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Conic Section

CONIC SECTION

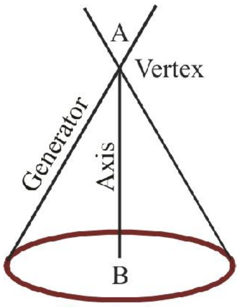
1. CONIC SECTIONS

A conic section, or conic is the locus of a point which moves in a plane so that its distance from a fixed point is in a constant ratio to its perpendicular distance from a fixed straight line.

- The fixed point is called the **Focus.**
- The fixed straight line is called the **Directrix.**
- The constant ratio is called the **Eccentricity** denoted by e.
- The line passing through the focus & perpendicular to the directrix is called the **Axis.**
- A point of intersection of a conic with its axis is called a **Vertex.**

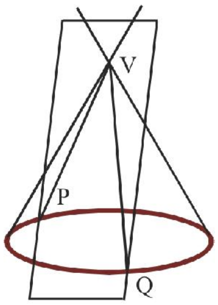
2. SECTION OF RIGHT CIRCULAR CONE BY DIFFERENT PLANES

A right circular cone is as shown in the fig. 1



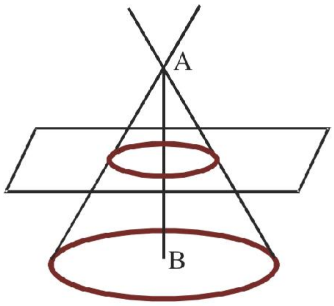
(Fig. 1)

(i) Section of a right circular cone by a plane passing through its vertex is a pair of straight lines passing through the vertex as shown in fig. 2



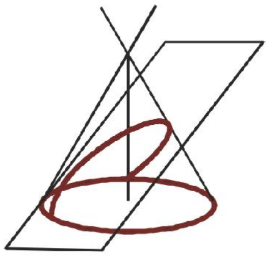
(Fig. 2)

(ii) Section of a right circular cone by a plane parallel to its base is a circle as shown in the fig. 3.

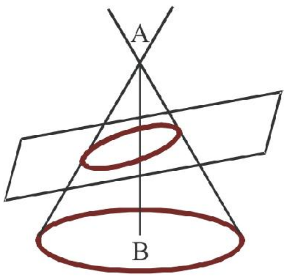


(Fig. 3)

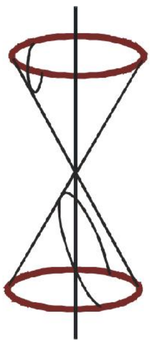
(iii) Section of a right circular cone by a plane parallel to a generator of the cone is a parabola as shown in the fig. 4.


(Fig. 4)

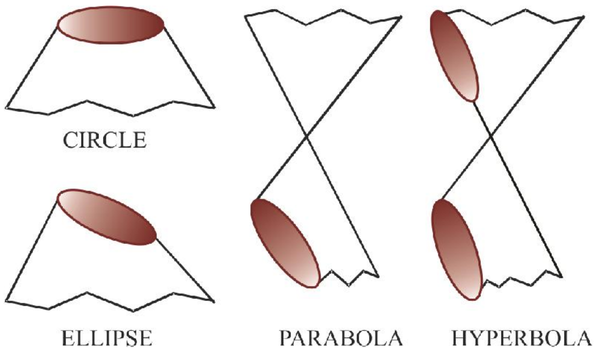
(iv) Section of a right circular cone by a plane neither parallel to any generator of the cone nor perpendicular or parallel to the axis of the cone is an ellipse or hyperbola as shown in the fig. 5 & 6.



(Fig. 5)


(Fig. 6)

3 D View :



3. GENERAL EQUATION OF A CONIC : FOCAL DIRECTRIX PROPERTY

The general equation of a conic with focus (p, q) & directrix $lx + my + n = 0$ is :

$(l^2 + m^2)[(x - p)^2 + (y - q)^2] = e^2(lx + my + n)^2 \equiv$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

4. DISTINGUISHING VARIOUS CONICS

The nature of the conic section depends upon the position of the focus S w.r.t. the directrix & also upon the value of the eccentricity e. Two different cases arise.

Case (I) When The Focus Lies On The Directrix.

In the case $\Delta \equiv abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ & the general equation of a conic represents a pair of straight lines if :

$e > 1 \equiv h^2 > ab$, the lines will be real and distinct intersecting at S.

$e = 1 \equiv h^2 \geq ab$, the lines will coincident.

$e < 1 \equiv h^2 < ab$, the lines will be imaginary.

Case (II) When The Focus Does Not Lie On Directrix.

a parabola : $e = 1$, $\Delta \neq 0$, $\mathbf{h^2 = ab}$

an ellipse : $0 < e < 1$; $\Delta \neq 0$, $\mathbf{h^2 < ab}$

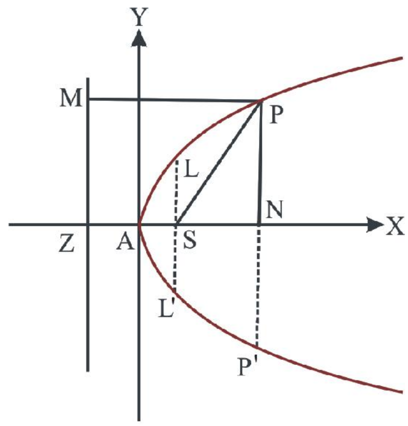
a hyperbola : $e > 1$; $\Delta \neq 0$, $\mathbf{h^2 > ab}$

rectangular hyperbola : $e > 1$; $\Delta \neq 0$, $\mathbf{h^2 > ab; a + b = 0}$

PARABOLA

5. DEFINITION AND TERMINOLOGY

A parabola is the locus of a point, whose distance from a fixed point (focus) is equal to perpendicular distance from a fixed straight line (directrix).



$y^2 = 4ax$

Four standard forms of the parabola are

$\mathbf{y^2 = 4ax \; ; \; y^2 = -4ax \; ; \; x^2 = 4ay \; ; \; x^2 = -4ay}$

For parabola $y^2 = 4ax$:

(i) Vertex is (0, 0) (ii) Focus is (a, 0)

(iii) Axis is $y = 0$ (iv) Directrix is $x + a = 0$

Focal Distance : The distance of a point on the parabola from the focus.

Focal Chord : A chord of the parabola, which passes through the focus.

Double Ordinate : A chord of the parabola perpendicular to the axis of the symmetry.

Latus Rectum : A double ordinate passing through the focus or a focal chord perpendicular to the axis of parabola is called the Latus Rectum (L.R.).

For $y^2 = 4ax$. $\Rightarrow$ Length of the latus rectum = 4a

$\Rightarrow$ ends of the latus rectum are L(a, 2a) & L' (a, –2a).

Note...

(i) Perpendicular distance from focus on directrix = half the latus rectum

(ii) Vertex is middle point of the focus & the point of intersection of directrix & axis.

(iii) Two parabolas are said to be equal if they have the same latus rectum.

6. POSITION OF A POINT RELATIVE TO A PARABOLA

The point $(x_1\ y_1)$ lies outside, on or inside the parabola $y^2 = 4ax$ according as the expression $\mathbf{y_1^2 - 4ax_1}$ is **positive**, **zero** or **negative**.

7. LINE & A PARABOLA

The line $y = mx + c$ meets the parabola $y^2 = 4ax$ in :

- two real points if $a > mc$
- two coincident points if $a = mc$
- two imaginary points if $a < mc$

$\Rightarrow$ **condition of tangency is, $c = a/m$.**

Length of the chord intercepted by the parabola on the line $y = mx + c$ is :

$$\left(\frac{4}{m^2}\right)\sqrt{a(1+m^2)(a-mc)}$$

Note..

Length of the focal chord making an angle α with the x-axis is $4a\operatorname{cosec}^2\alpha$.

8. PARAMETRIC REPRESENTATION

The simplest & the best form of representing the co-ordinates of a point on the parabola is $\mathbf{(at^2, 2at)}$ i.e. the equations $\mathbf{x = at^2}$ & $\mathbf{y = 2at}$

together represents the parabola $y^2 = 4ax$, t being the parameter.

The equation of a chord joining t_1 & t_2 is

$\mathbf{2x - (t_1 + t_2)y + 2at_1t_2 = 0}$.

Note..

If t_1 and t_2 are the ends of a focal chord of the parabola $y^2 = 4ax$ then $\mathbf{t_1t_2 = -1}$.

Hence the co-ordinates at the extremities of a focal chord can be taken as : $(at^2, 2at)$ & $\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$

9. TANGENTS TO THE PARABOLA $y^2 = 4ax$

(i) $\mathbf{yy_1 = 2a(x + x_1)}$ at the point (x_1, y_1) ;

(ii) $\mathbf{y = mx + \frac{a}{m}}$ $(m \neq 0)$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

(iii) $\mathbf{ty = x + at^2}$ at point $(at^2, 2at)$.

Note..

Point of intersection of the tangents at the point t_1 & t_2 is $[at_1t_2, a(t_1 + t_2)]$.

10. NORMALS TO THE PARABOLA $y^2 = 4ax$

(i) $\mathbf{y - y_1 = \frac{-y_1}{2a}}$ $(x - x_1)$ at (x_1, y_1)

(ii) $\mathbf{y = mx - 2am - am^3}$ at point $(am^2, -2am)$

(iii) $\mathbf{y + tx = 2at + at^3}$ at point $(at^2, 2at)$.

Note..

(i) Point of intersection of normals at t_1 & t_2 are, $a(t_1^2 + t_2^2 + t_1t_2 + 2)\ ;\ -at_1t_2(t_1 + t_2)$

(ii) If the normals to the parabola $y^2 = 4ax$ at the point t_1 meets the parabola again at the point t_2 then $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$

(iii) If the normals to the parabola $y^2 = 4ax$ at the points t_1 & t_2 intersect again on the parabola at the point 't_3' then $t_1t_2 = 2$; $t_3 = -(t_1 + t_2)$ and the line joining t_1 & t_2 passes through a fixed point $(-2a, 0)$.

11. PAIR OF TANGENTS

The equation to the pair of tangents which can be drawn from any point $(x_1\ y_1)$ to the parabola $y^2 = 4ax$ is given by: $\mathbf{SS_1 = T^2}$ where :

$S \equiv y^2 - 4ax$; $S_1 = y_1^2 - 4ax_1$; $T \equiv yy_1 - 2a(x + x_1)$.

12. DIRECTOR CIRCLE

Locus of the point of intersection of the perpendicular tangents to a curve is called the director circle. For parabola $y^2 = 4ax$ it's equation is

$x + a = 0$ which is parabola's own directrix.

13. CHORD OF CONTACT

Equation to the chord of contact of tangents drawn from a point $P(x_1, y_1)$ is **$yy_1 = 2a(x + x_1)$** ; (i.e., T = 0)

Note..

The area of the triangle formed by the tangents from the point (x_1, y_1) & the chord of contact is $(y_1^2 - 4ax_1)^{3/2} \div 2a$.

14. CHORD WITH A GIVEN MIDDLE POINT

Equation of the chord of the parabola $y^2 = 4ax$ whose middle point is : $(x_1\ y_1)$ is :

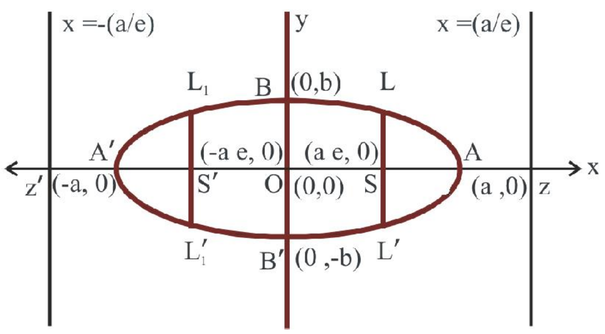
$$y - y_1 = \frac{2a}{y_1}(x - x_1) \equiv T = S_1.$$

ELLIPSE

15. STANDARD EQUATION AND DEFINITIONS

Standard equation of an ellipse referred to its principal axes along the co-ordinate axes is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{where} \quad a > b \quad \& \quad b^2 = a^2(1 - e^2).$$



Eccentricity : $e = \sqrt{1 - \frac{b^2}{a^2}}$, $(0 < e < 1)$

Foci : $S \equiv (a\,e, 0)$ & $S' \equiv (-a\,e, 0)$

Equations of Directrices : $x = \frac{a}{e}$ & $x = -\frac{a}{e}$

Major Axis : The line segment A'A in which the foci S' & S lie is of length 2a & is called the major axis (a > b) of the ellipse. Point of intersection of major axis with directrix is called the foot of the directrix (Z).

Minor Axis : The y-axis intersects the ellipse in the points $B' \equiv (0, -b)$ and $B \equiv (0, b)$. The line segment B'B of length 2b (b < a) is called the minor axis of the ellipse.

Principal Axis : The major and minor axis together are called principal axis of the ellipse.

Vertices : $A' \equiv (-a, 0)$ & $A \equiv (a, 0)$.

Focal Chord : A chord which passes through a focus is called a focal chord.

Double Ordinate : A chord perpendicular to the major axis is called a double ordinate.

Latus Rectum : The focal chord perpendicular to the major axis is called the latus rectum.

Length of latus rectum (LL′) :

$$\frac{2b^2}{a} = \frac{(\text{minor axis})^2}{\text{major axis}} = 2a(1 - e^2)$$

= 2e (distance from focus to the corresponding directrix).

Centre :

The point which bisects every chord of the conic drawn through it is called the centre of the conic. $C \equiv (0, 0)$

the origin is the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

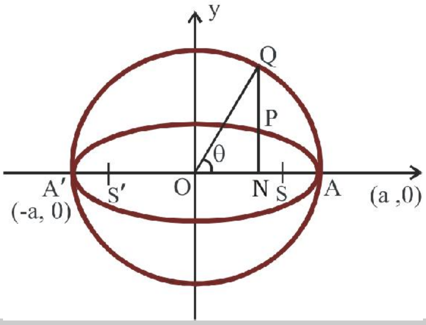
Note..

(i) If the equation of the ellipse is given as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and nothing is mentioned then the rule is to assume that a > b.

(ii) If b > a is given, then the y-axis will become major axis and x-axis will become the minor axis and all other points and lines will change accordingly.

16. AUXILIARY CIRCLE/ECCENTRIC ANGLE

A circle described on major axis as diameter is called the **auxiliary circle.** Let Q be a point on the auxiliary circle $x^2 + y^2 = a^2$ such that QP produced is perpendicular to the x-axis then P & Q are called as the **Corresponding Points** on the ellipse and the auxiliary circle respectively. 'θ' is called the **Eccentric Angle** of the point P on the ellipse ($-\pi < \theta \le \pi$).



Note..

$$\frac{\ell(PN)}{\ell(QN)} = \frac{b}{a} = \frac{\text{Semi major axis}}{\text{Semi major axis}}$$

Note..

If from each point of a circle perpendiculars are drawn upon a fixed diameter then the locus of the points dividing these perpendiculars in a given ratio is an ellipse of which the given circle is the auxiliary circle.

17. PARAMETRIC REPRESENTATION

The equation **$x = a\cos\theta$ & $y = b\sin\theta$** together represent the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Where θ is a parameter. Note that if

$P(\theta) \equiv (a\cos\theta, b\sin\theta)$ is on the ellipse then;

$Q(\theta) \equiv (a\cos\theta, a\sin\theta)$ is on the auxiliary circle.

The equation to the chord of the ellipse joining two points with eccentric angles α and β is given by

$$\frac{x}{a}\cos\frac{\alpha+\beta}{2}+\frac{y}{b}\sin\frac{\alpha+\beta}{2}=\cos\frac{\alpha-\beta}{2}.$$

18. POSITION OF A POINT W.R.T. AN ELLIPSE

The point p (x_1, y_1) lies outside, inside or on the ellipse according as ;

$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1 > 0$ (outside)

$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1 < 0$ (inside)

$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1 = 0$ (on)

19. LINE AND AN ELLIPSE

The line $y = mx + c$ meets the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in two points real, coincident or imaginary according as c^2 is < = or > $a^2m^2 + b^2$.

Hence $y = mx + c$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if **$c^2 = a^2m^2 + b^2$**.

20. TANGENTS

(a) Slope form $\mathbf{y = mx \pm \sqrt{a^2m^2+b^2}}$ is tangent to the ellipse for all values of m.

(b) Point form $\mathbf{\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1}$ is tangent to the ellipse at $(x_1\ y_1)$.

(c) Parametric for $\mathbf{\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1}$ is tangent to the ellipse at the point $(a\cos\theta, b\sin\theta)$.

Note...

(i) There are two tangents to the ellipse having the same m, i.e. there are two tangents parallel to any given direction. These tangents touches the ellipse at extremities of a diameter.

(ii) Point of intersection of the tangents at the point α & β is :

$$\left(a\frac{\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, b\frac{\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}\right)$$

(iii) The eccentric angles of point of contact of two parallel tangents differ by π.

21. NORMALS

(i) Equation of the normal at $(x_1\ y_1)$ is

$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2 = a^2e^2.$$

(ii) Equation of the normal at the point $(a\cos\theta, b\sin\theta)$ is;

$\mathbf{ax\ sec\theta - by\ cosec\theta = (a^2 - b^2)}$.

(iii) Equation of a normal in terms of its slope 'm' is

$$y = mx - \frac{(a^2 - b^2)m}{\sqrt{a^2 + b^2m^2}}$$

22. DIRECTOR CIRCLE

Locus of the point of intersection of the tangents which meet at right angles is called the **Director Circle.** The equation to this locus is $x^2 + y^2 = a^2 + b^2$ i.e. a circle whose centre is the centre of the ellipse & whose radius is the length of the line joining the ends of the major & minor axis.

Note...

Pair of tangents, Chord of contact, Pole & Polar, Chord with a given Middle point are to be interpreted as they are in Parabola/Circle.

23. DIAMETER (NOT IN SYLLABUS)

The locus of the middle points of a system of parallel chords with slope 'm' of an ellipse is a straight line passing through the centre of the ellipse, called its diameter and has the equation $y = -\frac{b^2}{a^2m}x$

Note...

All diameters of ellipse passes through its centre.

IMPORTANT HIGHLIGHTS

Referring to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(a) If P be any point on the ellipse with S & S' as to foci then $l(SP) + l(S'P) = 2a$.

(b) The tangent & normal at a point P on the ellipse bisect the external and internal angles between the focal distances of P. This refers to the well known reflection property of the ellipse which states that rays from one focus are reflected through other focus & vice-versa. Hence we can deduce that the straight lines joining each focus to the foot of the perpendicular from the other focus upon the tangent at any point P meet on the normal PG and bisects it where G is the point where normal at P meets the major axis.

(c) The product of the length's of the perpendicular segments from the foci on any tangent to the ellipse is b^2 and the feet of these perpendiculars lie on its auxiliary circle and the tangents at these feet to the auxiliary circle meet on the ordinate of P and that the locus of their point of intersection is a similar ellipse as that of the original one.

(d) The portion of the tangent to an ellipse between the point of contact & the directrix subtends a right angle at the corresponding focus.

(e) If the normal at any point P on the ellipse with centre C meet the major and minor axes in G & g respectively & if CF be perpendicular upon this normal then :

(i) PF. PG = b^2

(ii) PF. Pg = a^2

(iii) PG. Pg = SP. S'P

(iv) CG. CT = CS^2

(v) locus of the mid point of Gg is another ellipse having the same eccentricity as that of the original ellipse.

[Where S and S' are the foci of the ellipse and T is the point where tangent at P meet the major axis]

* The circle on any focal distance as diameter touches the auxiliary circle. Perpendiculars from the centre upon all chords which join the ends of any perpendicular diameters of the ellipse are of constant length.

* If the tangent at the point P of a standard ellipse meets the axis in T and t and CY is the perpendicular on it from the centre then :

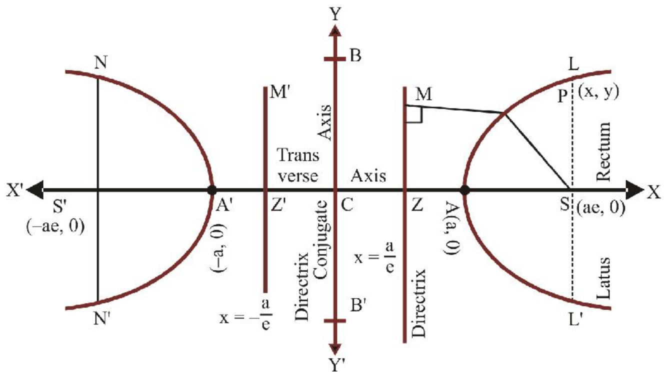
(i) T t. PY = $a^2 - b^2$ and

(ii) least value of T t is a + b.

HYPERBOLA

The **Hyperbola** is a conic whose eccentricity is greater thatn unity (e > 1).

24. STANDARD EQUATION & DEFINITION (S)



Standard equation of the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

where $\mathbf{b^2 = a^2 (e^2 - 1)}$.

