# 11 Waves

## Section-A : JEE Advanced/ IIT-JEE

**A**
**1.** $A(2\pi\nu), A(2\pi\nu)^2$ **2.** 0.125 m **3.** 1 : 1 **4.** 240 Hz **5.** 0.5 $ms^{-1}$
**6.** $f$ **7.** 6 Hz

**B**
**1.** F **2.** T **3.** F

**C**
**1.** (c) **2.** (c) **3.** (a) **4.** (c) **5.** (a) **6.** (a)
**7.** (a) **8.** (d) **9.** (d) **10.** (b) **11.** (b) **12.** (c)
**13.** (b) **14.** (a) **15.** (b) **16.** (b) **17.** (b) **18.** (c)
**19.** (d) **20.** (a) **21.** (a) **22.** (a) **23.** (a) **24.** (b)
**25.** (a) **26.** (b)

**D**
**1.** (a,b,c,d) **2.** (b) **3.** (a, c) **4.** (a) **5.** (a, c) **6.** (c)
**7.** (a,b,d) **8.** (b, c) **9.** (b, c) **10.** (b) **11.** (a, b, c) **12.** (b)
**13.** (b, c) **14.** (a, c) **15.** (b, c, d) **16.** (a,b,c,d) **17.** (a, b, c) **18.** (a,c,d)
**19.** (a,d) **20.** (b, d) **21.** (b, c) **22.** (a, b) **23.** (d) **24.** (a, c, d) **25.** (a,b,c)

**E**
**1.** 1650 Hz **2.** 70 m/s **3.** 3.33 cm; 163 Hz **4.** 8 **5.** 27.04 N

**6.** 0.75 m/s **7.** 11 Hz **8.** **(i)** 3.46 cm **(ii)** 0, 15, 30 **(iii)** zero **(iv)** $2\sin\left(96\pi t+\frac{\pi x}{15}\right)$ and $-2\sin\left(96\pi t-\frac{\pi x}{15}\right)$

**9.** 1.5 m/s **10.** **(i)** $z_1$ and $z_2$; $\frac{(2n+1)\pi}{2K}$ where $n = 0, 1, 2, ...$ **(ii)** $z_1$ and $z_3$; $\frac{(2n+1)\pi}{K}$

**11.** **(i)** 599 Hz **(ii)** 0.935 km, 621 Hz **12.** 438.7 Hz, 257.3 Hz

**13.** **(a)** $\frac{2\pi}{a}, \frac{b}{2\pi}$ **(b)** $y = -0.8A\cos(ax-bt)$ **(c)** 1.8 Ab, 0

**(d)** $y = -1.6A\sin ax\sin bt + 0.2A\cos(ax+bt)$ $\left[n+\frac{(-1)^n}{2}\right]\frac{\pi}{a}$, $-X$ direction **14.** **(i)** $\frac{2\pi}{10^3}$ sec. **(ii)** $\frac{\pi}{2}\times 10^{-3}$ sec.

**15.** 403.3 Hz to 484 Hz **16.** $\frac{(v+v_m)\times 2v_b f}{v^2-v_b^2}$

**17.** **(a)** $\frac{15}{16}$ m **(b)** $\frac{\Delta P_0}{\sqrt{2}}$ **(c)** equal to mean pressure **(d)** $P_0+\Delta P_0,\ P_0-\Delta P_0$ **18.** **(a)** 0.14 s **(b)** 2.0 cm, 1.5 cm

**19.** $\frac{-dH}{dt} = (1.11\times 10^{-2})\sqrt{H}$, 43 sec. **20.** **(a)** $1.007\times 10^5$ Hz, **(b)** $1.03\times 10^5$ Hz **21.** **(a)** $\frac{400}{189}$ **(b)** $\frac{3}{4}$

**22.** 336 m/s **23.** $\frac{\pi^2 Ta^2}{4\ell}$ **24.** 30 m/s **25.** $y = 0.1\sin\left[30t \pm \frac{3}{2}x \pm \phi\right]$

**F**
**1.** A → q; B → p; C → r **2.** A → p, t; B → p, s; C → q, s; D → q, r

**G**
**1.** (a) **2.** (c) **3.** (d) **4.** (b) **5.** (a) **6.** (a)

**I**
**1.** 5 **2.** 7 **3.** 5 **4.** 3

## Section-B : JEE Main/ AIEEE

**1.** (b) **2.** (c) **3.** (b) **4.** (b) **5.** (b) **6.** (a)
**7.** (a) **8.** (c) **9.** (b) **10.** (d) **11.** (c) **12.** (c)
**13.** (a) **14.** (a) **15.** (b) **16.** (a) **17.** (b) **18.** (a)
**19.** (d) **20.** (a) **21.** (a) **22.** (b) **23.** (c) **24.** (d)
**25.** (a) **26.** (b)

## Section-A JEE Advanced/ IIT-JEE

### A. Fill in the Blanks

**1.** Since $y = A\sin(\omega t - kx)$
Displacement amplitude $= A$ (Max displacement)

Particle velocity, $v = \frac{dy}{dt} = A\omega\cos(\omega t - kx)$

$\therefore$ Velocity amplitude $= A\omega = 2\pi\nu A$

Particle acceleration

$$\text{Acc} = \frac{dv}{dt} = -A\omega^2\sin(\omega t - kx)$$

$\therefore$ Acceleration (Max acc) amplitude $= A\omega^2 = 4\pi^2\nu^2 A$

**2.** $c = \nu\lambda \quad \therefore \lambda = \frac{c}{\nu} = \frac{330}{660} = 0.5\,\text{m}$

The rarefaction will be at a distance of

$$\frac{\lambda}{4} = \frac{0.5}{4} = 0.125\,\text{m}$$

**3.** $y_1 = 10\sin(3\pi t + \pi/4)$ ...(i)

$y_2 = 5\sin 3\pi t + 5\sqrt{3}\cos 3\pi t$ ...(ii)

$\therefore\ y_2 = 5 \times 2\left[\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right] = 10\sin(3\pi t + \pi/3)$

The ratio of amplitudes is $10 : 10 = 1 : 1$

**4.** $$\frac{\nu_1}{\nu_2} = \frac{\frac{1}{2\ell}\sqrt{\frac{50.7\times 8}{m}}}{\frac{1}{2\ell}\sqrt{\frac{43.2\times g}{m}}} \Rightarrow \nu_2 = \nu_1\sqrt{\frac{43.2}{50.7}} = 260\sqrt{\frac{43.2}{50.7}} = 240\,\text{Hz.}$$

**5.** As $y = \frac{1}{(1+x)^2}$

At $t = 0$ and $x = 0$, we get $y = 1$.
Also at $t = 2$ and $x = 1$, again $y = 1$
The wave pulse has travelled a distance of 1m in 2 sec.

$$\therefore\quad v = \frac{1}{2} = 0.5\,\text{ms}^{-1}$$

**6.** In figure (i)

$$\frac{\lambda}{2} = \ell \Rightarrow \lambda = 2\ell$$

and $f = \frac{c}{\lambda} = \frac{c}{2\ell}$

In figure (ii)

$$\Rightarrow \frac{\lambda'}{4} = \frac{\ell}{2} \Rightarrow \lambda' = 2\ell$$

and $f' = \frac{c}{\lambda'} = \frac{c}{2\ell} = f$

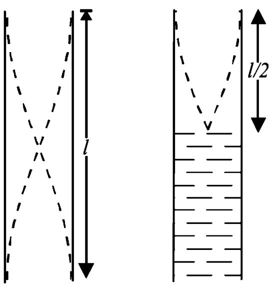
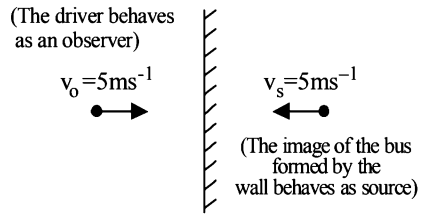


Fig (i) Fig (ii)

**7.** The first frequency that driver of bus hears is the original frequency of 200 Hz. The second frequency that driver hears is the frequency of sound reflected from the wall. The two frequencies of sound heard by driver is
(a) Original frequency (200 Hz.)
(b) Frequency of sound reflected from the wall ($\nu'$)




The frequency of sound reflected from the wall

$$\nu' = \nu\left[\frac{v + v_0}{v - v_s}\right] \Rightarrow \nu' = 200\left[\frac{342+5}{342-5}\right] \approx 206\,\text{Hz.}$$

$\therefore$ Frequency of beats $= \nu' - \nu = 6\,\text{Hz.}$

### B. True/ False

**1.** The intensity of sound at a given point is the energy per second received by a unit area perpendicular to the direction of propagation.

$$I = \frac{1}{2}\rho V\omega^2 A^2$$

Also intensity varies as distance from the point source as

$$I \propto \frac{1}{r^2}$$

**NOTE :** None of the parameters are changing in case of a clear night or a clear day.
Therefore the intensity will remain the same.

**2.** Speed of sound waves in water is greater than in air.

**3.** **NOTE :** If the sound reaches the observer after being reflected from a stationary surface and the medium is also stationary, the image of the source will become the source of reflected sound.
Thus in both the cases, one sound coming directly from the source and the other coming after reflection will have the same apparent frequency (Since velocity of source w.r.t. observer is same in both the cases). Therefore no beats will be heard.

### C. MCQs with ONE Correct Answer

**1.** **(c)** **Case (i)** Here $\frac{\lambda}{2} = \ell \quad \therefore \quad f = \frac{v}{\lambda} = \frac{v}{2\ell}$ ...(i)

**Case (ii)** Here $\frac{\lambda'}{4} = \frac{\ell}{2} \Rightarrow \lambda' = 2\ell \quad \therefore f = \frac{v}{\lambda'} = \frac{v}{2\ell} = f$

**2.** **(c)** **NOTE :** Stationary wave is produced when two waves travel in opposite direction.
Now, $y = a\cos(kx - \omega t) - a\cos(kx + \omega t)$
$\therefore$ $y = 2a\sin kx\sin\omega t$ is equation of stationary wave which gives a node at $x = 0$.

**3.** **(a)** *In air* : $T = \text{mg} = \rho Vg$

$$\therefore\quad f = \frac{1}{2\ell}\sqrt{\frac{\rho Vg}{m}} \quad \text{...(i)}$$

*In water* : $T = mg$ – upthrust

$$= V\rho g - \frac{V}{2}\rho_\omega g = \frac{Vg}{2}(2\rho - \rho_\omega)$$

$\therefore \quad f' = \frac{1}{2\ell}\sqrt{\frac{\frac{Vg}{2}(2\rho-\rho_\omega)}{m}} = \frac{1}{2\ell}\sqrt{\frac{Vg\rho}{m}}\sqrt{\frac{(2\rho-\rho_\omega)}{2\rho}}$

$\frac{f'}{f} = \sqrt{\frac{2\rho-\rho_\omega}{2\rho}} \quad f' = f\left(\frac{2\rho-\rho_\omega}{2\rho}\right)^{1/2}$

$= 300\left[\frac{2\rho-1}{2\rho}\right]^{1/2}$ Hz

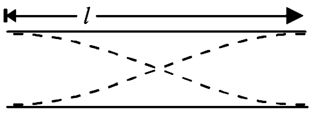
**4. (c)** Comparing it with
$y(x, t) = A\cos(\omega t + \pi/2)\cos kx$
If $kx = \pi/2$, a node occurs;
$\therefore \quad 10\pi x = \pi/2 \Rightarrow x = 0.05$ m
If $kx = \pi$, an antinode occurs
$\Rightarrow \quad 10\pi x = \pi \Rightarrow x = 0.1$ m
Also speed of wave
$= \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5$ m/s and $\lambda = 2\pi/k = 2\pi/10\pi = 0.2$ m

**5. (a)** Velocity of sound by a stretched string $v = \sqrt{\frac{T}{m}}$

$\frac{v}{v'} = \sqrt{\frac{T}{T'}} \quad v' = v\sqrt{\frac{T'}{T}} = v\sqrt{\frac{1.5x}{x}} = 1.22\, v$

**6. (a)** **For both end open**

$\frac{2\lambda_1}{4} = \ell \Rightarrow \lambda_1 = 2\ell$



$v_1 = \frac{c}{\lambda_1} = \frac{c}{2\ell}$ ... (i)

**For one end closed**

For third harmonic $\frac{3\lambda_2}{4} = \ell \Rightarrow \lambda_2 = \frac{4\ell}{3}$

$v_2 = \frac{c}{\lambda_2} = \frac{3c}{4\ell}$ ... (ii)

Given $v_2 - v_1 = 100$
From (i) and (ii)

$\frac{v_2}{v_1} = \frac{3/4}{1/2} = \frac{3}{2}$

On solving, we get $v_1 = 200$ Hz.

