**Q1: NTA Test 01 (Single Choice)**

The value of $\int_{-1}^{1}(x-[x])dx$ (where [.] denotes greatest integer function) is

(A) 0

(B) 1

(C) 2

(D) None of these

**Q2: NTA Test 02 (Single Choice)**

For $x \in R,\ x \neq 0$, if $y(x)$ is a differentiable function such that $x\int_{1}^{x} y(t)dt = (x+1)\int_{1}^{x} ty(t)dt$, then $y(x)$ equals (where $C$ is a constant)

(A) $Cx^3\ e^{\frac{1}{x}}$

(B) $\frac{C}{x^2}\ e^{-\frac{1}{x}}$

(C) $\frac{C}{x}\ e^{-\frac{1}{x}}$

(D) $\frac{C}{x^3}\ e^{-\frac{1}{x}}$

**Q3: NTA Test 03 (Single Choice)**

The integral $\int_{-1/2}^{1/2}\left([x]+\ell n\left(\frac{1+x}{1-x}\right)\right)dx$ is equal to ($[x]$ is the greatest integer $\leq x$)

(A) $-\frac{1}{2}$

(B) 1

(C) $2\ell n\left(\frac{1}{2}\right)$

(D) 0

**Q4: NTA Test 04 (Single Choice)**

If $I_1 = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx,\ I_2 = \int_0^{\pi} x\sin^4 x dx$ then, $I_1 \colon I_2$ is equal to

(A) $3:4$

(B) $1:2$

(C) $4:3$

(D) $2:3$

**Q5: NTA Test 05 (Single Choice)**

The value of the integral $\int_{0}^{1}\sqrt{\frac{1-x}{1-x}}dx$ is

(A) $-1$

(B) $1$

(C) $\frac{\pi}{2}-1$

(D) $\frac{\pi}{2}+1$

**Q6: NTA Test 06 (Single Choice)**

The value of the integral $\int_{4}^{10}\frac{[x^2]}{[x^2-28x+196]+[x^2]}dx$, where $[x]$ denotes the greatest integer less than or equal to $x$, is

(A) $\frac{1}{3}$

(B) 6

(C) 7

(D) 3

**Q7: NTA Test 07 (Single Choice)**

If $f(x) = \sin\left(\lim_{t\to 0}\frac{2x}{\pi}\cot^{-1}\frac{x}{t^2}\right)$, then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\,dx$ is equal to (where, $x \neq 0$)

(A) $-2$

(B) $-1$

(C) 0

(D) 2

**Q8: NTA Test 08 (Single Choice)**

The value of the integral $\int_{-a}^{a}\frac{e^x}{1+e^x}dx$ is

(A) $e^{a2}$

(B) $a$

(C) $e^{-a2}$

(D) $\frac{a}{2}$

**Q9: NTA Test 09 (Numerical)**

The value of $\left[\int_{-\pi}^{\pi} \sqrt{\frac{|\sin y|}{1+\tan^2 y}} dy\right]$ (where $[x]$ is greatest integer function) is

**Q10: NTA Test 10 (Single Choice)**

Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$. Then, which one of the following is true?

(A) $I > \frac{2}{3}$ and $J < 2$

(B) $I > \frac{2}{3}$ and $J > 2$

(C) $I < \frac{2}{3}$ and $J < 2$

(D) $I < \frac{2}{3}$ and $J > 2$

**Q11: NTA Test 11 (Single Choice)**

The value of $\lim_{n \to \infty} \sum_{r=1}^{r=4n} \frac{\sqrt{n}}{\sqrt{r}\left(3\sqrt{r}+4\sqrt{n}\right)^2}$ is equal to

(A) $\frac{1}{8}$

(B) $\frac{1}{10}$

(C) $\frac{1}{6}$

(D) $\frac{1}{9}$

**Q12: NTA Test 12 (Numerical)**

The value of the integral $\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{\left((x+1)^2(1-x)^6\right)^{\frac{1}{4}}} dx$ is

**Q13: NTA Test 13 (Numerical)**

Given $f(x) = \begin{cases} x|x| & \text{for } x \leq -1 \\ [x+1]+[1-x] & \text{for } -1 < x < 1 \\ -x|x| & \text{for } x \geq 1 \end{cases}$, where [.] denotes the greatest integer function. If $I = \int_{-2}^{2} f(x)\, dx$, then $|3I| =$

**Q14: NTA Test 14 (Single Choice)**

The value of $\int_0^{\pi} \left(\sum_{r=0}^{3} a_r \cos^{3-r} x \sin^r x\right) dx$ depends upon

(A) $a_1$ and $a_2$

(B) $a_0$ and $a_3$

(C) $a_2$ and $a_3$

(D) $a_1$ and $a_3$

**Q15: NTA Test 15 (Single Choice)**

Let the function F be defined as $F(x) = \int_1^x \frac{e^t}{t} dt$, $x > 0$, then the value of the integral $\int_1^x \frac{e^t}{t+a} dt$, where $a > 0$, is

(A) $e^a [F(x) - F(1+a)]$

(B) $e^{-a} [F(x+a) - F(a)]$

(C) $e^a [F(x+a) - F(1+a)]$

(D) $e^{-a} [F(x+a) - F(1+a)]$

**Q16: NTA Test 16 (Single Choice)**

If $f(x)$ is a continuous function and $\int_{x^2}^{x^4} t^3 f(t) dt = \sin 2\pi x$, then $f(1)$ is equal to

(A) 1

(B) $-1$

(C) $\pi$

(D) $-\pi$

**Q17: NTA Test 17 (Single Choice)**

The value of $I = \int_{-1}^{1} [x \sin(\pi x)] dx$ is (where $[\cdot]$ denotes the greatest integer function)

(A) $\pi$

(B) $2\pi$

(C) 0

(D) $-\pi$

**Q18: NTA Test 18 (Single Choice)**

The value of the integral $I=\int_{1}^{2} t^{[\{t\}]+t}(1+\ln t)\, dt$ is equal to

($[.]$ and $\{.\}$ denotes the greatest integer and fractional part function respectively)

(A) 0

(B) 1

(C) 2

(D) 3

**Q19: NTA Test 18 (Numerical)**

If $I_n=\int_{0}^{n\pi} \max\left(|\sin x|, \left|\sin^{-1}(\sin x)\right|\right) dx$, then $I_2+I_4$ has the value $\frac{\lambda\pi^2}{2}$; where $\lambda$ is

**Q20: NTA Test 19 (Single Choice)**

The value of the integral $I=\int_{0}^{\frac{\pi}{4}}[\sin x+\cos x](\cos x-\sin x) dx$ is equal to

(where, $[.]$ denotes the greatest integer function)

(A) $\sqrt{2}$

(B) $2\sqrt{2}$

(C) 1

(D) $\sqrt{2}-1$

**Q21: NTA Test 20 (Single Choice)**

If $f(k-x)+f(x)=\sin x$, then the value of integral $I=\int_{0}^{k} f(x) dx$ is equal to