Eccentricity (e) : $e^2 = 1 + \frac{b^2}{a^2} = 1 + \left(\frac{C.A}{T.A}\right)^2$

(C.A $\rightarrow$ Conjugate Axis;

T.A $\rightarrow$ Transverse Axis)

Foci : S $\equiv$ (ae, 0) & S' $\equiv$ (–ae, 0).

Equations of Directrix : $x = \frac{a}{e}$ & $x = -\frac{a}{e}$

Transverse Axis : The line segment A'A of length 2a in which the foci S' & S both lie is called the transverse axis of the hyperbola.

Conjugate Axis : The line segment B'B between the two points B' $\equiv$ (0, –b) & B $\equiv$ (0, b) is called as the conjugate axis of the hyperbola.

Principal Axes : The transverse & conjugate axis together are called Principal Axes of the hyperbola.

Vertices : A $\equiv$ (a, 0) & A' $\equiv$ (–a, 0)

Focal Chord : A chord which passes through a focus is called a focal chord.

Double Ordinate : A chord perpendicular to the transverse axis is called a double ordinate.

Latus Rectum (*l*) L : The focal chord perpendicular to the transverse axis is called the latus rectum.

$$\ell = \frac{2b^2}{a} = \frac{(C.A.)^2}{T.A.} = 2a\,(e^2 - 1)$$

Note...

l (L.R.) = 2 e (distance from focus to directrix).

Centre : The point which bisects every chord of the conic drawn through it is called the centre of the conic. C $\equiv$ (0, 0) the origin is the centere of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Note...

Since the fundamental equation to the hyperbola only differs from that to the ellipse in having $-b^2$ instead of b^2 it will be found that many propositions for the hyperbola are derived from those for the ellipse by simply changing the sign of b^2.

25. RECTANGULAR OR EQUILATERAL HYPERBOLA

The particular kind of hyperbola in which the lengths of the transverse and conjugate axis are equal is called an Equilateral Hyperbola. Note that the **eccentricity of the rectangular hyperbola is** $\sqrt{2}$.

26. CONJUGATE HYPERBOLA

Two hyperbolas such that transverse & conjugate axes of one hyperbola are respectively the conjugate & the transverse axes of the other are called Conjuagate Hyperbolas of each other.

e.g. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ & $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are **conjugate hyperbolas** of each other.

Note...

(a) If e_1 and e_2 are the eccentricities of the hyperbola and its conjugate then $e_1^{-2} + e_2^{-2} = 1$.

(b) The foci of a hyperbola and its conjugate are concyclic and form the vertices of a square.

(c) Two hyperbolas are said to be similiar if they have the same eccentricity.

(d) Two similiar hyperbolas are said to be equal if they have same latus rectum.

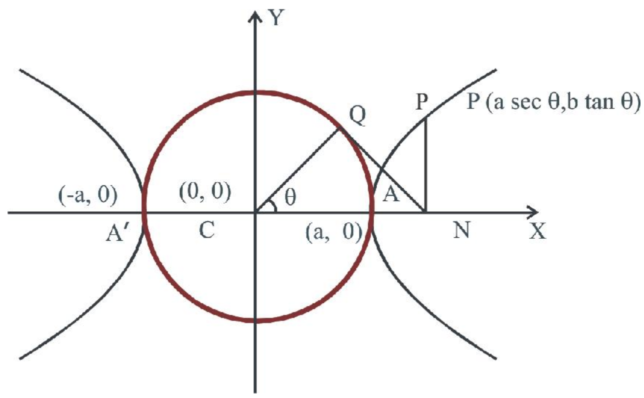
(e) If a hyperbola is equilateral then the conjugate hyperbola is also equilateral.

27. AUXILIARY CIRCLE

A circle drawn with centre C & T.A. as a diameter is called the **Auxiliary Circle** of the hyperbola. Equation of the auxiliary circle is $x^2 + y^2 = a^2$.

Note from the figure that P & Q are called the **"Corresponding Points"** on the hyperbola and the auxiliary circle.

In the hyperbola any ordinate of the curve does not meet the circle on AA' as diameter in real points. There is therefore no real eccentric angle as in the case of the ellipse.



28. PARAMETRIC REPRESENTATION

The equation **x = a sec θ & y = b tan θ** together represents the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ where θ is a parameter.

Note that if $P(\theta) \equiv (a \sec\theta, b \tan \theta)$ is on the hyperbola then ;

$Q(\theta) \equiv (a \cos \theta, a \tan \theta)$ is on the auxiliary circle.

The equation to the chord of the hyperbola joining two points with eccentric angles α and β is given by

$$\frac{x}{a}\cos\frac{\alpha-\beta}{2}-\frac{y}{b}\sin\frac{\alpha+\beta}{2}=\cos\frac{\alpha+\beta}{2}.$$

29. POSITION OF A POINT 'P' W.R.T. A HYPERBOLA

The quantity $S_1=\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}=1$ is positive, zero or negative according as the point (x_1, y_1) lies inside, on or outside the curve.

30. LINE AND A HYPERBOLA

The straight line $y = mx + c$ is a secant, a tangent or passes outside the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ according as : c^2 > or = or < $a^2m^2 - b^2$, respectively.

31. TANGENTS

(i) **Slope Form :** $y = mx \pm \sqrt{a^2m^2 - b^2}$ can be taken as the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(ii) **Point Form :** Equation of tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(x_1\ y_1)$ is

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

(iii) **Parametric Form :** Equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(a \sec \theta, b \tan \theta)$

$$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1.$$

Note...

(i) Point of intersection of the tangents at θ_1 & θ_2 is :

$$x = a\frac{\cos\frac{\theta_1 - \theta_2}{2}}{\cos\frac{\theta_1 + \theta_2}{2}},\ y = b\tan\left(\frac{\theta_1 + \theta_2}{2}\right)$$

(ii) If $|\theta_1 + \theta_2| = \pi$, then tangents at these points $(\theta_1\ \&\ \theta_2)$ are parallel.

(iii) There are two parallel tangents having the same slope m. These tangents touches the hyperbola at the extremities of a diameter.

32. NORMALS

(i) The equation of the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point P (x_1, y_1) on it is

$$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2 = a^2e^2$$

(ii) The equation of the normal at the point P $(a \sec \theta, b \tan \theta)$ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2 = a^2e^2$.

(iii) Equation of a normal in terms of its slope 'm' is

$$y = mx - \frac{(a^2 + b^2)m}{\sqrt{a^2 - b^2m^2}}.$$

Note...

Equation to the chord of contact, polar, chord with a given middle point, pair of tangents from an external point is to be interpreted as in parabola/circle.

33. DIRECTOR CIRCLE

The locus of the intersection point of tangents which are at right angles is known as the Director Circle of the hyperbola. The equation to the director circle is :

$x^2 + y^2 = a^2 - b^2$.

If $b^2 < a^2$ this circle is real.

If $b^2 = a^2$ (rectangular hyperbola) the radius of the circle is zero and it reduces to a point circle at the origin. In this case the centre is the only point from which the tangents at right angles can be drawn to the curve.

If $b^2 > a^2$, the radius of the circle is imaginary, so that there is no such circle & so no pair of tangents at right angle can be drawn to the curve.

34. DIAMETER (NOT IN SYLLABUS)

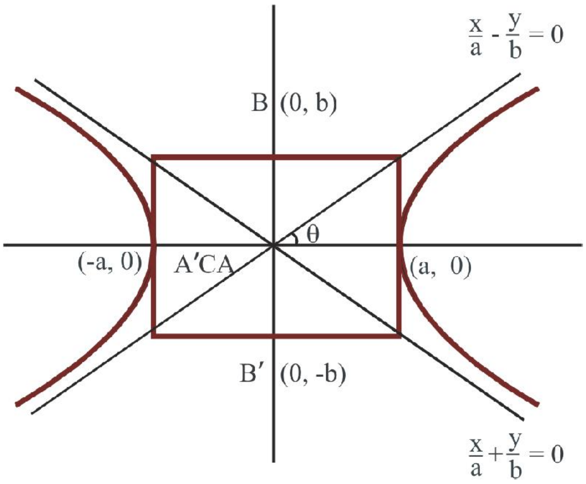
The locus of the middle points of a system of parallel chords with slope 'm' of an hyperbola is called its diameter. It is a straight line passing through the centre of the hyperbola and has the equation $y = +\frac{b^2}{a^2m}x$

Note...

All diameters of the hyperbola passes through its centre.

35. ASSYMPTOTES (NOT IN SYLLABUS)

Definition : If the length of the perpendicular let fall from a point on a hyperbola to a straight line tends to zero as the point on the hyperbola moves to infinity along the hyperbola, then the straight line is called the Asymptote of the hyperbola.



Equation of Asymptote : $\frac{x}{a} + \frac{y}{b} = 0$ and $\frac{x}{a} - \frac{y}{b} = 0$

Pairs of Asymptotes : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

Note...

(i) A hyperbola and its conjugate have the same asymptote.

(ii) The equation of the pair of asymptotes different form the equation of hyperbola (or conjugate hyperbola) by the constant term only.

(iii) The asymptotes pass through the centre of the hyperbla and are equally inclined to the transverse axis of the hyperbola. Hence the bisectors of the angles between the asymptotes are the principle axes of the hyperbola.

(iv) The asymptotes of a hyperbola are the diagonals of the rectangle formed by the lines drawn through the extremities of each axis parallel to the other axis.

(v) Asymptotes are the tangent to the hyperbola from the centre.

(vi) A simple method to find the co-ordinates of the centre of the hyperbola expressed as a general equation of degree 2 should be remembered as :

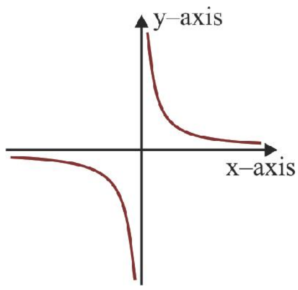
Let f(x y) = 0 represents a hyperbola.

Find $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$. Then the point of intersection of $\frac{\partial f}{\partial x} = 0$ & $\frac{\partial f}{\partial y} = 0$ gives the centre of the hyperbola.

36. RECTANGULAR HYPERBOLA ($xy = c^2$)

It is referred to its asymptotes as axes of co-ordinates.

Vertices : (c, c) and (–c, –c) ;



Foci : $(\sqrt{2}c, \sqrt{2}c)$ & $(-\sqrt{2}c, -\sqrt{2}c)$,

Directrices : $x + y = \pm \sqrt{2}c$

Latus Rectum (l) : $l = 2\sqrt{2}c$ = T.A. = C.A.

Parametric equation $\mathbf{x = ct, y = c/t}$, $t \in R - \{0\}$

Equation of a chord joining the points $P(t_1)$ & $Q(t_2)$ is $\mathbf{x + t_1t_2y = c(t_1 + t_2)}$.

Equation of the tangent at P $(x_1\ y_1)$ is $\frac{x}{x_1} + \frac{y}{y_1} = 2$ and at P(t) is $\frac{x}{t} + ty = 2c$.

Equation of the normal at P(t) is $xt^3 - yt = c(t^4 - 1)$.

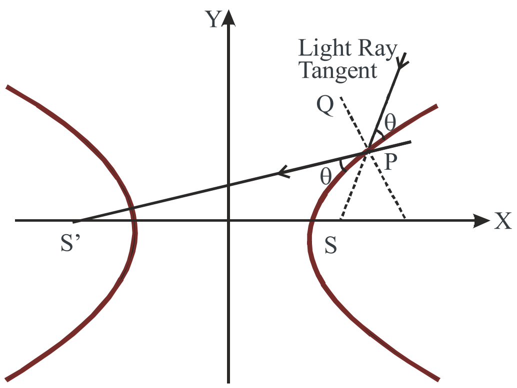
Chord with a given middle point as (h, k) is kx + hy = 2hk.

IMPORTANT HIGHLIGHTS

(i) Locus of the feet of the perpendicular drawn from focus of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ upon any tangent is its auxiliary circle i.e. $x^2 + y^2 = a^2$ & the product of the feet of these perpendiculars is b^2.

(ii) The portion of the tangent between the point of contact & the directrix subtends a right angle at the corresponding focus.

(iii) The tangent & normal at any point of a hyperbola bisect the angle between the focal radii. This spell the reflection property of the hyperbola as **"An incoming light ray"** aimed towards one focus is reflected from the outer surface of the hyperbola towards the other focus. It follows that if an ellipse and a hyperbola have the same foci, they cut at right angles at any of their common point.



Note that the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & the hyperbola $\frac{x^2}{a^2-k^2}+\frac{y^2}{k^2-b^2}=1$ $(a > k > b > 0)$ are confocal and therefore orthogonal.

(iv) The foci of the hyperbola and the points P and Q in which any tangent meets the tangents at the vertices are concyclic with PQ as diameter of the circle.

(v) If from any point on the asymptote a straight line be drawn perpendicular to the transverse axis, the product of the segments of this line, intercepted between the point and the curve is always equal to the square of the semi conjugate axis.

(vi) Perpendicular from the foci on either asymptote meet it in the same points as the corresponding directrix and the common points of intersection lie on the auxiliary circle.

(vii) The tangent at any point P on a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ with centre C, meets the asymptotes in Q and R and cuts off a Δ CQR of constant area equal to ab from the asymptotes and the portion of the tangent intercepted between the asymptote is bisected at the point of contact. This implies that locus of the centre of the circle circumscribing the Δ CQR in case of a rectangular hyperbola is the hyperbola itself and for a standard hyperbola the locus would be the curve, $4(a^2x^2 - b^2y^2) = (a^2 + b^2)^2$.

(viii) If the angle between the asymptote of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is 2θ then the eccentricity of the hyperbola is $\sec\theta$.

(ix) A rectangular hyperbola circumscribing a triangle also passes through the orthocentre of this triangle.

If $\left(ct_i, \frac{c}{t_i}\right)$ $i = 1, 2, 3$ be the angular points P, Q, R then orthocentre is $\left(\frac{-c}{t_1t_2t_3}, -ct_1\,t_2\,t_3\right)$.

(x) If a circle and the rectangular hyperbola $xy = c^2$ meet in the four points, t_1, t_2, t_3 and t_4, then

(a) $t_1\ t_2\ t_3\ t_4 = 1$

(b) the centre of the mean position of the four points bisects the distance between the centre of the circle through the points t_1, t_2, and t_3 is :

$$\left\{\frac{c}{2}\left(t_1+t_2+t_3+\frac{1}{t_1t_2t_3}\right), \frac{c}{2}\left(\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+t_1+t_2+t_3\right)\right\}$$

SOLVED EXAMPLES

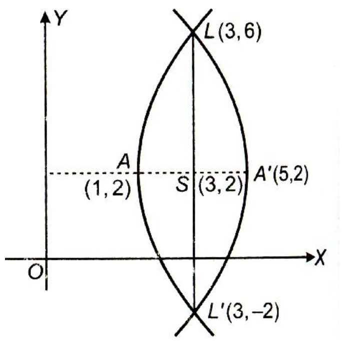
PARABOLA

Example - 1

Find the equation of the parabola with latus rectum joining the points (3, 6) and (3, –2)

Sol. Slope of (3, 6) and (3, –2) is $\frac{-2-6}{3-3} = \infty$ since latus rectum is perpendicular to axis. Hence axis parallel to x-axis. The equation of the two possible parabolas will be of the form

$(y - k)^2 = \pm 4a (x - h)$ ... (1)



Since latus rectum $= \sqrt{(3-3)^2 + (6+2)^3} = 8$

∴ $4a = 8$

$a = 2$

∴ From (1), $(y - k)^2 = \pm 8 (x - h)$

Since (3, 6) and (3, –2) lie on the parabola, then

$(6 - k)^2 = \pm 8 (3 - h)$ ... (2)

and $(-2 - k)^2 = \pm 8 (3 - h)$ ... (3)

Solving (2) and (3) we get

$k = 2$

From (2) $16 = \pm 8 (3 - h)$,

∴ $h = 3 \pm 2$

∴ $h = 5, 1$

Hence values of (h, k) are (5, 2) and (1, 2). The required parabolas are

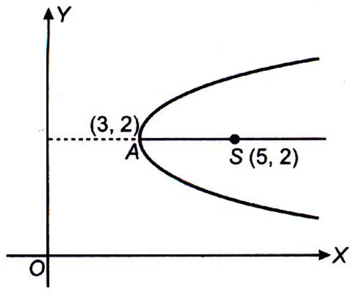
$(y - 2)^2 = 8 (x - 5)$ and $(y - 2)^2 = -8 (x - 1)$

Example - 2

Find the equation of the parabola with its vertex at (3, 2) and its focus at (5, 2).

Sol. Let Vertex A (3, 2) and focus is S (5, 2)

Slope of AS $= \frac{2-2}{5-3} = 0$ (which is parallel to x-axis)



Hence axis of parabola parallel to x-axis.

The equation is of the form

$(y - k)^2 = 4a (x - h)$

or $(y - 2)^2 = 4a (x - 3)$

as (h, k) is the vertex (3, 2)

a = distance between the focus and the vertex

$= \sqrt{(5-3)^2 + (2-2)^2}$

$= 2$

Hence the required equation is

$(y - 2)^2 = 8 (x - 3)$

or $y^2 - 8x - 4y - 28 = 0$

Example - 3

Show that line $x \cos \alpha + y \sin \alpha = p$ touches the parabola $y^2 = 4ax$ if $p \cos \alpha + a \sin^2 \alpha = 0$ and that the point of contact is $(a \tan^2 \alpha, -2a \tan \alpha)$.

Sol. The given line is

$x \cos \alpha + y \sin \alpha = p$

$\Rightarrow$ $y = -x \cot \alpha + p \operatorname{cosec} \alpha$

Comparing this line with $y = mx + c$

∴ $m = -\cot \alpha$ and $c = p \operatorname{cosec} \alpha$

since the given line touches the parabola

$\therefore \quad c = \frac{a}{m} \quad \Rightarrow \quad cm = a$

$\Rightarrow \quad (p \operatorname{cosec} \alpha)(-\cot \alpha) = a$

$\Rightarrow \quad a \sin^2 \alpha + p \cos \alpha = 0$

and point of contact is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ i.e.

$\left(\frac{a}{\cot^2 \alpha}, -\frac{2a}{\cot \alpha}\right) \Rightarrow (a \tan^2 \alpha, -2a \tan \alpha)$.

Example - 4

Prove that the line $\frac{x}{\ell} + \frac{y}{m} = 1$ touches the parabola $y^2 = 4a(x + b)$ if $m^2(l + b) + al^2 = 0$.

Sol. The given parabola is

$y^2 = 4a(x + b)$ ...(i)

Vertex of this parabola is (–b, 0).

Now shifting (0, 0) at (–b, 0) then

$x = X + (-b)$ and $y = Y + 0$

$\Rightarrow \quad x + b = X$ and $y = Y$ ...(ii)

from (i), $Y^2 = 4aX$ ...(iii)

and the line $\frac{x}{\ell} + \frac{y}{m} = 1$ reduces to $\frac{X - b}{\ell} + \frac{y}{m} = 1$

$\Rightarrow \quad Y = m\left(1 - \frac{X - b}{\ell}\right)$

$\Rightarrow \quad Y = \left(-\frac{m}{\ell}\right)X + m\left(1 + \frac{b}{\ell}\right)$ ...(iv)

The line (iv) will touch the parabola (iii), if

$m\left(1 + \frac{b}{\ell}\right) = \frac{a}{\left(-\frac{m}{\ell}\right)} \quad \Rightarrow$

$\frac{m^2}{\ell}\left(1 + \frac{b}{\ell}\right) = -a$

$\Rightarrow \quad m^2(l + b) + al^2 = 0$

Alternative Method :

Then given line and parabola are

$\frac{x}{\ell} + \frac{y}{m} = 1$ ...(i)

and $y^2 = 4a(x + b)$ ...(ii)

respectively.

Substituting the value of x from (i),

i.e., $x = l\left(1 - \frac{y}{m}\right)$ in (ii)

then $y^2 = 4a\left\{\ell\left(1 - \frac{y}{m}\right) + b\right\}$

$\Rightarrow \quad y^2 + \frac{4a\ell}{m} y - 4a(l + b) = 0$ ...(iii)

Since the line (i), touches the parabola (ii) then the roots of equation (iii) are equal

$\therefore \quad \left(\frac{4a\ell}{m}\right)^2 - 4.1\{-4a(l + b)\} = 0$

$\Rightarrow \quad \frac{a\ell^2}{m^2} + (l + b) = 0$

$\Rightarrow \quad al^2 + m^2(l + b) = 0$

$\Rightarrow \quad m^2(l + b) + al^2 = 0.$

Example - 5

(a) Find the equation of the tangents drawn to $y^2 + 12x = 0$ from the point (3, 8).

(b) Find the equation of tangents to the parabola $y^2 = 4x + 5$ which is parallel to the line $y = x + 7$.

Sol. **(a)** $y^2 + 12x = 0 \quad \Rightarrow \quad y^2 = -12x.$

$\Rightarrow \quad a = -3.$

Let tangent be $y = mx - \frac{3}{m}$.

