

Ionic Equilibrium

CONCEPT OF ACIDS AND BASES

Section - 1

Concept of Acids and Bases

Three main concepts in defining the acids and bases are :

1. The Arrhenius concept
2. Bronsted-Lowry concept
3. Lewis concept

1. The Arrhenius concept :

According to this concept, an acid and a base can be defined as follows :

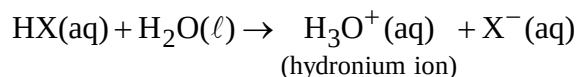
Acid : It is a substance that produces hydrogen ions (H^+) in water.

Base : It is a substance that produces hydroxyl ions (OH^-) in water.

HCl , H_2SO_4 , HNO_3 etc. are the examples of Arrhenius acids while NaOH , KOH , $\text{Mg}(\text{OH})_2$ etc. are the examples of Arrhenius bases.

Let us represent an acid as HX and a base as BOH .

The ionization of acid as (HX) can be represented by the following equation :



or simply, $\text{HX}(\text{aq}) \rightarrow \text{H}^+(\text{aq}) + \text{X}^-(\text{aq})$

Hydronium ion (H_3O^+) is used to represent a hydrated H^+ ion i.e., H^+ , ion surrounded by water molecules.

Note : The Arrhenius concept of acids and bases is only limited to aqueous solutions.

Limitations of Arrhenius Concept

- (i) It is applicable only to aqueous solutions. For the acidic or basic properties, the presence of water is absolutely necessary. Dry HCl shall not act as an acid.
- (ii) The concept does not explain acidic or basic properties of acids and bases in non-aqueous solvents respectively.
- (iii) It fails to explain the acidic nature of the non-protic compounds such as SO_2 , NO_2 , CO_2 , P_2O_5 etc. which do not have OH in the molecules to furnish H^+ ions.

- (iv) It fails to explain the basic nature of compounds like NH_3 , Na_2CO_3 etc. which do not have OH in the molecules to furnish OH^- ions.
- (v) It fails to explain the acidic nature of certain salts such as AlCl_3 in aqueous solution.

2. Bronsted-Lowry Acids and Bases

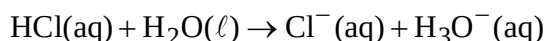
According to this concept, an acid and a base can be defined as follows :

Acid : It is a substance that can donate a proton.

Base : It is a substance that can accept a proton.

Example :

When HCl is dissolved in water, it donates a proton to H_2O which behaves as a base.



Other examples of Bronsted Lowry acids (underlined) are :

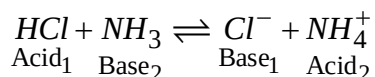
- (a) $\underline{\text{NH}_4^+}(\text{aq}) + \text{H}_2\text{O}(\ell) \rightleftharpoons \text{NH}_3(\text{aq}) + \text{H}_3\text{O}^+(\text{aq})$
- (b) $\underline{\text{HSO}_4^-}(\text{aq}) + \text{H}_2\text{O}(\ell) \rightarrow \text{SO}_4^{2-}(\text{aq}) + \text{H}_3\text{O}^+(\text{aq})$
- (c) $\underline{\text{HCl}}(\text{aq}) + \text{NH}_3(\text{aq}) \rightleftharpoons \text{Cl}^-(\text{aq}) + \text{NH}_4^+(\text{aq})$
- (d) $\underline{\text{HCO}_3^-}(\text{aq}) + \text{NH}_3(\text{aq}) \rightleftharpoons \text{CO}_3^{2-}(\text{aq}) + \text{NH}_4^+(\text{aq})$

Some examples of Bronsted Lowry bases (underlined) are :

- (a) $\underline{\text{O}^{2-}}(\text{aq}) + \text{H}_2\text{O}(\ell) \rightleftharpoons \text{OH}^-(\text{aq}) + \text{OH}^-(\text{aq})$
- (b) $\underline{\text{NH}_3}(\text{aq}) + \text{H}_2\text{O}(\ell) \rightleftharpoons \text{NH}_4^+(\text{aq}) + \text{OH}^-(\text{aq})$
- (c) $\underline{\text{CO}_3^{2-}}(\text{aq}) + \text{H}_2\text{O}(\ell) \rightleftharpoons \text{HCO}_3^-(\text{aq}) + \text{OH}^-(\text{aq})$

- Note :**
- This theory requires that an acid must contain transferable hydrogen and it offers great freedom in defining what constitutes base.
 - When an acid has donated its proton, the remaining portion of the molecule or ion is a base.
 - When a base accepts a proton, it forms an acid.
 - The base must have an unshared pair of electrons so as to accept a proton.

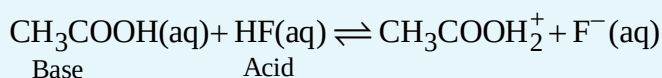
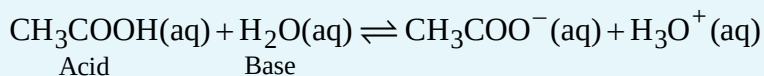
The base formed from an acid is known as the *conjugate base of the acid*. Correspondingly, the acid formed from a base is called the *conjugate acid of the base*.



In the above reaction, Cl^- is the conjugate base of HCl and NH_4^+ is the conjugate acid NH_3 .

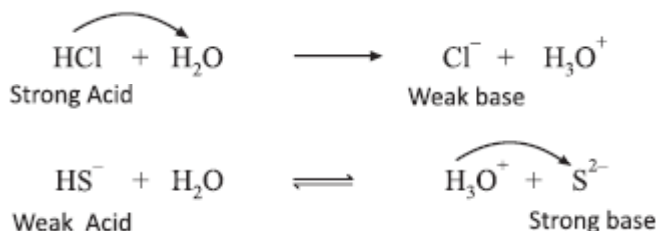
- Note :**
- (i) The conjugate acid always has one more proton than its conjugate base.
 - (ii) To obtain a conjugate acid of a compound, add a proton (H^+) from it and to obtain the conjugate base of the same, remove a proton to the compound, e.g., Conjugate acid of NH_3 is NH_4^+ while the conjugate base of NH_3 is NH_2^- .
 - (iii) This concept has the advantage that it can be applied to a solvent other than water, having the tendency to accept or lose a proton.
 - (iv) The terms acid and base are comparative. A substance can behave as an acid in one solvent and as a base in another.

For example : Acetic acid (CH_3COOH) behaves as an acid in water but as a base in HF.



Strength of Bronsted-Lowry Acid and Bases :

The strength of an acid or base is measured by its tendency to lose or gain proton. A strong acid is a substance which loses a proton easily to a base. Consequently, the conjugate base of a strong is a weak base.



The ability of an acid to lose a proton is experimentally measured by its equilibrium constant known as K_a . The larger the value of K_a , the more complete reaction or higher the concentration of H_3O^+ and the stronger is the acid. Similarly: For bases, we have the equilibrium constant, K_b which determines the extent of the completion of the reaction.

Acid-Base Chart Containing Some Common Conjugate Acid-Base Pairs

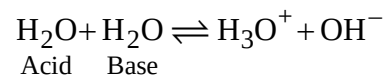
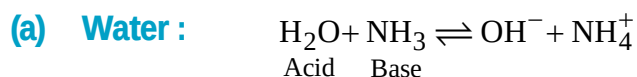
	Acid	Conjugate Base	K_a	pK_a
Increasing order of acid strength ↑	HI	I^-	10^{11}	-11
	$HClO_4$	ClO_4^-	10^{10}	-10
	HBr	Br^-	10^9	-9
	HCl	Cl^-	10^7	-7
	H_2SO_4	HSO_4^-	10^2	-2
	H_3O^+	H_2O	1	0.0
	H_2SO_3	HSO_3^-	1.5×10^{-2}	1.81
	HSO_4^-	SO_4^{2-}	1.2×10^{-2}	1.92
	H_3PO_4	$H_2PO_4^-$	7.5×10^{-3}	2.12
	HF	F^-	3.5×10^{-4}	3.45
↑ Increasing order of basic strength	CH_3COOH	CH_3COO^-	1.8×10^{-5}	4.7
	H_2S	HS^-	1.0×10^{-7}	7.0
	NH_4^+	NH_3	5.6×10^{-10}	9.3
	HCN	CN^-	4.0×10^{-10}	9.4
	H_2O	OH^-	1.0×10^{-14}	14.0

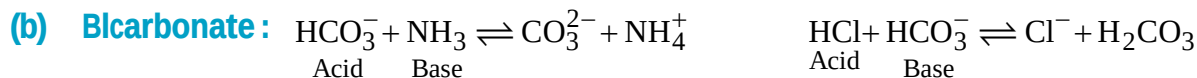
Note : In general, solvents can be of four types :

- Protophilic :** Solvents having a tendency to accept protons. For example water, alcohol, liq. Ammonia etc.
- Protogenic :** Solvents having a tendency to donate protons. For example water, liq. HF, liq. HCl etc.
- Amphiprotic :** Solvents having a tendency to accept or donate. For example water, liq. ammonia etc.
- Aprotic :** Solvents which neither accept nor donate protons. For example benzene, carbon tetrachloride etc.

Amphoteric Compounds :

The compounds which can act either as acids or as bases, H_2O , NH_3 and CH_3COOH are some of the examples.

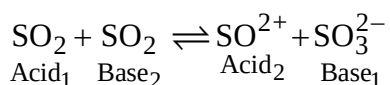




Note : The reaction $\text{H}_2\text{O} + \text{H}_2\text{O} \rightleftharpoons \text{H}_3\text{O}^+ + \text{OH}^-$ is known as *auto-ionization of water*.

Limitations of Bronsted Concept :

- (i) A substance is termed as an acid or base if it reacts with some other substance, i.e., if it donates proton to other substance, it is an acid and if it accepts proton from substance, it is a base.
- (ii) There are number of acid-base reaction in which no proton transfer takes place, e.g.,

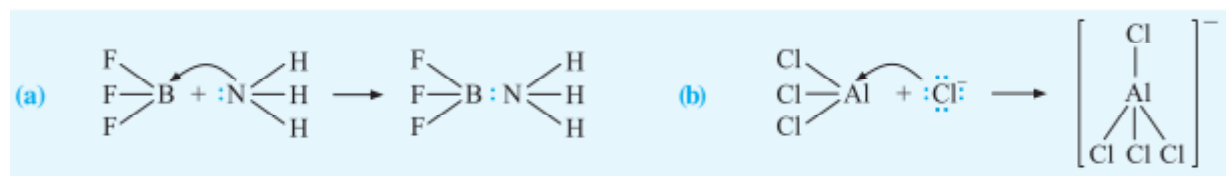


Thus, the protonic definition cannot be used to explain the reaction occurring in nonprotonic solvents such as COCl_2 , SO_2 , N_2O_4 etc.

3. Lewis Acid and Bases

Acid : It is a substance that can form a covalent bond by accepting a shared pair of electrons.

Base : It is a substance that possess at least one unshared pair of electrons.



Monoprotic Acids : Acids that give up one proton per molecule.

Polyprotic Acids : Acids that can give up more than one proton per molecule.

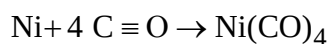
Substance that are bases in the Bronsted system are also bases according to the Lewis concept. However the Lewis definition of an acid considerably expands that number of substances that are classified as acid. A Lewis acid must have an empty orbital capable of receiving the electron pair of the base.

Lewis acids include molecules or atoms that have incomplete octets. For example molecules like BF_3 , AlCl_3 etc., act as Lewis Acid.

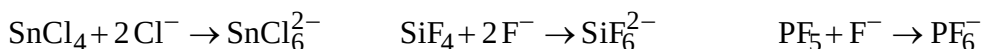
Many simple cations can act as Lewis acids :



Some metal atoms can function as acids in the formation of compounds such as:



Compounds that have central atoms capable of expanding their valence shells are Lewis acids in reactions in which this expansion occurs.



Some compounds have an acidic site because of one or more multiple bonds in the molecule.

BASICS OF IONIC EQUILIBRIUM

Section - 2

The type of equilibrium studied in *Chemical Equilibrium* is known as *molecular equilibrium* (the involvement of molecules only). Now, we will study reversible reactions involving formation of ions in water under *Ionic Equilibrium*.

Ionic Equilibrium is the study of equilibrium in the reactions where formation of ions takes place in aqueous solution.

Ions in solution arises mainly from two ways :

- When solute is ionic compound i.e., strong electrolyte (*a compound whose aqueous solution conducts electricity*) e.g. NaCl, MgCl₂, KBr, NaNO₃, NaOH etc. In such compounds, ions maintain their identity even in solid state. As such solutes are dissolved in water, ions get separated. In solution, whole of electrolyte is ionised i.e., ionisation is almost 100% complete.
- When solute is polar covalent compound and reacts with water to form ions. These may be *strong or weak electrolytes* depending upon their respective degree of ionizations (α) e.g., HCl, NH₃, H₂SO₄, HNO₃, CH₃COOH, HCN, NH₄OH etc.

Dissociation (or ionisation) of *strong electrolytes* and some *weak electrolytes* such as HNO₃, HCl, H₂SO₄ etc. is nearly complete in dilute solution i.e. almost 100% of electrolyte is converted to ions *or* one can say that degree of ionisation (α) is nearly 1.

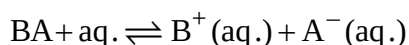
Let BA be an electrolyte and dissolved in water. It ionises as follows : $\text{BA} + \text{aq.} \rightleftharpoons \text{B}^+(\text{aq.}) + \text{A}^-(\text{aq.})$

Case - I : ($\alpha \sim 1$)

The extent of forward reaction is very high (i.e. such reactions are nearly complete). So, it is of no use to study equilibrium in such reactions.

Case - II : ($\alpha \ll 1$)

In aqueous solution of these compounds, only slight amount of reactant (i.e. electrolyte) undergoes dissociation and there exists an equilibrium between ionised molecules and unionized molecules (ions). Such electrolytes are weak electrolytes.



unionised molecules $\rightleftharpoons$ ionised molecules (ions)

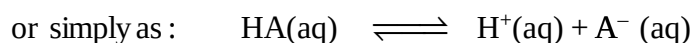
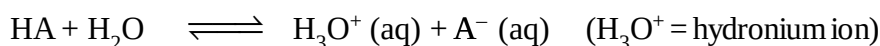
Weak Electrolytes are categorized into three types for easy understanding :

- Weak Acids :** CH_3COOH , HCN (and all organic acids)
- Weak Bases :** NH_4OH and all organic bases
- Sparingly soluble salts :** AgCl , PbCl_2 , $\text{Ag}_2\text{C}_2\text{O}_4$, MnS , H_2S , PbS etc.

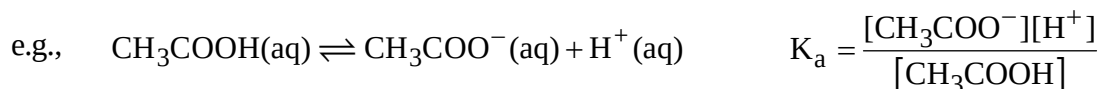
Weak Acids (HA) and Weak Bases (BOH)

Weak Acids :

Let HA be a monoprotic (monobasic) acid whose equilibrium is to be studied. The aqueous solution of HA can be studied in either of two ways :



The equilibrium constant for the above reaction is known as *ionisation constant* for an acid (K_a).
[] = concentration in mol/L



Note : The ion A^- formed due to the ionization of HA is known as its Conjugate base. Thus, to find a conjugate base of any acid, simply remove a Proton (H^+) ion from that acid.

$\Rightarrow$ Conjugate base of CH_3COOH is CH_3COO^- ; of H_2PO_4^- is HPO_4^{2-} and so on.

Lets study the equilibrium of a weak acid HA in an aqueous solution as follows :

If 'c' be the molar concentration of weak acid HA and ' α ' be its degree of dissociation (i.e. the fraction of total concentration which exist in ionic state), then : $\text{HA}(\text{aq}) \rightleftharpoons \text{H}^+ (\text{aq}) + \text{A}^- (\text{aq})$

Moles	HA	H^+	A^-
Initial	c	0	0
at equilibrium	$c - c\alpha$	$c\alpha$	$c\alpha$

$$\Rightarrow K_a = \frac{[\text{H}^+][\text{A}^-]}{[\text{HA}]} = \frac{(c\alpha)(c\alpha)}{(c - c\alpha)} = \frac{c\alpha^2}{1 - \alpha}$$

Also, $K_a = \frac{[\text{H}^+][\text{A}^-]}{[\text{HA}]} = \frac{[\text{H}^+]^2}{c - [\text{H}^+]}$ ($\because [\text{H}^+] = [\text{A}^-] = c\alpha$)

Ionic Equilibrium

For weak acids having $\alpha \ll 1$, we can take $1 - \alpha \approx 1$

$$\Rightarrow K_a = c \alpha^2 \quad \Rightarrow \quad \alpha = \sqrt{\frac{K_a}{c}}$$

- Note :** (i) In general, if $K_a < 10^{-5}$ and 'c' is fairly high (generally 0.1 M or 0.01 M) then $1 - \alpha$ can be taken as unity.
If c is very low (i.e. for very dilute solutions), α is higher. In that case, we can't take $1 - \alpha$ as 1. In such cases we have to solve a quadratic equation in α . i.e. $(1 - \alpha) K_a = c \alpha^2$.
- (ii) Basically, if decision on whether to go for approximation or not, is not easy, then just use approximation and solve but remember that you should always validate your approximation before reaching to a final answer. We will see more on this in illustrations and examples.

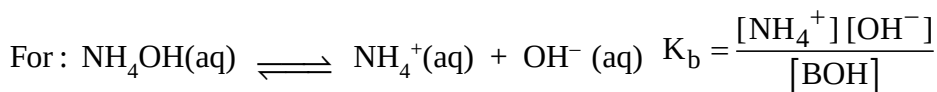
Weak Bases :

Let BOH be the weak base, its aqueous solution is represented as follows :



The equilibrium constant for the above equilibrium is known as ionisation constant of a base (K_b).

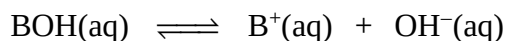
$$K_b = \frac{[\text{B}^+][\text{OH}^-]}{[\text{BOH}]} \quad [] = \text{concentration in mol/L}$$



- Note :** Similar to the concept of Conjugate base, there is a concept of Conjugate acid. e.g. NH_4^+ is a conjugate acid of base NH_3 . Thus, to find a conjugate acid of any base, simply add a Proton (H^+) ion to that.
- H_2PO_4^- : Its conjugate base is $\text{H}_2\text{PO}_4^{2-}$ and its conjugate acid is H_3PO_4
 - H_2O : Its Conjugate base is OH^- and its conjugate acid is H_3O^+

A substance is known as amphiprotic if it can both donate or accept a proton e.g. , H_2O

Now, if 'c' be the molar concentration of BOH and ' α ' be its degree of dissociation, then :



Moles	BOH	B ⁺	OH ⁻
Initial	c	0	0
at equilibrium	c - cα	cα	cα

$$\Rightarrow K_b = \frac{[\text{B}^+][\text{OH}^-]}{[\text{BOH}]} = \frac{(c\alpha)(c\alpha)}{(c - c\alpha)} = \frac{c\alpha^2}{1 - \alpha}$$

$$\text{Also, } K_b = \frac{[B^+][OH^-]}{[BOH]} = \frac{[OH^-]^2}{c - [OH^-]} \quad (\because [B^+] = [OH^-] = c\alpha)$$

For weak bases having $\alpha \ll 1$, $1 - \alpha \approx 1$

$$\Rightarrow K_b = c \alpha^2 \quad \text{or} \quad \alpha = \sqrt{\frac{K_b}{c}}$$

Note : At a given temperature, K_a and K_b are directly proportional to α i.e. comparing K_a of different acids (K_b for bases), one can compare their strengths or comparing ' α ' of different acids (or bases) at a given temperature and given concentrations, acids strengths (base strengths) can be compared.

For example in case of bases :

$$\frac{\alpha_1}{\alpha_2} = \sqrt{\frac{K_{b1}}{K_{b2}}} \quad (\text{two bases having same concentrations})$$

$$\text{or } K_b = c_1 \alpha_1^2 = c_2 \alpha_2^2 \quad (\text{two samples of same base have same value of } K_b)$$

$$\Rightarrow \frac{\alpha_1}{\alpha_2} = \sqrt{\frac{c_2}{c_1}}$$

In the above relations, it has been assumed that the degree of dissociation is small as compared to unity.