(A) $\cos k$

(B) $2\cos^2\left(\frac{k}{2}\right)$

(C) $\sin^2\left(\frac{k}{2}\right)$

(D) $\sin k$

**Q22: NTA Test 21 (Single Choice)**

If $f(1+x)=f(1-x)\ (\forall x\in R)$, then the value of the integral $I=\int_{-7}^{9} \frac{f(x)}{f(x)+f(2-x)} dx$ is

(A) 0

(B) 2

(C) 8

(D) 10

**Q23: NTA Test 22 (Single Choice)**

Let $I_1=\int_{0}^{1} \frac{|\ln x|}{x^2+4x+1} dx$ and $I_2=\int_{1}^{\infty} \frac{\ln x}{x^2+4x+1} dx$, then

(A) $I_1=I_2$

(B) $I_1>I_2$

(C) $I_1+I_2=0$

(D) $I_1=2I_2$

**Q24: NTA Test 23 (Single Choice)**

The value of $\lim_{n\to\infty} \sum_{r=1}^{n}\left(\frac{2r}{n^2}\right) e^{\frac{r^2}{n^2}}$ is equal to

(A) $e$

(B) $2e$

(C) $e-2$

(D) $e-1$

**Q25: NTA Test 23 (Single Choice)**

The value of $\int_0^{\frac{\pi}{2}}\left(\cos 2x \cos 2^2 x \cos 2^3 x \cos 2^4 x\right) dx$ is equal to

(A) 0

(B) $\frac{1}{2}$

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{4}$

**Q26: NTA Test 24 (Single Choice)**

If $I_1 = \int_{-1}^{2} x\sin(x(1-x))dx$ and $I_2 = \int_{-1}^{2} \sin(x(1-x))dx$, then $\frac{I_1}{I_2}$ is equal to

(A) 2

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{1}{3}$

**Q27: NTA Test 25 (Single Choice)**

The value of $\int_0^{12\pi}([\sin t] + [-\sin t])dt$ is equal to (where, $[.]$ denotes the greatest integer function)

(A) $12\pi$

(B) $-12\pi$

(C) $-10\pi$

(D) $-6\pi$

**Q28: NTA Test 26 (Single Choice)**

The value of $\int_0^{\pi/2} sgn\left(\sin^2 x - \sin x + \frac{1}{2}\right)dx$ is equal to, (where, $sgn(x)$ denotes the signum function of $x$ )

(A) 0

(B) 1

(C) $\pi$

(D) $\frac{\pi}{2}$

**Q29: NTA Test 26 (Single Choice)**

Let $I_1 = \int_1^{\frac{\pi}{2}} \frac{dt}{1+t^6}$ and $I_2 = \int_0^{\frac{\pi}{2}} \frac{x\cos x dx}{1+(x\sin x+\cos x)^6}$, then

(A) $2I_1 = I_2$

(B) $I_1 = 2I_2$

(C) $I_1 = I_2$

(D) $I_1 = I_2 = 0$

**Q30: NTA Test 27 (Numerical)**

If $f(x) = \min(|x-1|, \ |x|, \ |x+1|)$, then the value of $6\left(\int_{-1}^{1} f(x)dx\right)$ is

**Q31: NTA Test 28 (Single Choice)**

Let $I_1 = \int_0^1 e^{x^2}dx$ and $I_2 = \int_0^1 2x^2e^{x^2}dx$, then the value of $I_1 + I_2$ is equal to

(A) 1

(B) 2

(C) $e$

(D) $e^2$

**Q32: NTA Test 28 (Single Choice)**

The value of $\int_{-1}^{1} \cot^{-1}\left(\frac{x+x^3+x^5}{x^4+x^2+1}\right)dx$ is equal to

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{3\pi}{4}$

(D) $\pi$

**Q33: NTA Test 29 (Single Choice)**

If $\int_{\frac{-1}{\sqrt{3}}}^{1/\sqrt{3}} \frac{x^4}{1-x^4}\cos^{-1}\left(\frac{2x}{1+x^2}\right)dx = k\int_0^{1/\sqrt{3}} \frac{x^4}{1-x^4}dx$, then the value of k is equal to

(A) $\pi$

(B) $2\pi$

(C) -$\pi$

(D) $3\pi$

**Q34: NTA Test 30 (Single Choice)**

The value of $\int_{-1}^{1}\left(\sin^{-1} x + \frac{x^5+x^3-1}{\cos^2 x}\right)dx$ is equal to

(A) tan 1

(B) 0

(C) $2\tan 1$

(D) $-2\tan 1$

**Q35: NTA Test 31 (Single Choice)**

Let $\mathrm{I} = \int\limits_0^{24\pi} \{\sin x\} dx$, then the value of 2I is equal to (where, $\{.\}$ denotes the fractional part function)

(A) $10\pi$

(B) $24\pi$

(C) $12\pi$

(D) $4\pi$

**Q36: NTA Test 32 (Single Choice)**

The value of $\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}} e^{\sec^2 x} \frac{\sin x}{\cos^3 x} dx$ is equal to

(A) $\frac{1}{2}e^4$

(B) $\frac{1}{2}e^{\frac{4}{3}}$

(C) $\frac{1}{2}\left(e^4 - e^{\frac{4}{3}}\right)$

(D) $\frac{1}{2}\left(e^2 - 1\right)$

**Q37: NTA Test 33 (Single Choice)**

The value of the integral $\int\limits_{-3\pi}^{3\pi} \left|\sin^3 x\right| dx$ is equal to

(A) $\pi$

(B) $8\pi$

(C) 1

(D) 8

**Q38: NTA Test 34 (Single Choice)**

The value of $\int\limits_{\pi}^{2\pi} [2\sin x]\, dx$ is equal to (where [.] represents the greatest integer function)

(A) $-\pi$

(B) $\frac{5\pi}{3}$

(C) $\frac{-5\pi}{3}$

(D) $-2\pi$

**Q39: NTA Test 35 (Single Choice)**

If $\mathrm{a} = \int\limits_0^1 \frac{\cos(\sin x)}{\sec x} \mathrm{d}x$, then the value of $\mathrm{a}^2 + \cos^2(\sin 1)$ is equal to

(A) 0

(B) 1

(C) $\sin(1)$

(D) $\sin(\sin 1)$

**Q40: NTA Test 36 (Single Choice)**

The value of $\int\limits_3^6 \frac{\sqrt{(36-\mathrm{x}^2)^3}}{\mathrm{x}^4} \mathrm{dx}$ is equal to

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{4}$

**Q41: NTA Test 38 (Single Choice)**

Consider $A = \int\limits_0^1 \frac{dx}{1+x^3}$, then $A$ satisfies

(A) $A > \frac{\pi}{4}$

(B) $A < \frac{\pi}{4}$

(C) $A = \frac{\pi}{4}$

(D) $A = \frac{\pi}{6}$

**Q42: NTA Test 39 (Single Choice)**

If $A_n = \int\limits_0^{n\pi} |\sin x| dx,\ \forall n \in N$, then $\sum_{r=1}^{10} A_r$ is equal to

