be determined by the velocity–time curve.

(a) Instantaneous acceleration can be obtained by the slope of the tangent at any point corresponding to a particular time on the curve.

(b) Displacement during a time interval is obtained from the area enclosed by velocity–time graph and time axis for a time interval.

(c) For a uniformly accelerated motion, velocity–time graph is a straight line.

(d) For constant velocity (i.e. acceleration is zero), the graph obtained is a straight line AB parallel to x-axis (time).

(e) For constant acceleration, the graph obtained is oblique.

5.3 Acceleration–Time Graph

(a) Change in velocity for a given time interval is the area enclosed between acceleration–time graph and time axis.

(b) For constant acceleration, the obtained graph is a straight line parallel to x-axis, i.e. time (t).

(c) If the acceleration is non-uniform, then the graph is oblique.

Inference: Displacement–time graph for uniformly accelerated or retarded motion is a parabola. Since, for constant acceleration, then relation between displacement and time is: $s = ut \pm \frac{1}{2}at^2$ which is quadratic in nature. Thus, displacement–time graph will be parabolic in nature.

Illustration 21: Acceleration–time graph of a particle moving in a straight line is shown in Fig. 2.31. At time t=0, velocity of the particle is 2 m/s. Find velocity at the end of the 4th second. **(JEE MAIN)**

Sol: The area enclosed by the acceleration-time graph between t = 0 and t = 4s will give the change in velocity in this time interval.

dv = a dt

or change in velocity = area under a-t graph

Hence $v_f - v_i = \frac{1}{2}(4)(4) = 8\text{m/s}$; $\therefore\ v_f = v_i + 8 = (2+8)\text{m/s} = 10\text{m/s}$

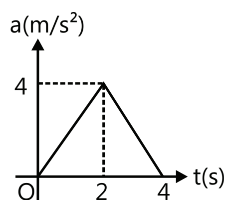
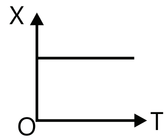
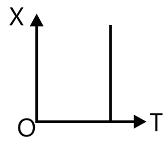
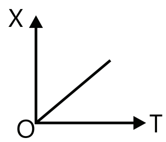
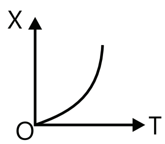
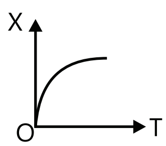
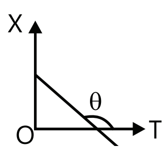
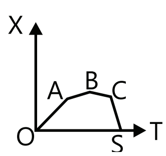
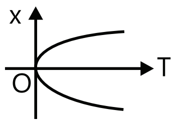
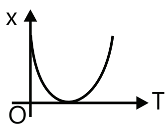
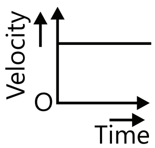
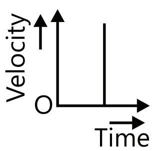
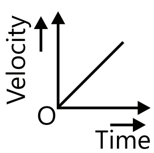
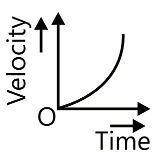
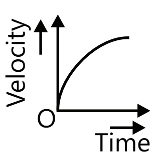
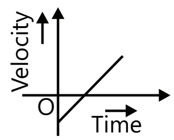
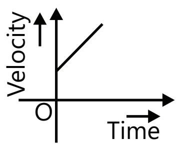
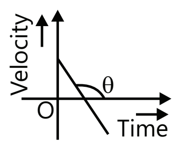
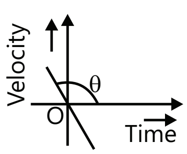
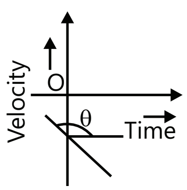


Figure 2.31

Various Position–time graphs and their interpretation	
 **Figure 2.32**	$\theta = 0^\circ$, so v = 0 i.e. line parallel to time axis represents that the particle is at rest
 **Figure 2.33**	$\theta = 90^\circ$,so $v = \infty$ i.e. line perpendicular to time axis represents that particle is changing its position with constant time. Hence, particle possesses infinite velocity (which is not possible practically).
 **Figure 2.34**	θ = constant, so v = constant, a=0 i.e. line with constant slope represents uniform velocity of the particle.
 **Figure 2.35**	θ is increasing, so v is increasing and a is positive. i.e. line bending towards position axis x represents increase in velocity of particle. Hence, the particle possesses acceleration.
 **Figure 2.36**	θ is decreasing, so v is decreasing and a is negative i.e. line bending towards time axis t represents decreasing velocity of the particle. Hence, the particle possesses retardation.
 **Figure 2.37**	θ is constant but $>90^\circ$, so v will be constant but negative i.e. line with negative slope represents that particle returns to the point of reference (i.e. negative displacement).
 **Figure 2.38**	Straight line segments of different slopes represent that velocity of the body is different for different intervals of time.

 **Figure 2.39**	At one point the particle has two positions, which is not possible.
 **Figure 2.40**	Particle moves towards origin initially and after that moves away from origin.
Various Velocity–time graphs and their interpretation	
 **Figure 2.41**	θ = 0°, a = 0, v = constant i.e. line parallel to time axis represents that the particle is moving with constant velocity.
 **Figure 2.42**	θ = 90°, a = ∞, v is increasing i.e. line perpendicular to time axis represents that particle is increasing in velocity, but there is no change in time. Hence the particle possesses infinite acceleration (which is not possible practically).
 **Figure 2.43**	θ =constant, so a = constant and v is increasing uniformly with time i.e. the slope represents uniform acceleration of the particle.
 **Figure 2.44**	θ is increasing, so acceleration is increasing i.e. line bending towards velocity axis represents increasing acceleration of the body.
 **Figure 2.45**	θ is decreasing, so acceleration is decreasing i.e. line bending towards time axis represents the decreasing acceleration in the body.

 **Figure 2.46**	θ is constant and <90° i.e. acceleration is constant and positive but initial velocity of the particle is negative.
 **Figure 2.47**	θ is constant and <90° i.e. acceleration is constant and positive but initial velocity of the particle is positive.
 **Figure 2.48**	θ is constant and >90° i.e. acceleration is constant and negative but initial velocity of the particle is positive.
 **Figure 2.49**	θ is constant and >90° i.e. acceleration is constant and negative but initial velocity of the particle is zero.
 **Figure 2.50**	θ is constant and >90° i.e. acceleration is constant and negative but initial velocity of the particle is negative.

CONCEPTS

The following are the lists of motions that are not possible practically:

- Slopes of v-t or s-t graphs can never be infinite at any point, because infinite slope of v-t graph means infinite acceleration. Similarly, infinite slope of s-t graph means infinite velocity. Hence, the graphs shown here are not possible.
- At a particular time, two values of velocities v_1 and v_2 or displacements S_1 and S_2 are not possible. Hence, the following graphs shown here are not possible.

GV Abhinav (JEE 2012 AIR 329)

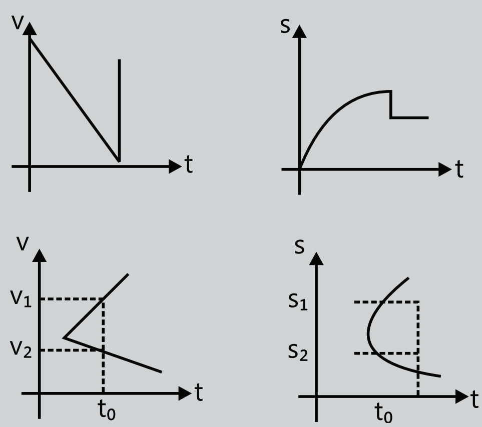


Figure 2.51

Illustration 22: At t = 0, a particle is at rest at origin. For the first 3 s the acceleration is 2 ms^{-2} and for the next 3 s acceleration is -2 ms^{-2}. Find the acceleration versus time, velocity versus time and position versus time graphs.

(JEE ADVANCED)

Sol: The area enclosed by the acceleration-time graph and the time-axis between t = 0 and t = t gives the change in velocity in this time interval. Similarly the area enclosed by the velocity-time graph and the time-axis between t = 0 and t = t will give the change in displacement in this time interval.

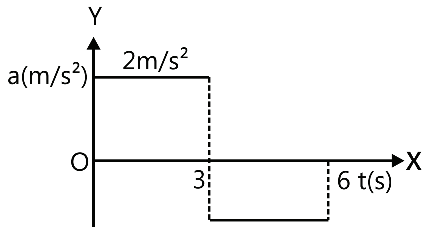


Figure 2.52

Given for the first 3 s acceleration is 2 ms^{-2} and for next 3 s acceleration is -2 ms^{-2}. Hence acceleration–time graph is as shown in the Fig. 2.52.

The area enclosed between a–t curve and t-axis gives change in velocity for the corresponding interval. Also at t=0, v=0, hence final velocity at t=3 s will increase to 6 ms^{-1}. In next 3 s the velocity will decrease to zero. Hence the velocity–time graph is as shown in figure.

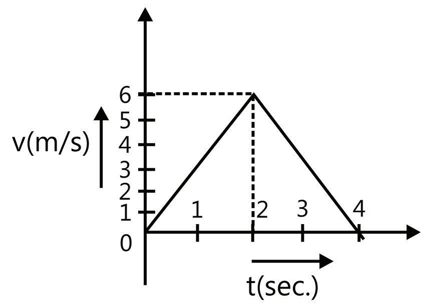


Figure 2.53

Note that due to constant acceleration v-t curves are taken as straight line.

Now for x-t curve, we will use the fact that area enclosed between v-t curve and time axis gives displacement for the corresponding interval. Hence displacement in the first 3 s is 4.5 m and in next 3 s is 4.5 m. Also the x–t curve will be of parabolic nature as the motion has a constant acceleration. Therefore, x–t curve is as shown in figure.

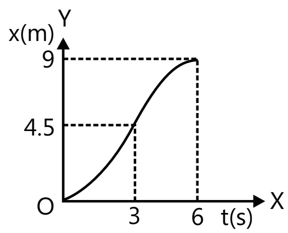


Figure 2.54

Illustration 23: The graph in the Fig. 2.55 shows the velocity of a body plotted as a function of time.

(a) Find the instantaneous acceleration at t = 3 s, 7 s, 10 s, and 13 s.

(b) Find the distance travelled by the body in the first 5 s, 9 s, and 14 s

(c) Find the total distance covered by the body during motion.

(d) Find the average velocity of the body during motion.

(JEE ADVANCED)

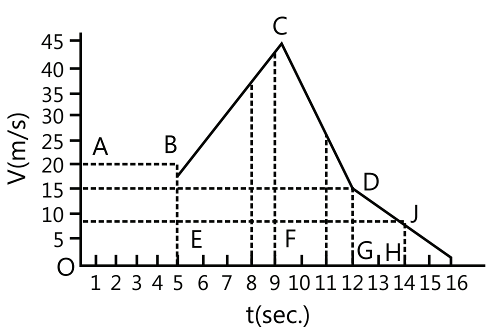


Figure 2.55

Sol: The area enclosed by the acceleration-time graph and the time-axis between t = 0 and t = t gives the change in velocity in this time interval. Similarly the area enclosed by the velocity-time graph and the time-axis between t = 0 and t = t will give the change in displacement in this time interval.

(a) Acceleration at t = 3 s

When the particle travels from point A to B for the first 5 s, the body moves with a constant velocity. Hence, the acceleration is zero.

Acceleration at t = 7 s.

When the particle travels from point B to C for the interval 5 to 9 s, the acceleration is uniform.

$$a = \frac{45-20}{(9-5)} = 6.25 m/s^2$$

Hence acceleration at t=7 s is 6.25 m/s^2. The acceleration at t = 10 s and 13 s are respectively -10 m/s^2 and -3.75 $m.s^2$.

(b) The distance covered in t seconds is the area enclosed by the curve in t seconds on velocity–time graph. The distance covered by the body in 5 s = Area of rectangle ABEO = 20 x 5 = 100 m

The distance covered in first 9 s = The area of the figure ABCFO = Area ABEO + Area EBCF

$$= 100 + \frac{1}{2}(20+45)x4 = 100 + 130 = 230 \text{ m.}$$

The distance covered by the body in first 14 s = Area [(ABCFO) + (CDGF) + DJHG)]

$$= 230 + \frac{1}{2}(45+15)x3 + \frac{1}{2}(15+7.5)x2 = 230 + 90 + 22.5 = 342.5 \text{ m.}$$

(c) The distance covered by the body during the entire motion $= 342.5 + \frac{1}{2}7.5x2 = 350 \text{ m/s.}$

(d) Average velocity for the motion $= \frac{350}{16} m/s = 21.9 m/s.$

6. RELATIVE MOTION

(a) Motion of an object is dependent on observation.

(b) Motion is a relative term.

(c) An observation of motion is always with respect to frame of reference.

6.1 Types of Frames of Reference

(a) **Inertial frames of reference:** It is defined as the frame of reference with uniform velocity (both in magnitude and direction). Thus, acceleration is also zero, i.e. $\vec{v} = \text{constant}$ and $\vec{a} = 0$.

(b) **Non-Inertial frames of reference**: It is defined as the frames of reference with non-uniform velocity (either magnitude or direction is not constant). Thus, acceleration is also non-zero, i.e. $\vec{a} \neq 0$.

6.2 Relative Velocity (Introduction to motion in 2D)

Consider two bodies A and B travelling with velocities V_{AO} and V_{BO}, respectively, with respect to origin O, then the relative velocity of B with respect to an observer A, V_{BA}, is given as follows:

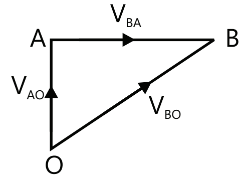


Figure 2.56

$V_{BA} = V_{BO} - V_{AO}$

Similarly, $V_{AB} = V_{AO} - V_{BO}$

Thus the relative velocity of any two bodies moving from the same origin is equal to the vector difference of their velocities.

The relative rate of change of V_{BA} gives relative acceleration of B with respect to A and is given by $a_{BA} = a_{BO} - a_{AO}$ and $a_{AB} = a_{AO} - a_{BA}$

Fact: Distance between two objects with respect to is independent of the reference frame.

If 'x' is the minimum distance between the two objects at time 't' then in any frame of reference the minimum distance of the objects remains constant at time 't'.

CONCEPTS

To find the relative velocity of an object A (say) w.r.t to object B (say), inverse (change the direction of) the velocity vector of object B and then add it to velocity of A.

Anurag Saraf (JEE 2011 AIR 226)

Illustration 24: A man whose velocity in still water is 5 m/s swims from point A to B (100 m down-stream of A) and back to point A. Velocity of the river is 3 m/s. Find the time taken in going down-stream and upstream and the average speed of the man during the motion. **(JEE MAIN)**

Sol: The velocity of man in ground frame is the vector sum of the velocity of river and the velocity of man in river frame. In going down-stream, the magnitude of velocity of river and the magnitude of velocity of man in river frame are added to get the magnitude of velocity of man in ground frame. In going up-stream, the magnitude of velocity of river is subtracted from the magnitude of velocity of man in river frame to get the magnitude of velocity of man in ground frame.

During down-stream, velocity of the man $= \overrightarrow{V_m} = \overrightarrow{V_{mw}} + \overrightarrow{V_w} = 3 + 5 = 8 \text{ m/s}$ Time taken during down-stream = 100/8 =12.5 s

During upstream, velocity of the man $= \overrightarrow{V_m}' = \overrightarrow{V_{mw}} + \overrightarrow{V_w}$ = - 5 + 3 = - 2 m/s.

Time taken during upstream = 100/2 = 50 s

Average speed = $\dfrac{200}{62.5} = 3.2 \text{ m/s}$

Illustration 25: Yashwant started moving with constant speed 10 m/s to catch the bus. When he was 40 m away from the bus, it started moving away from him with acceleration of 2 m/s^2. Find whether Yashwant catches the bus or not. If yes, at what time he catches the bus. If no, then find the minimum distance between the bus and him. **(JEE MAIN)**

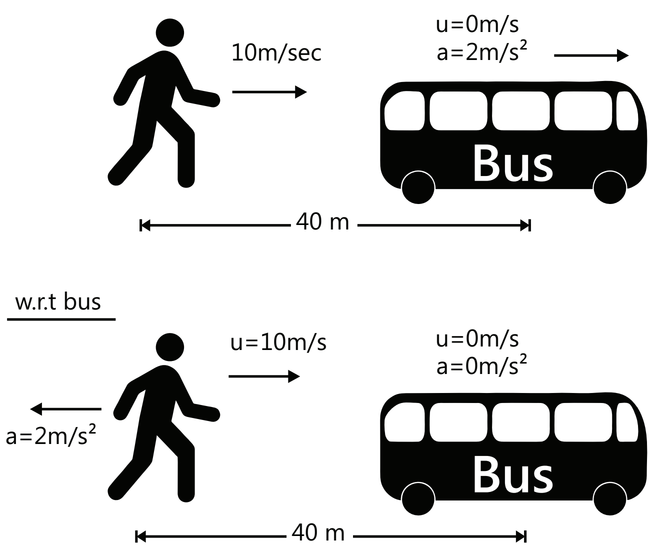


Figure 2.57

Sol: This problem is best solved in the reference frame of bus. In this frame the initial velocity of Yashwant is towards the bus (assumed positive) and the acceleration of Yashwant is in the opposite direction i.e. away from the bus (assumed negative).

Yashwant moves with the initial velocity of 10 m/s and acceleration of 2 m/s^2.

Assuming Yashwant never catches the bus, let us find the distance at which his velocity becomes zero.

We know $v^2 = u^2 + 2as$

Here v = 0; u = 10 m/s; a = -2 m/s^2; $\therefore\ 0^2 = (10)^2 + 2(-2)\ s \Rightarrow s = \frac{100}{4} = 25m$

Our assumption was right, since Yashwant travels only 25 m in the positive direction in bus reference frame.

So, minimum distance = 40 – 25 = 15 m.

6.3 Applications of relative velocity

Relative motion is widely used in two- and three-dimensional motions. The four types of problems arising based on relative motion are as follows:

(a) Problems on minimum distance between two bodies in motion

(b) River–boat problems

(c) Aircraft–wind problems

(d) Rain problems

(a) Minimum distance between two bodies in motion: Minimum distance between two moving bodies or the time taken when one body overtakes the other can be solved easily by the principle of relative motion. Here we consider one body to be at rest and other body to be in relative motion of the other body. By combining two problems into one, the solution becomes easy. Following examples will illustrate the statement.

Illustration 26: Car A and car B start moving simultaneously in the same direction along the line joining them. Car A moves with a constant acceleration a = 4 m/s^2, while car B moves with a constant velocity v = 1 m/s. At time t = 0, car A is 10 m behind car B. Find the time when car A overtakes car B. **(JEE MAIN)**

Sol: This problem is best solved in the reference frame of any one of the two cars (say car B). In this frame the initial velocity of car A is in the direction away from car B (assumed negative) and the acceleration of car A is in the direction towards the car B (assumed positive).

Given: $u_A=0$, $u_B=1m/s$, $a_A = 4\ m/s^2$ and $a_B = 0$

Assuming car B to be at rest, we have

$u_{AB} = u_A - u_B = 0 - 1 = -1m/s$

$a_{AB} = a_A - a_B = 4 - 0 = 4m/s^2$

Now, the problem can be solved easily as follows:

Substituting the proper values in equation

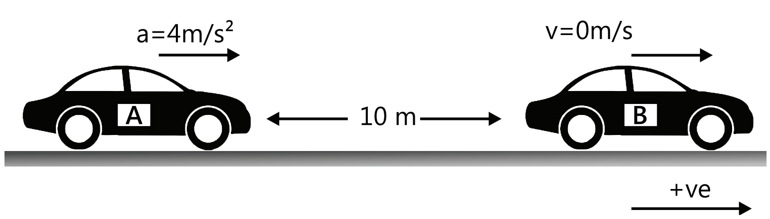


Figure 2.58

$s = ut + \frac{1}{2}at^2$, we get $10 = -t + \frac{1}{2}(4)(t^2)$ or, $2t^2 - t - 10 = 0$ or, $t = \frac{1 \pm \sqrt{1+80}}{4} = \frac{1 \pm \sqrt{81}}{4}$

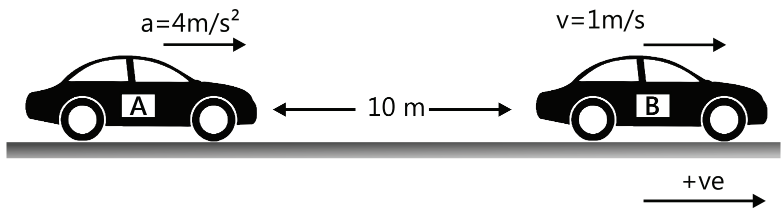


Figure 2.59

$= \frac{1 \pm 9}{4}$ or $t = 2.5$ s and -2 s Ignoring the negative value, the desired time is 2.5 s.

(b) River–boat problem

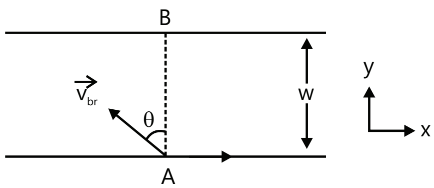


Figure 2.60

We come across the following three terms:

$\vec{v_r}$ = absolute velocity of river

$\vec{v_{br}}$ = velocity of boatman with respect to river or velocity of boatman in still water and $\vec{v_b}$ = absolute velocity of boatman

Here, it is important to note that $\vec{v_{br}}$ is the velocity of boatman with which he steers and $\vec{v_b}$ is the actual velocity of boatman relative to ground.

Further, $\vec{v_b} = \vec{v_{br}} + \vec{v_r}$ Now, let us derive some standard results and their special cases.

A boatman starts from point A on one bank of a river with velocity $\vec{v_{br}}$ in the direction shown in Fig. 2.60.

River is flowing along positive x-direction with velocity $\vec{v_r}$.

Width of the river is w, then

Therefore, $v_{bx} = v_{rx} + v_{brx} = v_r - v_{br}\sin\theta$ and $v_{by} = v_{ry} + v_{bry\,=\,0} + v_{br}\cos\theta$

Now, time taken by the boatman to cross the river is:

$$t = \frac{w}{v_{br}\cos\theta} \qquad \text{... (i)}$$

Further, displacement along x-axis when he reaches on the other bank (also called drift) is:

$$x = v_{bx}t = (v_r - v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta}$$

$$x = (v_r - v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta} \qquad \text{... (ii)}$$

Or the three special cases are:

(i) Condition when the boatman crosses the river in shortest interval of time

From Equation (i) we can see that time (t) will be minimum when θ = 0°, i.e. the boatman should steer his boat perpendicular to the river current.

Also, $t_{mim} = \frac{w}{v_{br}}$ as $\cos\theta = 1$

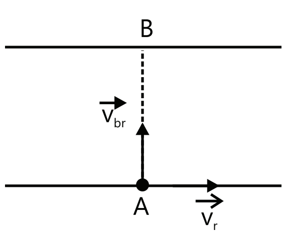


Figure 2.61

(ii) Condition when the boatman wants to reach point B, i.e. at a point just opposite from where he started

In this case, the drift (x) should be zero. Therefore x = 0

Or $(v_r - v_{br}\sin\theta)\frac{w}{v_{br}\cos\theta} = 0$ or $v_r = v_{br}\sin\theta$

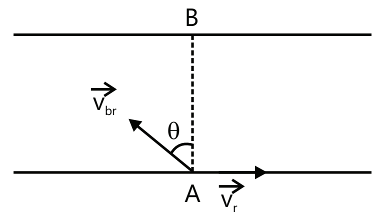


Figure 2.62

$\left(\frac{v_r}{v_{br}}\right)$ or $\sin\theta = \frac{v_r}{v_{br}}$ or $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$

Hence, to reach point B, the boatman should row at an angle $\theta = \sin^{-1}$ upstream from AB. Further, since $\sin\theta \le 1$.

So, if $v_r \ge v_{br}$, the boatman can never reach point B. If $v_r = v_{br}$, $\sin\theta = 1$ or 90° it is quite impossible to reach B if $\theta = 90°$. Moreover it can be seen that $v_{by} = 0$ if $v_r = v_{br}$ and $\theta = 90°$. Similarly, if $v_r \ge v_{br}$, $\sin\theta > 1$ i.e. no such angle exists. Practically it is not possible to reach B if river velocity (v_r) is too high.