Self Ionisation of Water

Water ionises as follows : $H_2O(\ell) \rightleftharpoons H^+(aq) + OH^-(aq)$

The equilibrium constant here is defined in a different way, and is called as ionic product (K_w) of water and is given by :

$$\Rightarrow K_{w0} = [H^+][OH^-] \quad \text{where } K_w = K_a[H_2O]$$

K_a : ionisation constant of H_2O and $[H_2O]$ is constant at a given temperature

At 25°C (298 K) ; $K_w = 1.0 \times 10^{-14}$

Since pure water is neutral, $[H^+] = [OH^-] = \sqrt{K_w} = 10^{-7} \text{ M}$ at 25°C .

- If a strong acid is added to it, $[H^+]$ increases and hence $[OH^-] < 10^{-7} \text{ M}$ (at 25°C) and solution is said to be acidic.
- If a strong base is added to it, $[OH^-]$ increases and hence $[H^+]$ must decrease in order to keep K_w constant. Now $[OH^-] > 10^{-7} \text{ M}$ and solution is basic (or alkaline)

Note : The dissociation of water is an endothermic reaction. Thus, increasing the temperature will increase K_w of water and thereby increasing the $[OH^-]$ and $[H^+]$.

pH Scale :

Sorensen, a French chemist developed a scale to measure the acidity in terms of concentrations of H^+ in a solution. As defined by him :

pH of a solution is the negative logarithm to the base 10 of the concentration of H^+ ions which it contains.

(Basically “p” acts as a mathematical operator : $p(A) = -\log_{10} [A]$)

$$\Rightarrow \text{pH} = -\log_{10} [H^+] \quad \text{and} \quad \text{pOH} = \log_{10} [OH^-]$$

At 25°C, using this definition, a scale called pH Scale is developed as follows:

For pure H_2O ; $[H^+] = [OH^-] = 10^{-7} \text{ M}$

$$\text{pH} = -\log_{10} (10^{-7}) = 7 = \text{pOH} \quad \Rightarrow \quad \text{For pure water at } 25^\circ\text{C} ; \quad \text{pH} = \text{pOH} = 7$$

- (i) If $[H^+] > 10^{-7} \text{ M}$ in a solution i.e., solution is acidic then $\text{pH} < 7$
- (ii) If $[H^+] < 10^{-7} \text{ M}$ (or $[OH^-] > 10^{-7} \text{ M}$) i.e. solution is basic then $\text{pH} > 7$

Acidic Solution						H ₂ O	Basic Solution						
At 25°C (298K)	1	2	3	4	5	6	7	8	9	10	11	12	13
						pH							
						(Neutral)							
← acidity increases-----								----- basicity increases →					

In general, we talk about pH of a solution, whether it is basic or acidic.

For an aqueous solution : $\text{pH} + \text{pOH} = \text{pK}_w$ [Use $K_w = [H^+][OH^-]$ and take “ $\log_{10}$ on both sides]

$$\text{At } 25^\circ\text{C}, \quad \text{pH} + \text{pOH} = \text{pK}_w = 14 \quad [K_w = 1 \times 10^{-14} \text{ at } 25^\circ\text{C}]$$

Now one can very easily re-define pH of a basic solution as : $\text{pH} = 14 + \log_{10} [OH^-]$

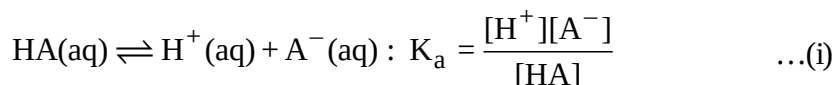
Note : “p” operator is generally applied on any variable whose value is < 1 . In most of the cases, we will use pK_w , pK_a , pK_b etc.

Also, we can see that as temperature increases, both pH and pOH of pure water will decrease since there will be an increase in $[H^+]$ and $[OH^-]$ due to increased dissociation.

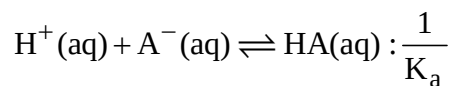
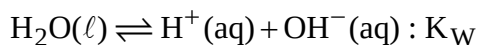
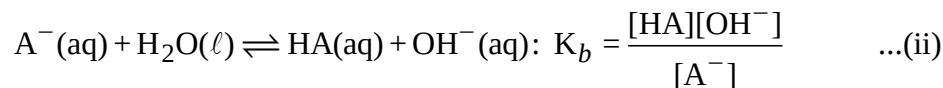
Dissociation constant (K_a and K_b) of conjugate ion of weak acid and weak base :

HA is a weak acid and A^- is its conjugate base.

Dissociation equilibrium of HA :



Dissociation equilibrium of conjugate base A^- :



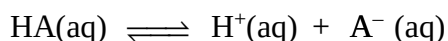
$$K_{a_{acid}} \times K_{b_{conjugate\ base}} = K_w \quad \Rightarrow \quad pK_{a(HA)} + pK_{b(A^-)} = pK_w$$

Similarly for dissociation constant of weak base (K_b) and dissociation constant of conjugate acid (K_a) we can write

$$K_{b_{weak\ base}} \times K_{a_{conjugate\ base}} = K_w \quad \Rightarrow \quad pK_{b(BOH)} + pK_{a(B^+)} = pK_w$$

pH of a Weak Acid and Weak Base :

Weak acid (HA)



For c molar HA, we have : $[H^+] = c\alpha \quad \Rightarrow \quad pH = -\log_{10} c\alpha$

$$\text{Also : } \alpha = \sqrt{\frac{K_a}{c}} \quad [\text{If the approximation } 1 - \alpha \approx 1 \text{ is valid}]$$

$$\Rightarrow \quad pH = -\log_{10} \sqrt{K_a c} = \frac{1}{2}(pK_a - \log_{10} c)$$

Weak base (BOH) :



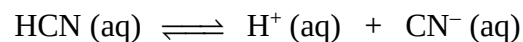
For c molar BOH, we have : $[OH^-] = c\alpha \quad \Rightarrow \quad pH = 14 + \log_{10} [OH^-] = 14 + \log_{10} c\alpha$

$$\text{Also : } \alpha = \sqrt{\frac{K_b}{c}} \quad [\text{If the approximation } 1 - \alpha \approx 1 \text{ is valid}]$$

$$\Rightarrow \quad pH = 14 + \log_{10} \sqrt{K_b c} = 14 - \frac{1}{2}(pK_b - \log_{10} c)$$

Illustration - 1 Determine the hydrogen ion concentration in 1.0 M solution of HCN, if its dissociation constant is 4.0×10^{-10}

SOLUTION :



$$\begin{array}{ccc} \text{C} & 0 & 0 \\ \text{C} - \text{C}\alpha & \text{C}\alpha & \text{C}\alpha \end{array}$$

$$[\text{H}^+] = \text{C}\alpha \text{ and } [\text{H}^+] = [\text{CN}^-]$$

$$\text{Use } K_a = \frac{[\text{H}^+]^2}{\text{C} - [\text{H}^+]} ; \text{ C} = 1.0 \text{ M}$$

Neglecting $[\text{H}^+]$ against c, as the given acid is very weak ($K_a = 4.0 \times 10^{-10}$) and c is quite high.

$$\text{i.e. } \text{C} - [\text{H}^+] \approx \text{C} = 1.0 \text{ M (given)}$$

$$\Rightarrow [\text{H}^+] = \sqrt{K_a \text{C}} = 2 \times 10^{-5} \text{ M}$$

Now, check the approximation : $\text{C} - [\text{H}^+] \approx \text{C}$

$$\Rightarrow \text{C} - [\text{H}^+] = 1 - 2 \times 10^{-5} \approx 1.0 \text{ M} = \text{C}$$

Thus, the answer obtained as above is correct.



Illustration - 2 What is the pH of a 0.2 M solution of acetic acid ? To what volume must one litre of this solution be diluted so that the pH of the resulting solution will be twice the original value ?

Given : $K_a = 2.0 \times 10^{-5}$

SOLUTION :



$$\begin{array}{ccc} \text{C} & 0 & 0 \\ \text{C} - \text{C}\alpha & \text{C}\alpha & \text{C}\alpha \end{array}$$

$$\Rightarrow K_a = \frac{[\text{H}^+]^2}{\text{C} - [\text{H}^+]} \approx \frac{[\text{H}^+]^2}{\text{C}}$$

(as K_a is too Low)

$$\text{Thus, } [\text{H}^+] = \sqrt{K_a \text{C}} = 2 \times 10^{-3} \text{ M} ;$$

$$\text{Also, } \alpha = \frac{[\text{H}^+]}{\text{C}} = 0.01 \Rightarrow \text{C} - [\text{H}^+] \approx \text{C}$$

$$\Rightarrow \text{pH} = 2.7 \quad (\text{pH} = -\log_{10}[\text{H}^+])$$

Now solution is diluted to make pH = 5.4 (twice the original)

Note : On dilution, α increases, so $1 - \alpha$ can't be taken as unity. So solve the quadratic in α .

$$\text{For pH} = 5.4, [\text{H}^+] = 4 \times 10^{-6} \text{ M}$$

(Doubling the pH value is simply equivalent to squaring the $[\text{H}^+]$)

$$\text{Using: } K_a = \frac{[\text{H}^+]^2}{\text{C} - [\text{H}^+]}$$

$$\Rightarrow \text{C} = [\text{H}^+] + \frac{[\text{H}^+]^2}{K_a} = 4.8 \times 10^{-6} \text{ M}$$

Check yourself that new degree of dissociation

$$\text{of acetic acid} = \alpha_{\text{new}} = \frac{[\text{H}^+]_{\text{new}}}{\text{C}_{\text{new}}} = 0.83$$

which clearly suggests that using approximations would have generated poor/wrong results.

Originally the solution was 0.2 M and 1 L, using molarity equation we have :

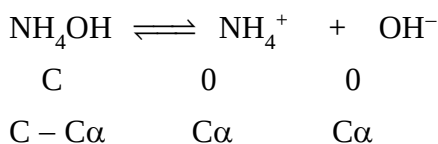
$$4.8 \times 10^{-6} \times V = 0.2 \times 1$$

$$(\text{C}_1 \text{V}_1 = \text{C}_2 \text{V}_2)$$

$$\Rightarrow V = 4.17 \times 10^4 \text{ L3}$$

Illustration - 3 Calculate $[H^+]$ and % dissociation of 0.1 M solution of ammonium hydroxide solution. The ionisation constant for NH_4OH is $K_b = 2.0 \times 10^{-5}$.

SOLUTION :



$$K_b = \frac{[NH_4^+][OH^-]}{[NH_4OH]} = \frac{C\alpha^2}{1-\alpha} \approx C\alpha^2$$

(Neglecting α , in denominator i.e. $1 - \alpha \sim 1$)

$$\Rightarrow \alpha = \sqrt{\frac{K_b}{C}} = \sqrt{2 \times 10^{-4}}$$

$$= 1.41 \times 10^{-2} = 1.41 \%$$

[Check that $1 - \alpha \sim 1$ holds]

$$\text{and } [OH^-] = C\alpha = 0.1 \times 1.41 \times 10^{-2} = 1.41 \times 10^{-3} \text{ M}$$

$$\Rightarrow [H^+] = \frac{K_w}{[OH^-]} = \frac{7.09 \times 10^{-12} \text{ M}}{1.41 \times 10^{-3} \text{ M}}$$

Illustration - 4 Calculate the pH of 10^{-8} M HCl solution.

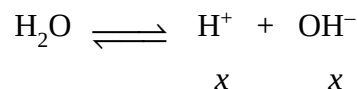
SOLUTION :

$$[HCl] = 10^{-8} \text{ M}$$

$$\Rightarrow [H^+] = 10^{-8} \text{ M}$$

$\Rightarrow \text{pH} = 8$ which is absolutely wrong since the solution must be acidic as it is an acidic solution i.e., $\text{pH} < 7$

The catch here is that since HCl has been added in water (which also dissociates) so we need to consider $[H^+]$ from dissociation of H_2O as well. In the earlier illustrations, we didn't consider this because $[H^+]$ from acid comes out to be much higher than that furnished by water but in this case we can not neglect the water contribution.



$$\Rightarrow K_w = [H^+][OH^-] = 10^{-14}$$

$$[H^+]_{\text{total}} = [H^+]_{\text{acid}} + [H^+]_{\text{water}} = 10^{-8} + x$$

$$\Rightarrow K_w = (x + 10^{-8})x = 10^{-14}$$

$$\Rightarrow x^2 + 10^{-8}x - 10^{-14} = 0$$

Solve the above quadratic equation to get :

$$x = 9.52 \times 10^{-8} \text{ M}$$

$$\text{Now, } [H^+]_{\text{total}} = 10^{-8} + 9.52 \times 10^{-8}$$

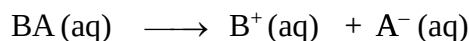
$$= 1.05 \times 10^{-7} \text{ M}$$

$$\Rightarrow \text{pH} = -\log_{10} [1.05 \times 10^{-7}] = 6.978$$

The addition of an ionic salt having a common ion (anion or cation) to weak acids or weak bases, suppresses their degree of dissociation (following *LeChatelier's principle*).

Weak Acids :

Let HA be a weak acid (like CH_3COOH , HCN etc) and $\text{B}^+ \text{A}^-$ be the ionic salt (100% dissociation in solution) containing common anion (A^-) that is added to the acid.



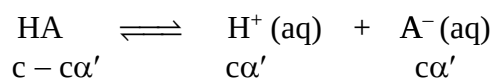
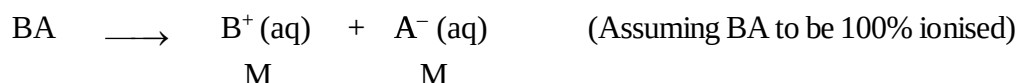
Now, the solution has excess of A^- ions. This means increasing concentration of products, in an equilibrium state, (following *LeChatelier's principle*) the reaction ($\text{HA} \rightleftharpoons \text{H}^+ + \text{A}^-$) must go in backward direction, in order to nullify the effect of added A^- ions. As a consequence, amount of H^+ in new equilibrium state will be less than before, or one can see that the degree of dissociation of acid (HA) is decreased.

Quantitative Aspect :

Consider a weak acid HA which dissociates as : $\text{HA}(\text{aq}) \rightleftharpoons \text{H}^+(\text{aq}) + \text{A}^-(\text{aq})$

Its degree of dissociation $= \alpha = \sqrt{\frac{K_a}{c}}$ [Assuming K_a to be small]

Let M molar BA be added to it and α' be its new degree of dissociation



Now in solution : $[\text{HA}] = c - c\alpha'$; $[\text{H}^+] = c\alpha'$

$$[\text{A}^-]_{\text{total}} = [\text{A}^-]_{\text{From HA}} + [\text{A}^-]_{\text{From BA}} = c\alpha' + M$$

$$K_a = \frac{[\text{H}^+][\text{A}^-]}{[\text{HA}]} = \frac{(c\alpha')(c\alpha' + M)}{(c - c\alpha')} = \frac{\alpha'(c\alpha' + M)}{1 - \alpha'}$$

$$\Rightarrow K_a = c\alpha'^2 + M\alpha' \quad (\text{Assuming } 1 - \alpha' \approx 1)$$

$$\Rightarrow K_a = M\alpha' \quad (\text{Neglecting } c\alpha'^2 \text{ in comparison to } M\alpha' \text{ as } \alpha' \ll 1 \Rightarrow \alpha'^2 \ll \alpha')$$

$$\Rightarrow \alpha' = \frac{K_a}{M}$$

By looking at the expressions of α and α' , we can clearly figure out that $\alpha' \gg \alpha$

Note : On similar lines, you can find α' for a weak base, BOH and adding B^+ ions to it.

Also, expression for K_a becomes : $K_a = \frac{[H^+][A^-]_{\text{ext.}}}{[HA]}$ where $[A^-]_{\text{ext.}}$ is the externally added salt

Buffer Solutions

A solution whose pH does not change very much when H^+ (H_3O^+) or OH^- are added to it is referred to as a *buffer solution*.

A *buffer solution* is prepared by mixing a weak acid and its salt having common anion (i.e. $HA + BA$ forms an acidic Buffer) or a weak base and its salt having common cation (i.e. $BOH + BA$ forms a Basic Buffer).

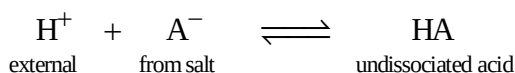
It can be prepared to have a desired value of pH by controlling the amounts of acids and their salts in case of acidic buffer and of bases and their salts in basic buffer.

Acidic buffer : $CH_3COOH + CH_3COONa$, $HCN + NaCN$

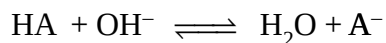
Basic buffer : $NH_4OH + NH_4Cl$

Note : See yourself that buffer solutions are actually conjugate acid-base pairs.

Consider an *acidic buffer* containing an acid HA and say common ions A^- . Now, any H^+ (or H_3O^+) added externally to this solution with in certain limits are neutralized by A^- ions as :



While, addition of OH^- ions externally (with in certain limits) are neutralised by acid HA as :



Hence in both the cases, effect of addition of H^+ or OH^- is almost compensated for (i.e. pH almost remains constant).

Such a system (may be acidic or basic) finds enormous use not only in industrial processes but also (most importantly) in biological reactions. Like the pH of normal blood is 7.4 and for good health and even for the survival, it should not change below 7.1 or greater than 7.7, the body maintains it through a buffer system made of carbonate and bicarbonate ions and $H_2PO_4^-$ and HPO_4^{2-} . Similarly, the pH of gastric juice is kept constant in order to operate good digestive functions.

pH of an Acidic Buffer

If [acid] = concentration of a weak acid and [salt] = concentration of the salt that is mixed with acid to make the buffer

We have : $[H^+] = \frac{K_a [HA]}{[A^-]_{\text{ext.}}} = \frac{K_a [\text{acid}]}{[\text{salt}]}$ [See the derivation in the Common Ion effect section]

$$\Rightarrow \text{pH} = \text{p}K_a + \log_{10} \frac{[\text{salt}]}{[\text{acid}]} \quad (\text{p}K_a = -\log_{10} K_a)$$

pH of a Basic Buffer

We have : $\text{pH} = 14 - \text{pOH}$

If [base] = concentration of weak base and [salt] = concentration of salt that is mixed to make the buffer

$$\Rightarrow [\text{OH}^-] = \frac{K_b [\text{BOH}]}{[\text{B}^+]_{\text{ext.}}} = \frac{K_b [\text{base}]}{[\text{salt}]}$$

$$\Rightarrow \text{pOH} = \text{p}K_b + \log_{10} \frac{[\text{salt}]}{[\text{base}]} \quad (\text{p}K_b = -\log_{10} K_b)$$

$$\Rightarrow \text{pH} = 14 - \text{p}K_b - \log_{10} \frac{[\text{salt}]}{[\text{base}]}$$

Note : The above equations representing the pH of a buffer are known as *Henderson's Equations*.

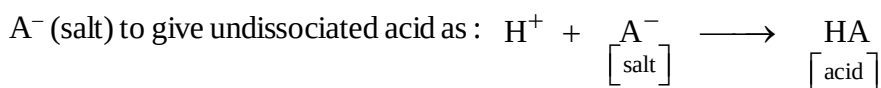
pH of a Buffer (When an acid or a base is added)

We have just discussed that addition of H^+ ions or OH^- ions to an acidic buffer (HA/A^-) does not appreciably changes the pH of buffer. Similarly, we can analyse the same for a basic buffer (BOH/B^+).

In actual, pH of a buffer solution changes by a small quantity. Let us calculate this change in pH quantitatively.

➤ Consider an acidic buffer HA/A^- where [salt] = $[\text{A}^-]$ and [acid] = $[\text{HA}]$

An acidic buffer is rich in A^- ions. Let us add x mole per litre of HCl to it. This added HCl (H^+) reacts with



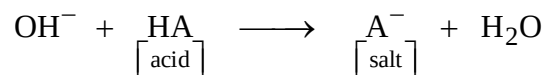
After adding $x \text{ M H}^+$ ions : $[\text{salt} - x]$ $[\text{acid} + x]$

$$\begin{aligned} \text{Now using Henderson's Equation : } \text{pH (original buffer)} &= \text{p}K_a + \log_{10} \frac{[\text{salt}]}{[\text{acid}]} \text{ and pH (new)} \\ &= \text{p}K_a + \log_{10} \frac{[\text{salt} - x]}{[\text{acid} + x]} \end{aligned}$$

$\Rightarrow$ pH of buffer decreases.

Change or difference in pH = pH (new) – pH (original)

Let us add x M NaOH to the buffer. This added NaOH (OH^- ions) react with acid (HA) to produce salt and H_2O .



