Sample Question Paper - 3
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
3. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
4. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

1. Find the values of k for which the following equation have real roots: $x^2 - 4kx + k = 0$ **[2]**

OR

Solve for x :
$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}; x \neq 1, -2, 2$

2. Three cubes whose edges measure 3 cm, 4 cm and 5 cm respectively to form a single cube. Find its edge. Also, find the surface area of the new cube. **[2]**

3. A survey was conducted by a group of students as a part of their environmental awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house. **[2]**

Number of plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Number of houses	1	2	1	5	6	2	3

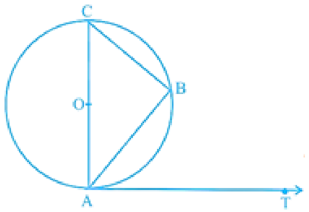
Which method did you use for finding the mean, and why?

4. Show that the sequence 9, 12, 15, 18, ... is an A.P. Find its 16th term and the general term. **[2]**

5. Write the median class for the following frequency distribution: **[2]**

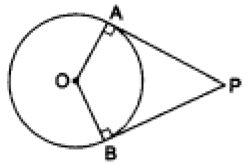
Class Interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Frequency	5	8	7	12	28	20	10	10

6. If AB is a chord of a circle with centre O. AOC is a diameter and AT is the tangent at A as shown in figure. Prove that $\angle$BAT = $\angle$ACB. **[2]**



OR

PA and PB are tangents to the circle with centre O from an external point P, touching the circle at A and B respectively. Show that the quadrilateral AOBP is cyclic.



Section B

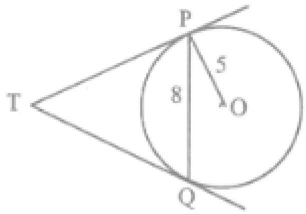
7. How many terms are there in the A.P. $-1, -\frac{5}{6}, -\frac{2}{3}, -\frac{1}{2}, \ldots, \frac{10}{3}$ **[3]**

8. As observed from the top of a 150 m tall light house, the angles of depression of two ships approaching it are 30° and 45°. If one ship is directly behind the other, find the distance between the two ships. **[3]**

OR

If the altitude of the Sun is 60^0, what is the height of a tower which casts a shadow of length 30 m?

9. In the given figure, PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at TP and TQ intersect at point T. Find the length of TP. **[3]**



10. Solve the quadratic equation by factorization: **[3]**

$(a+ b)^2x^2 - 4abx - (a-b)^2 = 0$.

Section C

11. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle. **[4]**

OR

Let ABC be a right triangle in which AB = 6cm, BC = 8cm and $\angle B = 90^\circ$. BD is the perpendicular from B on AC. The circle through B,C and D is drawn construct the tangents from A to this circle.

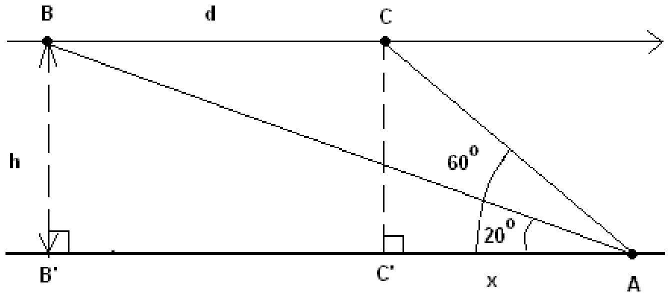
12. The following table gives the distribution of the life time of 400 neon lamps: **[4]**

Lite time (in hours)	**Number of lamps**
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86

3500-4000	74
4000-4500	62
4500-5000	48

Find the median life time of a lamp.