(iii) Shortest path

Distance travelled by the boatman when he reaches the opposite shore is $s = \sqrt{w^2 + x^2}$

Here, w = width of river which is constant. For s to be minimum, modulus of x (drift) should be minimum. Now two cases are possible

When $v_r < v_{br}$: In this case x = 0, when $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ or $s_{mim} = w$ at $\theta\ \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$

When $v_r > v_{br}$: In this case x is minimum, where $\left(\frac{dx}{d\theta}\right) = 0$; $\frac{d}{d\theta}\left\{\frac{w}{v_{br}\cos\theta}(v_r - v_{br}\sin\theta)\right\} = 0$

or $-v_{br} + v_r \sin\theta = 0$ or $\theta = \sin^{-1}\left(\frac{v_{br}}{v_r}\right)$

Now, at this angle we can find x_{min} and then s_{min}

$$s_{min} = w\left(\frac{v_r}{v_{br}}\right) \text{at}\ \theta = \sin^{-1}\left(\frac{v_{br}}{v_r}\right)$$

Illustration 27: A man rows a boat at 4 km/h in still water. If he is crossing a river with a 2 km/h current

(a) What will be the direction of the boat, if he wants to reach a point directly opposite the starting point on the other bank?

(b) With these conditions, how much time it will take for him to cross the river, given the width of river is 4 km?

(c) What will be the minimum time and what direction should he head to cross the river in shortest time?

(d) If he wants to row 2 km up the stream and back to the origin, what will be the time required? **(JEE MAIN)**

Sol: The velocity of boat in ground frame is the vector sum of the velocity of river and the velocity of boat in river frame. If the boat heads in the direction perpendicular to the direction of river flow in the frame of the river, it will cross the river in shortest time. But in doing so it will get drifted in the direction of the river flow as well, and thus will not reach the other bank directly opposite to the starting point.

(a) Given, that $v_{br} = 4$ km/h and $v_r = 2$ km/h ;

$$\therefore\ \theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right) = \sin^{-1}\left(\frac{2}{4}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

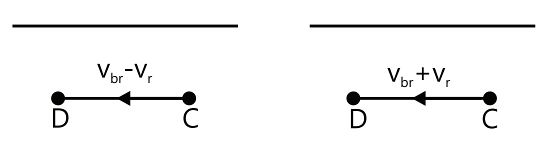


Figure 2.63(a)

To reach the point directly opposite to starting point, the boat should head at an angle of 30° with AB or 90° + 30° = 120° with the river flow.

(b) Time taken to cross the river w = width of river = 4 km

v_{br} = 4 km/h and θ = 30°; $t = \frac{4}{4\cos 30^o} = \frac{2}{\sqrt{3}}h$

(c) For shortest time θ = 0° and $t_{min} = \frac{w}{v_{br}\cos 30^o} = \frac{4}{4} = 1h$

Hence, he should incline his boat perpendicular to the current for crossing the river in shortest time of 1 h.

(d) $t = t_{CD} + t_{DC}$ or $t = \frac{CD}{v_{br-v_r}} + \frac{DC}{v_{br}+v_r} = \frac{2}{4-2} + \frac{2}{4+2} = 1 + \frac{1}{3} = \frac{4}{3}h$

(c) Aircraft wind problem: The only difference between this and the river boat is that $\overrightarrow{v_{br}}$ is replaced by $\overrightarrow{v_{aw}}$ (velocity of aircraft with respect to wind or velocity of aircraft in still air), $\overrightarrow{v_r}$ is replaced by $\overrightarrow{v_w}$ (velocity of wind) and $\overrightarrow{v_b}$ is replaced $\overrightarrow{v_a}$ (absolute velocity of aircraft) . Further, $\overrightarrow{v_a} = v_{aw} + v_{w.}$ Following example will illustrate it.

Illustration 28: An aircraft flies 400 km/h in still air. If 200√2 k/h wind is blowing from the south and the pilot wants to travel from point A to a point B, north east of A. Find the direction in which the aircraft is to be steered and time of journey if AB = 100 km. (**JEE MAIN)**

Sol: The velocity of aircraft in the ground frame is the vector sum of its velocity in the wind frame and the velocity of the wind. This velocity in ground frame is along the known direction A to B which is 45° east of north. The direction of wind is towards north. The direction of velocity in wind frame is un-known which can be found using triangle law of vector addition.

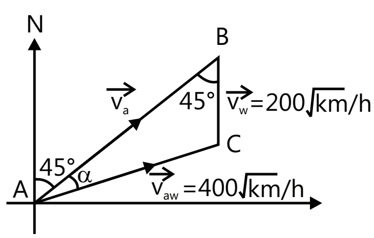


Figure 2.64

Give that $v_w = 200\sqrt{2}km/h$ v_{aw} = 400 km/h. $\vec{v}_a$ should be along AB or in north-east direction.

The direction of $\vec{v}_{aw}$ should be such that the resultant of $\vec{v}_w$ and $\vec{v}_{aw}$ is along AB or in north-east direction.

If $\vec{v}_{aw}$ makes an angle α with AB as shown in Fig. 2.64, then by applying sin law in triangle ABC,

$$\frac{AC}{\sin 45^o} = \frac{BC}{\sin\alpha} \quad \text{or} \quad \sin\alpha = \left(\frac{BC}{AC}\right)\sin 45^o = \left(\frac{200\sqrt{2}}{400}\right)\frac{1}{\sqrt{2}} = \frac{1}{2}; \ \therefore \alpha = 30^o$$

Therefore, the pilot should steer in a direction at an angle of (45° + α) or 75° from north towards east.

$$\text{Further, } \frac{[\vec{v}_a]}{\sin(180° - 45° - 30°)} = \frac{400}{\sin 45°}; \ |\vec{v}_o| = \frac{\sin 105^o}{\sin 45^o} \times (400)\frac{km}{h}$$

$$= \left(\frac{\cos 15^o}{\sin 45^o}\right)(400)\frac{km}{h} = \left(\frac{0.9659}{0.707}\right)(400)\frac{km}{h} = 546.47 \text{ km/h}$$

∴ The time for journey from A to B is $t = \frac{AB}{|\vec{v}_a|} = \frac{1000}{546.47}h$; t=1.83 h

(d) Rain problem: In these type of problems, we again come across three terms $\vec{v}_r, \vec{v}_m$ and $\vec{v}_{rm}$, Here

$\vec{v}_r$ = velocity of rain

$\vec{v}_m$ = velocity of man (it may be velocity of cyclist or velocity of motorist also)

and $\vec{v}_{rm}$ = velocity of rain with respect to man

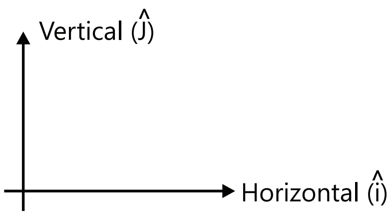


Figure 2.65

Here, $\vec{v}_{rm}$ is the velocity of rain which appears to the man. Now, let us take one example of this

Illustration 29: Rain appears to fall vertically to a man walking at a rate of 3km/h. At a speed of 6 km/h, it appears to meet him at an angle of 45^0 of vertical. Find out the speed of rain. **(JEE MAIN)**

Sol: This problem is best solved by using Cartesian coordinates. Take x-axis along the horizontal and y-axis vertically upwards. The velocity of man is along positive x-axis. The velocity of rain has both horizontal and vertical components. Express the velocity of man and rain in terms of unit vectors $\hat{i}$ and $\hat{j}$.

Let $\hat{i}$ and $\hat{j}$ be the unit vectors in horizontal and vertical directions, respectively.

Velocity of rain

$$\vec{v}_r = a\hat{i} + b\hat{j} \qquad \text{... (i)}$$

Then the speed of rain will be

$$|\vec{v}_r| = \sqrt{a^2 + b^2} \qquad \text{... (ii)}$$

In the first case, $\vec{v}_m = \text{velocity of man} = 3\hat{i}$

$\therefore\ \vec{v}_{rm} = \vec{v}_r - \vec{v}_m = (a-3)\hat{i} + b\hat{j}$ It seems to be in vertical direction. Hence, a – 3 = 0 or a = 3

In the second case $\vec{v}_m = 6\hat{i}$

$\therefore\ \vec{v}_m = (a-6)\hat{i} + b\hat{j} = -3\hat{i} + b\hat{j}$

This seems to be at 45^0 of vertical

Hence, $|b| = 3$

Therefore, from Eq. (ii) speed of rain is $|\vec{v}_r| = \sqrt{3^2 + 3^2} = 3\sqrt{2}\,\text{km/h}$

Alternative Solution:

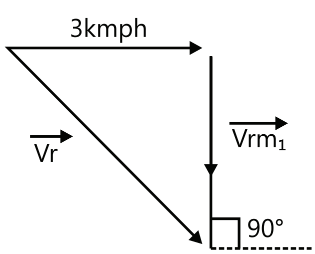


Figure 2.66

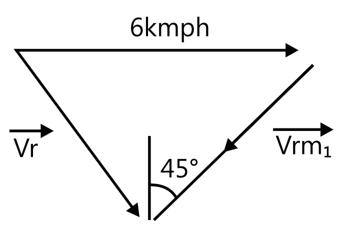


Figure 2.67

Combining these two we get

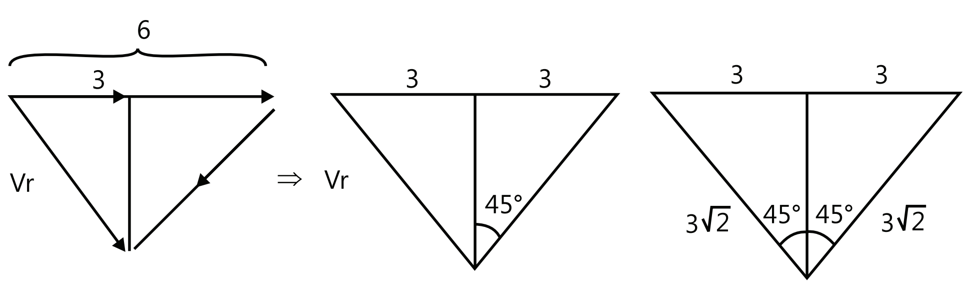


Figure 2.68

$|\vec{V}_r| = 3\sqrt{2}$ km/h

PROBLEM-SOLVING TACTICS

To avoid confusion while using signs in equations of motion, the following points need to be considered:

(a) Assuming any one direction to be positive, the other automatically becomes negative.

Generally, vertically up is considered as positive and right side is taken as positive.

(b) Write down the values of velocity, displacement and acceleration according to the sign convention.

(c) On completion of sign convention, then simply use the equations of motions.

FORMULAE SHEET

Position vector of point A with respect to O:

$\vec{r}_A = \overrightarrow{OA} = x_A\hat{i} + y_A\hat{j} + z_A\hat{k}$

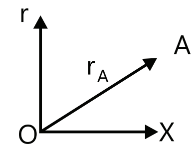


Figure 2.69

$\overrightarrow{AB} = \vec{r}_s - \vec{r}_A$

$\overrightarrow{AB} = (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$

For un-accelerated Motion: Distance = Speed x Time

Displacement = Velocity x Time

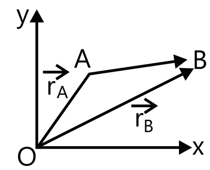


Figure 2.70

v_{av} = Average speed $= \frac{\Delta s}{\Delta t}$; $\vec{v}_{av}$ = Average Velocity $= \frac{\Delta \vec{r}}{\Delta t}$

$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$ and $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{d\vec{r}}{dt}$

(a) **Uniform motion is due to constant Velocity**

Average acceleration has the same direction as of change in velocity

$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$

(b) **One-dimensional Uniformly Accelerated Motion**

- $v = u + at$, $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$ and $v_{avg} = (v+u)/2$

(i) Maximum height attained by a particle, thrown upwards from ground with initial velocity u is $h = \frac{u^2}{2g}$

(c) **Displacement of particle in t^{th} second of its motion ,** $s_t = u + \frac{a}{2}(2t-1)$

- Time taken to reach maximum height = u/g

(i) Velocity of a particle (u=0) when it touches the ground when dropped from a height h is,

$v = \sqrt{2gh}$

(ii) In (b) time of collision with ground $t = \sqrt{2h/g}$

- If acceleration of particle is not constant, basic equations of velocity and acceleration are used, i.e.

(i) $\vec{v}=\frac{d\vec{s}}{dt}$ or $\vec{v}=\frac{d\vec{r}}{dt}$ (ii) $\vec{a}=\frac{d\vec{v}}{dt}$ (iii) $d\vec{s}=\vec{v}dt$ (iv) $d\vec{v}=\vec{v}dt$

- For one dimensional motion,

(i) $v=\frac{ds}{dt}$ (ii) $a=\frac{dv}{dt}=v\frac{dv}{ds}$

(iii) $ds=v\,dt$ and (iv) $dv=adt$ or $vdv=ads$

- If $z=\frac{dy}{dx}$ or $\frac{y}{x}$, the value of z at any point on x-y graph can be obtained by the slope of the graph at that point.

(i) slope of displacement–time graph gives velocity $\left(\text{as } v=\frac{ds}{dt}\right)$

(ii) slope of velocity–time graph gives acceleration $\left(\text{as } a=\frac{dv}{dt}\right)$

If z = yx, y (dx), or x (dy), the value of z between x_1 and x_2 or y_1 and y_2 can be obtained by the area of graph between x_1 and x_2 or y_1 and y_2

(iii) velocity–time graph gives displacement (as ds=v dt)

(iv) acceleration–time graph gives change in velocity (as dv= adt).

relative velocity of A with respect to B (written as $\vec{v}_{AB}$) is $\vec{v}_{AB}=\vec{v}_A-\vec{v}_B$

Similarly, relative acceleration of A with respect to B is $\vec{a}_{AB}=\vec{a}_A-\vec{a}_B$

$\vec{v}_{AB}=-\vec{v}_{BA}$ or $\vec{a}_{BA}=-\vec{a}_{AB}$

- In case of a one dimensional motion when we can treat the vectors as scalars by assigning positive to one direction and negative to another, the above equations can be written as $v_{AB}=v_A-v_B$ and $a_{AB}=a_A-a_B$

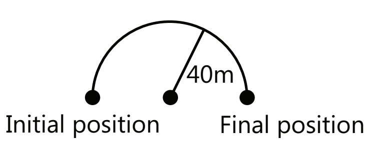
Solved Examples

JEE Main/Boards

Example 1: A person in his morning walk moves on a semicircular track of radius 40 m. Find the distance travelled and the displacement, when he starts from one end of the track and reaches the other end.

Sol: Distance is length of the path travelled. Displacement is the vector from initial point to final point.

The distance covered = length of the semicircular track



= π R = 3.14 x 40 m = 125.6 m

Displacement = Final position - initial position

= diameter of semicircular track

= 2 R = 2 x 40 = 80 m

Initial point to final point gives the direction of displacement.

Example 2: A man walks 2.5 km from his house to the market on a straight road with a speed of 5 km/h. He instantly turns back home with a speed of 7.5 km/h finding the market closed. Calculate the

(a) magnitude of average velocity and (b) the average speed of the man over the interval of time.

(i) 0 to 30 min (ii) 0 to 50 min

(iii) 0 to 40 min

Sol: Average speed is distance covered divided by time taken. Distance is length of the path travelled. Average velocity is displacement divided by time taken. Displacement is the vector from initial point to final point.

Distance between market and home = 2.5 km

Speed of man from home to market = 5 km/h

∴ Time taken by the man to reach the market

$$t_1 = \frac{\text{Distance}}{\text{Speed}};\ t_1 = \frac{2.5}{5} = \frac{1}{2}h = 30 \text{ min}$$

Speed of man during his return = 7.5 km/h

Time taken by man to return home

$$t_2 = \frac{2.5}{7.5} = \frac{1}{3}h = 20 \text{ min}$$

Total time taken by the man returning home = 30 + 20 = 50 min

(i) Over the interval 0 to 30 min:

During this time, man goes from home to market. Therefore, displacement s = 2.5 km.

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{2.5}{\frac{1}{2}} = 5 km/h$$

$$\text{Average speed} = \frac{\text{Distance}}{\text{Time}} = \frac{2.5}{\frac{1}{2}} = 5 km/h$$

(ii) For the time interval 0 to 50 min: In 50 min the man goes from home to market and return back

∴ Net displacement = zero

$$\therefore \text{Average velocity} = \frac{\text{Net displacement}}{\text{Total time}} = \frac{0}{50} = 0$$

Average speed

$$= \frac{\text{Total distance}}{\text{Total time}} = \frac{2.5+2.5}{50} = \frac{5}{50/60} = 6\frac{km}{h}$$

(iii) During the time interval 0 to 4 min: During first 30 min man goes home to market converting a distance 2.5 km in next 10 min the man is in path from market to home and comes a distance $= 7.5x\frac{10}{60} = 1.25 km$

Displacement = 2.5 – 1.25 = 1.25 km.

Distance = 2.5 + 1.25 = 3.75 km

Average velocity

$$= \frac{\text{Displacement}}{\text{time}} = \frac{1.25}{40x\frac{1}{60}} = 1.875 km/h$$

$$\text{Average speed} = \frac{\text{Distance}}{\text{time}} = \frac{3.75}{40x\frac{1}{60}} = 5.625 \text{ km/h}$$

Example 3: A particle moving with an initial velocity 2.5 m/s along the positive x direction accelerates uniformly at the rate 0.50 m/s^2. (i) Find the distance travelled in the first 2 sec. (ii) Calculate the time taken to reach the velocity of 7.5 m/s? (iii) Calculate the distance travelled in reaching the velocity 7.5 m/s?

Sol: This is the case of motion with uniform acceleration. Use the three equations of motion with uniform acceleration.

(i) We have, $x = ut + \frac{1}{2}at^2$

$$= (2.5m/s)(2s) + \frac{1}{2}(0.50 m/s^2)(2s)^2$$

= 5.0 m + 1.0 m 6.0 m

Since the particle does not turn back it is also the distance travelled.

(ii) We have, v = u + at

or 7.5 m/s = 2.5 m/s + (0.50 m/s^2) t

$$t = \frac{7.5m/s - 2.5m/s}{0.50m/s^2} = 10s$$

(iii) We have, $v^2 = u^2 + 2ax$

or , (7.5 m/s)^2 = (2.5 m/s) ^{2} +2.(0.50 m/s^2)x

$$\text{or, } x = \frac{(7.5m/s)^2 - (2.5m/s)^2}{2x0.50m/s^2} = 50m.$$

Example 4: A particle is projected vertically upwards with velocity 40 m/s. Find the displacement and distance travelled by the particle in

(i) 2s (ii) 4s (iii) 6s

Take g=10 m/s^2

Sol: Distance covered is equal to displacement if the object moves in a straight line and there is no change in direction of motion. If direction of motion changes, distance should be calculated separately for different parts of the path.

Here, due to upward motion, u is positive and due to downward motion, a is negative.

Velocity becomes zero at maximum height

Time taken to reach maximum height (t_0) = u/g;

$$t_0 = \left|\frac{u}{a}\right| = \frac{40}{10} = 4s \qquad \updownarrow^{+ve}_{-ve}$$

(i) t < t_0. Therefore, distance and displacement are equal. $d = s = ut + \frac{1}{2}at^2 = 40x2 - \frac{1}{2}x10x4 = 60m$

(ii) $t=t_0$, then distance and displacement are equal.

$$d = s = 40x4 - \frac{1}{2}x10x16 = 80m$$

(iii) $t>t_0$. Hence, d> s;

$$s = 40x6 - \frac{1}{2}x10x36 = 60m$$

$$\text{While } d = \left|\frac{u^2}{2_a}\right| + \frac{1}{2}|a(t-t_0)^2|$$

$$= \frac{(40)^2}{2x10} + \frac{1}{2}x10x(6-4)^2 = 100m$$

Example 5: Following information about an object's motion is given: $a = t^2$

Initial velocity = u

Find: (i) velocity (v) as a function of time.

(ii) Displacement (x) a function of time.

Sol: This is the case of motion with non-uniform acceleration. Acceleration is given as a function of time. Change in velocity can be found by integrating the expression for acceleration with respect to time. Displacement can be found by integrating the expression for velocity.

$$\text{(i) } a = \frac{dv}{dt} = t^2 \Rightarrow dv = t^2\,dt$$

$$\text{Integration we get: } \Rightarrow \int_u^v dv = \int_o^t t^2\,dt$$

$$\Rightarrow v = u = \frac{t^3}{3} \quad \Rightarrow v = \frac{t^3}{3} + 4$$

$$\text{(ii) and } \quad v = \frac{dx}{dt} = \frac{4+t^3}{3} \quad \Rightarrow dx = \left(\frac{4+t^3}{3}\right)dt$$

Integration we get

$$\Rightarrow \int_o^x dx = \int_o^t udt + \int_o^t \frac{t^3}{3}dt\,; \quad \Rightarrow x = ut + \frac{t^4}{12}$$

Example 6: The position of an object moving along x-axis is given by $x = 8.0 + 2.0t^2$, where x is in meter and t is in second. Calculate:

(i) the velocity at t = 0 and t = 2.0 sec.

(ii) average velocity between 2.0 sec and 4.0 sec.

Sol: Here the position is given as a function of time. Differentiate this expression w.r.t time to get velocity as a function of time. Average velocity is displacement divided by time taken.

$$\text{Velocity, } \quad v = \frac{dx}{dt}; \Rightarrow v_{avg} = \frac{x(t_2) - (x(t_1)}{t_2 - t_1}$$

Given, $x = 8.0 + 2.0\,t^2$

$$\text{(i) Velocity} \qquad v = \frac{dx}{dt} = \frac{d}{dt}(8.0 + 2.0t^2)$$

= 0 + 2.0 x 2t; v = 4t

Velocity at t = 0s, $(v)_{t-0}$ = 4 x 0 ms^{-1}

Velocity at t = 2s, $(v)_{t-2}$ =4 x 2 ms^{-1}

(ii) Average velocity

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{(x)_{t=4} - (x)_{t=2}}{4-2}$$

$$= \frac{(8.0 + 2t^2)_{t=4} - (8.0 + 2t^2)_{t=2}}{2}$$

$$= \frac{\{8 + 2x(4)^2\} - \{8 + 2x(2)^2\}}{2} = \frac{40-16}{2} = 12ms^{-1}$$

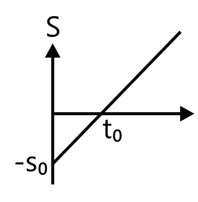
Example 7: The motion of two bodies A and B represented by two straight lines drawn on the same displacement-time graph, make angles 30° and 60° with time axis, respectively. Which body possesses greater velocity? What is the ratio of their velocities?

Sol: The slope of the displacement time graph at an instant gives the velocity at that instant.

The velocity of body = slope of displacement time graph. Therefore the line having greater slope has greater velocity, i.e. the body B has greater velocity. Ratio of their velocities,

$$\frac{v_A}{v_B}\frac{\tan 30^0}{\tan 60^0} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

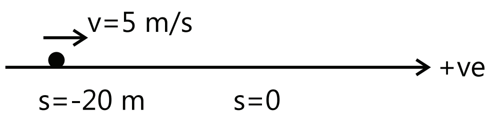
Example 8: Displacement–time graph of a particle moving in a straight line is as shown in Figure. State whether the motion is accelerated or not. Describe the motion in detail. Given s_0=20 m and t_0 = 4 s.



Sol: The slope of the displacement time graph at an instant gives the velocity at that instant. If the slope is constant, the velocity is uniform (zero acceleration). If the slope changes with time the motion is accelerated.

Slope of s–t graph is constant. Hence, velocity of particle is constant. Further at time t = 0, displacement of the particle from the mean position is $-s_0$ i.e. –20 m.

velocity of particle, $v = \text{slope} = \frac{s_0}{t_0} = \frac{20}{4} = 5\,m/s$



Motion of the particle is as shown in Fig. 2.72. At t = 0 is at –20 m and has a constant velocity of 5 m/s. At t_0 = 4 sec particle will pass through its mean position.

Example 9: Abhishek is moving with velocity $5\hat{i}+3\hat{j}$ and Amit with velocity $2\hat{i}+4\hat{j}-3\hat{k}$ Find

(i) Relative velocity of Abhishek w.r.t. Amit $(\vec{v}_{12})$

(ii) Relative velocity of Amit w.r.t. Abhishek $(\vec{v}_{21})$

Sol: Relative velocity of body 1 with respect to body 2 is obtained by vector sum of the velocity of body 1 and the negative of the velocity of body 2.

$$\text{Velocity of Abhishek}(\vec{v}_1) = 5\hat{i}+3\hat{j}$$

$$\text{Velocity of Amit}(\vec{v}_2) = 2\hat{i}+4\hat{j}-3\hat{k}$$

(i) $\vec{v}_{12} = \vec{v}_1 - v_2 = (5\hat{i}+3\hat{j})-(-2\hat{i}+4\hat{j}-3\hat{k}) = 7\hat{i}-\hat{j}+3\hat{k}$

(ii) $\vec{v}_{21} = \vec{v}_2 - \vec{v}_1 = (-2\hat{t}+4\hat{j}-3\hat{k})-(5\hat{i}-3\hat{j})$

$= -7\hat{t}+\hat{j}-3\hat{k} = -\vec{v}_{12}$

Note: $\vec{v}_{21} = -\vec{v}_{12}$

Example 10: Two trains, each of length 100 m, move in opposite direction along parallel lines at speeds 60 km/h and 30 km/h, respectively. If their accelerations are 30 cm/s^2 and 20 cm/s^2, respectively, then find the time they take to pass each other.