After adding x M OH^- ions : [acid - x] [salt + x]

$$\Rightarrow \text{pH (new)} = \text{pK}_a + \log_{10} \frac{[\text{salt} + x]}{[\text{acid} - x]} \Rightarrow \text{pH of buffer increases.}$$

Change in pH = pH (new) - pH (original)

Note : (i) In exactly similar manner, we can calculate the change in pH of a basic buffer (BOH/B^+). Try to get a relation like this for basic buffer. Remember, it is not to be used as standard result.

(ii) A buffer solution is assumed to be destroyed if on addition of strong acid or base, its pH changes by ± 1 unit i.e.,
 $\text{pH (new)} = \text{pK}_a \pm 1$ [if the initial pH of the buffer solution was pK_a].

This means the ratio $\frac{[\text{salt}]}{[\text{acid}]}$ OR $\frac{[\text{salt}]}{[\text{base}]} = 10$ or $1/10$.

Illustration - 5

To 1.0 L of a decimolar solution of acetic acid, how much dry sodium acetate be added (in moles) so as to decrease the concentration of H^+ ion to 1/10th of its previous value ? $K_a = 2.0 \times 10^{-5}$.

SOLUTION :



$$\Rightarrow [\text{H}^+] = c \alpha = \sqrt{K_a c} = \sqrt{2.0 \times 10^{-5} \times 0.1} = 1.41 \times 10^{-3} \text{ M} \quad [\text{Check yourself that approximations are valid}]$$

Note that when salt, CH_3COONa is added, the solution will behave just like an acidic Buffer solution.

$$\text{Using, Henderson equation : } \text{pH} = \text{pK}_a + \log_{10} \frac{[\text{salt}]}{[\text{acid}]} \Rightarrow [\text{H}^+] = K_a \frac{[\text{acid}]}{[\text{salt}]} \text{ or } [\text{salt}] = K_a \frac{[\text{acid}]}{[\text{H}^+]}$$

$$\text{Now, } [\text{H}^+]_{\text{new}} = \frac{1}{10} \times 1.41 \times 10^{-3} \text{ M} = 1.41 \times 10^{-4} \text{ M}$$

$$\Rightarrow [\text{salt}] = \frac{(2.0 \times 10^{-5})(0.1)}{1.41 \times 10^{-4}} = 0.0142 \text{ M}$$

$$\Rightarrow \text{moles of salt, sodium acetate} = 0.0142 (\equiv 14.2 \text{ mmol}) \text{ per 1.0 L is required. (V = 1.0 L)}$$

Illustration - 6

(a) A buffer solution of pH value = 4 is to be prepared, using CH_3COOH and H_3COONa . How much amount of sodium acetate is to be added to 1.0 L of M/10 acetic acid ? $K_a = 2.0 \times 10^{-5}$.

(b) What will be the pH if 0.01 mol of HCl is dissolved in the above Buffer solution ? Find the change in pH value.

(c) How will the pH be affected if 1.5 L of H_2O is added to above buffer ?

SOLUTION :

(a) For an acidic buffer containing acetic acid, CH_3COOH and sodium acetate, CH_3COONa , we have :

$$[\text{H}^+] = \frac{K_a [\text{acid}]}{[\text{salt}]}$$

Use this rather than: $\text{pH} = \text{p}K_a + \log_{10} \frac{[\text{salt}]}{[\text{acid}]}$

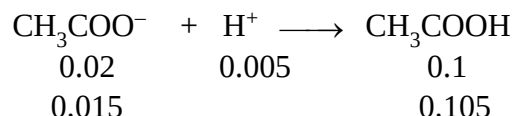
$[\text{CH}_3\text{COOH}] = 0.1 \text{ M}$;

$[\text{H}^+] = 10^{-4} \text{ M}$ and let $[\text{CH}_3\text{COONa}] = x \text{ M}$

$$\Rightarrow [\text{H}^+] = 10^{-4} = \frac{2 \times 10^{-5} \times 0.1}{x}$$

$\Rightarrow x = 0.02 \text{ moles}$ i.e. 0.02 moles of CH_3COONa is required.

(b) Now 0.01 mol of HCl ($[\text{H}^+] = 0.01 \text{ M}$) is added to 1 L of buffer, this will react with acetate ion (CH_3COO^-) as :



$\Rightarrow [\text{acid}] = 0.11 \text{ M}$ and $[\text{salt}] = 0.01 \text{ M}$ [V = 1.0 L]

$$\Rightarrow \text{pH}_{\text{new}} = \text{p}K_a + \log_{10} \frac{0.015}{0.105}$$

$= 4.7 + (-0.84) = 3.86$ [Use $\log_{10} 7 = 1.0.84$]

$\Rightarrow \text{Change in pH} = 3.86 - 4.0 = -0.14$

(c) Now, if 1.5 L of H_2O is added, it just increases the volume, thereby decreasing concentrations of both acid and salt. The ratio $\log_{10} \frac{[\text{salt}]}{[\text{acid}]}$ remains constant. Hence pH remains same.

Illustration - 7

How many of the following combination can act as buffer.

(i) $\text{HCl} + \text{NaOH}$

(ii) $\text{HCl} + \text{CH}_3\text{COO}^- \text{Na}^+$

(iii) $\text{H}_2\text{SO}_4 + \text{NaHSO}_4$

(iv) $\text{H}_2\text{CO}_3 + \text{NaOH}$

(v) $\text{NaOH} + \text{PhCOOH}$

(vi) $\text{HBr} + \text{NH}_4\text{OH}$

(vii) $\text{CH}_3\text{COOH} + \text{NH}_4\text{OH}$

(viii) $\text{NaOH} + \text{NH}_4\text{OH}$

(ix) $\text{HCl} + \text{CH}_3\text{COOH}$

(x) $\text{Borax}(\text{Na}_2\text{B}_4\text{O}_7) + \text{Boric acid}(\text{H}_3\text{BO}_3)$

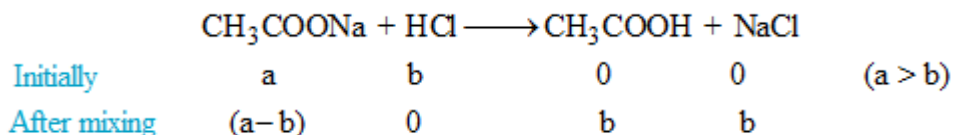
(xi) $\text{NaH}_2\text{PO}_4 + \text{Na}_2\text{HPO}_4$

(xii) $\text{Na}_2\text{CO}_3 + \text{NaHCO}_3$

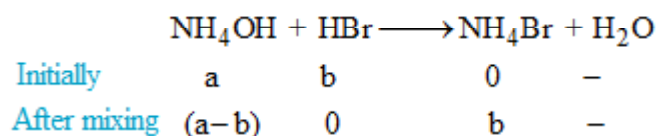
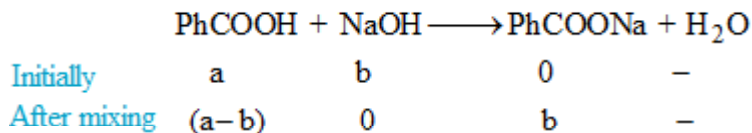
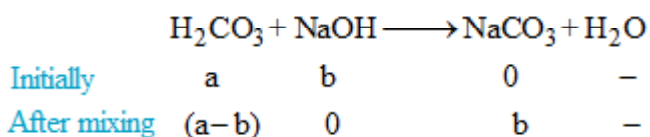
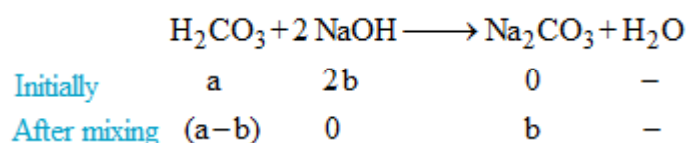
SOLUTION :

As we know that buffer is a mixture having weak acid and its salt with strong base or weak base and its salt with strong acid. Further a mixture of acid and base may also act as buffer depending on the nature of acid and base (strong or weak) and their moles taken for preparation of mixture,

because acid and base reacts to form salt. The combination (i) and (iii) can't acts as buffer because it does not contain weak electrolyte (weak acid or base) but combination (ii) can act as buffer if axcess of CH_3COONa is mixed with limited amount of HCl .



Resulting mixture contains weak acid (CH_3COOH) and its salt with strong base (CH_3COONa). Similarly combinations ($\text{H}_2\text{CO}_3 + \text{NaOH}$), ($\text{PhCOOH} + \text{NaOH}$), ($\text{NH}_4\text{OH} + \text{HBr}$) can also act as buffer.



The combinations(viii) and (ix) can't acts as buffer because salt component is missing.

Mixture of two salts of weak polyprotic acid can also acts as buffer because because in such cases we will visualize a mixture of acid and its conjugate base as salt.

NaHCO_3 [having weak acid HCO_3^-] and Na_2CO_3 [having conjugate base CO_3^{2-}] acts as buffer.

Hence the combination (ii), (iv), (v), (vi), (vii), (x), (xi) and (xii) can acts as buffer.

Illustration - 8 The pH of blood stream is maintained by a proper balance of H_2CO_3 and $NaHCO_3$ concentrations. What volume of 5M $NaHCO_3$ solution, should be mixed with 10 mL sample of blood which is 2M in H_2CO_3 in order to maintain a pH of 7.4 K_a for H_2CO_3 in blood is 4.0×10^{-7} ?

SOLUTION :

$$[H_2CO_3] \text{ in blood} = 2M$$

$$\text{Volume of blood} = 10 \text{ mL}$$

$$[NaHCO_3] = 5M$$

$$\text{Let volume of } NaHCO_3 \text{ used} = V \text{ mL}$$

$$[H_2CO_3] \text{ in mixture} = \frac{2 \times 10}{(V + 10)}$$

$$[NaHCO_3] \text{ in mixture} = \frac{(5 \times V)}{(V + 10)}$$

$$pH = pK_a + \log \frac{[\text{salt}]}{[\text{acid}]}$$

$$7.4 = \log 7.8 \times 10^7 + \log \frac{(5 \times V)/(V + 10)}{(2 \times 10)/(V + 10)}$$

$$V = 40 \text{ mL}$$

Illustration - 9 Consider a buffer solution containing 0.1 mole each of acetic acid and sodium acetate in 1.0 L of solution. 0.01 mole of NaOH is gradually added to this buffer solution. Calculate the new $[H^+]$ is the resulting solution. $[K_a = 2 \times 10^{-5}]$

SOLUTION :

pH of the solution is given by :

$$pH = pK_a + \log_{10} \frac{[\text{salt}]}{[\text{acid}]}$$

Initial pH of solution :

$$pH = pK_a + \log_{10} \frac{0.1}{0.1} = 4.7$$

When 0.01 M NaOH is added,

$$[\text{salt}] = 0.11 \text{ M and } [\text{acid}] = 0.09 \text{ M}$$

$\Rightarrow$ Final pH of solution :

$$pH = pK_a + \log_{10} \frac{0.11}{0.09} = 4.787$$

$$\Rightarrow [H^+] = K_a \frac{[\text{acid}]}{[\text{salt}]}$$

$$= 2 \times 10^{-5} \times \frac{0.09}{0.11} = 1.64 \times 10^{-5} \text{ M}$$

Illustration - 10 500 ml of 0.2 M aqueous solution of acetic acid is mixed with 500 ml of 0.2 M HCl at 25°C.

(i) Calculate the degree of dissociation of acetic acid in the resulting solution and pH of the solution.

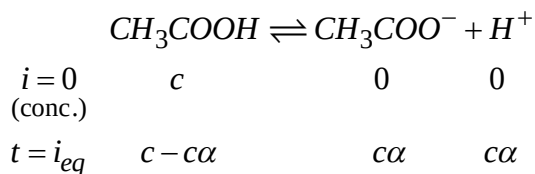
(ii) If 6g of NaOH is added to the above solution, determine the final pH. $[K_a \text{ acetic acid} = 2 \times 10^{-5}]$

SOLUTION :

$$(i) [CH_3COOH]_{\text{Just after mixing}} = \frac{0.2}{2} = 0.1M$$

$$[HCl]_{\text{just after mixing}} = \frac{0.2}{2} = 0.1M = [H^+]_{\text{From HCl}}$$

Note : Equal volumes added.



$$K_a = \frac{[CH_3COO^-][H^+]}{[CH_3COOH]} = \frac{c\alpha.(c\alpha + 0.1)}{c - c\alpha}$$

$$\left[\because [H^+]_{Total} = [H^+]_{From CH_3COOH} + [H^+]_{From HCl} \right]$$

$$\Rightarrow c\alpha + 0.1 \sim 0.1 ; c - c\alpha \sim c$$

[$\because \alpha$ will be small due to common ion effect]

$$\Rightarrow K_a \sim \alpha \times 0.1$$

$$\Rightarrow \alpha = \frac{K_a}{0.1} = 2 \times 10^{-4} \quad [\text{Check : } 1 - \alpha = 1 - 2 \times 10^{-4} \sim 1] \text{ and}$$

$$[H^+]_{Total} = c\alpha + 0.1 = 0.1 \times 2 \times 10^{-4} + 0.1 \sim 0.1M$$

$$\Rightarrow pH = 1$$

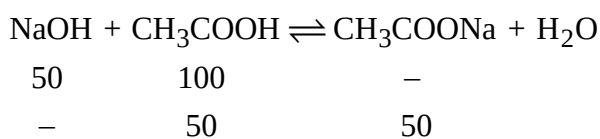
$$(ii) \quad 6\text{gm NaOH} \equiv \frac{6}{40/1} = 0.15 \text{ gmeq} \equiv 150 \text{ mmoles NaOH}$$

$$\text{Mmoles HCl} = 500 \times 0.2 = 100 \text{ mmoles.}$$

NaOH will first react with HCl and if there is a leftover then it will react with CH_3COOH

$$\Rightarrow \text{Mmoles NaOH left} = 150 - 100 = 50$$

These will react with CH_3COOH as :



$\Rightarrow$ Formation of an acidic buffer

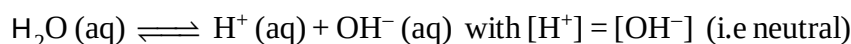
$$\begin{aligned}
 \Rightarrow pH &= pK_a + \log_{10} \frac{[\text{Salt}]}{[\text{Acid}]} \\
 &= 4.7 + \log_{10} \frac{50 / V_{Total}}{50 / V_{Total}} = 4.7
 \end{aligned}$$

HYDROLYSIS OF SALTS

Section - 4

It involves the study of the reactions between a salt and water to yield an acid and a base. The salts are generally ionised in water. Here we will deal with salts which are almost fully soluble (not sparingly soluble salts).

Water in the presence of ionic impurities is visualized as follows :

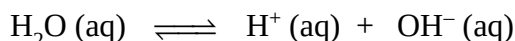
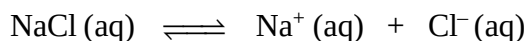


Now, when a salt is put in the water, there is an interaction between ions of salt and ions of water. The phenomenon of interaction of cations and anions of the salts with H^+ and OH^- ions furnished by water so that there is some net change in the pH of the resulting solution is known as *Hydrolysis of a salt*.

Four cases have been discussed here to understand the behavior of various type of salts.

1. Salts of strong acids and strong bases [SA-SB]

When such salts are dissolved in water, they do not undergo hydrolysis. They simply ionise and fail to change the H^+ ion concentration of solution and hence the pH of the solution remains same. e.g. NaCl , CaCl_2 , BaCl_2 , KCl , NaNO_3 , Na_2SO_4 , $\text{Ca}(\text{NO}_3)_2$



The final pH of the solution remains 7 at 25°C .

(If NaOH and HCl are formed, they neutralise each other to keep the pH same)

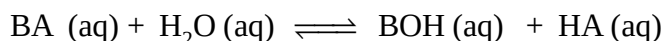
2. Salts of weak acids and strong bases [WA-SB]

Such salts give alkaline solutions in water. Some of such salts are : CH_3COONa , Na_2CO_3 , K_2CO_3 , KCN etc . For our discussion, we consider CH_3COONa (sodium acetate) in water. When CH_3COONa is put in water, it completely ionises to give CH_3COO^- (acetate) ions and Na^+ ions.

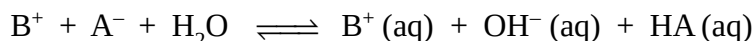
Now acetate ions (CH_3COO^-) absorb some H^+ ions from weakly dissociated H_2O molecules to form undissociated CH_3COOH . Na^+ remains in ionic state in water.

Now for K_w (ionic product) of water to remain constant, H_2O further ionises to produce more H^+ and OH^- ions. H^+ ions are taken up by CH_3COO^- ions leaving OH^- ions in excess and hence an alkaline solution.

Let BA represents such a salt. As it is put in water ;



BA dissociates into ions and BOH being strong base also ionises.



So, the net reaction is : $\text{A}^- (\text{aq}) + \text{H}_2\text{O} \rightleftharpoons \text{OH}^- (\text{aq}) + \text{HA} (\text{aq})$ [Anionic Hydrolysis]

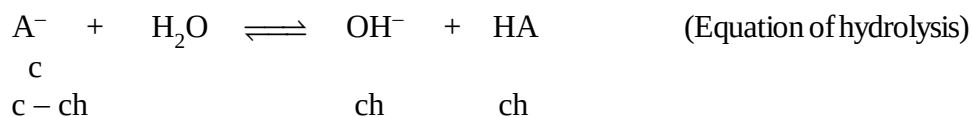
The above equation is known as *equation of hydrolysis* and equilibrium constant for such reactions is called as *Hydrolysis Constant* (K_h).

$$K_h = \frac{[\text{OH}^-][\text{HA}]}{[\text{A}^-]}$$

Degree of Hydrolysis :

It is defined as the fraction of total salt that has undergone hydrolysis on attainment of equilibrium. It is denoted by h .

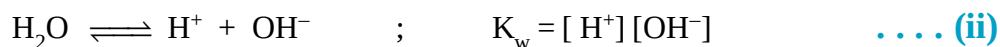
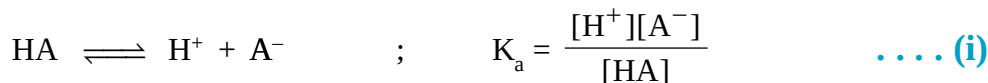
Let c be the concentration of salt and h be its degree of hydrolysis.



$$K_h = \frac{[\text{OH}^-][\text{HA}]}{[\text{A}^-]} = \frac{(ch)(ch)}{c - ch} = \frac{ch^2}{1 - h}$$

$$\Rightarrow K_h = ch^2 \quad [\text{assuming } h \ll 1] \quad \Rightarrow \quad h = \sqrt{\frac{K_h}{c}}$$

Now considering, dissociations of weak acid HA and H_2O



$$\Rightarrow K_h = \frac{K_w}{K_a} \quad \text{(Subtracting (i) from (ii), we get equation of hydrolysis)}$$

- Note :**
- Weaker is the acid, greater is the hydrolysis constant.
 - Degree of hydrolysis increases when 'c' decreases i.e. with dilution.

Now substituting the value of K_h in expression of h ,

$$\Rightarrow \quad h = \sqrt{\frac{K_w}{K_a c}}$$

pH of solution :

pH of a basic solution is given as :

$$\text{pH} = 14 + \log [\text{OH}^-] \quad \text{and} \quad [\text{OH}^-] = ch = \sqrt{K_h c}$$

Substituting for K_h , we get :

$$[\text{OH}^-] = \sqrt{\frac{K_w c}{K_a}} \quad \Rightarrow \quad \text{pH} = 14 + \log_{10} \sqrt{\frac{K_w c}{K_a}}$$

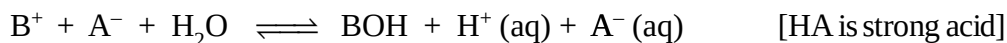
$$\Rightarrow \quad \text{pH} = \frac{1}{2} (\text{p}K_w + \text{p}K_a + \log_{10} c) \quad \text{or} \quad \text{pH (at } 25^\circ\text{C)} = 7 + \frac{1}{2} (\text{p}K_a + \log_{10} c)$$

3. Salts of strong acids and weak bases [SA-SB]

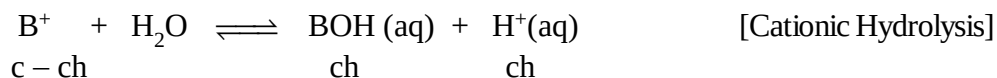
Such salts give acidic solutions in water. Some of such salts are : NH_4Cl , ZnCl_2 , FeCl_3 etc. For the purpose of discussion, we will consider hydrolysis of NH_4Cl .