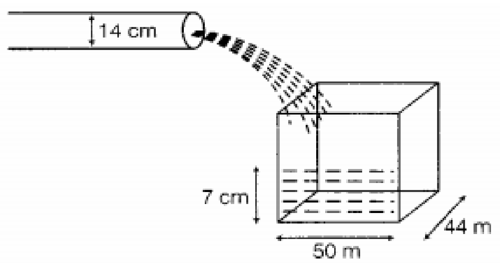
13. Mr. Vinod is a pilot in Air India. During the Covid-19 pandemic, many Indian passengers were stuck at Dubai Airport. The government of India sent special aircraft to take them. Mr. Vinod was leading this operation. He is flying from Dubai to New Delhi with these passengers. His airplane is approaching point A along a straight line and at a constant altitude h. At 10:00 am, the angle of elevation of the airplane is 20^{o} and at 10:01 am, it is 60^{o}. **[4]**



i. What is the distance 'd' is covered by the airplane from 10:00 am to 10:01 am if the speed of the airplane is constant and equal to 600 miles/hour?
ii. What is the altitude 'h' of the airplane? (round answer to 2 decimal places).

14. A farmer used to irrigate his land during summer on a regular basis to grow his crops and save them from dry weather. To irrigate his land he built a tube well in his field. The tube well has a rectangular tank and a pipe that is used to fill this tank. The dimensions of this tube well system are: **[4]**

Water is flowing at the rate of 5 km/hr through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide.



After reading the above given information, answer the following questions:

i. The volume of the water flowing through the cylindrical pipe in x hours.
ii. Determine the time in which the level of the water in the tank will rise by 7 cm.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

Section A

1. A quadratic equation $ax^2 + bx + c = 0$ ..(1)
has equal roots when discriminant is 0.
Discriminant (D) is given by: b^2-4ac
Given equation: $x^2 - 4kx + k = 0$
On comparing with [1] we have,
$a = 1,\ b = -4k\ and\ c = k$
The given quadratic equation has equal root
$\Rightarrow D = 0$ we have;
$\Rightarrow b^2 - 4ac = 0$
$\Rightarrow b^2 = 4ac$
Substitute the given values we have;
$(-4k)^2 = 4 \times 1 \times k$
Simplify:
$16k^2 = 4k$
Divide both sides by $4k$ we have;
$4k = 1\ or\ k = \frac{1}{4}$
$\therefore$ the value of k is $\frac{1}{4}$.

OR

Given, $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$
$\Rightarrow \frac{x^2+3x+2+x^2-3x+2}{x^2+x-2} = \frac{4x-8-2x-3}{x-2}$
$\Rightarrow (2x^2 + 4)(x - 2) = (2x - 11)(x^2 + x - 2)$
or, $5x^2 + 19x - 30 = 0$
$\Rightarrow 5x^2 + 25x - 6x - 30 = 0$
$\Rightarrow (x+5)(5x-6) = 0$
$\Rightarrow x = -5, \frac{6}{5}$

2. Let x cm be the edge of the new cube. Then,
Volume of the new cube = Sum of the volumes of three cubes.
$\Rightarrow x^3 = 3^3 + 4^3 + 5^3 = 27 + 64 + 125$
$\Rightarrow x^3 = 216$
$\Rightarrow x^3 = 6^3 \Rightarrow x = 6\text{cm}$
$\therefore$ Edge of the new cube is 6 cm long.
Surface area of the new cube = $6x^2 = 6 \times (6)^2\ cm^2 = 216\ cm^2$

3.

Number of plants	**Number of houses(f_i)**	**Class mark (x_i)**	**f_ix_i**
0-2	1	1	1
2-4	2	3	6
4-6	1	5	5
6-8	5	7	35
8-10	6	9	54
10-12	2	11	22
12-14	3	13	39
Total	$\sum f_i = 20$		$\sum f_ix_i = 162$

$$\therefore \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$ ... Using direct method because numerical values of x_i and f_i are small

$= \frac{162}{20}$

= 8.1 plants

We have used direct method for finding the mean because numerical values of x_i and f_i are small.

4. We have, (12 - 9) = (15 -12) = (18 - 15) = 3
   Therefore, the given sequence is an A.P. with common difference d=3
   a = First term = 9
   So 16^{th} term =9+(16-1)×3
   =9+45=54
   Also T_n= a+(n-1)×d
   =9+(n-1)×3=9+3n-3
   Or T_n =3n+6

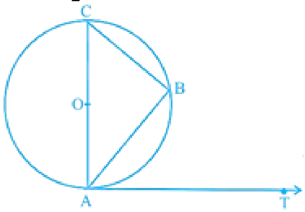
5.

