FACTORISATION

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FACTORISATION

Factors : If an algebraic expression is written as the product of numbers or algebraic expressions, then each of these numbers and expressions are called the factors of the given algebraic expression and the algebraic expression is called the product of these exrpressions.

Factorisation : The process of writing a given algebraic expression as the product of two or more factors is called factorization.

Ex.1 Write down all possible factors of $3x^2y$.

Sol. We have,

$3x^2y = 1 \times 3x^2y = 3 \times x^2y = 3x \times xy = 3xy \times x$

$= x^2 \times 3y = y \times 3x^2$

Thus, the possible factors of $3x^2y$ are

$1, 3x^2y, 3, x^2y, 3x, xy, 3xy, x, x^2, 3y, y, 3x^2$

Ex.2 Write down all possible factors of $12x^2$.

Sol. We have,

$12x^2 = 1 \times 12x^2 = 12 \times x^2 = 3 \times 4x^2 = 4 \times 3x^2 = 2 \times 6x^2 = 6 \times 2x^2$

$= 3x \times 4x = 6x \times 2x = 2 \times 3x \times 2x = 3 \times 2x \times 2x$

Thus, the possible factors of $12x^2$ are

$1, 12x^2, 12, x^2, 3, 4x^2, 4, 3x^2, 2, 6x^2, 6, 2x^2, 3x, 4x, 6x, 2x$

Greatest Common Factor (GCF) Or Highest Common Factor (HCF)

The greatest common factor of given monomials is the common factor having greastest coefficient and highest power of the variables.

The following step-wise procedure will be helpful to find the GCF of two or more monomials.

Step I Obtain the given monomials.

Step II Find the numerical coefficient of the each monomial and their greatest common factor (GCF/HCF).

Step III Find the common literals appearing in the given monomials.

Step IV Find smallest power of each common literal.

Step V Write a monomial of common literals with smallest powers obtained in step IV.

Step VI The required GCF is the product of the coefficient obtained in step II and the monomial obtained in step V.

Ex.3 Find the greatest common factors of the monomials $21a^3b^7$ and $35a^5b^5$.

Sol. The numerical coefficients of the given monomials are 21 and 35

The greatest common factor of 21 and 35 is 7
The common literals appearing in the given monomials are a and b

The smallest power of 'a' in the two monomials = 3

The smallest power of 'b' in the two monomials = 5

The monomial of common literals with smallest powers = a^3b^5

$\therefore$ The greatest common factor = $7a^3b^5$

Ex.4 Find the greatest common factors of the monomials $14x^2y^3$, $21x^2y^2$, $35x^4y^5z$.

Sol. The numerical coefficients of the given monomials are 14, 21, and 35

The greatest common factor of 14, 21 and 35 is 7

The common literals appearing in the three monomials are x and y

The smallest power of 'x' in the three monomials = 2

The smallest power of 'y' in the three monomials = 2

The monomial of common literals with smallest powers = x^2y^2

Hence, the greatest common factor = $7x^2y^2$

> **Factorisation of Algebraic Expression when a common Monomial Factor Occurs in each Term**
>
> In order to factorise algebraic expressions consisting of a common monomial factors of each term we use the following step-wise procedure.
>
> **Step I** Obtain the algebraic expression.
>
> **Step II** Find the greatest common factor (GCF/HCF) of its terms.
>
> **StepIII** Express each term of the given expression as the product of the GCF and the quotient when it is divided by the GCF.
>
> **Step IV** Use the distributive property of multiplication over addition to express the given algebraic expression as the product of the GCF and the quotient of the given expression by the GCF.

Ex.5 Factorise each of the following algebraic expressions:

(i) $3x + 15$ (ii) $2x^2 + 5x$

(iii) $3x^2y - 6xy^2$ (iv) $6x^3 + 8x^2y$

Sol. (i) The greatest common factor of the terms namely, 3x and 15 of the expression 3x + 15 is 3. Also, $3x = 3 \times x$ and $15 = 3 \times 5$.

$\therefore 3x + 15 = 3(x + 5)$

(ii) The greatest common factor of the terms $2x^2$ and $5x$ of the expression $2x^2 + 5x$ is x. Also, $2x^2 = 2x \times x$ and $5x = 5 \times x$.

$\therefore 2x^2 + 5x = 2x \times x + 5 \times x$

$= (2x + 5)x$

(iii) Clearly, 3xy is the greatest common factor of the terms $3x^2y$ and $6xy^2$ of the binomial $3x^2y - 6xy^2$. Also, $3x^2y = 3xy \times x$ and $6xy^2 = 3xy \times 2y$

$\therefore 3x^2y - 6xy^2 = 3xy \times x - 3xy \times 2y$

$= 3xy(x - 2y)$

(iv) Clearly, $2x^2$ is the GCF of the terms $6x^3$ and $8x^2y$ of the given binomial $6x^3 + 8x^2y$. Also,

$6x^3 = 2x^2 \times 3x$ and $8x^2y = 2x^2 \times 4y$.

$\therefore 6x^3 + 8x^2y$

$= 2x^2 \times 3x + 2x^2 \times 4y$

$= 2x^2(3x + 4y)$

Ex.6 Factorise :

(i) $7(2x + 5) + 3(2x + 5)$

(ii) $(x + 2)y + (x + 2)x$

(iii) $5a(2x + 3y) - 2b(2x + 3y)$

(iv) $8(5x + 9y)^2 + 12(5x + 9y)$

Sol. We have,

(i) $7(2x + 5) + 3(2x + 5) = (7 + 3)(2x + 5)$

[Taking (2x + 5) common]

$= 10(2x + 5)$

(ii) $(x + 2)y + (x + 2)x = (x + 2)(y + x)$

[Taking (x + 2) common]

(iii) $5a(2x + 3y) - 2b(2x + 3y)$

$= (2x + 3y)(5a - 2b)$

[Taking (2x + 3y) common]

(iv) $8(5x + 9y)^2 + 12(5x + 9y) = 4(5x + 9y)\{2(5x + 9y) + 3\}$

$= 4(5x + 9y)(10x + 18y + 3)$

Ex.7 Factorise :

(i) $(y - x)a + (x - y)b$

(ii) $9(a - 2b)^2 + 6(2b - a)$

(iii) $(x - 2y)^2 - 4x + 8y$

(iv) $2a + 6b - 3(a + 3b)^2$

Sol. We have,

(i) $(y - x)a + (x - y)b = -(x - y)a + (x - y)b$

[Taking (–1) common from (y – x)]

$= (x - y)(-a + b)$ [Taking (x – y) common]

$= (x - y)(b - a)$ [$\because -a + b = b - a$]

(ii) $9(a - 2b)^2 + 6(2b - a)$

$= 9(a - 2b)^2 - 6(a - 2b)$

[$\because 2b - a = -(a - 2b)$]

$= 3(a - 2b)\{3(a - 2b) - 2\}$

[Taking 3(a – 2b) common]

$= 3(a - 2b)(3a - 6b - 2)$

(iii) $(x - 2y)^2 - 4x + 8y = (x - 2y)^2 - 4(x - 2y)$

[Taking –4 common from –4x + 8y]

$= (x - 2y)\{(x - 2y) - 4\}$

[Taking (x – 2y) common]

$= (x - 2y)(x - 2y - 4)$

(iv) $2a + 6b - 3(a + 3b)^2 = 2(a + 3b) - 3(a + 3b)^2$

[Taking 2 common from 2a + 6b]

$= (a + 3b)\{2 - 3(a + 3b)\}$

[Taking (a + 3b) common]

$= (a + 3b)(2 - 3a - 9b)$

Factorisation by Grouping the Terms

Ex.8 Factorise :