When NH_4Cl is put in water, it completely ionises in water to give NH_4^+ and Cl^- ions. NH_4^+ ions combine with OH^- ions furnished by weakly dissociated water to form NH_4OH (weak base). Now for keeping K_w constant, water further ionises to give H^+ and OH^- ions, where OH^- ions are consumed by NH_4^+ ions leaving behind H^+ ions in solution to give an acidic solution.

Let BA be one of such salts. When it is put into water, the reaction is as follows.

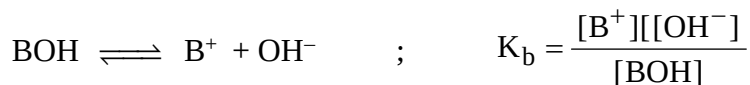


The net reaction of hydrolysis is :



$$\Rightarrow \quad K_h = \frac{[\text{BOH}][\text{H}^+]}{[\text{B}^+]} = \frac{(ch)(ch)}{c - ch} \frac{ch^2}{1 - h^2} \approx ch^2 \quad (h \ll 1) \quad \Rightarrow \quad h = \sqrt{\frac{K_h}{c}}$$

Considering ionisation of weak base BOH and H_2O .



From expressions for K_h , K_b and K_w , we have : (As done in previous case)

$$K_h = \frac{K_w}{K_b} \Rightarrow h = \sqrt{\frac{K_w}{K_b c}}$$

pH of solution :

Now, $\text{pH} = -\log [\text{H}^+]$

$$\text{and } [\text{H}^+] = ch = c \sqrt{\frac{K_h}{c}} = \sqrt{K_h c} \quad \Rightarrow \quad [\text{H}^+] = \sqrt{\frac{K_w c}{K_b}}$$

$$\Rightarrow \text{pH} = -\log_{10} \sqrt{\frac{K_w c}{K_b}}$$

$$\Rightarrow \text{pH} = \frac{1}{2}(\text{p}K_w - \text{p}K_b - \log_{10} c) \quad \text{or} \quad \text{pH (at } 25^\circ\text{C)} = 7 - \frac{1}{2}(\text{p}K_b + \log_{10} c)$$

Try Your Self :

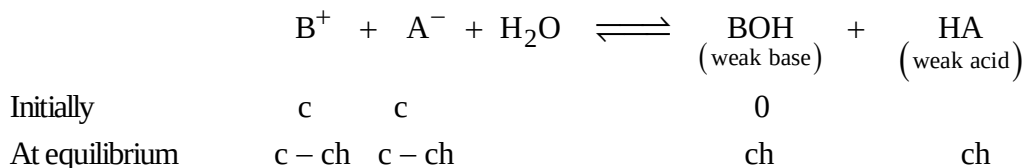
1. Calculate pH of M/100 solution of CH_3COONa at 25°C where K_a of CH_3COOH is 2.0×10^{-5} .
[Ans : 8.37]
2. Calculate pH of M/100 solution of NH_4Cl at 25°C where K_b for NH_4OH is 2.0×10^{-5} .
[Ans : 5.63]
3. In above two cases, find the degree of hydrolysis taking $K_w = 10^{-14}$.

Note : The results of above two cases of hydrolysis of salts can be used directly.

4. Salts of weak acids and weak bases [WA-WB]

Let us consider ammonium acetate ($\text{CH}_3\text{COONH}_4$) for our discussion. Both NH_4^+ ions and CH_3COO^- ions react respectively with OH^- and H^+ ions furnished by water to form NH_4OH (weak base) and CH_3COOH (acetic acid).

Let BA represents such a salt.



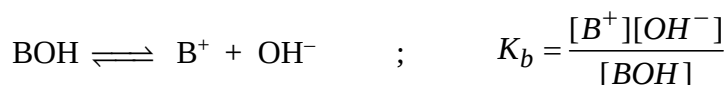
$$\Rightarrow K_h = \frac{[\text{BOH}][\text{HA}]}{[\text{B}^+][\text{A}^-]} = \frac{(ch)(ch)}{(c - ch)^2}$$

$$\Rightarrow K_h = \frac{h^2}{(1 - h)^2} ; \quad \text{Taking square root on both sides to get :}$$

$$h = \frac{\sqrt{K_h}}{1 + \sqrt{K_h}} \quad (\text{Here, } 1 - h \neq 0; \text{ Think why?})$$

Hence the degree of hydrolysis of such salts is independent of concentration of salt solution.

Now considering the dissociation of both weak base and acid.





Combining K_h , K_b , K_a and K_w , we have

$$K_h = \frac{K_w}{K_a K_b} \quad \text{and} \quad h = \sqrt{\frac{K_h}{1 + \sqrt{K_h}}} \quad [\text{Cation - Anionic Hydrolysis}]$$

(Observe that $1 - h \neq 1$)

pH of Solution :

$$\text{Consider, } K_a = \frac{[\text{H}^+][\text{A}^-]}{[\text{HA}]} \quad \Rightarrow \quad [\text{H}^+] = K_a \frac{[\text{HA}]}{[\text{A}^-]}$$

Since, base and acids are weaker, hence,

$$[\text{BOH}] = [\text{HA}] \quad \Rightarrow \quad [\text{B}^+] = [\text{A}^-]$$

$$\Rightarrow K_h = \frac{[\text{BOH}][\text{HA}]}{[\text{B}^+][\text{A}^-]} = \frac{[\text{HA}]^2}{[\text{A}^-]^2} \quad \Rightarrow \quad [\text{H}^+] = K_a \sqrt{K_h} = \sqrt{\frac{K_w K_a}{K_b}}$$

$$\text{pH} = -\log [\text{H}^+] = -\log \sqrt{\frac{K_w K_a}{K_b}}$$

$$\Rightarrow \text{pH} = \frac{1}{2}(\text{p}K_w + \text{p}K_a - \text{p}K_b) \quad \text{or} \quad \text{at } 25^\circ\text{C, } \text{pH} = 7 + \frac{1}{2}(\text{p}K_a - \text{p}K_b)$$

Illustrating the concept :

What is the pH of a 0.50 M aqueous NaCN solution ? $\text{p}K_b$ of CN^- is 4.70.

SOLUTION :

NaCN is a salt of weak acid and strong base. The pH of its aqueous solution is given by :

$$\text{pH (at } 25^\circ\text{C)} = 7 + \frac{1}{2} (\text{p}K_a + \log_{10} c)$$

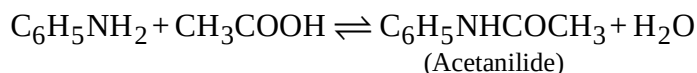
Remember that for any acid and its conjugate base or vice-versa : $\text{p}K_a + \text{p}K_b = \text{p}K_w$
e.g. For acid (say HA), $\text{p}K_a$ corresponds to HA and $\text{p}K_b$ corresponds to its conjugate base : A^-

Thus, $\text{p}K_a$ of HCN = $14 - 4.7 = 9.3$

$$\text{So, } \text{pH (at } 25^\circ\text{C)} = 7 + \frac{1}{2} (\text{p}K_a + \log_{10} c) = 7 + \frac{1}{2} (9.3 + \log_{10} 0.5) = 11.5$$

Illustration - 11 Calculate the degree of hydrolysis of a mixture of aniline and acetic acid each of them being 0.01 M. K_a of acetic acid = 1.8×10^{-5} and K_b (aniline) = 4.5×10^{-10} . Also calculate pH of the mixture.

SOLUTION :



Now when we mix equal concentrations of aniline and acetic acid, they will neutralise each other to form acetanilide (salt). The salt is of weak base and weak acid, so hydrolysis takes place. The degree of hydrolysis of the salt of weak acid and weak base is given by :

$$h = \frac{\sqrt{K_h}}{1 + \sqrt{K_h}}$$

$$\text{Here, } K_h = \frac{K_w}{K_a \cdot K_b} = \frac{10^{-14}}{1.8 \times 10^{-5} \times 4.5 \times 10^{-10}} = 1.23$$

$$\Rightarrow h = 0.52$$

However the pH of solution can be calculated by using the standard result :

$$\text{pH} = 7 + \frac{1}{2}(\text{p}K_a - \text{p}K_b)$$

$$\Rightarrow \text{pH} = 7 + \frac{1}{2} \left[-\log_{10} 1.8 \times 10^{-5} - \left(-\log_{10} 4.5 \times 10^{-10} \right) \right]$$

$$\Rightarrow \text{pH} = 7 + \frac{1}{2} \log_{10} \frac{4.5 \times 10^{-10}}{1.8 \times 10^{-5}} = 4.7$$

- Note :**
- Solution in this case is acidic. In such salts, pH depends upon which of two i.e. weak acid or weak base is stronger than the other, i.e. by comparing K_a and K_b ; one can theoretically get an idea about the pH of solution.
 - The formula $\text{pH} = 7 + \frac{1}{2}(\text{p}K_a - \text{p}K_b)$ is applicable even when 'h' is large as no approximation of 'h' comparison with unity, has been taken in the derivation (see derivation yourself).



Illustration - 12 2.5 mL of 0.4 M weak mono-acidic base ($K_b = 1 \times 10^{-12}$ at 25°C) is titrated with $2/15$ M HCl in water at 25°C . Find the concentration of H^+ ions at equivalence point. ($K_w = 1 \times 10^{-14}$ at 25°C)

SOLUTION :

First find the volume of HCl required to reach the equivalence point.

meq. of Base = meq of HCl

$$\Rightarrow 2.5 \times (0.4 \times 1) = \left(\frac{2}{15} \times 1 \right) V_{\text{HCl}}$$

$$\Rightarrow V_{\text{HCl}} = 7.5 \text{ mL}$$

The net volume of the solution at the equivalent point = $V_{\text{base}} + V_{\text{HCl}} = 2.5 + 7.5 = 10 \text{ mL}$

$$\Rightarrow [\text{salt}] = c = \frac{0.4 \times 2.5}{10} = 0.1 \text{ M}$$

pH of an aqueous solution of such a salt is given by :

$$\text{pH} = 7 - \frac{1}{2}(\text{p}K_b + \log_{10} c) = 7 - \frac{1}{2}(12 + \log_{10} 0.1) = 1.5$$

$$\Rightarrow [\text{H}^+] = 10^{-1.5} = \frac{1}{10\sqrt{10}} \approx 3.2 \times 10^{-2} \text{ M}$$

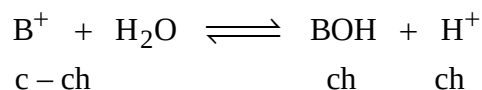
Please note that the above solution is Incorrect. (Why?)

Actually, $\text{pH} = 7 - \frac{1}{2}(\text{p}K_b + \log_{10} c)$ is valid only when $1 - h \approx 1$

To check, calculate h using :

$$h = \sqrt{\frac{K_h}{c}} = \sqrt{\frac{K_w}{K_b c}} = \sqrt{\frac{10^{-14}}{10^{-12} \times 0.1}} = \sqrt{0.1} \quad \Rightarrow \quad 1 - \sqrt{0.1} \neq 1$$

So we have solve from basics as follows :



$$\Rightarrow K_h = \frac{[\text{BOH}][\text{H}^+]}{[\text{B}^+]} = \frac{(\text{ch})(\text{ch})}{c - \text{ch}} \quad \Rightarrow \quad K_h = \frac{K_w}{K_b} = 10^{-2} = \frac{[\text{H}^+]^2}{c - [\text{H}^+]} \quad (\text{ch} = [\text{H}^+])$$

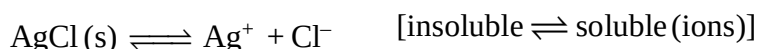
$$\Rightarrow [\text{H}^+] = -\frac{10^{-2} + \sqrt{10^{-4} + 4 \times 10^{-3}}}{2} = 2.7 \times 10^{-2} \text{ M}$$

SOLUBILITY PRODUCT (K_{sp}) AND PRECIPITATION OF SALTS

Section - 5

Here, we will study those salts which are generally considered as insoluble in water. (e.g. AgCl, BaSO₄, Ag₂S, PbS etc) Actually no insoluble salt is actually completely insoluble, they have a very small solubility. Consider a vessel containing water with a sufficient amount of AgCl being put into it. A very small amount of it dissolves in water (*because generally salts are composed of ions or contain polarised atoms*). The small amount which is soluble is almost 100% ionised.

The situation achieved at this stage is *Dynamic* in nature; with some of ions (soluble) being solidifying and some solid always dissolving. Thus a state of equilibrium exists (like in earlier cases). This means at a given temperature, the concentration of Ag⁺ ions and Cl⁻ ions in aqueous solution remains constant. The following equilibrium exists. In this equilibrium, the equilibrium state is known as *Saturated state*.

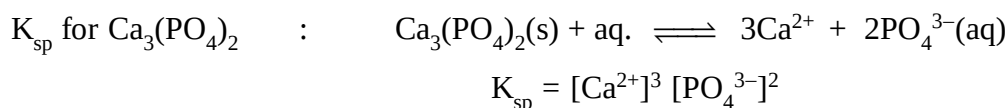
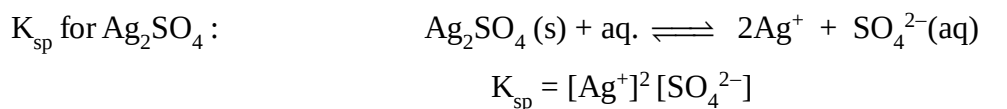


The equilibrium constant for this reaction is known as *solubility product* (K_{sp}) and is written as :

$$K_{sp} = [\text{Ag}^+] [\text{Cl}^-]$$

K_{sp} is expressed in concentration units. The reaction coefficient, Q in such system is known as ionic product (I.P)

- Note :**
- The solubility of a salt in water depends upon other substances (ions) present in water (at that time) and the solubility product is a constant value (at a given temperature) and is independent of presence of any other ion present. Like other equilibrium constants such as : K_p , K_c , K_a and K_b , it is also constant at a certain temperature and can change only when temperature changes.
 - The salts like AgCl are regarded as sparingly soluble salts. Some of them are : Ag₂SO₄, Ag₂CO₃, PbCl₂, PbCrO₄, PbS, ZnS, Al(OH)₃, Ca₃(PO₄)₂, BaSO₄, CaSO₄, MnS etc.



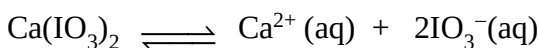
- Note :** In the solubilities of sparingly soluble salts, we will deal with mainly two types of problems : one to find out the solubility in water if K_{sp} of a salt is given and vice-versa. Second, to find out solubility of a salt in presence of other (common) ions in water.

Illustration - 13 If K_{sp} of calcium iodate $\text{Ca}(\text{IO}_3)_2 = 6.3 \times 10^{-7}$ at 18°C , find the solubility of salt in (i) gm/L (ii) mol/L in water at 18°C .

SOLUTION :

In questions, solubility is sometime asked (or given) to be expressed in gm/L.

Now the following equilibrium is established :



Let x mole/L be solubility of $\text{Ca}(\text{IO}_3)_2$ in water

$$\Rightarrow [\text{Ca}^{2+}] = x \text{ M} \quad \text{and} \quad [\text{IO}_3^-] = 2x \text{ M}$$

$$\text{Now, } K_{sp} = [\text{Ca}^{2+}] [\text{IO}_3^-]^2$$

$$\Rightarrow 6.3 \times 10^{-7} = (x) (2x)^2$$

$$\Rightarrow 4x^3 = 6.3 \times 10^{-7}$$

$$\Rightarrow x = 5.4 \times 10^{-3} \text{ M.}$$

Now answer is to expressed in gm/L

$$\begin{aligned} \Rightarrow \text{solubility} &= x \times M_0 \\ &= (5.4 \times 10^{-3}) \times 390 = 2.106 \text{ g/L} \end{aligned}$$

Illustration - 14 The K_{sp} of AgCl at 25°C is 1.5×10^{-10} . Find the solubility of AgCl (in gm/L) in an aqueous solution containing 0.01 M AgNO_3 .

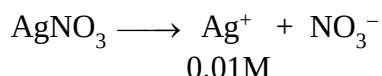
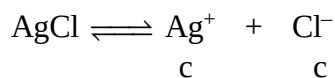
SOLUTION :

Now this is quite similar to finding the degree of dissociation of a weak acid or weak base in presence of its common ion (Le Chatelier's principle).

Here 0.01 M AgNO_3 will give 0.01 M Ag^+ ions in solution before the addition of AgCl .

(AgNO_3 is 100% soluble in water).

Let the solubility be c mol/L in AgNO_3 .



In solution :

$$[\text{Ag}^+] = c + 0.01 \quad \text{and} \quad [\text{Cl}^-] = c$$

Now K_{sp} is a constant and is independent of presence of any ion in water.

$$\Rightarrow K_{sp} = [\text{Ag}^+][\text{Cl}^-] = (0.01 + c)(c) = 1.50 \times 10^{-10}$$

(Now assuming $0.01 + c \sim 0.01$ as $c \ll 1$ for sparingly soluble salt)

$$\Rightarrow c = \frac{K_{sp}}{0.01} = 1.5 \times 10^{-8} \text{ M}$$

[Check that $c + 0.01 \approx 0.01 \text{ M}$]

$$\text{or solubility} = (1.5 \times 10^{-8}) \times 143.5 = 2.15 \times 10^{-6} \text{ g/L}$$

Note : The solubility of AgCl in water containing Ag^+ (a common ion) is much less than in pure water.

Illustration - 15 The solubility of BaSO_4 in water is $2.33 \times 10^{-4} \text{ gm/100 mL}$. Calculate the % age loss in weight when 0.2 gm of BaSO_4 is washed with

(a) 1 L of water (b) 1 L of 0.01 N H_2SO_4 . [$M_0 \text{ BaSO}_4 = 233 \text{ gm/mol}$]

SOLUTION :

(a) Solubility is in general expressed in gm/L,

$$\text{so solubility of } \text{BaSO}_4 = 2.33 \times 10^{-3} \text{ g/L}$$

Loss in weight of BaSO_4 = amount of BaSO_4 soluble.

$$\Rightarrow \% \text{ loss} = \frac{2.33 \times 10^{-3}}{0.2} \times 100 = 1.16 \%$$

(b) 0.01 N $\text{H}_2\text{SO}_4 \equiv 0.01 \text{ N SO}_4^{2-}$ ions
 $\equiv 0.005 \text{ M SO}_4^{2-}$ ions

Now presence of SO_4^{2-} prior to washing BaSO_4 will suppress the solubility of BaSO_4 (due to common ion effect). The suppression will be governed by K_{sp} value of BaSO_4 . So first calculate K_{sp} of BaSO_4 .

Solubility of BaSO_4 in fresh water

$$= 2.33 \times 10^{-3} \text{ g/L}$$

$$\equiv \frac{2.33 \times 10^{-3}}{233} \text{ mol/L} = 10^{-5} \text{ M}$$

$$K_{\text{sp}} = [\text{Ba}^{2+}] [\text{SO}_4^{2-}] = (10^{-5})^2 = 10^{-10}$$

Now let x be solubility in mol/L in H_2SO_4

$$\Rightarrow [\text{Ba}^{2+}] \text{ in solution} = x \text{ mol/L}$$

$$\text{and } [\text{SO}_4^{2-}] \text{ in solution} = (x + 0.005) \text{ mol/L}$$

$$\text{Ionic product} = [\text{Ba}^{2+}] [\text{SO}_4^{2-}]$$

$$= (x)(x + 0.005)$$

K_{sp} = Ionic Product at equilibrium (saturation)

$$\Rightarrow 1.0 \times 10^{-10} = (x)(x + 0.005)$$

Assuming x to be a small number

$$x + 0.005 \sim 0.005$$

$$\Rightarrow x = \frac{10^{-10}}{0.005} = 2 \times 10^{-8} \text{ mol/L}$$

$$= 2 \times 10^{-8} \times 233 \text{ g/L}$$

$$= 4.66 \times 10^{-6} \text{ g/L}$$

$$\Rightarrow 4.66 \times 10^{-6} \text{ gm of } \text{BaSO}_4 \text{ are washed away.}$$

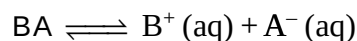
$$\Rightarrow \% \text{ loss} = \frac{4.66 \times 10^{-6} \times 100}{0.2} = 2.33 \times 10^{-3} \%$$

Note : Third type of problem are related to the precipitation of salts (sparingly soluble). To understand and solve such problems, let us read following article carefully.

Precipitation of salts

Now we will study (application of K_{sp}) how to determine whether or not a precipitate will form when we mix known concentrations of ions which combine to give a sparingly soluble salt. To understand and solve such problems let us analyse the expression of solubility product.