Sol: This problem is best solved in the reference frame of one of the trains. Find the initial relative velocity of one train with respect to the other, the relative acceleration of one train with respect to the other and the relative displacement of one train with respect to other as they pass each other. Use the second equation of motion with constant acceleration to find the required time.

The relative displacement of the trains is

S=100 + 100 = 200 M.

The initial velocity u of one train relative to the other train

$$=(60+30)\text{km/h} = 90\times\frac{1000}{3600}\text{m/s} = 90\times\frac{5}{18}\text{m/s} = 25\text{ m/s}$$

Relative acceleration, a = (30+20)cm/s^2 = 0.5m/s^2

If "t" is the time taken to cross each other,

$$s = ut + \frac{1}{2}at^2;\ 200 = 25t + \frac{1}{2}\times 0.5t^2$$

$0.5\,t^2 + 50\,t - 400 = 0$

$$t = \frac{-50 \pm \sqrt{2500+4\times 400\times 0.5}}{2\times 0.5} = -50 \pm \sqrt{3300}$$

As negative t is ignored,

$$t = -50 + \sqrt{3300} = -50 + 57.44$$

t = 7.44 sec

JEE Advanced/Boards

Example 1: Two cars started simultaneously towards each other from towns A and B which are 480 km apart. It took first car travelling from A to B 8 hours to cover the distance and second car travelling from B to A 12 hours. Determine the distance (in km) from town A where the cars meet. Assuming that both the cars travelled with constant speed.

Sol: This problem is best solved in the reference frame of one of the cars. Find the initial relative velocity of one car with respect to the other. The time elapsed before the cars meet is equal to the initial distance between the two cars divided by the relative velocity.

$$\text{Velocity of car from } A = \frac{480}{8} = 60\,\text{km/hour}$$

$$\text{Velocity of car from } B = \frac{480}{12} = 40\,\text{km/hour}$$

$$\therefore t = \frac{480}{60+40} = 4.8\text{ hour}$$

The distance $s = v_A \times t = 60 \times 4.8 = 288$ km

Example 2: An engine driver running a train at full speed suddenly applies brakes and shuts off steam. The train then travels 24 m in the first second and 22 m in the next second. Assuming that the brakes produce a constant retardation, find

(i) Original speed of the train

(ii) The time elapsed before it comes to rest

(iii) The distance travelled during the interval

(iv) If the length of the train is 44 m,

find the time that the train takes to pass an observer standing at a distance 100 m ahead of the train at the time when the brake was applied.

Sol: The retardation of train is constant, so we can use the equations of motion with uniform acceleration. The acceleration is taken with a negative sign.

(i) The distance covered by the body in nth second,

$$S_n = u + \frac{a}{2}(2n-1)$$

$$S_1 = 24 = u - \frac{a}{2}(2-1) = u - \frac{a}{2}$$

$$S_2 = 22 = u - \frac{a}{2}(4-1) = u - \frac{3a}{2}$$

Subtracting, $2 = \frac{3a}{2} - \frac{a}{2}; a = 2m/s^2$

$$u = 24 + \frac{a}{2} = 24 + 1 = 25\ m/s$$

(ii) Time t taken by the train before coming to rest,

v=u-at or 0 = 25 – 2t or t =12.5 sec.

(iii) If S is the distance before the train comes to rest i.e.

v=0; 0 = $u^2 - 2aS$; $S = \frac{u^2}{2a} = \frac{(25)^2}{2x2} = 156.25m.$

(iv) The time t taken by the train to cover a distance of 100 m is given by $S = ut - \frac{1}{2}at^2$,

$100 = 25t - \frac{1}{2}x2xt^2$; t^2 -25t + 100 = 0.

(t-20) (t-5) = 0.; t=20, t=5,

t = 20, is not possible as the train takes only 12.5 second to stop. Therefore t=5 second Time t′ taken by the train to cover a distance of 100 m plus length of the train, i.e, 44m, is given by $S = 100 + 44 = ut' - \frac{1}{2}at'^2$

$$25t' - \frac{1}{2}x2xt'^2 - 144 = 0;\ t'^2 - 25t' + 144 = 0$$

$$t'^2 - 16t' + 9t + 144 = 0;\ (t-16)(t'-9) = 0$$

$$t = 16\ s, t' = 9\ s$$

∴ Time taken by the train to pass the observer = 9 – 5 = 4 second.

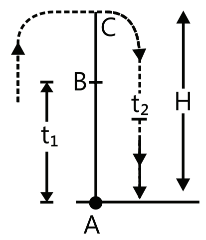
Example 3: A particle is projected vertically upwards from a point A on the ground. It takes a time t_1 to reach a point B at a height h above the ground as it continues to move, it takes a further time t_2 to reach the ground. Find

(i) The height h

(ii) The maximum height reached

(iii) The velocity of the particle at half the maximum height.

Sol: (i) Find the initial velocity u in terms of t_1 and t_2. Use the equations of motion with uniform acceleration. The acceleration due to gravity is taken with a negative sign.



Let u be initial velocity.

Total time of flight from A to B and from B to C to A$= t_1 + t_2 = \frac{2u}{g}$; $u = \frac{g(t_1 + t_2)}{2}$

$$h = ut_1 - \frac{1}{2}gt_1^2 = \frac{g}{2}(t_1 + t_2)t_1 - \frac{1}{2}gt_1^2 = \frac{gt_1t_2}{2}$$

(ii) Maximum height reached, AC = H $= \frac{u^2}{2g}$

$$= \frac{g^2(t_1 + t_2)^2}{4x2g} = \frac{g(t_1 + t_2)^2}{8}$$

(iii) Let v be velocity at height $\frac{H}{2}$,

$$v^2 = u^2 - 2g\frac{H}{2} = u^2 - gH$$

$$= \frac{g^2(t_1 + t_2)^2}{4} - \frac{g^2(t_1 + t_2)^2}{8} = \frac{g^2}{8}(t_1 + t_2)^2$$

$$v = \frac{g}{2\sqrt{2}}(t_1 + t_2)$$

Example 4: If v(s) = s^2 + s where s is displacement. Find acceleration when displacement is 1 m.

Sol: Differentiate the expression for velocity with respect to time to get the expression for acceleration.

We know $a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \left(\frac{dv}{ds}\right)(v)$

$$\Rightarrow a = v\frac{dv}{ds};\quad \therefore \frac{dv}{ds} = (2s) + (1)$$

and $\left(\frac{dv}{ds}\right)_{s=1m} = 3m$; $v(s) = s^2 + s$

$\therefore v(1) = (1)^2 + 1 = 2$; $\therefore a(s = 1m) = (2)x(3) = 6\ m/sec^2$

Example 5: A point mass moves along a straight line with a deceleration n which is equal to $K\sqrt{v}$ where K is a positive constant and v is the velocity of the particle. The velocity of the point mass at t = 0 is equal to v_0. Find the distance it will travel before it stops and the time it will take to cover this distance.

Sol: In the expression for acceleration separate the variables and integrate to get the desired quantity.

Acceleration $= \frac{dv}{dt} = -K\sqrt{v}$; $\frac{dv}{-\sqrt{v}} = Kdt.$

Let t_0 be the time which the particle takes to come to a stop.

Integrating

$$\int_0^{t_0} Kdt = -\int_{v_0}^{0} v^{-\frac{1}{2}} dv = \int_0^{v_0} v^{-\frac{1}{2}} dv = [2v^{1/2}]_0^{v_0}$$

$$Kt_0 = 2v_0^{\frac{1}{2}} \text{ or } t_0 = \frac{2v_0^{1/2}}{k}$$

$$\frac{dv}{dt} = \left(\frac{dv}{ds}\right)\left(\frac{ds}{dt}\right) = v\frac{dv}{ds};\ \therefore v\frac{dv}{ds} = -K\sqrt{v}$$

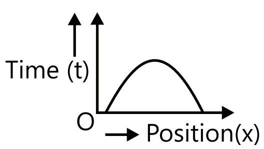
$$\sqrt{v}\,dv = -Kds$$

Let s_0 be the dist nce covered when the velocity decrease from v_0 to zero.

Integrating, $\int_{v_0}^{0} \sqrt{v}\,dv = -\int_0^{s_0} Kds = -Ks_0$

or $\left[\frac{v^{3/2}}{3/2}\right]_{v_0}^{0} = -\left[\frac{v_0^{3/2}}{\frac{3}{2}}\right] = -Ks_0 ;\ \therefore s_0 = \frac{2{v_0}^{3/2}}{3k}$

Example 6: Is the variation of position, shown in Figure. observed in nature?



Sol: Time never decreases in a reference frame.

No, since with increase of position x, time first increase and then decrease, which is impossible (Time always increase)

Example 7: A particle start from rest with constant acceleration for 100 s, then move with constant velocity for 5 min. Finally, particle retards uniformly and come to rest in 150 s.

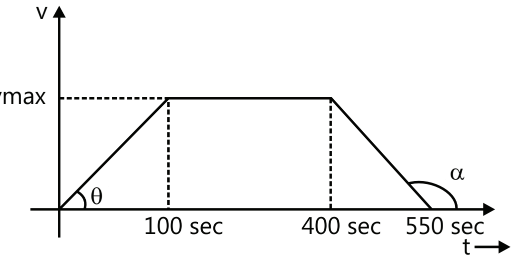
(i) Draw v-t graphs

(ii) If total distance travelled by the particle is 4250 m then find maximum speed.

(iii) Also, find the value of acceleration and retardation.

Sol: Area under the v-t graph in the given time interval is equal to displacement of the particle in the given time interval.

(i)



(ii) Distance travelled = Area under v – t graph

$$= \left[\frac{1}{2}(100)v_{max}\right] + \left[(400-100)\ (v_{max})\right] + \left[\frac{1}{2}(550-400)v_{max}\right]$$

$$\Rightarrow 4250 = \frac{1}{2} x 100 x v_{max} + 300 v_{max} + \frac{1}{2} x 150 v_{max}$$

$$\Rightarrow 4250 = v_{max} \Rightarrow v_{max} = 10 m/s$$

(iii) $a = \frac{\Delta v}{\Delta t}$

For $t = 0$ to 100 sec $\Rightarrow a = \frac{10-0}{100-0} = 0.1\ m/s^2$

For $t = 100$ to 400 sec $\Rightarrow a = \frac{10-10}{400-1000} = 0$

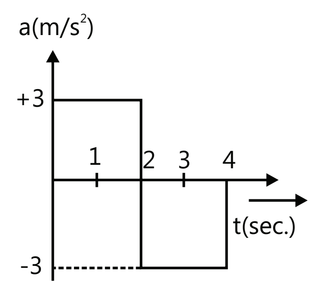
For t = 400 sec to 500 sec (retardation)

$\Rightarrow a^- = \frac{0-10}{550-400} = -\frac{10}{150} \Rightarrow 0.06\ m/s^2$

Example 8: A particle starts from rest at time t = 0 and undergoes acceleration a, as shown in the Figure.

(i) Draw a neat sketch showing the velocity of the particle as a function of time during the interval 1 to 4 seconds, indicating each second on the abscissa.

(ii) Draw a neat sketch showing the displacement of the particle as a function of time during 0 to 2 second. In both the cases, explain the various steps.



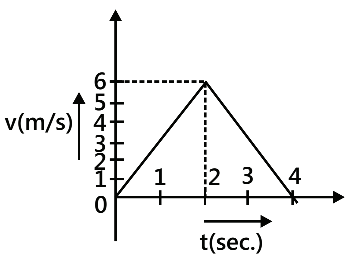
Sol: Area under the a-t graph in the given time interval is equal to the change in velocity of the particle in the given time interval. Thus v-t graph can be plotted if the initial velocity is known. Area under the v-t graph in the given time interval is equal to the displacement of the particle in the given time interval.

(i) The velocity is given by the area enclosed during the time interval; and the velocity is constant from 0 to 2 sec.

At t = 1 sec., velocity = 3 m/s

At t = 2 sec., velocity = 6 m/s.

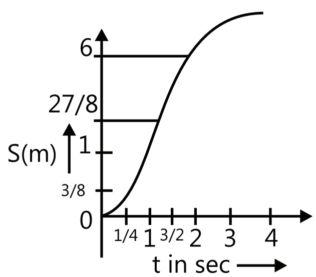
At this instant the acceleration becomes negative, so the velocity starts decreasing uniformly.



At t = 3 sec., velocity = 6 -3 = 3 m/s

At t = 4 sec., velocity = 6 – 6 = 0 m/s.

(ii) The particle starts from rest, and the acceleration a is constant from 0 to 2 sec.



$S = \frac{1}{2}at^2$ The graph between S and t will be parabola,

$$\text{At } t = \frac{1}{2}\text{sec}, S_{1/2} = \frac{1}{2} \text{x} 3 \text{x} \frac{1}{4} = \frac{3}{8}\text{m}$$

$$\text{At } t = 1\text{sec}, S_{1/2} = \frac{1}{2} \text{x} 3 \text{x} 1 = \frac{3}{2}\text{m}$$

$$\text{At } t = \frac{3}{2} \text{ sec } S_{1/2} = \frac{1}{2} \text{x} 3 \text{x} \frac{9}{4} = \frac{27}{8}\text{m}$$

$$\text{At } t = 2 \text{ sec } S_2 = \frac{1}{2} \text{x } 3 \text{ x } 4 = 6\text{m}$$

Beyond t = 4s, the acceleration becomes negative, the curvature of the graph becomes opposite at this instant.

In the interval between t = 2s and t = 3s,

$$\text{distance travelled} = 6 \text{ x } 1 - \frac{1}{2} \text{x} 3 \text{x} = 4.5\text{m}$$

At t = 3, S3 = 6 + 4 ½ = 10.5 m

At t = 4, S4 = 6 + [6 x 2- ½ x 3 x 4] = 12 m

Example 9: A ball is dropped from a height of 19.6 m above the ground. It rebounds from the ground and raises itself up to the same height. Take the starting point as the origin and vertically downward as the positive X-axis. Draw approximate plots of x versus t, v versus t and a versus t. Neglect the small interval during which the ball was in contact with the ground.

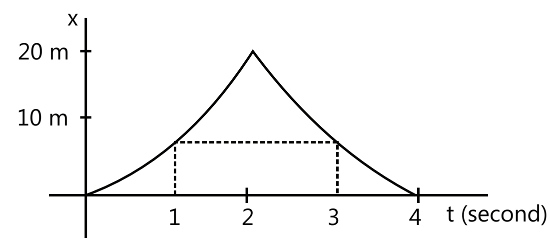
Sol: From the first equation of motion for constant acceleration, plot the v-t graph. From the second equation of motion for constant acceleration, plot the x-t graph.

Since the acceleration of the ball during the contact is different from 'g', we have to treat the downward motion and the upward motion separately.

For the downward motion: a = g = 9.8 m/s^2,

$$x = ut + \frac{1}{2}at^2 = (4.9 m/s^2)t^2$$

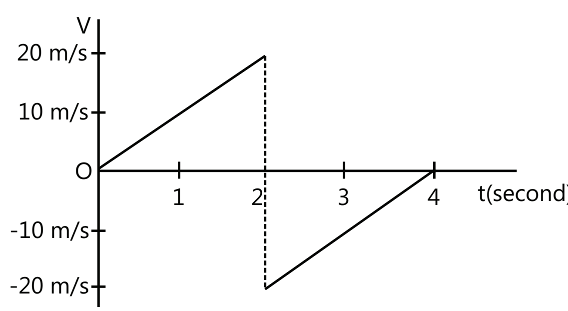
The ball reaches the ground when x = 19.6 m. This gives t = 2 s. After that it moves up, x decreases and at t = 4 s, x becomes zero, the ball reaching the initial point.



We have t = 0, x = 0

t = 1s,	x=4.9 m
t= 2s	x =19.6 m
t = 3s	x = 4.9 m
t = 4s	x = 0

Velocity: During the first two seconds



$v = u + at = (9.8 \text{ m/s}^2)*t$

at t = 0 $\quad$ v = 0

at t = 1 s, $\quad$ v = 9.8 m/s

at t = 2 s, $\quad$ v = 19.6 m/s

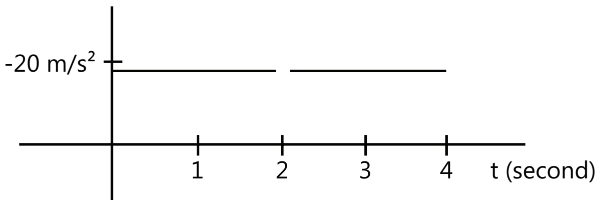
During the next two seconds the ball goes upward, velocity is negative, magnitude decreasing and at t = 4 s, v = 0. Thus

at t = 2 s, $\quad$ v = -19.6 m/s

at t = 3 s, $\quad$ v = -9.8 m/s

at t = 4 s, v = 0.

At t = 2 s there is an abrupt change in velocity from 19.6 m/s to -19.6 m/s. In fact this change in velocity takes place over a small interval during which the ball remains in contact with the ground.

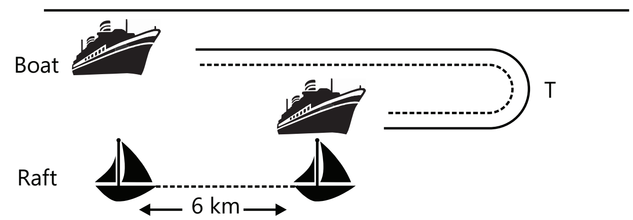


Acceleration: The acceleration is constant 9.8 m/s^2 throughout the motion (except at t=2s).

Example 10: Boat is moving down the stream and crosses a raft at t = 0 sec. After 1 hour boat turns and again crosses the raft at a point 6 km from the initial position of raft. Find the velocity of river assuming duty of engine remain constant.

Note: raft is an object which floats on the river i.e. it has zero velocity w.r.t. river.

Sol: Relative to the raft the boat is moving with constant speed. Distance travelled downstream relative to raft is equal to the distance travelled upstream relative to the raft. Total time of travel of boat is 2 h.



V_r = velocity of river w.r.t ground

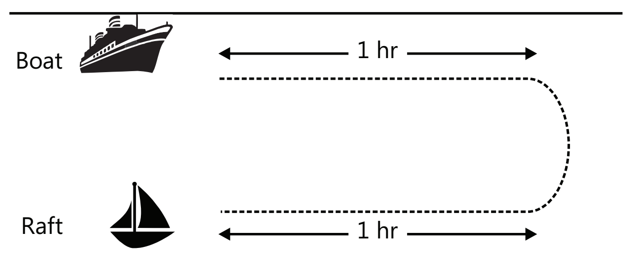
V_b = velocity of boat w.r.t ground

Let the time taken from starting to the end = T

Velocity of raft w.r.t. ground = V_r

Distance travelled by raft in time T = 6 km

$\Rightarrow V_r T = 6$ km



W.r.t river, since boat travels for 1 h downstream and since, velocity of boat remains constant and raft doesn't move w.r.t river so, boat will again take 1 hour to reach the raft back.

$\therefore$ T = 1 + 1 = 2 h

On substituting value of T in (i) we get

$V_r (2) = 6$ km; $\Rightarrow V_r = 3$ km/h

JEE Main/Boards

Exercise 1

Q.1 A body starting from rest has an acceleration of 20 ms^{-2}. Calculate the distance travelled by it in 6th second.

Q.2 A train was moving at rate of 36 kmh-1. When the brakes were applied, it comes to rest in a distance of 200 m. Calculate the retardation produced in the train.

Q.3 A body covers 12 m in 2nd second and 20 m in 4th second. Find what distance the body will cover in 4 seconds after the 5th second.

Q.4 A racing car moving with constant acceleration covers two successive kilometers in 30 s and 20 s respectively. Find the acceleration of the car.

Q.5 Two cars start off to race with velocities 2 m/s and

4 m/s travel in straight line with uniform acceleration $2m/s^2$ and a m/s^2 respectively. What is the length of the path if they reach the final point at the same time?

Q.6 Brakes are applied to a train travelling at 72 kmh-1. After passing over 200 m, its velocity is raced to 36 kmh-1. At the same rate of retardation, how much further will it go before it is brought to rest?

Q.7 On turning a corner, a motorist rushing at 44 ms-1 finds a child on the road 100 m ahead. He instantly stops the engine and applies the brakes so as to stop it within 1 m of the child. Calculate time required to stop it.

Q.8 A body starting from rest, was observed to cover 20 m in 1 second and 40 m during the next second. How far had it travelled before the first observation was taken?

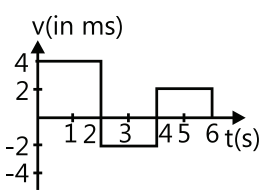
Q.9 An automobiles starts from rest and accelerates uniformly for 30 seconds to a speed of 72 km h^{-1}. It then moves with a uniform velocity and it is finally brought to rest in 50 m with a constant retardation. If the total distance travelled is 950 m, find the acceleration, the retardation and total time taken.

Q.10 From the top of a tower 100 m in height a ball is dropped and at the same instant another ball is projected vertically upwards from the ground so that it just reaches the top of tower. At what height do the two balls pass one another?

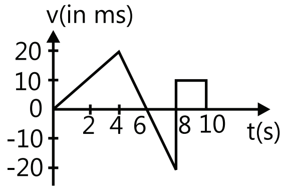
Q.11 A body falling from rest was observed to fall through 78.4 m in 2 seconds. Find how long had it been falling before it was observed?

Q.12 A stone is dropped from a balloon at an altitude of 300 m. How long will the stone take to reach the ground if (i) the balloon is ascending with a velocity of 5 ms^{-1}. (ii) the balloon is descending with a velocity of 5 ms^{-1}. (iii) the balloon is stationary?

Q.13 The velocity–time graph of a body moving in a straight line is shown in Fig. Find the displacement and the distance travelled by the body in 6 seconds.



Q.14 The velocity–time graph of a particle moving along a straight line is as shown in Fig. Calculate the distance covered between t = 0 to t = 10 second. Also find displacement in time 0 to 10 seconds.



Q.15 The position of an object moving along x-axis is given by $x=a+bt^2$, where a = 8.5 m and b = 2.5 ms^{-2} and and t is measured in second. What is the velocity at t=0 s and t=2.0 s? What is the average velocity between t = 2.0 s and t = 4.01 s?

Q.16 The displacement x (in m) of a body varies with time t (in sec) as $x = -(2/3)t^2 + 16t + 2$. How long does the body take to come to rest?

Q.17 The height y and the distance x along the horizontal, for a body projected in the vertical plane are given by $y = 8t - 5t^2$ and x = 6 t. What is initial velocity of the body?

Q.18 The displacement of a particle along X-axis is given by $x = 3 + 8t + 7t^2$. Obtain its velocity and acceleration at t = 2s.

Q.19 The relation between time t and distance x is $t = \alpha x^2 + \beta x$ where α and β are constants. Show that retardation is $2\alpha \upsilon$, where υ is the instantaneous velocity.

Q.20 The acceleration 'a' in ms^{-2} of a particle is given by $a = 3t^2 + 2t + 2$, where t is the time. If the particle starts out with a velocity $\upsilon = 2\ ms^{-1}$ at t = 0, then find the velocity at the end of 2s.

Q.21 A tennis ball is dropped onto the floor from a height of 4.0 ft. It rebounds to a height of 3.0 ft. If the ball was in contact with the floor for 0.010 s, what was the average acceleration during contact?

Q.22 A particle starts from rest with zero initial acceleration. the acceleration increases uniformly with time. Find the time average and distance average of velocity upto a certain instant when the velocity becomes v.

Q.23 A particle moves along a straight line such that its displacement x from a fixed point on the line at time t is given by $x^2 = at^2 + 2bt + c$ Find acceleration as a function of displacement x.

Q.24 A ball is dropped from a height of 19.6 m above the ground. It rebounds from the ground and raises itself up to the same height. Take the starting point as the origin and vertically downward as the positive X-axis. Draw approximate plots of x versus t, v versus t and a versus t. Neglect the small interval during which the ball was in contact with ground.