(A) 100

(B) 110

(C) 55

(D) 105

**Q43: NTA Test 40 (Single Choice)**

If $\int\limits_0^{\infty} \frac{\sin x}{x} dx = k$, then the value of $\int\limits_0^{\infty} \frac{\sin^3 x}{x} dx$ is equal to

(A) $k$

(B) $\frac{k}{2}$

(C) $\frac{k}{4}$

(D) $2k$

**Q44: NTA Test 41 (Numerical)**

Let $f: R \rightarrow R$ is a function defined as $f(x) = \begin{cases} |x - [x]| & : [x] \text{ is odd} \\ |x - [x+1]| & : [x] \text{ is even} \end{cases}$, where [.] denotes the greatest integer function, then $\int_{-2}^{4} f(x)\, dx$ is equal to

**Q45: NTA Test 42 (Single Choice)**

The value of $\int_0^{\infty} \frac{dx}{1+x^4}$ is equal to

(A) $\frac{\pi}{2\sqrt{2}}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{\sqrt{2}}$

(D) $2\pi\sqrt{2}$

**Q46: NTA Test 43 (Single Choice)**

Consider $\mathrm{I}(\alpha) = \int\limits_{\alpha}^{\alpha^2} \frac{dx}{x}$ (where $\alpha > 0$), then the value of $\sum_{r=2}^{5} \mathrm{I}(r) + \sum_{k=2}^{5} \mathrm{I}\left(\frac{1}{k}\right)$ is

(A) 0

(B) 1

(C) ln 2

(D) ln 4

**Q47: NTA Test 44 (Single Choice)**

Consider $A = \int\limits_0^{\frac{\pi}{4}} \frac{\sin(2x)}{x} dx$, then

(A) $A > \frac{\pi}{2}$

(B) $A = \frac{\pi}{2}$

(C) $A < \frac{\pi}{2}$

(D) $A > \pi$

**Q48: NTA Test 45 (Single Choice)**

The value of $\int_0^{\frac{\pi}{3}} \log\left(1 + \sqrt{3}\tan x\right) dx$ is equal to

(A) $\pi \log 2$

(B) $\frac{\pi}{2} \log 2$

(C) $\frac{\pi}{3} \log 2$

(D) $\frac{\pi}{4} \log 2$

**Q49: NTA Test 46 (Single Choice)**

The value of the integral $\mathrm{I} = \int\limits_0^{\pi} [|\sin x| + |\cos x|] dx$, (where [.] denotes the greatest integer function) is equal to

(A) 1

(B) 2

(C) $\pi$

(D) $2\pi$

**Q50: NTA Test 47 (Numerical)**

If the value of the integral $\mathrm{I} = \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}} \max(\sin x, \tan x) dx$ is equal to $\ln k$, then the value of $k^2$ is equal to

**Q51: NTA Test 48 (Single Choice)**

The value of $\lim_{n \to \infty}\left(\frac{1}{2n}+\frac{1}{2n+1}+\frac{1}{2n+2}+.....+\frac{1}{4n}\right)$ is equal to

(A) $e^2$

(B) $\ln 2$

(C) $\ln 4$

(D) $3\ln 2$

# Answer Keys

| | | |
|---|---|---|
| **Q1:** (B) | **Q2:** (D) | **Q3:** (A) |
| **Q4:** (C) | **Q5:** (C) | **Q6:** (D) |
| **Q7:** (B) | **Q8:** (B) | **Q9:** 2 |
| **Q10:** (C) | **Q11:** (B) | **Q12:** 2 |
| **Q13:** 8 | **Q14:** (D) | **Q15:** (D) |
| **Q16:** (C) | **Q17:** (C) | **Q18:** (D) |
| **Q19:** 3 | **Q20:** (D) | **Q21:** (C) |
| **Q22:** (C) | **Q23:** (A) | **Q24:** (D) |
| **Q25:** (A) | **Q26:** (B) | **Q27:** (B) |
| **Q28:** (D) | **Q29:** (C) | **Q30:** 3 |
| **Q31:** (C) | **Q32:** (D) | **Q33:** (A) |
| **Q34:** (D) | **Q35:** (B) | **Q36:** (C) |
| **Q37:** (D) | **Q38:** (C) | **Q39:** (B) |
| **Q40:** (C) | **Q41:** (A) | **Q42:** (B) |
| **Q43:** (B) | **Q44:** 3 | **Q45:** (A) |
| **Q46:** (A) | **Q47:** (C) | **Q48:** (C) |
| **Q49:** (C) | **Q50:** 2 | **Q51:** (B) |

# Solutions

**Q1: (B) 1**

$$\int_{-1}^{1}(x-[x])\,dx=\int_{-1}^{1}x dx-\int_{-1}^{1}[x]\,dx$$

$$=\left[\frac{x^2}{2}\right]_{-1}^{1}-\left[\int_{-1}^{0}[x]\,dx+\int_{0}^{1}[x]\,dx\right]$$

$$=\frac{1}{2}[1-1]-\left[\int_{-1}^{0}(-1)\,dx+\int_{0}^{1}0.dx\right]$$

$$\left[\begin{array}{l}\text{If } -1\le x<0,\ [x]=-1\\ \text{If } \quad 0\le x<1,\ [x]=0\end{array}\right]$$

$$=0-[-x]_{-1}^{0}-0=0-[-0-(1)]=1$$

**Q2: (D)** $\frac{C}{x^3}e^{-\frac{1}{x}}$

$$x\int_1^x y(t)dt = x\int_1^x ty(t)dt + \int_1^x ty(t)dt$$

Differentiate w.r.t. $x$

$$\int_1^x y(t)dt + xy(x) = \int_1^x ty(t)dt + x[xy(x)] + xy(x)$$

$$\int_1^x y(t)dt = \int_1^x ty(t)dt + x^2y(x)$$

Differentiate again w.r.t. $x$

$$y(x) = xy(x) + 2x\,y(x) + x^2y'(x)$$

$$(1-3x)y(x) = x^2y'(x)$$

$$\frac{y'(x)}{y(x)} = \frac{1-3x}{x^2}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1-3x}{x^2}$$