For a salt (sparingly soluble) when dissolved in water :



At equilibrium (saturation): $K_{\text{sp}} = [\text{B}^+] [\text{A}^-]$

Ionic Equilibrium

When we mix ions or if there be two or more ions in water, we define reaction coefficient (Q) called as *ionic-product (I.P.)*, giving the products of ions in water (ions of soluble salts and other common ions).

I.P. is product of ionic concentration due to ions already present in water or from a salt. It may be and may not be equal to K_{sp} . To illustrate it more clearly, consider a case when 500 ml of 0.005 M solution of AgNO_3 is added to 500 ml of 0.001 M solution of KCl . Now in solution (mixture), there are Ag^+ , NO_3^- , K^+ and Cl^- ions. The concentration of $[\text{Ag}^+] = [\text{NO}_3^-] = 0.005/2 = 0.0025 \text{ M}$ (*equal volumes are mixed*) and $[\text{K}^+] = [\text{Cl}^-] = \frac{0.001}{2} = 0.0005 \text{ M}$ as equal volumes of two solutions are mixed.

Now, we know that Ag^+ will react with Cl^- ions to form AgCl since :

$\text{AgCl} \rightleftharpoons \text{Ag}^+ + \text{Cl}^-$ is a reversible reaction with a high tendency towards left (solidifying or precipitating).

Now, question is, whether AgCl will be formed or not (precipitation of Ag^+ and Cl^- as AgCl) and if it formed, how much of it will be formed ? For this we define some rules.

1. If Ionic Product (IP) $> K_{sp}$; precipitation takes place till I.P. equals K_{sp} .
2. If Ionic Product $< K_{sp}$; a precipitate will not be formed and the solution will be *unsaturated*.
3. If Ionic Product $= K_{sp}$; a precipitate will not be formed and the solution is saturated in that salt.
(or we can say that solution is at a critical stage, when precipitation just begins, but actually has not occurred)

In present case, $\text{I.P.} = [\text{Ag}^+][\text{Cl}^-] = (0.0025)(0.0005) = 1.25 \times 10^{-7}$

(Only for the salt which is sparingly soluble not for KNO_3)

Ionic product $> K_{sp}$ in this case ($K_{sp} \text{ AgCl} = 1.56 \times 10^{-10}$) which mean precipitation takes place.

Illustration - 16 When 15mL of 0.05 M AgNO_3 is mixed with 45.0 mL of 0.03 M K_2CrO_4 , predict whether precipitation of Ag_2CrO_4 occurs or not ? K_{sp} of $\text{Ag}_2\text{CrO}_4 = 1.9 \times 10^{-12}$.

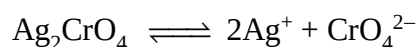
SOLUTION :

First find the concentrations of Ag^+ and CrO_4^{2-} ions in the resulting mixture.

$$[\text{Ag}^+] = \frac{15 \times 0.05}{15 + 45} = 1.25 \times 10^{-2} \text{ M}$$

$$[\text{CrO}_4^{2-}] = \frac{45 \times 0.03}{15 + 45} = 2.25 \times 10^{-2} \text{ M}$$

The ionic product for Ag_2CrO_4 is given as follows.



$$\begin{aligned} \text{Ionic product (I.P.)} &= [\text{Ag}^+]^2[\text{CrO}_4^{2-}] \\ &= (1.25 \times 10^{-2})^2 (2.25 \times 10^{-2}) \\ &= 3.51 \times 10^{-6} > K_{sp} \end{aligned}$$

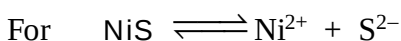
Hence precipitation occurs.

Illustration - 17 The concentration of Ni^{2+} ions in a given NiS solution is 2.0×10^{-6} moles/L. Find the minimum S^{2-} ions necessary to cause precipitation of NiS . The solubility product, K_{sp} of $\text{NiS} = 1.4 \times 10^{-14}$.

SOLUTION :

Note : Solubility product, K_{sp} gives an idea of maximum possible concentration of an ion which can be retained in solution.

K_{sp} = ionic product (I.P.) for saturated solution



$$\text{Now, } K_{sp} = [\text{Ni}^{2+}][\text{S}^{2-}]$$

$$\Rightarrow [\text{S}^{2-}] = \frac{K_{sp}}{[\text{Ni}^{2+}]}$$

It is the maximum permissible concentration of S^{2-} ion in solution.

➤ For $[\text{S}^{2-}] = \frac{K_{sp}}{[\text{Ni}^{2+}]}$, NiS will just start to precipitate and

➤ If $[\text{S}^{2-}] > \frac{K_{sp}}{[\text{Ni}^{2+}]}$, NiS will precipitate.

This means $\frac{K_{sp}}{[\text{Ni}^{2+}]}$ is minimum concentration of S^{2-} ion above which precipitation occurs.

$$\Rightarrow [\text{S}^{2-}]_{\min} = \frac{1.4 \times 10^{-14}}{2 \times 10^{-6}} = 7 \times 10^{-9} \text{ M}$$

Illustration - 18 A solution contains 0.01 M each of CaCl_2 and SrCl_2 . A 0.005 M solution of SO_4^{2-} is slowly added to the given solution.

- Which substance begins to precipitate first ?
- If H_2SO_4 is continuously added, determine when will other salt be precipitated ?
- When second salt starts to precipitate, find the concentration of cation of first salt.

Assume that CaCl_2 and SrCl_2 are 100% ionised and volume of the solution remains constant.

$$K_{sp} \text{ of } \text{CaSO}_4 = 1.3 \times 10^{-4} \text{ and } K_{sp} \text{ of } \text{SrSO}_4 = 3.2 \times 10^{-7}.$$

SOLUTION :

The solution contains 0.01 M Ca^{2+} ion and 0.01 M Sr^{2+} ions (100% ionization). Now SO_4^{2-} ions are added gradually and slowly.....

- First calculate the minimum $[\text{SO}_4^{2-}]$ ions required to start precipitation of CaSO_4 and SrSO_4

$$[\text{SO}_4^{2-}]_{\text{Min. for } \text{CaSO}_4} = \frac{K_{sp} [\text{CaSO}_4]}{[\text{Ca}^{2+}]}$$

$$= \frac{1.3 \times 10^{-4}}{0.01} = 1.3 \times 10^{-2} \text{ M}$$

$$[\text{SO}_4^{2-}]_{\text{Min. for } \text{SrSO}_4} = \frac{K_{sp} [\text{SrSO}_4]}{[\text{Sr}^{2+}]}$$

$$= \frac{3.2 \times 10^{-7}}{0.01} = 3.2 \times 10^{-5} \text{ M}$$

Since $[\text{SO}_4^{2-}]$ for SrSO_4 is much lower than that required for CaSO_4 , so SrSO_4 will precipitate FIRST.

- Now, if i.e., SO_4^{2-} ions are continuously added, at some instant, its concentration will become equal to that minimum required for precipitating out Ca^{2+} ions.

Ionic Equilibrium

Hence CaSO_4 will start precipitating if $[\text{SO}_4^{2-}]$
 $= 1.3 \times 10^{-2} \text{ M}$.

(c) Now, find the concentration of Sr^{2+} ions
 when $[\text{SO}_4^{2-}] = 1.3 \times 10^{-2} \text{ M}$ in the solution.

$$[\text{Sr}^{2+}] = \frac{K_{\text{sp}} \text{ SrSO}_4}{[\text{SO}_4^{2-}]} = \frac{3.2 \times 10^{-7}}{1.3 \times 10^{-2}}$$

$$= 2.46 \times 10^{-5} \text{ M}$$

$$\left[2.46 \times 10^{-5} \text{ M} \lll 0.01 \text{ M} \right]$$

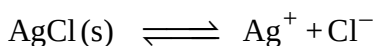
Can you observe that when CaSO_4 starts to precipitate, SrSO_4 is almost precipitated out as the value of Sr^{2+} is very very low?

Note : The above illustration represents a case of selective precipitation (or separation of metals cations) by adding a common anion. Observe that there is a vast difference in the K_{sp} values of two sulphates, and hence in the minimum values of sulphate ion for precipitating out the two metals ions.

Illustration - 19 How much the concentration of Ag^+ ions in a saturated solution of AgCl diminish if such an amount of HCl is added to it that the concentration of Cl^- ions in the solution becomes equal to 0.03 M ? Also find the amount of AgCl precipitated at the given concentration. K_{sp} of $\text{AgCl} = 1.8 \times 10^{-10}$.

SOLUTION :

HCl is added to a solution containing Ag^+ ions in saturated solution. First find the concentration of Ag^+ ion in this solution.



$\Rightarrow K_{\text{sp}} = [\text{Ag}^+][\text{Cl}^-] = x^2$ where x is solubility of AgCl in mol/L

$$\begin{aligned} \Rightarrow [\text{Ag}^+] &= \sqrt{K_{\text{sp}}} = \sqrt{1.8 \times 10^{-10}} \\ &= 1.34 \times 10^{-5} \text{ M} \end{aligned}$$

When HCl is added, the ionic product of AgCl approaches the K_{sp} value of AgCl , the precipitation of Ag^+ ions will occur. As ionic product increases (i.e. becomes greater than K_{sp} value), an appreciable amount of AgCl precipitates out, and precipitation continues till ionic product (I.P.) equals solubility product (K_{sp}).

$$\text{I.P.} = [\text{Ag}^+][\text{Cl}^-] = K_{\text{sp}}$$

$$\Rightarrow [\text{Ag}^+] = \frac{K_{\text{sp}}}{[\text{Cl}^-]} = \frac{1.8 \times 10^{-10}}{(0.03)} = 6.0 \times 10^{-9} \text{ M}$$

Now this is the amount of Ag^+ ions left un-precipitated.

$\Rightarrow [\text{Ag}^+]$ diminishes in the solution by

$$\frac{6.0 \times 10^{-9}}{1.34 \times 10^{-5}} = \left(\frac{1}{2233} \right) \text{ times}$$

The concentration of AgCl precipitated out of the solution
 $= [\text{Ag}^+]_{\text{initial}} - [\text{Ag}^+]_{\text{left}}$

$$= 1.34 \times 10^{-5} - 6.0 \times 10^{-9} \text{ M}$$

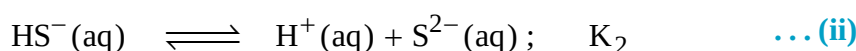
It means almost whole of AgCl is precipitated out of the solution at $[\text{Cl}^-] = 0.03 \text{ M}$.

Precipitation through H_2S

The most typical technique used in precipitating the metals ions or selectively precipitating (separation) the two metal ions, is passing H_2S gas in a solution of metal ion (s) whose acidity is externally controlled (or preset by adding HCl). First consider the ionisation (equilibrium) of H_2S gas in an aqueous solution and then in an acidic solution.

➤ Passing $H_2S(g)$ in an aqueous solution :

H_2S is a dibasic acid. We have to visualize two stage ionisation for it as follows:



K_1 and K_2 are Ist and IInd ionisation constants respectively for $H_2S(aq)$. Since $K_2 \ll K_1$ (due to common ion effect), we usually neglect the concentration of H^+ ions from second equilibrium.

$$\Rightarrow [H^+]_{\text{in Solution}} = [H^+]_{\text{from } H_2S} + [H^+]_{\text{from } HS^-} \approx [H^+]_{\text{from } H_2S}$$

$$\text{Now, } K_1 = \frac{[H^+][HS^-]}{[H_2S]} \quad \text{and} \quad K_2 = \frac{[H^+][S^{2-}]}{[HS^-]}$$

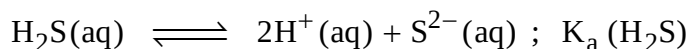
From (i), $[H^+] = [HS^-]$ and neglecting $[H^+]$ from (ii), we have from second ionisation constant :

$$K_2 = \frac{[H^+][S^{2-}]}{[HS^-]} \approx [S^{2-}] \quad \text{or} \quad [S^{2-}] \approx K_2$$

So if we pass H_2S in an aqueous solution, sulphide ion concentration is simply equal to the value of second ionisation constant. We can determine whether precipitation of metal sulphide will occur or not by comparing ionic product (I.P.) with solubility constant (K_{sp}).

➤ Passing $H_2S(g)$ in an acidic solution :

Now if we pass H_2S in an acidic solution of a metal ion till saturation, we simply neglect the concentration of H^+ ion from H_2S (both from first and second ionisations) totally as the concentration of H^+ from acid (strong acid added externally) is very high. In that case we can simply add two equilibriums (i) and (ii) to get :



$$K_a = K_1 \times K_2 = \frac{[H^+]^2[S^{2-}]}{[H_2S]}$$

Note that in a saturated solution of H_2S at $25^\circ C$, $[H_2S]$ is constant at 0.1 M.

$$\Rightarrow K_a \times (0.1 \text{ M}) = [H^+]^2 [S^{2-}]$$

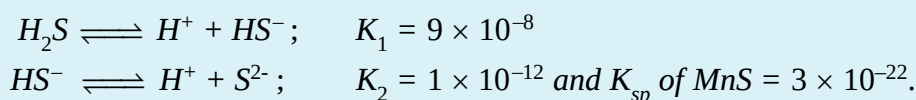
$$([H^+][S^{2-}] = K_{sp} \text{ of } H_2S)$$

$$\Rightarrow [S^{2-}] = \frac{K_a \times (0.1 \text{ M})}{[H^+]^2} = \frac{K_1 \times K_2 \times (0.1 \text{ M})}{[H^+]^2}$$

In this way, we now know the concentration of sulphide ion in an acidic solution, so accordingly we can plan precipitation of metal ions. The important aspect of this method is that we can always increase or decrease the concentration of sulphide ion by controlling the amount of acid.



Illustration - 20 Calculate the maximum possible concentration of Mn^{2+} in water that is saturated with H_2S (which is 0.1 M at 300K) and maintained at $pH = 3$ with HCl . The equilibrium constant(s) for dissociation of H_2S are :



SOLUTION :

Note that solution contains S^{2-} ions (from H_2S).

So Mn^{2+} will react with S^{2-} ions to give a precipitate of MnS (being sparingly soluble salt). Hence the concentration of S^{2-} at saturation will determine the maximum possible concentration of Mn^{2+} ions in solution that will not be precipitating. So first calculate $[S^{2-}]$ at saturation.

Calculate K_a for dissociation of H_2S .

$$K_a \text{ of } H_2S = K_1 \times K_2 = 9 \times 10^{-20}$$

$$H_2S \rightleftharpoons 2H^+ + S^{2-}$$

$$\Rightarrow K_a = \frac{[H^+]^2[S^{2-}]}{[H_2S]} \Rightarrow [S^{2-}] = \frac{K_a [H_2S]}{[H^+]^2}$$

Note that pH of solution is maintained at 3 (externally).

$$\Rightarrow [H^+] = 10^{-3} \text{ M}$$

$$\Rightarrow [S^{2-}] = \frac{9 \times 10^{-20} \times 0.1}{(10^{-3})^2} = 9 \times 10^{-15}$$

In a saturated solution, ionic product = K_{sp}

Now ionic product = $[Mn^{2+}][S^{2-}] = [Mn^{2+}](9 \times 10^{-15})$

$$\Rightarrow [Mn^{2+}](9 \times 10^{-15}) = 3 \times 10^{-22}$$

$$\Rightarrow [Mn^{2+}] = 3.33 \times 10^{-8} \text{ M}$$



ACID BASE TITRATIONS

Section - 6

pH of Mixtures of Acids and Bases

Let us take x millimoles of acid (HA) and y millimoles of base (BOH). Note that acid is monobasic and base is monoacidic.

(a) Strong acid and strong base

- If $x = y$, then complete neutralization takes place and we get $x (= y)$. Millimoles of salt (BA) of strong acid and strong base which means no hydrolysis takes place and pH of solution = 7.
- If $x > y$, then there is an excess of strong acid and resulting solution is acidic with millimoles of acid left in excess = $x - y$. Now if V (in cc) be the volume of mixture, then :

$$\Rightarrow M = \frac{x-y}{V}. \text{ Now calculate pH using the equation } \text{pH} = -\log_{10} [\text{H}^+]$$

- If $x < y$, then there is an excess of strong base and resulting solution is basic with millimoles of base left in excess = $y - x$. Now if V (in cc) be the volume of mixture, then :

$$\Rightarrow M = \frac{y-x}{V}. \text{ Now calculate pH using the equation } \text{pH} = 14 - \log_{10} [\text{OH}^-]$$

(b) Strong base and weak acid

- If $x = y$, first of all neutralization takes place to give $x (= y)$ millimoles of salt (BA). The salt will now undergo *hydrolysis* to give an alkaline solution. Calculate pH by using the standard result :

$$\text{pH} = 7 + \frac{1}{2}(\text{pK}_a + \log_{10} c) \quad ; \quad c \text{ is concentration expressed in M (mol/L)}$$

- If $x > y$, there is excess of weak acid whose millimoles = $x - y$ and y millimoles of salt is formed. This will give an *acidic buffer solution*. Calculate pH of buffer solution using *Henderson's Equation*.

$$\text{pH} = \text{pK}_a + \log_{10} \frac{[\text{salt}]}{[\text{acid}]} = \text{pK}_a + \log_{10} \frac{y}{x-y}$$

- If $x < y$, the solution in this case contains excess of strong base whose millimoles are $y - x$.

$$M = \frac{y-x}{V} \quad \text{Calculate pH using the equation } \text{pH} = 14 - \log_{10} [\text{OH}^-]$$

(c) Strong acid and weak base

- If $x = y$, first of all complete neutralisation takes place to produce $x (= y)$ millimoles of salt (BA). The salt (BA) is of strong acid and weak base, hence *hydrolysis* takes place to give an acidic solution. Calculate its pH by using standard result.

$$\text{pH} = 7 - \frac{1}{2}(\text{pK}_b + \log_{10} c) \quad ; \quad c : \text{mol/L}$$

- If $x > y$, then solution contain excess of strong acid whose millimoles = $x - y$.

$$M = \frac{x-y}{V} \quad \text{Now calculate pH using the equation } \text{pH} = -\log_{10} [\text{H}^+]$$

- If $x < y$, then there is an excess of weak base whose millimoles are $y - x$ and millimoles of salt (BA) are x . This will give a *basic buffer solution*. Calculate the pH by using Henderson's Equation.

$$\text{pH} = 14 - \text{pK}_b - \log_{10} \frac{[\text{salt}]}{[\text{base}]} = 14 - \text{pK}_b - \log_{10} \frac{x}{y-x}$$

(d) Weak acid and weak base

- If $x = y$, neutralisation takes place completely with the formation of $x (= y)$ millimoles of salt (BA) of weak acid and weak base. So hydrolysis takes place. Calculate the pH by using :

$$\text{pH} = 7 + \frac{1}{2}(\text{pK}_a - \text{pK}_b)$$

Indicators

Definition : An “indicator” is a dye (or substance) which change color over a short pH range.

Example: Methyl orange changes color in the pH range of 3.1 to 4.5

Indicator	1	2	3	4	5	6	7	8	9	10	11	12	13
Methyl Orange			← Pink →	3.1 – 4.5					Yellow				
Methyl Red			← Pink →		4.2 – 6.2					Yellow			
Phenolphthalein					Colourless				8.3 – 10		← Pink →		
Thymolphthalein					Colourless					9.3 – 10.5	← Pink →		
Bromothymol Blue				Orange		6.0 – 7.5				Blue			
Bromocresol Green			Yellow	3.8 – 4.6						Blue			

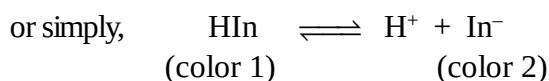
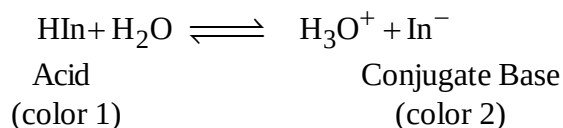
pH Range and color change of some common Acid-base indicators

End Point : It is the point at which the reaction between an acid and a base is “observed” to be complete i.e. the pH at which an indicator changes its color.

Equivalence Point: It is the point at which the reaction between an acid and a base is “theoretically” complete (as per the balanced equation for the reaction).

- Note :**
- At “equivalence point”, meq. of acid = meq. of base but the “end point” can only be detected by measuring the pH at different points of time during the titration. It is usually more convenient to add an indicator to the solution and detect the “observed end” of the reaction through a color change in the solution.
 - An indicator is usually a weak acid or a weak base for acid-base titration. The color change occurs when protons are transferred to form a conjugate base or a conjugate acid of a different color.

Let us represent an indicator as HIn (Weak Acid)



Necessary property of Indicator : Color of ionised form must be remarkably different from that of unionized form.