Class Interval	**Frequency**	**C.F**
0 - 10	5	5
10 - 20	8	13
20 - 30	7	20
30 - 40	12	32
40 - 50	28	60
50 - 60	20	80
60 - 70	10	90
70 - 80	10	100
Total	N = Σf_i = 100	

Here N = 100, then $\frac{N}{2}$ = 50

which lies in the class 40 - 50 ...(∵ 32 < 50 < 60)

∴ Required Median class interval is 40 - 50

6. 

Given: Chord AB, diameter AOC and tangents at A of a circle with centre O.

To prove: ∠BAT = ∠ACB

Proof: Radius OA and tangent AT at A are perpendicular.

∴ ∠OAT = 90^o (radius at the point of contact of tangent is perpendicular)

⇒ ∠BAT = 90^o - ∠BAC .....(i)

AOC is diameter.

∴ ∠B = 90

⇒ ∠C + ∠BAC = 90^o

⇒ ∠C = 90^o - ∠BAC ...(ii)

From (i) and (ii), we get

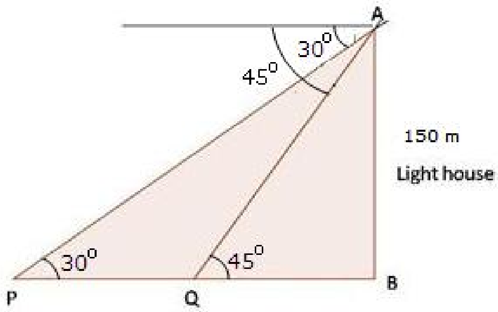
∠BAT = ∠ACB. Hence, proved.

OR

Given: PA and PB are tangents to the circle with centre O from an external point P.
T Prove: Quadrilateral AOBP is cyclic.
Since tangents to a circle is perpendicular to the radius,
∴ OA ⊥ AP and OB ⊥ BP.
⇒ ∠OAP = 90° and ∠OBP = 90°
⇒ ∠OAP + ∠OBP = 90° + 90° = 180° .... (1)
In quadrilateral OAPB,
∠OAP + ∠APB + ∠AOB + ∠OBP = 360°
⇒ (∠APB + ∠AOB) + (∠OAP + ∠OBP) = 360°
⇒ ∠APB + ∠AOB + 180° = 360° [From (1)]
⇒ ∠APB + ∠AOB = 180° .... (2)
From (1) and (2), the quadrilateral AOBP is cyclic.

Section B

7. Here, A.P is $-1, -\frac{5}{6}, -\frac{2}{3}, -\frac{1}{2}, \ldots, \frac{10}{3}$
The first term (a) = -1
The last term (a_n) = $\frac{10}{3}$
Now,
common difference (d) = a_1 - a
= $-\frac{5}{6} - (-1)$
= $-\frac{5}{6} + 1$
= $\frac{-5+6}{6}$
= $\frac{1}{6}$
Thus, using the formula for nth term, viz. a_n = a + (n-1) b, we get
$\frac{10}{3} = -1 + (n-1)\frac{1}{6}$
$\frac{10}{3} + 1 = \frac{1}{6}n - \frac{1}{6}$
$\frac{13}{3} + \frac{1}{6} = \frac{1}{6}n$
Further solving for n, we get
$\frac{26+1}{6} = \frac{1}{6}n$
$n = \frac{27}{6}(6)$
Thus, n = 27
Therefore, the number of terms present in the given A.P is 27

8.


Height of light house $AB = 150m$
In ΔABQ
$\tan 45^\circ = \frac{AB}{BQ}$
$\Rightarrow 1 = \frac{150}{BQ}$
$BQ = 150m$
In ΔABP
$\tan 30^\circ = \frac{AB}{PB}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{150}{PB}$
$\Rightarrow PB = 150\sqrt{3} = 150 \times 1.73 = 259.5m$

$\therefore$ Distance between two ships PQ = PB - BQ

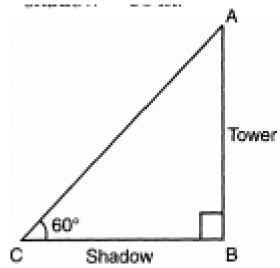
$= 259.5 - 150$

$= 109.5m$

OR

Let AB be the tower whose height be h m.