(i) $ax + bx + ay + by$

(ii) $ax^2 + by^2 + bx^2 + ay^2$

(iii) $a^2 + bc + ab + ac$

(iv) $ax - ay + bx - by$

Sol. We have,

(i) $ax + bx + ay + by = (ax + bx) + (ay + by)$

[Grouping the terms]

$= (a + b)x + (a + b)y$

$= (a + b)(x + y)$ [Taking (a + b) common]

(ii) $ax^2 + by^2 + bx^2 + ay^2$

$= ax^2 + bx^2 + ay^2 + by^2$

[Re-arranging the terms]

$= (a + b)x^2 + (a + b)y^2$

$= (a + b)(x^2 + y^2)$ [Taking (a + b) common]

(iii) $a^2 + bc + ab + ac = (a^2 + ab) + (ac + bc)$

[Re-grouping the terms]

$= a(a + b) + (a + b)c$

$= (a + b)(a + c)$ [Taking (a + b) common]

(iv) $ax - ay + bx - by = a(x - y) + b(x - y)$

$= (a + b)(x - y)$ [Taking (x – y) common]

Ex.9 Factorise each of the following expression:

(i) $a^3x + a^2(x - y) - a(y + z) - z$

(ii) $(x^2 + 3x)^2 - 5(x^2 + 3x) - y(x^2 + 3x) + 5y$

Sol. (i) We have,

$a^3x + a^2(x - y) - a(y + z) - z$

$= a^3x + a^2x - a^2y - ay - az - z$

$= (a^3x + a^2x) - (a^2y + ay) - (az + z)$

$= a^2x(a + 1) - ay(a + 1) - z(a + 1)$

$= (a + 1)(a^2x - ay - z)$

(ii) $(x^2 + 3x)^2 - 5(x^2 + 3x) - y(x^2 + 3x) + 5y$

$= (x^2 + 3x)\{(x^2 + 3x) - 5\} - y\{(x^2 + 3x) - 5\}$

$= (x^2 + 3x - 5)(x^2 + 3x - y)$

Factorisation of Binomial Expressions expressible as the difference of two squares

Ex.10 Factorise :

(i) $9a^2 - 16b^2$ (ii) $36a^2 - (x - y)^2$

(iii) $80a^2 - 45b^2$ (iv) $(3a - b)^2 - 9c^2$

Sol. We have,

(i) $9a^2 - 16b^2 = (3a)^2 - (4b)^2$

$= (3a + 4b)(3a - 4b)$

[Using : $(a^2 - b^2) = (a + b)(a - b)$]

(ii) $36a^2 - (x - y)^2 = (6a)^2 - (x - y)^2$

$= \{6a + (x - y)\}\{6a - (x - y)\}$

[Using : $a^2 - b^2 = (a + b)(a - b)$]

$= (6a + x - y)(6a - x + y)$

(iii) $80a^2 - 45b^2 = 5(16a^2 - 9b^2)$

$= 5\{(4a)^2 - (3b)^2\}$

$= 5(4a + 3b)(4a - 3b)$

[Using : $a^2 - b^2 = (a + b)(a - b)$]

(iv) $(3a - b)^2 - 9c^2 = (3a - b)^2 - (3c)^2$

$= \{(3a - b) + 3c\}\{(3a - b - 3c)\}$

$= (3a - b + 3c)(3a - b - 3c)$

Ex.11 Factorise :

(i) $16a^2 - \frac{25}{4a^2}$ (ii) $16a^2b - \frac{b}{16a^2}$

(iii) $100(x + y)^2 - 81 (a + b)^2$

(iv) $(x - 1)^2 - (x - 2)^2$

Sol. We have,

(i) $16a^2 - \frac{25}{4a^2}$

$= (4a)^2 - \left(\frac{5}{2a}\right)^2 = \left(4a + \frac{5}{2a}\right)\left(4a - \frac{5}{2a}\right)$

(ii) $16a^2b - \frac{b}{16a^2} = b\left(16a^2 - \frac{1}{16a^2}\right)$

$= b\left\{(4a)^2 - \left(\frac{1}{4a}\right)^2\right\}$

$= b\left(4a + \frac{1}{4a}\right)\left(4a - \frac{1}{4a}\right)$

(iii) $100(x + y)^2 - 81 (a + b)^2$

$= \{10 (x + y)\}^2 - \{9(a + b)\}^2$

$= \{10(x + y) + 9 (a + b)\} \{10 (x + y) - 9 (a + b)\}$

$= (10x + 10y + 9a + 9b) (10x + 10y - 9a - 9b)$

(iv) $(x - 1)^2 - (x - 2)^2$

$= \{(x - 1) + (x - 2)\}\{(x - 1) - (x - 2)\}$

$= (2x - 3) (x - 1 - x + 2)$

$= (2x - 3) \times 1$

$= 2x - 3$

Ex.12 Factorise each of the following algebraic expression:

(i) $x^4 - 81y^4$ (ii) $2x^5 - 2x$

(iii) $3x^4 - 243$ (iv) $2 - 50x^2$

(v) $x^8 - y^8$ (vi) $a^{12}x^4 - a^4x^{12}$

Sol. (i) $x^4 - 81y^4 = (x^2)^2 - (9y^2)^2$

$= (x^2 - 9y^2) (x^2 + 9y^2)$

$= \{x^2 - (3y)^2\} (x^2 + 9y^2)$

$= (x - 3y) (x + 3y) (x^2 + 9y^2)$

(ii) $2x^5 - 2x = 2x (x^4 - 1)$

$= 2x \{(x^2)^2 - 1^2\}$

$= 2x (x^2 - 1) (x^2 + 1)$

$= 2x (x - 1) (x + 1) (x^2 + 1)$

(iii) $3x^4 - 243 = 3 (x^4 - 81)$

$= 3\{(x^2)^2 - 9^2\} = 3(x^2 - 9) (x^2 + 9)$

$= 3(x^2 - 3^2) (x^2 + 9) = 3(x + 3) (x - 3) (x^2 + 9)$

(iv) $2 - 50x^2 = 2\{1 - 25x^2\}$

$= 2\{1^2 - (5x)^2\} = 2(1 - 5x) (1 + 5x)$

(v) $x^8 - y^8 = \{(x^4)^2 - (y^4)^2\}$

$= (x^4 - y^4) (x^4 + y^4)$

$= \{(x^2)^2 - (y^2)^2\}(x^4 + y^4)$

$= (x^2 - y^2) (x^2 + y^2) (x^4 + y^4)$

$= (x - y) (x + y) (x^2 + y^2) (x^4 + y^4)$

$= (x - y) (x + y) (x^2 + y^2)$

$\{(x^2)^2 + (y^2)^2 + 2x^2y^2 - 2x^2y^2\}$

$= (x - y) (x + y) (x^2 + y^2) \left\{(x^2 + y^2)^2 - \left(\sqrt{2}xy\right)^2\right\}$

$= (x - y) (x + y) (x^2 + y^2) \left(x^2 + y^2 - \sqrt{2}xy\right)$

$\left(x^2 + y^2 + \sqrt{2}xy\right)$

(vi) $a^{12}x^4 - a^4x^{12} = a^4x^4 \left(a^8 - x^8\right)$

$= a^4x^4 \{(a^4)^2 - (x^4)^2\} = a^4x^4 (a^4 + x^4) (a^4 - x^4)$

$= a^4x^4 (a^4 + x^4) \{(a^2)^2 - (x^2)^2\}$

$= a^4x^4 (a^4 + x^4) (a^2 + x^2) (a^2 - x^2)$

$= a^4x^4 (a^4 + x^4) (a^2 + x^2) (a + x) (a - x)$

Factorisation of Algebraic Expressions expressible as a perfect square

(i) $a^2 + 2ab + b^2 = (a + b)^2 = (a + b) (a + b)$

(ii) $a^2 - 2ab + b^2 = (a - b)^2 = (a - b) (a - b)$

Ex.13 Factorise :