Now, K_{In} = Equilibrium Constant for the dissociation of the indicator = $\frac{[\text{H}^+][\text{In}^-]}{[\text{HIn}]}$

$$\Rightarrow \text{pH} = \text{p}K_{\text{In}} + \log_{10} \frac{[\text{In}^-]}{[\text{HIn}]}$$

Note : ➤ An indicator changes color over a pH range (the so called “*transition range*”). It is the range over which color change in the solution is detectable by the “*observer*”.

➤ The smaller the transition range, the better the indicator in detecting “*equivalence point*”. (*Equivalence Point* $\neq$ *End Point*)

With indicator in which both the forms, ionized and unionized form are colored, generally one colored is visible if the ratio of the concentration of the two forms is 10 : 1.

Thus, when only the color of un-ionized (HIn) form is seen: $\frac{[\text{HIn}]}{[\text{In}^-]} = \frac{10}{1}$

$$\Rightarrow \text{pH}_1 = \text{p}K_a + \log_{10} \frac{1}{10} = \text{p}K_a - 1$$

and when only the color of ionized (In^-) form is seen: $\frac{[\text{In}^-]}{[\text{HIn}]} = \frac{10}{1}$

$$\Rightarrow \text{pH}_2 = \text{p}K_a + \log_{10} \frac{10}{1} = \text{p}K_a + 1$$

Thus, pH change required in going from one color to other is $\Delta\text{pH} = \text{pH}_2 - \text{pH}_1 = 2$

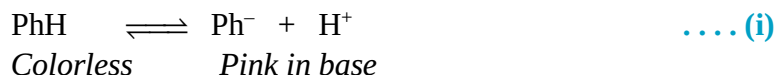
In the midway between the two colors (i.e. in the transition range), one can assume that $[\text{HIn}] = [\text{In}^-]$

$$\Rightarrow \text{pH} = \text{p}K_a + \log_{10} 1 = \text{p}K_a \quad \Rightarrow \quad \text{pH} = \text{p}K_a$$

➤ The $\text{p}K_a$ of the indicator should be close to the pH of the equivalence point

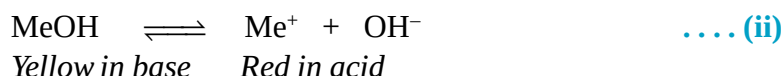
For weak-base indicators, same transition range is observed and $\text{pOH} = \text{p}K_b$ at the midway.

- (i) **Phenolphthalein** : It is a weak acid represented by PhH.



- Phenolphthalein is colorless in acid solution and pink in basic solution.
- When a strong base like NaOH is added, it displaces the equilibrium (I) towards left direction and there is a color change.

- (ii) **Methyl Orange** : It is a weak base represented as MeOH.



- When a strong acid is added, equilibrium (II) is shifted towards right and color change to red is detected..

Illustrating the concept :

- (a) What is the equivalence point of 0.1 M NH_4Br solution ?

$$[(\text{pK}_b)_{\text{NH}_4\text{OH}} = 4.74]$$

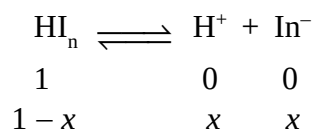
When a strong acid reacts with a strong base, equivalence point is the neutralisation point (i.e pH of the solution = 7 if both of acid and base have same meq.) but when a strong acid reacts with a weak base or vice-versa, pH at the equivalence point $\neq 7$ as the salt formed will get hydrolysed and pH of the solution > 7 or < 7 at the equivalence point. NH_4Br is a salt of Strong acid and Weak base.

$$\Rightarrow \text{pH} = 7 - \frac{1}{2}(\text{pK}_b + \log_{10} c) = 7 - \frac{1}{2}(4.74 + \log_{10} 0.1) = 5.13$$

- (b) What will be the color of the methyl red indicator in the solution of NH_4Br in part (a) ? At $\text{pH} < 3.1$, the indicator methyl red is colored pink, at $\text{pH} > 6.3$, it is yellow and at the intermediate values of the pH it is orange.

Since the pH of NH_4Br solution is 5.13 and it lies in the intermediate range of methyl orange, its color will be orange. ($3.1 < \text{pH} < 6.3$)

- (c) An acid indicator has $\text{pK}_a = 5.3$. In a certain solution, this indicator is found to be 80% ionised. What is the pH of the solution ?



Indicator is 80% ionized in acid form : $\Rightarrow x = 0.8$

$$\Rightarrow \text{pH} = \text{pK}_a + \log_{10} \frac{[\text{In}^{-1}]}{[\text{HIn}]} \quad \Rightarrow \text{pH} = 5.3 + \log_{10} \frac{0.8}{0.2} = 5.9$$

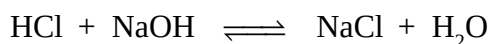
★★ Acid Base titration curves :

Titration Curve : It is a curve in which pH of the solution is titrated against volume of the titrate added.

(i) Strong Acid and Strong Base :

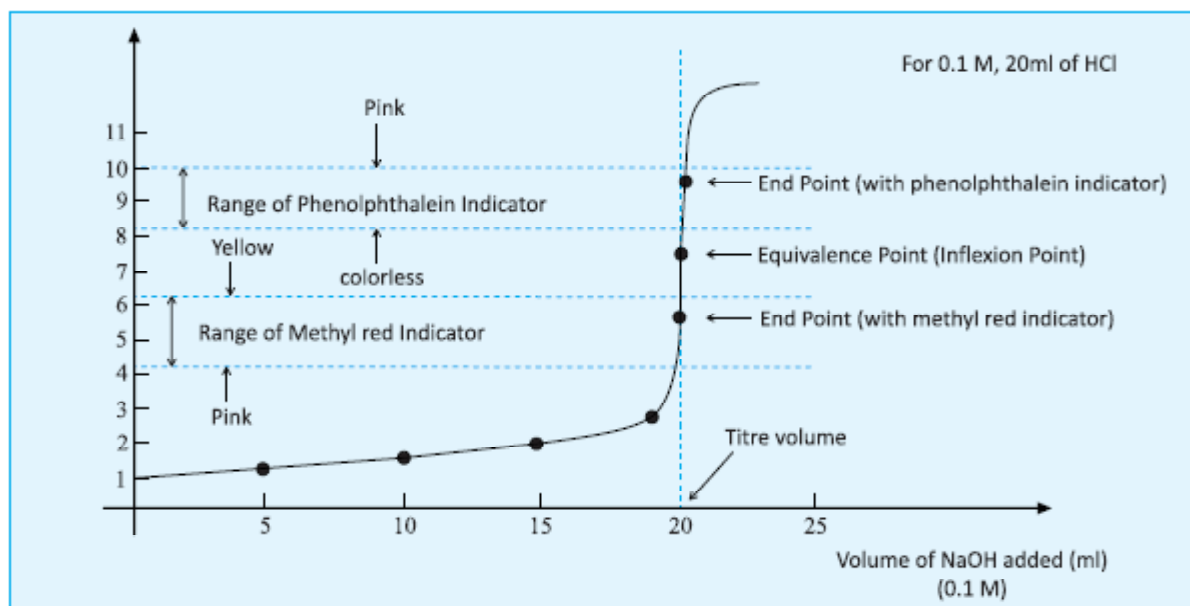


In this case, both the titrants are completely ionized.



As OH^- is added to an acid solution, $[\text{H}^+]$ decreases and $[\text{OH}^-]$ increases $\Rightarrow$ pH goes on increasing. As the equivalence point is reached, $[\text{H}^+]$ is rapidly reduced. Above this point, $[\text{OH}^-]$ increases rapidly from 10^{-7} M and after that pH of the solution remains fairly constant. Thus, there is an inflexion point at the equivalence point.

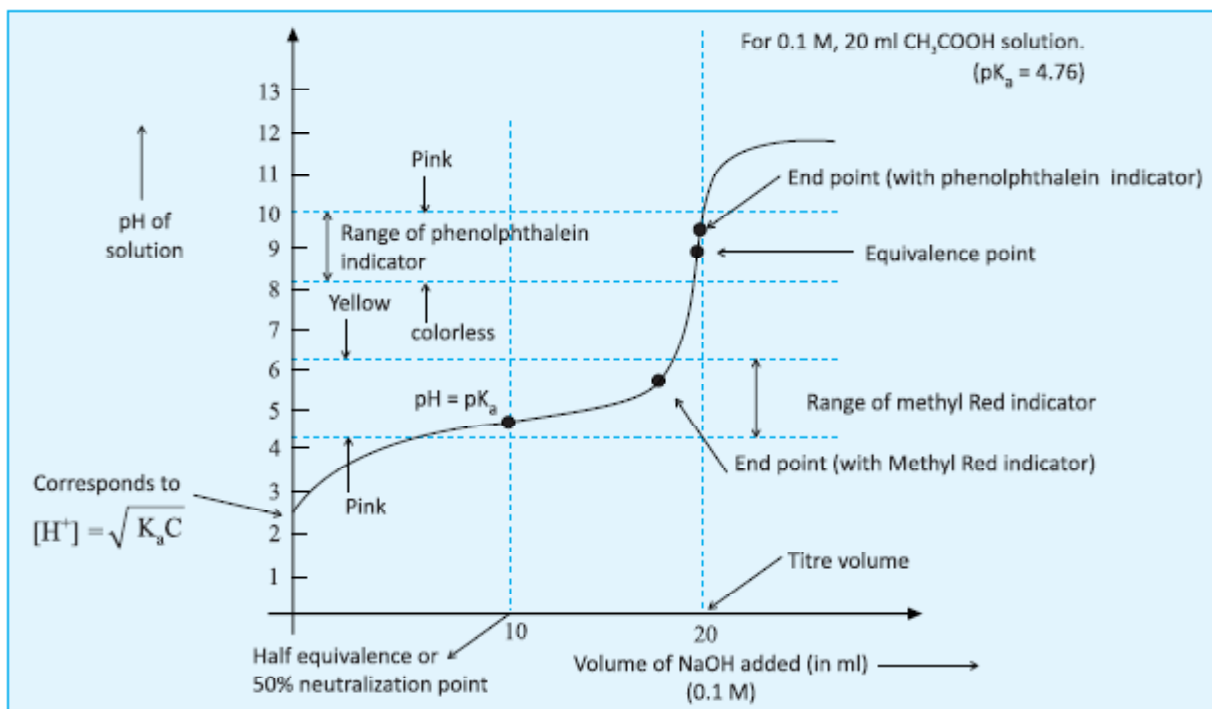
The difference in the volume of NaOH solution between the *end point* and the *equivalence point* is not significant for most of the commonly used indicators as there is a large change in the pH value around the equivalence point and most of them change their color across this pH change.



(ii) Weak Acid and Strong Base :



Initially, when a few drops of OH^- are added to the weak acid, it forms an acidic buffer $\text{CH}_3\text{COOH}/\text{CH}_3\text{COONa}$ and thus, pH of the solution changes slowly. At the equivalence point, solution will be alkaline. If OH^- addition is continued after the equivalence point, $[\text{OH}^-]$ will increase rapidly.



$$(\text{pH}) \text{ at equivalence point} = 7 + \frac{1}{2} (4.76 + \log_{10} c) = 7 + \frac{1}{2} (4.76 + \log_{10} 0.05) = 8.72$$

[Here c = concentration of the salt formed at equivalence point]

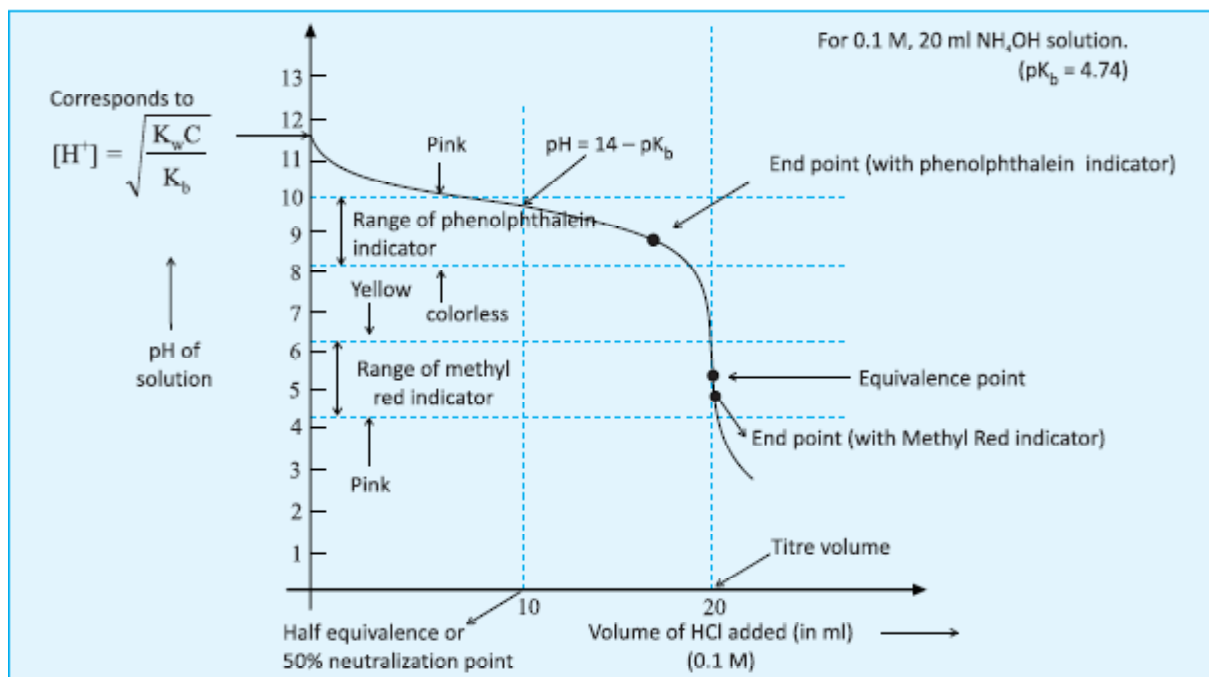
Thus, methyl red, methyl orange are inappropriate for use in the detection of the end point between a weak acid and a strong base.

Also note that the buffer capacity of the buffer solution will be maximum at 50% equivalence point i.e. when the volume of NaOH is 10 ml.

(iii) Strong Acid and Weak Base :

(HCl) (NH_4OH)

Initially, when a few drops of H^+ are added to the weak acid, it forms a basic buffer $\text{NH}_4\text{OH}/\text{NH}_4\text{Cl}$ and thus, pH of the solution changes slowly. At the equivalence point, solution will be alkaline. If H^+ addition is continued after the equivalence point, $[\text{H}^+]$ will increase rapidly.



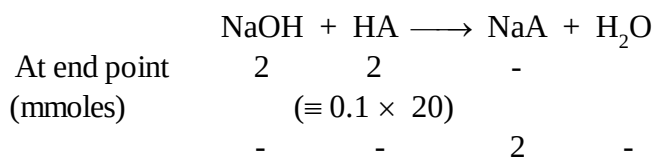
Thus, phenolphthalein, thymolphthalein are quite inappropriate to be used as an indicator to detect the end point between a strong acid and weak base neutralization.

Also note that the buffer capacity of the buffer solution will be maximum at 50% equivalence point i.e. when the volume of HCl is 10 ml.

Note : The selection of an indicator therefore is determined by the pH of the solution at the equivalence point.

Illustration - 21 0.1 M NaOH is titrated with 0.1 M, 20 ml HA till the end point. $K_a(\text{HA}) = 6 \times 10^{-6}$ and degree of dissociation of HA is negligible (small) as compared to unity. Calculate the pH of the resulting solution at the end point [Use $\log_{10} 6 \sim 0.8$]

SOLUTION :



Note : 20ml of NaOH is required for the complete neutralisation of HA.

NaA is a salt of strong base and weak acid. Thus, will undergo hydrolysis and solution will become basic.

$$\text{Here } c = [\text{NaA}] = \frac{2}{20 + 20} = 0.05 \text{ M}$$

$$\text{and } \text{pK}_a = -\log_{10} (6 \times 10^{-6}) = 5.2$$

$$\text{pH}_{\text{at end point}} = 7 + \frac{1}{2} (\text{pK}_a + \log_{10} c)$$

$$= 7 + \frac{1}{2} (5.2 + \log_{10} 0.05) = 8.95$$

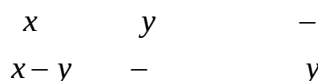
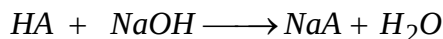


Illustration - 22 The equivalence point in a titration of 40.0 mL of a solution of a weak monoprotic acid occurs when 35.0 mL of a 0.10 M NaOH solution has been added. The pH of the solution is 5.5 after the addition of 2.0 mL of NaOH solution. What is the dissociation constant of the acid ?

SOLUTION :

Let x millimoles of HA are taken initially. Find x :

Consider the equation of neutralization :



At equivalent point, millimoles of acid = millimoles of acid = NaOH [i.e. $x = y$]

$\Rightarrow$ millimoles HA = $x = 3.5$ [35 ml of 0.1 M NaOH]

At pH = 5.5, millimoles of NaOH added = $2 = y$
[20 ml of 0.1 M NaOH]

$\Rightarrow$ millimoles of HA left $x - y = 1.5$

and millimoles of NaA formed = 2

Now such a solution will behave as an acidic buffer, whose pH is given as :

$$[H^+] = \frac{K_a [\text{acid}]}{[\text{salt}]} \Rightarrow 10^{-5.5} = K_a = \frac{1.5}{2}$$

$$\Rightarrow K_a = 4.22 \times 10^{-6} [10^{-5.5} = \sqrt{10} \times 10^{-6}]$$

SIMULTANEOUS EQUILIBRIUM

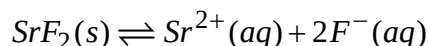
Section - 7

When two salts having a common ion (either cation or anion) are together in water, then their respective solubilities are not independent of each other.

Illustrating the concept :

Consider a solution containing two salt : CaF_2 ($K_{sp} = 3.4 \times 10^{-11}$) and SrF_2 ($K_{sp} = 2.9 \times 10^{-9}$). Compare their K_{sp} values. Let us assume that most of F^- ion concentration in the saturated solution is from SrF_2 , as its K_{sp} is much higher than that of CaF_2 . It means that first SrF_2 will establish its equilibrium (as if there is no CaF_2), then CaF_2 will dissolve in presence of ions furnished only by SrF_2 .

Let the solubility of SrF_2 be x mol/L.



$$\Rightarrow [Sr^{2+}] = x \text{ and } [F^-] = 2x$$

$$K_{sp} = [Sr^{2+}][F^-]^2 = (x)(2x)^2 = 4x^3$$

$$\Rightarrow x = \left(\frac{K_{sp}}{4} \right)^{1/3} = 9 \times 10^{-4}$$

Now the solubility of CaF_2 is determined in presence of $9 \times 10^{-4} \text{ M F}^-$ ions. It means neglect the contribution of F^- ions from CaF_2 .

$$K_{sp} = [\text{Ca}^{2+}][\text{F}^-]^2 \Rightarrow [\text{Ca}^{2+}] = \frac{K_{sp}}{[\text{F}^-]^2} = \frac{3.4 \times 10^{-11}}{(2 \times 9 \times 10^{-4})^2} = 1.0 \times 10^{-5} \text{ M}$$

Hence the solubility of CaF_2 is $1.0 \times 10^{-5} \text{ mol/L}$.

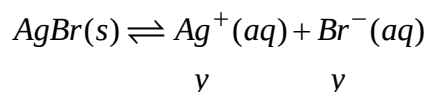
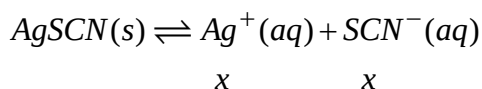
Note : Let us check our assumption. F^- ions from CaF_2 is twice the amount of Ca^{2+} ions i.e., $2.0 \times 10^{-5} \text{ M}$, whereas, F^- ions from SrF_2 is $2 \times 9.0 \times 10^{-4}$ i.e. $1.8 \times 10^{-3} \text{ M}$, which is much higher than $2.0 \times 10^{-5} \text{ M}$.

Illustration - 23 Calculate the simultaneous solubilities of AgSCN and AgBr .

$$K_{sp}(\text{AgSCN}) = 1.0 \times 10^{-12} \quad ; \quad K_{sp}(\text{AgBr}) = 5.0 \times 10^{-13}$$

SOLUTION :

In this case, please note that the K_{sp} values of two salts are very similar. So the concentration of Ag^+ ions (the common ion) can not be calculated from a single salt alone and we have to consider the equilibrium of the two salts simultaneously. Let the simultaneous solubilities of AgSCN and AgBr be x and y respectively in mol/L .