Since tangent passes through (3, 8), $8 = 3m - \frac{3}{m}$

$\Rightarrow 3m^2 - 8m - 3 = 0 \Rightarrow \quad (m - 3)(3m + 1) = 0$

$\Rightarrow m = 3, m = -\frac{1}{3} \Rightarrow$ Tangent are $y = 3x - 1$

and $y = -\frac{x}{3} + 9.$

(b) Any line || to given is $y = 2x + c$.

If it is tangent to the parabola then it will meet it in two coincident points.

Eliminating x, we get : $y^2 - 5 = 2(y - \lambda)$

$\Rightarrow y^2 - 2y + 2\lambda - 5 = 0$. Roots are equal.

$\Rightarrow b^2 - 4ac = 0 \Rightarrow 4 - 4(2\lambda - 5) = 0$

$\Rightarrow \lambda = 3 \Rightarrow y = -\frac{x}{3} + 9.$

Example - 6

Show that the locus of a point, such that two of the three normals drawn from it to the parabola $y^2 = 4ax$ are perpendicular is $y^2 = a(x - 3a)$.

Sol. Let $P \equiv (x_1, y_1)$ be the point from where normals AP, BP, CP are drawn to $y^2 = 4ax$.

Let $y = mx - 2am - am^3$ be one of these normals.

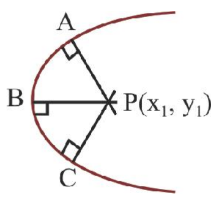
P lies on it $\Rightarrow y_1 = mx_1 - 2am - am^3$.

Slopes m_1, m_2, m_3 of AP, BP, CP are roots of the cubic

$y_1 = mx_1 - 2am - am^2$.

$\Rightarrow am^3 + (2a - x_1)m + y_1 = 0 \Rightarrow m_1 + m_2 + m_3 = 0$

$$\Rightarrow m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a - x_1}{a}$$



$$\Rightarrow m_1m_2m_3 = -\frac{y_1}{a}$$

As two of the three normals are perpendicular, we take $m_1m_2 = -1$. (i.e. we assume AP perpendicular BP)

To get the locus, we have to eliminate m_1, m_2, m_3,

$$m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a - x_1}{a}$$

$$\Rightarrow -1 + m_3(-m_3) = \frac{2a - x_1}{a}$$

$$\Rightarrow -1 - \left(\frac{+y_1}{a}\right)^2 = \frac{2a - x_1}{a}$$

[using $m_1m_2m_3 = -y_1/a$ and $m_1m_2 = -1$]

$\Rightarrow a^2 + y_1^2 = -2a^2 + ax_1$

$\Rightarrow y_1^2 = a(x_1 - 3a)$

$\Rightarrow y^2 = a(x - 3a)$ is the required locus.

Example - 7

Find the equation of common tangent to the circle $x^2 + y^2 = 8$ and parabola $y^2 = 16x$.

Sol. Let $ty = x + at^2$ (where $a = 4$) be a tangent to parabola which also touches circle.

$\Rightarrow ty = x + 4at^2$ and $x^2 + y^2 = 8$

have only one common solution.

$\Rightarrow (ty - 4t^2)^2 + y^2 = 8$

has equal roots as a quadratic in y.

$\Rightarrow (1 + t^2)y^2 - 8t^3y + 16t^4 - 8 = 0$ has equal roots.

$\Rightarrow 64t^6 = 64t^6 + 64t^4 - 32 - 32t^2$

$\Rightarrow t^2 + 1 - 2t^4 = 0$

$\Rightarrow t^2 = 1, -1/2$

$\Rightarrow t = \pm 1$

$\Rightarrow$ the common tangents are

$y = x + 4$ and $y = -x - 4$.

Example - 8

Find the equation of the common tangents to the parabola $y^2 = 4ax$ and $x^2 = 4by$.

Sol. The equation of any tangent in terms of slope (m) to the parabola $y^2 = 4ax$ is

$$y = mx + \frac{a}{m} \quad \text{...(i)}$$

If this line is also tangent to the parabola $x^2 = 4ay$ then (i) meets $x^2 = 4by$ in two coincident points.

Substituting the value of y from (i) in $x^2 = 4by$ we get

$$x^2 = 4b\left(mx + \frac{a}{m}\right) \Rightarrow \quad x^2 - 4bmx - \frac{4ab}{m} = 0$$

The roots of this quadratic are equal provided "$B^2 = 4AC$"

i.e., $(-4bm)^2 = 4.1.\left(\frac{-4ab}{m}\right)$

$\Rightarrow \quad 16b^2m^3 + 16ab = 0, \ m \neq 0$

$\Rightarrow \quad m^3 = -a/b \quad \therefore \quad m = -a^{1/3}/b^{1/3}$

Substituting the value of m in (i) the required equation is

$$y = \frac{a^{1/3}}{b^{1/3}}x - \frac{ab^{1/3}}{a^{1/3}}$$

$$\Rightarrow \quad y = -\frac{a^{1/3}}{b^{1/3}}x - a^{2/3}b^{1/3}$$

$$\Rightarrow \quad a^{1/3}x + b^{1/3}y + a^{2/3}b^{2/3} = 0$$

Example - 9

Find the locus of the point of intersection of the tangents to the parabola $y^2 = 4ax$ which include as 45°.

Sol. $P \equiv (at_1^2, 2at_1)$ and $Q \equiv (at_2^2, 2at_2)$. $T \equiv (x_1, y_1)$

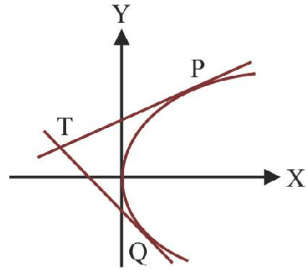
$\Rightarrow \quad x_1 = a\,t_1t_2 \quad$ ...(i) $\quad$ and $\quad y_1 = a\,(t_1 + t_2)$.

As $\underline{|PTQ} = 45°$, $\tan 45° = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

$$= \left|\frac{\frac{1}{t_1} - \frac{1}{t_2}}{1 + \frac{1}{t_1t_2}}\right| = \left|\frac{t_2 - t_1}{1 + t_1t_2}\right|$$

$\Rightarrow \quad (t_2 - t_1)^2 = (1 + t_1t_2)^2$

$\Rightarrow \quad (t_2 + t_1)^2 - 4t_1t_2 = (1 + t_1t_2) \quad$ ...(iii)



Replace values of $t_1 + t_2$ and $t_1\,t_2$ from (i) and (ii) in (iii) to get

$$\frac{y_1^2}{a^2} - 4\frac{x_1}{a} = \left(1 + \frac{x_1}{a}\right)^2$$

$\Rightarrow \quad$ Required locus $\equiv y^2 - 4ax = (x + a)^2$

Example - 10

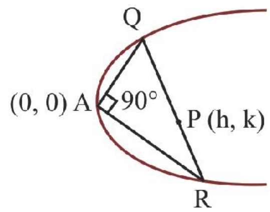
Find the locus of the mid points of the chords of the parabola $y^2 = 4ax$ which subtend a right angle at the vertex of the parabola.

Sol. Let P (h, k) be mid point of a chord QR of the parabola $y^2 = 4ax$, then equation of chord QR is

$T = S_1$

$\Rightarrow \quad yk - 2a\,(x + h) = k^2 - 4ah$

$\Rightarrow \quad yk - 2ax = k^2 - 2ah \quad$ ...(i)



If A is the vertex of the parabola. For combined equation of AQ and AR, Making homogenous of $y^2 = 4ax$ with the help of (i)

$\therefore \quad y^2 = 4\,ax\,.\,1$

$\therefore \quad y^2 = 4ax\left(\frac{yk - 2ax}{k^2 - 2ah}\right)$

$\Rightarrow \quad y^2(k^2 - 2ah) - 4akxy + 8a^2x^2 = 0$

Since $\angle QAR = 90°$.

$\therefore \quad$ Co–efficient of x^2 + Co–efficient of $y^2 = 0$

$\Rightarrow \quad k^2 = 2ah + 8a^2 = 0$

Hence the locus is P (h, k) is $y^2 - 2ax + 8a^2 = 0$.

Example - 11

Show that the locus of the middle points of normal chords of the parabola $y^2 = 4ax$ is

$$y^4 - 2a(x - 2a)y^2 + 8a^4 = 0.$$

Sol. Equation of the normal chord at any point $(at^2, 2at)$ of the parabola $y^2 = 4ax$ is

$$y + tx = 2at + at^3 \quad \text{...(i)}$$

But if $M(x_1, y_1)$ be its middle point, its equation must be also

$$T = S_1$$

$$yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$$

$$\Rightarrow \quad yy_1 - 2ax = y_1^2 - 2ax_1 \quad \text{...(ii)}$$

As Equations (i) and (ii) are identical. Comparing them

$$\frac{1}{y_1} = \frac{t}{-2a} = \frac{2at + at^3}{y_1^2 - 2ax_1}$$

From first two relations $t = -\dfrac{2a}{y_1}$ ...(iii)

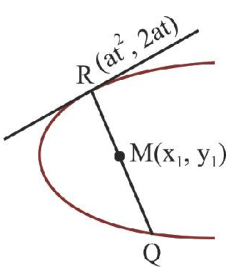
From last two relations

$$\frac{t}{-2a} = \frac{2at + at^3}{y_1^2 - 2ax_1}$$

$$\Rightarrow \quad \frac{y_1^2 - 2ax_1}{-2a} = 2a + at^2$$

$$\Rightarrow \quad \frac{y_1^2 - 2ax_1}{-2a} = 2a + a\left(\frac{-2a}{y_1}\right)^2 \quad \text{[from equation (iii)]}$$

$$\Rightarrow \quad \frac{y_1^2 - 2ax_1}{-2a} = \frac{2ay_1^2 + 4a^3}{y_1^2}$$

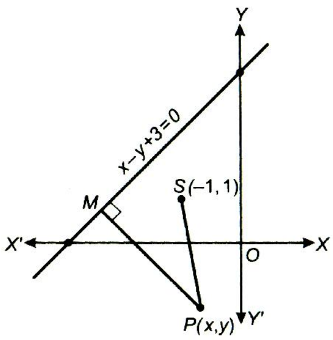


ELLIPSE

Example - 12

Find the equation of an ellipse whose focus is $(-1, 1)$, eccentricity is $\frac{1}{2}$ and the directrix is $x - y + 3 = 0$

Sol. Let P (x, y) be any point on the ellipse whose focus is S (–1, 1) and the directrix is $x - y + 3 = 0$. Draw PM perpendicular from P (x, y) on the directrix $x - y + 3 = 0$. Then by definition



$$SP = ePM$$

$$\Rightarrow \quad (SP)^2 = e^2(PM)^2$$

$$\Rightarrow \quad (x+1)^2 + (y-1)^2 = \frac{1}{4}\left\{\frac{x - y + 3}{\sqrt{2}}\right\}^2$$

$$\Rightarrow \quad 8(x^2 + y^2 + 2x - 2y + 2)$$

$$= x^2 + y^2 + 9 - 2xy + 6x - 6y$$

$$\Rightarrow \quad 7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$$

which is the required equation of the ellipse.

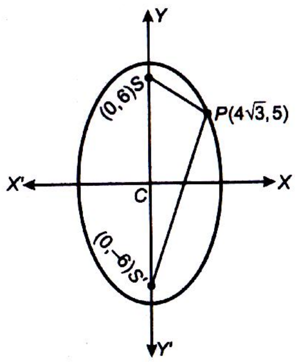
Example - 13

Find the lengths and equations of the focal radii drawn from the point $(4\sqrt{3}, 5)$ on the ellipse $25x^2 + 16y^2 = 1600$

Sol. The equation of the ellipse is

$25x^2 + 16y^2 = 1600$

or $\dfrac{x^2}{64} + \dfrac{y^2}{100} = 1$



Here b > a

$a^2 = 64,\ b^2 = 100$

$a^2 = b^2(1 - e^2)$

$\therefore$ $64 = 100(1 - e^2)$

$\Rightarrow$ $e = 3/5$

Let $P(x_1, y_1) \equiv (4\sqrt{3}, 5)$

be a point on the ellipse then SP and S′P are the focal radii

$\therefore$ $SP = b - ey_1$ and $S'P = b + ey_1$

$\therefore$ $SP = 10 - \dfrac{3}{5} \times 5$ and $S'P = 10 + \dfrac{3}{5} \times 5$

$\Rightarrow$ SP = 7 and S′P = 13

Also S is (0, be)

i.e., $\left(0, 10 \times \dfrac{3}{5}\right)$ i.e., $(0, 6)$

and S′ is (0, –be)

i.e., $\left(0, -10 \times \dfrac{3}{5}\right)$

i.e., (0, –6)

$\therefore$ Equation of SP is

$$y - 5 = \frac{6-5}{0-4\sqrt{3}}(x - 4\sqrt{3})$$

$$-4\sqrt{3}y + 20\sqrt{3} = x - 4\sqrt{3}$$

or $x + 4\sqrt{3}\,y - 24\sqrt{3} = 0$

and equation of S′P is

$\therefore$ $y - 5 = \dfrac{-6-5}{0-4\sqrt{3}}(x - 4\sqrt{3})$

$\Rightarrow$ $-4\sqrt{3}y + 20\sqrt{3} = -11x + 44\sqrt{3}$

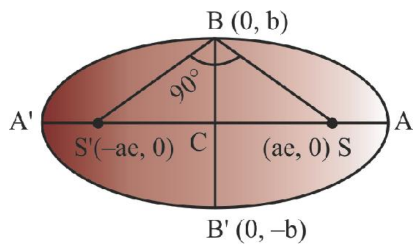
or $11x - 4\sqrt{3}y - 24\sqrt{3} = 0$

Example - 14

If the angle between the straight lines joining foci and the end of minor axis of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is 90°, find its eccentricity.

Sol. The equation of the ellipse is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

The ends of minor axis are B (0, b) and B′ (0, –b). If the eccentricity of the ellipse is e, then the foci are S (ae, 0) and S′ (–ae, 0).



$\therefore$ Slope of BS is $m_1 = \dfrac{b-0}{0-ae} = -\dfrac{b}{ae}$

and Slop of BS′ is $m_2 = \dfrac{b-0}{0+ae} = \dfrac{b}{ae}$

$\therefore$ The angle between BS and BS′ is 90°,

$\therefore$ $m_1m_2 = -1$ $\Rightarrow$ $-\dfrac{b}{ae} \times \dfrac{b}{ae} = -1$

$\Rightarrow$ $b^2 = a^2e^2$

$\Rightarrow$ $a^2(1 - e^2) = a^2e^2$ $\Rightarrow$ $1 - e^2 = e^2$

$\Rightarrow$ $2e^2 = 1$ $\therefore$

$e = \dfrac{1}{\sqrt{2}}$.

Example - 15

For what value of λ does the line $y = x + \lambda$ touches the ellipse $9x^2 + 16y^2 = 144$.

Sol. Equation of ellipse is $9x^2 + 16y^2 = 144$

$$\Rightarrow \quad \frac{x^2}{16} + \frac{y^2}{9} = 1$$

Comparing this with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

then we get $a^2 = 16$ and $b^2 = 9$ and comparing the line $y = x + \lambda$ with $y = mx + c$

$\therefore \quad m = 1$ and $c = \lambda$

If the line $y = x + \lambda$ touches the ellipse

$9x^2 + 16y^2 = 144$, then

$c^2 = a^2m^2 + b^2$

$\Rightarrow \quad \lambda^2 = 16 \times 1^2 + 9$

$\Rightarrow \quad \lambda^2 = 25$

$\therefore \quad \lambda = \pm 5$

Example - 16

If the normal at a point P(θ) to the ellipse $\frac{x^2}{14} + \frac{y^2}{15} = 1$ intersect it again at Q (2θ). Show that $\cos\theta = -2/3$.

Sol. The equation of normal at P(θ) is :

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$$

As $Q \equiv (a \cos 2\theta, b \sin 2\theta)$ lies on it, we can have :

$$\frac{a}{\cos\theta}(a \cos 2\theta) - \frac{b}{\sin\theta}(b \sin 2\theta) = a^2 - b^2$$

$$\Rightarrow \quad a^2 \frac{(2\cos^2\theta - 1)}{\cos\theta} - 2b^2 \cos\theta = a^2 - b^2$$

Put $a^2 = 14$, $b^2 = 5$ in the above equation to get :

$14(2 \cos^2\theta - 1) - 10 \cos^2\theta = 9 \cos\theta$

$\Rightarrow \quad 18\cos^2\theta - 9\cos\theta - 14 = 0$

$\Rightarrow \quad (6 \cos \theta - 7)(3 \cos \theta + 2) = 0$

$\Rightarrow \quad \cos\theta = 7/6$ (reject) or $\cos\theta = -2/3$

Hence $\cos\theta = -2/3$.

Example - 17

Show that the locus of the foot of the perpendicular drawn from the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ on any tangent is $(x^2 + y^2)^2 = a^2 x^2 + b^2 y^2$.

Sol. Let the tangent be $y = mx + \sqrt{a^2m^2 + b^2}$.

Draw CM perpendicular to tangent and let $M \equiv (x_1, y_1)$.

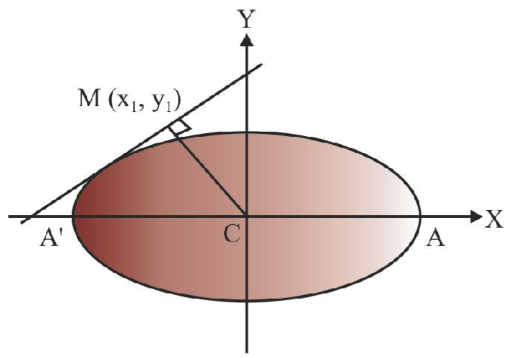
M lies on tangent,

$$\Rightarrow \quad y_1 = mx_1 + \sqrt{a^2m^2 + b^2} \quad \text{...(i)}$$

Slope (CM) = $-1/m$

$$\Rightarrow \quad \frac{y_1}{x_1} = -\frac{1}{m} \quad \Rightarrow \quad m = -\frac{x_1}{y_1} \quad \text{...(ii)}$$

Replace the value of m from (ii) into (i) and take square to get :



$$(x_1^2 + y_1^2)^2 = a^2x_1^2 + b^2y_1^2$$

Hence the required locus is :

$$(x^2 + y^2)^2 = a^2 x^2 + b^2 y^2.$$

Example - 18

A tangent to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touches it at a point P in the first quadrant and meets the axes in A and B respectively. If P divides AB is 3 : 1, find the equation of tangent.

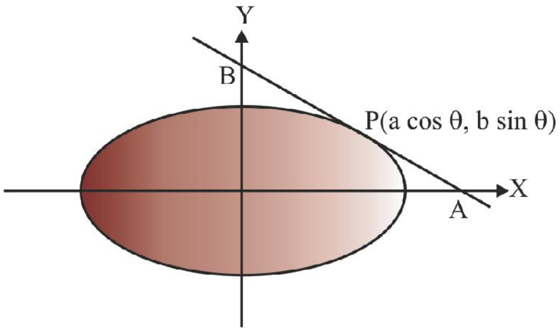
Sol. Let the coordinates of the point P ≡ (a cosθ, b sinθ)

$\Rightarrow$ the equation of the tangent at P is :

$$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1 \quad ...(i)$$

$\Rightarrow$ The coordinates of the points A and B are :

$$A \equiv \left(\frac{a}{\cos\theta},0\right) \text{ and } B \equiv \left(0,\frac{b}{\sin\theta}\right)$$



By section formula, the coordinates of P are

$$\left(\frac{a}{4\cos\theta},\frac{3b}{4\sin\theta}\right) \equiv (a\cos\theta, b\sin\theta)$$

$$\Rightarrow \frac{a}{4\cos\theta} = a\cos\theta \quad \text{and} \quad \frac{3b}{4\sin\theta} = b\sin\theta$$

$$\Rightarrow \cos\theta = \pm\frac{1}{2} \quad \text{and} \quad \sin\theta = \pm\frac{\sqrt{3}}{2}$$

$\Rightarrow$ θ = 60°

For equation of tangent, replace the value of θ in (i)

$\Rightarrow$ The equation of tangent is : $\frac{x}{a}+\frac{\sqrt{3}y}{b}=2.$

Example - 19

A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P and Q. Prove that the tangents at P and Q of the ellipse $x^2 + 2y^2 = 6$ are at right angles.

Sol. Chord of contact of

$$\frac{x^2}{6}+\frac{y^2}{3}=1 \text{ is } \frac{hx}{6}+\frac{ky}{3}=1 \quad ...(i)$$

Equation of any tangent to

$$\frac{x^2}{4}+\frac{y^2}{1}=1 \text{ is } \frac{x}{2}\cos\theta + y\sin\theta = 1. \quad ...(ii)$$

Compare (i) and (ii), eliminate θ and get locus of (h, k)

i.e. $x^2 + y^2 = 9$ (i.e. $a^2 + b^2$)

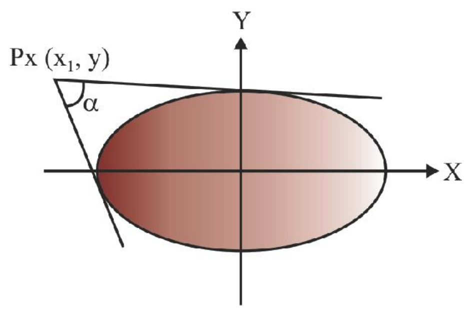
i.e. director circle of 2nd ellipse.