Q.25 A train travelling at 72 km/h is checked by track repairs. It retards uniformly for 200 m covering the next 400 m at constant speed and accelerates uniformly to 72 km/h in a further 600 m. If the time at constant lower speed is equal to the sum of the times taken in retarding and accelerating. Find the total time taken.

Q.26 A point traversed half the distance with a velocity v_0. The remaining part of the distance was covered with velocity v_1 for half the time, and with velocity v_2 for the other half of the time. Find the men velocity of the point averaged over the whole time of motion.

Q.27 A person sitting on the top of a tall building is dropping balls at regular intervals of one second. Find the positions of the 3rd, 4th and 5th ball when the 6th ball is being dropped.

Q.28 A stone is dropped from a balloon going up with a uniform velocity of 5.0 m/s. if the balloon was 50 m high when the stone was dropped, find its height when the stone hits the ground.

Take $g = 10\ m/s^2$.

Exercise 2

Single Correct Choice Type

Q.1 An object is moving along the x axis with position as a function of time given by x = x(t). Point O is at x = 0. The object is definitely moving towards O when

(A) dx/dt < 0 (B) dx/>dt>0

(C) $d(x^2)/dt < 0$ (D) $d(x^2)/dt > 0$

Q.2 A particle starts moving rectilinearly at time t = 0 such as that its velocity 'v' changes with time 't' according to the equation $v = t^2 - t$ where t is in seconds and v is in m/s. The time interval for which the particle retards is

(A) t < 1/2 (B) 1/2 < t <1

(C) t> 1 (D) t < ½ and t>1

Q.3 An object is tossed vertically into the air with an initial velocity of 8 m/s. Using the sign convention upwards as positive, how does the vertical component of the acceleration a_y of the object (after leaving the hand) vary during the flight of the object?

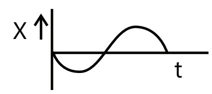
(A) On the way up $a_y > 0$, on the way down $a_y > 0$

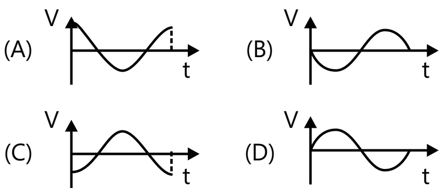
(B) On the way up $a_y < 0$, on the way down $a_y < 0$

(C) On the way up $a_y > 0$, on the way down $a_y < 0$

(D) on the way up $a_y < 0$, on the way down $a_y < 0$

Q.4 If position time graph of a particle is since curve as shown, What will be its v-t graph?





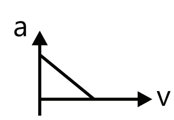
Q.5 A man moves in x-y plane along the path shown. At what point is his average velocity vector in the same direction as his instantaneous velocity vector. The man starts from point P.

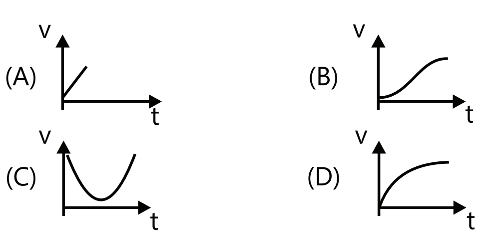
(A) A (B) B (C) C (D) D

Q.6 The greatest acceleration or deceleration that a train may have is a. The minimum time in which the train reach from one station to the other separated by a distance D is

(A) $\sqrt{\frac{d}{a}}$ (B) $\sqrt{\frac{2d}{a}}$ (C) $\frac{1}{2}\sqrt{\frac{d}{a}}$ (D) $2\sqrt{\frac{d}{a}}$

Q.7 Acceleration versus velocity graph of a particle moving in a straight line starting from rest is as shown in figure. The corresponding velocity–time graph would be





Q.8 Suppose a player hits several baseballs, which baseball will be in the air for the longest time?

(A) The one with the farthest range.

(B) The one which reaches maximum height.

(C) The one with the greatest initial velocity.

(D) The one leaving the bat at 45° with respect to the ground.

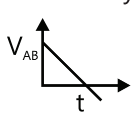
Q.9 A ball is thrown from a point on ground at some angle of projection. At the same time a bird starts from a point directly above this point of projection at a height h horizontally with speed u. Given that in its flight ball just touches the bird at one point. Find the distance on ground where ball strikes.

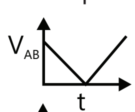
(A) $2u\sqrt{\frac{h}{g}}$ (B) $u\sqrt{\frac{2h}{g}}$ (C) $2u\sqrt{\frac{2h}{g}}$ (D) $u\sqrt{\frac{h}{g}}$

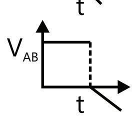
Q.10 It takes one minute for a passenger standing on an escalator to reach the top. If the escalator does not move it takes him 3 minute to walk up. How long will it take for the passenger to arrive at the top if he walks up the moving escalator?

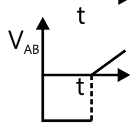
(A) 30 sec (B) 45 sec (C) 40 sec (D) 35 sec

Q.11 A body A is thrown vertically upwards with such a velocity that it reaches a maximum height of h. simultaneously another body B is dropped from height h. It strikes the ground and does not rebound. The velocity of A relative to B v/s time graph is best represented by: (upward direction is positive)

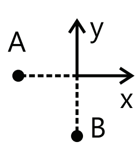
(A) 

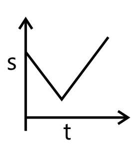
(B) 

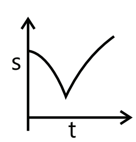
(C) 

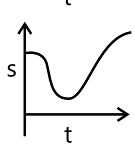
(D) 

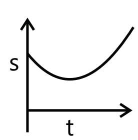
Q.12 Particle A and B are moving with constant velocities along x and y axis respectively, the graph of separation between them with time is



(A) 

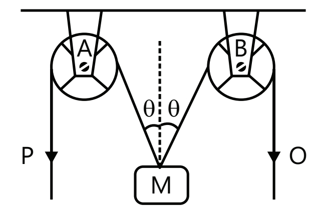
(B) 

(C) 

(D) 

Previous Years' Questions

Q.1 In the arrangement shown in the Figure. the ends P and Q of an unstretchable string move downwards with uniform speed U. Pulleys A and B are fixed.



Mass M moves upwards with s speed ***(1982)***

(A) 2U cos θ (B) U/cos θ

(C) 2U/cos θ (D) U cos θ

Q.2 A particle is moving eastwards with a velocity of 5 m/s. In 10 s the velocity changes to 5 m/s northwards. The average acceleration in this time is ***(1982)***

(A) Zero

(B) $\frac{1}{\sqrt{2}} m/s^2$ towards north-east

(C) $\frac{1}{\sqrt{2}} m/s^2$ towards north-west

(D) $\frac{1}{2} m/s^2$ towards north

Q.3 A river is flowing from west to east at a speed of 5 m/min. A man on the south bank of the river, capable of swimming at 10 m/min in still water to swim across the river in the shortest time. He should swim in a direction. ***(1983)***

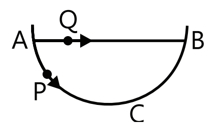
(A) Due north (B) 30° east of north

(C) 30° west of north (D) 60° east of north

Q.4 A boat which has a speed of 5 km/h still crosses a river of width 1 km along the shortest possible path in 15 min. The velocity of the river water in km/h is ***(1988)***

(A) 1 (B) 3 (C) 4 (D) $\sqrt{41}$

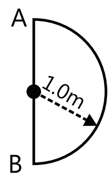
Q.5 A particle P is sliding down a frictionless hemispherical bowl. It passes the point A at t = 0. At this instant of time, the horizontal component of its velocity is v. A bead Q of the same mass as P is ejected from A at t = 0 along the horizontal string mass AB, with the speed v. Friction between the bead and the string may be neglected. Let t_p and t_Q be the respective times taken by P and Q to reach the point B. Then ***(1993)***



(A) $t_p < t_Q$ (B) $t_p = t_Q$

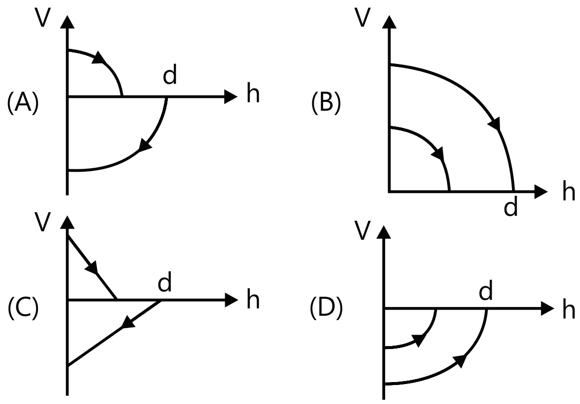
(C) $t_p > t_Q$ (D) $\frac{t_p}{t_Q}\frac{\text{length of arc ACB}}{\text{length of chord AB}}$

Q.6 In 1.0 s, a particle goes from point A to point B, moving in a semicircle (see the Figure). The magnitude of the average velocity is ***(1999)***

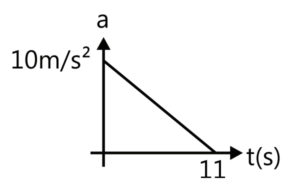


(A) 3.14 m/s (B) 2.0 m/s (C) 1.0 m/s (D) Zero

Q.7 A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height d/2. Neglecting subsequent motion and air resistance, its velocity v varies with height h above the ground as ***(2000)***



Q.8 A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the Figure. The maximum speed. The maximum speed of the particle will be ***(2004)***



(A) 110 m/s (B) 55 m/s

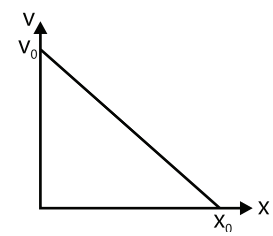
(C) 550 m/s (D) 660 m/s

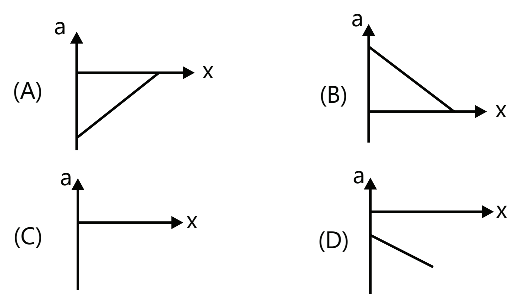
Q.9 A small block slides without friction down an inclined plane starting from rest. Let s_n be the distance travelled from $t = n-1$ to $t = n$. Then $\frac{s_n}{s_n+1}$ is ***(2004)***

(A) $\frac{2n-1}{2n}$ (B) $\frac{2n+1}{2n-1}$

(C) $\frac{2n-1}{2n+1}$ (D) $\frac{2n}{2n+1}$

Q.10 The given graph shows the variation of velocity with displacement. Which on of the graph given below correctly represents the variation of acceleration with displacement. ***(2005)***





Q.11 From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is: ***(2014)***

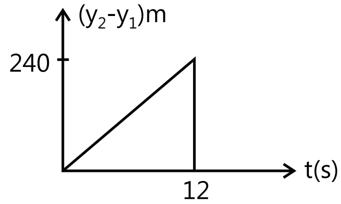
(A) $2gH = nu^2(n-2)$ (B) $gH = (n-2)u^2$

(C) $2gH = n^2u^2$ (D) $gH = (n-2)^2u^2$

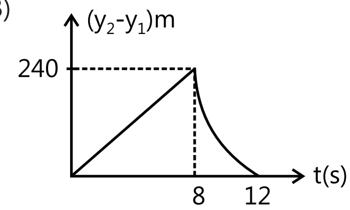
Q.12 Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?

(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\ m/s^2$) (The figures are schematic and not drawn to scale) ***(2015)***

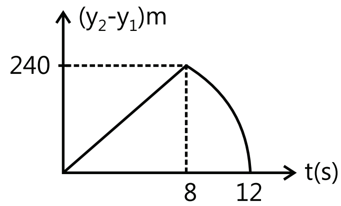
(A)



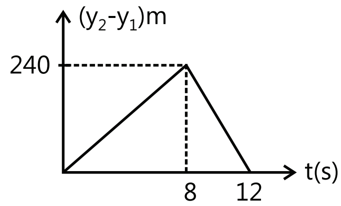
(B)



(C)



(D)



JEE Advanced/Boards

Exercise 1

Q.1 A car moving along a straight highway with a speed of 126 km/h is brought to stop with in a distance of 200 m. what is the retardation of the car (assumed uniform) and how long does it take for the car to stop?

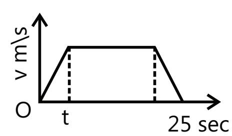
Q.2 A police van moving on a highway with a speed of 30 km/h fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km/h, if the muzzle speed of bullet is 150 m/s, with what speed does the bullet hit the thief's car.

Q.3 What is the ratio of the distance travelled by a body falling freely from rest during first, second and third second of its fall.

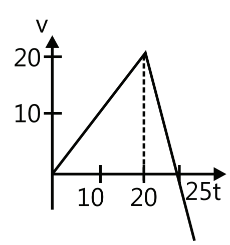
Q.4 At a distance L = 400 m from the traffic light breaks are applied to locomotive moving at a velocity v=54 km/hr. Determine the position of the locomotive relative to the traffic light 1 minute after the application of the breaks if its acceleration is $-0.3 m/s^2$

Q.5 An object moving with uniform acceleration has a velocity of 12 cm/s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is -5.00 cm, what is its acceleration?

Q.6 The velocity–time graph of the particle moving along the straight line is shown. The rate of acceleration and deceleration is constant and it is equal to $5 ms^{-2}$, If the average velocity during the motion is $20 ms^{-2}$, then the final value of t.



Q.7 The figure shows the v-t graph of a particle moving in the straight line. Find the time when particle returns to the starting point.



Q.8 A stone is dropped from a height h. simultaneously another stone is thrown up from the ground with such a velocity that it can reach a height of 4h. Find the time when two stones cross each other.

Q.9 A glass wind screen whose inclination with the vertical can be changed is mounted on a car. The car moves horizontally with a speed of 2 m/s. At what angle α with the vertical should the wind screen be placed so that the rain drops falling vertically downwards with velocity 6 m/s strikes the wind screen perpendicularly?

Q.10 Two particles are moving along two straight lines, in the same plane, with the same speed=20 cm/s. The angle between the two lines is 60°, and their intersection point is O. At a certain moment, the two particles are located at distances 3m and 4m from O, and are moving towards O. Find the shortest distance between them subsequently?

Q.11 A point mass starts moving in a straight line with a constant acceleration a. At a time t, after the beginning of motion, the acceleration changes sign, remaining the same magnitude. Determine the time t from the beginning of motion in which the point mass returns to the initial position?

Q.12 For $\left(\frac{1}{m}\right)^{th}$ of the distance between two stations, a train is uniformly accelerated and for $\left(\frac{1}{n}\right)^{th}$ of the distance, it is uniformly retarded. It starts from rest at one station and comes to rest at another. Find the ratio of its maximum velocity to its average velocity?

Q.13 The velocity of a particle moving in the positive direction of the x axis varies as v= α $v\sqrt{x}$, where α is positive constant. Assuming that at the moment t=0 the particle was located at the point x=0, find: (i) the time dependence of the velocity and acceleration of the particle.

(ii) the mean velocity of the particle averaged over the time that the particle takes to cover the first s meter of the path.

Q.14 A boy standing on a long railroad car throws a ball straight upwards. The car is moving on the horizontal road with an acceleration of 1 m/s^2 and projection velocity in the vertical direction is 9.8 m/s. How far behind the boy will the ball fall on the car?

Q.15 A person walks up a stalled escalator in 90 s. When standing on the same escalator, now moving, he is carried up to 60 s. How much time would it take him to walk up the moving escalator?

Q.16 Two trains of lengths 180 m are moving on parallel tracks. If they move in the same direction then they cross each other in 15 s, and if they move in opposite directions then they cross in 7.5 seconds, then calculate their velocities.

Q.17 At the instant the traffic light turns green, an automobile starts with a constant acceleration a_x of 6.0 ft/s^2, at the same instant a truck, travelling with a constant speed of 30 ft/s, overtakes and passes the automobile.

(i) how far beyond the straight point will automobile overtake the truck?

(ii) How fast will the automobile be travelling at that instant? (It is instructive to plot qualitative graph of x versus t for each vehicle.)

Q.18 Two bodies moves in a straight line towards each other at initial velocities v^1 and v^2 and constant accelerations a^1 and a^2 directed against the corresponding velocities at the initial instant.

What must be the maximum initial separation l_{max} between the bodies for which they meet during motion? Motion in which the point mass returns to the initial position?

Q.19 An ant runs from an ant-hill in a straight line so that its velocity is inversely proportional to the distance from the center of the ant-hill. When the ant is at point A at a distance $l_1 = 1m$ from the center of the ant-hill, its velocity $v_1 = 2cm/s$, what time will it take ant to run from point A to point B, which is at a distance $l_2 = 2m$ from the center of the ant-hill?

Q.20 Distance between two points A and B is 33 m. A particle P starts from B with velocity of 1m/s along AB with an acceleration of $2m/s^2$. Simultaneously another particle Q starts from A with a velocity of 9 m/s in the same direction AB and has an acceleration $1m/s^2$ in the direction AB. Find whether Q will be able to catch P.

Exercise 2

Multiple Correct Choice Type

Q.1 A particle moves with constant speed v along a regular hexagon ABCDEF in the same order. Then the magnitude of the average velocity for its motion from A to

(A) F is v/5 (B) D is v/3

(C) C is $v\sqrt{3}/2$ (D) B is v

Q.2 A particle moving with a speed v changes direction by an angle θ, without change in speed.

(A) The change in the magnitude of its velocity is zero.

(B) The change in the magnitude of its velocity is 2vsin (θ/2).

(C) The magnitude of change in velocity is 2vsin (θ/2).

(D) The magnitude of change in velocity is 2v (1 - cosθ).

Q.3 A particle has initial velocity 10 m/s. It moves due to constant retarding force along the line of velocity which produces a retardation of 5 m/s^2. Then

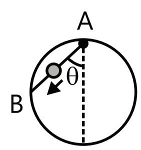
(A) the maximum displacement in the direction of initial velocity is 10 m.

(B) the distance travelled in first 3 seconds is 7.5 m.

(C) the distance travelled in the first 3 seconds is 12.5 m.

(D) the distance travelled in the first 3 seconds is 17.5 m.

Q.4 A bead is free to slide down a smooth wire tightly stretched between points A and B on a vertical circle. If the bead starts from rest at A, the highest point on the circle



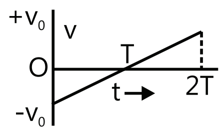
(A) Its velocity v on arriving at B is proportional to $\cos\theta$

(B) Its velocity v on arriving at B is proportional to $\tan\theta$

(C) Time to arrive at B is proportional to $\cos\theta$

(D) Time to arrive at B is independent of θ.

Q.5 The figure shows the velocity (v) of a particle plotted against time (t)



(A) The particle changes its direction of motion at some point.

(B) The acceleration of the particle remains constant.

(C) The displacement of the particle is zero.

(D) The initial and final speed of the particle are the same.

Q.6 An observer moves with a constant speed along the line joining two stationary objects. He will observe that the two objects

(A) Have the same speed.

(B) Have the same velocity.

(C) Move in the same direction.

(D) Move in opposite directions.

Q.7 A man on a rectilinearly moving cart, facing the direction of motion, throws a ball straight up with respect to himself

(A) The ball will always return to him.

(B) The ball will never return to him.

(C) The ball will return to him if the cart moves with constant velocity.

(D) The ball will fall behind him if the cart moves with some positive acceleration.

Assertion Reasoning Type

(A) Statement-I is true, statement-II, is true and statement-II is correct explanation for statement-I

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

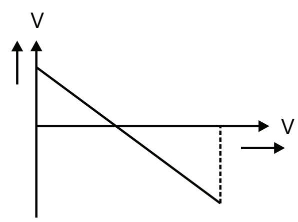
Q.8 Statement-I: Positive acceleration in rectilinear motion of a body does not imply that the body is speeding up

Statement-II: Both the acceleration and velocity are vectors.

Q.9 Statement-I: A particle having zero acceleration must have constant speed.

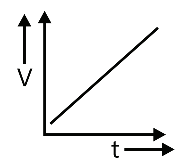
Statement-II: A particle having zero acceleration must have zero acceleration.

Q.10 Statement-I: A student performed an experiment by moving a certain block in a straight line. The velocity position graph cannot be as shown.



Statement-II: When a particle is at its maximum in rectilinear motion its its velocity must be zero.

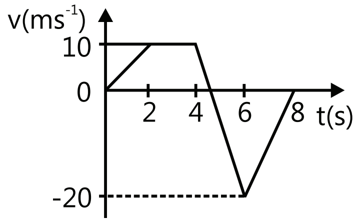
Q.11 Statement-I: If the velocity time graph of a body moving in a straight line is as shown here, the acceleration of the body must be constant



Statement-II: The rate of change of quantity which is constant is always zero.

Comprehension Type

Paragraph 1: The figure shows a velocity–time graph of a particle moving along a straight line



Q.12 Choose the incorrect statement. The particle comes to rest at

(A) t = 0 s (B) t = 5 s

(B) t = 8 s (C) None of these

Q.13 Identify the region in which the rate of change of velocity $\left|\frac{\Delta \vec{v}}{\Delta t}\right|$ of the particle is maximum

(A) 0 to 2s (B) 2 to 4s

(C) 4 to 6s (D) 6 to 8 s

Q.14 If the particle starts from the position

x_0=- 15 m, then its position at t=2s will be

(A) -52m (B) 5 m (C) 10 m (D) 15 m

Q.15 The maximum displacement of the particle is

(A) 33.3 m (B) 23.3 m

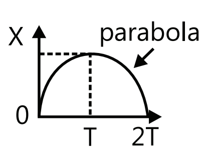
(B) 18.3 m (C) Zero

Q.16 The total distance travelled by the particle is

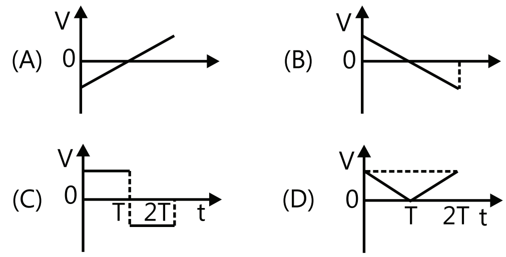
(A) 66.7 m (B) 51.6 m

(B) Zero (C) 36.6 m

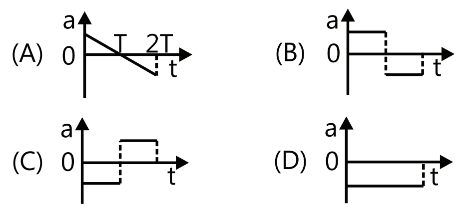
Paragraph 2: The x-t graph of the particle moving along a straight line is shown in the figure



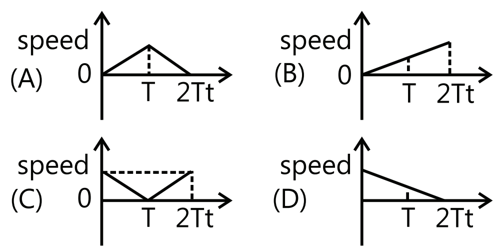
Q.17 The v-t graph of the particle is correctly shown by



Q. 18 The a-t graph of the particle is correctly shown by

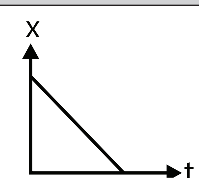
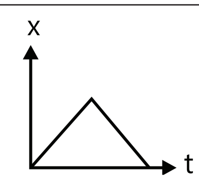
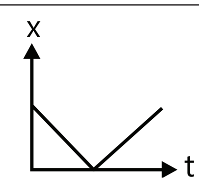


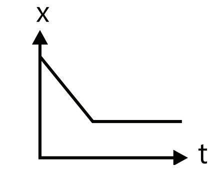
Q. 19 The speed–time graph of the particle is correctly shown by



Match the Columns

Q.20 Column I shows position versus time graph for an object and column II shows possible graphs.

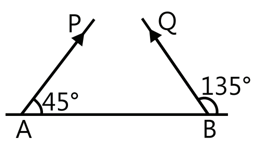
Column I	Column II
(A) 	(p) A ball rolls along the floor towards the origin
(A) 	(q) A ball rolled towards a wall at the origin, then ball rebounds.
(A) 	(r) A ball rolling away from the origin; hits a wall and bounces straight back.