(i) $x^2 + 8x + 16$ (ii) $4a^2 - 4a + 1$

Sol. We have,

(i) $x^2 + 8x + 16 = x^2 + 2 \times x \times 4 + 4^2$

$= (x + 4)^2$ [Using : $a^2 + 2ab + b^2 = (a + b)^2$]

$= (x + 4)(x + 4)$

(ii) $4a^2 - 4a + 1 = (2a)^2 - 2 \times 2a \times 1 + (1)^2$

$= (2a - 1)^2$ [Using : $a^2 - 2ab + b^2 = (a - b)^2$]

$= (2a - 1)(2a - 1)$

Ex.14 Factorise :

(i) $4x^2 + 12xy + 9y^2$ (ii) $x^4 - 10x^2y^2 + 25y^4$

(iii) $a^4 - 2a^2b^2 + b^4$

Sol. We have,

(i) $4x^2 + 12xy + 9y^2 = (2x)^2 + 2 \times 2x \times 3y + (3y)^2$

$= (2x + 3y)^2$

$= (2x + 3y)(2x + 3y)$

(ii) $x^4 - 10x^2y^2 + 25y^4$

$= (x^2)^2 - 2 \times x^2 \times 5y^2 + (5y^2)^2$

$= (x^2 - 5y^2)^2$

$= (x^2 - 5y^2)(x^2 - 5y^2)$

(iii) $a^4 - 2a^2b^2 + b^4 = (a^2)^2 - 2 \times a^2 \times b^2 + (b^2)^2$

$= (a^2 - b^2)^2$

$= \{(a - b)(a + b)\}^2 = (a - b)^2 (a + b)^2$

Ex.15 Factorise each of the following expressions:

(i) $x^2 - 2xy + y^2 - x + y$

(ii) $4a^2 + 12ab + 9b^2 - 8a - 12b$

(iii) $a^2 + b^2 - 2(ab - ac + bc)$

Sol. (i) $x^2 - 2xy + y^2 - x + y = (x^2 - 2xy + y^2)$

$= (x - y)^2 - (x - y)$

$= (x - y)\{(x - y) - 1\}$

$= (x - y)(x - y - 1)$

(ii) $4a^2 + 12ab + 9b^2 - 8a - 12b$

$= (2a)^2 + 2 \times 2a \times 3b + (3b)^2 - 4(2a + 3b)$

$= (2a + 3b)^2 - 4(2a + 3b) = (2a + 3b)(2a + 3b - 4)$

(iii) $a^2 + b^2 - 2(ab - ac + bc)$

$= a^2 + b^2 - 2ab + 2ac - 2bc = (a - b)^2 + 2c(a - b)$

$= (a - b)\{(a - b) + 2c\} = (a - b)(a - b + 2c)$

Ex.16 Factorise each of the following expressions:

(i) $x^2 + 2xy + y^2 - a^2 + 2ab - b^2$

(ii) $25x^2 - 10x + 1 - 36y^2$

(iii) $1 - 2ab - (a^2 + b^2)$

Sol. (i) $x^2 + 2xy + y^2 - a^2 + 2ab - b^2$

$= (x^2 + 2xy + y^2) - (a^2 - 2ab + b^2)$

$= (x + y)^2 - (a - b)^2$

$= \{(x + y) + (a - b)\}\{(x + y) - (a - b)\}$

$= (x + y + a - b)(x + y - a + b)$

(ii) $25x^2 - 10x + 1 - 36y^2$

$= (5x)^2 - 2 \times 5x \times 1 + 1^2 - (6y)^2$

$= (5x - 1)^2 - (6y)^2$

$= (5x - 1 + 6)(5x - 1 - 6y)$

(iii) $1 - 2ab - (a^2 + b^2) = 1 - (2ab + a^2 + b^2)$

$= 1 - (a + b)^2$

$= \{1 + (a + b)\}\{1 - (a + b)\}$

$= (1 + a + b)(1 - a - b)$

Ex.17 Factorise:

(i) $x^2 + 8x + 15$ (ii) $x^4 + x^2 + 1$ (iii) $x^4 + 4$

Sol. We have,

(i) $x^2 + 8x + 15 = (x^2 + 8x + 16) - 1$

[Replacing 15 by 16 – 1]

$= \{(x)^2 + 2 \times x \times 4 + 4^2\} - 1 = (x + 4)^2 - 1^2$

$= \{x + 4 + 1\}\{(x + 4) - 1\}$

$= (x + 5)(x + 3)$

(ii) $x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2$

[Adding and subtracting x^2]

$= (x^4 + 2x^2 + 1) - x^2$

$= \left((x^2)^2 + 2 \times x^2 \times 1 + 1^2\right) - x^2$

$= (x^2 + 1)^2 - x^2 = \{(x^2 + 1) + x\}\{(x^2 + 1) - x\}$

$= (x^2 + x + 1)(x^2 - x + 1)$

(iii) $x^4 + 4 = x^4 + 4x^2 + 4 - 4x^2$

[Adding and subtracting $4x^2$]

$= \{(x^2)^2 + 2 \times x^2 \times 2 + 2^2\} - 4x^2$

$= (x^2 + 2)^2 - (2x)^2$

$= \{(x^2 + 2) + 2x\}\{(x^2 + 2) - 2x\}$

$= (x^2 + 2x + 2)(x^2 - 2x + 2)$

Factorisation of Quadratic Polynomials in one varibale

Algorithm :

Step I Obtain the quadratic polynomial $x^2 + px + q$.

Step II Obtain p = coefficient of x and, q = constant term.

Step III Find the two numbers a and b such that a + b = p and ab = q.

Step IV Split up the middle term as the sum of two terms ax and bx.

Step V Factorise the expression obtained in step IV by grouping the term.

Ex.18 Factorise each of the following expressions:

(i) $x^2 + 6x + 8$ (ii) $x^2 + 4x - 21$

(iii) $x^2 - 7x + 12$

Sol. (i) In order to Factorise $x^2 + 6x + 8$, we find two numbers p and q such that p + q = 6 and pq = 8.

Clearly, 2 + 4 = 6 and 2 × 4 = 8

We now split the middle term 6x in the given quadratic as 2x + 4x.

$\therefore x^2 + 6x + 8 = x^2 + 2x + 4x + 8$

$= (x^2 + 2x) + (4x + 8)$

$= x(x + 2) + 4(x + 2)$

$= (x + 2)(x + 4)$

(ii) In order to Factorise $x^2 + 4x - 21$, we have to find two numbers p and q such that

p + q = 4 and pq = – 21

Clearly, 7 + (–3) = 4 and 7 × –3 = 21.

We now split the middle term 4x of $x^2 + 4x - 21$ as 7x – 3x.

$\therefore x^2 + 4x - 21 = x^2 + 7x - 3x - 21$

$= (x^2 + 7x) - (3x + 21)$

$= x(x + 7) - 3(x + 7)$

$= (x + 7)(x - 3)$

(iii) In order to Factorise $x^2 - 7x + 12$ we have to find two numbers p and q such that p + q = –7 and pq = 12.

Clearly, – 3 – 4 = – 7

and – 3 × – 4 = 12.

We now split the middle term – 7x of the given quadratic as – 3x – 4x.