Example - 20

Find the locus of a point from which the two tangents to the ellipse are inclined at an angle α.

Sol. Equation of tangent of slope m is

$$\equiv y = mx + \sqrt{a^2m^2+b^2} \quad ...(i)$$



Point P ≡ (x_1, y_1) lies on (i)

$$\Rightarrow y_1 = mx_1 + \sqrt{a^2m^2+b^2}$$

$$\Rightarrow m^2(x_1^2-a^2) - 2x_1y_1 m + (y_1^2-b^2) = 0$$

Let roots be m_1 and m_2

$$\Rightarrow m_1 + m_2 = \frac{2x_1y_1}{x_1^2-a^2}$$

and $m_1 m_2 = \dfrac{y_1^2 - b^2}{x_1^2 - a^2}$

$$\tan\alpha = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$\Rightarrow$ $\tan^2\alpha\,(1 + m_1 m_2)^2 = (m_1 + m_2)^2 - 4 m_1 m_2$ ...(ii)

Using (ii),

$$\tan^2\alpha\left(1 + \frac{y_1^2 - b^2}{x_1^2 - a^2}\right)^2 = \frac{4x_1^2 y_1^2}{(x_1^2 - a^2)^2} - 4\,\frac{y_1^2 - b^2}{x_1^2 - a^2}$$

$\Rightarrow$ Locus is $\tan^2\alpha\,(x^2 + y^2 - a^2 - b^2)^2$
$= 4\,[x^2b^2 + a^2y^2 - a^2b^2]$

Example - 21

Prove that in general four normals can be drawn to an ellipse from any point and the sum of the eccentric angles of the feet of these normal is equal to an odd multiple of two right angles.

Sol. Equation of Normal $\equiv \dfrac{ax}{\cos\theta} - \dfrac{by}{\sin\theta} = a^2 - b^2$

As it passes through (h, k) $\equiv \dfrac{ah}{\cos\theta} - \dfrac{bk}{\sin\theta} = a^2 - b^2$

Replace $\cos\theta = \dfrac{1-t^2}{1+t^2}$, $\sin\theta = \dfrac{2t}{1+t^2}$, where $t = \tan\dfrac{\theta}{2}$

$\Rightarrow$ $bk\,t^4 + 2\,(ak + a_2 - b_2)\,t_3 + 2\,(ak - a_2 + b_2)\,t - bk = 0$

It roots are $\tan\dfrac{\theta r}{2}$, r = 1, 2, 3, 4

$$\tan\left(\frac{\theta_1}{2} + \frac{\theta_2}{2} + \frac{\theta_3}{2} + \frac{\theta_4}{2}\right) = \frac{S_1 - S_3}{1 - S_2 + S_4} = \infty = \tan\frac{\pi}{2}$$

$$\left(\text{as } S_2 = 0, S_4 = -\frac{bk}{bk} = -1\right)$$

$\therefore$ $\dfrac{\theta_1 + \theta_2 + \theta_3 + \theta_4}{2} = n\pi + \dfrac{\pi}{2}$

$\Rightarrow$ $\theta_1 + \theta_2 + \theta_3 + \theta_4 = (2n + 1)$

HYPERBOLA

Example - 22

Find the eccentricity of the hyperbola whose latus rectum is half of its transverse axis.

Sol. Let the equation of hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Then transverse axis = 2a and latus-rectum = $\dfrac{2b^2}{a}$

According to question $\dfrac{2b^2}{a} = \dfrac{1}{2}(2a)$

$\Rightarrow$ $2b^2 = a^2$ $\quad(\because b^2 = a^2(e^2 - 1))$

$\Rightarrow$ $2a^2(e^2 - 1) = a^2$

$\Rightarrow$ $2e^2 - 2 = 1$

$\Rightarrow$ $e^2 = \dfrac{3}{2}$

$\therefore$ $e = \sqrt{\dfrac{3}{2}}$

Hence the required eccentricity is $\sqrt{\dfrac{3}{2}}$.

Example - 23

Obtain the equation of a hyperbola with co-ordinate axes as principal axes given that the distances of one of its vertices from the faci are 9 and 1 units.

Sol. Let equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{... (1)}$$

If vertices are A (a, 0) and A′ (–a, 0) and foci are S (ae, 0) and S′ (–ae, 0)

Given l (S′A) = 9 and l (SA) = 1

$\Rightarrow$ a + ae = 9 and ae – a = 1

or a (1 + e) = 9 and a (e – 1) = 1

$\therefore$ $\dfrac{a(1+e)}{a(e-1)} = \dfrac{9}{1}$

$\Rightarrow$ $1 + e = 9e - 9 \Rightarrow e = \dfrac{5}{4}$

$\because$ a (1 + e) = 9

$\therefore \quad a\left(1+\frac{5}{4}\right)=9$

$\Rightarrow \quad a = 4$

$$b^2 = a^2 (e^2 - 1) = 16\left(\frac{25}{16} - 1\right)$$

$\therefore \quad b^2 = 9$

From (1) equation of hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Example - 24

Show that the equation $7y^2 - 9x^2 + 54x - 28y - 116 = 0$ represents a hyperbola. Find the co–ordinates of the centre, length of transverse and conjugate axes, eccentricity, latus rectum, co–ordinates of foci and vertices, equations of the directrices of the hyperbola.

Sol. We have $7y^2 - 9x^2 + 54x - 28y - 116 = 0$

$\Rightarrow \quad 7(y^2 - 4y) - 9(x^2 - 6x) - 116 = 0$

$\Rightarrow \quad 7(y^2 - 4y + 4) - 9(x^2 - 6x + 9) = 116 + 28 - 81$

$\Rightarrow \quad 7(y - 2)^2 - 9(x - 3)^2 = 63$

$$\Rightarrow \quad \frac{(y-2)^2}{9} - \frac{(x-3)^2}{7} = 1$$

$$\Rightarrow \quad \frac{Y^2}{9} - \frac{X^2}{7} = 1 \text{ [where } X = x - 3 \text{ and } Y = y - 2]$$

This equation represents conjugate hyperbola. Comparing it with

$$\frac{Y^2}{b^2} - \frac{X^2}{a^2} = 1 \text{ we get } b^2 = 9 \text{ and } a^2 = 7$$

$\therefore \quad b = 3$ and $a = \sqrt{7}$.

Centre : $X = 0, Y = 0$.

i.e., $x - 3 = 0, y - 2 = 0 \quad \therefore \quad$ Centre is (3, 2)

Length of transverse axis :

Length of transverse axis = 2b = 6.

Length of conjugate axis :

Length of conjugate axis = $2a = 2\sqrt{7}$.

Eccentricity : The eccentricity e is given by

$$e = \sqrt{1+\frac{a^2}{b^2}} = \sqrt{1+\frac{9}{7}} = \frac{4}{\sqrt{7}}$$

Length of latus rectum :

The length of latus rectum = $\frac{2a^2}{b}$.

$$= \frac{2(7)}{3} = \frac{14}{3}.$$

Foci : The co–ordinates of foci are $(0, \pm be)$

$\therefore \quad X = 0, Y = \pm be$

$$\Rightarrow \quad x - 3 = 0, y - 2 = \pm 3 \times \frac{4}{\sqrt{7}}.$$

$$\Rightarrow \quad \left(3, 2 \pm \frac{12}{\sqrt{7}}\right).$$

Vertices : The co–ordinates of vertices are $(0, \pm b)$.

$\Rightarrow \quad X = 0, Y = \pm b$

$\Rightarrow \quad x - 3 = 0, y - 2 = \pm 3$

$\Rightarrow \quad (3, 2 \pm 3)$

$\Rightarrow \quad$ vertices are (3, 5) and (3, –1)

Equation of directrices :

The equation of directrices are

$$Y = \pm \frac{3}{4/\sqrt{7}}$$

$$\Rightarrow \quad y - 2 = \frac{3\sqrt{7}}{4}$$

$$\Rightarrow \quad y = \left(2 \pm \frac{3\sqrt{7}}{4}\right).$$

Example - 25

For what value of c does the line $y = 2x + c$ touches the hyperbola $16x^2 - 9y^2 = 144$?

Sol. Equation of hyperbola is

$$16x^2 - 9y^2 = 144 \quad \Rightarrow \quad \frac{x^2}{9} - \frac{y^2}{16} = 1$$

comparing this with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get $a^2 = 9$, $b^2 = 16$.

and comparing this line $y = 2x + c$ with $y = mx + c$.

$\therefore$ $m = 2$ and $c = 1$

If the line $y = 2x + 1$ touches the hyperbola

$16x^2 - 9y^2 = 144$ then $c^2 = a^2m^2 - b^2$

$\Rightarrow$ $c^2 = 9\,(2)^2 - 16 = 36 - 16 = 20$

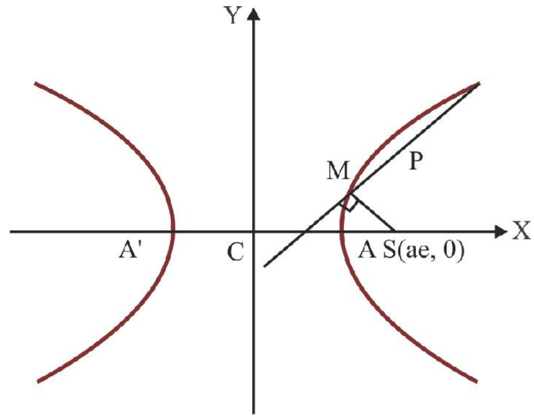
$\therefore$ $c = \pm\, 2\sqrt{5}$.

Example - 26

Find the locus of the foot of the perpendicular drawn from focus S of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to any tangent.

Sol. Let the tangent be $y = mx + \sqrt{a^2m^2 - b^2}$.

Let M (x_1, y_1) be the foot of perpendicular SM drawn to the tangent from focus S (ae, 0).



Slope (SM) × Slope (PM) = – 1

$\Rightarrow$ $\left(\frac{y_1 - 0}{x_1 - ae}\right) m = -1 \qquad x_1 + my_1 = ae$ ...(i)

As M lies on tangent, we also have

$$y_1\ mx_1 + \sqrt{a^2m^2 - b^2}$$

$\Rightarrow$ $-mx_1 + y_1 = \sqrt{a^2m^2 - b^2}$ ...(ii)

We can now eliminate m from (i) and (ii).

Substituting value of m from (i) in (ii) leads to a lot of simplification and hence we avoid this step.

By squaring and adding (i) and (ii), we get :

$$x_1^2(1+m^2) + y_1^2(1+m^2) = a^2e^2 + a^2m^2 - b^2$$

$$(x_1^2 + y_1^2)(1+m^2) = a^2(1+m^2)$$

$\Rightarrow$ $x_1^2 + y_1^2 = a^2$

$\Rightarrow$ Required Locus is : $x^2 + y^2 = a^2$.

Note : M lies on the auxiliary circle.

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Parabola

Basic

1. The name of the conic represented by the equation $x^2 + y^2 - 2xy + 20x + 10 = 0$ is

(a) a hyperbola (b) an ellipse
(c) a parabola (d) circle

2. The equation $\lambda x^2 + 4xy + y^2 + \lambda x + 3y + 2 = 0$ represents a parabola, if λ is

(a) –4 (b) 4
(c) 0 (d) none of these

3. The name of the curve described parametrically by the equations $x = t^2 + t + 1$, $y = t^2 - t + 1$ is

(a) a circle (b) an ellipse
(c) a hyperbola (d) a parabola

4. The equation of the parabola whose focus is (–1, 1) and directrix is $4x + 3y - 24 = 0$ is

(a) $9x^2 + 16y^2 - 24xy + 242x + 94y - 526 = 0$

(b) $16x^2 + 9y^2 - 24xy + 242x + 94y - 526 = 0$

(c) $2x^2 - 23y^2 + 7xy + 32x + 17y + 40 = 0$

(d) none of these

Standard And Shifted Parabola

5. The focus of the parabola is (1, 1) and the tangent at the vertex has the equation $x + y = 1$. Then :

(a) equation of the parabola is $(x - y)^2 = 2(x + y - 1)$

(b) equation of the parabola is $(x - y)^2 = 4(x + y - 1)$

(c) the co-ordinates of the vertex are (½, ½)

(d) length of the latus rectum is $2\sqrt{2}$

6. The coordinates of an end-point of the latus-rectum of the parabola $(y-1)^2 = 4(x+1)$ are

(a) (0, –3) (b) (0, –1)
(c) (0, 1) (d) (1, 3)

7. The length of the latus-rectum of the parabola $x = ay^2 + by + c$ is

(a) $\frac{a}{4}$ (b) $\frac{a}{3}$

(c) $\frac{1}{a}$ (d) $\frac{1}{4a}$

8. Coordinates of the focus of the parabola $x^2 - 4x - 8y - 4 = 0$ are

(a) (0, 2) (b) (2, 1)
(c) (1, 2) (d) (–2, –1)

9. If the parabola $y^2 = 4ax$ passes through the point (–3, 2), then the length of its latus rectum is

(a) 2/3 (b) 4/3
(c) 1/3 (d) 4

10. If focus of a parabola is (2, 0) and one extremity of latus rectum is (2, 2), then its equation is

(a) $y^2 = 4(3 - x)$ (b) $y^2 = 4x - 4$
(c) both (a) and (b) (d) none of these

11. Equation of parabola which has its axis along x-axis and which passes through the points (3, 2) and (–2, –1) is

(a) $5y^2 = 3x + 11$ (b) $y^2 = 3x - 1$
(c) $y^2 = x + 3$ (d) none of these

12. The equation of the directrix of the parabola $y^2 = 12x$ is –

(a) $x + 3 = 0$ (b) $y + 3 = 0$
(c) $x - 3 = 0$ (d) $y - 3 = 0$

13. The equation of the latus rectum of the parabola $x^2 = -12y$ is–

(a) $y = 3$ (b) $x = 3$
(c) $y = -3$ (d) $x = -3$

14. Vertex, focus, latus rectum, length of the latus rectum and equation of directrix of the parabola $y^2 = 4x + 4y$ are

(a) (1, 2), (0, 2), y = 0, 4, x = –2
(b) (–1, 2), (0, 2), x = 0, 4, x = –2
(c) (–1, 2), (1, 2), x = 0, 4, x = 2
(d) (–1, 2), (0, 2), y = 0, 2, y = –2

15. Given the two ends of the latus rectum, the maximum number of parabolas that can be drawn, is

(a) 1 (b) 2
(c) 0 (d) inifinite

16. If the vertex = (2, 0) and the extremities of the latusrectum are (3, 2) and (3, –2), then the equation of the parabola is

(a) $y^2 = 2x - 4$ (b) $x^2 = 2y - 8$
(c) $y^2 = 4x - 8$ (d) none of these

17. The equation of the parabola whose axis is parallel to y-axis and which passes through the points (0, 4), (1, 9) and (–2, 6) is given by

(a) $2y^2 + 3y - x + 4 = 0$ (b) $3x^2 + 2x + y - 4 = 0$

(c) $2x^2 + 3x - y + 4 = 0$ (d) none of the above

18. The equation of the parabola having its axis parallel to x-axis and which passes through the points (1, 2), (–1,3) and (–2, 1) is

(a) $5y^2 + 2x - 21y + 20 = 0$ (b) $5y^2 - 2x - 21y + 20 = 0$

(c) $5x^2 - 2x - 21y - 20 = 0$ (d) none of the above

Parametric Form and Focal Chord :

19. The parametric equation of the curve $(y-2)^2 = 12(x-4)$ are-

(a) $6t, 3t^2$ (b) $2 + 3t, 4 + t^2$

(c) $4 + 3t^2, 2 + 6t$ (d) None of these

20. The point on $y^2 = 4ax$ nearest to the focus has its abscissa

(a) $x = -a$ (b) $x = a$

(c) $x = \frac{3}{2}$ (d) $x = 0$

21. The parametric equation of a parabola is $x = t^2 + 1$, $y = 2t + 1$. The Cartesian equation of its directrix is

(a) $x = 0$ (b) $x + 1 = 0$

(c) $y = 0$ (d) none of these

22. Any point on the parabola whose focus is (0, 1) and the directrix is $x + 2 = 0$ is given by

(a) $(t^2 + 1, 2t-1)$ (b) $(t^2 + 1, 2t + 1)$

(c) $(t^2, 2t)$ (d) $(t^2 - 1, 2t + 1)$

23. The length of a focal chord of the parabola $y^2 = 4ax$ at a distance b from the vertex is c. Then,

(a) $2a^2 = bc$ (b) $a^3 = b^2c$

(c) $ac = b^2$ (d) $b^2c = 4a^3$

24. A double ordinate of the parabola $y^2 = 8px$ is of length 16p. The angle subtended by it at the vertex of the parabola is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) none of these

25. The latus rectum of a parabola whose focal chord is PSQ such that SP = 3 and SQ = 2 is given by

(a) $\frac{24}{5}$ (b) $\frac{12}{5}$

(c) $\frac{6}{5}$ (d) none of these

26. The length of the chord of the parabola $x^2 = 4ay$ passing through the vertex and having slope $\tan\alpha$, is

(a) $4a \operatorname{cosec}\alpha \cot\alpha$ (b) $4a \tan\alpha \sec\alpha$

(c) $4a \cos\alpha \cot\alpha$ (d) $4a \sin\alpha \tan\alpha$

27. If b and c are the lengths of the segments of any focal chord of parabola $y^2 = 4ax$, then the length of the semi- latus- rectum is

(a) $\frac{b+c}{2}$ (b) $\frac{bc}{b+c}$

(c) $\frac{2bc}{b+c}$ (d) $\sqrt{bc}$

Position of Point w.r.t. Parabola :

28. For the parabola $y^2 = 8x$ point (2, 5) is

(a) inside the parabola

(b) Focus

(c) outside the parabola

(d) On the parabola

29. The equation of a parabola is $y^2 = 4x$ P (1, 3) and Q (1, 1) are two points in the xy-plane. Then, for the parabola

(a) P and Q are exterior points

(b) P is an interior point while Q is an exterior point

(c) P and Q are interior points

(d) P is an exterior point while Q is an interior point

30. The point (a, 2a) is an interior point of the region bounded by the parabola $y^2 = 16x$ and the double ordinate through the focus. Then, a belongs to the open interval

(a) $a < 4$ (b) $0 < a < 4$

(c) $0 < a < 2$ (d) $a > 4$

Position of Line w.r.t. Parabola :

31. The line $y = mx + 1$ is a tangent to the parabola $y^2 = 4x$ if

(a) $m = 1$ (b) $m = 4$
(c) $m = 2$ (d) $m = 3$

32. If the line $2x - 3y = k$ touches the parabola $y^2 = 6x$, then the value of k is

(a) 27/4 (b) –81/4
(c) –7 (d) –27/4

33. If the line $x + y - 1 = 0$ touches the parabola $y^2 = kx$, then the value of k is

(a) 4 (b) –4
(c) 2 (d) –2

34. If $y = 2x - 3$ is a tangent to the parabola $y^2 = 4a\left(x - \frac{1}{3}\right)$, then a is equal to

(a) 1 (b) –1
(c) $\frac{14}{3}$ (d) $-\frac{14}{3}$

35. If two tangents drawn from the point (α, β) to the parabola $y^2 = 4x$ be such that the slope of one tangent is double of the other then

(a) $\beta = \frac{2}{9}\alpha^2$ (b) $\alpha = \frac{2}{9}\beta^2$
(c) $2\alpha = 9\beta^2$ (d) none of these

36. The angle between the tangents drawn to the parabola $y^2 = 12x$ from the point $(-3, 2)$ is

(a) 90° (b) 60°
(c) 30° (d) 45°

37. The tangent to the parabola $y^2 = 16x$, which is perpendicular to a line $y - 3x - 1 = 0$ is

(a) $3y + x + 36 = 0$ (b) $3y - x - 36 = 0$
(c) $x + y - 36 = 0$ (d) $x - y + 36 = 0$

38. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(a) $x = 1$ (b) $2x + 1 = 0$
(c) $x = -1$ (d) $2x - 1 = 0$

39. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is

(a) $\pi/6$ (b) $\pi/4$
(c) $\pi/3$ (d) $\pi/2$

40. The equations of common tangents to $y^2 = 4ax$ and $(x + a)^2 + y^2 = a^2$ are

(a) $y = \left(\frac{x}{\sqrt{3}} + a\right)$ (b) $y = \pm\left(\sqrt{3}x + \frac{a}{\sqrt{3}}\right)$
(c) $y = \pm\left(\frac{x}{\sqrt{3}} + \sqrt{3}\,a\right)$ (d) none of these

Ellipse

Basic :

41. The equation $\frac{x^2}{10 - a} + \frac{y^2}{4 - a} = 1$ represents an ellipse, if

(a) $a < 4$ (b) $a > 4$
(c) $4 < a < 10$ (d) $a > 10$

42. The equation to the ellipse, whose focus is the point (–1, 1), whose directrix is the straight line $x - y + 3 = 0$, and whose eccentricity is $\frac{1}{2}$ is