Integrating on both sides

$$\Rightarrow log\,y = -\frac{1}{x} - 3log\,x + c$$

$$\Rightarrow log\left(y\,x^3\right) = -\frac{1}{x} + c$$

$$\Rightarrow yx^3 = e^{-\frac{1}{x}+c}$$

$$\Rightarrow y = \frac{e^{-\frac{1}{x}+c}}{x^3}$$

$$\Rightarrow y = \frac{C}{x^3}e^{-\frac{1}{x}}$$

**Q3: (A)** $-\frac{1}{2}$

Let, $I = \int_{-1/2}^{1/2}\left([x] + \ell n\left(\frac{1+x}{1-x}\right)\right)dx$

$$= \int_{-1/2}^{1/2}[x]\,dx + \int_{-1/2}^{1/2}\ell n\left(\frac{1+x}{1-x}\right)dx$$

$$= \int_{-1/2}^{0} -1dx + \int_{0}^{1/2} 0dx + 0 \qquad \left[\because\ \log\left(\frac{1+x}{1-x}\right)\text{ is an odd function}\right]$$

$$= [-x]_{-1/2}^{0} = 0 - \left(\frac{1}{2}\right) = -\frac{1}{2}$$

**Q4: (C)** $4:3$

Given, $I_1 = \int_0^\pi \frac{(\pi-x)\sin(\pi-x)}{1+\left(\cos(\pi-x)\right)^2}dx \quad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$$= \pi\int_0^\pi \frac{\sin x}{1+\cos^2 x}dx - \int_0^\pi \frac{x\sin x}{1+\cos^2 x}dx$$

$2I_1 = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx = 2\pi \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$

$\Rightarrow\ I_1 = \pi \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx = \pi \int_0^1 \frac{dt}{1+t^2}\ (t = \cos x)$

$= \left[\pi\ \tan^{-1} t\right]_0^1 = \frac{\pi^2}{4}$

$I_2 = \int_0^{\pi} (\pi - x)\sin^4 x dx$

$= \pi \int_0^{\pi} \sin^4 x dx - I_2$

$\Rightarrow\ 2I_2 = 2\pi \int_0^{\pi/2} \sin^4 x dx = 2\pi \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$

$\Rightarrow\ I_2 = \frac{3}{16}\pi^2$

Therefore, $I_1 : I_2 = \frac{1}{4} : \frac{3}{16} = 4 : 3$

**Q5: (C)** $\frac{\pi}{2} - 1$

$I = \int\limits_0^1 \left[\sqrt{\frac{1-x}{1+x}} \times \frac{\sqrt{1-x}}{\sqrt{1-x}}\right]\ dx$ (rationalising the denominator)

$= \int\limits_0^1 \frac{1-x}{\sqrt{1-x^2}} dx$

$= \int\limits_0^1 \frac{1}{\sqrt{1-x^2}} dx - \int\limits_0^1 \frac{x\,dx}{\sqrt{1-x^2}}$

$\Rightarrow\ I = \left[\sin^{-1} x\right]_0^1 - \frac{1}{2} \int\limits_0^1 \frac{2x}{\sqrt{1-x^2}} dx$

$= \left[\frac{\pi}{2} - 0\right] + \frac{1}{2}\left[2\sqrt{1-x^2}\right]_0^1$

$= \frac{\pi}{2} + [0 - 1]$

$= \frac{\pi}{2} - 1$

**Q6: (D)** 3

Let $I = \int\limits_4^{10} \frac{[x^2]}{[x^2-28x+196]+[x^2]} dx$ ........(i)

Use $\int\limits_a^b f(x)dx = \int\limits_a^b f(a+b-x)dx$

$I = \int\limits_4^{10} \frac{\left[(14-x)^2\right]}{[x^2]+\left[(14-x)^2\right]}\ dx$ ......(ii)

(i) + (ii)

we get $2I = \int\limits_4^{10} \frac{\left[(14-x)^2\right]+[x^2]}{[x^2]+\left[(14-x)^2\right]}\ dx$

$\Rightarrow 2I = \int\limits_4^{10} dx$

$\Rightarrow 2I = 6$

$\Rightarrow I=3$

**Q7: (B)** $-1$

Let $y=\lim\limits_{t\to 0}\frac{2x}{\pi}\cot^{-1}\frac{x}{t^2}$

Case-I : when $x>0$ then $y=\frac{2x}{\pi}\lim\limits_{t\to 0}\cot^{-1}\frac{x}{t^2}=\frac{2x}{\pi}\times 0=0$

Case-II : when $x<0$ then $y=\frac{2x}{\pi}\lim\limits_{t\to 0}\cot^{-1}\frac{x}{t^2}=\frac{2x}{\pi}\times\pi=2x$

$f(x)=\begin{cases}\sin 0 & x>0\\ \sin 2x & x<0\end{cases}$

Now, $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\,dx=\int_{-\frac{\pi}{2}}^{0}\sin 2x\,dx+\int_0^{\frac{\pi}{2}}0\;dx=-\left(\frac{\cos 2x}{2}\right)_{-\frac{\pi}{2}}^{0}=-\frac{1}{2}(1-(-1))=-1$

**Q8: (B)** $a$

$\int_{-a}^{a}\frac{e^x}{1+e^x}dx=\int_0^a\left(\frac{e^x}{1+e^x}+\frac{e^{-x}}{1+e^{-x}}\right)dx\left(\because\int_{-a}^{a}f(x)\,dx=\int_0^a\left(f(x)+f(-x)\right)dx\right)$

$=\int_0^a\left(\frac{e^x}{1+e^x}+\frac{1}{1+e^x}\right)dx$

$=\int_0^a dx=a$

**Q9: 2**

$I=\int_{-\pi}^{\pi}\sqrt{\frac{|\sin x|}{1+\tan^2 x}}dx=4\int_0^{\pi/2}\sqrt{\sin x}\cos x dx$

$\Rightarrow I=\frac{8}{3}\left[\sin^{\frac{3}{2}}\frac{\pi}{2}-0\right]=\frac{8}{3}$

$[I]=2$

**Q10: (C)** $I<\frac{2}{3}$ and $J<2$

Since, $I=\int_0^1\frac{\sin x}{\sqrt{x}}dx<\int_0^1\frac{x}{\sqrt{x}}dx$

as in $x\in(0,1),\ x>\sin x$

$I<\int_0^1\sqrt{x}dx=\frac{2}{3}\left[x^{3/2}\right]_0^1\Rightarrow I<\frac{2}{3}$

For, $x\in(0,1),\ \frac{\cos x}{\sqrt{x}}<\frac{1}{\sqrt{x}}$

Hence, $J=\int_0^1\frac{\cos x}{\sqrt{x}}dx<\int_0^1 x^{-\frac{1}{2}}dx=2\Rightarrow J<2$

**Q11: (B)** $\frac{1}{10}$

$\lim\limits_{n\to\infty}\sum\limits_{r=1}^{r=4n}\left(\frac{1}{\sqrt{r/n}\left(3\sqrt{\frac{r}{n}}+4\right)^2}\right)\cdot\frac{1}{n}$

$= \int_0^4 \frac{1}{\sqrt{x}\left(3\sqrt{x}+4\right)^2} dx$

Put, $3\sqrt{x} + 4 = t$

$\frac{3}{2\sqrt{x}} dx = dt \Rightarrow \frac{1}{\sqrt{x}} dx = \frac{2}{3} dt$

when $x = 0$ then $t = 4$

when $x = 4$ then $t = 10$

$= \frac{2}{3} \int_4^{10} \frac{1}{t^2} dt = \frac{2}{3}\left(-\frac{1}{t}\right)_4^{10}$