**7. (a)** $v = \frac{dy}{dt} = -A\omega\cos(kx - \omega t) \quad \therefore \quad v_{max} = A\omega$

**8. (d)** $n_1 = n_0\frac{340}{340-34} = \frac{10}{9}n_0;$

$n_2 = n_0\frac{340}{340-17} = \frac{20}{19}n_0; \quad \frac{n_1}{n_2} = \frac{10}{9}\times\frac{19}{20} = \frac{19}{18}$

**9. (d)** $n_1 = \frac{1}{2\ell}\sqrt{\left(\frac{T}{4\pi r^2\rho}\right)}$ and $n_2 = \frac{1}{4\ell}\sqrt{\left(\frac{T}{\pi r^2\rho}\right)}$

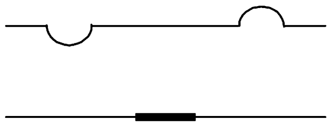
$n = \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T}{m}}$ [where $\frac{\lambda}{2}$ = length of string]

$\therefore \quad \frac{n_1}{n_2} = 2\times\frac{1}{2} = 1 \left[\because m = \frac{\text{mass}}{\text{length}} = \frac{\rho\times A\times \text{length}}{\text{length}} = \rho A\right]$

**10. (b)** **KEY CONCEPT :** $C_{rms} = \sqrt{\left(\frac{\gamma RT}{M}\right)}$ Here $C_{rms} \propto \sqrt{\frac{1}{m}}$;

$\therefore \quad \frac{C_{rms_1}}{C_{rms_2}} = \sqrt{\left(\frac{m_2}{m_1}\right)}$

**11. (b)**



After two seconds pulses will overlap each other.
**NOTE :** According to superposition principle the string will not have any distortion and will be straight. Hence there will be no P.E. The total energy will be only kinetic.

**12. (c)** $E \propto A^2\nu^2$ where $A$ = amplitude and $\nu$ = frequency.
Also $\omega = 2\pi\nu \Rightarrow \omega \propto \nu$
**In case 1 :** Amplitude $= A$ and $\nu_1 = \nu$
**In case 2 :** Amplitude $= A$ and $\nu_2 = 2\nu$

$\therefore \quad \frac{E_2}{E_1} = \frac{A^2\nu_2^2}{A^2\nu_1^2} = 4 \Rightarrow E_2 = 4E_1$

**13. (b)** Using the formula $n' = n\left(\frac{v_A + v}{v}\right)$

$\frac{v_A + v}{v} = \frac{5.5}{5}$ and $\frac{V_B + V}{V} = \frac{6}{5} \Rightarrow \frac{v_B}{v_A} = 2$

**14. (a)** $f_0 = \frac{5}{2\ell}\sqrt{\frac{9g}{\mu}} = \frac{3}{2\ell}\sqrt{\frac{Mg}{\mu}}$

$\Rightarrow M = 25$ kg

**NOTE :** Using the formula of a vibrating string,

$f = \frac{p}{2\ell}\sqrt{\frac{T}{\mu}}$ where $p$ = number of loops.

In each case, the wire vibrates, in resonance with the same tuning fork. Frequency of wire remains same while $p$ and $T$ change.

$\therefore \quad \frac{p_1}{2\ell}\sqrt{\frac{T_1}{\mu}} = \frac{p_2}{2\ell}\sqrt{\frac{T_2}{\mu}}$ or $p_1\sqrt{T_1} = p_2\sqrt{T_2}$

or $\sqrt{\frac{T_2}{T_1}} = \frac{p_1}{p_2}$

$\sqrt{\frac{M\times g}{9\times g}} = \frac{5}{3}$ or $M = \frac{5\times5\times9}{3\times3}$ or M = 25 kg.

**15. (b)** $f_1$ = frequency of the police car heard by motorcyclist,
$f_2$ = frequency of the siren heard by motorcyclist.

$f_1 = \frac{330-v}{330-22}\times176; \; f_2 = \frac{330+v}{330}\times165;$

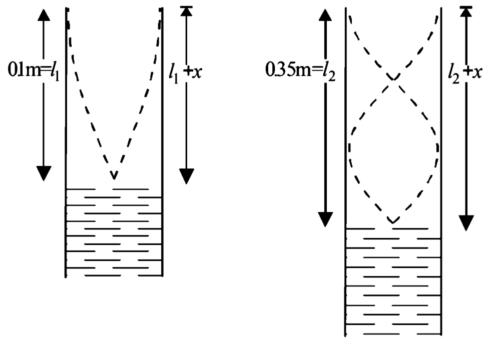
$\because \quad f_1 - f_2 = 0 \Rightarrow v = 22$ m/s

**16. (b)** $\ell_1 + x = \frac{\lambda}{4}$ or, $\lambda = 4(\ell_1 + x)$

$(\ell_2 + x) = \frac{3\lambda}{4}$ or $\lambda = \frac{4}{3}(\ell_2 + x)$

$\therefore \quad v_1 = \frac{v}{\lambda_1} = \frac{v}{4(\ell_1 + x)} \quad \therefore \quad v_2 = \frac{v}{\lambda_2} = \frac{3v}{4(\ell_2 + x)}$

Given $v_1 = v_2$




or, $\dfrac{v}{4(\ell_1 + x)} = \dfrac{3v}{4(\ell_2 + x)}$ or, $x = 0.025\,\text{m}$

**17. (b)** Frequency of first overtone in closed pipe,

$$\nu = \frac{3v}{4\ell_1}\sqrt{\frac{P}{\rho_1}} \quad \text{... (i)}$$

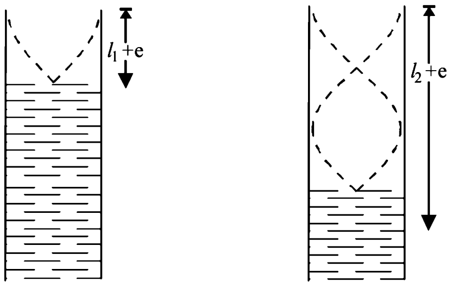
Frequency of first overtone in open pipe,

$$\nu' = \frac{1}{\ell_2}\sqrt{\frac{P}{\rho_2}} \quad \text{... (ii)}$$

From equation (i) and (ii)

$$\Rightarrow \ell_2 = \frac{4}{3}\ell_1\sqrt{\frac{\rho_1}{\rho_2}}$$

**18. (c)**




**For first resonance** $\ell_1 + e = \dfrac{\lambda}{4}$

**For second resonance** $\ell_2 + e = \dfrac{3\lambda}{4}$

But $v = \nu\lambda$

$\therefore\ v = \nu\dfrac{4}{3}(\ell_2 + e) \quad \Rightarrow \ell_2 + e = \dfrac{3v}{4\nu}$ ...(i)

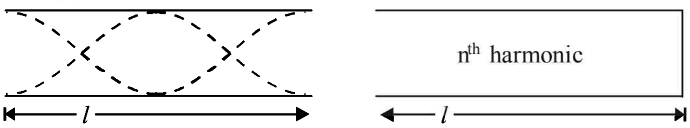
$\therefore\ v = \nu\, 4(\ell_1 + e) \quad \Rightarrow \ell_1 + e = \dfrac{v}{4\nu}$ ... (ii)

Subtracting (i) and (ii),

$v = 2\nu(\ell_2 - \ell_1) \ \therefore\ \Delta v = 2\nu(\Delta\ell_2 + \Delta\ell_1)$

$= 2 \times 512 \times (0.1 + 0.1)\,\text{cm/s} = 204.8\,\text{cm/s}$

**19. (d)**




$\lambda = \ell$

$$\therefore\ f_1 = \frac{v}{\lambda} = \frac{v}{\ell} \quad \text{... (i)}$$

$$\therefore\ f_2 = \frac{v}{\lambda} = \frac{nv}{4\ell} \quad \text{... (ii)}$$

Here $n$ is a odd number. From (i) and (ii)

$f_2 = \dfrac{n}{4}f_1$, For first resonance, $n = 5, f_2 = \dfrac{5}{4}f_1$

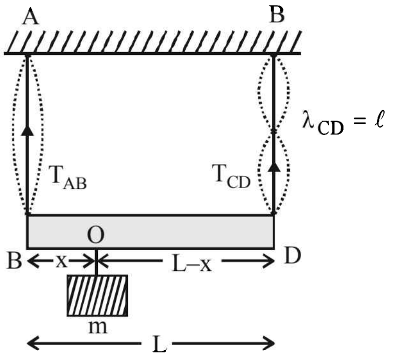
**20. (a)** Frequency of Ist harmonic of $AB = \dfrac{1}{2\ell}\sqrt{\dfrac{T_{AB}}{m}}$

Frequency of 2nd harmonic of $CD = \dfrac{1}{\ell}\sqrt{\dfrac{T_{CD}}{m}}$

Given that the two frequencies are equal.

$$\therefore\ \frac{1}{2\ell}\sqrt{\frac{T_{AB}}{m}} = \frac{1}{\ell}\sqrt{\frac{T_{CD}}{m}} \Rightarrow \frac{T_{AB}}{4} = T_{CD}$$

$\Rightarrow T_{AB} = 4T_{CD}$ ... (i)




For rotational equilibrium of massless rod, taking torque about point $O$.

$T_{AB} \times x = T_{CD}(L - x)$ ... (ii)

For translational equilibrium,

$T_{AB} + T_{CD} = mg$ ... (iii)

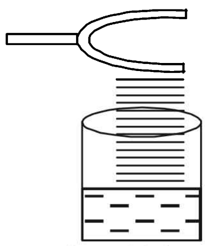
On solving, (i) and (iii), we get

$T_{CD} = \dfrac{mg}{5} \ \therefore\ T_{AB} = \dfrac{4mg}{5}$

Substituting these values in (ii), we get

$$\frac{4mg}{5} \times x = \frac{mg}{5}(L - x) \Rightarrow x = \frac{L}{5}$$

**21. (a)** As shown in the figure, the fringes of the tuning fork are kept in a vertical plane.



**22. (a)** Since the wave is sinusoidal moving in positive x-axis the point will move parallel to y-axis therefore options (c) and (d) are ruled out. As the wave moves forward in positive X-direction, the point should move upwards i.e. in the positive Y-direction. Therefore correct option is a.

**23. (a)** The frequency ($\nu$) produced by the air column is given by

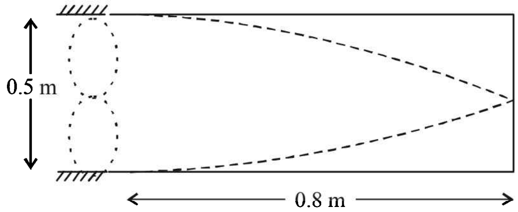
$$\lambda \times \nu = v \Rightarrow \nu = \frac{v}{\lambda}$$

Also, $\dfrac{3\lambda}{4} = \ell = 75\text{cm} = 0.75\,\text{m}$

$\therefore \lambda = \frac{4\times0.75}{3} \Rightarrow v = \frac{340\times3}{4\times0.75} = 340$ Hz

$\therefore$ The frequency of vibrating string = 340. Since this string produces 4 beats/sec with a tuning fork of frequency n therefore n = 340 + 4 or n = 340 – 4. With increase in tension, the frequency produced by string increases. As the beats/sec decreases therefore n = 340 + 4 = 344 Hz.