(a) $7x^2 + 2xy + 7y^2 + 10x - 10y + 7 = 0$
(b) $x^2 + 2xy + 10x - 10y + 3 = 0$
(c) $3x^2 + xy + 10x - 10y + 3 = 0$
(d) None of these

43. The equation of a conic section whose focus is at (–1, 0), directrix is the line $4x - 3y + 2 = 0$ and eccentricity is $\frac{1}{\sqrt{2}}$, is

(a) $34x^2 + 24xy + 41y^2 + 84x + 12y + 46 = 0$
(b) $41x^2 + 24xy + 34y^2 + 84x + 12y + 46 = 0$
(c) $34x^2 + 24xy + 41y^2 + 12x + 84y + 46 = 0$
(d) $34x^2 + 24xy + 41y^2 + 84x + 12y + 81 = 0$

44. A line of fixed length a + b moves so that its ends are always on two fixed perpendicular straight lines; then the locus of a point, which divides this line into portions of length a and b, is a/an

(a) ellipse (b) parabola

(c) straight line (d) none of these

45. The curve represented by x = 2 (cos t + sin t), y = 5 (cos t – sin t) is

(a) a circle (b) a parabola

(c) an ellipse (d) a hyperbola

Shifted Ellipse and Geometrical Properties of Ellipse :

46. The centre of the ellipse $8x^2 + 6y^2 - 16x + 12y + 13 = 0$ is at

(a) (1, 1) (b) (–1, 1)

(c) (1, –1) (d) none of these

47. The equation of the latus rectum of the ellipse $9x^2 + 4y^2 - 18x - 8y - 23 = 0$ are

(a) $y = \pm\sqrt{5}$ (b) $y = -\sqrt{5}$

(c) $y = 1 \pm \sqrt{5}$ (d) $x = -1 \pm \sqrt{5}$

48. If the latus rectum of an ellipse is half of its minor axis, its eccentricity is

(a) 3/4 (b) 1/4

(c) 1/2 (d) $\sqrt{3}/2$

49. Equation of ellipse whose minor axis is equal to the distance between the foci and whose latus rectum is 10, is given by (take origin as centre and major axis along x-axis)

(a) $2x^2 + y^2 = 100$ (b) $x^2 + 2y^2 = 100$

(c) $2x^2 + y^2 = 50$ (d) none of these

50. The length of the latus rectum of an ellipse is one third of the major axis, its eccentricity would be

(a) 2/3 (b) $\sqrt{(2/3)}$

(c) $1/\sqrt{3}$ (d) $1/\sqrt{2}$

51. Equation of the ellipse whose foci are (4, 0) and (–4, 0) and e = 1/3 is

(a) $x^2/9 + y^2/8 = 16$ (b) $x^2/8 + y^2/9 = 16$

(c) $x^2/9 + y^2/8 = 32$ (d) none of these

52. An ellipse has its centre at (1, –1) and semi major axis equal to 8. If this ellipse passes through the point (1, 3), its eccentricity is equal to

(a) $1/\sqrt{2}$ (b) 1/2

(c) $\sqrt{3}/2$ (d) none of these

53. An ellipse has OB as semi-minor axis, F and F′ are its foci and the angle FBF is a right angle. Then the eccentricity of the ellipse is

(a) 1/2 (b) 1/4

(c) $1/\sqrt{2}$ (d) none of these

54. S and T are the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and B is an end of the minor axis. If STB is an equilateral triangle, the eccentricity of the ellipse is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

Parametric Form and Auxiliary Circle :

55. The equation of auxiliary circle of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ is

(a) $x^2 + y^2 = 9$ (b) $x^2 + y^2 = 16$

(c) $x^2 + y^2 = 25$ (d) none of these

56. Parametric equation of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is

(a) x = 4 cos θ, y = 3 sin θ

(b) x = 3 cos θ, y = 3 sin θ

(c) x = 4 cos θ, y = 4 sin θ

(d) x = 3 cos θ, y = 4 sin θ

57. The parametric representation of a point on the ellipse whose foci are (–1,0) and (7, 0) and eccentricity $\frac{1}{2}$ is

(a) $(3 + 8\cos\theta, 4\sqrt{3}\sin\theta)$ (b) $(8\cos\theta, 4\sqrt{3}\sin\theta)$

(c) $(3 + 4\sqrt{3}\cos\theta, 8\sin\theta)$ (d) none of these

Position of Point w.r.t. Ellipse :

58. Let E be the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and C be the circle $x^2 + y^2 = 9$. Let P and Q be the points (1, 2) and (2, 1) respectively. Then

(a) Q lies inside C but outside E
(b) Q lies outside both C and E
(c) P lies inside both C and E
(d) P lies inside C but outside E

59. The number of real tangents that can be drawn to the ellipse $3x^2 + 5y^2 = 32$ passing through (3, 5) is

(a) 0 (b) 1
(c) 2 (d) infinite

Position of Line w.r.t. Ellipse and Equation of Tangent :

60. The number of values of c such that the straight line y = 4x + c touches the curve $\frac{x^2}{4}+y^2=1$ is

(a) 0 (b) 1
(c) 2 (d) infinite

61. The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line y = mx + c intersect in real points only, if

(a) $a^2m^2 < c^2 - b^2$ (b) $a^2m^2 > c^2 + b^2$
(c) $a^2m^2 \geq c^2 - b^2$ (d) $c \geq b$

62. The line $y = 2t^2$ intersects the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ in real points, if

(a) $|t| \leq 1$ (b) $|t| < 2$
(c) $|t| > 1$ (d) $|t| \geq 1$

63. The equation of the tangents to the ellipse $4x^2 + 3y^2 = 5$ which are parallel to the line y = 3x + 7 are

(a) $y=3x\pm\sqrt{\frac{155}{3}}$ (b) $y=3x\pm\sqrt{\frac{155}{12}}$
(c) $y=3x\pm\sqrt{\frac{95}{12}}$ (d) none of these

64. Equation of tangents to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, which are perpendicular to the line 3x + 4y = 7, are

(a) $4x-3y=\pm 6\sqrt{5}$ (b) $4x-3y=\pm\sqrt{12}$
(c) $4x-3y=\pm\sqrt{2}$ (d) $4x-3y=\pm 1$

65. Equation of the tangents to the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ which are parallel to the line x + y + 1 = 0, are

(a) x + y + 5 = 0 (b) x + y + 6 = 0
(c) 2x + y − 5 = 0 (d) x + y − 6 = 0

66. Equation of the tangent to the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ perpendicular to the line 2x + y + 7 = 0, is

(a) x − 2y + 4 = 0 (b) x − 2y + 5 = 0
(c) x − 2y − 2 = 0 (d) x − 2y − 5 = 0

67. The angle between pair of tangents drawn to the ellipse $3x^2 + 2y^2 = 5$ from the point (1, 2) is

(a) $\tan^{-1}\frac{12}{5}$ (b) $\tan^{-1}\frac{6}{\sqrt{5}}$
(c) $\tan^{-1}\frac{12}{\sqrt{5}}$ (d) $\tan^{-1}\frac{\sqrt{12}}{5}$

68. The equation of the ellipse which passes through origin and has its foci at the points (1, 0) and (3, 0) is -

(a) $3x^2 + 4y^2 = x$ (b) $3x^2 + y^2 = 12x$
(c) $x^2 + 4y^2 = 12x$ (d) $3x^2 + 4y^2 = 12x$

Hyperbola

Basics

69. The locus of the point of intersection of the lines $\sqrt{3}x-y-4\sqrt{3}k=0$ and $\sqrt{3}kx+ky-4\sqrt{3}=0$ for different values of k is-

(a) Ellipse (b) Parabola
(c) Circle (d) Hyperbola

70. Find the equation of the hyperbola whose directrix is 2x + y = 1, focus (1, 2) and eccentricity $\sqrt{3}$.

(a) $7x^2 - 2y^2 + 12xy - 2x + 14y - 22 = 0$
(b) $7x^2 - 2y^2 + 2xy - 2x + 14y - 22 = 0$
(c) $7x^2 - 2y^2 + xy - 14x + 2y - 22 = 0$
(d) none of the above

Shifted Hyperbola and Equation of Hyperbola

71. The foci of the hyperbola $3(y-1)^2 - 4(x-2)^2 = 12$ are

(a) $(0, \sqrt{7})$ (b) $(2, 1+\sqrt{7})$

(c) $(2, 1-\sqrt{7})$ (d) $(0, -\sqrt{7})$

72. The eccentricity of the conic represented by $x^2 - y^2 - 4x + 4y + 16 = 0$ is

(a) 1 (b) $\sqrt{2}$

(c) 2 (d) 1/2

73. The equation $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ represents a hyperbola

(a) the length of whose transverse axis is $4\sqrt{3}$

(b) the length of whose conjugate axis is 4

(c) whose centre is $(-1, 2)$

(d) whose eccentricity is $\sqrt{\frac{19}{3}}$

74. The equation $9x^2 - 16y^2 - 18x + 32y - 151 = 0$ represents a hyperbola -

(a) The length of the transverse axes is 4

(b) Length of latus rectum is 9

(c) Equation of directrix is $x = \frac{21}{5}$ and $x = -\frac{11}{5}$

(d) None of these

75. Equation of the hyperbola with eccentricity 3/2 and foci at $(\pm 2, 0)$ is

(a) $x^2/4 - y^2/9 = 4/9$ (b) $x^2/9 - y^2/4 = 4/9$

(c) $x^2/4 - y^2/9 = 1$ (d) none of these

76. If latus rectum of the hyperbola is half of its transverse axis, then its eccentricity is

(a) 3/2 (b) $\sqrt{3}/2$

(c) $\sqrt{(3/2)}$ (d) none of these

77. The foci of a hyperbola coincide with the foci of the ellipse $x^2/25 + y^2/9 = 1$. If eccentricity of the hyperbola is 2, then its equation is :

(a) $x^2 - 3y^2 - 12 = 0$ (b) $3x^2 - y^2 - 12 = 0$

(c) $x^2 - y^2 - 4 = 0$ (d) none of these

78. If the foci of the elipse $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then the value of b^2 is

(a) 4 (b) 9

(c) 16 (d) 12

79. The eccentricity of the hyperbola whose latusrectum is 8 and conjugate axis is equal to half the distance between the foci, is

(a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$

(c) $\frac{2}{\sqrt{3}}$ (d) none of these

Position of Point, Position of Line & Eq. of Tangent :

80. The position of point $(5, -4)$ relative to hyperbola $9x^2 - y^2 = 1$ is

(a) inside the hyperbola (b) outside the hyperbola

(c) on the hyperbola (d) none of the above

81. The value of m for which $y = mx + 6$ is a tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{49} = 1$ is

(a) $\sqrt{\frac{17}{20}}$ (b) $\sqrt{\frac{20}{17}}$

(c) $\sqrt{\frac{3}{20}}$ (d) $\sqrt{\frac{20}{3}}$

82. If m_1 and m_2 are the gradients of tangents to hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ which passes through (6, 2), then

(a) $m_1 + m_2 = \frac{24}{11}$ (b) $m_1 m_2 = \frac{20}{11}$

(c) $m_1 + m_2 = \frac{48}{11}$ (d) $m_1 m_2 = \frac{11}{20}$

83. The equations of the tangents to the hyperbola $x^2 - 4y^2 = 36$ which are perpendicular to the line $x - y + 4 = 0$ are

(a) $y = -x \pm 3\sqrt{3}$ (b) $y = -x \pm 2$

(c) $y = -x \pm \sqrt{5}$ (d) none of these

Conjugate and Rectangular Hyperbola :

84. The eccentricity of the conjugate hyperbola of the hyperbola $x^2 - 3y^2 = 1$ is

(a) 2 (b) $\frac{2}{\sqrt{3}}$

(c) 4 (d) $\frac{4}{3}$

85. If the eccentricity of a hyperbola is $\sqrt{3}$, then the eccentricity of its conjugate hyperbola is

(a) $\sqrt{2}$ (b) $\sqrt{3}$

(c) $\sqrt{\frac{3}{2}}$ (d) $2\sqrt{3}$

86. If e_1 and e_2 are the eccentricities of a hyperbola $3x^2 - 3y^2 = 25$ and its conjugate, then

(a) $e_1^2 + e_2^2 = 2$ (b) $e_1^2 + e_2^2 = 4$

(c) $e_1 + e_2 = 4$ (d) $e_1 + e_2 = \sqrt{2}$

87. If e and e′ be the eccentricities of a hyperbola and its conjugate then the value of $\frac{1}{e^2} + \frac{1}{e'^2} =$

(a) 0 (b) 1

(c) 2 (d) 4

88. If e and e_1 are the eccentricities of the hyperbolas $xy = c^2$ and $x^2 - y^2 = a^2$, then $(e + e_1)^2$ is equal to

(a) 2 (b) 4

(c) 6 (d) 8

89. If $5x^2 + \lambda y^2 = 20$ represents a rectangular hyperbola, then λ =

(a) 5 (b) – 5

(c) 4 (d) –4

90. A ray emanating from the point $(-\sqrt{41}, 0)$ is incident on the hyperbola $16x^2 - 25y^2 = 400$ at the point P with abscissa –10. Then the equation of the reflected ray after first reflection and point P lies in second quadrant is

(a) $4\sqrt{3}\,x - (10 - \sqrt{41})\,y + 4\sqrt{123} = 0$

(b) $4\sqrt{3}\,x + (10 - \sqrt{41})\,y - 4\sqrt{123} = 0$

(c) $4\sqrt{3}\,x + (10 - \sqrt{41})\,y + 4\sqrt{123} = 0$

(d) $4\sqrt{3}\,x - (10 - \sqrt{41})\,y - 4\sqrt{123} = 0$

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are **(2002)**

(a) $x = \pm (y + 2a)$ (b) $y = \pm (x + 2a)$

(c) $x = \pm (y + a)$ (d) $y = \pm (x + a)$

2. Locus of mid point of the portion between the axes of $x \cos \alpha + y \sin \alpha = p$ where p is constant is **(2002)**

(a) $x^2 + y^2 = \dfrac{4}{p^2}$ (b) $x^2 + y^2 = 4p^2$

(c) $\dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{1}{p^2}$ (d) $\dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{4}{p^2}$

3. The foci of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{b^2} = 1$ and the hyperbola $\dfrac{x^2}{144} - \dfrac{y^2}{81} = \dfrac{1}{25}$ coincide. Then the value of b^2 is **(2003)**

(a) 5 (b) 7

(c) 9 (d) 1

4. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then **(2003)**

(a) $t_2 = -t_1 + \dfrac{2}{t_1}$ (b) $t_2 = t_1 - \dfrac{2}{t_1}$

(c) $t_2 = t_1 + \dfrac{2}{t_1}$ (d) $t_2 = -t_1 - \dfrac{2}{t_1}$

5. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is **(2004)**

(a) $\left(\dfrac{-9}{8}, \dfrac{9}{2}\right)$ (b) $(2, -4)$

(c) $(2, 4)$ (d) $\left(\dfrac{9}{8}, \dfrac{9}{2}\right)$

6. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passess through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then **(2004)**

(a) $d^2 + (2b - 3c)^2 = 0$ (b) $d^2 + (3b - 2c)^2 = 0$

(c) $d^2 + (2b + 3c)^2 = 0$ (d) $d^2 + (3b - 2c)^2 = 0$

7. The eccentricity of an ellipse, with its centre at the origin, is 1/2. If one of the directrices is $x = 4$, then the equation of hte ellipse is **(2004)**

(a) $4x^2 + 3y^2 = 12$ (b) $3x^2 + 4y^2 = 12$

(c) $x^2 + y^2 = 1$ (d) $4x^2 + 3y^2 = 1$

8. Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is **(2005)**

(a) $x^2 - 4y + 2 = 0$ (b) $x^2 + 4y + 2 = 0$

(c) $y^2 + 4x + 2 = 0$ (d) $y^2 - 4x + 2 = 0$

9. An ellipse has OB as semi minor axis, F and F′ its foci and the angle FBF′ is a right angle. Then the eccentricity of the ellipse is **(2005)**

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{\sqrt{2}}$

(c) $\dfrac{1}{\sqrt{3}}$ (d) $\dfrac{1}{4}$

10. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ is **(2005)**

(a) a circle (b) an ellipse

(c) a hyperbola (d) a parabola

11. The locus of the vertices of the family of parabolas $y = \dfrac{a^3x^2}{3} + \dfrac{a^2x}{2} - 2a$ is **(2006)**

(a) $xy = \dfrac{105}{64}$ (b) $xy = \dfrac{3}{4}$

(c) $xy = \dfrac{35}{16}$ (d) $xy = \dfrac{64}{105}$

12. In an ellipse, the distance between its focii is 6 and minor axis is 8. Then its eccentricity is **(2006)**

(a) 3/5 (b) 1/2

(c) 4/5 (d) $1/\sqrt{5}$

13. A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is 1/2. Then the length of the semi–major axis is **(2008)**

(a) 5/3 (b) 8/3

(c) 2/3 (d) 4/3

14. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is : **(2014)**

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$

(c) $\frac{3}{2}$ (d) $\frac{1}{8}$

15. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is : **(2014)**

(a) $(x^2+y^2)^2 = 6x^2 - 2y^2$ (b) $(x^2-y^2)^2 = 6x^2 + 2y^2$

(c) $(x^2-y^2)^2 = 6x^2 - 2y^2$ (d) $(x^2+y^2)^2 = 6x^2 + 2y^2$

16. Let a and b be my two numbers satisfying $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{4}$

Then, the foot of perpendicular from the origin on the variable line, $\frac{x}{y}+\frac{y}{b}=1$, lies on **(2014/Online Set–1)**

(a) a hyperbola with each semi-axis = $\sqrt{2}$.

(b) a hyperbola with each semi-axis = 2.

(c) a circle of radius = 2

(d) a circle of radius = $\sqrt{2}$

17. If OB is the semi-minor axis of an ellipse, F_1 and F_2 are its foci and the angle between F_1B and F_2B is a righ angle, then the square of the eccentricity of the ellipse is: **(2014/Online Set–1)**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{4}$

18. A stair-case of length l rests against a vertical wall and a floor of a room. Let P be a point on the stair-case, nearer to its end on the wall, that divides its length in the ratio 1 : 2. If the stair-case begins to slide on the floor, then the locus of P is: **(2014/Online Set–2)**

(a) an ellipse of accentricity $\frac{1}{2}$

(b) an ellipse of eccentricity $\frac{\sqrt{3}}{2}$

(c) a circle of radius $\frac{l}{2}$

(d) a circle of radius $\frac{\sqrt{3}}{2}l$

19. Let L_1 be the length of the common chord of the curves $x^2 + y^2 = 9$ and $y^2 = 8x$, and L_2 be the length of the latus recutm of $y^2 = 8x$, then: **(2014/Online Set–2)**

(a) $L_1 > L_2$ (b) $L_1 = L_2$

(c) $L_1 < L_2$ (d) $\frac{L_1}{L_2} = \sqrt{1}$

20. Let P $(3\sec\theta, 2\tan\theta)$ and Q $(3\sec\phi, 2\tan\phi)$ where $\theta+\phi=\frac{\pi}{2}$, be two distinct point on the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$. Then the ordinate of the point of intersection of the normals at P and Q is: **(2014/Online Set–2)**

(a) $\frac{11}{3}$ (b) $-\frac{11}{3}$

(c) $\frac{13}{2}$ (d) $-\frac{13}{2}$

21. Two tangents are drawn from a point (-2, -1) to the curve, $y^2 = 4x$. If α is the angle between them, then $|\tan\alpha|$ is equal to : **(2014/Online Set–3)**

(a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) 3

22. The minimum area of a triangle formed by any tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{81}=1$ and the co-ordinate axes is: **(2014/Online Set–3)**

(a) 12 (b) 18

(c) 26 (d) 36

23. A chord is drawn through the focus of the parabola $y^2=6x$ such that its distance from the vertex of this parabola is $\frac{\sqrt{5}}{2}$, then its slope can be : **(2014/Online Set–4)**

(a) $\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{2}{\sqrt{5}}$ (d) $\frac{2}{\sqrt{3}}$

24. The tangent at an extremity (in the first quadrant) of latus rectum of the hyperbola $\frac{x^2}{4}-\frac{y^2}{5}=1$, meets x-axis and y-axis at A and B respectively. Then $(OA)^2-(OB)^2$, where O is the origin, equals : **(2014/Online Set–4)**

(a) $-\frac{20}{9}$ (b) $\frac{16}{9}$

(c) 4 (d) $-\frac{4}{3}$

25. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is: **(2015)**

(a) $y^2 = 2x$ (b) $x^2 = 2y$

(c) $x^2 = y$ (d) $y^2 = x$

26. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is: **(2015)**

(a) $\frac{27}{2}$ (b) 27

(c) $\frac{27}{4}$ (d) 18

27. If the tangent to the conic, $y - 6 = x^2$ at (2, 10) touches the circle, $x^2 + y^2 + 8x - 2y = k$ (for some fixed k) at a point (α, β); then (α, β) is : **(2015/Online Set–1)**

(a) $\left(-\frac{7}{17},\frac{6}{17}\right)$ (b) $\left(-\frac{6}{17},\frac{10}{17}\right)$

(c) $\left(-\frac{4}{17},\frac{1}{17}\right)$ (d) $\left(-\frac{8}{17},\frac{2}{17}\right)$

28. Let PQ be a double ordinate of the parabola, $y^2 = -4x$, where P lies in the second quadrant, if R divides PQ in the ratio 2 : 1 then the locus of R is: **(2015/Online Set–2)**

(a) $3y^2 = -2x$ (b) $3y^2 = 2x$

(c) $9y^2 = 4x$ (d) $9y^2 = -4x$

29. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y+6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is : **(2016)**

(a) $x^2 + y^2 - x + 4y - 12 = 0$

(b) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

(c) $x^2 + y^2 - 4x + 9y + 18 = 0$

(d) $x^2 + y^2 - 4x + 8y + 12 = 0$

30. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is : **(2016)**

(a) $\frac{4}{\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $\frac{4}{3}$

31. Let a and b respectively be the semi-transverse and semi-conjugate axes of a hyperbola whose eccentricity satisfies the equation $9e^2 - 18e + 5 = 0$. If S(5, 0) is a focus and $5x = 9$ is the corresponding directrix of this hyperbola, then $a^2 - b^2$ is equal to : **(2016/Online Set–1)**

(a) 7 (b) –7

(c) 5 (d) –5

32. A hyperbola whose transverse axis is along the major axis of the conic, $\frac{x^2}{3}+\frac{y^2}{4}=4$ and has vertices at the foci of this conic. If the eccentricity of the hyperbola is $\frac{3}{2}$, then which of the following points does **NOT** lie on it?