$\therefore x^2 - 7x + 12 = x^2 - 3x - 4x + 12$

$= (x^2 - 3x) - (4x - 12)$

$= x(x - 3) - 4(x - 3)$

$= (x - 3)(x - 4)$

Ex.19 Factorise each of the following quadratic polynomials :

(i) $x^2 - 23x + 132$

(ii) $x^2 - 21x + 108$

(iii) $x^2 + 5x - 36$

Sol. (i) In order to Factorise $x^2 - 23x + 132$, we have to find numbers p and q such that p + q = –23 and pq = 132.

Clearly, – 12 – 11 = –23 and –12 × –11 = 132.

We now split the middle term – 23x of $x^2 - 23x + 132$ as – 12x – 11x

$\therefore x^2 - 23x + 132 = x^2 - 12x - 11x + 132$

$= (x^2 - 12x) - (11x - 132)$

$= x(x - 12) - 11(x - 12)$

$= (x - 12)(x - 11)$

(ii) In order to Factorise $x^2 - 21x + 108$, we have to find two numbers such that their sum is – 21 and the product 108.

Clearly, –21 = –12 – 9 and – 12 × – 9 = 108
So, we split the middle term – 21x as – 12x – 9x

$\therefore x^2 - 21x + 108 = x^2 - 12x - 9x + 108$

$= (x^2 - 12x) - (9x - 108)$

$= x(x - 12) - 9(x - 12)$

$= (x - 12)(x - 9)$

(iii) In order to Factorise $x^2 + 5x - 36$, we have to find two numbers p and q such that p + q = 5 and pq = – 36.

Clearly, 9 + (–4) = 5 and 9 × –4 = –36.

So, we write the middle term 5x of

$x^2 + 5x - 36$ as 9x – 4x.

$\therefore\ x^2 + 5x - 36 = x^2 + 9x - 4x - 36$

$= (x^2 + 9x) - (4x + 36) = x(x + 9) - 4(x + 9)$

$= (x + 9)(x - 4)$

Factorisation of Quadratic Polynomials of Theorem $ax^2 + bx + c,\ a \neq 1$

Procedure:

Step I Obtain the quadratic trinomial $ax^2 + bx + c$

Step II Obtain a = coefficient of x^2, b = coefficient of x and c = constant terms.

Step III Find the product of the coefficient of x^2 and the constant term i.e. ac.

Step IV Split up the coefficient of x i.e. b into two parts whose sum is b and product ac and write the middle term as the sum of two terms.

Step V Factorise the expression obtained in step IV by grouping the term. Factors so obtained will be the required factors of the given quadratic trinomial.

Ex.20 Factorise :

(i) $2x^2 + 5x + 3$ (ii) $6x^2 + 5x - 6$

(iii) $6x^2 - 13x + 6$ (iv) $-2x^2 - 3x + 2$

Sol. (i) The given expression is $2x^2 + 5x + 3$

Here, coefficient of x^2 =2, coeffcient of x = 5, and constant term = 3.

We shall now split up the coefficient of the middle term i.e. 5 into two parts such that their sum is 5 and product equal to the product of coefficient of x^2 and constant term i.e. 2 × 3 = 6. Clearly, 2 + 3 = 5 and 2 × 3 = 6. So, we replace the middle term 5x by 2x + 3x.

Thus, we have

$2x^2 + 5x + 3 = 2x^2 + 2x + 3x + 3$

$= (2x^2 + 2x) + (3x + 3) = 2x(x + 1) + 3(x + 1)$

$= (x + 1)(2x + 3)$

(ii) The given expression is $6x^2 + 5x - 6$

Here, coefficient of x^2 = 6, coefficient of x = 5, constant term = – 6

We shall now split up the coefficient of x i.e., 5 into two parts such that their sum is equal to coefficient of x i.e., 5 and product equal to the product of coefficient of x^2 and constant term i.e., 6 × – 6 = – 36.

Clearly, 9 + (–4) = 5 and 9 × – 4 = – 36. So, we replace the middle term 5x by 9x – 4x.

Thus, we have

$6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3) = (2x + 3)(3x - 2)$

(iii) The given expression is $6x^2 - 13x + 6$.

Here, coefficient of x^2 = 6, coefficient of x = – 13, and constant term = 6.

We shall now split up the coefficient of x i.e. – 13 into two parts whose sum is –13 and product equal to product of the coefficient of x^2 and constant term i.e, 6× 6 = 36. Clearly, –4, –9 = –13 and –4 × –9 = 36. So, we write the middle term – 13x as – 4x – 9x

Thus, we have

$6x^2 - 13x + 6 = 6x^2 - 4x - 9x + 6$

$= 2x(3x - 2) - 3(3x - 2) = (3x - 2)(2x - 3)$

(iv) The given expression is $-2x^2 - 3x + 2$.

Here, coefficient of x^2 = – 2, coefficient of x = –3 and constant term = 2.

We shall now split up the coefficient of the middle term i.e. – 3 into two parts such that their sum is – 2 and the product is equal to the product of the coefficient of x^2 and constant term i.e. – 2 × 2 = –4.

Clearly, – 4 + 1 = – 3 and – 4 × 1 = –4.

So, we write the middle term – 3x as – 4x + x.

Thus, we have

$-2x^2 - 3x + 2 = -2x^2 - 4x + x + 2$

$= -2x(x + 2) + 1(x + 2) = (x + 2)(-2x + 1)$

Ex.21 Factorise:

(i) $12x^2 - 23xy + 10y^2$ (ii) $12x^2 + 7xy - 10y^2$

Sol. (i) The given expression is $12x^2 - 23xy + 10y^2$

Here, coefficient of x^2 = 12, coefficient of x = –23y, and constant term = $10y^2$.

Now, we split up the coefficient of the middle term i.e., –23y into two parts whose sum is –23y and product equal to the product of the coefficient of x^2 and constant term i.e., $12 \times 10y^2 = 120y^2$.

Clearly, –15y – 8y = – 23y and –15y × –8y = $120y^2$

So,we replace the middle term

$-23xy$ by $-15xy -8xy$.

Thus, we have

$12x^2 - 23xy + 10y^2 = 12x^2 - 15xy - 8xy + 10y^2$

$= 3x(4x - 5y) - 2y(4x - 5y) = (4x - 5y)(3x - 2y)$

(ii) The given expression is $12x^2 + 7xy - 10y^2$

Here, coefficient of $x^2 = 12$, coefficient of $x = 7y$ and constant term $= -10y^2$.

We shall now split up the coefficient of the middle term i.e. $7y$ into two parts whose sum is $7y$ and product equal to the product of the coefficient of x^2 and constant term i.e. $12 \times -10y^2 = -120y^2$.

Clearly, $15y - 8y = 7y$ and $15y \times -8y = -120y^2$

So, we replace the middle term $7xy$ by $15xy - 8xy$

Thus, we have

$12x^2 + 7xy - 10y^2 = 12x^2 + 15xy - 8xy - 10y^2$

$= 3x(4x + 5y) - 2y(4x + 5y) = (4x + 5y)(3x - 2y)$

Factorisation of Quadratic Polynomials by using the method of completing the perfect square

Procedure:

Step I Obtain the quadratic polynomial. Let the polynomial be $ax^2 + bx + c$, where $a \neq 0$.

Step II Make the coefficient of x^2 unity by dividing and multiplying throughout by it, if it is not unity i.e., write

$$ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

Step III Add and subtract square of half of the coefficient of x i.e., write

$$ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

$$= a\left\{x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right\}$$

Step IV Write first three terms as the square of a binomial and simplify last two terms i.e., write

$ax^2 + bx + c$

$$= a\left\{x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right\}$$

$$= a\left\{\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a^2}\right)\right\}$$

Step V Factorise last step obtained in step IV by using $a^2 - b^2 = (a - b)(a + b)$ to get desired factors.