**24.** (b) Frequency of 2nd harmonic of string = Fundamental frequency produced in the pipe




$$\therefore\ 2\times\left[\frac{1}{2l_1}\sqrt{\frac{T}{\mu}}\right] = \frac{v}{4l_2}$$

$$\therefore\ \frac{1}{0.5}\sqrt{\frac{50}{\mu}} = \frac{320}{4\times0.8}$$

$\therefore \mu = 0.02$ kg m$^{-1}$

The mass of the string = $\mu\, l_1$

= 0.02 × 0.5 kg = 10g

**25.** **(a)** $f' = f\left[\frac{v+v_o}{v-v_s}\right]$

Here v = 320 m/s (given)

$$v_o = v_s = 36\times\frac{5}{18} = 10 \text{ m/s}$$

$$f' = 8\left[\frac{320+10}{320-10}\right] = 8\times\frac{33}{31} \approx 8.5\text{ kHz}$$

**26.** **(b)** Considering the end correction [$e = 0.3\,D$], we get

$$L + e = \frac{\lambda}{4} \quad \Rightarrow L = \frac{\lambda}{4} - e$$

$$= \frac{336\times100}{512\times4} - 0.3\times4 = 15.2 \text{ cm} \quad \left[\because \lambda = \frac{v}{\upsilon}\right]$$

## D. MCQs with ONE or MORE THAN ONE Correct

**1.** **(a,b,c,d)** $y = 10^{-4}\sin(60t + 2x)$

Comparing the given equation with the standard wave equation travelling in negative $x$-direction

$y = a\sin(\omega t + kx)$

we get amplitude $a = 10^{-4}$m

Also, $\omega = 60$ rad/s $\therefore\ 2\pi f = 60 \Rightarrow f = \frac{30}{\pi}$Hz

Also, $k = 2 \Rightarrow \frac{2\pi}{\lambda} = 2 \Rightarrow \lambda = \pi$ m

We know that $v = f\lambda = \frac{30}{\pi}\times\pi = 30$ m/s

**2.** **(b)** $y = y_0 \sin 2\pi\left[ft - \frac{x}{\lambda}\right]$

$$\therefore\ \frac{dy}{dt} = \left[y_0\cos 2\pi\left(ft - \frac{x}{\lambda}\right)\right]\times 2\pi f$$

or, $\left[\frac{dy}{dt}\right]_{max} = y_0 \times 2\pi f$

Given that the maximum particle velocity is equal to four times the wave velocity ($c = f\times\lambda$)

$\therefore\ y_0 \times 2\pi f = 4(f\times\lambda)\ \therefore\ \lambda = \frac{\pi y_0}{2}$

**3.** **(a,c)** The wavelengths possible in an air column in a pipe which has one closed end is

$$\lambda = \frac{4\ell}{(2n+1)} \quad \text{So, } c = \nu\lambda$$

$$300 = 264\times\frac{4\ell}{2n+1}$$

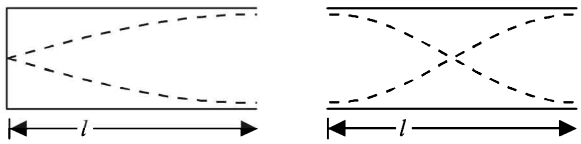
$\nu = 264$ Hz as it is in resonance with a vibrating turning fork of frequency 264 Hz.

$$\ell = \frac{330\times(2n+1)}{264\times4}$$

For $n = 1$, $\ell = 0.3125$ m = 31.25 cm

For $n = 2$, $\ell = 0.9375$ m = 93.75 cm

**4.** **(a)**




$\frac{\lambda}{4} = \ell$ (Fundamental mode) $\quad\therefore\ \lambda = 4\ell$

$\frac{\lambda'}{2} = \ell$ (Fundamental mode) $\quad\therefore\ \lambda' = 2\ell$

$\therefore\ \nu = \frac{c}{\lambda} = \frac{c}{4\ell} = 512$ Hz (given)

and $\nu' = \frac{c}{\lambda'} = \frac{c}{2\ell} = 2\left(\frac{c}{4\ell}\right) = 2\times512 = 1024$ Hz.

**5.** **(a,c)** For wave motion, the differential equation is

$$\frac{\partial^2 y}{\partial t^2} = \left(\text{constant}\ \frac{\omega^2}{k^2}\right)\frac{\partial^2 y}{\partial x^2}$$

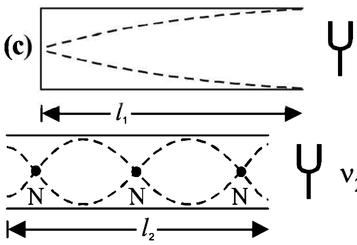
or $\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$ ....(i)

**NOTE :** The wave motion is characterized by the two conditions

$f(x, t) = f(x, t + T)$ ....(ii)

$f(x, t) = f(x + \lambda, t)$ ....(iii)

**6.** **(c)**




$\nu_1 = \frac{v}{4\ell_1}$ for first harmonic

$\nu_2 = \frac{3v}{2\ell_2}$ for third harmonic

$\because\ \nu_1 = \nu_2\ \therefore\ \frac{v}{4\ell_1} = \frac{3v}{2\ell_2} \Rightarrow \frac{\ell_1}{\ell_2} = \frac{1}{6}$

**7.** **(a,b,d)** $\therefore\ \nu = \frac{v}{4\ell}, \frac{3v}{4\ell}, \frac{5v}{4\ell}, ..... = 80, 240, 400....$

**8.** **(b,c)** $y = A\sin(10\pi x + 15\pi t + \pi/3)$

The standard equation of a wave travelling in $-X$ direction is $y = A\sin\left[\frac{2\pi}{\lambda}(vt + x) + (\phi)\right]$

$$\Rightarrow y = A\sin\left[\frac{2\pi v}{\lambda}t + \frac{2\pi}{\lambda}x + \phi\right]$$

Comparing it with the given equation, we find

$$\frac{2\pi v}{\lambda} = 15\pi \text{ and } \frac{2\pi}{\lambda} = 10\pi$$

$$\Rightarrow \lambda = \frac{1}{5} = 0.2\text{ m and } v = \frac{15\pi}{2\pi} \times \frac{1}{5} = 1.5\text{ m/s}$$

**9.** **(b,c)** As, $f = \frac{1}{2\pi}\sqrt{\frac{T}{\mu}}$ $\quad\therefore\quad f \propto \sqrt{T}$

Given that $T_1 > T_2$ $\quad\therefore\quad f_1 > f_2$

Initially beat frequency $(f_1 - f_2) = 6$.

The beat frequency remains unchanged which is possible when $f_2$ increases and $f_1$ decreases. Thus $T_2$ increases and $T_1$ decreases.

**10.** **(b)** $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000\,t) = 2\left(2\cos^2\frac{t}{2}\sin 1000t\right)$

$= 2[\cos t + 1]\sin 1000t$
$= 2\cos t \sin 1000t + 2\sin 1000t$
$= \sin 1000t + \sin 999t + 2\sin 1000t$

**11.** **(a, b, c)** Frequency of reflected wave is $f'' = f\left(\frac{c+v}{c-v}\right)$

$$\Rightarrow \text{Beat freq.} = f'' - f = \frac{2v}{c-1}.$$

Wavelength of reflected wave $= \frac{c}{f''} = \frac{c(c-v)}{f(c+v)}$

**12.** **(b)** **KEY CONCEPT :** The time required for constructive interference equal to the time period of a wave pulse.

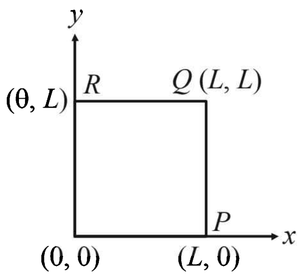
For a string frequency $f = \frac{1}{2\ell}\sqrt{\frac{F}{m}}$

$\therefore$ Time period, $T = 2\ell\sqrt{\frac{m}{F}}$

$$F = 1.6\text{ N},\ m = \frac{\text{mass}}{\text{length}} = \frac{10^{-2}}{0.4} = 2.5 \times 10^{-2}$$

$$\therefore\quad T = 2 \times 0.4\sqrt{\frac{2.5 \times 10^{-2}}{1.6}} = 0.1\text{ sec.}$$

**13.** **(b,c)** Due to the clamping of the square plate at the edges, its displacements along the $x$ and $y$ axes will individually be zero at the edges. Only the choices (b) and (c) predict these displacements correctly. This is because $\sin 0 = 0$.




Option (a) :

$u(x, y) = 0$ at $x = L, y = L$

$u(x, y) \neq 0$ at $x = 0, y = 0$

Option (b) :

$u(x, y) = 0$ at $x = 0, y = 0\ [\because \sin 0 = 0]$

$u(x, y) = 0$ at $x = L, y = L\ [\because \sin\pi = 0]$

Option (c) :

$u(x, y) = 0$ at $x = 0, y = 0\ [\because \sin 0 = 0]$

$u(x, y) = 0$ at $x = L, y = L\ [\because \sin\pi = 0, \sin 2\pi = 0]$

Option (d) :

$u(x, y) = 0$ at $y = 0, y = L\ [\because \sin 0 = 0, \sin\pi = 0]$

$u(x, y) \neq 0$ at $x = 0, x = L\ [\because \cos 0 = 1, \cos 2\pi = 1]$

**14.** **(a,c)**

**NOTE :** For a transverse sinusodial wave travelling on a string, the maximum velocity is $a\omega$.

But maximum velocity is $\frac{v}{10} = \frac{10}{10} = 1\text{ m/s}$

$$\therefore\quad a\omega = 1 \quad\Rightarrow\quad 10^{-3} \times 2\pi v = 1$$

$$\Rightarrow\quad v = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi}\text{Hz}$$

$$\therefore\quad \lambda = \frac{v}{v} = \frac{10}{10^3/2\pi} = 2\pi \times 10^{-2}\text{ m}$$

**15.** **(b,c,d)** $y = \frac{0.8}{(4x+5t)^2 + 5} = \frac{0.8}{16\left[x + \frac{5}{4}t\right]^2 + 5}$ ... (1)

We know that equation of moving pulse is

$y = f(x + vt)$ ... (2)

On comparing (1) and (2), we get

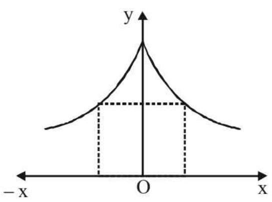
$$v = \frac{5}{4}\text{ms}^{-1} = \frac{2.5}{2}\text{ms}^{-1}$$

So, the wave will travel a distance of 2.5 m in 2 sec.

$$\because\quad y = \frac{0.8}{(4x+5t)^2 + 5}$$

At

$x = 0, t = 0, y = \frac{0.8}{5} = 0.16\text{ m}$




$\therefore$ maximum displacement is 0.16 m

The graph for the given equation is drawn. This is symmetric about $y$-axis.

**16.** **(a,b,c,d)** In the wave motion $y = a(kx - \omega t)$, $y$ can represent electric and magnetic fields in electromagnetic waves and displacement and pressure in sound waves.

**17.** **(a,b,c)** Standing waves are produced by two similar waves superposing while travelling in opposite direction.

This can happen in case (a), (b) and (c).

**18.** **(a,c,d)** For a plane wave, intensity (energy crossing per unit area per unit time) is constant at all points.

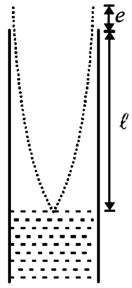
But for a spherical wave, intensity at a distance r from a point source of power $(P)$, is given by $I = \frac{P}{4\pi r^2}$

$\Rightarrow \; I \propto \frac{1}{r^2}$

But the **total intensity** of the spherical wave over the spherical surface centered at the source remains constant at all times.

**NOTE :** For line source $I \propto \frac{1}{r}$

**19. (a, d)**




At second resonance the length of air column is more as compared to first resonance. Now, longer the length of air column, more is the absorption of energy and lesser is the intensity of sound heard.

As shown in the figure, the length of the air column at the first resonance is somewhat shorter than $\frac{1}{4}$th of the wavelength of the sound in air due to end correction.

$$\ell + e = \frac{\lambda}{4} \qquad \therefore \quad \ell = \frac{\lambda}{4} - e$$

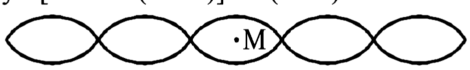
**20. (b, d)**

When sound pulse is reflected through a rigid boundary (closed end of a pipe), no phase change occurs between the incident and reflected pulse i.e., a high pressure pulse is reflected as a high pressure pulse.

When a sound pulse is reflected from open end of a pipe, a phase change of a radian occurs between the incident and the reflected pulse. A high pressure pulse is reflected as a low pressure pulse.