(2016/Online Set–2)

(a) $(0,2)$ (b) $(\sqrt{5}, 2\sqrt{2})$

(c) $(\sqrt{10}, 2\sqrt{3})$ c (d) $(5, 2\sqrt{3})$

33. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x = -4$, then the equation of the normal to it at $\left(1,\frac{3}{2}\right)$ is: **(2017)**

(a) $2y-x=2$ (b) $4x-2y=1$

(c) $4x+2y=7$ (d) $x+2y=4$

34. A hyperbola passes through the point $P(\sqrt{2},\sqrt{3})$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point: **(2017)**

(a) $\left(3\sqrt{2},2\sqrt{3}\right)$ (b) $\left(2\sqrt{2},3\sqrt{3}\right)$

(c) $\left(\sqrt{3},\sqrt{2}\right)$ (d) $\left(-\sqrt{2},-\sqrt{3}\right)$

35. The locus of the point of intersection of the straight lines,

$tx-2y-3t=0$

$x-2ty+3=0\ (t\in R)$, is : **(2017/Online Set–1)**

(a) an ellipse with eccentricity $\frac{2}{\sqrt{5}}$

(b) an ellipse with the length of major axis 6

(c) a hyperbola with eccentricity $\sqrt{5}$

(d) a hyperbola with the length of conjugate axis 3

36. If the common tangent to the parabola $x^2 = 4y$ and the circle, $x^2 + y^2 = 4$ intersect at the point P, then the distance of P from the origin, is : **(2017/Online Set–1)**

(a) $\sqrt{2}+1$ (b) $2\left(3+2\sqrt{2}\right)$

(c) $2\left(\sqrt{2}+1\right)$ (d) $3+2\sqrt{2}$

37. Consider an ellipse, whose centre is at the origin and its major axis is along the x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area (in sq. units) of the quadrilateral inscribed in the ellipse, with the vertices as the vertices of the ellipse, is :

(2017/Online Set–1)

(a) 8 (b) 32

(c) 80 (d) 40

38. The eccentricity of an ellipse having centre at the origin, axes along the co-ordinate axes and passing through the points (4, –1) and (–2, 2) is : **(2017/Online Set–2)**

(a) $\frac{1}{2}$ (b) $\frac{2}{\sqrt{5}}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{\sqrt{3}}{4}$

39. If $y = mx + c$ is the normal at a point on the parabola $y^2 = 8x$ whose focal distance is 8 units, then $|c|$ is equal to:

(2017/Online Set–2)

(a) $2\sqrt{3}$ (b) $8\sqrt{3}$

(c) $10\sqrt{3}$ (d) $16\sqrt{3}$

40. If the tangent at (1, 7) to the curve $x^2 = y-6$ touches the circle $x^2+y^2+16x+12y+c=0$ then the value of c is : **(2018)**

(a) 95 (b) 195

(c) 185 (d) 85

41. Tangents are drawn to the hyperbola $4x^2-y^2=36$ at the points P and Q. If these tangents intersect at the point T (0, 3) then the area (in sq. units) of ΔPTQ is : **(2018)**

(a) $36\sqrt{5}$ (b) $45\sqrt{5}$

(c) $54\sqrt{3}$ (d) $60\sqrt{3}$

42. Tangent and normal are drawn at P (16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is: **(2018)**

(a) $\frac{4}{3}$ (b) $\frac{1}{2}$

(c) 2 (d) 3

43. Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3, then the equation of the common tangent to the two parabolas is : **(2018/Online Set–1)**

(a) $4(x+y)+3=0$ (b) $3(x+y)+4=0$

(c) $8(2x+y)+3=0$ (d) $x+2y+3=0$

44. If the tangents drawn to the hyperbola $4y^2 = x^2 + 1$ intersect the co-ordinate axes at the distinct points A and B, then the locus of the mid point of AB is : **(2018/Online Set–1)**

(a) $x^2 - 4y^2 + 16x^2y^2 = 0$

(b) $x^2 - 4y^2 - 16x^2y^2 = 0$

(c) $4x^2 - y^2 + 16x^2y^2 = 0$

(d) $4x^2 - y^2 - 16x^2y^2 = 0$

45. If β is one of the angle between the normals to the ellipse, $x^2 + 3y^2 = 9$ at the points $\left(3\cos\theta, \sqrt{3}\sin\theta\right)$ and $\left(-3\sin\theta, \sqrt{3}\cos\theta\right); \theta \in \left(0, \frac{\pi}{2}\right)$; then $\frac{2\cot\beta}{\sin 2\theta}$ is equal to : **(2018/Online Set–1)**

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\sqrt{2}$ (d) $\frac{\sqrt{3}}{4}$

46. The curve satisfying the differential equation, $(x^2 - y^2)\,dx + 2xy\,dy = 0$ and passing through the point (1, 1) is : **(2018/Online Set–2)**

(a) A circle of radius one

(b) A hyperbola

(c) An ellipse

(d) A circle of radius two

47. Tangents drawn from the point (-8, 0) to the parabola $y^2 = 8x$ touch the parabola at P and Q. If F is the focus of the parabola, then the area of the triangle PFQ (in sq. units) is equal to : **(2018/Online Set–2)**

(a) 24 (b) 32

(c) 48 (d) 64

48. A normal to the hyperbola, $4x^2 - 9y^2 = 36$ meets the co-ordinate axes x and y at A and B, respectively. If the parallelogram OABP (O being the origin) is formed, then the locus of P is : **(2018/Online Set–2)**

(a) $4x^2 + 9y^2 = 121$ (b) $9x^2 + 4y^2 = 169$

(c) $4x^2 - 9y^2 = 121$ (d) $9x^2 - 4y^2 = 169$

49. The locus of the point of intersection of the lines, $\sqrt{2}x - y + 4\sqrt{2}k = 0$ and $\sqrt{2}kx + ky - 4\sqrt{2} = 0$ (k is any non-zero real parameter), is : **(2018/Online Set–3)**

(a) an ellipse whose eccentricity is $\frac{1}{\sqrt{3}}$.

(b) an ellipse with length of its major axis $8\sqrt{2}$

(c) a hyperbola whose eccentricity is $\sqrt{3}$

(d) a hyperbola with length of its transverse axis

50. Let P be a point on the parabola, $x^2 = 4y$. If the distance of P from the centre of the circle, $x^2+y^2+6x+8=0$ is minimum, then the equation of the tangent to the parabola at P, is : **(2018/Online Set–3)**

(a) $x+4y-2=0$ (b) $x-y+3=0$

(c) $x+y+1=0$ (d) $x+2y=0$

51. If the length of the latus rectum of an ellipse is 4 units and the distance between a focus and its nearest vertex on the major axis is $\frac{3}{2}$ units, then its eccentricity is : **(2018/Online Set–3)**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{9}$

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Single Answer Type Questions

1. The angle subtended by double ordinate of length 8a at the vertex of the parabola $y^2 = 4ax$ is

(a) 45° (b) 90°

(c) 60° (d) 30°

2. The length of the chord of parabola $x^2 = 4ay$ passing through the vertex and having slope $\tan \alpha$ is

(a) $4a \operatorname{cosec} \alpha \cot \alpha$ (b) $4a \tan \alpha \sec \alpha$

(c) $4a \cos \alpha \cot \alpha$ (d) $4a \sin \alpha \tan \alpha$

3. If the line $x + my + am^2 = 0$ touches the parabola $y^2 = 4ax$, then the point of contact is

(a) $(am^2, -2am)$ (b) $\left(\frac{a}{m^2}, \frac{-2a}{m}\right)$

(c) $(-am^2, -2am)$ (d) The line does not touch

4. A circle described on any focal chord of the parabola, $y^2 = 4ax$ as its diameter will touch :

(a) the axis of the parabola

(b) the directrix of the parabola

(c) the tangent drawn at the vertex of the parabola

(d) latus rectum

5. The locus of the middle points of the focal chords of the parabola, $y^2 = 4x$ is :

(a) $y^2 = x - 1$ (b) $y^2 = 2(x - 1)$

(c) $y^2 = 2(1 - x)$ (d) none of these

6. The length of the side of an equilateral triangle inscribed in the parabola, $y^2 = 4x$ so that one of its angular point is at the vertex is :

(a) $8\sqrt{3}$ (b) $6\sqrt{3}$

(c) $4\sqrt{3}$ (d) $2\sqrt{3}$

7. The condition that the line, $x. \cos\theta + y. \sin \theta = p$ touches the parabola, $y^2 = 4a (x + a)$ is :

(a) $a - p \cos \theta = 0$ (b) $a + p \cos \theta = 0$

(c) $a \cos \theta - p = 0$ (d) $a \cos \theta + p = 0$

8. If the normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ cuts the parabola again at $(aT^2, 2aT)$, then

(a) $-2 \le T \le 2$ (b) $T \in (-\infty, -8) \cup (8, \infty)$

(c) $T^2 < 8$ (d) $T^2 \ge 8$

9. If the straight line $x + y = 1$ is a normal to the parabola $x^2 = ay$, then the value of a is

(a) 4/3 (b) 1/2

(c) 3/4 (d) 1/4

10. A normal is drawn to the parabola $y^2 = 4ax$ at the point $(2a, -2\sqrt{2}\, a)$ then the length of the normal chord, is

(a) $4\sqrt{2}a$ (b) $6\sqrt{2}\, a$

(c) $4\sqrt{3}\, a$ (d) $6\sqrt{3}\, a$

11. Which of the following lines, is a normal to the parabola $y^2 = 16\, x$

(a) $y = x - 11 \cos \theta - 3 \cos 3\theta$

(b) $y = x - 11 \cos \theta - \cos 3\theta$

(c) $y = (x - 11) \cos \theta + \cos 3\theta$

(d) $y = (x - 11) \cos \theta - \cos 3\theta$

12. If the distance of 2 points P and Q on parabola $y^2 = 4ax$ from the focus are 4 and 9 respectively, then the distance of the point of intersection of tangents at P and Q from the focus is

(a) 8 (b) 6

(c) 5 (d) 13

13. If the normals at two points P, Q of the parabola, $y^2 = 4x$ intersect at a third point R on the parabola, then the product of the ordinates of P & Q is :

(a) 4 (b) 6

(c) 16 (d) 8

14. The normal chord at a point 't' on the parabola $y^2 = 4ax$ subtends a right angle at the vertex. Then t^2 is equal to :

(a) 3 (b) 1

(c) 4 (d) 2

15. The locus of the foot of the perpendiculars drawn from the vertex on a variable tangent to the parabola $y^2 = 4ax$ is :

(a) $x (x^2 + y^2) + ay^2 = 0$ (b) $y (x^2 + y^2) + ax^2 = 0$

(c) $x (x^2 - y^2) + ay^2 = 0$ (d) none of these

16. Length of the focal chord of the parabola $y^2 = 4ax$ at a distance p from the vertex is

(a) $\frac{2a^2}{p}$ (b) $\frac{a^3}{p^2}$

(c) $\frac{4a^3}{p^2}$ (d) $\frac{p^3}{a}$

17. If P, Q, R are three co-normal points on the parabola $y^2 = 4ax$ then the centroid of the triangle PQR always lies on :

(a) the x-axis

(b) the y-axis

(c) the line y = x

(d) the directrix of the parabola

18. Two tangents to the parabola $y^2 = 4ax$ make angle α_1 and α_2 with the x-axis. The locus of their point of intersection if $\frac{\cot\alpha_1}{\cot\alpha_2} = 2$ is :

(a) $2y^2 = 9$ ax (b) $4y^2 = 9$ ax

(c) $y^2 = 9$ ax (d) none of these

19. From an external point P, pair of tangent lines are drawn to the parabola, $y^2 = 4x$. If θ_1 and θ_2 are the inclinations of these tangents with the axis of x such that, $\theta_1 + \theta_2 = \frac{\pi}{4}$, then the locus of P is :

(a) $x - y + 1 = 0$ (b) $x + y - 1 = 0$

(c) $x - y - 1 = 0$ (d) $x + y + 1 = 0$

20. If $(t^2, 2t)$ is one end of a focal chord of the parabola, $y^2 = 4x$ then the length of the focal chord will be :

(a) $\left(t+\frac{1}{t}\right)^2$ (b) $\left(t+\frac{1}{t}\right)\sqrt{\left(t^2+\frac{1}{t^2}\right)}$

(c) $\left(t-\frac{1}{t}\right)\sqrt{\left(t^2+\frac{1}{t^2}\right)}$ (d) none of these

21. The equation of common tangent to the parabola, $y^2 = 2x$ and $x^2 = 16y$ is :

$Ax + By + C = 0$, where A, B, C $\in$ I then A = ____ ,

B = ____ , C = _____ :

(a) 1, 2, 2 (b) 2, 2, 1

(c) 2, 1, 2 (d) none of these

22. A tangent to the parabola $x^2 + 4ay = 0$ cuts the parabola $x^2 = 4by$ at A and B the locus of the mid point of AB is :

(a) $(a + 2b)\, x^2 = 4\, b^2y$ (b) $(b + 2a)\, x^2 = 4\, b^2y$

(c) $(a + 2b)\, y^2 = 4\, b^2x$ (d) $(b + 2x)\, x^2 = 4\, a^2y$

23. The equation of the other normal to the parabola $y^2 = 4ax$ which passes through the intersection of those at (4a, –4a) & (9a, –6a) is :

(a) $5x - y + 115a = 0$ (b) $5x + y - 135a = 0$

(c) $5x - y - 115a = 0$ (d) $5x + y + 115 = 0$

24. The distance of a point on the ellipse $\frac{x^2}{6}+\frac{y^2}{2}=1$ from the centre is 2. The eccentric angle of the point is

(a) $\pm\frac{\pi}{2}$ (b) $\pm\pi$

(c) $\frac{\pi}{2}, \frac{3\pi}{4}$ (d) $\pm\frac{\pi}{4}$

25. The equation of tangents to the ellipse $9x^2 + 16y^2 = 144$ which pass through the point (2, 3) -

(a) y = 3 (b) x + y = 2

(c) x – y = 3 (d) y = 3; x + y = 5

26. If the points of intersection of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, and $\frac{x^2}{p^2}+\frac{y^2}{q^2}=1$ form quadrilateral ABCD such that AC & BD be the conjugate diameter of first ellipse, then -

(a) $\frac{x^2}{p^2}+\frac{y^2}{q^2}=2$ (b) $\frac{a^2}{p^2}+\frac{b^2}{q^2}=1$

(c) $\frac{a}{p}+\frac{b}{q}=1$ (d) $\frac{a^2}{p^2}+\frac{b^2}{q^2}=2$

27. If $\tan\theta_1 \tan\theta_2 = -\frac{a^2}{b^2}$, then the chord joining two point θ_1 and θ_2 on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ will subtend a right angle at

(a) Focus (b) Centre

(c) End of the major axes (d) End of minor axes

28. The line $x = at^2$ meets the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in the real points if -

(a) $|t| < 2$ (b) $|t| \le 1$

(c) $|t| > 1$ (d) None of these

29. The eccentric angles of the extremities of latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(a) $\tan^{-1}\left(\frac{\pm ae}{b}\right)$ (b) $\tan^{-1}\left(\frac{\pm be}{b}\right)$

(c) $\tan^{-1}\left(\frac{\pm b}{ae}\right)$ (d) $\tan^{-1}\left(\frac{\pm a}{be}\right)$

30. Product of the perpendiculars from the foci upon any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(a) b (b) a

(c) a^2 (d) b^2

31. The equation of the common tangents to the parabola $y^2 = 8x$ and the hyperbola $3x^2 - y^2 = 3$ is-

(a) $2x \pm y + 1 = 0$ (b) $x \pm y + 1 = 0$

(c) $x \pm 2y + 1 = 0$ (d) $x \pm y + 2 = 0$

32. The area of a triangle formed by the lines $x - y = 0$, $x + y = 0$ and any tangent to the hyperbola $x^2 - y^2 = a^2$ is

(a) a^2 (b) $2a^2$

(c) $3a^2$ (d) $4a^2$

33. The locus of the mid point of the chords of the circle $x^2 + y^2 = a^2$, which are tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

(a) $x^2 + y^2 = a^2 - b^2$

(b) $(x^2 + y^2)^2 = a^2 - b^2$

(c) $(x^2 + y^2)^2 = a^2x^2 - b^2y^2$

(d) $(x^2 + y^2)^2 = a^2 + b^2$

34. The circles on focal radii of a parabola as diameter touch

(a) the tangent at the vertex (b) the axis

(c) the directrix (d) none of these

35. The ends of a line segment are P (1, 3) and Q (1, 1). R is a point on the line segment PQ such that PR : QR = 1 : λ. If R is in interior point of a parabola $y^2 = 4x$, then

(a) $\lambda \in (0, 1)$ (b) $\lambda \in \left(-\frac{3}{5}, 1\right)$

(c) $\lambda \in \left(\frac{1}{2}, \frac{3}{5}\right)$ (d) none of these

36. If y_1, y_2 are the ordinates of two points P and Q on the parabola and y_3 is the ordinate of the point of intersection of tangents at P and Q, then

(a) y_1, y_2, y_3 are in A.P. (b) y_1, y_3, y_2 are in A.P.

(c) y_1, y_2, y_3 are in G.P. (d) y_1, y_3, y_2 are in G.P.

37. The two parabola $y^2 = 4ax$ and $y^2 = 4c(x - b)$ cannot have a common normal, other than the axis unless, if

(a) $\frac{a-b}{b}>2$ (b) $\frac{b}{a-c}>2$

(c) $\frac{b}{a+b}>2$ (d) None of these

38. If the normals drawn from any point to the parabola $y^2 = 4ax$ cut the line $x = 2a$ in points whose ordinates are in arithmetic progression, then tangents of the angles which the normals makes with the axis, are

(a) in A.P. (b) in G.P.

(c) in H.P. (d) None of these

39. If the normals from any point to the parabola $x^2 = 4y$ cuts the line $y = 2$ in points whose abscissae are in A.P., then the slopes of the tangents at the three conormal points are in

(a) A.P. (b) G.P.