Column I	Column II
(A) 	(s) An object rolling towards the origin and suddenly stops.
	(t) A book at rest on a table.

Previous Years' Questions

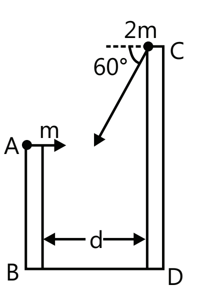
Q.1 Particles P and Q of mass 20 g and 40 g respectively are simultaneously projected from points A and B on the ground. The initial velocities of P and Q makes 45° and 135° angles respectively with the horizontal AB as shown in the Fig. Each particle has an initial speed of 49 m/s. The separation AB is 245 m.

Both particles travel in the same vertical plane and undergo a collision. After the collision, P retraces its path. Determine the position Q where it hits the ground. How much time after collision does the particle Q take to reach the ground? (Take $g = 9.8\ m/s^2$) ***(1982)***



Q.2 A body falling freely from a given height H hits an inclined plane in its path at a height h. As a result of this impact the direction of the velocity of the body becomes horizontal. For what value of (h/H) the body will take maximum time to reach the ground? ***(1986)***

Q.3 Two towers AB and CD are situated a distance d apart as shown in Figure. AB is 20 m high and CD is 30 m high from the ground. An object of mass m is thrown from the top of AB horizontally with a velocity of 10 m/s towards CD. Simultaneously other object of mass 2m is thrown from top of CD at an angle of 60° to the horizontal towards AB with the same magnitude of initial velocity as that of the first object. The two objects moves in the same vertical plane, collide in mid-air and stick to each other. ***(1994)***



(i) Calculate the distance d between the towers.

(ii) Find the position where the objects hit the ground.

Q.4 Two guns situated on the top of a hill of height 10 m fire one shot each with same speed $5\sqrt{3}$ m/s at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at point P$(g = 10 m/s^2)$ find

(i) the time interval between the firings and

(ii) the coordinates of the point P. Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x-y plane. ***(1996)***

Q.5 A cart is moving along x-direction with a velocity of 4 m/s. A person on the cart throws a stone with a velocity of 6 m/s relative to himself. In the frame of reference of the cart the stone is thrown in y-z plane making an angle of 30° with vertical z-axis. At the highest point of its trajectory the stone hits an object of equal mass hung vertically from the branch of a tree by means of a string of length L. A completely inelastic collision occurs, in which the stone gets embedded in the object. Determine (g=9.8 m/s^2) ***(1997)***

(i) the speed of the combined mass immediately after the collision with respect to an observer on the ground.

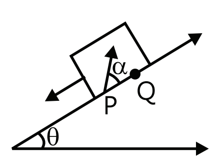
(ii) the length L of the string such that the tension in the string becomes zero when the string becomes horizontal during the subsequent motion of the combined mass.

Q.6 A particle of mass 10^{-2} kg is moving along the positive x-axis under the influence of a force F(x) $= -k/2x^2$ where $k = 10^{-2} Nm^2$. At time t=0 it is at x=0.1 m and its velocity v=0.

(i) Find its velocity when it reaches x=0.5 m.

(ii) Find the time at which it reaches x=0.25 m. ***(1998)***

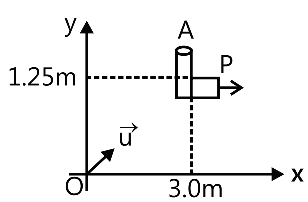
Q.7 A large heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to box is u and the direction of projection makes an angle α with the bottom as shown in the Figure. ***(1998)***



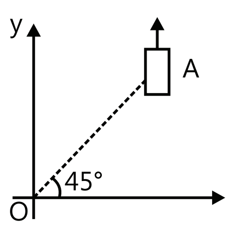
(i) Find the distance along the bottom of the box between the point of projection P and point Q where the particle lands (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

(ii) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected.

Q.8 An object A is kept fixed at the point x=3 and y=1.25 m on a plank P raised above the ground. At time t=0 the plank starts moving, along the +x-direction with an acceleration 1.5 m/s^2. At the same instant a stone is projected from the origin with velocity $\vec{u}$ as shown. A stationery person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in x-y plane, Find $\vec{u}$ and the time after which the stone hits the object. (Take g=10 m/s^2) ***(2000)***



Q.9 On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along a straight line parallel to the y-axis with a constant velocity of $(\sqrt{3}-1)$ m/s. At a particular instant when the line OA makes an angle of $45°$ with the x-axis, a ball is thrown along the surface from origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley. ***(2002)***



(i) The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in the frame.

(ii) Find the speed of the ball with respect to the surface, if $\phi = 4\theta/3$.

Q.10 A train is moving along a straight line with a constant acceleration a. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of $60°$ to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train in m/s^2, is. ***(2011)***

Assertion Reasoning Type

Mark your answer as

(A) If Statement-I is true, statement-II is true: statement-II is the correct explanation for statement-I.

(B) If Statement-I is true, statement-II is true: statement-II is not a correct explanation for statement-I.

(C) If Statement-I is true: statement-II is false.

(D) If Statement-I is false: statement-II is true.

Q.11 Statement-I For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.

Statement-II If the observer and the object are moving at velocities $\vec{v_1}$ and $\vec{v_2}$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\vec{v_2}-\vec{v_1}$ ***(2008)***

Q.12 A particle of mass m moves on the x-axis as follows: it starts from rest at t=0 from the point x-0, and comes to rest at t=1 at the point x=1. No other information is available about its motion at intermediate times (0<t<1). If α denotes the instantaneous acceleration of the particle, then ***(1993)***

(A) α cannot remain positive for all t in the interval $0 \le t \le 1$

(B) $|\alpha|$ cannot exceeds 2 at any point in its path

(C) $|\alpha|$ must be ≥ 4 at some point or points in its path

(D) α must change sign during the motion, but no other assertion can be made with information given.

Q.13 The coordinates of a particle moving in a plane are given by x(t)= acos (pt) and y(t)=bsin(pt) where a,b (<a) and p are positive constants of appropriate dimensions. Then

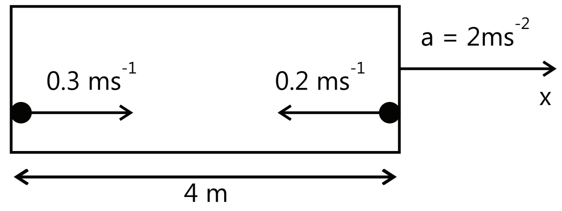
(A) the path of the particle is an ellipse

(B) the velocity and acceleration of the particle are normal to each other at t= $\pi/2p$

(C) the acceleration of the particle is always directed towards a focus.

(D) the distance travelled by the particle in time interval t=0 to t= $\pi/2p$ is a.

14. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in -x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is ***(2014)***



Questions

JEE Main/Boards

Exercise 1

Q. 9 Q.12 Q.15
Q.25 Q.27

Exercise 2

Q.3 Q.4 Q.7
Q.12

Previous Years' Questions

Q.1 Q.5

JEE Advanced/Boards

Exercise 1

Q.2 Q.5 Q.7
Q.9 Q.10 Q.17

Exercise 2

Q.4 Q.17 Q.18
Q.19 Q20

Previous Years' Questions

Q.1 Q.3 Q.5
Q.8

Answer Key

JEE Main/Boards

Exercise 1

Q.1 110m

Q.2 0.25 ms^{-2}

Q.3 136 m

Q.4 2/3 ms^{-2}

Q.5 24m

Q.6 66.67m

Q.7 4.5 s

Q.8 2.5 m

Q.9 2/3 ms^{-2}, 4 ms^{-2}, 65 sec

Q.10 75 m from ground

Q.11 3 sec

Q.12 (i) 8.36 s (ii) 7.33 s (iii) 7.82 s

Q.13 8m, 16m

Q.14 100m, 60m

Q.15 0 ms^{-1}; 10 ms^{-1}; 15 ms^{-1}

Q.16 12 second

Q.17 10 ms^{-2}

Q.18 36 ms^{-1}; 14 ms^{-1};

Q.20 18 ms^{-1}

Q.21 Approximately, 3000 ft/s^2 (in the upward direction)

Q.22 $< v >_{time} = 3v/5$

Q.23 $\frac{ac - b^2}{x^3}$

Q.25 2 min

Q.26 $\frac{2v_0(v_1 + v_2)}{v_1 + v_2 + 2v_0}$

Q.27 45 m, 20 m, 5 m

Q.28 68.5 m

Exercise 2

Single Correct Choice Type

Q.1 C	**Q.2** B	**Q.3** D	**Q.4** C	**Q.5** C	**Q.6** D
Q.7 D	**Q.8** B	**Q.9** C	**Q.10** B	**Q.11** C	**Q.12** D

Previous Years' Questions

Q.1 B	**Q.2** C	**Q.3** A	**Q.4** B	**Q.5** A	**Q.6** B
Q.7 A	**Q.8** B	**Q.9** C	**Q.10** A	**Q.11** A	**Q.12** B

JEE Advanced/Boards

Exercise 1

Q.1 11.43 sec,-3.06 ms^{-2}

Q.2 105 m/s

Q.3 1:3:5

Q.4 25m

Q.5 $-16\, cm/s^2$

Q.6 5 s

Q.7 36.2 sec

Q.8 $\sqrt{h/8g}$

Q.9 $\tan^{-1}(3)$

Q.10 $50\sqrt{3}$cm

Q.11 $t = t_1(2 + \sqrt{2})$

Q.12 $\left[1 + \frac{1}{m} + \frac{1}{n}\right] : 1$

Q.13 (i) $v = \frac{\alpha^2 t}{2}, a = \frac{\alpha^2}{2}$ (ii) $\overrightarrow{v} = \frac{\alpha\sqrt{S}}{2}$

Q.14 2 m

Q.15 36 s

Q.16 36 m/s, 12 m/s

Q.17 (i) 300 ft (ii) 60 ft/s

Q.18 $l_{max} = \frac{(v_1 + v_2)^2}{2(a_1 + a_2)}$

Q.19 t=75 s

Q.20 Q can not catch P

Exercise 2

Multiple Correct Choice Type

Q.1 A, C, D **Q.2** A, C **Q.3** A, C **Q.4** A, D **Q.5** A, B, C, D

Q.6 A, B, C **Q.7** C, D

Assertion Reasoning Type

Q.8 A **Q.9** D **Q.10** A **Q.11** B

Comprehension Type

Paragraph 1: **Q.12** B **Q.13** C **Q.14** A **Q.15** A **Q.16** A

Paragraph 2: **Q.17** B **Q.18** D **Q.19** C

Match the Columns

Q.20 A $\rightarrow$ p, s; B $\rightarrow$ r ; C $\rightarrow$ q ; D $\rightarrow$ s

Previous Years' Questions

Q.1 Just midway between A and B, 3.53 s

Q.2 $\frac{1}{2}$

Q.3 (i) Approximately 17.32 m (ii) 11.55 m from B

Q.4 (i) 1s (ii) $(5\sqrt{3}m, 5m)$

Q.5 (i) 2.5 m/s (ii) 0.32 m

Q.6 (i) -0.1 m/s (ii) 1.48 s

Q.7 (i) $\frac{u^2 \sin 2\alpha}{g\cos\theta}$ (ii) $\frac{u\ \cos(\alpha+\theta)}{\cos\theta}$(down the plane)

Q.8 $\overrightarrow{u} = (3.75\hat{i} + 6.25\hat{j})m/s, 1s$

Q.9 (i) 45° (ii) 2 m/s

Q.10 5

Q.11 B

Q.12 A, C

Q.13 A ,B, C

Solutions

JEE Main/Boards

Exercise 1

Sol 1: We use the formula $S_n = u + a\left(n - \frac{1}{2}\right)$

u = 0 (body starts from rest)

a = 20 ms^{-1}

n = 6

$S_n = 0 + 20\left(6 - \frac{1}{2}\right)$ = 110 m

Sol 2: We use the formula $v^2 - u^2 = 2as$

$u = 36km\ h^{-1} = 36 \times \frac{5}{18} ms^{-1} = 10\ ms^{-1}$

v = 0

s = 200m

$0^2 - 10^2 = 2a.200$

$a = -0.25\ ms^{-2}$

$\therefore$ Retardation is 0.25 ms^{-2}

Sol 3: $S_n = u + a\left(n - \frac{1}{2}\right)$

$12 = u + a\left(2 - \frac{1}{2}\right) \Rightarrow 12 = u + \frac{3}{2}a$ ... (i)

$20 = u + a\left(4 - \frac{1}{2}\right) \Rightarrow 20 = u + \frac{7}{2}a$ ... (ii)

Eq (2) – Eq (1)

$\Rightarrow 20 - 12 = \left(u + \frac{7}{2}a\right) - \left(u + \frac{3}{2}a\right)$

$\Rightarrow 8 = \frac{7-3}{2} = 2a \;\Rightarrow\; a = \frac{8}{2} = 4ms^{-2}$

Substituting in (i)

$12 = u + \frac{3}{2}.4$

$12 = u + \frac{3}{2}.4 = 12–6\ ms^{-1} = 6\ ms^{-1}$

$\Delta s = s_{t_2} - s_{t_1} = \left[u(t_2) + \frac{1}{2}at_2^{\,2}\right] - \left[u(t_1) + \frac{1}{2}at_1^{\,2}\right]$

$\Delta s = u(t_2 - t_1) + \frac{1}{2}a(t_2^{\,2} - t_1^{\,2})$

$\Delta s = s_{t_2} - s_{t_1} = \left[u(t_2) + \frac{1}{2}at_2^{\,2}\right] - \left[u(t_1) + \frac{1}{2}at_1^{\,2}\right]$

$\Delta s = u(t_2 - t_1) + \frac{1}{2}a(t_2^{\,2} - t_1^{\,2})$

$t_2 = a,\;\; t_1 = 5$

$\Delta s = u(9-5) + \frac{1}{2}a(9^2 - 5^2) = 6(9-5) + \frac{1}{2}4(9^2 - 5^2)$

$= 6(4) + 2(56)$

$\Delta s = 136m$

∴ Distance covered by body in 4 seconds after 5th second is 136 m

Sol 4: $v_1^2 - u_1^2 = 2as.$ ... (i)

$v_1 = u_1 + a(30);$ –30 seconds travel

$v_2^2 - u_2^2 = 2as.$ ... (ii)

$v_2 = u_2 + a(20);$ –20 seconds travel

Here $v_1 = u_2$

∴ adding (i), (ii)

$v_1^2 - u_1^2 = 4as$

$\boxed{(v_2 - u_1)(v_2 + u_1) = 4as}$ ... (iii)

$v_2 = u_2 + a(20)$

$u_2 = v_1 = u_1 + a(30)$

$\Rightarrow v_2 = u_1 + a(30) + a(20) = u_1 + 50a$

Substituting in (iii)

$(u_1 + 50a - u_1)(u_1 + u_1 + 50a) = 4as$

$50a(2u_1 + 50a) = 4as$

$\Rightarrow 50(2u_1 + 50a) = 4s$

$s = 1000m$

$\Rightarrow 50(2u_1 + 50a) = 4 \times 1000$

$\Rightarrow 2u_1 + 50a = \frac{4000}{50} = 80$

$\Rightarrow 2u_1 + 50a = 80$

Dividing by 2 on both sides

$\boxed{u_1 + 25a = 40}$ ... (iv)

$v_1^2 - u_1^2 = 2as$

$(v_1 - u_1)(v_1 + u_1) = 2as$

$v_1 = u_1 + 30a \quad , \quad s = 1000$

$(u_1 + 30a - u_1)(u_1 + 30a + u_1) = 2a \times 1000$

$30a(2u_1 + 30a) = 2000a$

$2u_1 + 30a = \frac{200}{3}$

$\boxed{u_1 + 15a = \frac{100}{3}}$ ... (v)

Subtracting (iv)–(v)

$u_1 + 25a - (u_1 + 15a) = 40 - \frac{100}{3}$

$10a = \frac{2.0}{3}$

$a = \frac{2}{3}$

acceleration of car is $\frac{2}{3}ms^{-2}$

Sol 5: $s_1 = u_1t + \frac{1}{2}a_1t^2$ ....for car 1

$s_2 = u_2t + \frac{1}{2}a_2t^2$ ....for car 2

$s_1 = s_2$

$\therefore u_1t+\frac{1}{2}a_1t^2=u_2t+\frac{1}{2}at^2$

$u_1=2ms^{-1}$ $a_1=2ms^{-2}$

$u_2=4ms^{-1}$ $a_2=1ms^{-2}$

$\Rightarrow 2(t)+\frac{1}{2}2t^2=4(t)+\frac{1}{2}1(t^2)$

$\Rightarrow (4-2)t=\frac{1}{2}(2-1)t^2 \Rightarrow 2t=\frac{1}{2}t^2$

$\Rightarrow 2=\frac{1}{2}t \Rightarrow t=4s$

$\Rightarrow s_1=u_1t+\frac{1}{2}a_1t^2=2(4)+\frac{1}{2}2(4)^2=8+16=24m$

Length of path is 24m

Sol 6: We know that $v^2-u^2=2as$, Given that

$u=72kmh^{-1}=20ms^{-1}$, $v=36kmh^{-1}=10ms^{-1}$

$s=200m$, $\Rightarrow 10^2-20^2=2a(200)$

$\Rightarrow a=\frac{100-400}{2\times 200}=\frac{-3}{4}ms^{-2}$

$s_2=\frac{v_2^2-u_2^2}{2a}$ (Distance travelled further)

$v_2=0$

$u_2=10ms^{-1}$

$\Rightarrow s_2=\frac{0-10^2}{2\times\left(\frac{-3}{4}\right)}=\frac{-100}{\frac{-3}{2}}=\frac{200}{3}$=66.7 m

Sol 7: Since he stops 1m from child, distance travelled is (100–1)=99m

So, we have, S = 99m, u = 44 ms^{-1}

$v^2-u^2=2as$

$v=0$

$\Rightarrow a=\frac{-(44)^2}{2.100}=\frac{-u^2}{2s} \Rightarrow t=\frac{v-u}{a}=\frac{0-u}{-\frac{u^2}{2s}}=\frac{u.2s}{u^2}=\frac{2s}{u}$

$t=\frac{2\times 99}{44}=4.5s$

Sol 8: $s=u+a\left(t-\frac{1}{2}\right)$

$\because$ body starts from rest, u=0

$20=0+a\left(t-\frac{1}{2}\right)$

$20=a\left(t_1-\frac{1}{2}\right)$ ... (i)

$40=a\left(t_2-\frac{1}{2}\right)$, Also $t_2=t_1+1$

(body travels 40 m in here 1 second)

$\Rightarrow 40=a\left(t_1+1-\frac{1}{2}\right)$

$\Rightarrow 40=a\left(t_1+\frac{1}{2}\right)$ ... (ii)

Subtract (ii) – (i)

$40-20=a\left(t_1+\frac{1}{2}\right)-a\left(t_1-\frac{1}{2}\right)$

$20=a\left(t_1+\frac{1}{2}\right)-a\left(t_1-\frac{1}{2}\right)$

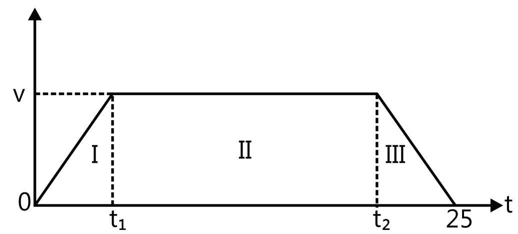
$a=20ms^{-2}$

Substitute in (i)

$20=20\left(t-\frac{1}{2}\right)\Rightarrow t=\frac{1}{2}s$

$S=\frac{1}{2}at^2=\frac{1}{2}.20.\left(\frac{1}{2}\right)^2=2.5m$

Sol 9:



$t_1=30s$

$v=72kmh^{-1}=72\times\frac{5}{18}=20ms^{-1}$

$a=\frac{v}{t_1}=\frac{20}{30}=\frac{2}{3}ms^{-2}$

Let distance travelled in acceleration be s_1

$\Rightarrow s_1=\frac{1}{2}at^2=\frac{1}{2}.\frac{2}{3}.30^2=300m$

Let distance travelled in uniform velocity and retardation be s_2,s_3 respectively

$s_3=50m$

Total distance travelled s=950m

$\Rightarrow s_2 = s - (s_1 + s_3) = 950 - (300 + 50) = 600$

$t_2 = \frac{s_2}{v} = \frac{600}{20} = 30s$

Retardation $r = \frac{v^2}{2s_3} = \frac{20^2}{2 \times 50} = 4ms^{-2}$

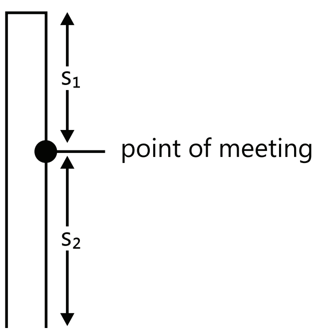
$\Rightarrow t_3 = \frac{v}{r} = \frac{20}{4} = 5s$

$\therefore$ Total time $t = t_1 + t_2 + t_3 = 30 + 30 + 5 = 65s$

$\therefore$ Acceleration is $\frac{2}{3}ms^{-2}$, Retardation is $4\,ms^{-2}$

Total time taken is 65 s.

Sol 10:



For ball thrown from top, let distance travelled be s, time taken t,

$s_1 = \frac{1}{2}gt^2$

For ball thrown from bottom, let initial velocity be v, given it just reaches top.

$\Rightarrow v^2 = 2gs \qquad (s = 100m)$

$\Rightarrow v = \sqrt{2g(100)} \quad \Rightarrow v = 10\sqrt{2g}\ ms^{-1}$

Let s_2 be distance travelled by ball 2, when it meets ball 1,

$s_2 = vt - \frac{1}{2}gt^2 \quad = 10\sqrt{2g}\ t - \frac{1}{2}gt^2$

$\boxed{s_1 + s_2 = 100m}$

$\therefore 100 = \frac{1}{2}gt^2 + 10\sqrt{2}gt - \frac{1}{2}gt^2$

$t = \frac{100}{10\sqrt{2g}} = \frac{10}{\sqrt{2g}}$

$s_2 = vt - \frac{1}{2}gt^2$

$= 10\sqrt{2g}\left(\frac{10}{\sqrt{2g}}\right) - \frac{1}{2}g.v\left(\frac{10}{\sqrt{2g}}\right)^2$

$= 100 - \frac{1}{2}g.\frac{100}{2g} = 100 - 25 = 75m$

$\therefore$ Balls meet at 75 m height

Sol 11: $\Delta s = \frac{1}{2}g\left(t_2^2 - t_1^2\right)\ t_2 = 2 + t_1$

Δs is distance between travelled between t_2 and t_1.