Ex.22 Factorise $y^2 + 6y + 8$ by using the method of completing the square.

Sol. Here, coefficient of y^2 is unity. So, we add and subtract the square of the half of coefficient of y.

$\therefore\ y^2 + 6y + 8 = y^2 + 6y + 3^2 - 3^2 + 8$

$$\left[\text{Adding and subtracting} \left(\frac{6}{2}\right)^2 = 3^2\right]$$

$= (y^2 + 6y + 3^2) - 1$

$= (y + 3)^2 - 1^2$ [By completing the square]

$= \{(y + 3) - 1\}\{(y + 3) + 1\}$

[Using : $a^2 - b^2 = (a - b)(a + b)$]

$= (y + 2)(y + 4)$

Ex.23 Factorise: $4y^2 - 8y + 3$

Sol. We have, $4y^2 - 8y + 3$

$= 4\left\{y^2 - 2y + \frac{3}{4}\right\}$ [Making coefficient of y^2 as 1]

$= 4\left\{y^2 - 2y + 1^2 - 1^2 + \frac{3}{4}\right\}$

$\left[\text{Adding and subtracting}\left(\frac{1}{2}\text{Coeff. of y}\right)^2 \text{ i.e., } 1^2\right]$

$= 4\left\{(y^2 - 2y + 1^2) - \frac{1}{4}\right\}$

$= 4\left\{(y-1)^2 - \left(\frac{1}{2}\right)^2\right\}$ [Completing the square]

$= 4\left[\left\{(y-1) - \frac{1}{2}\right\}\left\{(y-1) + \frac{1}{2}\right\}\right]$

[Using $a^2 - b^2 = (a - b)(a + b)$]

$= 4\left(y - 1 - \frac{1}{2}\right)\left(y - 1 + \frac{1}{2}\right)$

$= 4\left(y - \frac{3}{2}\right)\left(y - \frac{1}{2}\right) = 4\left(\frac{2y-3}{2}\right)\left(\frac{2y-1}{2}\right)$

$= (2y - 3)(2y - 1)$

Ex.24 Factorise : $6 - x - 2x^2$

Sol. We have,

$6 - x - 2x^2 = -2x^2 - x + 6$

$= -2\left(x^2 + \frac{1}{2}x - 3\right)$

$\left[\text{Dividing and multiplying by } -2 \text{ i.e., the coeff. of } x^2\right]$

$= -2\left\{x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 3\right\}$

$\left[\text{Adding and subtracting}\left(\frac{1}{2}\text{Coeff. of x}\right)^2 \text{ i.e., } \left(\frac{1}{a}\right)^2\right]$

$= -2\left[\left\{x^2 + 2 \times \frac{1}{4} \times x + \left(\frac{1}{4}\right)^2\right\} - \left\{\frac{1}{16} + 3\right\}\right]$

$= -2\left\{\left(x + \frac{1}{4}\right)^2 - \frac{49}{16}\right\} = -2\left\{\left(x + \frac{1}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right\}$

$= -2\left\{\left(x + \frac{1}{4}\right) - \frac{7}{4}\right\}\left\{\left(x + \frac{1}{4}\right) + \frac{7}{4}\right\}$

$= -2\left(x + \frac{1}{4} - \frac{7}{4}\right)\left(x + \frac{1}{4} + \frac{7}{4}\right)$

$= -2\left(x - \frac{3}{2}\right)(x + 2) = (-2x + 3)(x + 2)$

➤ POLYNOMIALS

Polynomials : An algebraic expression in which the variables involved have only non-negative integral powers, is called a polynomial.

Degree of a polynomial in one variable: In a polynomial in one variable, the highest power of the variable is called degree.

Degree of a polynomial in two variable: In a polynomial in more than one variable the sum of the powers of the variables in each term is computed and the highest sum so obtained is called the degree of the polynomial.

Constant Polynomial : A polynomial consisting of a constant term only is called a constant polynomial. The degree of a constant polynomial is zero.

Linear Polynomial : A polynomial of degree 1 is called a linear polynomial.

Quadratic Polynomial : A polynomial of degree 2 is called a quadratic polynomial.

Cubic Polynomial : A polynomial of degree 3 is called a cubic polynomial.

Biquadratic Polynomials : A polynomial of degree 4 is called a biquadratic polynomial.

Eg : $\frac{2}{3}x^2 - \frac{3}{2}x^2 + x - 5$ is a polynomial in variables x whereas $\frac{1}{2}x^3 - 3x^2 + 5x^{1/2} + x - 1$ is not a polynomial, because it contains a term $5x^{1/2}$ which contains $\frac{1}{2}$ as the power of variable x, which is not a non-negative integer.

Eg : $3 - 2x^2 + 4x^2y + 8y - \frac{5}{3}xy^2$ is a polynomial in two variables x and y.

Eg : (i) $2x + 3$ is a polynomial in x of degree 1.

(ii) $2x^2 - 3x + \frac{7}{5}$ is polynomial in x of degree 2.

(iii) $\frac{2}{3}a^2 - \frac{7}{2}a^2 + 4$ is a polynomial in a dgree 3.

Eg : $3x^4 - 2x^3y^2 + 7xy^3 - 9x + 5y + 4$ is a polynomial in x and y of degree 5, whereas $\frac{1}{2} - 3x + 7x^2y - \frac{3}{4}x^2y^2$ is a polynomial of degree 4 in x and y.

Eg : $2 - \frac{3}{4}x$, $\frac{1}{2} + \frac{3}{5}y$,

2 + 3a etc. are linear polynomials.

Eg : $2x^2 - 3x + 4$, $2 - x + x^2$,

$2y^2 - \frac{3}{2}y + \frac{1}{4}$ are quadratic polynomials.

Eg : $x^3 - 7x + 2x - 3$,

$2 + \frac{1}{2}y - \frac{3}{2}y^2 + 4y^3$ are cubic polynomial.

Eg : $3x^4 - 7x^3 + x^2 - x + 9$,

$4 - \frac{2}{3}x^2 + \frac{3}{5}x^4$ are biquadratic polynomials.

Division of a monomial by a monomial

While dividing a monomial by a monomial, we follow the following two rules:

Rule-1 Coefficient of the quotient of two monomial is equal to the quotient of their coefficients.

Rule-2 The variable part in the quotient of two monomials is equal to the quotient of the variables in the given monomials.

Ex.25 Divide :

(i) $12x^3y^3$ by $3x^2y$ (ii) $-15a^2bc^3$ by 3ab

Sol. (i) We have,

$$\frac{12x^3y^2}{3x^2y} = \frac{12 \times x \times x \times x \times y \times y}{3 \times x \times x \times y}$$

$$= 4 \times x \times y = 4xy$$

(ii) We have,

$$\frac{-15a^2bc^3}{3ab} = \frac{-15 \times a \times a \times b \times c \times c \times c}{3 \times a \times b}$$

$$= -5ac^3$$

Division of a Polynomial by a Monomial

Step I Obtain the polynomial (dividend) and the monomial (divisor).

Step II Arrange the terms of the dividend in descending order of their degrees. For example, write

$7x^2 + 4x - 3 + 5x^3$ as $5x^2 + 7x^2 + 4x - 3$.

Step III Divided each term of the plynomial by the given monomial by using the rules of division of a monomial by a monomial.