$= -\frac{2}{3}\left(\frac{1}{10} - \frac{1}{4}\right)$

$= \frac{1}{10}$

**Q12: 2**

$$\int_0^{\frac{1}{2}} \frac{(1+\sqrt{3})dx}{\left[(1+x)^2(1-x)^6\right]^{\frac{1}{4}}}$$

$$\int_0^{\frac{1}{2}} \frac{(1+\sqrt{3})dx}{(1+x)^2\left[\frac{(1-x)^6}{(1+x)^6}\right]^{\frac{1}{4}}}$$

Put $\frac{1-x}{1+x} = t \Rightarrow \frac{-2dx}{(1+x)^2} = dt$

$$I = \int_1^{\frac{1}{3}} \frac{(1+\sqrt{3})dt}{-2t^{\frac{6}{4}}} = \frac{-(1+\sqrt{3})}{2} \times \left|-\frac{2}{\sqrt{t}}\right|_1^{\frac{1}{3}} = (1+\sqrt{3})(\sqrt{3}-1) = 2$$

**Q13: 8**

$f(x) = -x^2$ for $x \leq -1$,
$f(x) = 1$ for $-1 < x < 0$,
$f(x) = 2$ for $x = 0$,
$f(x) = 1$ for $0 < x < 1$ and
$f(x) = -x^2$ for $x \geq 1$
$\Rightarrow f(x)$ is even
$\therefore I = 2\int_0^2 f(x)\,dx = 2\int_0^1 f(x)\,dx + 2\int_1^2 f(x)\,dx = 2\int_0^1 (1)\,dx + 2\int_1^2 -x^2 dx = \frac{-8}{3}$
$\therefore |3I| = |-8| = 8$

**Q14: (D) $a_1$ and $a_3$**

Let, $I = \int_0^{\pi} \sum_{r=0}^{3} a_r \cos^{3-r} x \sin^r x\, dx$

$= \int_0^{\pi} a_0 \cos^3 x\, dx + \int_0^{\pi} a_1 \cos^2 x \sin x dx$

$+ \int_0^{\pi} a_2 \cos x \sin^2 x\, dx + \int_0^{\pi} a_3 \sin^3 x\, dx$

Since, $\int_0^{2a} f(x) dx$

$= \begin{cases} 2\int_0^a f(x)dx & \text{, if } f(2a-x) = f(x) \\ 0 & \text{, if } f(2a-x) = -f(x) \end{cases}$

$\therefore$ Integral Ist and IIIrd become zero.

$\therefore$The given integral depends upon $a_1$and $a_3$.

**Q15: (D)** $e^{-a}\left[F\left(x+a\right)-F\left(1+a\right)\right]$

$F\left(x\right)=\int_1^x \frac{e^t}{t}dt$

$I=\int_1^x \frac{e^t}{t+a}dt$

Let, $t+a=y \Rightarrow dt=dy$

Also, $t=1 \Rightarrow y=1+a$ and $t=x \Rightarrow y=x+a$

$\therefore \ I=\int_{1+a}^{x+a} \frac{e^{y-a}}{y}dy$

$=e^{-a}\int_{1+a}^{x+a} \frac{e^y}{y}dy$

$=e^{-a}\int_{1+a}^{x+a} \frac{e^t}{t}dt$

$=e^{-a}\left[F\left(x+a\right)-F\left(1+a\right)\right]$

**Q16: (C)** $\pi$

Differentiating using Leibnitz rule, we get,
$x^{12}f\left(x^4\right)4x^3-x^6f\left(x^2\right)2x=2\pi\cos\pi x$
Putting $x=1$
$\Rightarrow 4f\left(1\right)-2f\left(1\right)=2\pi$
$\Rightarrow 2f\left(1\right)=2\pi \Rightarrow f\left(1\right)=\pi$

**Q17: (C)** 0

$I=2\int_0^1 [x\sin(\pi x)]dx$

Now, $x\sin(\pi x)\in(0,1)$

as $x\in(0,1)$

$\therefore [x\sin(\pi x)]=0$

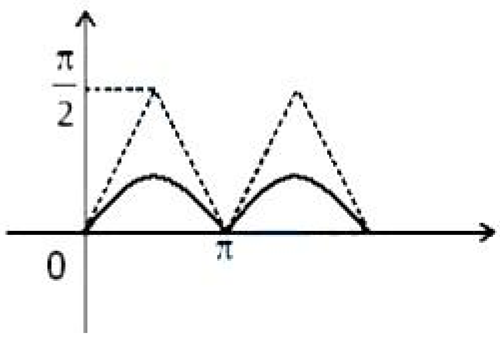
$\therefore I=2\int_0^1 0dx=0$

**Q18: (D)** 3

As $[\{t\}] = 0$, we get,

$$I = \int_1^2 t^t\,(1 + \ln t)dt$$
$$= \int_1^2 d\,(t^t)$$
$$= [t^t]_1^2$$
$$= 2^2 - 1^1 = 3$$

**Q19: 3**




max $(|\sin x|, |\sin^{-1}(\sin x)|)$ is $|\sin^{-1}(\sin x)|$
Hence, $I_n = n\left(\frac{1}{2}\cdot\pi\frac{\pi}{2}\right)$
$= \frac{n\pi^2}{4}$ (area of triangle)
$\therefore I_2 + I_4 = \frac{6\pi^2}{4}$

**Q20: (D)** $\sqrt{2} - 1$

Let, $\sin x + \cos x = t$

$$\Rightarrow I = \int_1^{\sqrt{2}} [t]dt = \int_1^{\sqrt{2}} 1dt = \sqrt{2} - 1$$

**Q21: (C)** $\sin^2\left(\frac{k}{2}\right)$

Applying property of $(a + b - x)$ and adding, we get,

$$2\mathrm{I} = \int_0^k f(x) + f(k - x)dx = \int_0^k \sin x dx = 1 - \cos k$$

**Q22: (C)** 8

As $f(1 + x) = f(1 - x) \xrightarrow{x \to x - 1} f(x) = f(2 - x)$
$\therefore$ Using $(a + b - x)$; $2\mathrm{I} = \int_{-7}^{9} \frac{f(x) + f(2 - x)}{f(2 - x) + f(x)}dx \Rightarrow \mathrm{I} = 8$

**Q23: (A)** $\mathrm{I}_1 = \mathrm{I}_2$

In $\mathrm{I}_2$ substitute $x = \frac{1}{t} \Rightarrow dx = -\frac{dt}{t^2}$
So, $\mathrm{I}_2 = -\int_1^0 \frac{\ln t}{t^2 + 4t + 1}\cdot t^2\left(\frac{-dt}{t^2}\right) = \int_0^1 \frac{(-\ln t)}{t^2 + 4t + 1}dt = \mathrm{I}_1$ $\{$as $|\ln t| = -\ln t$ in $(0,1)\}$