**21. (b, c)**

y = [0.01 sin (62.8x)] cos (628 t).



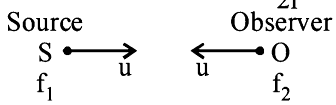

$$\text{Length of string} = 5 \times \frac{\lambda}{2} = 5 \times \frac{1}{20} = 0.25 \text{ m}$$

$$\left[\because \frac{2\pi}{\lambda} = 62.8\right]$$

The midpoint M is an antinode and has the maximum displacement = 0.01 m

$$\text{The fundamental frequency} = \frac{v}{2l} = \frac{\omega / k}{2l} = 20\text{Hz}$$

**22. (a,b)**




**Case1:** Wind blows from observer to source

$$f_2 = f_1\left[\frac{(V-w)+u}{(V-w)-u}\right] \qquad \therefore f_2 > f_1$$

**Case2 :** Wind blows from source to observer

$$f_2 = f_1\left[\frac{(V+w)+u}{(V+w)-u}\right] \qquad \therefore f_2 > f_1$$

(a) and (b) are correct options

**23. (d)** Here, $v = \frac{v}{4l} = \sqrt{\frac{\gamma RT}{M \times 10^{-3}}} \times \frac{1}{4l} \Rightarrow v = v \times 4l$

$\Rightarrow \quad v = 336.7$ m/s to 346.5 m/s

**For monatomic gas** $\gamma = 1.67$

$$v = \sqrt{\frac{\gamma RT}{M \times 10^{-3}}} = \sqrt{100\gamma RT} \times \sqrt{\frac{10}{M}}$$

$$= \sqrt{167RT} \times \sqrt{\frac{10}{M}} = 640\sqrt{\frac{10}{M}}$$

**For Neon** M = 20 and $\sqrt{\frac{10}{20}} = \frac{7}{10}$

$\therefore \; v = 640 \times \frac{7}{10} = 448 \text{ ms}^{-1}$

$\therefore$ (a) is incorrect

**For Argon** M = 36, $\sqrt{\frac{10}{36}} = \frac{17}{32}$

$\therefore \; v = 640 \times \frac{17}{32} = 340 \text{ ms}^{-1}$

$\therefore$ (d) is the correct option.

**For diatomic gas $\gamma = 1.4$**

$$v = \sqrt{140RT}\sqrt{\frac{10}{M}} = 590 \times \sqrt{\frac{10}{M}}$$

**For Oxygen** $\sqrt{\frac{10}{32}} = \frac{9}{16}$

$\therefore \; v = 590 \times \frac{9}{16} = 331.87 \text{ ms}^{-1}$

$\therefore$ (c) is incorrect

**For Nitrogen** $\sqrt{\frac{10}{28}} = \frac{3}{5}$

$\therefore \; v = 590 \times \frac{3}{5} = 354 \text{ ms}^{-1}$

$\therefore$ (b) is incorrect

**24. (a, c, d)**

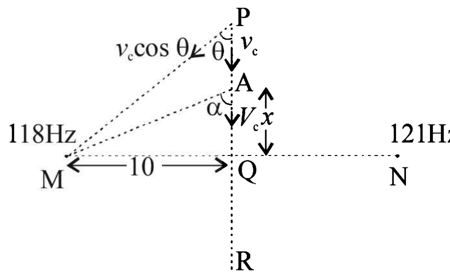
Clearly in the given situation a displacement node is present at $x = 0$ and a displacement antinode is present at $x = 3$ m.

Therefore, $y = 0$ at $x = 0$ and $y = \pm A$ at $x = 3$ m.

The velocity $v = \frac{\omega}{k} = 100 \text{ ms}^{-1}$.

a, c and d are the correct options which satisfy the above conditions.

**25. (a,b, c)**




$$v_P = 121 - 118 = \left[\frac{v + v\cos\theta}{v}\right]$$

$$v_\theta = 121 - 118 = 3$$

$$v_R = (121-118)\left[\frac{v - v_c\cos\theta}{v}\right]$$

$\therefore$ $v_P + v_R = 2v_Q \Rightarrow (A)$ is correct option

In general when the car is passing through A

$$v = 3\left[\frac{v + v_c\cos\alpha}{v}\right] \quad \ldots(i)$$

$$\therefore \frac{dv}{d\alpha} = -3\left[\frac{v_c\sin\alpha}{v}\right] \quad \left|\frac{dv}{d\alpha}\right| \text{ is max when } \sin\alpha = 1$$

i.e., $\alpha = 90°$ (at Q)

$\Rightarrow$ (b) is correct option.

$$\text{From (i)} \ \frac{dv}{dt} = \frac{3v_c}{v}(-\sin\alpha)\frac{d\alpha}{dt} \quad \ldots(ii)$$

$$\text{Also} \ \tan\alpha = \frac{10}{x} \ \therefore \ \sec^2\alpha\frac{d\alpha}{dt} = -\frac{10}{x^2}\frac{dx}{dt}$$

$$\therefore \frac{d\alpha}{dt} = \frac{-10v}{x^2\sec^2\alpha} \quad \ldots(iii)$$

From (ii) & (iii)

$$\frac{dv}{dt} = -\frac{3v_c}{v}\sin\alpha \times \left(\frac{-10v}{x^2\sec^2\alpha}\right) = \frac{30V_c\sin\alpha}{x^2\sec^2\alpha}$$

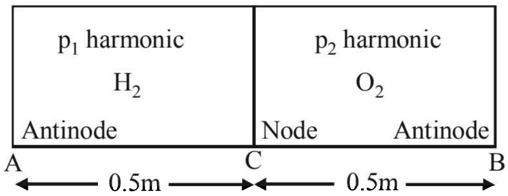
$$\therefore \frac{dv}{dt} = \frac{30v_c\sin\alpha}{(10\cot\alpha)^2\sec^2\alpha} = 0.3v_c\sin^3\alpha \text{. At } \alpha = 90°$$

$$\frac{dv}{dt} = max$$

$\therefore$ (c) is the correct option

## E. Subjective Problems

**1.** It is given that $C$ acts as a node. This implies that at $A$ and $B$ antinodes are formed. Again it is given that the frequencies are same.




$$\Rightarrow \frac{v_1}{4\ell}\times p_1 = \frac{v_2}{4\ell}\times p_2 \text{ or } \frac{p_1}{p_2} = \frac{v_1}{v_2} = \frac{3}{11}$$

or, $11p_1 = 3p_2$

This means that the third harmonic in $AC$ is equal to 11th harmonic in $CB$.

Now, the fundamental frequency in $AC$

$$= \frac{v_1}{4\ell} = \frac{1100}{4\times 0.5} = 550\,\text{Hz}$$

and the fundamental frequency in $CB$

$$= \frac{v_2}{4\ell} = \frac{300}{4\times 0.5} = 550\,\text{Hz}$$

$\therefore$ Frequency in $AC = 3\times 550 = 1650$ Hz

and frequency in $CB = 11\times 150 = 1650$ Hz

**2.** **(a)** Using the formula of the coefficient of linear expansion of wire, $\Delta\ell = \ell\alpha\Delta\theta$ we get

$$F = YA\alpha\Delta\theta$$

Speed of transverse wave is given by

$$v = \sqrt{\frac{F}{m}} \quad \left[\text{where } m = \text{mass per unit length} = \frac{Al\rho}{\ell} = A\rho\right]$$

$$= \sqrt{\frac{YA\alpha\Delta\theta}{A\rho}} = \sqrt{\frac{Y\alpha\Delta\theta}{\rho}}$$

$$= \sqrt{\frac{1.3\times10^{11}\times1.7\times10^{-5}\times20}{9\times10^3}} = 70\,\text{m/s}$$

**3.** **Tube open at both ends :**

(a) $$v = \frac{v}{2(\ell+0.6D)} \quad \therefore 320 = \frac{320}{2(0.48+0.6\times D)}$$

$$0.48 + 0.6D = 0.5 \Rightarrow 0.6D = 0.02$$

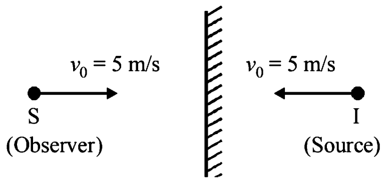
$$\Rightarrow D = \frac{0.02}{60}\times 100\,\text{cm} = 3.33\,\text{cm}$$

**Tube closed at one end :**

$$v = \frac{v}{4(\ell+0.3D)} = \frac{320}{4(0.48+0.3\times0.033)}$$

$\approx 163$ Hz

**4.**




**NOTE :** If the sound reaches the observer after being reflected from a stationary surface and the medium is also stationary, the image of the source in the reflecting surface will become the source of the reflected sound.

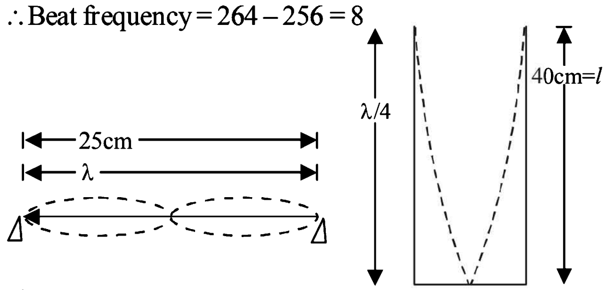
$$v' = v\left[\frac{c - v_0}{c - v_s}\right]$$

$v_0$, $v_s$ are + ve if they are directed from source to the observer and – ve if they are directed from observer to source.

$$v' = 256\left[\frac{330-(-5)}{330-5}\right] = 264\,\text{Hz}$$

$\therefore$ Beat frequency $= 264 - 256 = 8$

**5.**




First Overtone

$$\text{Mass of string per unit length} = \frac{2.5\times10^{-3}}{0.25} = 0.01\,\text{kg/m}$$

$$\therefore \text{Frequency, } v_s = \frac{1}{\lambda}\sqrt{\frac{T}{m}} = \frac{1}{0.25}\sqrt{\frac{T}{0.01}} \quad \ldots(i)$$

**Fundamental frequency**

$$\therefore \frac{\lambda}{4} = 0.4 \Rightarrow \lambda = 1.6\,\text{m}$$

$$\therefore v_T = \frac{c}{\lambda_T} = \frac{320}{1.6} = 200\,\text{Hz} \quad \ldots(ii)$$

Given that 8 beats/second are heard. The beat frequency decreases with the decreasing tension. This means that beat frequency decreases with decreasing $\nu_s$. So beat frequency is given by the expression.

$$\nu = \nu_s - \nu_T$$

$$\therefore \quad 8 = \frac{1}{0.25}\sqrt{\frac{T}{0.01}} - 200 \Rightarrow T = 27.04\,\text{N}$$

**6.** **KEY CONCEPT :** The velocity of wave on the string is given by the formula

$$v = \sqrt{\frac{T}{m}}$$

where $T$ is the tension and $m$ is the mass per unit length. Since the tension in the string will increase as we move up the string (as the string has mass), therefore the velocity of wave will also increase. ($m$ is the same as the rope is uniform)

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{2\times 9.8}{8\times 9.8}} = \frac{1}{2} \quad \therefore \quad v_2 = 2v_1$$

Since frequency remains the same

$$\therefore \quad \lambda_2 = 2\lambda_1 = 2\times 0.06 = 0.12\,\text{m}$$

**7.** **KEY CONCEPT :** Using the formula of the coefficient of linear expansion,

$$\Delta\ell = \ell\alpha \times \Delta\theta$$

Also, $Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{\Delta\ell/\ell} = \frac{T/A}{\alpha A\theta} \quad \therefore T = YA\,\alpha\,\Delta\theta$

The frequency of the fundamental mode of vibration.

$$\nu = \frac{1}{2\ell}\sqrt{\frac{T}{m}} = \frac{1}{2\ell}\sqrt{\frac{YA\alpha\,\Delta\theta}{m}}$$

$$= \frac{1}{2\times 1}\sqrt{\frac{2\times 10^{11}\times 10^{-6}\times 1.21\times 10^{-5}\times 20}{0.1}} = 11\,\text{Hz}$$

**8.** (i) Here amplitude, $A = 4\sin\left(\frac{\pi x}{15}\right)$

At $x = 5$ m

$$A = 4\sin\left(\frac{\pi\times 5}{15}\right) = 4\times 0.866 = 3.46\,\text{cm}$$