At equilibrium :

$$[\text{Ag}^+] = x + y; \quad [\text{SCN}^-] = x; \quad [\text{Br}^-] = y$$

$$[\text{Ag}^+][\text{Br}^-] = K_{sp} \text{ AgBr}$$

$$\text{and} \quad [\text{Ag}^+][\text{SCN}^-] = K_{sp} \text{ AgSCN}$$

$$\text{Note that : } [\text{Ag}^+] = [\text{Br}^-] + [\text{SCN}^-]$$

[This is an Electrical charge neutrality equation]

$$[\text{Ag}^+] = \frac{K_{sp} \text{ AgBr}}{[\text{Ag}^+]} + \frac{K_{sp} \text{ AgSCN}}{[\text{Ag}^+]}$$

$$\Rightarrow [\text{Ag}^+] = \sqrt{K_{sp} \text{ AgBr} + K_{sp} \text{ AgSCN}} \\ = x + y = 1.22 \times 10^{-6} \quad \dots \text{(i)}$$

$$\text{Also, } \frac{[\text{Br}^-]}{[\text{SCN}^-]} = \frac{y}{x} = \frac{K_{sp} \text{ AgBr}}{K_{sp} \text{ AgSCN}} = 0.5 \quad \dots \text{(ii)}$$

Using (i) and (ii), we get :

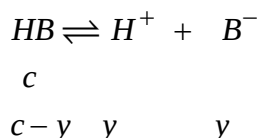
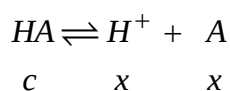
$$x = 8.0 \times 10^{-7}; \quad y = 4.0 \times 10^{-7}$$

Illustration - 24 Two weak monobasic organic acids HA and HB have dissociation constants as 3.0×10^{-5} and 1.5×10^{-5} respectively at 25°C . If 500 ml of 1 M solutions of each of these two acids are mixed to produce 1 litre of mixed acid solution, what is the pH of the resulting solution ?

SOLUTION :

Note that K_a of two acids is nearly same. In such cases, we have to consider H^+ from both HA and HB simultaneously. The concentration of HA and HB in the mixture = 5.0 M

[equal volume are mixed] = cM



Now, $[H^+]_{\text{final}} = [H^+]_{\text{from HA}} + [H^+]_{\text{from HB}}$

$$\Rightarrow K_{aHA} = \frac{(x+y)x}{c-x} \text{ and } K_{aHB} = \frac{(x+y)y}{c-y}$$

As K_a of both acid $\sim 10^{-5}$ and H^+ from one acid acts as common ion for other's dissociation, x and y are very less as compared to c.

$$\Rightarrow c-x \approx c \text{ and } c-y \approx c$$

$$\Rightarrow K_{a(HA)} \approx \frac{(x+y)x}{c} \text{ and}$$

$$K_{a(HB)} \approx \frac{(x+y)y}{c}$$

Divide the two expressions to get :

$$\frac{1}{2} = \frac{x}{y} \Rightarrow x = 2y$$

$$\text{Substitute for } y = \frac{1}{2}x \text{ in } K_{a(HA)} = \frac{x^2 + xy}{c}$$

$$\Rightarrow 3.0 \times 10^{-5} = \frac{x^2 + 0.5x^2}{0.5}$$

$$\Rightarrow x = \sqrt{10} \times 10^{-3} \text{ M}$$

$$\text{and } y = \frac{\sqrt{10}}{2} \times 10^{-3} \text{ M}$$

$$[H^+] = x + y = \frac{3\sqrt{10}}{2} \times 10^{-3} \text{ M}$$

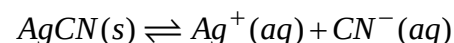
$$pH = -\log_{10} \left(\frac{3\sqrt{10}}{2} \times 10^{-3} \right) \Rightarrow pH = 2.32$$

Illustration - 25 Determine the solubility of AgCN in a buffer solution maintained at pH = 3.

$$K_{sp}(\text{AgCN}) = 2.0 \times 10^{-16} ; K_a(\text{HCN}) = 6.0 \times 10^{-10}$$

SOLUTION :

AgCN is a sparingly soluble salt in aqueous medium.

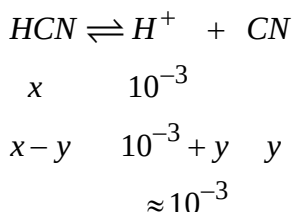
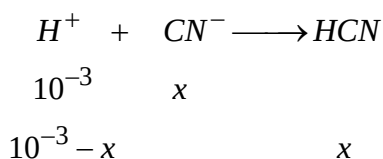


Let the solubility of AgCN be x M. Thus,

$[\text{Ag}^+] = x\text{M}$ and $[\text{CN}^-] = x\text{M}$ from salt but it will not be equal to x finally.

The CN^- ions will react with H^+ in the solution to form HCN. (a weakly dissociated acid) and we have to assume that in solution $[\text{CN}^-] = y\text{M}$ finally which will be decided by the dissociation of HCN as

explained below : $\text{AgCN}(s) \rightleftharpoons \text{Ag}^+(aq) + \text{CN}^-(aq)$



We have two equations now :

$$K_{sp}(AgCN) = [Ag^+][CN^-] = xy = 2 \times 10^{-16}$$

and

$$K_a(HCN) = \frac{[H^+][CN^-]}{[HCN]} = \frac{10^{-3}y}{x-y} = 6 \times 10^{-10}$$

Solve the equation to get :

$$x = [Ag^+] = \sqrt{\frac{K_{sp}}{K_a}([H^+] + K_a)} = 1.82 \times 10^{-5} \text{ M}$$

Note that in the solution :

$$[Ag^+] = [CN^-] + [HCN]$$

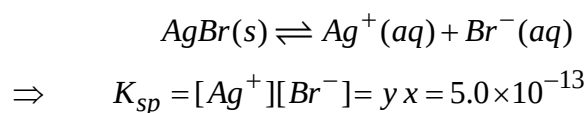
(Electrical charge neutrality equation)

Note : In this example, since the dissociation constant of acid is very low and pH of the solution is fairly high, we could have assumed that $[H^+]$ and K_a in the expression for x .

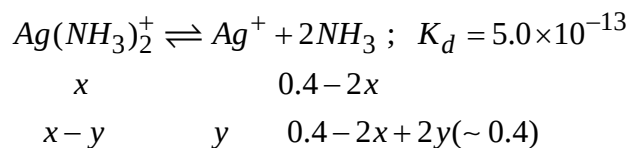
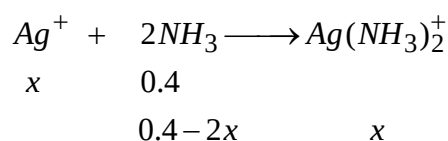
Illustration - 26 How much AgBr could dissolve in 1.0 L of 0.4 M NH_3 ? Assume that $Ag(NH_3)_2^+$ is the only complex formed. Given: The dissociation constant for $Ag(NH_3)_2^+ \rightleftharpoons Ag^+ + 2NH_3$; $K_d = 6.0 \times 10^{-8}$ and $K_{sp}(AgBr) = 5.0 \times 10^{-13}$.

SOLUTION :

Let solubility of AgBr be x M. Thus, $[Br^-] = x$ M
but $[Ag^+] \neq x$ M since it will react with NH_3 to form a complex and thus, its concentration will be decided by the dissociation of the complex. So, let $[Ag^+] = y$ M.



Since the formation constant (K_f) of the complex is very high, assume that whole of Ag^+ formed is consumed.



$$K_d = \frac{[Ag^+][NH_3]^2}{[Ag(NH_3)_2^+]} = \frac{y(0.4 - 2x + 2y)^2}{x - y} = 6 \times 10^{-8}$$

Ionic Equilibrium

Assuming $x - y \approx x$ since K_d is low and $\ll 0.4$, we get :

$$K_d = \frac{y(0.4)^2}{x}$$

Solving for x : $x = 1.15 \times 10^{-3} \text{ M}$
(Verify the approximation yourself)

Illustration - 27 HN_3 (hydroazoic acid) is a weak acid dissociating as : $\text{HN}_3 \rightleftharpoons \text{H}^+ + \text{N}_3^-$. Find the concentration of Ag^+ ions, if excess of solid AgN_3 is added to a solution maintained at $\text{pH} = 4$. The ionization constant K_a of HN_3 is 2.0×10^{-5} . The solubility of AgN_3 in pure water is found to be $5.4 \times 10^{-3} \text{ M}$ at 25°C .

SOLUTION :

AgN_3 is a sparingly soluble salt, dissociating in water as : $\text{AgN}_3 \rightleftharpoons \text{Ag}^+ + \text{N}_3^-$

$$K_{sp} = [\text{Ag}^+][\text{N}_3^-]$$

Since solubility of AgN_3 in water is 5.4×10^{-3} ,

$$K_{sp} = (5.4 \times 10^{-3})^2 = 2.92 \times 10^{-5}$$

Now we have to find the solubility of AgN_3 in solution having $\text{pH} = 4$.

Let solubility of AgN_3 be $x \text{ M}$ at $\text{pH} = 4$.

$$\Rightarrow [\text{Ag}^+] = x \text{ M}, \text{ but } [\text{N}_3^-] \neq x \text{ M}$$

($[\text{N}_3^-]$ will be decided by dissociation of HN_3).

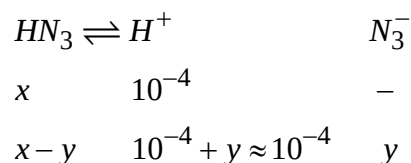
Let $[\text{N}_3^-] = y \text{ M}$

First, assume that whole of N_3^- ($x \text{ M}$) formed from

AgN_3 reacts with H^+ ions to form HN_3 .

$$\Rightarrow [\text{HN}_3] = x \text{ M}$$

Now HN_3 dissociates as follows :



[As pH is maintained at 4 hence $[\text{H}^+] = 10^{-4} \text{ M}$]

Now we have a simultaneous equilibrium in aqueous solution involving dissociation of AgN_3 and HN_3 .

$$K_{sp} \text{ of } \text{AgN}_3 = [\text{Ag}^+][\text{N}_3^-] = xy \quad \dots \text{(i)}$$

$$K_a \text{ of } \text{HN}_3 = \frac{[\text{H}^+][\text{N}_3^-]}{[\text{HN}_3]} = \frac{[\text{H}^+]y}{(x - y)} \quad \dots \text{(ii)}$$

Solving (i) and (ii) simultaneously, we have :

$$x = [\text{Ag}^+] = \sqrt{\frac{K_{sp}}{K_a} ([\text{H}^+] + K_a)}$$

Substitute the values of K_{sp} , K_a and $[\text{H}^+]$ to get $x = 0.0132 \text{ mol/L}$

Note : In this example, since the dissociation constant of acid is quite high and pH of the solution is quite low, we can not assumed that $x - y \approx x$. You can check the same by comparing $[\text{H}^+]$ and K_a in the expression for x .

★★ SUBJECTIVE SOLVED EXAMPLES

Example - 1 A solution contains a mixture of Ag^+ (0.1M) and Hg_2^{2+} (0.10 M) which are separated by selective precipitation. Calculate maximum concentration of Iodide ion at which one of them gets precipitated almost completely. What % age of that metal ion is precipitated ?
 ($K_{sp} \text{ AgI} = 8.5 \times 10^{-17} \text{ M}$; $K_{sp} \text{ HgI}_2 = 2.5 \times 10^{-26}$)

SOLUTION :

First determine, which ion starts precipitating. first

$$[\text{I}^-]_{\text{Min. for AgI}} = \frac{K_{sp} \text{ of AgI}}{[\text{Ag}^+]} \\ = \frac{8.5 \times 10^{-17}}{0.1} = 8.5 \times 10^{-16} \text{ M}$$

$$[\text{I}^-]_{\text{Min. for Hg}_2\text{I}_2} = \sqrt{\frac{K_{sp} \text{ of Hg}_2\text{I}_2}{[\text{Hg}_2^{2+}]}} \\ = \sqrt{\frac{2.5 \times 10^{-26}}{0.1}} = 5 \times 10^{-13} \text{ M}$$

This means that AgI will be precipitated first as $[\text{I}^-]$ required for AgI is less.

But when $[\text{I}^-]$ reaches $5 \times 10^{-13} \text{ M}$ then precipitation of Hg_2I_2 also starts.

So $[\text{Ag}^+]$ left at that stage is given as :

$$[\text{Ag}^+] = \frac{K_{sp} \text{ of AgI}}{[\text{I}^-]_{\text{Hg}_2\text{I}_2}} \\ = \frac{8.5 \times 10^{-17}}{5.0 \times 10^{-13}} = 1.7 \times 10^{-4} \text{ M}$$

% of Ag^+ left un-precipitated

$$= \frac{1.7 \times 10^{-4}}{0.1} \times 100 = 0.17\%$$

$\Rightarrow$ % age of Ag^+ = 99.83 %

This means when Hg_2I_2 starts precipitating, Ag^+ is almost precipitated.

Example - 2 Carbonic acid, H_2CO_3 , is a diprotic acid for which $K_1 = 10^{-7}$ and $K_2 = 10^{-11}$. Which solution will produce a pH closest to 9 ?

(A) 0.1M H_2CO_3

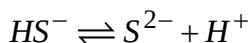
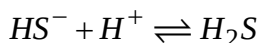
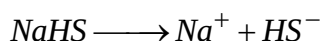
(B) 0.1M Na_2CO_3

(C) 0.1 M NaHCO_3

(D) 0.1M NaHCO_3 and 0.1 M Na_2CO_3

SOLUTION :

Amphiprotic salt is a salt that can donate proton well as can accept a proton. For example : NaHS , NaHCO_3 etc.



Ionic Equilibrium

pH of amphiprotic salt of weak acid is calculated by using following expression.

$$pH = \frac{1}{2} [pK_{a_1} + pK_{a_2}]$$

$$pH = \frac{1}{2} [pK_{a_1} + pK_{a_2}] = \frac{1}{2} [7 + 11] = 9$$

Example - 3

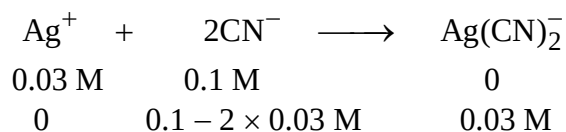
For the reaction $Ag(CN)_2^- \rightleftharpoons Ag^+ + 2CN^-$, in an aqueous medium, the equilibrium constant K_C at $25^\circ C$ is 4×10^{-19} . Calculate the Ag^+ ion concentration in the solution which was originally 0.1 M in KCN and 0.03 M $AgNO_3$.

SOLUTION :

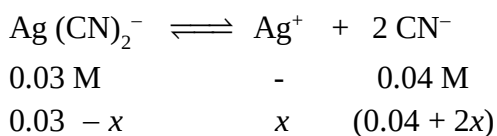
0.1 M KCN and 0.03 M $AgNO_3$ are mixed.

Since K_{eq} for $Ag^+ + 2CN^- \rightleftharpoons Ag(CN)_2^-$ is very-

very high $\left(K = \frac{1}{4} \times 10^{19} \right)$, first assume that whole of Ag^+ is converted to $Ag(CN)_2^-$.



Now, Let 0.03 M $Ag(CN)_2^-$ ($K_{eq} = 4 \times 10^{-19}$) to dissociate as follows :



$$K_C = \frac{[Ag^+][CN^-]^2}{[Ag(CN)_2^-]} = 4 \times 10^{-19}$$

$$\begin{aligned} \Rightarrow [Ag^+] &= \frac{K_C [Ag(CN)_2^-]}{[CN^-]^2} \\ &= \frac{4 \times 10^{-19} [0.03 - x]}{(0.04 + 2x)^2} \end{aligned}$$

Solve the above equation by assuming 'x' to be very small.

$$\begin{aligned} \Rightarrow [Ag^+] &= \frac{4 \times 10^{-19} [0.03]}{(0.04)^2} = 7.5 \times 10^{-18} M \\ &\quad \text{[Verify the approximation yourself]} \end{aligned}$$

Example - 4

Calculate $[NH_4^+]$ (derived from NH_4Cl) needed to prevent $Mg(OH)_2$ from precipitating in a litre of solution which contains 0.2 mole of ammonia and 0.001 mole of Mg^{2+} ions. The ionisation constant of ammonia is 2.0×10^{-5} and K_{sp} of $Mg(OH)_2$ is 1.0×10^{-11} .

SOLUTION :

Note that aq. ammonia is NH_4OH .

First check whether we need to add NH_4Cl .

$$[OH^-]_{\text{From } NH_3} = \sqrt{K_b c} = 2 \times 10^{-3} M$$

$$\begin{aligned} \Rightarrow \text{I.P.}_{Mg(OH)_2} &= 10^{-3} \times (2 \times 10^{-3})^2 \\ &= 4 \times 10^{-9} > K_{sp} \end{aligned}$$

Thus, precipitation will take place if NH_4Cl is not added and hence, NH_4Cl is required to be added so as to suppress the dissociation of NH_4OH . As a result, $[\text{OH}^-]$ will decrease.

Now, calculate the concentration of OH^- ions in equilibrium with Mg^{2+} ions from K_{sp} of $\text{Mg}(\text{OH})_2$.

$$K_{sp} \text{ of } \text{Mg}(\text{OH})_2 = [\text{Mg}^{2+}][\text{OH}^-]^2$$

$$\Rightarrow [\text{OH}^-] = \sqrt{\frac{K_{sp}}{[\text{Mg}^{2+}]}} = \sqrt{\frac{1.0 \times 10^{-11}}{0.001}} = 1.0 \times 10^{-4}$$

If $[\text{OH}^-] \leq 1.0 \times 10^{-4} \text{M}$, Mg^{2+} ions will not be precipitated. So, maximum concentration of OH^- in the solution should be $1.0 \times 10^{-4} \text{M}$. As $[\text{OH}^-]$ is derived from dissociation of ammonia.

$$\text{NH}_4\text{OH} \rightleftharpoons \text{NH}_4^+ + \text{OH}^-$$

$$\Rightarrow [\text{NH}_4^+] = \frac{K_b[\text{NH}_4\text{OH}]}{[\text{OH}^-]}$$

$$\Rightarrow [\text{NH}_4^+]_{\min} = \frac{(2.0 \times 10^{-5})(0.2)}{[1.0 \times 10^{-4}]}$$

$$= 4.0 \times 10^{-2} \text{M}$$

For this much concentration of NH_4^+ ions, Mg^{2+} will not be precipitated.

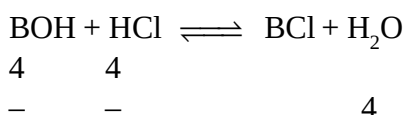
Example - 5

When a 40 mL of a 0.1 M weak base, BOH is titrated with 0.10 M HCl, the pH of the solution at the end point is 5.5. What will be the pH if 10 mL of 0.10 M NaOH is added to the resulting solution?

SOLUTION :

At the end point, pH = 5.5 (i.e. an acidic solution). It means salt formed at the end of neutralisation undergoes hydrolysis.

For 4 millimoles (40×0.1) of weak base, 4 millimoles of acid is required at the end point.



$\Rightarrow$ millimoles of salt (BCl) formed = 4

$\Rightarrow [\text{BCl}] = 4/V$ where $V = V_{\text{HCl}} + V_{\text{base}}$

At the end point : $4 = 0.10 \times V_{\text{HCl}}$

$\Rightarrow V_{\text{HCl}} = 40 \text{ mL}$

$\Rightarrow V = 40 + 40 = 80 \text{ mL}$

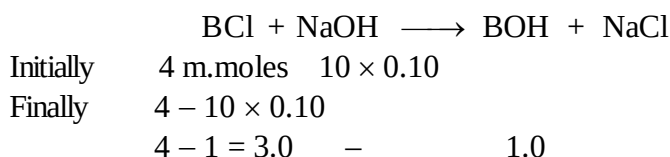
$\Rightarrow [\text{BCl}] = 4 / 80 = 0.05 \text{ M}$

Note that BCl is a salt of strong acid and weak base.

$$\Rightarrow [H^+] = \sqrt{\frac{K_w c}{K_b}}$$

$$\Rightarrow K_b = \frac{K_w c}{[H^+]^2} = \frac{(10^{-14}) \times (0.05)}{(10^{-5.5})^2} = 5.0 \times 10^{-5}$$

When 10 mL of 0.12 M NaOH is added :



It means a basic buffer containing 3.0 m.moles of BCl and 1.0 m.moles of BOH is formed. Find the pH by using Henderson's equation for basic buffer.

$$\text{pOH} = \text{p}K_b + \log \frac{[\text{BCl}]}{[\text{BOH}]}$$

$$\Rightarrow \text{pOH} = -\log(5 \times 10^{-5}) + \log \frac{[3.0/V]}{[1.0/V]}$$

$$\Rightarrow \text{pOH} = 4.77 \quad (\text{p}K_b = 4.3)$$

$$\Rightarrow \text{pH} = 14 - \text{pOH} = 9.22$$

Example - 6

It is given that 0.001 mol each of Cd^{2+} and Fe^{2+} ions are contained in 1.0 L of 0.02M HCl solution. This solution is now saturated with H_2S gas at 25°C .