Ex.26 Divide :

(i) $9m^5 + 12m^4 - 6m^2$ by $3m^2$

(ii) $24x^3y + 20x^2y^2 - 4xy$ by 2xy

Sol. (i) We have,

$$\frac{9m^5 + 12m^4 - 6m^2}{3m^2} = \frac{9m^5}{3m^2} + \frac{12m^4}{3m^2} - \frac{6m^2}{3m^2}$$

$$= 3m^3 + 4m^2 - 2$$

(ii) We have,

$$\frac{24x^3y + 20x^2y^2 - 4xy}{2xy}$$

$$= \frac{24x^3y}{2xy} + \frac{20x^2y^2}{2xy} - \frac{4xy}{2xy}$$

$$= 12x^2 + 10xy - 2$$

Division of a Polynomial by a Binomial by using long division

Step I Arrange the terms of the dividend and divisor in descending order of their degrees.

Step II Divide the first term of the dividend by the first term of the divisor to otbain the first term of the quotient.

Step III Multiply the divisor by the first term of the quotient and subtract the result from the dividend to obtain the remainder.

Step IV Consider the remainder (if any) as dividend and repeat step II to obtain the second term of the quotient.

Step V Repeat the above process till we obtain a remainder which is either zero or a polynomial of degree less than that of the divisor.

Ex.27 Divide $6 + x - 4x^2 + x^3$ by $x - 3$.

Sol. We go through the following steps to perform the division:

Step I We write the terms of the dividend as well as of divisor in descending order of their degress. Thus, we write

$= 6 + x - 4x^2 + x^3$ as $x^3 - 4x^2 + x + 6$ and $x - 3$ as $x - 3$

Step II We divide the first term x^3 of the dividend by the first term x of the divisor and obtain $\frac{x^3}{x} = x^2$ as the first term of the quotient.

Step III We multiply the divisor $x - 3$ by the first term x of the quotient and subtract the result from the dividend $x^3 - 4x^2 + x + 6$. We obtain $- x^2 + x + 6$ as the remainder.

$$\begin{array}{r|l}
 & x^2 - x - 2 \\
\hline
x - 3 & x^3 - 4x^2 + x + 6 \\
 & x^3 - 3x^2 \\
 & - \quad + \\
\hline
 & -x^2 - x + 6 \\
 & -x^2 + 3x \\
 & + \quad - \\
\hline
 & -2x + 6 \\
 & -2x + 6 \\
 & + \quad - \\
\hline
 & 0
\end{array}$$

Step IV We take $-x^2 + x + 6$ as the new dividend and repeat step II to obtain the second term $\left(\frac{x^2}{x} = \right) - x$ of the quotient.

Step V We multiply the divisor $x - 3$ by the second term $- x$ of the quotient and subtract the result $-x^2 + 3x$ from the new dividend. We obtain $- 2x + 6$ as the remainder.

Step VI Now we treat $- 2x + 6$ as the new dividend and divide its first term $- 2x$ by the first term x of the divisor to obtain $\frac{-2x}{x} = -2$ as the third term of the quotient.

Step VII We multiply the divisor $x - 3$ and the third term -2 of the quotient and subtract the result $- 2x + 6$ from the the new dividend. We obtain 0 as the remainder.

Thus, we can say that

$(6 + x - 4x^2 + x^3) \div (x - 3)$

$= x^2 - x - 2$

or, $\frac{6 + x - 4x^2 + x^3}{x - 3}$

$= x^2 - x - 2$

The above procedure is displaced on the right side of the above step.

Note :

In the above example, the remainder is zero. So, we can say that $(x - 3)$ is a factor of $6 + x - 4x^2 + x^3$.

Ex.28 Divide : $x^3 - 6x^2 + 11x - 6$ by $x^2 - 4x + 3$

Sol. On dividing, we get

$$\begin{array}{r|l}
 & x - 2 \\
\hline
x^2 - 4x + 3 & x^3 - 6x^2 + 11x - 6 \\
 & x^3 - 4x^2 + 3x \\
 & - \quad + \quad - \\
\hline
 & -2x^2 + 8x - 6 \\
 & -2x^2 + 8x - 6 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}$$

$\therefore \ x^3 - 6x^2 + 11x - 6 = (x - 2)(x^2 - 4x + 3)$

Ex.29 Using division show that $3y^2 + 5$ is factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$.

Sol. On dividing $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$ by $3y^2 + 5$, we obtain

$$\begin{array}{r|l}
 & 2y^3 + 5y^2 + 2y - 7 \\ \hline
3y^2 + 5 & 6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35 \\
 & 6y^5 \qquad\quad + 10y^3 \\
 & - \qquad\qquad - \\ \hline
 & 15y^4 + 6y^3 + 4y^2 + 10y - 35 \\
 & 15y^4 \qquad + 25y^2 \\
 & - \qquad\qquad - \\ \hline
 & 6y^3 - 21y^2 + 10y - 35 \\
 & 6y^3 \qquad\quad + 10y \\
 & - \qquad\qquad - \\ \hline
 & -21y^2 - 35 \\
 & -21y^2 - 35 \\
 & + \qquad + \\ \hline
 & 0
\end{array}$$

Since the remainder is zero. Therefore, $3y^2 + 5$ is a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$.

> **Division Algorithm:**
>
> We know that if a number is divided by another number, then
>
> Dividend = Divisor × Quotient + Remainder
>
> Similarly, if a polynomial is divided by another polynomial, then
>
> Dividend = Divisor × Quotient + Remainder
>
> This is generally known as the division algorithm.

Ex.30 What must be subtracted from

$8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$.

Sol. We know that

Dividend = Quotient × Divisor + Remainder

$\Rightarrow$ Dividend – Remainder = Quotient × Divisor

Clearly, R.H.S of the above result is divisble by the divisor. Therefore, L.H.S. is also divisible by the divisor. Thus, if we subtract remainder from the dividend, then it will be exactly divisible by the divisior.

Dividing $8x^4 + 14x^3 - 2x^2 + 7x - 8$ by $4x^2 + 3x - 2$, we get

$$\begin{array}{r|l}
 & 2x^2 + 2x - 1 \\ \hline
4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 7x - 8 \\
 & 8x^4 + 6x^3 - 4x^2 \\
 & - \quad - \qquad + \\ \hline
 & 8x^3 + 2x^2 + 7x - 8 \\
 & 8x^3 + 6x^2 - 4x \\
 & - \quad - \qquad + \\ \hline
 & -4x^2 + 11x - 8 \\
 & -4x^2 - 3x + 2 \\
 & + \quad + \quad - \\ \hline
 & 14x - 10
\end{array}$$

$\therefore$ Quotient = $2x^2 + 2x - 1$ and,

Remainder = $14x - 10$

Thus, if we subtract the remainder $14x - 10$ from $8x^4 + 14x^3 - 2x^2 + 7x - 8$, it will be divisible by $4x^2 + 3x - 2$

Ex.31 Find the values of a and b so that

$x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$.

Sol. If $x^4 + x^3 + 8x^2 + ax + b$ is exactly divisible by $x^2 + 1$, then the remainder should be zero.

On dividing, we get

$$\begin{array}{r|l}
 & x^2 + x + 7 \\ \hline
x^2 + 1 & x^4 + x^3 + 8x^2 + ax + b \\
 & x^4 \qquad + x^2 \\
 & - \qquad - \\ \hline
 & x^3 + 7x^2 + ax + b \\
 & x^3 \qquad + x \\
 & - \qquad - \\ \hline
 & 7x^2 + x(a-1) + b \\
 & 7x^2 \qquad\quad + 7 \\
 & - \qquad\qquad - \\ \hline
 & x(a-1) + b - 7
\end{array}$$

$\therefore$ Quotient = $x^2 + x + 7$ and,

Remainder = $x(a - 1) + b - 7$

Now, Remainder = 0

$\Rightarrow x(a - 1) + (b - 7) = 0$

$\Rightarrow x(a - 1) + (b - 7) = 0x + 0$

$\Rightarrow a - 1 = 0$ and $b - 7 = 0$

[Comparing coefficients of x and constant terms]

$\Rightarrow$ a = 1 and b = 7

Ex.32 Divide $x^4 - x^3 + x^2 + 5$ by $(x + 1)$ and write the quotient and remainder.