(ii) Nodes are the position where $A = 0$

$$\therefore \quad \sin\left(\frac{\pi x}{15}\right) = 0 = \sin n\pi \quad \therefore \quad x = 15\,n$$

where $n = 0, 1, 2$ $\qquad x = 15\,\text{cm}, 30\,\text{cm}, 60\,\text{cm}, \ldots$

(iii) $y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$

$$v = \frac{dy}{dt} = 4\sin\left(\frac{\pi x}{15}\right)[-96\pi\sin(96\pi t)]$$

At $x = 7.5$ cm, $t = 0.25$ cm

$$v = 4\sin\left(\frac{\pi\times 7.5}{15}\right)[-96\pi\sin(96\pi\times 0.25)]$$

$$= 4\sin\left(\frac{\pi}{2}\right)[-96\pi\sin(24\pi)] = 0$$

(iv) $y = 4\sin\left(\frac{\pi x}{15}\right)\cos[96\pi t]$

$$= 2\left[2\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)\right]$$

$$= 2\left[\sin\left(96\pi t + \frac{\pi x}{15}\right) - \sin\left(96\pi t - \frac{\pi x}{15}\right)\right]$$

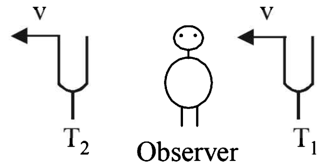
$$= 2\sin\left(96\pi t + \frac{\pi x}{15}\right) - 2\sin\left(96\pi t - \frac{\pi x}{15}\right)$$

$$= y_1 + y_2$$

where $y_1 = 2\sin\left(96\pi t + \frac{\pi x}{15}\right)$

and $y_2 = -2\sin\left(96\pi t - \frac{\pi x}{15}\right)$

**9.** The apparent frequency from tuning fork $T_1$ as heard by the observer will be




$$\nu_1 = \frac{c}{c - v}\times\nu \qquad \ldots\text{(i)}$$

where $c$ = velocity of sound
$v$ = velocity of turning fork

The apparent frequency from tuning fork $T_2$ as heard by the observer will be

$$\nu_2 = \frac{c}{c + v}\times\nu \qquad \ldots\text{(ii)}$$

Given $\nu_1 - \nu_2 = 3$

$$\therefore \quad c\times\nu\left[\frac{1}{c - v} - \frac{1}{c + v}\right] = 3 \quad \text{or,} \quad 3 = \frac{c\times\nu\times 2v}{c^2 - v^2}$$

Since, $v << c$ $\qquad \therefore \; 3 = \frac{c\times\nu\times 2v}{c^2}$

$$\therefore \quad v = \frac{3\times 340\times 340}{340\times 340\times 2} = 1.5\,\text{m/s}$$

**10.** (i) **KEY CONCEPT :** When two progressive waves having same amplitude and period, but travelling in opposite direction with same velocity superimpose, we get standing waves.

The following two equations qualify the above criteria and hence produce standing wave

$$z_1 = A\cos(kx - \omega t)$$
$$z_2 = A\cos(kx + \omega t)$$

The resultant wave is given by $z = z_1 + z_2$

$$\Rightarrow \quad z = A\cos(kx - \omega t) + A\cos(kx + \omega t)$$
$$= 2A\cos kx\cos\omega t$$

The resultant intensity will be zero when

$$2A\cos kx = 0$$

$$\Rightarrow \quad \cos kx = \cos\frac{(2n+1)}{2}\pi$$

$$\Rightarrow \; kx = \frac{2n+1}{2}\pi \quad \Rightarrow x = \frac{(2n+1)\pi}{2k}$$

where $n = 0, 1, 2, ...$

(ii) The transverse waves

$z_1 = A\cos(kx - \omega t)$

$z_3 = A\cos(ky - \omega t)$

Combine to produce a wave travelling in the direction making an angle of 45° with the positive $x$ and positive $y$ axes.

The resultant wave is given by $z = z_1 + z_3$

$z = A\cos(kx - \omega t) + A\cos(ky - \omega t)$

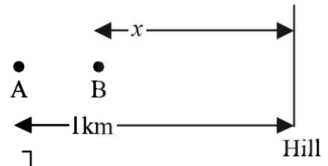
$$\Rightarrow \; z = 2A\cos\frac{(x-y)}{2}\cos\left[\frac{k(x+y) - 2\omega t}{2}\right]$$

The resultant intensity will be zero when

$$2A\cos\frac{k(x-y)}{2} = 0 \quad \Rightarrow \quad \cos\frac{k(x-y)}{2} = 0$$

$$\Rightarrow \; \frac{k(x-y)}{2} = \frac{2n+1}{2}\pi \quad \Rightarrow \quad (x-y) = \frac{(2n+1)}{k}\pi$$

**11.** (i) The frequency of the whistle as heard by observer on the hill




$$n' = n\left[\frac{v + v_m}{v + v_m - v_s}\right]$$

$$= 580\left[\frac{1200 + 40}{1200 + 40 - 40}\right] = 599 \text{ Hz}$$

(ii) Let echo from the hill is heard by the driver at $B$ which is at a distance $x$ from the hill.

The time taken by the driver to reach from $A$ to $B$

$$t_1 = \frac{1-x}{40} \qquad ...(i)$$

The time taken by the echo to reach from hill

$t_2 = t_{AH} + t_{HB}$

$$t_2 = \frac{1}{(1200+40)} + \frac{x}{(1200-40)} \qquad ...(ii)$$

where $t_{AH}$ = time taken by sound from $A$ to $H$ with velocity $(1200 + 40)$

$t_{HB}$ = time taken by sound from $H$ to $B$ with velocity $1200 - 40$

From (i) and (ii)

$$t_1 = t_2 \Rightarrow \frac{1-x}{40} = \frac{1}{1200+40} + \frac{x}{1200-40}$$

$\Rightarrow \; x = 0.935$ km

The frequency of echo as heard by the driver can be calculated by considering that the source is the acoustic image.

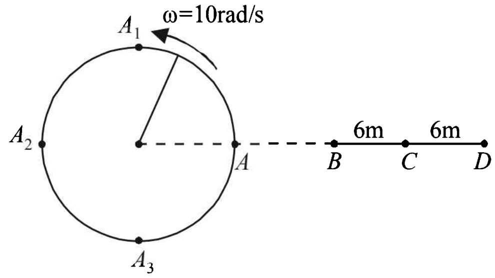
$$n'' = n\left[\frac{(v - v_m) + v_s}{(v - v_m) - v_0}\right]$$

$$= 580\left[\frac{(1200-40)+40}{(1200-40)-40}\right] = 621 \text{ Hz}$$

**12.** The angular frequency of the detector $= 2\pi\nu$

$$= 2\pi \times \frac{5}{\pi} = 10 \text{ rad/s}$$

The angular frequency of the detector matches with that of the source.




$\Rightarrow$ When the detector is at $C$ moving towards $D$, the source is at $A_1$ moving leftwards. It is in this situation that the frequency heard is minimum

$$\nu' = \nu\left[\frac{v - v_0}{v + v_s}\right] = 340 \times \frac{(340-60)}{(340+30)} = 257.3\text{Hz}$$

Again when the detector is at $C$ moving towards $B$, the source is at $A_3$ moving rightward. It is in this situation that the frequency heard is maximum.

$$\nu'' = \nu\left[\frac{v + v_0}{v - v_s}\right] = 340 \times \frac{(340+60)}{(340-30)} = 438.7\text{Hz}$$

**13.** **(a) KEY CONCEPT :** Use the equation of a plane progressive wave which is as follows.

$$y = A\cos\left(\frac{2\pi}{\lambda}x + 2\pi\nu t\right)$$

The given equation is

$y_1 = A\cos(ax + bt)$

On comparing, we get $\dfrac{2\pi}{\lambda} = a \;\Rightarrow \lambda = \dfrac{2\pi}{a}$

Also, $2\pi\nu = b$

$$\Rightarrow \; \nu = \frac{b}{2\pi}$$

**(b)** Since the wave is reflected by an obstacle, it will suffer a phase difference of $\pi$. The intensity of the reflected wave is 0.64 times of the incident wave.

Intensity of original wave $I \propto A^2$

Intensity of reflected wave $I' = 0.64\,I$

$\Rightarrow \; I' \propto A'^2 \Rightarrow 0.64\,I \propto A'^2$

$\Rightarrow \; 0.64\,A^2 \propto A'^2 \Rightarrow A' \propto 0.8A$

So the equation of resultant wave becomes

$y_2 = 0.8A\cos(ax - bt + \pi) = -0.8\,A\cos(ax - bt)$

**(c) KEY CONCEPT :** The resultant wave equation can be found by superposition principle

$y = y_1 + y_2$

$= A\cos(ax + bt) + [-0.8\,A\cos(ax - bt)]$

The particle velocity can be found by differentiating the above equation

$$v = \frac{dy}{dt} = -Ab\sin(ax + bt) - 0.8\,Ab\sin(ax - bt)$$

$= -Ab\,[\sin(ax + bt) + 0.8\sin(ax - bt)]$

$= -Ab\,[\sin ax\cos bt + \cos ax\sin bt$

$+ 0.8\sin ax\cos bt - 0.8\cos ax\sin bt]$

$v = -Ab\,[1.8\sin ax\cos bt + 0.2\cos ax\sin bt]$

The maximum velocity will occur when $\sin ax = 1$ and $\cos bt = 1$ under these condition $\cos ax = 0$ and $\sin bt = 0$

$\therefore \; |v_{max}| = 1.8\,Ab$

Also, $|v_{min}| = 0$

**(d)** $y = [A\cos(ax+bt)] - [0.8A\cos(ax-bt)]$
$= [0.8A\cos(ax+bt) + 0.2A\cos(ax+bt)] - [0.8A\cos(ax-bt)]$
$= [0.8A\cos(ax+bt) - 0.8A\cos(ax-bt)] + 0.2A\cos(ax+bt)]$

$$= 0.8A\left[-2\sin\left\{\frac{(ax+bt)+(ax-bt)}{2}\right\}\sin\left\{\frac{(ax+bt)-(ax-bt)}{2}\right\}\right] 0.2A\cos(ax+bt)]$$

$\Rightarrow$ $y = -1.6A\sin ax \sin bt + 0.2A\cos(ax+bt)$
where $(-1.6A\sin ax\sin bt)$ is the equation of a standing wave and $0.2A\cos(ax+bt)$ is the equation of travelling wave.
The wave is travelling in $-x$ direction.
**NOTE :** Antinodes of the standing waves are the positions where the amplitude is maximum,

i.e., $\sin ax = 1 = \sin\left[n\pi + (-1)^n\frac{\pi}{2}\right]$

$\Rightarrow$ $x = \left[n + \frac{(-1)^n}{2}\right]\frac{\pi}{a}$

**14.** Let the two radio waves be represented by the equations
$y_1 = A\sin 2\pi\nu_1 t$
$y_2 = A\sin 2\pi\nu_2 t$
The equation of resultant wave according to superposition principle
$y = y_1 + y_2 = A\sin 2\pi\nu_1 t + A\sin 2\pi\nu_2 t$
$= A[\sin 2\pi\nu_1 t + \sin 2\pi\nu_2 t]$

$$= A\times 2\sin\frac{(2\pi\nu_1+2\pi\nu_2)t}{2}\cos\frac{(2\pi\nu_1+2\pi\nu_2)t}{2}$$

$= 2A\sin\pi(\nu_1+\nu_2)t\cos\pi(\nu_1-\nu_2)t$
where the amplitude $A' = 2A\cos\pi(\nu_1-\nu_2)t$
Now, intensity $\propto$ (Amplitude)$^2$
$\Rightarrow$ $I \propto A'^2$
$I \propto 4A^2\cos^2\pi(\nu_1-\nu_2)t$
The intensity will be maximum when
$\cos^2\pi(\nu_1-\nu_2)t = 1$
or, $\cos\pi(\nu_1-\nu_2)t = 1$
or, $\pi(\nu_1-\nu_2)t = n\pi$

$\Rightarrow$ $\frac{(\omega_1-\omega_2)}{2}t = n\pi$ or, $t = \frac{2n\pi}{\omega_1-\omega_2}$

$\therefore$ Time interval between two maxima

or, $\frac{2n\pi}{\omega_1-\omega_2} - \frac{2(n-1)\pi}{\omega_1-\omega_2}$ or, $\frac{2\pi}{\omega_1-\omega_2} = \frac{2\pi}{10^3}$ sec