**Q24: (D)** $e - 1$

$\lim_{n\to\infty} \sum_{r-1}^{n} 2\left(\frac{r}{n}\right)e^{\frac{r^2}{n^2}}\left(\frac{1}{n}\right) = \int_0^1 2xe^{x^2}dx = \left(e^{x^2}\right)_0^1 = e-1$

**Q25: (A)** 0

Let the value of integral is $I$, then,

$I = \int_0^{\frac{\pi}{2}} \left(\cos 2x \cos 2^2 x \cos 2^3 x \cos 2^4 x\right)dx$

$I = \int_0^{\frac{\pi}{2}} -\cos 2x \cos 2^2 x \cos 2^3 x \cos 2^4 x dx$ (applying $f(x) = f(a+b-x)$)

Adding, we get,

$2I = 0 \Rightarrow I = 0$

**Q26: (B)** $\frac{1}{2}$

$I_1 = \int_{-1}^{2} x\sin(x(1-x))dx$

$I_1 = \int_{-1}^{2} (1-x)\sin((1-x)x)dx$

$I_1 = \int_{-1}^{2} \sin(x(1-x))dx - \int_{-1}^{2} x\sin((1-x)x)dx$

$I_1 = I_2 - I_1$

$2I_1 = I_2$

$\frac{I_1}{I_2} = \frac{1}{2}$

**Q27: (B)** $-12\pi$

We know that, $[x] + [-x] = \begin{cases} -1 & , \ x \notin Z \\ 0 & , \ x \in Z \end{cases}$

So, $\int_0^{12\pi} ([\sin t] + [-\sin t])dt$

$= 6\int_0^{2\pi} ([\sin t] + [-\sin t])dt$

$= 6\left(\int_0^{\frac{\pi}{2}} (-1)dt + \int_{\frac{\pi}{2}}^{\pi} (-1)dt + \int_{\pi}^{\frac{3\pi}{2}} (-1)dt + \int_{\frac{3\pi}{2}}^{2\pi} (-1)dt\right)$

$= 6\left((-1)\left(\frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2}\right)\right)$

$= -12\pi$

**Q28: (D)** $\frac{\pi}{2}$

$\Rightarrow \sin^2 x - \sin x + \frac{1}{2} = \left(\sin x - \frac{1}{2}\right)^2 + \frac{1}{4} > 0 \ \forall x \in \left(0, \frac{\pi}{2}\right)$

$\therefore \ sgn\left(\sin^2 x - \sin x + \frac{1}{2}\right) = 1$

Thus, $I = \int_0^{\frac{\pi}{2}} 1dx = (x)_0^{\frac{\pi}{2}} = \frac{\pi}{2}$

**Q29: (C)** $I_1 = I_2$

In $I_2$,

let, $x\sin x + \cos x = t$

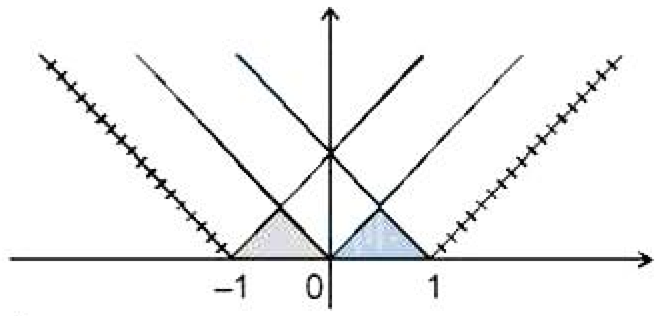
$\Rightarrow (x\cos x + \sin x - \sin x)dx = dt$ or $x\cos x dx = dt$

$\therefore I_2 = \int_1^{\frac{\pi}{2}} \frac{dt}{1+t^6} = I_1$

Also, $I_1$ & $I_2$ are both positive as

$\frac{1}{1+t^6} > 0 \ \forall t \in \left(1, \frac{\pi}{2}\right)$

**Q30: 3**




$\int_{-1}^{1} f(x)dx =$ shaded area

$= 2\left(\frac{1}{2}\text{base} \cdot \text{height}\right)$

$= 2\left(\frac{1}{2} \cdot 1 \cdot \frac{1}{2}\right) = \frac{1}{2}$

**Q31: (C)** $e$

$I_2 = \int_0^1 2x^2 e^{x^2} dx$

$= \int_0^1 \left(2xe^{x^2}\right) x dx$

Using integration by parts,

$I_2 = \left[xe^{x^2}\right]_0^1 - \int_0^1 e^{x^2} dx$

$I_2 + I_1 = e$

**Q32: (D)** $\pi$

$\frac{x+x^3+x^5}{x^4+x^2+1}$ is an odd function, let it be equal to $\lambda$,

So,

$I(say) = \int_0^1 \left(\cot^{-1}(\lambda) + \cot^{-1}(-\lambda)\right) dx = \pi \int_0^1 dx = \pi$

**Q33: (A)** $\pi$

Let $I = \int_{\frac{1}{\sqrt{3}}}^{1/\sqrt{3}} \left(\frac{x^4}{1-x^4}\right) \cos^{-1}\left(\frac{2x}{1+x^2}\right) dx$

$= \int_{\frac{-1}{\sqrt{3}}}^{1/\sqrt{3}} \left(\frac{x^4}{1-x^4}\right)\left(\frac{\pi}{2} - \sin^{-1}\left(\frac{2x}{1+x^2}\right)\right) dx$

$= \frac{\pi}{2} \int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{x^4}{1-x^4} dx - \underbrace{\int_{-1/\sqrt{3}}^{1.\sqrt{3}} \left(\frac{x^4}{1-x^4}\right) \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx}_{=0, \text{ as integrand is an odd function}}$

$= \frac{\pi}{2} \cdot 2 \int_0^{1/\sqrt{3}} \frac{x^4}{1-x^4} dx \quad \left(\text{as } \frac{x^4}{1-x^4} \text{ is an even function}\right)$

Hence, $k = \pi$

**Q34: (D)** $-2\tan 1$

Let, $I = \int_{-1}^{1} \left(\sin^{-1} x + \frac{x^5+x^3-1}{\cos^2 x}\right) dx$

$I = \int_{-1}^{1} \frac{-1}{\cos^2 x} dx$

$I = -\int_{-1}^{1} \sec^2 x dx$

$I = [-\tan x]_{-1}^{1}$

$I = -2\tan 1$

**Q35: (B)** $24\pi$

Applying $(a+b-x)$ property,

$$I = \int_0^{24\pi} \{\sin(24\pi - x)\}dx$$
$$= \int_0^{24\pi} \{-\sin x\}dx$$
Adding the two integrals, we get,
$$2I = \int_0^{24\pi} (\{\sin x\} + \{-\sin x\})dx$$
Now, $\{\alpha\} + \{-\alpha\} = \begin{cases} 0; & \alpha \in I \\ 1; & \alpha \notin I \end{cases}$