- (a) Determine whether or not each of these ions will be precipitated as sulphide ?
 (b) How much Cd^{2+} ions remain in the solution at equilibrium ?

$$K_1(\text{H}_2\text{S}) = 1.0 \times 10^{-7} ; K_2(\text{H}_2\text{S}) = 1.0 \times 10^{-14} ; K_{sp}(\text{CdS}) = 8 \times 10^{-27} ; K_{sp}(\text{FeS}) = 3.7 \times 10^{-19}$$

SOLUTION :

Note that two salts are of same type (i.e., AB type) and K_{sp} of CdS is lower than that of FeS. This means CdS will precipitate first if at all any precipitation takes place.

Calculate the minimum concentration of Sulphide ion required to initiate the precipitation of each of the metal sulphide.

$$[\text{S}^{2-}]_{\text{Min for CdS}}$$

$$= \frac{K_{sp} \text{ CdS}}{[\text{Cd}^{2+}]} = \frac{8 \times 10^{-27}}{0.001} = 8 \times 10^{-24} \text{ M}$$

$$[\text{S}^{2-}]_{\text{Min for FeS}}$$

$$= \frac{K_{sp} \text{ FeS}}{[\text{Fe}^{2+}]} = \frac{3.7 \times 10^{-19}}{0.001} = 3.7 \times 10^{-16} \text{ M}$$

Now calculate the sulphide ion concentration in the saturated solution, using :

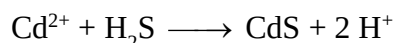
$$K_a = K_1 \times K_2 = \frac{[\text{H}^+]^2 [\text{S}^{2-}]}{[\text{H}_2\text{S}]} = 10^{-21}$$

$$\Rightarrow [\text{S}^{2-}] = \frac{10^{-21} \times 0.1}{0.02^2} = 2.5 \times 10^{-19} \text{ M}$$

$$\Rightarrow [\text{Cd}^{2+}]_{\text{Left}} = \frac{8 \times 10^{-27}}{2.5 \times 10^{-19}} = 3.2 \times 10^{-8} \text{ M}$$

Thus, only CdS will get precipitated.

To calculate the remaining concentration of Cd^{2+} ion (more accurately) ion in the solution, first assume that whole of the Cd^{2+} has been precipitated as :



$$\text{Thus, } [\text{H}^+]_{\text{new}} = 0.02 + 0.001 \times 2 = 0.022 \text{ M}$$

$$\Rightarrow [\text{S}^{2-}]_{\text{At new equilibrium}} = \frac{10^{-21} \times 0.1}{0.022^2} = 2.07 \times 10^{-19} \text{ M}$$

$$\Rightarrow [\text{Cd}^{2+}]_{\text{Left}} = \frac{8 \times 10^{-27}}{2.07 \times 10^{-19}} = 3.86 \times 10^{-8} \text{ M}$$

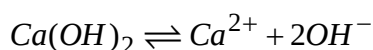
Note : FeS will not be precipitated at all.

Example - 7

The solubility product, K_{sp} of $\text{Ca}(\text{OH})_2$ at 25°C is 4.0×10^{-6} . A 500 mL of saturated of $\text{Ca}(\text{OH})_2$ is mixed with equal volume of 0.4 M NaOH solution. How much $\text{Ca}(\text{OH})_2$ in milligrams is precipitated ?

SOLUTION :

First find the concentration of Ca^{2+} ions at saturation using K_{sp} of $\text{Ca}(\text{OH})_2$.



$$\Rightarrow K_{sp} = [\text{Ca}^{2+}][\text{OH}^-]^2$$

$$\text{Let } [Ca^{2+}] = x \text{ mol / L} \quad \Rightarrow \quad [OH^-] = 2x$$

$$\Rightarrow K_{sp} = 4x^3 \text{ or } x = \sqrt[3]{\frac{K_{sp}}{4}} = 0.01 M$$

$$\Rightarrow [Ca^{2+}] = 0.01 M \text{ and } [OH^-] = 0.02 M$$

As equal volumes of saturated solution and 0.4 M NaOH are mixed :

$$[Ca^{2+}] = \frac{0.01}{2} = 5 \times 10^{-3} M$$

$$\text{and } [OH^-]_{Total} = \frac{0.02}{2} + \frac{0.4}{2} = 0.21 M$$

$$[\because [OH^-]_{Total} = [OH^-]_{NaOH} + [OH^-]_{From Ca(OH)_2}]$$

$$\text{Now, calculate I. } P_{Ca(OH)_2} = [Ca^{2+}][OH^-]^2 = (5 \times 10^{-3})(0.21)^2 = 2.2 \times 10^{-4} > K_{sp}$$

Since concentration of OH^- is quite high, Ca^{2+} will be precipitated till a new saturation state is reached.

Let at new saturated state. $[OH^-] = 0.21 M$ (Assuming no change in $[OH^-]$)

$$[Ca^{2+}]_{left} = \frac{K_{sp}}{[OH^-]^2} = \frac{(4 \times 10^{-6})}{(0.21)^2} = 9.07 \times 10^{-5} M$$

$$\Rightarrow [Ca^{2+}]_{precipitated} = 5 \times 10^{-3} - 9.07 \times 10^{-5} M = 4.91 \times 10^{-3} M$$

$$\Rightarrow \text{amount of } Ca^{2+} = 4.91 \times 10^{-3} \times 74 \times 10^3 \text{ mg / L} = 363.3 \text{ mg / L}$$

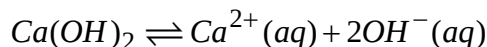
Check the approximation : $[OH^-]_{left} \sim 0.21 M$. How ??

Find out the $[OH^-]_{used} = 2[Ca^{2+}]_{used}$ (see Reaction's Stoichiometry)

$$= 2 \times 4.91 \times 10^{-3} M = 9.8 \times 10^{-3} M$$

$$\Rightarrow [OH^-]_{left} = 0.21 - 9.8 \times 10^{-3} M = 0.2 M \approx 0.21 M$$

Alternate Approach :



$$t = t_{eq} : \text{ concentration:} \quad \begin{array}{cc} 0.01 & 0.02 \end{array} \quad I.P. = K_{sp}$$

$$t = 0(\text{new}) \text{ concentration:} \quad \begin{array}{cc} 5 \times 10^{-3} & 0.21 \end{array} \quad I.P. > K_{sp}$$

(After mixing equal volumes with NaOH)

$$t = t_{eq}(\text{new}) \quad \begin{array}{cc} 5 \times 10^{-3} - x & 0.21 - 2x \end{array} \quad I.P. = K_{sp}$$

Ionic Equilibrium

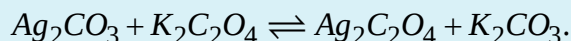
Now, $I.P. = K_{sp}$ at equilibrium

$$(5 \times 10^{-3} - x)(0.21 - 2x)^2 = 4 \times 10^{-6} \Rightarrow \text{This equation in 'x' is clearly a cubic.}$$

To solve the above equation approximately, assume $0.21 - 2x \sim 0.21$ which is what has been done in the previous approach.

Example - 8

The solubility product of $Ag_2C_2O_4$ at $25^\circ C$ is 1.20×10^{-11} . A solution of $K_2C_2O_4$ containing 0.15 moles in 500 ml water is mixed with excess of Ag_2CO_3 till the following equilibrium established :

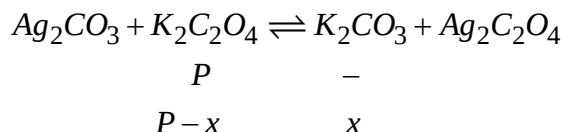


At equilibrium, the solution contains 0.03 mole of K_2CO_3 . Assuming that the degree of dissociation of $K_2C_2O_4$ and K_2CO_3 to be equal, calculate the solubility product of Ag_2CO_3 . [Take 100% ionisation of $K_2C_2O_4$ and K_2CO_3]

SOLUTION :

$$[K_2C_2O_4]_{\text{initial}} = \frac{0.15}{0.5} = 0.30 M = P$$

$$\text{and } [K_2CO_3]_{\text{final}} = \frac{0.03}{0.5} = 0.06 M = x$$



1 mol $K_2CO_3 \equiv$ 1 mole $K_2C_2O_4$ and since both are completely ionized :

Now find $[Ag^+]$ at equilibrium as :

$$\begin{aligned} [Ag^+] &= \sqrt{\frac{K_{sp} \text{ of } Ag_2C_2O_4}{[C_2O_4^{2-}]}} \\ &= \sqrt{\frac{1.20 \times 10^{-11}}{P - x}} = \sqrt{\frac{1.20 \times 10^{-11}}{0.3 - 0.06}} \\ &= 7.07 \times 10^{-6} M \end{aligned}$$

$$[CO_3^{2-}]_{\text{final}} = x = 0.06 M$$

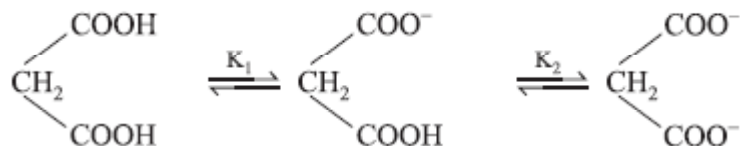
$$\text{Now, } K_{sp} \text{ of } Ag_2CO_3 = [Ag^+]^2 [CO_3^{2-}]$$

$$\begin{aligned} \Rightarrow K_{sp} \text{ of } Ag_2CO_3 &= (7.07 \times 10^{-6})^2 \times 0.06 \\ &= 3 \times 10^{-12} \end{aligned}$$

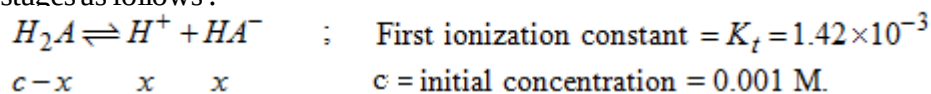
Example - 9 Malonic acid is an organic dibasic acid like H_2S having first ionization constant, $K_1 = 1.42 \times 10^{-3}$ and second ionization constant, $K_2 = 2.0 \times 10^{-6}$. Compute the divalent malonate ion concentration in :

- (a) 0.001 M Malonic acid (b) in a solution that is 0.0001 M in malonic acid and 0.0004 M HCl.
 (c) in a solution that is 0.0001 M in malonic acid and 0.1 M HCl.

SOLUTION :



- (a) Consider malonic acid to be H_2A , where malonate ion is A^{2-} . For dibasic acids, we consider ionization in two stages as follows :



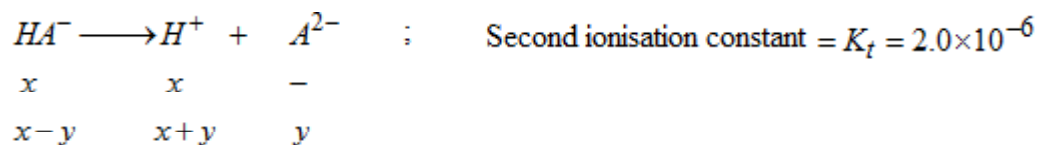
$$\Rightarrow K_1 = \frac{[H^+][HA^-]}{[H_2A]} = \frac{x^2}{c-x} = 1.42 \times 10^{-3}$$

$$\text{Solving the quadratic, } x^2 + 1.42 \times 10^{-3}x - 1.42 \times 10^{-6} = 0 \Rightarrow x = 6.75 \times 10^{-4} \text{ M}$$

Note that $K_1 \gg K_2$. so concentration of H^+ in solution is considered only from dissociation i.e.,

$$[H^+] = x \text{ M (neglect } H^+ \text{ concentration from second ionization).}$$

Consider second ionisation :



$$\Rightarrow K_2 = \frac{[H^+][A^{2-}]}{[HA^-]} = \frac{(x+y)y}{x-y}$$

$$\text{Note : } [H^+] [H^+] = x+y = x \quad [HA^-] = x-y = x \quad (y \ll x)$$

$$\Rightarrow K_2 = y = [A^{2-}] = 2.0 \times 10^{-6} \text{ M}$$

Note : Usually, $[A^{2-}]$ for H_2A (dibasic acid) can be approximately taken to be K_2 .

(b) Here $= 10^{-4} M$, $[H^+] = 4 \times 10^{-4} M$ (From HCl)

$$\Rightarrow K_1 = \frac{[H^+][HA]}{[H_2A]} = \frac{(4 \times 10^{-4} + x)(x)}{10^{-4} - x} = 1.42 \times 10^{-3}$$

Important: Note that, we can not go for approximation in this case, since the concentration of HCl is so low that H^+ is considered both from HCl and molonic acid.

Solving the quadratic, $x^2 + 1.82 \times 10^{-3}x - 1.42 \times 10^{-7} = 0 \Rightarrow x = 7.5 \times 10^{-5} M$

Now consider, second ionisation and substitute for value of x.

$$K_2 = \frac{[H^+][A^{2-}]}{[HA]} = \frac{(4 \times 10^{-4} + x)}{x} = 2.0 \times 10^{-6}$$

$$\Rightarrow \frac{(4 \times 10^{-4} \times 7.5 \times 10^{-5})y}{7.5 \times 10^{-5}} = 2.0 \times 10^{-6}$$

$$\Rightarrow y = [A^{2-}] = 3.2 \times 10^{-7} M$$

(c) Here $c = 10^{-4} M$, $[H^+] = 0.1 M$

$$\Rightarrow K_a = K_1 K_2 = \frac{[H^+]^2 [A^{2-}]}{[H_2A]} = \frac{(0.1)^2 [A^{2-}]}{10^{-4}} = 2.84 \times 10^{-9}$$

$$\Rightarrow [A^{2-}] = 2.84 \times 10^{-11} M$$

Important: Note that, we will do the approximation in this case, since the concentration of HCl is so high that H^+ is considered from malonic acid is negligible. Recall the ionization of H_2S (g) in an acidic solution.

Example - 10

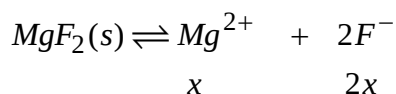
Calculate the $[F^-]$ in a solution saturated with respect to MgF_2 and SrF_2 .

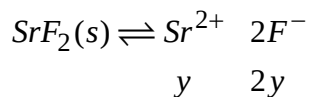
$$K_{sp}(MgF_2) = 6.0 \times 10^{-9}; K_{sp}(SrF_2) = 3.0 \times 10^{-9}$$

SOLUTION :

In this case, please note that the K_{sp} values of two salts are very similar. So the concentration of F^- ions (the common ion) can not be calculated from a single salt alone.

Let the simultaneous solubilities of MgF_2 and SrF_2 be x and y respectively in mol/L.





At eq : $[\text{F}^{-}] = 2x + 2y$; $[\text{Mg}^{2+}] = x$; $[\text{Sr}^{2+}] = y$

$$\Rightarrow [\text{Mg}^{2+}][\text{F}^{-}]^2 = K_{sp \text{ MgF}_2}$$

$$\text{and } [\text{Sr}^{2+}][\text{F}^{-}]^2 = K_{sp \text{ SrF}_2}$$

Note that : $[\text{F}^{-}] = 2([\text{Mg}^{2+}] + [\text{Sr}^{2+}])$ [This is an Electrical charge neutrality equation]

$$[\text{F}^{-}] = 2 \left(\frac{K_{sp \text{ MgF}_2}}{[\text{F}^{-}]^2} + \frac{K_{sp \text{ SrF}_2}}{[\text{F}^{-}]^2} \right)$$

$$\Rightarrow [\text{F}^{-}] = \sqrt[3]{2(K_{sp \text{ MgF}_2} + K_{sp \text{ SrF}_2})} = 2.62 \times 10^{-3} M$$

THINGS TO REMEMBER

1. Weak Electrolyte

$$(a) \quad \text{Weak Acid : } K_a = \frac{[H^+]^2}{c - [H^+]} = \frac{c\alpha^2}{1 - \alpha} \quad ; \quad c = [H^+] + \frac{[H^+]^2}{K_a}$$

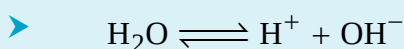
$$\text{If } 1 - \alpha \sim 1, \text{ then } K_a = c\alpha^2 \text{ and } [H^+] = \sqrt{K_a c}$$

$$(b) \quad \text{Weak Base : } K_b = \frac{[OH^-]^2}{c - [OH^-]} = \frac{c\alpha^2}{1 - \alpha} \quad ; \quad c = [OH^-] + \frac{[OH^-]^2}{K_b}$$

$$\text{If } 1 - \alpha \sim 1, \text{ then } K_b = c\alpha^2 \text{ and } [OH^-] = \sqrt{K_b c}$$

➤ For any acid – conjugate base pair, $K_a \cdot K_b = K_w$

➤ “ α ” of weak electrolytes increase on dilution.



$$K_w = [H^+][OH^-] \quad \text{or} \quad pH + pOH = pK_w$$

K_w increases as T increases and decreases as T decreases.

2. Buffer solution :

$$(a) \quad \text{Acidic : } HA + A^- \Rightarrow pH = pK_a + \log_{10} \frac{[\text{salt}]}{[\text{acid}]}$$

$$(b) \quad \text{Basic : } BOH + B^+ \Rightarrow pOH = pK_b + \log_{10} \frac{[\text{salt}]}{[\text{base}]}$$

➤ Buffer capacity of the buffer solution is maximum at $pH = pK_a$ or $pOH = pK_b$.

➤ pH of a buffer solution does not change on dilution.

3. Hydrolysis :

➤ Salts of SA – SB do not get hydrolysed.

➤ Salts of SA – WB on hydrolysis give acidic solution.

$$K_h = \frac{ch^2}{1 - h} = \frac{K_w}{K_b} \quad ; \quad pH = \frac{1}{2} [pK_w - (pK_b + \log_{10} c)]$$

➤ Salts of WA – SB on hydrolysis give basic solution.

$$K_h = \frac{ch^2}{1 - h} = \frac{K_w}{K_a} \quad ; \quad pH = \frac{1}{2} [pK_w + (pK_a + \log_{10} c)]$$

- Salts of WA – SB on hydrolysis may give acidic, basic or neutral solution.

$$K_h = \frac{h^2}{(1-h)^2} = \frac{K_w}{K_a \cdot K_b} ; \quad pH = \frac{1}{2} [pK_w + (pK_a - pK_b)]$$

K_h : hydrolysis constant ; h : degree of hydrolysis

4. Sparingly soluble salts and Precipitation :

- Solubility of a sparingly soluble salt is expressed in mol / L or gm/L
- Solubility (in Mol/L) of any salt $A_x B_y$ in pure water is given by : Solubility = $\left[\frac{(K_{sp})}{x^x \cdot y^y} \right]^{1/(x+y)}$
- Solubility of a salt decreases in the presence of a common ion.
In general, we can neglect the contribution of common ion from the sparingly soluble salt.

$$\text{e.g. solubility of AgCl in pM NaCl} = \frac{K_{sp} \text{AgCl}}{[Cl^-]} = \frac{K_{sp} \text{AgCl}}{p}$$

- **Selective precipitation :** When difference in the minimum concentrations of common cation/anion required to start the precipitation is very high.
To check, always calculate the conc. of the cation/anion left in the solution when the second cation/anion starts to precipitate, and so on.....
- **Simultaneous Solubility :** Solubility of two or more salts having a common cation / anion when their K_{sp} values are comparable. Solve the governing equations simultaneously.

5. Use of Indicators :

- Transition range of an indicator : $pK_{in} - 1$ to $pK_{in} + 1$
- For SA - SB titration, almost every indicator is effective.
- For SA - WB and WA - SB, indicator selection is important and depends on the pH at equivalence point.

6. Miscellaneous :

- For dissociation of H_2A , $[A^{2-}] \approx K_2$; For H_3A , $[A^{3-}] \approx K_3$ and so on.
- For any polyprotic acid, $K_{a1} \gg K_{a2} \gg K_{a3}$ and so on.
- Solubility of a salt can be increased by adding a substance that makes a stable complex / compound with any of the cation / anion in the salt.
e.g. AgCl dissolves more in aq. NH_3 solution due to the formation of $Ag(NH_3)_2^+$ and AgCN dissolves more in an acidic solution due to the formation of HCN (weak electrolyte)