Time interval between two successive maximas is $2\pi\times 10^{-3}$ sec
(ii) For the detector to sense the radio waves, the resultant intensity $\geq 2A^2$
$\therefore$ Resultant amplitude $\geq \sqrt{2}A$
or, $2A\cos\pi(\nu_1-\nu_2)t \geq \sqrt{2A}$

or, $\cos\pi(\nu_1-\nu_2)t \geq \frac{1}{\sqrt{2}}$ or, $\cos\left[\frac{(\omega_1-\omega_2)t}{2}\right] \geq \frac{1}{\sqrt{2}}$

The detector lies idle when the values of $\cos\left[\frac{(\omega_1-\omega_2)t}{2}\right]$ is between 0 and $\frac{1}{\sqrt{2}}$

$\therefore$ $\frac{(\omega_1-\omega_2)t}{2}$ is between $\frac{\pi}{2}$ and $\frac{\pi}{4}$

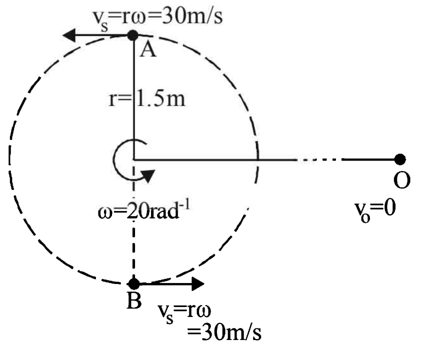
$\therefore$ $t_1 = \frac{\pi}{\omega_1-\omega_2}$ and $t_2 = \frac{\pi}{2(\omega_1-\omega_2)}$

$\therefore$ The time gap $= t_1 - t_2$

$$= \frac{\pi}{\omega_1-\omega_2} - \frac{\pi}{2(\omega_1-\omega_2)} = \frac{\pi}{2(\omega_1-\omega_2)}$$

$$= \frac{\pi}{2}\times 10^{-3}\text{ sec}$$

**15.** The whistle which is emitting sound is being rotated in a circle.




$r = 1.5$ m (given); $\omega = 20$ rads$^{-1}$ (given)
We know that
$v = r\omega = 1.5\times 20 = 30$ ms$^{-1}$
When the source is instantaneously at the position $A$, then the frequency heard by the observer will be

$$\nu' = \nu\left[\frac{v}{v-v_s}\right] = 440\left[\frac{330}{330-30}\right] = 484\text{ Hz}$$

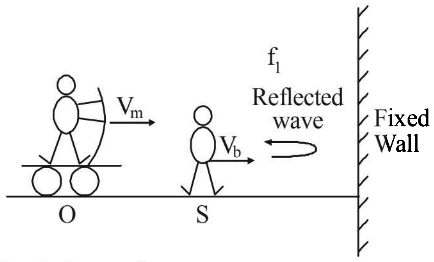
When the source is instantaneously at the position $B$, then the frequency heard by the observer will be

$$\nu'' = \nu\left[\frac{v}{v+v_s}\right] = 440\left[\frac{330}{330+30}\right] = 403.3\text{ Hz}$$

Hence the range of frequencies heard by the observer is 403.3 Hz to 484 Hz.

**16.** **KEY CONCEPT :** Motorist will listen two sound waves. One directly from the sound source and other reflected from the fixed wall. Let the apparent frequencies of these two waves as received by motorist are $f'$ and $f''$ respectively.
**For Direct Sound :** $V_m$ will be positive as it moves towards the source and tries to increase the apparent frequency. $V_b$ will be taken positive as it move away from the observer and hence tries to decrease the apparent frequency value.

$$f' = \frac{v+v_m}{v+v_b}f \quad ...(1)$$




**For reflected sound :**
For sound waves moving towards stationary observer (i.e. wall), frequency of sound as heard by wall

$$f_1 = \frac{v}{v - v_b} f$$

After reflection of sound waves having frequency $f_1$ fixed wall acts as a stationary source of frequency $f_1$ for the moving observer i.e. motorist. As direction of motion of motorist is opposite to direction of sound waves, hence frequency $f''$ of reflected sound waves as received by the motorist is

$$f'' = \frac{v + v_m}{v} f_1 = \frac{v + v_m}{v - v_b} f \qquad \text{... (2)}$$

Hence, beat frequency as heard by the motorist

$$\Delta f = f'' - f' = \left(\frac{v + v_m}{v - v_b} - \frac{v + v_m}{v + v_b}\right) f$$

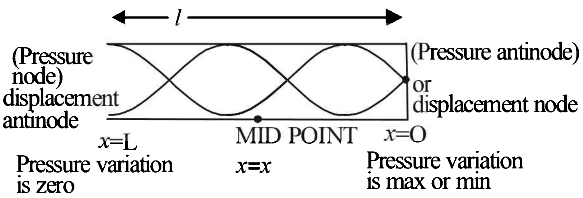
or, $$\Delta f = \frac{2v_b(v + v_m) f}{v^2 - v_b^2}$$

**17.** (a) For second overtone as shown,

$$\frac{5\lambda}{4} = \ell \quad \therefore \lambda = \frac{4\ell}{5}$$

Also, $\quad v = \nu\lambda$

$$\Rightarrow 330 = 440 \times \frac{4\ell}{5} \Rightarrow \ell = \frac{15}{16}\text{m.}$$




(b) **KEY CONCEPT :** At any position $x$, the pressure is given by

$$\Delta P = \Delta P_0 \cos kx \cos \omega t$$

Here amplitude $A = \Delta P_0 \cos kx = \Delta P_0 \cos \frac{2\pi}{\lambda} x$

For $x = \frac{15}{2 \times 16} = \frac{15}{32}$ m $\qquad$ (mid point)

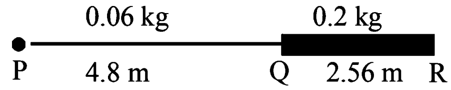
$$\text{Amplitude} = \Delta P_0 \cos\left[\frac{2\pi}{(330/440)} \times \frac{15}{32}\right] = \frac{\Delta P_0}{\sqrt{2}}$$

(c) At open end of pipe, pressure is always same i.e. equal to mean pressure $\because \Delta P = 0, P_{max} = P_{min} = P_0$

(d) At the closed end :
Maximum Pressure $= P_0 + \Delta P_0$
Minimum Pressure $= P_0 - \Delta P_0$

**18.** (a) Mass per unit length of $PQ$

$$m_1 = \frac{0.06}{4.8} \text{ kg/m}$$




Mass per unit length of $QR$, $m_2 = \frac{0.2}{2.56}$ kg/m

Velocity of wave in $PQ$ is

$$v_1 = \sqrt{\frac{T}{m_1}} = \sqrt{\frac{80}{0.06/4.8}} = 80\,\text{ms}^{-1} \,[\because T = 80\text{ N given}]$$

Velocity of wave in $QR$ is

$$v_2 = \sqrt{\frac{T}{m_2}} = \sqrt{\frac{80}{0.2/2.56}} = 32\text{ m/s}$$

$\therefore$ Time taken for the wave to reach from $P$ to $R$

$$= t_{PQ} + t_{QR}$$
$$= \frac{4.8}{80} + \frac{2.56}{32} = 0.14\text{s}$$

(b) When the wave which initiates from $P$ reaches $Q$ (a denser medium) then it is partly reflected and partly transmitted.
In this case the amplitude of reflected wave

$$A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) A_i \qquad \text{... (i)}$$

where $A_i$ = amplitude of incident wave.
Also amplitude of transmitted wave is

$$A_t = \left(\frac{2v_2}{v_1 + v_2}\right) A_i \qquad \text{... (ii)}$$

From (i), (ii)
Therefore, $A_t = 2$ cm and $A_r = -1.5$ cm.

**19.** Speed of sound, $v = 340$ m/s.

Let $\ell_0$ be the length of air column corresponding to the fundamental frequency. Then

$$\frac{v}{4\ell_0} = 212.5$$

or $$\ell_0 = \frac{v}{4(212.5)} = \frac{340}{4(212.5)} = 0.4\text{ m.}$$

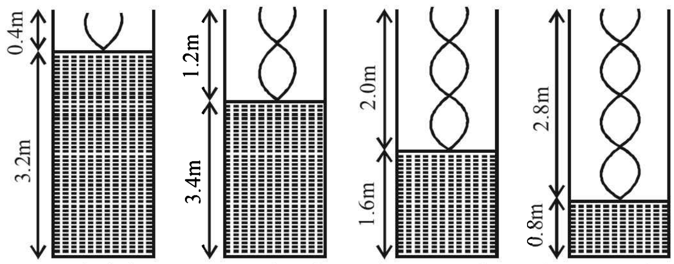
**NOTE :** In closed pipe only odd harmonics are obtained.
Now, let $\ell_1, \ell_2, \ell_3, \ell_4$, etc. be the lengths corresponding to the 3rd harmonic, 5th harmonic, 7th harmonic etc. Then

$$3\left(\frac{v}{4\ell_1}\right) = 212.5 \Rightarrow \ell_1 = 1.2\text{ m;}$$

$$5\left(\frac{v}{4\ell_2}\right) = 212.5 \Rightarrow \ell_2 = 2.0\text{ m}$$

$$7\left(\frac{v}{4\ell_3}\right) = 212.5 \Rightarrow \ell_3 = 2.8\text{ m;}$$

$$9\left(\frac{v}{4\ell_4}\right) = 212.5 \Rightarrow \ell_4 = 3.6\text{ m}$$




or heights of water level are (3.6 – 0.4) m, (3.6 – 1.2) m, (3.6 – 2.0) m and (3.6 – 2.8) m.
Therefore heights of water level are 3.2 m, 2.4 m, 1.6 m and 0.8 m.
Let $A$ and a be the area of cross-sections of the pipe and hole respectively. Then

$A = \pi(2\times10^{-2})^2 = 1.26\times10^{-3}\ m^{-2}$
and $a = \pi(10^{-3})^2 = 3.14\times10^{-6}\ m^2$

Velocity of efflux, $v = \sqrt{2gH}$
Continuity equation at 1 and 2 gives,

$$a\sqrt{2gH} = A\left(\frac{-dH}{dt}\right)$$

Therefore, rate of fall of water level in the pipe,

$$\left(\frac{-dH}{dt}\right) = \frac{a}{A}\sqrt{2gH}$$

Substituting the values, we get

$$\frac{-dH}{dt} = \frac{3.14\times10^{-6}}{1.26\times10^{-3}}\sqrt{2\times10\times H}$$

$$\Rightarrow \frac{-dH}{dt} = (1.11\times10^{-2})\sqrt{H}$$

Between first two resonances, the water level falls from 3.2 m to 2.4 m.

$$\therefore \frac{dH}{\sqrt{H}} = -1.11\times10^{-2}\,dt$$

$$\Rightarrow \int_{3.2}^{2.4}\frac{dH}{\sqrt{H}} = -(1.11\times10^{-2})\int_0^t dt$$

$$\Rightarrow 2[\sqrt{2.4}-\sqrt{3.2}] = -(1.1\times10^{-2}).t$$

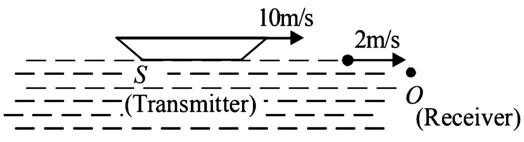
$$\Rightarrow t \approx 43\ \text{second}$$

**20.** **KEY CONCEPT :** The question is based on Doppler's effect where the medium through which the sound is travelling is also in motion.
By Doppler's formula

$$v' = v\left[\frac{c+v_m \pm v_0}{c+v_m \pm v_s}\right] \quad ...(1)$$

**NOTE :** Sign convention for $V_m$ is as follows :
If medium is moving from $s$ to $O$ then + ve and vice versa.
Similarly $v_0$ and $v_s$ are positive if these are directed from $S$ to $O$ and vice versa.