Sol. We have,

$x^4 - x^3 + x^2 + 5 = x^3(x + 1) - 2x^2(x + 1) + 3x(x + 1) - 3(x + 1) + 8$

$= (x + 1)(x^3 - 2x^3 + 3x - 3) + 8$

Hence, Quotient = $x^3 - 2x^2 + 3x - 3$ and, Remainder = 8.

Ex.33 Divide $12x^3 - 8x^2 - 6x + 10$ by $(3x - 2)$. Also, write the quotient and the remainder.

Sol. We have,

$12x^3 - 8x^2 - 6x + 10$

$= 4x^2(3x - 2) - 2(3x - 2) + 6$

$= \{4x^2(3x - 2) - 2(3x - 2)\} + 6$

$= (3x - 2)(4x^2 - 2) + 6$

Hence, Quotient = $4x^2 - 2$ and, Remainder = 6.

Ex.34 Divide $6x^3 - x^2 - 10x - 3$ by $(2x - 3)$.

Sol. We have,

$6x^3 - x^2 - 10x - 3$

$= 3x^2(2x - 3) + 4x(2x - 3) - 1(2x - 3) - 6$

$= \{3x^2(2x - 3) + 4x(2x - 3) - 1(2x - 3)\} - 6$

$= (2x - 3)(3x^2 + 4x - 1) - 6$

Hence, Quotient = $3x^2 + 4x - 1$ and,

Remainder = – 6.

Division of Polynomials by using Factorisation

Ex.35 Divide:

(i) $35a^2 + 32a - 99$ by $7a - 9$

(ii) $ax^2 + (b + ac)x + bc$ by $x + c$

Sol. (i) We have,

$35a^2 + 32a - 99$

$= 35a^2 + 77a - 45a - 99$

$= 7a(5a + 11) - 9(5a + 11) = (5a + 11)(7a - 9)$ ...(i)

$\therefore (35a^2 + 32a - 99) \div (7a - 9)$

$= \dfrac{35a^2 + 32a - 99}{7a - 9}$

$= \dfrac{(5a + 11)(7a - 9)}{(7a - 9)} = 5a + 11$ [Using (i)]

[Just as numbers, we cancel common factor (7a – 9) in numerator and denominator]

(ii) We have,

$ax^2 + (b + ac)x + bc$

$= (ax^2 + bx) + (acx + bc)$

$= x(ax + b) + c(ax + b) = (ax + b)(x + c)$ ...(i)

$\therefore \left(ax^2 + (b + ac)x + bc\right) \div (x + c)$

$= \dfrac{ax^2 + (b + ac)x + bc}{(x + c)}$

$= \dfrac{(ax + b)(x + c)}{(x + c)}$ [Using (i)]

$= ax + b$ [Canceling common factor (x + c) in numerator and denominator]

Ex.36 Divide: $a^{12} + a^6b^6 + b^{12}$ by $a^6 - a^3b^3 + b^6$

Sol. We have,

$a^{12} + a^6b^6 + b^{12}$

$= a^{12} + 2a^6b^6 + b^{12} - a^6b^6$

[Adding and subtracting a^6b^6]

$= (a^6 + b^6)^2 - (a^3b^3)^2$

$= (a^6 + b^6 - a^3b^3)(a^6 + b^6 + a^3b^3)$

$= (a^6 - a^3b^3 + b^6)\,(a^6 + a^3b^3 + b^6)$ ...(i)

$$\therefore \frac{a^{12} + a^6b^6 + b^{12}}{a^6 - a^3b^3 + b^6}$$

$$= \frac{(a^6 - a^3b^3 + b^6)(a^6 + a^3b^3 + b^6)}{(a^6 - a^3b^3 + b^6)}$$

$= a^6 + a^3b^3 + b^6$

[Canceling $a^6 - a^3b^3 + b^6$ from N^r and D^r]

EXERCISE # 1

Q.1 Factorise:
(i) $12x^3y^4 + 16x^2y^5 - 4x^5y^2$
(ii) $18a^3b^2 + 36ab^4 - 24a^2b^3$

Q.2 Factorise:
(i) $(x + y)(2x + 3y) - (2x + 3y) - (x + y)(x + 1)$
(ii) $(x + y)(2a + b) - (3x - 2y)(2a + b)$

Q.3 Factorise :
(i) $x^2 + xy + 8x + 8y$
(ii) $15xy - 6x + 10y - 4$
(iii) $n - 7 + 7lm - lmn$

Q.4 Factorise:
(i) $a^2 + 2a + ab + 2b$
(ii) $x^2 - xz + xy - xz$

Q.5 Factorise each of the following expressions:
(i) $a^2 - b + ab - a$
(ii) $xy - ab + bx - ay$
(iii) $6ab - b^2 + 12ac - 2bc$
(iv) $a(a + b - c) - bc$
(v) $a^2x^2 + (ax^2 + 1)x + a$
(vi) $3ax - 6ay - 8by + 4bx$

Q.6 Factorise:
(i) $x^3 - 2x^2y + 3xy^2 - 6y^3$
(ii) $6ab - b^2 + 12ac - 2bc$

Q.7 Factorise :
(i) $x^4 - y^4$ (ii) $16x^4 - 81$
(iii) $x^4 - (y + z)^4$ (iv) $2x - 32x^5$
(v) $3a^4 - 48b^4$ (vi) $81x^4 - 121x^2$

Q.8 Factorise each of the following algebraic expressions:
(i) $16(2x - 1)^2 - 25z^2$
(ii) $4a^2 - 9b^2 - 2b - 3b$
(iii) $x^2 - 4x + 4y - y^2$
(iv) $3 - 12(a - b)^2$
(v) $x(x + z) - y(y + z)$
(vi) $a^2 - b^2 - a - b$

Q.9 Factorise :
(i) $4x^2 - 4xy + y^2 - 9z^2$
(ii) $16 - x^2 - 2xy - y^2$
(iii) $x^4 - (x - z)^4$

Q.10 Factorise :
(i) $4(x + y)^2 - 28y(x + y) + 49y^2$
(ii) $(2a + 3b)^2 + 2(2a + 3b)(2a - 3b) + (2a - 3b)^2$

Q.11 Factorise each of the following expressions:
(i) $9x^2 - 4y^2$
(ii) $36x^2 - 12x + 1 - 25y^2$
(iii) $a^2 - 1 + 2x - x^2$

Q.12 Factorise:
(i) $9 - a^6 + 2a^3b^3 - b^6$
(ii) $x^{16} - y^{16} + x^8 + y^8$

Q.13 Factorzie: $(2x + 3y)^2 - 5(2x + 3y) - 14$

Q.14 Factorise: $3m^2 + 24m + 36$

Q.15 Divide:
(i) $6x^4yz - 3xy^3z + 8x^2yz^4$ by $2xyz$
(ii) $\frac{2}{3}a^2b^2c^2 + \frac{4}{3}ab^2c^3 - \frac{1}{5}ab^3c^2$ by $\frac{1}{2}abc$

Q.16 Divide the polynomial $2x^4 + 8x^3 + 7x^2 + 4x + 3$ by $x + 3$.

Q.17 Divide $10x^4 + 17x^3 - 62x^2 + 30x - 3$ by $2x^2 + 7x - 1$

Q.18 Divide $3y^5 + 6y^4 + 6y^3 + 7y^2 + 8y + 9$ by $3y^3 + 1$ and verify that
Dividend = Divisor × Quotient + Remainder

Q.19 Divide $16x^4 + 12x^3 - 10x^2 + 8x + 20$ by $4x - 3$. Also, write the quotient and remainder.