As, $\sin x$ takes integral values only at discrete points,

Hence,
$$2I = \int_0^{24\pi} 1dx = [x]_0^{24\pi} = 24\pi - 0$$
$$= 24\pi$$

**Q36: (C)** $\frac{1}{2}\left(e^4 - e^{\frac{4}{3}}\right)$

Let, $I = \int e^{\sec^2 x} \frac{\sin x}{\cos^3 x} dx$

$\Rightarrow I = \int e^{\sec^2 x} \tan x \cdot \sec^2 x dx$

$\Rightarrow I = \int e \cdot e^{\tan^2 x} \tan x \sec^2 dx$

Substituting, $\tan^2 x = t$

$2 \tan x \sec^2 x dx = dt$

$\Rightarrow I = \frac{1}{2} \int e \cdot e^t dt$

$I = \frac{e}{2} e^t + C$

$I = \frac{e}{2} e^{\tan^2 x} + C$

$= \frac{e^{\sec^2 x}}{2} + C$

$I = \frac{1}{2} e^{\sec^2 x} \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{1}{2}\left(e^4 - e^{\frac{4}{3}}\right)$

**Q37: (D)** 8

The period of the function $|\sin^3 x|$ is $\pi$, thus $I = 6 \int_0^{\pi} \sin^3 x dx$

$\Rightarrow I = 6 \int_0^{\pi} \frac{3 \sin x - \sin(3x)}{4} dx \ \left(\text{As } \ (\sin(3x) = 3 \sin x - 4\sin^3 x)\right)$

$\Rightarrow I = \frac{6}{4} \int_0^{\pi} (3 \sin x - \sin(3x))dx$

$\Rightarrow I = \frac{3}{2}\left([-3 \cos x]_0^{\pi} + \left[\frac{\cos(3x)}{3}\right]_0^{\pi}\right)$

$= \frac{3}{2}\left(3 - (-3) + \left(\frac{-1}{3} - \frac{1}{3}\right)\right)$

$= \frac{3}{2}\left(6 - \frac{2}{3}\right) = \frac{3}{2} \times \frac{16}{3} = 8$

**Q38: (C)** $\frac{-5\pi}{3}$

Let, $I = \int_{\pi}^{2\pi} [2 \sin x]dx$

For $\pi \le x \le \frac{7\pi}{6}$ or $\frac{11\pi}{6} \le x \le 2\pi$

$-1 \le 2 \sin x < 0$

$\Rightarrow \quad [2 \sin x] = -1$

For $\frac{7\pi}{6} \le x < \frac{11\pi}{6}$

$-2 \leq 2\sin x < -1$

$\Rightarrow \quad [2\sin x] = -2$

$$\therefore \quad I = \int_{\pi}^{7\pi/6} -1dx + \int_{7\pi/6}^{11\pi/6} -2dx + \int_{11\pi/6}^{2\pi} -1\,dx$$

$$= \left(-\tfrac{7\pi}{6} + \pi\right) + 2\left(-\tfrac{11\pi}{6} + \tfrac{7\pi}{6}\right) + \left(-2\pi + \tfrac{11\pi}{6}\right)$$

$$= -\tfrac{\pi}{6} - \tfrac{8\pi}{6} - \tfrac{\pi}{6} = -\tfrac{10\pi}{6} = \tfrac{-5\pi}{3}$$

**Q39: (B)** 1

$a = \int_0^1 \cos(\sin x)\cos x\mathrm{dx}$

Let $\sin x = \mathrm{t}$

$\Rightarrow \cos x\mathrm{dx} = \mathrm{dt}$

$\therefore \mathrm{a} = \int_0^{\sin 1} \cos t\mathrm{dt} = (\sin t)_0^{\sin 1}$

$= \sin(\sin 1) - \sin 0$

$= \sin(\sin 1)$

$\therefore \mathrm{a}^2 + \cos^2(\sin 1) = \sin^2\theta + \cos^2\theta = 1; \quad (\theta = \sin 1)$

**Q40: (C)** $\frac{\pi}{3}$

Let, $\mathrm{I} = \int_3^6 \frac{\sqrt{(36-\mathrm{x}^2)^3}}{\mathrm{x}^4}\mathrm{dx}$

Substituting, $\mathrm{x} = 6\sin\theta$

$\mathrm{dx} = 6\cos\theta\mathrm{d}\theta$

$\mathrm{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\left(\sqrt{36\cos^2\theta}\right)^3}{6^4\sin^4\theta} 6\cos\theta\mathrm{d}\theta$

$\mathrm{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{\cos^4\theta}{\sin^4\theta}\right)\mathrm{d}\theta$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot^2\theta\left(\mathrm{cosec}^2\theta - 1\right)\mathrm{d}\theta$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot^2\theta\mathrm{cosec}^2\theta\mathrm{d}\theta - \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot^2\theta\mathrm{d}\theta$

$= -\left[\frac{\cot^3\theta}{3}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\mathrm{cosec}^2\theta - 1\right)\mathrm{d}\theta$

$= \frac{-1}{3}\left(0 - 3\sqrt{3}\right) + [\cot\theta]_{\frac{\pi}{6}}^{\frac{\pi}{2}} + (\theta)_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= \sqrt{3} + \left[0 - \sqrt{3}\right] + \left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \frac{\pi}{3}$

**Q41: (A)** $A > \frac{\pi}{4}$

As, $x \in (0,1) \Rightarrow x^2 > x^3$

$\Rightarrow 1 + x^2 > 1 + x^3$ or $\frac{1}{1+x^2} < \frac{1}{1+x^3}$

Thus, $\int_0^1 \frac{dx}{1+x^2} < \int_0^1 \frac{dx}{1+x^3}$

i.e., $\left[\tan^{-1}x\right]_0^1 < A$

$\Rightarrow A > \tan^{-1}1 - \tan^{1}0$

$\Rightarrow A > \frac{\pi}{4}$

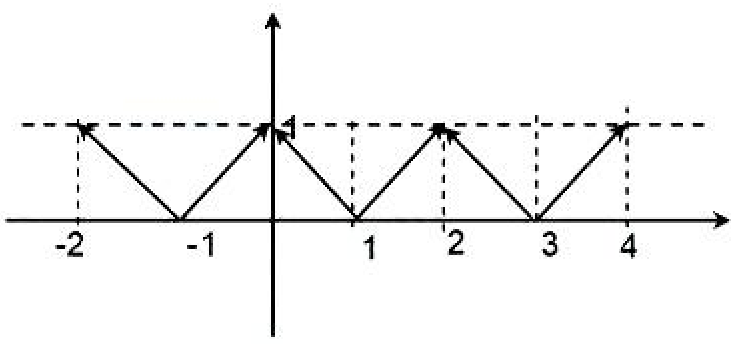
**Q42: (B)** 110

As the period of $y=|\sin x|$ is $\pi$, hence

$A_n=n\int\limits_0^{\pi}|\sin x|dx$
$=n\int\limits_0^{\pi}\sin x dx$
$=n[-\cos x]_0^{\pi}$
$=2n$
$\therefore \sum_{r=1}^{10}A_r=\sum_{r=1}^{10}2r$
$=2\left(1+2+\ldots .+10\right)$
$=2\times\frac{10\times11}{2}=110$