$\Rightarrow \Delta s = \frac{1}{2}g(t_2 - t_1)(t_2 + t_1)$

$= \frac{1}{2}g(t_1 + 2 - t_1)(t_1 + 2 + t_1)$

$= \frac{1}{2}g(2)(2t_1 + 2)$

$\Rightarrow \Delta s = g(2t_1 + 2)$ and we are given that $\Delta s = 78.4$

$\Rightarrow 78.4 = g(2t_1 + 2)$

$2t_1 + 2 = \frac{78.4}{g} = \frac{78.4}{9.8} = 8 \qquad (g = 9.8)$

$2t_1 = 8 - 2$

$t_1 = 3s$

$\therefore$ Time travelled by ball before it was observed is 3s

Sol 12: (i) Given that h = 300 m. We know that $h = ut + \frac{1}{2}gt^2$

If balloon is ascending $\Rightarrow$ initial velocity of stone is $-5ms^{-1}$

$\Rightarrow 300 = -5(t) + \frac{1}{2}(9.8)t^2 \Rightarrow 4.9t^2 - 5t - 300 = 0$

It is a quadratic equation in t

$\therefore t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; b=–5, a = 4.9, c=–300

$\Rightarrow t = \frac{-(-5) \pm \sqrt{5^2 - 4(4.9)(-300)}}{2(4.9)} = \frac{5 \pm \sqrt{25 + 5880}}{9.8}$

$\Rightarrow t = \frac{5 \pm 76.844}{9.8}$

For solution to be real, t>0

$t = \frac{5 + 76.844}{9.8} = 8.36s$

(ii) If balloon is descending $\Rightarrow$ initial velocity of stone is $6\ ms^{-1}$

$\Rightarrow 300 = 5t + \frac{1}{2}9.8t^2 \Rightarrow 4.9t^2 + 5t - 300 = 0$

$\Rightarrow t = \frac{-5 \pm \sqrt{5^2 - 4(4.9) - (300)}}{2(4.9)}$

For t > 0

$\Rightarrow t = \frac{-5 + \sqrt{25 + 5880}}{9.8} = 7.33$ s

(iii) If balloon is stationary $\Rightarrow$ initial velocity is 0 ms^{-1}

$\Rightarrow 300 = \frac{1}{2}.9.8t^2 \Rightarrow t = \sqrt{\frac{300 \times 2}{9.8}} = 7.82$ s

Sol 13: Displacement travelled equals to area under v–t graph

$\therefore S = \sum vt$ =4(2)+(–2)(2)+2(2) = 8m

Displacement = 8 ms^{-1}

Distance travelled $\sum |vt|$

=|4(2)|+|(–2)(2)|+|2(2)| = 8+4+4 =16m

Sol 14: Displacement is area under v–t graph for $0 \le t \le 6$

$s_1 = \frac{1}{2} \times 6 \times 20$ (area of triangle = $\frac{1}{2}$x base x height)

=60m

For 6 < t < 8, $s_2 = \frac{1}{2}(2)(-20)$ =–20m

For 8 < t < 10, $s_3 = 2 \times 10 = 20$m

Displacement = $\sum_{i=1}^{3} s_i$

$= s_1 + s_2 + s_3$ =60–20+20 = 60 m

Distance= $\sum_{i=1}^{3} |s_i|$

$= |s_1| + |s_2| + |s_3|$ =|60|+|(–20)|+|20|

= 60+20+20 = 100m

$\therefore$ Distance travelled is 100 m and displacement is 60 m

Sol 15: $x(t) = a + bt^2$

$v = \frac{dx}{dt} = \frac{d}{dt}\left(a + bt^2\right) = 2bt$

V(t)=2bt

v(0)=0

v(2)=2b(2)=4b=4(2.5) = 10 ms^{-1}

v(2)= 10 ms^{-1}

Average velocity = $\frac{\text{displacement}}{\text{time}}$

$= \frac{x(4.01) - x(2)}{4.01 - 2} = \frac{a + b(4.01)^2 - (a + b(2)^2)}{4.01 - 2}$

$= \frac{b}{2.01}\left((4.01)^2 - 2^2\right) = \frac{2.5(16.0801 - 4)}{2.01}$

= 15. 025 $ms^{-1} \cong 15$ ms^{-1}

$\therefore$ Average velocity= 15 ms^{-1}

Sol 16:

$v = \frac{dx}{dt} = \frac{d}{dt}\left(\frac{-2}{3}t^2 + 16t + 2\right) = -2\left(\frac{2}{3}\right)t + 16$

$\therefore v(t) = -\frac{4}{3}t + 16$

When body comes to rest, v(t)=0

$\Rightarrow \frac{-4}{3}t + 16 = 0 \Rightarrow 16 = \frac{4}{3}t \Rightarrow t = 12$

$\therefore$ Body takes 12 seconds to come to rest.

Sol 17: $v_y = \frac{dy}{dt} = \frac{d}{dt}(8t - 5t^2) = 8 - 10t$

$v_y(0) = 8\ ms^{-1}$

$v_x = \frac{dx}{dt} = \frac{d}{dt}(6t) = 6\ ms^{-1}$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{8^2 + 6^2} = 10$

$\therefore$ Initial velocity is 10 ms^{-1}

Sol 18: $x = 3 + 8t + 7t^2$

Velocity (v) $= \frac{dx}{dt} = \frac{d}{dt}(3 + 8t + 7t^2)$

$v(t) = 8t + 14t$

$v(2) = 8 + 14(2) = 36\ ms^{-1}$

$a = \frac{dv}{dt} = \frac{d}{dt}(8 + 14t) = 14ms^{-2}$

$\therefore$ Velocity = 36 ms^{-1}

Acceleration =14 ms^{-2}

Body is having a constant acceleration.

Sol 19: Given that $t = \alpha x^2 + \beta x$

$$\Rightarrow \frac{dt}{dx} = 2\alpha x + \beta$$

$$\Rightarrow \frac{dx}{dt} = \frac{1}{\left(\frac{dt}{dx}\right)} = \frac{1}{2\alpha x + \beta}$$

$$\Rightarrow \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = \frac{dv}{dx}.(v)$$

$$\Rightarrow v = \frac{1}{2\alpha x + \beta}$$

$$\Rightarrow \frac{dv}{dx} = \frac{-1}{(2\alpha x + \beta)^2}.(2\alpha)$$

$$\therefore \frac{dv}{dt} = \frac{-2\alpha}{(2\alpha x + \beta)^2} v \Rightarrow = -2\alpha (v)^2 (v)$$

$a = -2\alpha v^3$

Retardation = $-a = -(-2\alpha v^3) = 2\alpha v^3$

$\therefore$ Retardation $= 2\alpha v^3$

Sol 20: $v = \int a.dt = \int (3t^2 + 2t + 2).dt$

$v(t) = t^3 + t^2 + 2t + c$

Given $v(0) = 2\ ms^{-1}$

$v(0) = 0^3 + 0^2 + 2(0) + C = C$

$\Rightarrow C = 2\ ms^{-1}$

$\Rightarrow v(t) = t^3 + t^2 + 2t + 2$

$\Rightarrow v(t) = 2^3 + 2^2 + 2(2) + 2 = 18 ms^{-1}$

$\therefore$ velocity at the end of 2s is $18 ms^{-1}$

Sol 21: Let u be the velocity upon reaching ground

$u = \sqrt{2gh_1}$ $(h_1 = 4\ ft)$

Let v be the velocity upon rebounding from ground

$v = \sqrt{2gh_2}$ $(h_2 = 3\ ft)$

But v is in upward direction, u downward.

So for sign convention let's take upward positive

$\Rightarrow u = -\sqrt{2gh_1}$

Time t = 0.01s

$$a = \frac{v-u}{t} = \frac{\sqrt{2gh_2} - \left(-\sqrt{2gh_1}\right)}{t} = \frac{\sqrt{2g}\left(\sqrt{h_2} + \sqrt{h_1}\right)}{t}$$

g = (9.8)0.3 = 2. 94 (1 ft = 0.3 m)

$$\therefore a = \frac{\sqrt{2(2.94)}\left(\sqrt{4} - \sqrt{3}\right)}{0.01} \cong 3000 ft/s^2$$

Sol 22: Let $a = c_1 t$

(since uniformly increasing from zero)

$v = \int a.dt = \int c_1 t.dt$

$$v(t) = \frac{c_1 t^2}{c} + c_2$$

$v(0) = c_2$

Given v(0)=0 (body starting from rest)

$$\Rightarrow c_2 = 0 \Rightarrow v(t) = \frac{c_1 t^2}{2}$$

$$x = \int v.dt = \int \frac{c_1 t^2}{2}.dt$$

$$x(t) = \frac{c_1 t^3}{6} + c_3 \ (c_3 \text{ some constant})$$

Time average of velocity is given by

$$\langle v \rangle_6 = \frac{\int v.dt}{\int dt} = \frac{\int_0^{t_0} \frac{c_1 t^2}{2}.dt}{\int_0^t dt} = \frac{\left.\frac{c_1 t^3}{6} + c_3\right|_0^{t_0}}{t\big|_0 t}$$

$$= \frac{\frac{c_1 t_0^3}{6} + c_3 - \left(\frac{c_1 (0)^3}{6} + c_3\right)}{t_0 - 0} = \frac{\frac{c_1 t_0^3}{6}}{t_0} = \frac{c_1 t_0^2}{6}$$

We have $v(t_0) = v$

$$\frac{c_1 t_0^2}{2} = v \Rightarrow c_1 t_0^2 = 2v$$

$$\therefore \langle v \rangle_t = \frac{2v}{6} = \frac{v}{3}$$

$$\therefore \langle v \rangle_t = \frac{v}{3}$$

Distance average of velocity is given by

$$\langle v \rangle_x = \frac{\int v.dx}{\int dx}$$

$$x(t) = \frac{c_1 t^3}{6} + c_3 \Rightarrow dx = \frac{c_1 t^2}{2}.dt$$

$$\therefore \langle v \rangle_x = \frac{\int_0^{t_0} \frac{c_1 t^2}{2} \cdot \frac{c_1 t^2}{2} . dt}{\int_0^{t_0} \frac{c_1 t^2}{2} . dt}$$

$$= \frac{\frac{c_1^{\ 2}}{4^2} \int_0^{t_0} t^4 . dt}{\frac{c_1}{2} \int_0^{t_0} t^2 . dt} = \frac{c_1}{2} . \frac{\left. \frac{t^5}{5} \right|_0^{t_0}}{\left. \frac{t^3}{3} \right|_0^{t_0}} = \frac{c_1}{2} . \frac{\frac{t_0^{\ 5}}{5}}{\frac{t_0^{\ 3}}{3}}$$

$$= \frac{3}{10} c_1 \ t_0^{\ 2} = \frac{3}{10} . 2v$$

$$\langle v \rangle_x = \frac{3}{5} v$$

Distance average of velocity is $\frac{3}{5}v$

Sol 23: $x^2 = at^2 + 2bt + c$

Differentiating by 't' on both sides

$$\Rightarrow 2x\frac{dx}{dt} = 2at + 2b \Rightarrow 2xv = 2at + 2b$$

$$xv = at + b \qquad \text{... (i)}$$

Differentiating by 't' on both sides

$$\Rightarrow x.\frac{dv}{dt} + v\left(\frac{dx}{dt}\right) = a$$

$$x.a_c + v(v) = a \qquad (a_c = \text{acceleration})$$

$$\Rightarrow a_c = \frac{a - v^2}{x} \qquad \text{... (ii)}$$

Coming back to (i)

$$v = \frac{at + b}{x}$$

Squaring on both sides

$$v^2 = \left(\frac{at + b}{x}\right)^2 = \frac{a^2t^2 + 2a + b + b^2}{x^2}$$

$$v^2 = \frac{a(at^2 + 2abt) + b^2}{x^2} \qquad \text{... (iii)}$$

$$x^2 = at^2 + 2bt + c$$

$$at^2 + 2bt = x^2 - c$$

Substituting in (iii)

$$v^2 = \frac{a(x^2 - c) + b^2}{x^2}$$

Substituting in (ii)

$$a_c = \frac{a - \frac{\left(ax^2 - ac + b^2\right)}{x^2}}{x} = \frac{ax^2 - ax^2 + ac - b^2}{x^3}$$

$$\therefore \boxed{a_c = \frac{ac - b^2}{x^3}}$$

Sol 24: Lets take downward as positive

$a(t) = g$

$$h = \frac{1}{2}gt^2$$

h = 19. 6 m

g =9. 8 m

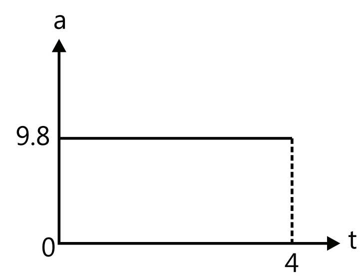
$$\Rightarrow t_1 = \sqrt{\frac{2h}{g}} \ (t_1 = \text{time of decent})$$

$$= \sqrt{\frac{2(19.16)}{9.8}} = 2 \text{ sec}$$

Time of ascent (t_2) = time of decent (t_1)

$\therefore\ t_2 = 2$ sec

Total time = $t_1 + t_2$ =2+2 = 4 sec



$v = gt \qquad 0 \le t < 2$

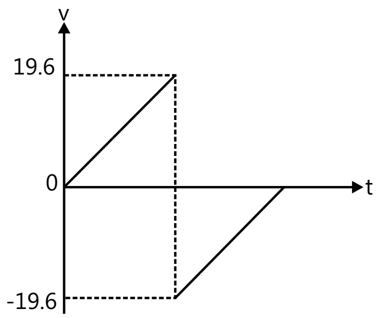
For ascent $u_{int} = -v(2) = -g(2)$

$u = 19.6 \text{ ms}^{-1}$

v = u+g(t–2) = 19. 6 + g(t–2); 2 < t < 4

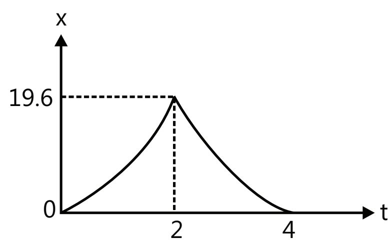
$v = gt; \qquad 0 \le t < 2$

$= 19.6 + g(t-2); \qquad 2 < t \le 4$



$S = \frac{1}{2}gt^2 \qquad 0 \le t < 2$

The motion of ball is symmetric about t=2 i.e., if we make t=4 as initial point, and reverse the motion of time ascent looks like decent.



Graph is symmetric about 2

$S = \frac{1}{2}gt^2 \qquad 0 \le t < 2$

$\frac{1}{2}g(4-t)^2 \qquad 2 < t \le 4$

Sol 25: Let r = retardation = $-a_1$

Let a = acceleration

Let t_1 = retardation time

t_2 = uniform velocity time

t_3 = acceleration time

Let v = uniform velocity

$t_2 = \frac{400}{v} \qquad \left(t = \frac{s}{v}\right)$

$t_1 = \frac{20-v}{a_1} \qquad \left(t = \frac{v-u}{a}\right)$

$t_3 = \frac{20-v}{a}$

$t_2 = t_1 + t_3$

$\Rightarrow \frac{400}{v} = \frac{20-v}{a_1} + \frac{20-v}{a}$

$\Rightarrow \frac{400}{v} = (20-v)\left(\frac{1}{a} + \frac{1}{a}\right)$ ... (i)

$200 = \frac{20^2 - v^2}{2a_1}$ ... (ii)

$\left(s = \frac{v^2 - u^2}{2a}\right)$

$600 = \frac{20^2 - v^2}{2a}$ ... (iii)

Divide (ii) by (iii)

$\Rightarrow \frac{200}{600} = \frac{\frac{20^2 - v^2}{2a_1}}{\frac{20^2 - v^2}{2a}} \Rightarrow \frac{1}{3} = \frac{a}{a_1} \Rightarrow a_1 = 3a$

Substitute in (i)

$\Rightarrow \frac{400}{v} = (20-v)\left(\frac{1}{3a} + \frac{1}{a}\right) \Rightarrow \frac{400}{v} = (20-v)\left(\frac{4}{3a}\right)$

$\Rightarrow (20-v)(v) = 300a$ ... (iv)

From eq (iii)

$600 = \frac{20^2 - v^2}{2a}$

$a = \frac{(20-v)(20+v)}{1200}$

Substitute in (iv)

$(20-v) = \frac{300(20-v)(20+v)}{1200}$

$\Rightarrow v = \frac{1}{4}(20+v) \Rightarrow 4v = 20 + v$

$\Rightarrow v = \frac{20}{3}\,ms^{-1} \Rightarrow t_2 = \frac{400}{\frac{20}{3}} = 60$ s

Total time=2 t_2 = 120 s = 2 min

Sol 26: Let v_0 velocity be for time t.

Let each of half time be t_n

$v_0 t = v_1 t_n + v_2 t_n \Rightarrow (v_1 + v_2)t_n = v_0 t$

$\Rightarrow t_n = \frac{v_0 t}{v_1 + v_2}$

Mean velocity = $\frac{\text{total distance}}{\text{total time}}$

Total distance = $2v_0 t$

Total time = $2th + t$

$$= t + \frac{2v_0 t}{v_1 + v_2} = t\left(1 + \frac{2v_0}{v_1 + v_2}\right) = t\left(\frac{v_1 + v_2 + 2v_0}{v_1 + v_2}\right)$$

Mean velocity

$$= \frac{2v_o t}{t\left(\frac{v_1 + v_2 + 2v_0}{v_1 + v_2}\right)} = \frac{2v_o(v_1 + v_2)}{v_1 + v_2 + 2v_0}$$

Sol 27: $s = \frac{1}{2}gt^2$ (s is distance from top)

Time of flight of 5th ball = 1 sec

Time of flight of 4th ball = 2 sec

Time of flight of 3rd ball = 3 sec

$$s = \frac{1}{2}g\left[1^2 \quad 2^2 \quad 3^2\right] = \frac{1}{2}g\left[1 \quad 4 \quad 9\right]$$

take $g = 10\ ms^{-2}$

$$s = 5\left[1 \quad 4 \quad 9\right] = \left[5 \quad 20 \quad 45\right]$$

$\therefore$ 3rd , 4th , 5th ball are at 45, 20 , 5 m from top

Sol 28: initial velocity of stone = 5 ms^{-2}

Height = 50 m

$$\Rightarrow 50 = -5(t) + \frac{1}{2}10t^2$$

$$\Rightarrow 5t^2 - 5t - 50 = 0 \ \Rightarrow t^2 - t - 10 = 0$$

$$\Rightarrow t = \frac{1 \pm \sqrt{1 + 4(10)(1)}}{2(1)} = \frac{1 \pm \sqrt{41}}{2}$$

for $t > 0 \, t = \frac{1 + \sqrt{41}}{2}$

Distance travelled by balloon in this time = 5(t)

Total height of balloon 50 + 5t

$$= 50 + 5\left(1 + \frac{\sqrt{41}}{2}\right) = 68.5 \text{ m}$$

Exercise 2

Single Correct Choice Type

Sol 1: (C) $\frac{d(x^2)}{dt} = 2x - \frac{dx}{dt} < 0$

$2x - v < 0$

$\because$ if $x > 0 \Rightarrow v < 0$

if $x , 0 \Rightarrow v > 0$

$\Rightarrow$ It is always pointing towards origin.

Sol 2: (B) $v = t^2 - t$; $\quad a = 2t - 1$

$a < 0 \quad 0 < t < \frac{1}{2}$

$a > 0 \quad \frac{1}{2} < t < 1$

$\because$ Retarding means coming back to original position,

For $\frac{1}{2} < t < 1$, body is trying to go back to original position.

Sol 3: (D) The acceleration on the ball is acceleration due to gravity which is always pointed downward, towards the earth.

Hence $a_y < 0$ always.

Sol 4: (C) Here x (t) = – sin (t)

$$v(t) = \frac{dx}{dt} = -\cos(t)$$

Sol 5: (C) Average velocity vector is along the direction of the line joining the instantaneous point and the starting point.

Instantaneous velocity vector is along the slope at the instantaneous point.

Both are same for point C.

Sol 6: (D) Here we use the symmetry of motion. i.e. from ending point if you go reverse in time, it looks like train is accelerating.

So here for minimum time, train should accelerate for half the distance and then decelerate.

$\therefore \frac{d}{2} = \frac{1}{2}at^2; \qquad t = \sqrt{\frac{d}{a}}$

$T = 2t; \qquad T = 2\sqrt{\frac{d}{a}}$

Sol 7: (D) The equation of line is

a + kV= C, where k is slope of line, C is y intercept.

$$a = \frac{dv}{dt}$$

$$\Rightarrow \frac{dv}{dt} = C - kv; \qquad \frac{dv}{C - kv} = dt$$

Intergrade on both sides

$$\int \frac{dv}{C-kv} = \int dt$$

$$-\frac{1}{k}\log(C-kv) = t$$

$$C-kv = e-kt$$

$$\Rightarrow v = \frac{C-e^{-kt}}{k}$$ it corresponds to option [D]

Sol 8: (B) Time of flight $= 2.\sqrt{\frac{2h}{g}}$

$\therefore T \propto \sqrt{h}$

Hence the one which reaches maximum height flies for longer time.

Sol 9: (C) Horizontal velocity of ball= velocity of bird=u

Time of flight $(t) = 2\sqrt{\frac{2h}{g}}$

$\therefore \text{Distance} = 2u\sqrt{\frac{2h}{g}}$

Sol 10: (B) We may use the formula

$$\frac{1}{T} = \frac{1}{t_1} + \frac{1}{t_2}$$

Sol 11: (C) Lets take upward as positive

For 0 < t < T

$V_A = V - gt$

$V_B = -gt$

$V_{AB} = V_A - V_B \quad = V - gt + gt$

$V_{AB} = V$

After T, ball B sticks to ground

$\therefore V_B = 0$

$\therefore V_{AB} = v - gt$

$\therefore V_A = v \qquad 0<t<T$

$\quad = v - gt \qquad T<t$

$\therefore$ Corresponding graph is C

Sol 12: (D) Distance $D(t) = \sqrt{(A-a_x t)^2 + (B-a_y t)^2}$

a_x, a_y are accelerations in x, y

Let $(A-a_x t)^2 + (B-a_y t)^2 = f(x)$

$D(t) = \sqrt{f(x)}$

f(x) is a quadratic equation

$f(x) > 0 \ \forall\ x$

$D'(t) = \frac{1}{2\sqrt{f(x)}}.f'(x); \qquad \frac{1}{2\sqrt{f(x)}} > 0 \ \forall\ x$

$D'(t) = g.f'(x)$

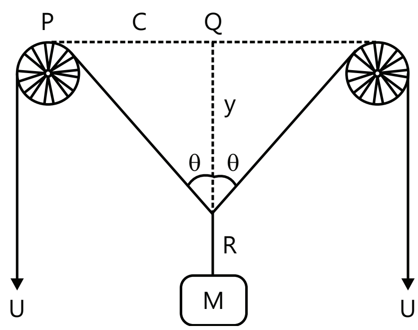
g is some positive variable.

Hence slope of D(t) is similar to f(x) which is quadratic equation.

So nearest graph is D.

Previous Years' Questions

Sol 1: (B)



In the right angle ΔPQR

$\ell^2 = c^2 + y^2$

Differentiating this equation with respect to time, we get

$$2\ell\frac{d\ell}{dt} = 0 + 2y\frac{dy}{dt}$$

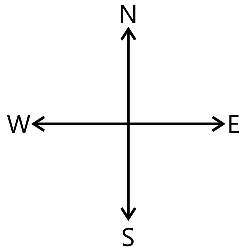
$$\text{Or } \left(-\frac{dy}{dt}\right) = \frac{\ell}{y}\left(-\frac{d\ell}{dt}\right)$$

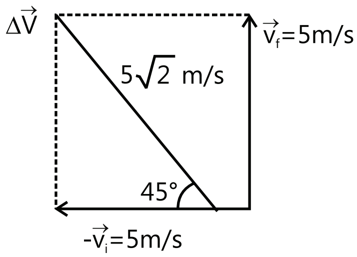
Here, $-\frac{dy}{dt} = v_M$

$\frac{\ell}{y} = \frac{1}{\cos\theta}$ and $\frac{-d\ell}{dt} = U$

Hence, $v_M = \frac{U}{\cos\theta}$

Sol 2: (C) $\overrightarrow{a_{av}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$



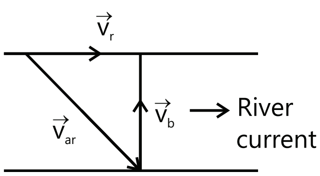


$\Delta\vec{v} = 5\sqrt{2}$ m/s in north–west direction.

$\vec{a}_{av} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$ m/s^2 (in north–west direction)

Sol 3: (A) To cross the river in shortest time one has to swim perpendicular to the river current.

Sol 4: (B) Shortest possible path comes when absolute velocity of boatman comes perpendicular to river current as shown in figure.



$$t = \frac{\omega}{v_b} = \frac{\omega}{\sqrt{v_{br}^2 - v_r^2}}$$

$$\therefore \frac{1}{4} = \frac{1}{\sqrt{25 - v_r^2}}$$

Solving this equation, we get v_r = 3 km/h

Sol 5: (A) For particle P, motion between AC will be an accelerated one while between CB a retarded one. But in any case horizontal component of its velocity will be greater than or equal to v. On the other hand, in case of particle Q, it is always equal to v. Horizontal displacement for both the particles are equal. Therefore, $t_P < t_Q$.

Sol 6: (B) $|\text{Average velocity}| = \left|\frac{\text{Displacement}}{\text{time}}\right|$

$$= \frac{AB}{\text{time}} = \frac{2}{1} = 2\text{m/s}$$

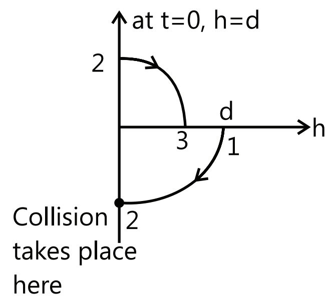
Sol 7: (A) (a) For uniformly accelerated/decelerated motion

$v^2 = u^2 \pm 2gh$

i.e., v–h graph will be a parabola (because equation is quadratic).

(b) Initially velocity is downwards (–ve) and then after collision it reverses its direction with lesser magnitude, i.e., velocity is upwards (+ve). Graph (a) satisfies both these conditions.

Note that time t = 0 corresponds to the point on the graph where h = d



Sol 8: (B) Area under acceleration–time graph gives the change in velocity.