**(a) Situation 1.**




Velocity of sound in water $c = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.088\times10^9}{10^3}}$

$c = 1445$ m/s; $v_m = +2$ m/s; $v_0 = 0$; $v_s = 10$ m/s

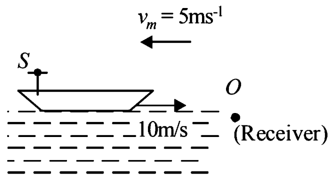
$$\therefore \quad v' = v\left[\frac{1445+2-0}{1445+2-10}\right] = v[1.007]$$

$$\text{Now, } v = \frac{c}{\lambda} = \frac{1445}{14.45\times10^{-3}} = 10^5\ \text{Hz}$$

$$\therefore \quad v' = 1.007\times10^5\ \text{Hz}$$

**(b) Situation 2.**

In air $c = \sqrt{\frac{\gamma RT}{M}} = 344$ m/s




Applying formula (1)

$$v' = v\left[\frac{344-5-0}{344-5-10}\right] = 1.03\times10^5\ \text{Hz}$$

**21.** (a) Second harmonic in pipe $A$ is

$$2(v_0)_A = 2\left[\frac{v}{2\ell}\right] = \frac{1}{\ell}\sqrt{\frac{\gamma_A RT}{M_A}}$$

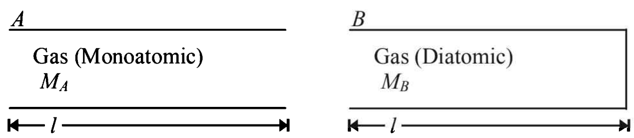
Third harmonic in pipe $B$ is

$$3(v_0)_B = 3\left[\frac{v}{4\ell}\right] = \frac{3}{4\ell}\sqrt{\frac{\gamma_B RT}{M_B}}$$

Given $v_A = v_B$

$$\frac{1}{\ell}\sqrt{\frac{\gamma_A RT}{M_A}} = \frac{3}{4\ell}\sqrt{\frac{\gamma_B RT}{M_B}}$$

$$\text{or,} \quad \frac{M_A}{M_B} = \frac{\gamma_A}{\gamma_B}\times\left(\frac{4}{3}\right)^2 = \frac{5/3}{7/5}\times\frac{16}{9} = \frac{400}{189}$$




$$\text{Now,} \quad \frac{(v_0)_A}{(v_0)_B} = \sqrt{\frac{\gamma_A}{\gamma_A}\times\frac{M_B}{M_B}} = \frac{3}{4}$$

**22.** **KEY CONCEPT :** In the fundamental mode

$$(\ell+0.6r) = \frac{\lambda}{4} = \frac{v}{4f} \Rightarrow v = 4f(\ell+0.6r) = 336\ \text{m/s.}$$

**23.** Here $\ell = \frac{\lambda}{2}$ or $\lambda = 2\ell$ Since, $k = \frac{2\pi}{\lambda} = \frac{2\pi}{2\ell} = \frac{\pi}{\ell}$

The amplitude of vibration at a distance $x$ from $x = 0$ is given by $A = a\sin kx$
Mechanical energy at $x$ of length $dx$ is

$$dE = \frac{1}{2}(dm)A^2\omega^2 = \frac{1}{2}(\mu dx)(a\sin kx)^2(2\pi v)^2$$

$= 2\pi^2\mu v^2 a^2 \sin^2 kx\, dx$

But $v = \nu\lambda$

$\therefore \quad \nu = \dfrac{v}{\lambda} \Rightarrow \nu^2 = \dfrac{v^2}{\lambda^2} = \dfrac{T/\mu}{4\ell^2} \qquad \left[\because v = \sqrt{T/\mu}\right]$

$\therefore \quad dE = 2\pi^2\mu \dfrac{T/\mu}{4\ell^2} a^2 \sin^2\left\{\left(\dfrac{\pi}{\ell}\right)x\right\}dx$

$\therefore$ Total energy of the string

$$E = \int dE = \int_0^{\ell} 2\pi^2\mu \frac{T/\mu}{4\ell^2} a^2 \sin^2\left(\frac{\pi x}{\ell}\right)dx$$

$$= \frac{\pi^2 T a^2}{4\ell}$$

**24.** Let the speed of the train be $v_T$

**While the train is approaching**

Let $\nu$ be the actual frequency of the whistle. Then

$$\nu' = \nu \frac{v_S}{v_S - v_T}$$

where $v_S$ = Speed of sound = 300 m/s (given)

$\nu'$ = 2.2 K Hz. = 2200 Hz (given)

$$\therefore \quad 2200 = \nu \frac{300}{300 - v_T} \qquad \text{... (i)}$$

**While the train is receding**

$$\nu'' = \nu \frac{v_S}{v_S + v_T}$$

Here, $\nu'$ = 1.8 KHz = 1800 Hz (given)

$$\therefore \quad 1800 = \nu \frac{300}{300 + v_T} \qquad \text{... (ii)}$$

Dividing (i) and (ii)

$$\frac{2200}{1800} = \frac{300}{300 - v_T} \times \frac{300 + v_T}{300} \quad \Rightarrow \quad v_T = 30 \text{ m/s}$$

**25.** **KEY CONCEPT :** The wave form of a transverse harmonic disturbance is

$y = a \sin(\omega t \pm kx \pm \phi)$

Given $v_{max} = a\omega = 3$ m/s ... (i)

$A_{max} = a\omega^2 = 90$ m/s$^2$ ... (ii)

Velocity of wave $v = 20$ m/s ... (iii)

Dividing (ii) by (i)

$$\frac{a\omega^2}{a\omega} = \frac{90}{3} \Rightarrow \omega = 30 \text{ rad/s} \quad \text{... (iv)}$$

Substituting the value of $\omega$ in (i), we get

$$a = \frac{3}{30} = 0.1\text{m} \qquad \text{... (v)}$$

$$\text{Now, } k = \frac{2\pi}{\lambda} = \frac{2\pi}{v/\nu} = \frac{\omega}{v} = \frac{30}{20} = \frac{3}{2} \qquad \text{... (vi)}$$

From (iv), (v) and (vi) the wave form is

$$y = 0.1 \sin\left[30t \pm \frac{3}{2}x \pm \phi\right]$$

## F. Match the Following

**1.** (A) Pitch q. frequency

(B) quality p. waveform

(C) loudness r. intensity

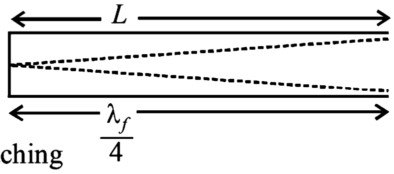
**2.** **A-p,t; B-p,s; C-q,s; D-q,r**

(A) **Pipe closed at one end**

Waves produced are longitudinal (sound waves)

$\dfrac{\lambda_f}{4} = L$

$\therefore \lambda_f = 4L$



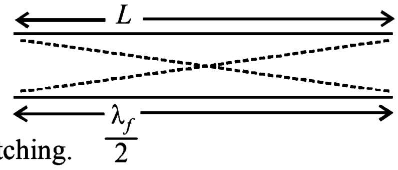

(p, t) are correct matching

(B) **Pipe open at both ends**

waves produced are longitudinal (sound waves)

$\dfrac{\lambda_f}{2} = L$

$\therefore \lambda_f = 2L$



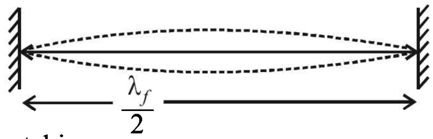

(p, s) are correct matching.

(c) **Stretched wire clamped at both ends**

Waves produced are transverse in nature. (waves on string)

$\dfrac{\lambda_f}{2} = L$

$\therefore \lambda_f = 2L$

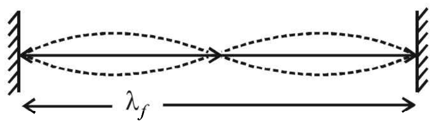



(q, s) are correct matching.

(D) **Stretched wave clamped at both ends & mid point**

Waves produced are transverse in nature (waves on string)

$\lambda_f = L$




(q, r) are correct matching.

## G. Comprehension Based Questions

**1. (a) 2. (c) 3. (d)**

The equations are $y_1 = A\cos(0.5\,\pi x - 100\,\pi t)$ and $y_2 = A\cos(0.46\,\pi x - 92\,\pi t)$ represents two progressive wave travelling in the same direction with slight difference in the frequency. This will give the phenomenon of beats.

Comparing it with the equation

$y = A\cos(kx - \omega t)$, we get

$\omega_1 = 100\,\pi \Rightarrow 2\pi f_1 = 100\,\pi \Rightarrow f_1 = 50$ Hz and

$$K_1 = 0.5\,\pi \Rightarrow \frac{2\pi}{\lambda_1} = 0.5\pi \Rightarrow \lambda_1 = 4\text{ m}$$

Wave velocity $= \lambda_1 f_1 = 200$ m/s [Alternatively use $v = \dfrac{\omega}{K}$]

$\omega_2 = 92\,\pi \Rightarrow 2\pi f_2 = 92\,\pi \Rightarrow f_2 = 46$ Hz

Therefore beat frequency $= f_1 - f_2 = 4$ Hz and

$$K_2 = 0.46\,\pi \Rightarrow \frac{2\pi}{\lambda_2} = 0.46\pi \Rightarrow \lambda_2 = \frac{200}{46}$$

$$\text{Wave velocity} = \frac{200}{46} \times 46 = 200 \text{ m/s}$$

**NOTE :** Wave velocity is same because it depends on the medium in which the wave is travelling.

Now, at $x = 0$,

$y_1 + y_2 = (A\cos 10\,\pi t) + (A\cos 92\,\pi t) = 0$

$\Rightarrow \cos 100\pi t = -\cos 92\pi t = \cos(-92\pi t)$

$= \cos[(2n+1)\pi - 92\pi t \Rightarrow t = \frac{2n+1}{192}$

when $t = 0, n = -\frac{1}{2}$ and when $t = 1,\ n = \frac{191}{2} = 95.2$

$\Rightarrow$ net amplitude is zero for $n$ = 96 times (the nearest answer).

4. (b) The speed of sound depends on the frame of reference of the observer.

5. (a) Since all the passengers in train $A$ are moving with a velocity of 20 m/s therefore the distribution of sound intensity of the whistle by the passengers in train $A$ is uniform.

6. (a) $\nu' = \nu_1\left[\frac{v - v_0}{v - v_s}\right] = 800\left[\frac{340-30}{340-20}\right] = 800 \times \frac{31}{32}$

$\nu'' = \nu_2\left[\frac{v - v_0}{v - v_s}\right] = 1120 \times \frac{31}{32}$

$\therefore\quad \nu'' - \nu' = (1120 - 800) \times \frac{31}{32} = 320 \times \frac{31}{32} = 310\,\text{Hz}.$

### I. Integer Value Correct Type

1. We know that, $\nu = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{0.5}{10^{-3}/0.2}} = 10\,\text{m/s}$

The wavelength of the wave established

$\lambda = \frac{v}{f} = \frac{10}{100} = 0.1m = 10\,\text{cm}$

$\therefore$ The distance between two successive nodes

$= \frac{\lambda}{2} = \frac{10}{2} = 5\text{cm}$

2. Let $v$ be the speed of sound and $v_c$ and $f_0$ the speed and frequency of car.

The frequency of sound reflected by the car is

$\therefore f_1' = f_0\left[\frac{v + v_c}{v - v_c}\right]$

Differentiating the above equation w.r.t. $v_c$, we get

$$\frac{d f_1'}{dv_c} = f_0\left[\frac{(v - v_c)\frac{d}{dv_c}(v + v_c) - (v + v_c)\frac{d}{dv_c}(v - v_c)}{(v - v_c)^2}\right]$$

$\therefore \frac{df_1'}{dv_c} = f_0\left[\frac{2v}{(v - v_c)^2}\right] = f_0\frac{2v}{v^2}\quad (\because v_c << v)$

$\therefore \frac{df_1'}{f_0} \times 100 = \frac{2}{v} \times dv_c$

$\therefore 0.012 = \frac{2 \times dv_c}{330}\qquad \therefore dv_c = 0.198\text{ m/s} \approx \textbf{7}\text{ km/h}$

3. Resultant amplitude, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$= \sqrt{4^2 + 3^2 + 2 \times 4 \times 3 \times \cos\frac{\pi}{2}} = \sqrt{16 + 9 + 0} = \mathbf{5}$

4. (3) $y = \sqrt{I_0}\left[\sin O + \sin\frac{\pi}{3} + \sin\frac{2\pi}{3} + \sin\pi\right]$

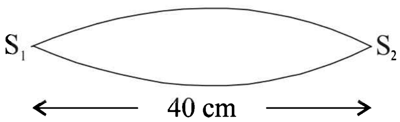
$y = \sqrt{I_0}\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right] = \sqrt{3}\sqrt{I_0}$

$\therefore\quad I_r = y^2 = 3I_0 \quad\Rightarrow\quad n = 3$

## Section-B JEE Main/ AIEEE

1. (b) This will happen for fundamental mode of vibration as shown in the figure. $S_1$ and $S_2$ are rigid support

Here $\frac{\lambda}{2} = 40\ \therefore \lambda = 80\,\text{cm}$




2. (c) **KEY CONCEPT :** The fundamental frequency for closed organ pipe is given by $\upsilon_c = \frac{v}{4\ell}$ and

For open organ pipe is given by $\upsilon_0 = \frac{v}{2\ell}$

$\therefore \frac{\upsilon_0}{\upsilon_c} = \frac{v}{2\ell} \times \frac{4\ell}{v} = \frac{2}{1}$

3. (b) A tuning fork produces 4 beats/sec with another tuning fork of frequency 288 cps. From this information we can conclude that the frequency of unknown fork is 288 + 4 cps or 288 – 4 cps i.e. 292 cps or 284 cps. When a little wax is placed on the unknown fork, it produces 2 beats/sec. When a little wax is placed on the unknown fork, its frequency decreases and simultaneously the beat frequency decreases confirming that the frequency of the unknown fork is 292 cps.