(c) H.P. (d) none of these

40. The points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, such that the tangent at that point makes equal angles with coordinate axes is

(a) $\left[\pm\frac{a^2}{\sqrt{a^2+b^2}}, \pm\frac{b^2}{\sqrt{a^2+b^2}}\right]$

(b) $\left[\pm\frac{b^2}{\sqrt{a^2+b^2}}, \pm\frac{a^2}{\sqrt{a^2+b^2}}\right]$

(c) $\left[\pm\frac{1}{\sqrt{a^2+b^2}}, \pm\frac{1}{\sqrt{a^2+b^2}}\right]$

(d) None of these

41. Tangent are drawn from the points on the line $x - y - 5 = 0$ to $x^2 + 4y^2 = 4$, then all the chords of contact pass through a fixed point, whose coordinate are

(a) $\left(\frac{4}{5}, -\frac{1}{5}\right)$ (b) $\left(\frac{4}{5}, \frac{1}{5}\right)$

(c) $\left(-\frac{4}{5}, \frac{1}{5}\right)$ (d) None of these

42. Tangents are drawn to $y^2 = 4ax$ from a variable point P moving on $x + a = 0$, then the locus of foot of perpendicular drawn from P on the chord of contact of P is

(a) $y = 0$ (b) $(x-a)^2 + y^2 = a^2$

(c) $(x-a)^2 + y^2 = 0$ (d) $y(x-a) = 0$

43. The eccentricity of the conic $4(2y - x - 3)^2 - 9(2x + y - 1)^2 = 80$ is

(a) $\frac{3}{13}$ (b) $\frac{\sqrt{13}}{3}$

(c) $\sqrt{13}$ (d) 3

44. The points (s) on the parabola $y^2 = 4x$ which are closest to the circle :

$x^2 + y^2 - 24y + 128 = 0$ is/are

(a) (0, 0) (b) $(2, 2\sqrt{2})$

(c) (4, 4) (d) none of these

45. If M is the foot of the perpendicular from a point P of a parabola $y^2 = 4ax$ to its directrix and SPM is an equilateral triangle, where S is the focus, then SP is equal to :

(a) a (b) 2a

(c) 3a (d) 4a

46. From the focus of the parabola, $y^2 = 8x$ as centre, a circle is described so that a common chord of the curves is equidistant from the vertex & focus of the parabola. The equation of the circle is :

(a) $(x - 2)^2 + y^2 = 9$ (b) $(x - 2)^2 + y^2 = 6$

(c) $(x - 2)^2 + y^2 = 4$ (d) $(x - 2)^2 + y^2 = 18$

47. IF the chord of contact of tangents from a point P to the parabola $y^2 = 4ax$ touches the parabola $x^2 = 4by$, the locus of P is :

(a) circle (b) parabola

(c) ellipse (d) hyperbola

48. AB, AC are tangents to a parabola $y^2 = 4ax$. p_1, p_2 and p_3 are the lengths of the perpendiculars from A, B and C respectively to any tangent to the curve expect at points B and C, then p_2, p_1, p_3 are in :

(a) A.P. (b) G.P.

(c) H.P. (d) none of these

49. If two normals to a parabola $y^2 = 4ax$ intersect at right angles then the chord joining their feet passes through a fixed point whose co-ordinates are :

(a) (–2a, 0) (b) (a, 0)

(c) (2a, 0) (d) none of these

50. The triangle PQR of area 'A' is inscribed in the parabola $y^2 = 4ax$ such that the vertex P lies at the vertex of the parabola and the base QR is a focal chord. The modulus of the difference of the ordinates of the points Q and R is :

(a) $\frac{A}{2a}$ (b) $\frac{A}{a}$

(c) $\frac{2A}{a}$ (d) $\frac{4A}{a}$

51. Three normals, none of which is the axis of parabola, drawn from any point to the parabola $y^2 = 4ax$, are cut to the line $x = 2a$ in points whose ordinates are in arithmetical progression. Then the tangents of the angles which the normals make the axis of the parabola are in :

(a) A.P. (b) G.P.

(c) H.P. (d) none of these

52. T is a point on the tangent to a parabola $y^2 = 4ax$ at its point P. TL and TN are the perpendiculars on the focal radius SP and the directrix of the parabola respectively. Then :

(a) SL = 2 (TN) (b) 3(SL) = 2 (TN)

(c) SL = TN (d) 2 (SL) = 3 (TN)

53. A circle is described whose centre is the vertex and whose diameter is three–quarters of the latus rectum of hte parabola $y^2 = 4ax$. If PQ is the common chord of the circle and the circle and the parabola and $L_1 L_2$ is the latus rectum, then the area of the trapezium $PL_1 L_2Q$ is

(a) $3\sqrt{2}\ a^2$ (b) $2\sqrt{2}\ a^2$

(c) $4\ a^2$ (d) $\left(\frac{2+\sqrt{2}}{2}\right) a^2$

54. From the point (15, 12) three normals are drawn to the parabola $y^2 = 4x$, then centroid of triangle formed by three co–normal points is

(a) $\left(\frac{16}{3}, 0\right)$ (b) (4, 0)

(c) $\left(\frac{26}{3}, 0\right)$ (d) (6, 0)

55. A ray of light travels along a line y = 4 and strikes the surface of a curve $y^2 = 4(x + y)$ then equation of the line along reflected ray travel is

(a) x = 0 (b) x = 2

(c) x + y = 4 (d) 2x + y = 4

56. Through the vertex O of the parabola $y^2 = 4ax$ two chords OP & OQ are drawn and the circles on OP & OQ as diameter intersect in R. If θ_1, θ_2 & ϕ are the angles made with the axis by the tangents at P & Q on the parabola & by OR then $\cot\theta_1 + \cot\theta_2$ is equal to

(a) $-2\tan\phi$ (b) $-2\tan(\pi - \phi)$

(c) 0 (d) $2\cot\phi$

57. Normals at three points P, Q, R at the parabola $y^2 = 4ax$ meet in a point A and S be its focus, if |SP|. |SQ| . |SR| = $\lambda(SA)^2$, then λ is equal to

(a) a^3 (b) a^2

(c) a (d) 1

58. A hyperbola has centre 'C' and one focus at $P(6,8)$. If its two directrices are $3x+4y+10=0$ and $3x+4y-10=0$ then $CP=$

(a) 14 (b) 8

(c) 10 (d) 6

59. Any ordinate MP of an ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ meets the auxiliary circle in Q, then locus of point of intersection of normals at P and Q to the respective curves, is

(a) $x^2 + y^2 = 8$ (b) $x^2 + y^2 = 34$

(c) $x^2 + y^2 = 64$ (d) $x^2 + y^2 = 15$

60. If P is a moving point in the xy–plane in such a way that perimeter of triangle PQR is 16 {where Q ≡ $(3, \sqrt{5})$, R ≡ $(7, 3\sqrt{5})$} then maximum area of triangle PQR is

(a) 6 sq. unit (b) 12 sq. unit

(c) 18 sq. unit (d) 9 sq. unit

61. If $\alpha + \beta = 3\pi$ then the chord joining the points α and β for the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through

(a) focus

(b) centre

(c) one of the end points of the transverse axis

(d) one of the end points of the conugates axis

62. The combined equation of the asymptotes of the hyperbola $2x^2 + 5xy + 2y^2 + 4x + 5y = 0$ is

(a) $2x^2 + 5xy + 2y^2 + 4x + 5y + 2 = 0$

(b) $2x^2 + 5xy + 2y^2 + 4x + 5y - 2 = 0$

(c) $2x^2 + 5xy + 2y^2 = 0$

(d) none of these

Multiple Type Questions

63. Consider a circle with its centre lying on the focus of the parabola, $y^2 = 2px$ such that it touches the directrix of the parabola. Then a point of intersection of the circle & the parabola is :

(a) $\left(\frac{p}{2}, p\right)$ (b) $\left(\frac{p}{2}, -p\right)$

(c) $\left(-\frac{p}{2}, p\right)$ (d) $\left(-\frac{p}{2}, -p\right)$

64. Let V be the vertex and L be the latus rectum of the parabola $x^2 = 2y + 4x - 4$. Then the equation of the parabola whose vertex is at V, latus rectum is L/2 and axis is perpendicular to the axis of the given parabola.

(a) $y^2 = x - 2$ (b) $y^2 = x - 4$

(c) $y^2 = 2 - x$ (d) $y^2 = 4 - x$

65. If equation of tangent at P, Q and vertex A of a parabola are 3x + 4y – 7 = 0, 2x + 3y – 10 = 0 and x – y = 0 respectively, then

(a) focus is (4, 5)

(b) length of latus ractum is $2\sqrt{2}$

(c) axis is x + y – 9 = 0

(d) vertex is $\left(\frac{9}{2}, \frac{9}{2}\right)$

66. If A & B are points on the parabola $y^2 = 4ax$ with vertex O such that OA perpendicular to OB & having lengths r_1 & r_2 respectively, then the value of $\frac{r_1^{4/3} r_2^{4/3}}{r_1^{2/3} + r_2^{2/3}}$ is

(a) $16a^2$ (b) a^2

(c) 4a (d) None of these

67. Let P, Q and R are three co–normal points on the parabola $y^2 = 4ax$. Then the correct statement(s) is/are

(a) algebraic sum of the slopes of the normals at P, Q and R vanishes

(b) algebraic sum of the ordinates of the points P, Q and R vanishes

(c) centroid of the triangle PQR lies on the axis of the parabola

(d) circle circumscribing the triangle PQR passes through the vertex of the parabola

68. The locus of the mid point of the focal radii of a varable point moving on the parabola, $y^2 = 4ax$ is a parabola whose

(a) Latus rectum is half the latus rectum of the original parabola

(b) Vertex is (a/2, 0)

(c) Directrix is y–axis

(d) Focus has the co–ordinates (a, 0)

69. If P is a point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose foci are S and S′. Let $\angle PSS' = \alpha$ and $\angle PS'S = \beta$, then

(a) PS + PS′ = 2a, if a > b

(b) PS + PS′ = 2b, if a < b

(c) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{1-e}{1+e}$

(d) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{\sqrt{a^2-b^2}}{b^2}[a - \sqrt{a^2-b^2}]$ when a > b

70. If the chord through the points whose eccentric angles are θ & φ on the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through a focus, then the value of tan (θ/2) tan (φ/2) is :

(a) $\frac{e+1}{e-1}$ (b) $\frac{e-1}{e+1}$

(c) $\frac{1+e}{1-e}$ (d) $\frac{1-e}{1+e}$

71. The equation, $3x^2 + 4y^2 - 18x + 16y + 43 = c$.

(a) cannot represent a real pair of straight lines for any value of c

(b) represents an ellipse, if c > 0

(c) represent empty set, if c < 0

(d) a point, if c = 0

72. If foci of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ coincide with the foci of $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and eccentricity of the hyperbola is 2, then

(a) $a^2 + b^2 = 16$

(b) there is no director circle to the hyperbola

(c) centre of the director circle is (0, 0)

(d) length of latus rectum of the hyperbola = 12

73. If (5, 12) and (24, 7) are the foci of a conic passing through the origin then the eccentricity of conic is

(a) $\sqrt{386}/12$ (b) $\sqrt{386}/13$

(c) $\sqrt{386}/25$ (d) $\sqrt{386}/38$

Assertion Reason

74. **Assertion :** If straight line x = 8 meets the parabola $y^2 = 8x$ at P & Q then PQ substends a right angle at the origin.

Reason : Double ordinate equal to twice of latus rectum of a parabola subtands a right angle at the vertex.

(a) A (b) B

(c) C (d) D

75. **Assertion :** Circum circle of a triangle formed by the lines x = 0, x + y + 1 = 0 & x – y + 1 = 0 also passes through the point (1, 0)

Reason : Circum circle of a triangle formed by three tangents of a parabola passes through its focus.

(a) A (b) B

(c) C (d) D

76. **Assertion :** The perpendicular bisector of the line segment joining the point (–a, 2 at) and (a, 0) is tangent to the parabola $y^2 = 4ax$, where t ∈ R

Reason : Number of parabolas with a given point as vertex and length of latus rectum equal to 4, is 2.

(a) A (b) B

(c) C (d) D

77. **Assertion :** Feet of perpendiculars drawn from foci of an ellipse $4x^2 + y^2 = 16$ on the line $2\sqrt{3}\,x + y = 8$ lie on the circle $x^2 + y^2 = 16$.

Reason : If perpendicular are drawn from foci of an ellipse to its any tangent then feet of these perpendiculars lie on dirctor circle of the ellipse.

(a) A (b) B

(c) C (d) D

78. **Assertion :** In a triangle ABC, if base BC is fixed and perimeter of the triangle is constant, then vertex A moves on an ellipse.

Reason : If sum of distances of a point 'P' from two fixed points is constant then locus of 'P' is a real ellipse.

(a) A (b) B

(c) C (d) D

Match the Column

79.

Column – I	Column – II
(A) Area of a triangle formed by the tangents drawn from a point (–2, 2) to the parabola $y^2 = 4(x + y)$ and their corresponding chord of contact is	(p) 8
(B) Length of the latusrectum of the conic $25\{(x-2)^2 + (y-3)^2\} = (3x + 4y - 6)^2$ is	(q) $4\sqrt{3}$
(C) If focal distance of a point on the parabola $y = x^2 - 4$ is 25/4 and points are of the form $(\pm\sqrt{a}, b)$ then value of a + b is	(r) 4
(D) Length of side of an equilateral triangle inscribed in a parabola $y^2 - 2x - 2y - 3 = 0$ whose one angular point is vertex of the parabola, is	(s) 24/5

80.

Column – I	Column – II
(A) If the mid point of a chord of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ is (0, 3), then length of the chord is $\frac{4k}{5}$, then k is	(p) 6
(B) If the line $y = x + \lambda$ touches the ellipse $9x^2 + 16y^2 = 144$, then the sum of values of λ is	(q) 8
(C) If the distance between a focus and corresponding directix of an ellipse be 8 and the eccentricity be 1/2, then length of the minor axis is $\frac{k}{\sqrt{3}}$, then k is	(r) 0
(D) Sum of distances of a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ from the foci	(s) 16

Passage Type Questions

Use the following passage, solve Q. 81 to Q. 83

PASSAGE–1

If the locus of the circumcentre of a variable triangle having sides y–axis, y = 2 and $lx + my = 1$, where (l, m) lies on the parabola $y^2 = 4ax$ is a curve C, then

81. Coordinates of the vertex of this curve C is

(a) $\left(2a, \frac{3}{2}\right)$ (b) $\left(-2a, -\frac{3}{2}\right)$

(c) $\left(-2a, \frac{3}{2}\right)$ (d) $\left(-2a, -\frac{3}{2}\right)$

82. The length of smallest focal chord of this curve C is :

(a) $\frac{1}{12a}$ (b) $\frac{1}{4a}$

(c) $\frac{1}{16a}$ (d) $\frac{1}{8a}$

83. The curve C is symmetric about the line :

(a) $y = -\dfrac{3}{2}$ (b) $y = \dfrac{3}{2}$

(c) $x = -\dfrac{3}{2}$ (d) $x = \dfrac{3}{2}$

Use the following passage, solve Q. 84 to Q. 86

PASSAGE–2

If P is a variable point and F_1 and F_2 are two fixed points such that $|PF_1 - PF_2| = 2a$. Then the locus of the point P is a hyperbola, with points F_1 and F_2 as the two focii ($F_1F_2 > 2a$).

If $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ is a hyperbola, then its conjugate hyperbola is $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = -1$. Let P(x, y) is a variable point such that

$$|\sqrt{(x-1)^2 + (y-2)^2} - \sqrt{(x-5)^2 + (y-5)^2}| = 3.$$

84. If the locus of the point P represents a hyperbola of eccentricity e, then the eccentricity e′ of the corresponding conjugate hyperbola is :

(a) $\dfrac{5}{3}$ (b) $\dfrac{4}{3}$

(c) $\dfrac{5}{4}$ (d) $\dfrac{3}{\sqrt{7}}$

85. Locus of intersection of two perpendicular tangents to the given hyperbola is

(a) $(x-3)^2 + \left(y - \dfrac{7}{2}\right)^2 = \dfrac{55}{4}$

(b) $(x-3)^2 + \left(y - \dfrac{7}{2}\right)^2 = \dfrac{25}{4}$

(c) $(x-3)^2 + \left(y - \dfrac{7}{2}\right)^2 = \dfrac{7}{4}$

(d) none of these

86. If origin is shifted to point $\left(3, \dfrac{7}{2}\right)$ and the axes are rotated through an angle θ is clockwise sense so that equation of given of given hyperbola changes to the standard form $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, then θ is :

(a) $\tan^{-1}\left(\dfrac{4}{3}\right)$ (b) $\tan^{-1}\left(\dfrac{3}{4}\right)$

(c) $\tan^{-1}\left(\dfrac{5}{3}\right)$ (d) $\tan^{-1}\left(\dfrac{3}{5}\right)$

Subjective Question

87. The equation to the parabola whose axis parallel to the y-axis and which passes through the points (0, 4), (1, 9) and (4, 5). If latus rectum of parabola is λ, then the value of 361λ must be

88. The locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio 1 : 2 is a parabola. If the vertex of parabola is (λ, μ), then the value of 729 $(\lambda + \mu)^2$ must be

89. Tangents are drawn to the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$ at ends of latusrecta. If the area of quadrilateral formed is λ sq unit, then the value of λ must be

90. If a circle cuts a rectangular hyperbola $xy = c^2$ in A, B, C and D and the parameters of these four points be t_1, t_2, t_3 and t_4 respectively, then the value of $16t_1t_2t_3t_4$ must be

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Single Answer Type Questions

1. The equation

$\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1, |r| < 1$ represents **(1981)**

(a) an ellipse (b) a hyperbola

(c) a circle (d) None of these

2. If $P = (x, y), F_1 = (3, 0), F_2 = (-3, 0)$

and $16x^2 + 25y^2 = 400$, then $PF_1 + PF_2$ equals **(1998)**

(a) 8 (b) 6

(c) 10 (d) 12

3. The number of values of c such that the straight line

$y = 4x + c$ touches the curve $\frac{x^2}{4} + y^2 = 1$ is **(1998)**

(a) 0 (b) 2

(c) 1 (d) ∞

4. Let $P(a \sec\theta, b \tan\theta)$ and $Q(a \sec\phi, b \tan\phi)$, where $\theta + \phi = \frac{\pi}{2}$, be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ If (h, k) is the point of the intersection of the normals at P and Q, then k is equal to **(1999)**

(a) $\frac{a^2+b^2}{a}$ (b) $-\left(\frac{a^2+b^2}{a}\right)$

(c) $\frac{a^2+b^2}{b}$ (d) $-\left(\frac{a^2+b^2}{b}\right)$

5. If $x = 9$ is the chord of contact of the hyperbola $x^2 - y^2 = 9$, then the equation of the corresponding pair of tangents is **(1999)**

(a) $9x^2 - 8y^2 + 18x - 9 = 0$ (b) $9x^2 - 8y^2 - 18x + 9 = 0$

(c) $9x^2 - 8y^2 - 18x - 9 = 0$ (d) $9x^2 - 8y^2 + 18x + 9 = 0$

6. The curve described parametrically by

$x = t^2 + t + 1, y = t^2 - t + 1$ represents **(1999)**

(a) a pair of straight lines (b) an ellipse

(c) a parabola (d) a hyperbola

7. If $x + y = k$ is normal to $y^2 = 12x$, then k is **(2000)**

(a) 3 (b) 9

(c) –9 (d) –3

8. If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the values of k is **(2000)**

(a) 1/8 (b) 8

(c) 4 (d) 1/4

9. The equation of the common tangent touching the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the x–axis is **(2001)**

(a) $\sqrt{3}\,y = 3x + 1$ (b) $\sqrt{3}\,y = -(x + 3)$

(c) $\sqrt{3}\,y = x + 3$ (d) $\sqrt{3}\,y = -(3x + 1)$

10. The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is **(2001)**

(a) $x = -1$ (b) $x = 1$

(c) $x = -3/2$ (d) $x = 3/2$

11. The locus of the mid point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix **(2002)**

(a) $x = -a$ (b) $x = -a/2$

(c) $x = 0$ (d) $x = a/2$

12. The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is **(2002)**

(a) $3y = 9x + 2$ (b) $y = 2x + 1$

(c) $2y = x + 8$ (d) $y = x + 2$

13. If $a > 2b > 0$, then positive value of m for which $y = mx - b\sqrt{1+m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is **(2002)**

(a) $\frac{2b}{\sqrt{a^2-4b^2}}$ (b) $\frac{\sqrt{a^2-4b^2}}{2b}$

(c) $\frac{2b}{a-2b}$ (d) $\frac{b}{a-2b}$

14. Tangent is drawn to ellipse $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ (where $\theta \in (0, \pi/2)$).

Then, the value of θ such that the sum of intercepts on axes made by this tangent is minimum, is **(2003)**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

15. The focal chord to $y^2 = 16x$ is tangent to $(x-6)^2 + y^2 = 2$, then the possible values of the slope of this chord, are **(2003)**

(a) $\{-1, 1\}$ (b) $\{-2, 2\}$

(c) $\{-2, 1/2\}$ (d) $\{2, -1/2\}$

16. For hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant with change in 'α' ? **(2003)**

(a) abscissae of vertices (b) abscissae of foci

(c) eccentricity (d) directrix

17. If the line $2x + \sqrt{6}y = 2$ touches the hyperbola $x^2 - 2y^2 = 4$, then the point of contact is **(2004)**

(a) $(-2, \sqrt{6})$ (b) $(-5, 2\sqrt{6})$

(c) $\left(\frac{1}{2}, \frac{1}{\sqrt{6}}\right)$ (d) $(4, -\sqrt{6})$

18. Axis of a parabola is $y = x$ and vertex and focus are at a distance $\sqrt{2}$ and $2\sqrt{2}$ respectively from the origin. Then equation of the parabola is **(2006)**

(a) $(x-y)^2 = 8(x+y-2)$

(b) $(x+y)^2 = 2(x+y-2)$

(c) $(x-y)^2 = 4(x+y-2)$

(d) $(x+y)^2 = 2(x-y+2)$

19. If e_1 is the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ and e_2 is the eccentricity of the hyperbola passing through the foci of the ellipse and $e_1e_2 = 1$, then equation of the hyperbola is **(2006)**

(a) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (b) $\frac{x^2}{16} - \frac{y^2}{9} = -1$