Q.20 Divide $8y^3 - 6y^2 + 4y - 1$ by $4y + 2$. Also, write the quotient and the remainder.

Q.21 Divide: $a^4 - b^4$ by $a - b$

Q.22 Divide: $x^{4a} + x^{2a}y^{2b} + 4y^{4b}$ by $x^{2a} + x^ay^b + y^{2b}$

ANSWER KEY

EXERCISE # 1

2. (i) $(x + y)(x + 3y - 1)$ (ii) $(-2x + 3y)(2a + b)$

4. (i) $(a + 2)(a + b)$ (ii) $(x + y)(x - z)$

5. (i) $(a + b)(a - 1)$ (ii) $(y + b)(x - a)$ (iii) $(b + 2c)(6a - b)$
(iv) $(a + b)(a - c)$ (v) $(x + a)(ax^2 + 1)$ (vi) $(3a + 4b)(x - 2y)$

6. (i) $(x - 2y)(x^2 + 3y^2)$ (ii) $(6a - b)(b + 2c)$

7. (i) $(x - y)(x + y)(x^2 + y^2)$ (ii) $(2x - 3)(2x + 3)(4x^2 + 9)$
(iii) $(x - y - z)(x + y + z)\{x^2 + (y + z)^2\}$ (iv) $2x(1 + 4x^2)(1 - 2x)(1 + 2x)$
(v) $3(a - 2b)(a + 2b)(a^2 + 4b^2)$ (vi) $x^2(9x - 11)(9x + 11)$

8. (i) $(8x - 5z - 4)(8x + 5z - 4)$ (ii) $(2a + 3b)(2a - 3b - 1)$
(iii) $(x - y)(x + y - 4)$ (iv) $3(1 + 2a - 2b)(1 - 2a + 2b)$
(v) $(x - y)(x + y + z)$ (vi) $(a + b)\{(a - b) - 1\}$

9. (i) $(2x - y + 3z)(2x - y - 3z)$ (ii) $(4 + x + y)(4 - x - y)$
(iii) $(2x^2 - 2xz + z^2)(2x - z)z$

10. (i) $(2x - 5y)^2$ (ii) $16a^2$

11. (i) $(3x + 2y)(3x - 2y)$
(ii) $(6x - 5y - 1)(6x + 5y - 1)$
(iii) $(a - 1 + x)(a + 1 - x)$

12. (i) $(a^3 - b^3 + 3)(-a^3 + b^3 + 3)$
(ii) $(x^8 + y^8)(x^8 - y^8 + 1)$
(iii) $(p + q - a + b)(p + q + a - b + 1)$

13. $(2x + 3y - 7)(2x + 3y + 2)$

14. $3(m + 2)(m + 6)$

15. (i) $3x^3 - \frac{3}{2}y^2 + 4xz^3$ (ii) $\frac{4}{3}abc + \frac{8}{3}bc^2 - \frac{2}{5}b^2c$

16. $(x + 3)(2x^3 + 2x^2 + x + 1)$

17. $(2x^2 + 7x - 1)(5x^2 - 9x + 3)$

20. $(4y + 2)\left(2y^2 - \frac{5}{2}y + \frac{9}{4}\right) - \frac{11}{2}$

21. $(a + b)(a^2 + b^2)$

22. $x^{2a} - x^a y^b + y^{2b}$

EXERCISE # 2

Q.1 If x and y are non-zero rational unequal numbers, then find the value of $\frac{(x+y)^2-(x-y)^2}{x^2y-xy^2}$

(A) $\frac{1}{xy}$ (B) $\frac{1}{x-y}$ (C) $\frac{4}{x-y}$ (D) $\frac{2}{x-y}$

Q.2 Let $\frac{a}{b}-\frac{b}{a}=x:y$. If $(x-y)=\left(\frac{a}{b}+\frac{b}{a}\right)$, then find the value of x -

(A) $\frac{a+b}{a}$ (B) $\frac{a+b}{b}$

(C) $\frac{a-b}{a}$ (D) None of these

Q.3 If (x – 2) is a factor of $(x^2 + 3qx - 2q)$, then find the value of q.

Q.4 If $x^3 + 6x^2 + 4x + k$ is exactly divisible by (x + 2), then find the value of k.

Q.5 Let $f(x) = x^3 - 6x^2 + 11x - 6$. Then, which one of the following is not factor of f(x) ?

(A) x – 1 (B) x – 2 (C) x + 3 (D) x – 3

Q.6 The polynomial $(x^4 - 5x^3 + 5x^2 - 10x + 24)$ has a factor as -

(A) x + 4 (B) x – 2

(C) x + 2 (D) None of these

Q.7 $(x^{29} - x^{25} + x^{13} - 1)$ is divisible by -

(A) both (x – 1) & (x + 1)

(B) (x – 1) but not by (x + 1)

(C) (x + 1) but not by (x – 1)

(D) neither (x – 1) nor (x + 1)

Q.8 Value of k for which (x – 1) is a factor of $(x^3 - k)$.

Q.9 Find the factors of $(8x^3 - 27y^3)$ -

(A) $(2x - 3y)(4x^2 + 9y^2 - 6xy)$

(B) $(2x - 3y)(4x^2 + 9y^2 + 6xy)$

(C) $(2x - 3y)(4x^2 - 9y^2 - 6xy)$

(D) $(2x - 3y)(4x^2 - 9y^2 + 6xy)$

Q.10 Find the factors of $(x^3 + y^3 + 2x^2 - 2y^2)$.

Q.11 Find the factors of $(x^3 - 5x^2 + 8x - 4)$.

Q.12 Find the factors of $(x^4 + 4)$.

Q.13 Find the factors of $(x + y)^3 - (x - y)^3$.

Q.14 If $(x^5 - 9x^2 + 12x - 14)$ is divided by (x – 3), then find the remainder.

Q.15 If $(x^{11} + 1)$ is divided by (x + 1), then find the remainder.

Q.16 Find the value of expression $(16x^2 + 24x + 9)$ for $x = -\frac{3}{4}$.

Q.17 Find the sum of $(x^2 + 1)$ and the reciprocal of $(x^2 - 1)$.

Q.18 Find the factors of $(x^2 - 1 - 2a - a^2)$.

Q.19 Find the factors of $(x^2 - 8x - 20)$.

Q.20 Find the factors of $(x^2 - xy - 72y^2)$.

Q.21 Find the factors of $(x^2 - 11xy - 60y^2)$.

Q.22 Find the factors of $(x^4 + x^2 + 25)$.

ANSWER KEY

EXERCISE # 2

1. $\frac{4}{x - y}$
2. None of these
3. -1
4. -8
5. $x + 3$
6. $x - 2$
7. $(x - 1)$ but not by $(x + 1)$
8. 1
9. $(2x - 3y)(4x^2 + 9y^2 + 6xy)$
10. $(x + y)(x^2 + y^2 + xy + 2x - 2y)$
11. $(x - 2)^2 (x - 1)$
12. $(x^2 + 2x + 2)(x^2 - 2x + 2)$
13. $2y(3x^2 + y^2)$
14. 184
15. 0
16. 0
17. $\frac{x^4}{x^2 - 1}$
18. $(x + a + 1)(x - a - 1)$
19. $(x - 10)(x + 2)$
20. $(x - 9y)(x + 8y)$
21. $(x - 15y)(x + 4y)$
22. $(x^2 + 5 + 3x)(x^2 + 5 - 3x)$