BC = shadow = 30 m.

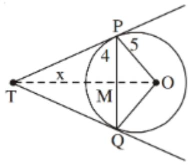


$\triangle ABC, \quad \frac{AB}{BC} = \tan 60^\circ$ In

$\Rightarrow \quad \frac{h}{30} = \sqrt{3}$

$h = 30\sqrt{3}\text{m}$

Hence, Height of tower = $30\sqrt{3}\text{m}$.

9. 

Join OT and OQ.

TP = TQ (As length of tangents from a point outside the circle is equal)

$\therefore$ TM $\perp$ PQ and bisects PQ

Hence PM = 4 cm

Therefore OM = $\sqrt{25 - 16}$ = $\sqrt{9}$ = 3 cm

Let TM = x

From $\triangle$PMT, $PT^2 = x^2 + 16$

From $\triangle$POT, $PT^2 = (x + 3)^2 - 25$

Hence $x^2 + 16 = x^2 + 9 + 6x - 25$

$\Rightarrow$ 6x = 32 $\Rightarrow$ x = $\frac{16}{3}$

Hence, $PT^2 = \frac{256}{9} + 16 = \frac{400}{9}$

$\therefore$ PT = $\frac{20}{3}$ cm = 6.667 cm

10. We have,

$(a+b)^2x^2 - 4abx - (a-b)^2 = 0$

In order to factorize $(a+b)^2x^2 - 4abx - (a-b)^2$, we have to find two numbers 'l' and 'm' such that.

$l + m = -4ab$ and $lm = -(a + b)^2(a - b)^2$

Clearly, $(a - b)^2 + [-(a + b)^2] = -4ab$ and $lm = -(a + b)^2(a - b)^2$

$\therefore l = (a - b)^2$ and $m = -(a + b)^2$

Now,

$(a+b)^2x^2 - 4abx - (a-b)^2 = 0$

$\Rightarrow (a+b)^2x^2 - (a+b)^2x + (a-b)^2x - (a-b)^2 = 0$

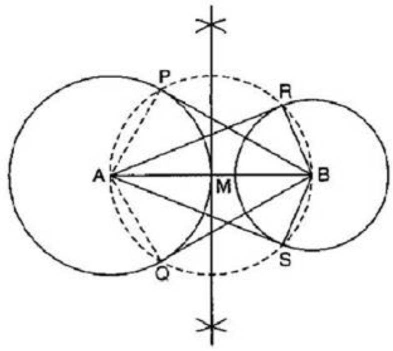
$\Rightarrow (a + b)^2x [x - 1] + (a - b)^2[x - 1] = 0$

$\Rightarrow (x - 1)[(a + b)^2x + (a - b)^2] = 0$

$\Rightarrow x - 1 = 0$ or $(a + b)^2x + (a - b)^2 = 0$

$\Rightarrow$ x = 1 or $x = -\frac{(a-b)^2}{(a+b)^2}$

Section C

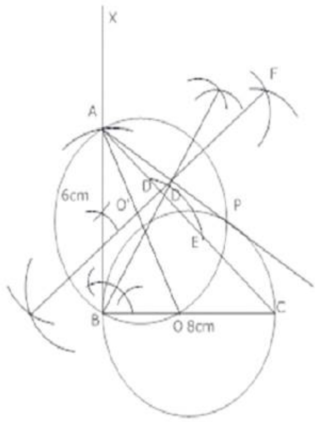
11. 

To construct: A line segment of length 8 cm and taking A as centre, to draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Also, to construct tangents to each circle from the centre to the other circle.

Steps of Construction :

i. Bisect BA. Let M be the mid-point of BA.
ii. Taking M as centre and MA as radius, draw a circle. Let it intersects the given circle at the points P and Q.
iii. Join BP and BQ.
Then, BP and BQ are the required two tangents from B to the circle with centre A.
iv. Again, Let M be the mid-point of AB.
v. Taking M as centre and MB as radius, draw a circle. Let it intersects the given circle at the points R and S.
vi. Join AR and AS.
Then, AR and AS are the required two tangents from A to the circle with centre B.
Justification: Join BP and BQ.
Then $\angle APB$ being an angle in the semicircle is 90°.
$\Rightarrow BP \perp AP$
Since AP is a radius of the circle with centre A, BP has to be a tangent to a circle with centre A. Similarly, BQ is also a tangent to the circle with centre A.
Again join AR and AS.
Then $\angle ARB$ being an angle in the semicircle is 90°.
$\Rightarrow AR \perp BR$
Since BR is a radius of the circle with centre B, AR has to be a tangent to a circle with centre B. Similarly, AS is also a tangent to the circle with centre B.