(c) $\frac{x^2}{9} - \frac{y^2}{25} = 1$ (d) None of these

20. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is **(2007)**

(a) $x^2\text{cosec}^2\theta - y^2\sec^2\theta = 1$

(b) $x^2\sec^2\theta - y^2\text{cosec}^2\theta = 1$

(c) $x^2\sin^2\theta - y^2\cos^2\theta = 1$

(d) $x^2\cos^2\theta - y^2\sin^2\theta = 1$

21. Consider a branch of the hyperbola

$x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$

with vertex at the point A. Let B be one of the end points of its latusrectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is **(2008)**

(a) $1 - \sqrt{\frac{2}{3}}$ sq unit (b) $\sqrt{\frac{3}{2}} - 1$ sq unit

(c) $1 + \sqrt{\frac{2}{3}}$ sq unit (d) $\sqrt{\frac{3}{2}} + 1$ sq unit

22. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is **(2009)**

(a) $\frac{31}{10}$ (b) $\frac{29}{10}$

(c) $\frac{21}{10}$ (d) $\frac{27}{10}$

23. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x–axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latusrectum of the given ellipse at the points **(2009)**

(a) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ (b) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{\sqrt{19}}{4}\right)$

(c) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (d) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

24. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is **(2014)**

(a) 3 (b) 6

(c) 9 (d) 15

25. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k ? **(2017)**

(a) $p = 2, h = 3, k = -4$

(b) $p = 5, h = 4, k = -3$

(c) $p = -1, h = 1, k = -3$

(d) $p = -2, h = 2, k = -4$

Passage

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B. **(2010)**

26. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(a) $2x - \sqrt{5}y - 20 = 0$ (b) $2x - \sqrt{5}y + 4 = 0$

(c) $3x - 4y + 8 = 0$ (d) $4x - 3y + 4 = 0$

27. Equation of the circle with AB as its diameter is

(a) $x^2 + y^2 - 12x + 24 = 0$

(b) $x^2 + y^2 + 12x + 24 = 0$

(c) $x^2 + y^2 + 24x - 12 = 0$

(d) $x^2 + y^2 - 24x - 12 = 0$

28. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1 : 3. Then, the locus of P is **(2011)**

(a) $x^2 = y$ (b) $y^2 = 2x$

(c) $y^2 = x$ (d) $x^2 = 2y$

29. Let P (6, 3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x-axis at (9, 0), then the eccentricity of the hyperbola is **(2011)**

(a) $\sqrt{\frac{5}{2}}$ (b) $\sqrt{\frac{3}{2}}$

(c) $\sqrt{2}$ (d) $\sqrt{3}$

Passage

Box 1 contains three cards bearing numbers 1,2,3 ; box 2 contains five cards bearing numbers 1,2,3,4,5 and box 3 contains seven cards bearing numbers 1,2,3,4,5,6,7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, $i = 1,2,3$. **(2014)**

30. The probability that $x_1 + x_2 + x_3$ is odd, is

(a) $\frac{29}{105}$ (b) $\frac{53}{105}$

(c) $\frac{57}{105}$ (d) $\frac{1}{2}$

31. The probability that x_1, x_2, x_3 are in an arithmetic progression, is

(a) $\frac{9}{105}$ (b) $\frac{10}{105}$

(c) $\frac{11}{105}$ (d) $\frac{7}{105}$

Passage

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$, for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant.

32. The orthocentre of the triangle F_1MN is

(a) $\left(-\frac{9}{10}, 0\right)$ (b) $\left(\frac{2}{3}, 0\right)$

(c) $\left(\frac{9}{10}, 0\right)$ (d) $\left(\frac{2}{3}, \sqrt{6}\right)$

33. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF_1NF_2 is

(a) 3 : 4 (b) 4 : 5

(c) 5 : 8 (d) 2 : 3

Passage

Columns 1, 2 and 3 contain conics, equations of tangents to the conics and points of contact, respectively.

Column 1	Column 2	Column 3
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2+1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2-1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2+1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

(2017)

34. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3} + 2y = 4$, then which of the following options is the only CORRECT combination ?

(a) (IV) (iii) (S) (b) (II) (iii) (R)

(c) (IV) (iv) (S) (d) (II) (iv) (R)

35. If a tangent to a suitable conic (Column 1) is found to be y = x + 8 and its point of contact is (8, 16), then which of the following options is the only CORRECT combination?

(a) (III) (i) (P) (b) (I) (ii) (Q)

(c) (II) (iv) (R) (d) (III) (ii) (Q)

36. For $a = \sqrt{2}$, if a tangent is drawn to suitable conic (Column 1) at the point of contact (–1, 1), then which of the following options is the only CORRECT combination for obtaining its equation?

(a) (II) (ii) (Q) (b) (I) (i) (P)

(c) (I) (ii) (Q) (d) (III) (i) (P)

Passage

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, i = 1, 2,3, 4, 5. But, on the examination day, the five students are randomly allotted the five seats.

(2018)

37. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1, and NONE of the remaining students gets the seat previously allotted to him/her is

(a) $\frac{3}{40}$ (b) $\frac{1}{8}$

(c) $\frac{7}{40}$ (d) $\frac{1}{5}$

38. For i = 1, 2, 3, 4, let Ti denote the event that the students S_i and $S_i + 1$ do NOT sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

(a) $\frac{1}{15}$ (b) $\frac{1}{10}$

(c) $\frac{7}{60}$ (d) $\frac{1}{5}$

Multiple Type Questions

39. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points P (x_1, y_1), Q(x_2, y_2), R(x_3, y_3) S (x_4, y_4), then **(1998)**

(a) $x_1 + x_2 + x_3 + x_4 = 0$ (b) $y_1 + y_2 + y_3 + y_4 = 0$

(c) $x_1x_2x_3x_4 = c^4$ (d) $y_1\, y_2\, y_3\, y_4 = c^4$

40. Equation of common tangent of $y = x^2$, $y = -x^2 + 4x - 4$ is **(2006)**

(a) y = 4 (x – 1) (b) y = 0

(c) y = – 4 (x – 1) (d) y = – 30x – 50

41. Let P(x_1, y_1) and Q(x_2, y_2), $y_1 < 0$, $y_2 < 0$, be the end points of the latusrectum of the ellipse $x^2 + 4y^2 = 4$. The equation of parabola with latusrectum PQ are **(2008)**

(a) $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$ (b) $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$

(c) $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$ (d) $x^2 - 2\sqrt{3}y = 3 - \sqrt{3}$

42. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose **(2009)**

(a) vertex is $\left(\frac{2a}{3},0\right)$ (b) directrix is $x=0$

(c) latusrectum is $\frac{2a}{3}$ (d) focus is (a, 0)

43. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal to that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then **(2009)**

(a) Equation of ellipse is $x^2 + 2y^2 = 2$

(b) The foci of ellipse are $(\pm 1, 0)$

(c) Equation of ellipse is $x^2 + 2y^2 = 4$

(d) The foci of ellipse are $(\pm \sqrt{2}, 0)$

44. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be **(2010)**

(a) $-\frac{1}{r}$ (b) $\frac{1}{r}$

(c) $\frac{2}{r}$ (d) $-\frac{2}{r}$

45. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), then L is given by **(2011)**

(a) $y-x+3=0$ (b) $y+3x-33=0$

(c) $y+x-15=0$ (d) $y-2x+12=0$

46. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be reciprocal to that of the ellipse $x^2+4y^2=4$. If the hyperbola passes through a focus of the ellipse, then **(2011)**

(a) the equation of the hyperbola is $\frac{x^2}{3}-\frac{y^2}{2}=1$

(b) a focus of the hyperbola is (2, 0)

(c) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(d) the equation of the hyperbola is $x^2 - 3y^2 = 3$

47. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P ? **(2015)**

(a) $\left(4,2\sqrt{2}\right)$ (b) $\left(9,3\sqrt{2}\right)$

(c) $\left(\frac{1}{4},\frac{1}{\sqrt{2}}\right)$ (d) $\left(1\sqrt{2}\right)$

48. Let E_1 and E_2 be two ellipses whose centres are at the origin. The major axes of E_1 and E_2 lie along the x-axis and the y-axis, respectively. Let S be the circle $x^2 + (y-1)^2 = 2$. The straight line $x + y = 3$ touches the curves S, E_1 and E_2 at P, Q and R respectively. Support that $PQ = PR = \frac{2\sqrt{2}}{3}$.

If e_1 and e_2 are the eccentricities of E_1 and E_2, respectively, then the correct expression(s) is (are) **(2015)**

(a) $e_1^2 + e_2^2 = \frac{43}{40}$ (b) $e_1e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$

(c) $\left|e_1^2 - e_2^2\right| = \frac{5}{8}$ (d) $e_1e_2 = \frac{\sqrt{3}}{4}$

49. Consider the hyperbola $H : x^2 - y^2 = 1$ and a circle S with centre $N(x_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H and S at P intersects the x-axis at point M. If (l, m) is the centroid of the triangle PMN, then the correct expression(s) is (are) : **(2015)**

(a) $\frac{dl}{dx_1} = 1-\frac{1}{3x_1^2}$ for $x_1 > 1$ (b) $\frac{dm}{dx_1} = \frac{x_1}{3\left(\sqrt{x_1^2-1}\right)}$ for $x_1 > 1$

(c) $\frac{dl}{dx_1} = 1+\frac{1}{3x_1^2}$ for $x_1 > 1$ (d) $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$

50. Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then **(2016)**

(a) $SP = 2\sqrt{5}$

(b) $SQ : QP = \left(\sqrt{5}+1\right) : 2$

(c) the x-intercept of the normal to the parabola at P is 6

(d) the slope of the tangent to the circle at Q is $\frac{1}{2}$

51. Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. Consider the ellipse whose center is at the origin O(0,0) and whose semi-major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE? **(2018)**

(a) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

(b) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

(c) The area of the region bounded by the ellipse between the lines

(d) The area of the region bounded by the ellipse between the lines

Matrix Match

52. Let $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a > b > 0$, be a hyperbola in the *xy*-plane whose conjugate axis LM subtends an angle of 60° at one of its vertices N. Let the area of the triangle LMN be $4\sqrt{3}$. **(2018)**

(A) The length of the conjugate axis of H is (P) 8

(B) The ecentricity of H is (Q) $\frac{4}{\sqrt{3}}$

(C) The distance between the foci of H is (R) $\frac{2}{\sqrt{3}}$

(D) The length of the latus rectum of H is (S) 4

SUBJECTIVE – QUESTIONS

53. Suppose that the normals drawn at three different points on the parabola $y^2 = 4x$ pass through the point (h, 0). Show that h > 2. **(1981)**

54. For any real t, $x = \frac{e^t + e^{-t}}{2}, y = \frac{e^t - e^{-t}}{2}$ is a point on the hyperbola $x^2 - y^2 = 1$. Find the area bounded by this hyperbola and the line joining its centre to the points corresponding to t_1 and $-t_1$. **(1982)**

55. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1, 2). **(1984)**

56. Through the vertex O of parabola $y^2 = 4x$, chords OP and OQ are drawn at right angles to one another. Show that for all positions of P, PQ cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of PQ. **(1994)**

57. Let d be the perpendicular distance from the centre of the ellipse $x^2 / a^2 + y^2 / b^2 = 1$ to the tangent drawn at a point P on the ellipse. If F_1 and F_2 are the two foci of the ellipse, then show that

$$(PF_1 - PF_2)^2 = 4a^2\left(1 - \frac{b^2}{d^2}\right)$$ **(1995)**

58. Show that the locus of a point that divide a chord of slope 2 of the parabola $y^2 = 4ax$ internally in the ratio 1 : 2 is a parabola. Find the vertex of this parabola. **(1995)**

59. Points A, B and C lie on the parabola $y^2 = 4ax$. The tangents to the parabola at A, B and C, taken in pairs, intersect at points P,Q and R. Determine the ratio of th areas of the triangle ABC and PQR. **(1996)**

60. From a point A common tangents are drawn to the circle $x^2 + y^2 = \frac{a^2}{2}$ and parabola $y^2 = 4ax$. Find the area of the quadrilateral formed by the common tangents, the chord of contact of the circle and the chord of contact of the parabola. **(1996)**

61. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P and Q. Prove that the tangents at P and Q of the ellipse $x^2 + 2y^2 = 6$ are at right angles. **(1997)**

62. The angle between a pair of tangents drawn from a point P to the parabola $y^2 = 4ax$ is 45°. Show that the locus of the point P is a hyperbola. **(1998)**

63. Find the coordinates of all the points P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, for which the area of the triangle PON is maximum, where O denotes the origin and N be the foot of the perpendicular from O to the tangent at P. **(1999)**

64. Let ABC be an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$. Suppose perpendiculars from A, B, C to the major axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a > b)$ meets the ellipse respectively at P, Q, R so that P, Q, R lie on the same side of the major axis as A, B, C respectively. Prove that, the normals to the ellipse drawn at the points P, Q and R are concurrent. **(2000)**

65. Let C_1 and C_2 be, respectively, the parabola $x^2 = y - 1$ and $y^2 = x - 1$. Let P be any point on C_1 and Q be any point on C_2. Let P_1 and Q_1 be the reflections of P and Q, respectively, with respect to the line y = x. Prove that P_1 lies on C_2, Q_1 lies on C_1 and $PQ \geq \min\{PP_1, QQ_1\}$. Hence or otherwise, determine points P_0 and Q_0 on the parabolas C_1 and C_2 respectivley such the $P_0Q_0 \leq PQ$ for all pairs of points (P, Q) with P on C_1 and Q on C_2. **(2000)**

66. Let P be a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, 0 < b < a$. Let the line parallel to y–axis passing through P meet the circle $x^2 + y^2 = a^2$ at the point Q such that P and Q are on the same side of x–axis. For two positive real numbers r and s, find the locus of the point R on PQ such that PR : RQ = r : s as P varies over the ellipse. **(2001)**

67. Prove that, in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse of the point of contact meet on the corresponding directrix. **(2002)**

68. Normals are drawn from the point P with slopes m_1, m_2, m_3 to the parabola $y^2 = 4x$. If locus of P with $m_1m_2 = \alpha$ is a part of the parabola itself, then find α. **(2003)**

69. At any point P on the parabola $y^2 - 2y - 4x + 5 = 0$ a tangent is drawn which meets the directrix at Q. Find the locus of point R, which divides QP externally in the ratio $\frac{1}{2} : 1$. **(2004)**

70. Find the equation of the common tangent in 1st quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$. Also find the length of the intercept of the tangent between the coordinate axes. **(2005)**

71. Tangents are drawn from any point on the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ to the circle $x^2 + y^2 = 9$. Find the locus of mid point of the chord of contact. **(2005)**

72. The line $2x + y = 1$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is..... **(2010)**

73. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latusrectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latusrectum. Then $\frac{\Delta_1}{\Delta_2}$ is.... **(2011)**

74. If the normal of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x - 3)^2 + (y + 2)^2 = r^2$, then the value of r^2 is **(2015)**

75. Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the line x + y + 4 = 0. If A and B are the points of intersection of C with the line y = –5, then the distance between A and B is **(2015)**

76. Suppose that the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ are $(f_1, 0)$ and $(f_2, 0)$ where $f_1 > 0$ and $f_2 < 0$. Let P_1 and P_2 be two parabolas with a common vertex at (0, 0) and with foci at $(f_1, 0)$ and $(2f_2, 0)$, respectively. Let T_1 be a tangent to P_1 which passes through $(2f_2, 0)$ and T_2 be a tangent to P_2 which passes through $(f_1, 0)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m_1^2}+m_2^2\right)$ is **(2015)**

ANSWER KEY

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (c)	2. (b)	3. (d)	4. (a)	5. (bcd)	6. (b)	7. (c)	8. (b)
9. (b)	10. (c)	11. (a)	12. (a)	13. (c)	14. (b)	15. (b)	16. (c)
17. (c)	18. (a)	19. (c)	20. (d)	21. (a)	22. (d)	23. (d)	24. (b)
25. (a)	26. (b)	27. (c)	28. (c)	29. (d)	30. (b)	31. (a)	32. (d)
33. (b)	34. (d)	35. (b)	36. (a)	37. (a)	38. (c)	39. (c)	40. (c)
41. (a)	42. (a)	43. (a)	44. (a)	45. (c)	46. (c)	47. (c)	48. (d)
49. (b)	50. (b)	51. (a)	52. (c)	53. (c)	54. (c)	55. (b)	56. (a)
57. (a)	58. (d)	59. (c)	60. (c)	61. (c)	62. (a)	63. (b)	64. (a)
65. (a)	66. (a)	67. (c)	68. (d)	69. (d)	70. (a)	71. (bc)	72. (b)
73. (d)	74. (c)	75. (d)	76. (c)	77. (b)	78. (c)	79. (c)	80. (a)
81. (a)	82. (ab)	83. (a)	84. (a)	85. (c)	86. (b)	87. (b)	88. (d)
89. (b)	90. (c)						

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. (b)	2. (d)	3. (b)	4. (d)	5. (d)	6. (c)	7. (b)	8. (d)
9. (b)	10. (c)	11. (a)	12. (a)	13. (b)	14. (b)	15. (d)	16. (c)
17. (a)	18. (b)	19. (c)	20. (d)	21. (d)	22. (d)	23. (a)	24. (a)
25. (b)	26. (b)	27. (d)	28. (d)	29. (d)	30. (b)	31. (b)	32. (d)
33. (b)	34. (b)	35. (d)	36. (c)	37. (d)	38. (c)	39. (c)	40. (a)
41. (b)	42. (c)	43. (a)	44. (b)	45. (a)	46. (a)	47. (c)	48. (d)
49. (d)	50. (c)	51. (b)					

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (b)	2. (b)	3. (a)	4. (b)	5. (b)	6. (a)	7. (b)	8. (d)
9. (a)	10. (d)	11. (d)	12. (b)	13. (d)	14. (d)	15. (a)	16. (c)
17. (a)	18. (a)	19. (c)	20. (a)	21. (a)	22. (a)	23. (b)	24. (d)
25. (d)	26. (d)	27. (b)	28. (b)	29. (c)	30. (d)	31. (a)	32. (a)
33. (c)	34. (a)	35. (b)	36. (b)	37. (b)	38. (a,b,c)	39. (a,b,c)	40. (a)
41. (a)	42. (c)	43. (b)	44. (c)	45. (d)	46. (a)	47. (d)	48. (d)
49. (b)	50. (c)	51. (b)	52. (c)	53. (d)	54. (c)	55. (a)	56. (a)
57. (c)	58. (c)	59. (c)	60. (b)	61. (b)	62. (a)	63. (a,b)	64. (a,c)
65. (a,b,c,d)	66. (a)	67. (a,b,c,d)	68. (a,b,c,d)	69. (a,b,c)	70. (a,b)	71. (a,b,c,d)	72. (a,b,d)
73. (a,d)	74. (a)	75. (a)	76. (c)	77. (c)	78. (c)		
79. A–r;B–s; C–p; D–q		80. A–q; B–r; C–s; D–q					
81. (c)	82. (d)	83. (b)	84. (c)	85. (d)	86. (b)		
87. (0228)	88. (0900)	89. (0027)	90. (0016)				

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (b)	2. (c)	3. (b)	4. (d)	5. (b)	6. (c)	7. (b)	8. (c)
9. (c)	10. (d)	11. (c)	12. (d)	13. (a)	14. (b)	15. (a)	16. (b)
17. (d)	18. (a)	19. (b)	20. (a)	21. (b)	22. (d)	23. (c)	24. (d)
25. (a)	26. (b)	27. (a)	28. (c)	29. (b)	30. (b)	31. (c)	32. (a)
33. (c)	34. (d)	35. (a)	36. (c)	37. (a)	38. (c)	39. (a,b,c,d)	40. (a,b)
41. (b,c)	42. (a,d)	43. (a,b)	44. (c,d)	45. (a,b,d)	46. (b,d)	47. (a, d)	48. (a, b)
49. (a, b, d)	50. (a,c,d)	51. (a,c)	52. (A–S; B–S; C–P; D–Q)		54. t_1	55. $x+y=3$	

56. $y^2=2(x-4)$ 58. $\frac{2}{9},\frac{8}{9}$ 59. 2 60. $\frac{15a^2}{4}$ 63. $\left(\frac{\pm a^2}{\sqrt{a^2+b^2}},\frac{\pm b^2}{\sqrt{a^2+b^2}}\right)$ 65. $P_0\left(\frac{1}{2},\frac{5}{4}\right),Q_0\left(\frac{5}{4},\frac{1}{2}\right)$

66. $\frac{x^2}{a^2}+\frac{y^2(r+s)^2}{(ar+bs)^2}=1$ 68. $a=2$ 69. $(x+1)(y-1)^2+4=0$ 70. $y=-\frac{2x}{\sqrt{3}}+4\frac{\sqrt{7}}{3},\frac{14\sqrt{3}}{3}$

71. $\frac{x^2}{9}-\frac{y^2}{4}=\frac{(x^2+y^2)^2}{81}$ 72. (2) 73. (2) 74. (2) 75. (4) 76. (4)

Dream on !!