Hence, $v_{max} = \frac{1}{2} \times 10 \times 11 = 55$ m/s

Sol 9: (C) Distance travelled in tth second is,

$s_1 = u + at - \frac{1}{2}a$

Given, u = 0

$$\therefore \frac{s_n}{s_{n+1}} = \frac{an - \frac{1}{2}a}{a(n+1) - \frac{1}{2}a} = \frac{2n-1}{2n+1}$$

Sol 10: (A) The v–x equation from the given graph can be written as

$$v = \left(-\frac{v_0}{x_0}\right)x + v_0 \qquad \text{... (i)}$$

$$a = \frac{dv}{dt} = \left(-\frac{v_0}{x_0}\right)\frac{dx}{dt} = \left(-\frac{v_0}{x_0}\right)v$$

Substituting v from Eq (i) we get,

$$a = \left(-\frac{v_0}{x_0}\right)\left[\left(-\frac{v_0}{x_0}\right)x + v_0\right]$$

$$a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

Thus, a–x graph is a straight line with positive slope and negative intercept.

Sol 11: (A) Time to reach the maximum height

$$t_1 = \frac{u}{g}$$

If t_2 be the time taken to hit the ground

$$-H = ut_2 - \frac{1}{2}gt_2^2$$

But t_2 = nt1 (given)

$$\Rightarrow -H = u\frac{nu}{g} - \frac{1}{2}g\frac{n^2u^2}{g^2} \Rightarrow 2gH = nu^2(n-2)$$

Sol 12: (B) $y_1 = 10t - \frac{1}{2}gt^2$ and $y_2 = 40t - \frac{1}{2}gt^2$

$y_2 - y_1 = 30t$ (straight line)

but stone with 10 m/s speed will fall first and the other stone is still in air. Therefore path will become parabolic till other stone reaches ground.

JEE Advanced/Boards

Exercise 1

Sol 1: Initial velocity of car =126 km/h

$$= 126 \times \frac{5}{18}ms^{-1} = 35ms^{-1}$$

Distance =200 m

$v^2 = 2rs \Rightarrow r = \frac{v^2}{2s} = \frac{35 \times 35}{2 \times 200} = 3.0625\ ms^{-2}$

$$E = \frac{v}{r} = \frac{35}{3.0625} = 11.43\,s$$

∴Retardation = 3.06 ms^{-2}

Time taken = 11.43 s

Sol 2: Velocity of police van (V_1) = 30 km /h $= \frac{30 \times 5}{18}ms^{-1}$

Velocity of bullet with respect to police van =150 ms^{-1}

Velocity of bullet with respect to ground = (V_1+150) ms^{-1}

Velocity of bullet with respect of thief

= Vel. w.r.t. ground – Vel. of thief =V_1+150–V_2

Velocity of thief (V_2) = 192 km/h $= 192 \times \frac{5}{18}ms^{-1}$

∴ Velocity with which bullet will hit thief

$$= V_1 + 150 - V_2 = 150 + 36 \times \frac{5}{18} - 192 \times \frac{5}{18}$$

$$= 150 + \frac{5}{18}(36 - 192) = 150 - 156 \times \frac{5}{18} = 105\ ms^{-1}$$

Sol 3: $S = \frac{1}{2}gt^2 \Rightarrow S \propto t^2$

Let S_1=distance travelled in 1 second similarly define S_2, S_3.

$$S_1 = \frac{1}{2}g(1)^2 = \frac{g}{2}$$

$$S_2 = \frac{1}{2}g(2)^2 = \frac{g}{2}(4)$$

$$S_3 = \frac{1}{2}g(3)^2 = \frac{g}{2}(9)$$

Distance travelled in 1st second $D_1 = \frac{g}{2}$

Distance travelled in 2nd second $D_2 = S_2 - S_1$

$$= \frac{g}{2}(4-1) = \frac{3g}{2}$$

Similarly $D_3 = S_3 - S_2 = \frac{g}{2}(9-4) = \frac{g}{2}(5)$

$$D_1 : D_2 : D_3 = \frac{g}{2} : \frac{3g}{2} : \frac{5g}{2} = 1:3:5$$

∴ Ratio is 1: 3: 5

Sol 4: Locomotive stops when V=0

given u = 54 km/hr=15 ms^{-1}

retardation (r)= 0.3 ms^{-2}

Hence $t = \frac{u}{r} = \frac{15}{0.3} = 50\,s$

Distance travelled

$$S = ut - \frac{1}{2}rt^2 = 15(50) - \frac{1}{2}(0.3)(50)^2 = 375m$$

Distance of traffic light (L) =400 m

Hence final distance of Locomotive=

L – S = 400–375=25 m

∴Locomotive is 25 m from traffic light.

Sol 5: initial position = 3 cm

Final position =–5 cm

Displacement = –5–(3)=–8 cm

Time =2 s

Initial velocity = 12 cms^{-1}

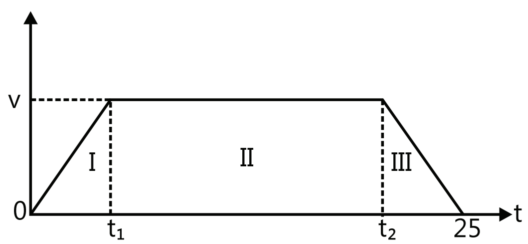
$$S = ut + \frac{1}{2}at^2$$

$$-8 = 12(2) + \frac{1}{2}a(2)^2$$

$$\Rightarrow a = -16cms^{-2}$$

Acceleration is $-16cms^{-2}$

Sol 6:



$$\text{Average Velocity} = \frac{\text{total Displacement}}{\text{total time}}$$

total time (t) = 25 s

total displacement = area under v–t. graph $= \int_0^{25} v.dt$

$V_0=5(t_1)$

Final acceleration equal declaration,

$V_0 - 5\ (25 - t_2)=0$

$\Rightarrow 5t_1–5(25–t_2) =0$

$\Rightarrow t_2=25–t_1$

For $0 < t < t_1$ area

$$= \int_0^{t_1} 5t.dt \quad (v = at) \quad = \frac{5t_1^2}{2}$$

For $t_1 < t < t_2$ area

$$= \int_{t_1}^{t_2} 5t_1.dt = 5t_1\left(t_2 - t_1\right) = 5t_1\left(25 - 2t_1\right)$$

Now is you observe the graph, region I ($0 < t < t_1$) is symmetric to region II (($t_2 < t < 25$)

$\therefore$ for $t_2 < t < 25$ area $= \frac{5}{2}t_1^2$

$\therefore$ total Displacement

$$= \frac{5}{2}t_1^2 + 5t_1(25 - 2t_1) + \frac{5}{2}t_1^2 = 5t_1^2 + 125t_1 - 10t_1^2$$

$$= 125t_1 - 5t_1^2$$

Avg velocity =20 ms^{-1}

$$\Rightarrow 20 = \frac{125t_1 - 5t_1^2}{25}$$

$$\Rightarrow 5t_1^2 - 125t_1 + 100 = 0$$

$$\Rightarrow (t_1 - 5)(t_1 - 20) = 0$$

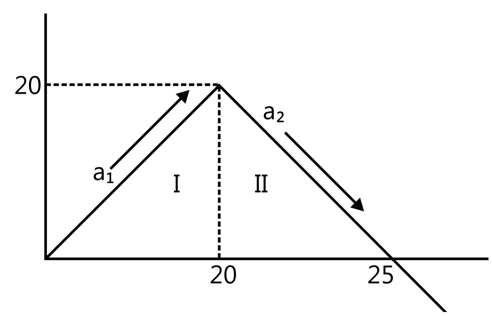
$$\Rightarrow t_1 = 5\ ,\ t_1 = 20$$

for $t_1=5$, $\Rightarrow t_2=20$ or $t_1=20$, $\Rightarrow t_2=5$

$\Rightarrow t_1 < t_2 \Rightarrow t_1=5$

Hence the value of t is 5 sec.

Sol 7:



$$a_2 = \frac{0 - 20}{25 - 20} \qquad \left(a = \frac{\Delta v}{t}\right)$$

$$a_2 = -4ms^{-2}$$

$\because$ Particle returns to initial position, its displacement is zero

$$\therefore \int v.dt = 0$$

for $0 < t < 20$, area $= \frac{1}{2} \times 20 \times 20 = 200m$

for t > 0 V = 20 – 4 (t –20)

$\because$ v = u –at and its starts for t =20 s

$$= \int_{20}^{t} [20 - 4(t - 20)].dt = \int_{20}^{t} (100 - 4t).dt$$

$$= 100(t - 20) - \frac{4}{2}(t^2 - 20^2) = 100t - 2t^2 - 1200$$

total area = 0

$\Rightarrow$ 200 + [100t –2t^2 –1200] = 0

$\Rightarrow$ 2t^2–100t +1000 =0

$\Rightarrow$ t^2–50t+500=0

$\Rightarrow$ t = 36.2 s

Sol 8: $\because$the stone can reach height (H) = 4h,

It initial velocity

$v = \sqrt{2(4h)g} \qquad \left(v = \sqrt{2Hg}\right)$

Let time of flight be t

Distance travelled by upper stone $D_1 = \frac{1}{2}gt^2$

Distance travelled by lower stone

$$D_2 = vt - \frac{1}{2}gt^2$$

$$D_1 + D_2 = h$$

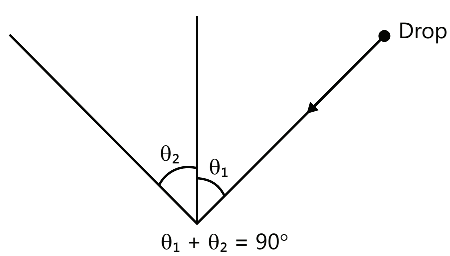
$$\Rightarrow \frac{1}{2}gt^2 - vt - \frac{1}{2}gt^2 = h$$

$$\Rightarrow h = vt \Rightarrow h = \sqrt{8hg}.t$$

$$\Rightarrow t = \frac{h}{\sqrt{8hg}} = \sqrt{\frac{h}{8g}}$$

Hence time when two stone cross is $\sqrt{\frac{h}{8g}}$.

Sol 9: Let vertical direction limit vector be $\hat{j}$, horizontal $\hat{i}$



V_{car} = 2 $\hat{i}$

V_{drops}=6 $\hat{j}$

V_{drops} w. r.t car = $V_{drops} - V_{car} = -2\hat{i} + 6\hat{j}$

$$\cos\theta_1 = \frac{\hat{j}.(-2\hat{i}+6\hat{j})}{|\hat{j}|.|-2\hat{i}+6\hat{j}|} = \frac{6}{\sqrt{2^2+6^2}.1} = \frac{6}{\sqrt{40}}$$

$$\Rightarrow \cos\theta_1 = \frac{3}{\sqrt{10}}$$

$$\Rightarrow \sin\theta_2 = \frac{1}{\sqrt{10}}$$

$$\Rightarrow \tan\theta_2 = 3$$

$$\Rightarrow \theta_2 = \tan^{-1}3$$

Hence wind stream is inclined at tan^{-1}3° with vertical.

Sol 10: for particle 1

S_1= (300 – 20t) cm

for particle 2

S_2 = (400– 20t) cm

Let S_1 have unit vector $\hat{i}$

$$\Rightarrow S_2 = \frac{1}{2}(400-20t)\hat{i} - \frac{\sqrt{3}}{2}(400-20t)\hat{j}$$

$$= (200-20t)\hat{i} - \sqrt{3}(200-10t)\hat{j}$$

Separation = $S_1 - S_2$

$$= [(300-20t)-(200-2t)]\hat{i} + \sqrt{3}(200-10t)\hat{j}$$

$$= (100-10t)\hat{i} + \sqrt{3}(200-10t)\hat{j}$$

$$= 10[(10-t)\hat{i} + \sqrt{3}(20-t)\hat{j}]$$

$$\text{distance} = 10\sqrt{(10-t)^2 + 3(20.t)^2}$$

$$d(t) = 10\sqrt{1300 - 140t + 4t^2}$$

for a quadratic equation, min occurs at $-\frac{b}{2a} = \frac{140}{2\times 8} = \frac{35}{2}$

$$d\left(\frac{35}{2}\right) = 10\sqrt{1300 - 140\left(\frac{35}{2}\right) + 4\left(\frac{35}{2}\right)^2} = 50\sqrt{3}\text{cm}$$

$\therefore$ min separation is $50\sqrt{3}$cm.

Sol 11: Final velocity (u) = at_1

Let it further travel for t seconds, acceleration a.

Let initial displacement be S_1.

$$S_1 = \frac{1}{2}at_1^2$$

Let further displacement be S_2.

$$S_2 = ut - \frac{1}{2}at^2 = at_1(t) - \frac{1}{2}at^2$$

S_1+S_2=0 $\because$ It comes back to initial position

$$\therefore \frac{1}{2}at_1^2 + at_1t - \frac{1}{2}at^2 = 0$$

$$\Rightarrow t^2 - 2t_1t - t_1^2 = 0$$

$$\Rightarrow t = \frac{2t_1 \pm \sqrt{8t_1^2}}{2} = (\pm\sqrt{2}+1)t_1$$

t>0

$$\therefore t_1 = (\sqrt{2}+1)t_1$$

Total Time = t + t_1

$$\text{Total Time} = (\sqrt{2}+2)t_1$$

Sol 12: Let max velocity = V, total distance = S

$$V^2 = 2a\frac{s}{m} \text{ ; a=acceleration}$$

$$\Rightarrow a = \frac{mv^2}{2s}$$

$$\text{Similarly retardation } r = \frac{nv^2}{2s}$$

Here we again used the principle, retardation is acceleration in reverse time.

$$\therefore \text{Time of acceleration } t_1 = \frac{v}{a} = \frac{2s}{mv}$$

$$\text{Time of deceleration } t_2 = \frac{v}{r} = \frac{2s}{nv}$$

$$\text{Time of uniform velocity } = \frac{1}{v}\left(s - \frac{s}{m} - \frac{s}{n}\right)$$

$$\text{Avg. Velocity} = \frac{s}{\frac{2s}{mv} + \frac{2s}{nv} + \frac{1}{v}\left(s - \frac{s}{m} - \frac{s}{n}\right)}$$

$$= \frac{V}{1 + \frac{1}{m} + \frac{1}{n}}$$

$$\frac{\text{Max Velocity}}{\text{Avg. Velocity}} = \left[1 + \frac{1}{m} + \frac{1}{n}\right] : 1$$

Sol 13: (i) $v = \alpha\sqrt{x}$

$$\frac{dv}{dt} = \frac{\alpha}{2\sqrt{x}}.\frac{dx}{dt} = \frac{\alpha}{2\sqrt{x}}.v$$

$$\frac{v}{\sqrt{x}} = \alpha$$

$$\therefore \frac{dv}{dt} = \frac{\alpha}{2}\alpha = \frac{\alpha^2}{2} = a$$

$$dv = \frac{\alpha^2}{2}dt$$

$$\Rightarrow v = \frac{\alpha^2}{2}t$$

$$\therefore a = \frac{\alpha^2}{2},\ v = \frac{\alpha^2}{2}t$$

(ii) $s = \int_0^t v.dt$

$$s = \int_0^t \frac{\alpha^2}{2}t.dt$$

$$s = \frac{\alpha^2t^2}{4} \Rightarrow t = \frac{2\sqrt{s}}{\alpha}$$

$$\text{Average velocity} = \frac{s}{t} = \frac{s}{\frac{2\sqrt{s}}{\alpha}} = \frac{\alpha\sqrt{s}}{2}$$

$$\therefore \text{Mean velocity } \langle v \rangle = \frac{\alpha\sqrt{s}}{2}$$

Sol 14: Time of flight $= 2\left(\frac{V}{g}\right)$

$$\text{dispalcement} = \frac{1}{2}at^2 = \frac{1}{2}.a.\frac{4v^2}{g^2} = \frac{2av^2}{g^2}$$

$$= \frac{2 \times 1 \times (9.8)^2}{(9.8)^2} = 2m$$

It falls 2m behind him.

Sol 15: Let distance be S

$$V_{man} = \frac{s}{90}$$

$$V_{escalator} = \frac{s}{60}$$

V_{man} on moving escalator w. r. t ground= V_{man}+ $V_{escalator}$

$$= s\left(\frac{1}{90} + \frac{1}{60}\right) = \frac{s}{36}$$

$$\text{Time} = \frac{s}{v} = \frac{s}{\frac{s}{36}} = 36$$

He can reach in 36 seconds.

Sol 16: Total distance to be covered= 180+180=360m

Let V_1, V_2 be velocities of trains

It they move in same direction, relative velocity = $V_1 - V_2$

Opposite direction, relative velocity = $V_1 + V_2$

$\Rightarrow V_1 - V_2 = \frac{360}{15} = 24$

$\Rightarrow V_1 + V_2 = \frac{360}{7.5} = 48$

$\Rightarrow V_1 = 36 m/s, V_2 = 12 m/s$

Sol 17: Let it overtake after time t

Distance travelled by truck = 30 t

Distance travelled by automobile $= \frac{1}{2} \times 6t^2 = 3t^2$

$3t^2 = 30t$

$\Rightarrow t = 10$ s

$\Rightarrow$ Distance = $3(10)^2 = 300$ ft

$V_{car} = 6t = 60$ ft s^{-1}

It overtakes it at 300 ft distance. Its velocity is 60 ft s^{-1}.

Sol 18: Relative velocity (V) = $V_1 + V_2$

Relative acceleration (a) = $-(a_1 + a_2)$

$0 - V^2 = 2$ as

$-(V_1 + V_2)^2 = -2(a_1 + a_2) I_{max}$

$\Rightarrow I_{max} = \frac{(V_1 + V_2)^2}{2(a_1 + a_2)}$

Sol 19: $v \propto \frac{1}{l}$

$\Rightarrow v = \frac{k}{l}$ (k = constant)

$v = 2\times10^{-2} \Rightarrow \ell = 1$

$\Rightarrow 2\times10^{-2} = k$

$\therefore v = \frac{2\times10^{-2}}{\ell}$

$v = \frac{d\ell}{dt}$

$\frac{d\ell}{dt} = \frac{2\times10^{-2}}{\ell}$

$\ell . d\ell = 2\times10^{-2}.dt$

$\int_1^2 \ell d\ell = \int_1^t 2\times10^{-2}.t.dt$

$\frac{\ell^2}{2}.\Big|_1^2 = 2\times10^{-2}.t\Big|_0^t$

$\frac{1}{2}(2^2 - 1^2) = 2\times10^{-2}.t$

$\Rightarrow t = \frac{3}{4}\times10^2 = 75s$

Sol 20: Let time of travel be t

Distance travelled by P

$S_1 = 1(t) + \frac{1}{2}.2t^2 = t + t^2$

Distance travelled by Q

$S_2 = 9(t) + \frac{1}{2}.1t^2 = 9t + \frac{t^2}{2}$

Hence $S_2 = S_1 + 33$

$\therefore 9t + \frac{t^2}{2} = 33 + t + t^2$

$\frac{t^2}{2} - 8t + 33 = 0$

$\Rightarrow t^2 - 16t + 66 = 0$

$b^2 - 4ac = 16^2 - 4(66) = 256 - 264 = -8$

$\because b^2 - 4ac < 0$

$\Rightarrow$ there is no real solution for t

$\Rightarrow$ Q can't catch P.

Exercise 2

Multiple Correct Choice Type

Sol 1: (A, C, D) Let time to travel along each side = t

Displacement A to F = vt

Time to travel from A to F = st

$\therefore$ Avg. Velocity $= \frac{vt}{st} = \frac{v}{s}$

Displacement by A to C = $\sqrt{3}$ Vt

Time for A to C = 2t

$\therefore$ Avg. Velocity $= \frac{\sqrt{3}vt}{2t} = \frac{\sqrt{3}}{2}v$

Displacement of A to B = vt

Time = t

$\therefore$ Avg. Velocity $= \frac{vt}{t} = v$

Sol 2: (A, C) The magnitude of velocity is same. Hence change in magnitude is zero.

Let initial velocity be V along: i–direction

Final velocity = $v\cos\theta\,\hat{i} + v\sin\theta\,\hat{j}$

Change in velocity = $v(\cos\theta - 1)\hat{i} + v\sin\theta\,\hat{j}$

Magnitude of change in velocity

$$= v\sqrt{(\cos\theta - 1)^2 + \sin^2\theta}$$

$$= v\sqrt{2(1-\cos\theta)} = 2v\sin\frac{\theta}{2}$$

Sol 3: (A, C) Maximum displacement $= \frac{v^2}{2r} = \frac{10^2}{2\times 5} = 10m$

In the problem, the particle goes to maximum displacement and comes back.

Displacement is minimum distance between initial and final point

$$d = 10(3) - \frac{1}{2}5.(3)^2 \quad = 7.5m$$

But distance is total path covered by body i.e $D = \sum |v|.dt$

Here the body come back 2.5 m from maximum displacement

$$\therefore \quad D = 10 + 2.5 \qquad = 12.5m$$

Sol 4: (A, D) Acceleration due to gravity along AB is

$a_{AB} = g\cos\theta; \quad v_B = g\cos\theta t \quad \therefore v_B \propto \cos\theta$

$AB = 2r\cos\theta$

$$t = \sqrt{\frac{2S}{a}} = \sqrt{\frac{2r\cos\theta}{g\cos\theta}} = \sqrt{\frac{2r}{g}}$$

$\therefore$ t is independent of θ.

Sol 5: (A, B, C, D)

- Particle changed its direction of motion at t = T
- Slope is constant. Hence acceleration is constant.
- Area under v–T graph is zero. So displacement is zero
- Speed is magnitude of velocity.

Speed=|velocity|

Which is same initial and final.

Sol 6: (A, B, C) It's velocity is v_1

He observes both moving with velocity –v.

Sol 7: (C, D) While throwing, the horizontal component of the velocity of ball (with respect to earth) is equal to velocity of cart.

Assertion Reasoning Type

Sol 8: (A) Statement–II supports statement–I

Sol 9: (D) Constant speed means, body can have different velocity as velocity is vector (change in direction). Hence body can accelerate.

Sol 10: (A) Statement–II supports statement–I

Sol 11: (B) Both statements are true, but Statement–II doesn't explain statement–I

Comprehension Type

Paragraph 1:

Sol 12: (B) V(t)=0 for t=0.8, 4.66 s (B)

Sol 13: (C) Slope is maximum for 4 to 6

$$\left|\frac{\Delta v}{\Delta t}\right| = 15$$

Sol 14: (A) Position = area under V–T graph + initial position.

Area under V–T graph for 0 < t < 2 = $\frac{1}{2}\times 10\times 2 = 10m$

$\therefore$ Position $= 10 + (-15) = -5m$

Sol 15: (A) The maximum displacement is at t = 4.66 as area under graph is positive.

$$\text{Displacement} = \frac{1}{2}\times 2\times 10 + 10\times 2 + \frac{1}{2}\times\frac{2}{3}\times 10$$
$$= 10 + 20 + 3.33 = 33.33m$$

Sol 16: (A) Total distance = $\sum |v| t$

$$= 33.33 + \left|\frac{1}{2}\times(-20)\times 3.33\right| = 33.33 + 33.33 = 66.7m$$

Paragraph 2:

Sol 17: (B) For

$0, t < T \quad \frac{dx}{dt} > 0$

$T < t < 2T \quad \frac{dx}{dt} < 0$

$$\frac{d^2x}{dt^2} < 0 \;\forall\; t \qquad \Rightarrow \frac{dv}{dt} < 0 \;\forall\; t$$

Sol 18: (D) $\frac{dv}{dt} < 0$

Sol 19: (C) Speed is |v|. Apply |v| to v–t graph.

Match the Columns

Sol 20: A → p,s; B → r; C → q; D → s

(A) $\frac{dx}{dt} < 0, \quad x > 0\ ,\ 0 < t < T$

⇒ Ball rolling towards origin. (p)

$\frac{dx}{dt} = 0 \qquad t > T$

So it suddenly stops (s)

(B) There is a sudden change in slope i.e. $\frac{dx}{dt}$

Hence it had bounced of something (r)

(C) There is a sudden change in slope. The particle was going to origin and went back (q)

(D) $\frac{dx}{dt} < 0,\ x > 0 \Rightarrow$ Object going towards origin.

$\frac{dx}{dt} = 0; \qquad t > T$

$\frac{dx}{dt} < 0; \qquad 0 < t < T$

⇒ Sudden change in velocity.