4. (b) To form a node there should be superposition of this wave with the reflected wave. The reflected wave should travel in opposite direction with a phase change of $\pi$. The equation of the reflected wave will be

$y = a\sin(\omega t + kx + \pi)$

$\Rightarrow y = -a\sin(\omega t + kx)$

5. (b) **KEY CONCEPT :** The frequency of a tuning fork is given by the expression

$$f = \frac{m^2k}{4\sqrt{3}\,\pi\ell^2}\sqrt{\frac{Y}{\rho}}$$

As temperature increases, $\ell$ increases and therefore f decreases.

6. (a) $y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right)$

But $y = A\sin(\omega t - kx + \phi)$

On comparing we get $\omega = 600$; k=2

$v = \frac{\omega}{k} = \frac{600}{2} = 300\ ms^{-1}$

**7. (a)** **KEY CONCEPT :** For a string vibrating between two rigid support, the fundamental frequency is given by

$$n = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\times 1}\sqrt{\frac{10\times 9.8}{9.8\times 10^{-3}}} = 50\,Hz$$

As the string is vibrating in resonance to a.c of frequency n, therefore both the frequencies are same.

**8. (c)** A tuning fork of frequency 256 Hz makes 5 beats/second with the vibrating string of a piano. Therefore the frequency of the vibrating string of piano is $(256 \pm 5)$ Hz ie either 261Hz or 251 Hz. When the tension in the piano string increases, its frequency will increases. Now since the beat frequency decreases, we can conclude that the frequency of piano string is 251Hz

**9. (b)** From equation given,

$$\omega = 100 \quad \text{and} \quad k = 20, \ v = \frac{\omega}{k} = \frac{100}{20} = 5\,m/s$$

**10. (d)** No. of beats heard when fork 2 is sounded with fork 1 $= \Delta n = 4$

Now we know that if on loading (attaching tape) an unknown fork, the beat frequency increases (from 4 to 6 in this case) then the frequency of the unknown fork 2 is given by,

$n = n_0 - \Delta n = 200 - 4 = 196\,Hz$

**11. (c)** $n' = n\left[\frac{v+v_0}{v}\right] = n\left[\frac{v+\frac{v}{5}}{v}\right] = n\left[\frac{6}{5}\right]$

$$\frac{n'}{n} = \frac{6}{5}; \ \frac{n'-n}{n} = \frac{6-5}{5}\times 100 = 20\%$$

**12. (c)** $v' = v\left[\frac{v}{v - v_s}\right] \Rightarrow 10000 = 9500\left[\frac{300}{300 - v}\right]$

$$\Rightarrow 300 - v = 300 \times 0.95 \Rightarrow v = 300 - 285 = 15\ ms^{-1}$$

**13. (a)** Given $\frac{nv}{2\ell} = 315$ and $(n+1)\frac{v}{2\ell} = 420$

$$\Rightarrow \frac{n+1}{n} = \frac{420}{315} \Rightarrow n = 3$$

Hence $3 \times \frac{v}{2\ell} = 315 \Rightarrow \frac{v}{2\ell} = 105$ Hz

Lowest resonant frequency is when $n = 1$

Therefore lowest resonant frequency = 105 Hz.

**14. (a)** We have, $L_1 = 10\log\left(\frac{I_1}{I_0}\right)$; $L_2 = 10\log\left(\frac{I_2}{I_0}\right)$

$$\therefore L_1 - L_2 = 10\log\left(\frac{I_1}{I_0}\right) - 10\log\left(\frac{I_2}{I_0}\right)$$

or, $\Delta L = 10\log\left(\frac{I_1}{I_0}\times\frac{I_0}{I_2}\right)$ or, $\Delta L = 10\log\left(\frac{I_1}{I_2}\right)$

or, $20 = 10\log\left(\frac{I_1}{I_2}\right)$ or, $2 = \log\left(\frac{I_1}{I_2}\right)$

or, $\frac{I_1}{I_2} = 10^2$ or, $I_2 = \frac{I_1}{100}$.

$\Rightarrow$ Intensity decreases by a factor 100.

**15. (b)** For first resonant length $v = \frac{v}{4\ell_1} = \frac{v}{4\times 18}$ (in winter)

For second resonant length

$v' = \frac{3v'}{4\ell_2} = \frac{3v'}{4x}$ (in summer) $\quad \therefore \frac{v}{4\times 18} = \frac{3v'}{4\times x}$

$\therefore x = 3\times 18\times\frac{v'}{v} \quad \therefore x = 54\times\frac{v'}{v}$ cm

$v' > v$ because velocity of light is greater in summer as compared to winter $(v \propto \sqrt{T})$

$\therefore x > 54$ cm

**16 (a)** $y(x,t) = 0.005\cos(\alpha x - \beta t)$ (Given)

Comparing it with the standard equation of wave

$y(x,t) = a\cos(kx - \omega t)$ we get

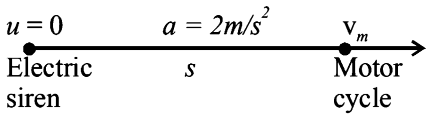
$k = \alpha$ and $\omega = \beta$

$$\therefore \frac{2\pi}{\lambda} = \alpha \quad \text{and} \quad \frac{2\pi}{T} = \beta$$

$$\therefore \alpha = \frac{2\pi}{0.08} = 25\pi \quad \text{and} \quad \beta = \frac{2\pi}{2} = \pi$$

**17. (b)** Maximum number of beats $= (\nu + 1) - (\nu - 1) = 2$

**18. (a)**




$v_m^2 - u^2 = 2as \Rightarrow v_m^2 = 2\times 2\times s \quad \therefore v_m = 2\sqrt{s}$

According to Doppler's effect

$$0.94v = v\left[\frac{330 - 2\sqrt{s}}{330}\right] \Rightarrow s = 98.01\,m$$

**19. (d)** $y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)}\right) - \frac{x}{0.50(m)}\right]$

But $y = a\sin(\omega t - kx)$

$$\therefore \omega = \frac{2\pi}{0.04} \Rightarrow v = \frac{1}{0.04} = 25\,Hz$$

$$k = \frac{2\pi}{0.50} \Rightarrow \lambda = 0.5\,m$$

$\therefore$ velocity, $v = \nu\lambda = 25\times 0.5$ m/s $= 12.5$ m/s

Velocity on a string is given by

$$v = \sqrt{\frac{T}{\mu}} \quad \therefore T = v^2\times\mu = (12.5)^2\times 0.04 = 6.25\,N$$

**20.** **(a)** Given wave equation is $y(x,t)=e^{\left(-ax^2+bt^2+2\sqrt{ab}\,xt\right)}$

$= e^{-[(\sqrt{a}x)^2+(\sqrt{b}t)^2+2\sqrt{a}\,x.\sqrt{b}\,t]} = e^{-(\sqrt{a}x+\sqrt{b}t)^2}$

$= e^{-\left(x+\sqrt{\frac{b}{a}}t\right)^2}$

It is a function of type $y=f(x+vt)$

$\Rightarrow$ Speed of wave $=\sqrt{\frac{b}{a}}$

**21.** **(a)** The fundamental frequency of open tube

$$\nu_0=\frac{v}{2l_0} \quad \ldots\text{(i)}$$

That of closed pipe

$$\nu_c=\frac{v}{4l_c} \quad \ldots\text{(ii)}$$

According to the problem $l_c=\frac{l_0}{2}$

Thus $\nu_c=\frac{v}{l_0/2}\Rightarrow \nu_c\frac{v}{2l} \quad \ldots\text{(iii)}$

From equations (i) and (iii)

$$\nu_0=\nu_c$$

Thus, $\nu_c=f$ ($\because \nu_0=f$ is given)

**22.** **(b)** Fundamental frequency,

$$f=\frac{v}{2\ell}=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}=\frac{1}{2\ell}\sqrt{\frac{T}{A\rho}}\left[\because v=\sqrt{\frac{T}{\mu}} \text{ and } \mu=\frac{m}{\ell}\right]$$

Also, $Y=\frac{T\ell}{A\Delta\ell}\Rightarrow \frac{T}{A}=\frac{Y\Delta\ell}{\ell}\Rightarrow f=\frac{1}{2\ell}\sqrt{\frac{\gamma\Delta\ell}{\ell\rho}}$ ....(i)

Putting the value of $\ell,\frac{\Delta\ell}{\ell}$, $\rho$ and $\gamma$ in eq$^n$. (i) we get,

$$f=\sqrt{\frac{2}{7}}\times\frac{10^3}{3} \quad \text{or, } f\approx 178.2\text{ Hz}$$

**23.** **(c)** Length of pipe = 85 cm = 0.85m

Frequency of oscillations of air column in closed organ pipe is given by,

$$f=\frac{(2n-1)v}{4L}$$

$$f=\frac{(2n-1)v}{4L}\le 1250$$

$$\Rightarrow \frac{(2n-1)\times 340}{0.85\times 4}\le 1250$$

$$\Rightarrow 2n-1\le 12.5\approx 6$$

**24.** **(d)** $f_1=f\left[\frac{v}{v-v_s}\right]=f\times\frac{320}{300}Hz$

$$f_2=f\left[\frac{v}{v+v_s}\right]=f\times\frac{320}{340}Hz$$

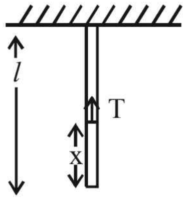
$$\left(\frac{f_2}{f_1}-1\right)\times 100=\left(\frac{300}{340}-1\right)\times 100\simeq 12\%$$

**25.** **(a)** We know that velocity in string is given by

$$v=\sqrt{\frac{T}{\mu}} \quad \text{...(I)}$$

where $\mu=\frac{m}{l}=\frac{\text{mass of string}}{\text{length of string}}$

The tension $T=\frac{m}{\ell}\times x\times g$ ..(II)



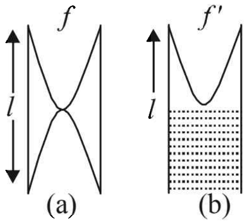

From (a) and (b)

$$\frac{dx}{dt}=\sqrt{gx}$$

$$x^{-1/2}dx=\sqrt{g}\,dt \quad \therefore \int_0^{\ell}x^{-1/2}dx-\sqrt{g}\int_0^{\ell}dt$$

$$2\sqrt{l}=\sqrt{g}\times t \quad \therefore t=2\sqrt{\frac{\ell}{g}}=2\sqrt{\frac{20}{10}}=2\sqrt{2}$$

**26.** **(b)**




The fundamental frequency in case (a) is $f=\frac{v}{2\ell}$

The fundamental frequency in case (b) is

$$f'=\frac{v}{4(\ell/2)}=\frac{u}{2\ell}=f$$