OR



Steps of construction:

i. Draw $\triangle ABC$ with BC = 8 cm, AB = 6 cm and $\angle B = 90^\circ$.
ii. Draw perpendicular BD from B to AC.
iii. Let O be the mid-point of BC. Draw a circle with centre O and radius OB = OC. This circle will pass through the point D.
iv. Join AO and bisect AO.
v. Draw a circle with centre O' and O'A as radius cuts the previous circle at B and P.
vi. Join AP, AP and AB are required tangents drawn from A to the circle passing through B, C and D.

12.

Life time	Number of lamps (f_i)	Cumulative frequency

1500-2000	14	14
2000-2500	56	14 + 56 = 70
2500-300	60	70 + 60 = 130
3000-3500	86	130 + 86 = 216
3500-4000	74	216 + 74 = 290
4000-4500	62	290 + 62 = 352
4500-5000	48	352 + 48 = 400
	400	

N = 400

Now we may observe that cumulative frequency just greater than $\frac{n}{2}$ (ie., $\frac{400}{2}$ = 200) is 216

Median class = 3000 - 3500

Median = l + $\left(\frac{\frac{n}{2}-cf}{f}\right) \times$ h

Here,

l = Lower limit of median class

F = Cumulative frequency of class prior to median class.

f = Frequency of median class.

h = Class size.

Lower limit (l) of median class = 3000

Frequency (f) of median class 86

Cumulative frequency (cf) of class preceding median class = 130

Class size (h) = 500

Median = 3000 + $\left(\frac{200-130}{86}\right) \times 500$

= 3000 + $\frac{70\times500}{86}$

= 3406.98

13. i. Time covered 10.00 am to 10.01 am = 1 minute = $\frac{1}{60}$ hour

Given: Speed = 600 miles/hour

Thus, distance d = 600 $\times \frac{1}{60}$ = 10 miles

ii. Now, tan 20$^{\mathrm{o}}$ = $\frac{BB'}{B'A}$ = $\frac{h}{10+x}$ ....eq(1)

And tan 60$^{\mathrm{o}}$ = $\frac{CC'}{C'A} = \frac{BB'}{C'A}$ = $\frac{h}{x}$

x = $\frac{h}{tan60^o}$ = $\frac{h}{\sqrt{3}}$

Putting the value of x in eq(1), we get,

tan 20$^{\mathrm{o}}$ = $\frac{h}{10+\frac{h}{\sqrt{3}}} = \frac{\sqrt{3}h}{10\sqrt{3}+h}$

0.364(10$\sqrt{3}$ + h) = $\sqrt{3}$ h

6.3 + 0.364 h = 1.732 h

1.368 h = 6.3

h = 4.6

Thus, the altitude 'h' of the airplane is 4.6 miles.

14. According to question,

Diameter of the pipe = 14 cm

Thus, Radius of the pipe = 7 cm

Now, the volume of the water flowing through the cylindrical pipe in 1 hour = π r^2 h

Clearly, the water column forms a cylinder whose radius r = $\frac{14}{2}\text{cm} = \frac{7}{100}\text{m}$

and Length = h = 5000x m

i. Volume of the water flowing through the cylindrical pipe in x hours

$= \pi r^2 h = \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 5000xm^3 = \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 5000x$

$= 77x \text{ m}^3$

ii. Volume of the water that falls into the tank in x hours = $50 \times 44 \times \frac{7}{100} = 154\text{m}^3$

Volume of the water flowing through the cylindrical pipe in x hours = Volume of the water that falls in the tank in x hours

$\Rightarrow$ 77x = 154

$\Rightarrow \quad x = \frac{154}{77} = 2$

Hence, the level of water in the tank will rise by 7 cm in 2 hours.