Also $\frac{dx}{dt} = 0 \Rightarrow$ ball stops (s)

Previous Years' Questions

Sol 1: (i) Range of both the particles is

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(49)^2 \sin 90°}{9.8}$$

By symmetry we can say that they will collide at highest point.

u cos 45° ucos45° ucos45° v

→ ← ← →

• • • •

20g 40g 20g 40g

Just before collision Just after collision

Let v be the velocity of Q just after collision. Then, from conservation of linear momentum, we have

20(u cos 45°) – 40(u cos 45°) = 40(v) – 20(u cos 450°)

∴ v = 0

i.e., particle Q comes to rest. So, particle Q will fall vertically downwards and will strike just midway between A and B.

(ii) Maximum height,

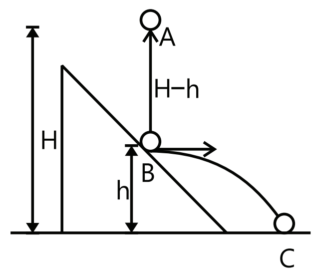
$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{(49)^2 \sin^2 45°}{2 \times 9.8} = 61.25 \text{ m}$$

Therefore, time taken by Q to reach the ground,

$$t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 61.25}{9.8}} = 3.53 \text{ s}$$

Sol 2: Let t_1 be the time from A to B and t_2 the time from B to C

Then



$$t_1 = \sqrt{\frac{2(H-h)}{g}} \text{ and } t_2 = \sqrt{\frac{2h}{g}}$$

Then, the total time

$$T = t_1 + t_2 = \sqrt{\frac{2}{g}}\,[\sqrt{H-h} + \sqrt{h}\,]$$

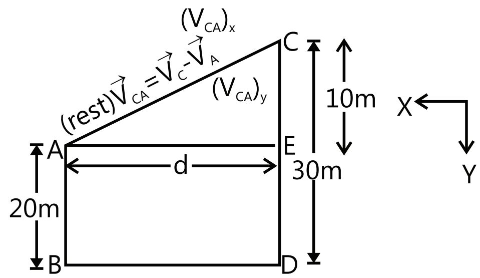
For t to be maximum $\frac{dt}{dh} = 0$

or $\sqrt{\frac{2}{g}}\left[\frac{-1}{2\sqrt{H-h}} + \frac{1}{2\sqrt{h}}\right] = 0$

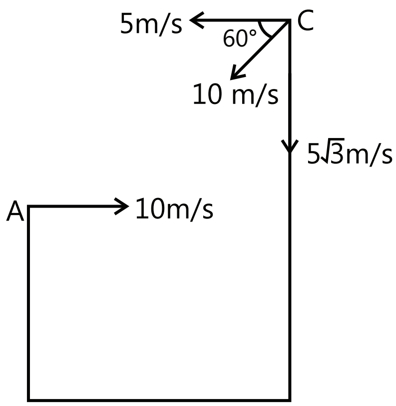
or $\frac{1}{\sqrt{h}} = \frac{1}{\sqrt{H-h}}$ or $2h = H$

$\therefore \frac{h}{H} = \frac{1}{2}$

Sol 3: (i) Acceleration of A and C both is 9.8 m/s^2 downwards. Therefore, relative acceleration between them is zero i.e., the relative motion between them will be uniform.



Now assuming A to be at rest, the condition of collision will be that $\vec{v}_{CA} = \vec{v}_C - \vec{v}_A$ = relative velocity of C w.r.t. A should be along CA.



$v_{Ax} = -10$ m/s

$v_{Ay} = 0$

$v_{Cx} = 5$ m/s

$v_{Cy} = 5\sqrt{3}$ m/s

$$\therefore \frac{(v_{CA})_y}{(v_{CA})_x} = \frac{CE}{AE} = \frac{10}{d}$$

$$\text{or } \frac{v_{Cy} - v_{Ay}}{v_{Cx} - v_{Ax}} = \frac{10}{d} \text{ or } \frac{5\sqrt{3} - 0}{5 - (-10)} = \frac{10}{d}$$

$\therefore$ d = $10\sqrt{3}$ m = 17.32 m

(ii) Time of collision, $t = \dfrac{AC}{|\vec{v}_{CA}|}$

$$|\vec{v}_{CA}| = \sqrt{(v_{CAx}^2) + (v_{CAy})^2}$$

$$= \sqrt{\{5 - (-10)\}^2 + \{5\sqrt{3} - 0\}^2} = 10\sqrt{3} \text{ m/s}$$

$$CA = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20 \text{ m}$$

$$\therefore t = \frac{20}{10\sqrt{3}} = \frac{2}{\sqrt{3}} \text{ s}$$

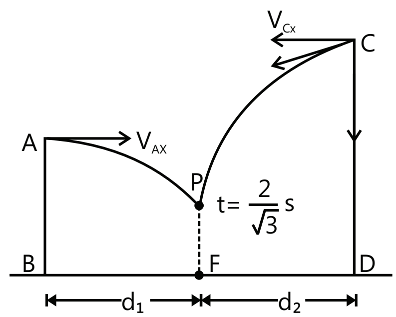
Horizontal (or x) component of momentum of A. i.e., $P_{Ax} = mv_{Ax} = -10$ m.

Similarly, x component of momentum of C, i.e.,

$P_{Cx} = (2m)v_{Cx} = (2m)(5) = +10$ m

Since, $P_{Ax} + P_{Cx} = 0$

i.e., x–component of momentum of combined mass after collision will also be zero, i.e., the combined mass will have momentum or velocity in vertical or y–direction only.



Hence, the combined mass will fall at point F just below the point of collision P.

Here $d_1 = |(v_{Ax})|\, t = (10)\dfrac{2}{\sqrt{3}} = 11.55$ m

$\therefore d_2 = (d - d_1) = (17.32 - 11.55) = 5.77$ m

d_2 should also be equal to

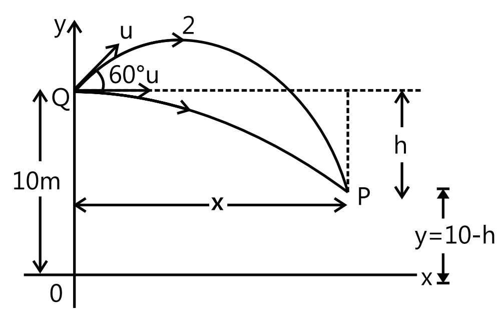
$$|v_{Cx}|\, t = (5)\left(\frac{2}{\sqrt{3}}\right) = 5.77 \text{ m}$$

Therefore, position from B is d_1 i.e., 11.55 m and from D is d_2 or 5.77 m.

Sol 4: $u = 5\sqrt{3}$ m/s

$$\therefore u\cos 60° = (5\sqrt{3})\left(\frac{1}{2}\right) \text{m/s} = 2.5\sqrt{3} \text{ m/s}$$

$$\text{and } u\sin 60° = (5\sqrt{3})\left(\frac{\sqrt{3}}{2}\right) \text{m/s} = 7.5 \text{ m/s}$$



Since, the horizontal displacement of both the shots are equal. The second should be fired early because it's horizontal component of velocity u cos 60° or 2. $5\sqrt{3}$

m/s is less than the other which is u or $5\sqrt{3}$ m/s.

Now let first shot takes t_1 time to reach the point P and the second t_2.

Then, $x = (u\cos 60°)t_2 = ut_1$

or $x = 2.5\sqrt{3}\,t_2 = 5\sqrt{3}\,t_1$ ... (i)

or $t_2 = 2t_1$ ... (ii)

and $h = |(u\sin 60°)t_2 - \frac{1}{2}gt_2^2| = \frac{1}{2}gt_1^2$

or $h = \frac{1}{2}gt_2^2 - (u\sin 60°)t_2 = \frac{1}{2}gt_1^2$

Taking $g = 10\ m/s^2$

$h = 5t_2^2 - 7.5\,t_2 = 5t_1^2$ ... (iii)

Substituting $t_2 = 2t_1$ in Eq. (iii), we get

$5(2t_1)^2 - 7.5(2t_1) = 5t_1^2$

or $15t_1^2 = 15t_1 \Rightarrow t_1 = 1$ s

and $t_2 = 2t_1 = 2$ s

$x = 5\sqrt{3}\,t_1 = 5\sqrt{3}$ m [From Eq. (i)]

and $h = 5t_1^2 = 5(1)^2 = 5$m [From Eq. (iii)]

$\therefore y = 10 - h = (10 - 5) = 5$ m

Hence,

(i) Time interval between the firings

$= t_2 - t_1 = (2 - 1)$ s

$\Delta t = 1$ s

(ii) Coordinates of point

$P = (x, y) = (5\sqrt{3}$ m, 5 m)

Sol 5: (i) Let $\hat{i}$, $\hat{j}$ and $\hat{k}$ be the unit vectors along x, y and z–directions respectively.

Given $\vec{v}_{cart} = 4\hat{i}$ m/s

$\therefore \vec{v}_{stone.\,cart} = (6\sin 30°)\hat{j} + (6\cos 30°)\hat{k}$

$= (3\hat{j} + 3\sqrt{3}\hat{k})$ m/s

$\therefore \vec{v}_{stone} = \vec{v}_{stone.\,cart} + \vec{v}_{cart}$

$= (4\hat{i} + 3\hat{j} + 3\sqrt{3}\,\hat{k})$ m/s

This is the absolute velocity of stone (with respect to ground). At highest point of its trajectory, the vertical component (z) of its velocity will become zero, whereas the x and y–components will remain unchanged

Therefore, velocity of stone at highest point will be

$\vec{v} = (4\hat{i} + 3\hat{j})$ m/s

or speed at highest point,

$v = |\vec{v}| = \sqrt{(4)^2 + ((3)^2}$ m/s = 5 m/s

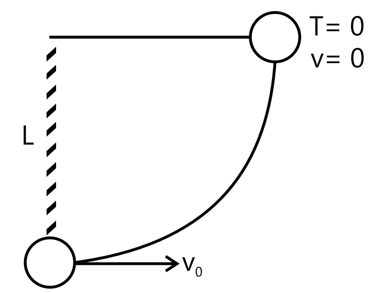
Now, applying law of conservation of linear momentum, let v_0 be the velocity of combined mass after collision.

Then, $mv = (2m)v_0$

$\therefore v_0 = \frac{v}{2} = \frac{5}{2}$ m/s = 2. 5 m/s

$\therefore$ Speed of combined mass just after collision is 2. 5 m/s

(ii) Tension in the string becomes zero at horizontal position. It implies that velocity of combined mass also becomes zero in horizontal position. Applying conservation of energy, we have



$\theta = v_0^2 - 2gL$

$\theta = v_0^2 - 2gL$

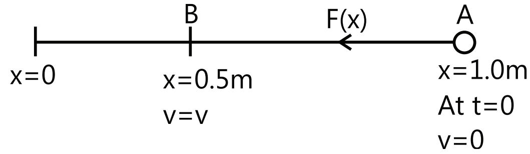
$\therefore L = \frac{v_0^2}{2g} = \frac{(2.5)^2}{2(9.8)} = 0.32$ m

Hence, length of the string is 0.32 m

Sol 6: (i) $F(x) = \frac{-k}{2x^2}$

k and x^2 both are positive.

Hence, F(x) is always negative.



Applying work–energy theorem between points A and B. Change in kinetic energy between A and B = work done by the force between A and B

$\therefore \frac{1}{2}mv^2 = \int_{x=1.0m}^{x=0.5m} F(dx) = \int_{1.0}^{0.5}\left(\frac{-k}{2x^2}\right)(dx) = \frac{-k}{2}\int_{1.0}^{0.5}\frac{dx}{x^2}$

$$= \frac{k}{2}\left(\frac{1}{x}\right)_{1.0}^{0.5} = \left(\frac{k}{2}\right)\left(\frac{1}{0.5} - \frac{1}{1.0}\right) = \frac{k}{2}$$

$$\therefore v = \pm\sqrt{\frac{k}{m}}$$

Substituting the values

$$v = \pm\sqrt{\frac{10^{-2}\ Nm^2}{10^{-2}\ kg}} = \pm 1\ m/s$$

Therefore, velocity of particle at x = 1.0 m is

v = – 1.0 m/s

Negative sign indicates that velocity is in negative x–direction.

(ii) Applying work–energy theorem between any intermediate value x = x, we get

$$\frac{1}{2}mv^2 = \int_{1.0}^{x}\frac{-k\,dx}{2x^2} = \frac{k}{2}\left(\frac{1}{2}\right)_{1.0}^{x} = \frac{k}{2}\left(\frac{1}{x} - 1\right)$$

$$\therefore v^2 = \frac{k}{m}\left(\frac{1}{x} - 1\right) \qquad \therefore v = \sqrt{\frac{1}{x} - 1} = \sqrt{\frac{1-x}{x}}$$

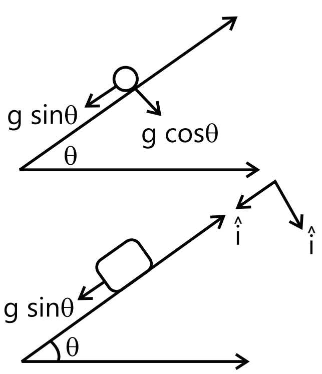
$$\frac{k}{m} = \frac{10^{-2}\lambda Nm^2}{10^{-2}kg}$$

$$\text{but } v = -\left(\frac{dx}{dt}\right) = \sqrt{\frac{1-x}{x}}$$

$$\therefore \int\sqrt{\frac{x}{1-x}}\,dx = -\int dt \quad \text{or} \quad \int_{1}^{0.25}\sqrt{\frac{x}{1-x}}\,dx = -\int_{0}^{x}dt$$

Solving this, we get t = 1.48 s

Sol 7: (i) Accelerations of particle and block are shown in figure.

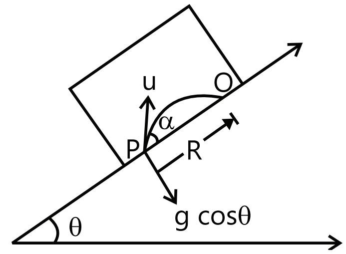


Acceleration of particle with respect to block

= acceleration of particle – acceleration of block

$= (g\sin\theta\ \hat{i} + g\cos\theta\ \hat{j}) - (g\sin\theta)\hat{i} = g\cos\theta\,\hat{j}$

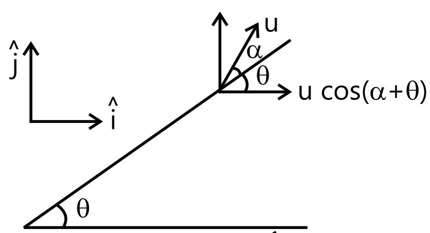
Now motion of particle with respect to block will be a projectile as shown

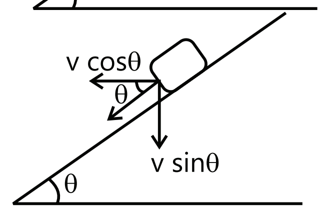


The only difference is, g will be replaced by g cos θ

$$\therefore PQ = \text{Range (R)} = \frac{u^2\sin 2\alpha}{g\cos\theta}$$

(ii) Horizontal displacement of particle with respect to ground is zero. This implies that initial velocity with respect to ground is only vertical, or there is no horizontal component of the absolute velocity of the particle.





Let v be the velocity of the block down the plane.

Velocity of particle

$= u\cos(\alpha + \theta)\,\hat{i} + u\sin(\alpha + \theta)\,\hat{j}$

Velocity of block $= -v\cos\theta\,\hat{i} - v\sin\theta\,\hat{j}$

∴ Velocity of particle with respect to ground

$= \{u\cos(\alpha + \theta) - v\cos q\}\,\hat{i}\ +$

$\{u\sin(\alpha + \theta) - v\sin q\}\,\hat{j}$

Now, as we said earlier that horizontal component of absolute velocity should be zero.

Therefore, $u\cos(\alpha + \theta) - v\cos\theta = 0$

$$\text{or } v = \frac{u\cos(\alpha+\theta)}{\cos\theta} \text{ (down the plane)}$$

Sol 8: Let t be the time after which the stone hits the object and θ be the angle which the velocity vector $\vec{u}$ makes with horizontal. According to question, we have following three conditions. Vertical displacement of stone is 1.25 m

$$1.25 = (u \sin\theta)t - \frac{1}{2}gt^2$$

where g = 10 m/s^2

or $(u \sin\theta)t = 1.25 + 5t^2$ ... (i)

Horizontal displacement of stone

= 3 + displacement of object A

$\therefore (u \cos\theta)t = 3 + \frac{1}{2}at^2$,

where a = 1.5 m/s^2

or $(u \cos\theta)t = 3 + 0.75\,t^2$ ... (ii)

Horizontal component of velocity (of stone) = vertical component (because velocity vector is inclined at 45° with horizontal)

$\therefore (u \cos\theta) = gt - (u \sin\theta)$ ... (iii)

(The right hand side is written $gt - u\sin\theta$ because the stone is in its downward motion. Therefore, $gt > u\sin\theta$. In upward motion $u\sin\theta > gt$)

Multiplying Eq. (iii) with t we can write

or $(u \cos\theta)t + (u \sin\theta)t = 10\,t^2$ ... (iv)

Now Eqs. (iv), (ii) and (i) gives

$4.25t^2 - 4.25 = 0$

or t = 1 s

Substituting t = 1 s in Eqs. (i) and (ii), we get

$u \sin\theta = 6.25$ m/s

or $u_y = 6.25$ m/s

and $u \cos\theta = 3.75$ m/s

or $u_x = 3.75$ m/s

therefore $\vec{u} = u_x\hat{i} + u_y\hat{j}$ m/s

or $\vec{u} = (3.75\hat{i} + 6.25\hat{j})$ m/s

Note: Most of the problems of projectile motion are easily solved by breaking the motion of the particle in two suitable mutually perpendicular directions, say x and y. Find u_x, u_y, a_x and a_y and then apply

$v_x = v_x + a_x t$; $s_y = u_y t + \frac{1}{2}a_y t^2$ etc.

Sol 9: (i) Let A stands for trolley and B for ball.

Relative velocity of B with respect to A ($\vec{v}_{BA}$) should be along OA for the ball to hit the trolley. Hence, $\vec{v}_{BA}$ will make an angle of 45° with positive x–axis.

Q = 45°

(ii) Let v = absolute velocity of ball.

$$\phi = \frac{4\theta}{3} = \frac{4}{3}(45°) = 60° \rightarrow \text{with x–axis}$$

$$\therefore \vec{v}_B = (v\cos\theta)\hat{i} + (v\sin\theta)\hat{j} = \frac{v}{2}\hat{i} + \frac{\sqrt{3}v}{2}\hat{j}$$

$$\vec{v}_A = (\sqrt{3} - 1)\hat{j}$$

$$\therefore \vec{v}_{BA} = \frac{v}{2}\hat{i} + \left(\frac{\sqrt{3}v}{2} - \sqrt{3} + 1\right)\hat{j}$$

Since $\vec{v}_{BA}$ is at 45°

$$\therefore \frac{v}{2} = \frac{\sqrt{3}v}{2} - \sqrt{3} + 1 \text{ or } v = 2 \text{ m/s}$$

Sol 10: $t = T = \frac{2u\sin\theta}{g} = \frac{2\times10\times\sin60°}{10} = \sqrt{3}$ s

Displacement of train in time t = $\frac{1}{2}at^2$

Displacement of boy with respect to train = 1.15 m

$\therefore$ Displacement of boy with respect to ground

$$= \left(1.15 + \frac{1}{2}at^2\right)$$

Displacement of ball with respect to ground = $(u \cos 60°)t$

To catch the ball back at initial height,

$$1.15 + \frac{1}{2}at^2 = (u\cos 60°)t$$

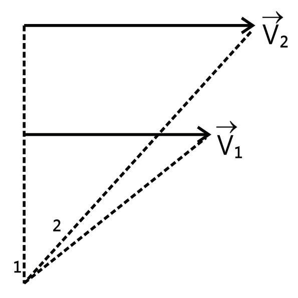
$$\therefore 1.15 + \frac{1}{2}a(\sqrt{3})^2 = 10 \times \frac{1}{2} \times \sqrt{3}$$

Solving this equation, we get

a = 5 m/s^2

$\therefore$ Answer is 5

Sol 11: (B)



$\theta_2 > \theta_1 \therefore \omega_2 > \omega_1$

Statement–II, is formula of relative velocity. But it does not explain statement–I correctly. The correct explanation of statement–I is due to visual perception of motion. The object appears to be moving faster, when its angular velocity is greater w.r.t. observer.

Sol 12: (A, C) Since, the body is at rest at x = 0 and x = 1. Hence, α cannot be positive for all time in the interval $0 \le t \le 1$.

Therefore, first the particle is accelerated and then retarded.

Now, total time t = 1 s (given)

Total displacement s = 1 m (given)

s = Area under v–t graph

$\therefore$ Height or $v_{max} = \frac{2s}{t} = 2$m/s is also fixed.

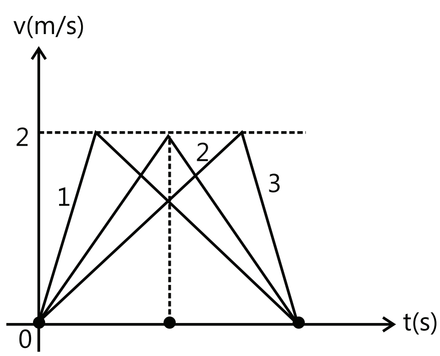
[Area or $s = \frac{1}{2} \times t \times v_{max}$]

If height and base are fixed, area is also fixed

In case 2 : Acceleration = Retardation = 4 m/s^2

In case 1 : Acceleration > 4m/s^2 while

Retardation < 4 m/s^2



While in case 3 : Acceleration < 4 m/s^2 and Retardation > 4 m/s^2

Hence, $|a| \ge 4$ at some point or points in its path.

Sol 13: (A, B, C)

$x = a \cos(pt) \Rightarrow \cos(pt) = \frac{x}{a}$ ... (i)

$y = b \sin(pt) \Rightarrow \sin(pt) = \frac{y}{b}$ ... (ii)

Squaring and adding Eqs. (i) and (ii), we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Therefore, path of the particle is an ellipse. Hence, option (a) is correct.From the given equations we can find

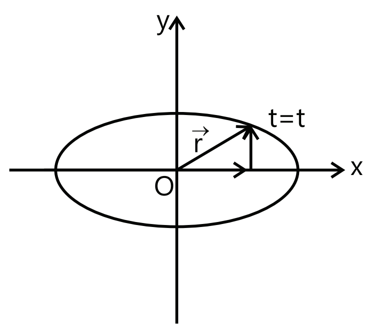
$$\frac{dx}{dt} = v_x = -ap \sin pt$$

$$\frac{d^2x}{dt^2} = a_x = -ap^2 \cos pt$$

$$\frac{dy}{dt} = v_y = bp \cos pt \text{ and}$$

$$\frac{d^2y}{dt^2} = a_y = -bp^2 \sin pt$$

At time $t = \frac{\pi}{2p}$ or $pt = \frac{\pi}{2}$



a_x and v_y become zero (because $\cos \frac{\pi}{2} = 0$)

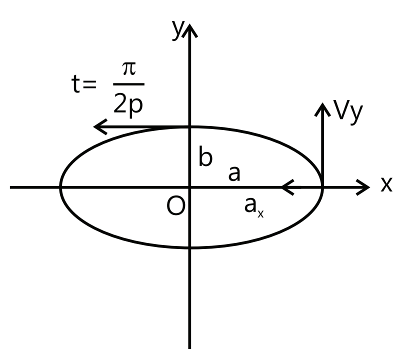
only v_x and a_y are left.

or we can say that velocity is along negative x–axis and acceleration along y–axis

Hence, at $t = \frac{\pi}{2p}$ velocity and acceleration of the particle are normal to each other. So, option (b) is also correct.

At t = t, position of the particle

$$\vec{r}(t) = x\hat{i} + y\hat{j} = a \cos pt\,\hat{i} + b \sin pt\,\hat{j}$$



t = 0
y = 0 = v_x = a_y
x = a
vy = by and
ax = – ap^2

and acceleration of the particle is

$\vec{a}(t) = a_x\hat{i} + a_y\hat{j}$

$= -p^2[a\cos pt\,\hat{i} + b\sin pt\,\hat{j}]$

$= -p^2[x\hat{i} + y\hat{j}] = -p^2\vec{r}(t)$

Therefore, acceleration of the particle is always directed towards origin and not any of the foci.

Hence, option (C) is wrong.

At t = 0, particle is at (a, 0) and at $t = \frac{\pi}{2p}$

particle is at (0, b). Therefore, the distance covered is one–fourth of the elliptical path not a.

Hence, option (D) is wrong.

Sol 14: Maximum displacement of the left ball from the left wall of the chamber is 2.25 cm, so the right ball has to travel almost the whole length of the chamber (4m) to hit the left ball. So the time taken by the right ball is 1.9 sec (approximately 2 sec)