**Q43: (B)** $\frac{k}{2}$
$\sin 3x=3\sin x-4\sin^3 x$
$\sin^3 x=\frac{3}{4}\sin x-\frac{1}{4}\sin 3x$
Now, $\int\limits_0^{\infty}\frac{\sin^3 x}{x}dx=\frac{3}{4}\int\limits_0^{\infty}\frac{\sin x}{x}dx-\frac{1}{4}\int\limits_0^{\infty}\frac{\sin 3x}{x}dx$
$=\frac{3k}{4}-\frac{1}{4}$ (let)
$\mathrm{I}=\int\limits_0^{\infty}\frac{\sin 3x}{x}dx$
Let $3x=t$
$dx=\frac{dt}{3}$
$\mathrm{I}=\int\limits_0^{\infty}\frac{\sin t}{\left(\frac{t}{3}\right)}\frac{dx}{3}=k$
Required value $=\frac{3k}{4}-\frac{k}{4}=\frac{k}{2}$

**Q44: 3**




We know that,
$x-[x]=\{x\}$
$\Rightarrow x-[x+1]=\{x\}-1$
Hence, $\int_{-2}^{4}f\left(x\right)dx=6.\frac{1}{2}\left(1.1\right)=3$

**Q45: (A)** $\frac{\pi}{2\sqrt{2}}$

Let, $\mathrm{I}=\int_0^{\infty}\left(\frac{x^2+1-x^2}{1+x^4}\right)dx$
$\mathrm{I}=\int_0^{\infty}\frac{x^2}{1+x^4}dx+\int_0^{\infty}\frac{1-x^2}{1+x^4}dx$
$\mathrm{I}=\mathrm{I}_1+\mathrm{I}_2$
$\mathrm{I}_2=\int_0^{\infty}\frac{\frac{1}{x^2}-1}{x^2+\frac{1}{x^2}}dx$
Let, $x+\frac{1}{x}=t$
$I_2=\int_{\infty}^{\infty}\frac{dt}{t^2-2}=0$
$I=\int_0^{\infty}\frac{x^2}{1+x^4}dx$ also $I=\int_0^{\infty}\frac{1}{1+x^4}dx$
So, $2I=\int_0^{\infty}\frac{1+x^2}{1+x^4}dx$
$I=\frac{1}{2}\int_0^{\infty}\frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$
Let, $x-\frac{1}{x}=t$
$\mathrm{I}=\frac{1}{2}\int_{-\infty}^{\infty}\frac{dt}{t^2+2}$

$\mathrm{I} = \frac{1}{2\sqrt{2}}\left|\tan^{-1}\frac{t}{\sqrt{2}}\right|_{-\infty}^{\infty}$
$= \frac{1}{2\sqrt{2}}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \frac{\pi}{2\sqrt{2}}$

**Q46: (A)** 0

$\mathrm{I}(\alpha) = [\ln x]_{\alpha}^{\alpha^2} = \ln\alpha^2 - \ln\alpha$
$= \ln\left(\frac{\alpha^2}{\alpha}\right)$
$= \ln\alpha$
Thus, $\sum_{r=2}^{5}\mathrm{I}(r) = \ln 2 + \ln 3 + \ln 4 + \ln 5$ and

$\sum_{r=2}^{5}\mathrm{I}\left(\frac{1}{k}\right) = \ln\left(\frac{1}{2}\right) + \ln\left(\frac{1}{3}\right) + \ln\left(\frac{1}{4}\right) + \ln\left(\frac{1}{5}\right) = -(\ln 2 + \ln 3 + \ln 4 + \ln 5)$
Hence, their sum equals to zero

**Q47: (C)** $A < \frac{\pi}{2}$

As $\sin\theta < \theta,\ \forall\theta \in \left(0, \frac{\pi}{2}\right)$
$\therefore \sin(2x) < 2x$
So, $A = 2\int\limits_{0}^{\frac{\pi}{4}} \frac{\sin(2x)}{2x}dx < 2\int\limits_{0}^{\frac{\pi}{4}} 1dx$
$\Rightarrow A < 2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$

**Q48: (C)** $\frac{\pi}{3}\log 2$

Let, $\mathrm{I} = \int_0^{\frac{\pi}{3}} \log\left(1+\sqrt{3}\tan x\right)dx$
$\mathrm{I} = \int_0^{\frac{\pi}{3}} \log\left[1+\sqrt{3}\tan\left(\frac{\pi}{3}-x\right)\right]dx$
$= \int_0^{\frac{\pi}{3}} \left[\log\left(1+\sqrt{3}\tan\left(\frac{\sqrt{3}-\tan x}{1+\sqrt{3}\tan x}\right)\right)\right]dx$
$= \int_0^{\frac{\pi}{3}} \log\left(\frac{1+\sqrt{3}\tan x+3-\sqrt{3}\tan x}{1+\sqrt{3}\tan x}\right)dx$
$\mathrm{I} = \int_0^{\frac{\pi}{3}} \left(\log 4 - \log\left(1+\sqrt{3}\tan x\right)\right)dx$
$\mathrm{I} = (\log 4)\left(\frac{\pi}{3}\right) - \mathrm{I}$
$\mathrm{I} = \frac{\pi}{6}\log 4 = \frac{\pi}{3}\log 2$

**Q49: (C)** $\pi$

Let, $y = |\sin x| + |\cos x|$
$\Rightarrow y^2 = 1 + |\sin(2x)| \in [1,2]$
$\Rightarrow y \in \left[1, \sqrt{2}\right]$
$\Rightarrow [|\sin x| + |\cos x|] = 1$
Thus, $\mathrm{I} = \int\limits_{0}^{\pi} 1 \cdot dx = \pi - 0$
$= \pi$

**Q50: 2**

For $x \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right),\ \tan x > \sin x$ (as $\cos x < 1$ )
$\therefore \max(\tan x, \sin x) = \tan x$
Thus, $\mathrm{I} = \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan x dx$
$= (-\ln|\cos x|)_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= -\ln\left(\frac{1}{2}\right) + \ln\left(\frac{1}{\sqrt{2}}\right)$
$= \ln\sqrt{2}$

**Q51: (B)** $\ln 2$
$\lim\limits_{n\to\infty} \sum_{r=0}^{2n} \frac{1}{2n+r} = \lim\limits_{n\to\infty} \frac{1}{n} \sum_{r=0}^{2n} \frac{1}{2+\frac{r}{n}}$

$= \int_0^2 \left(\frac{1}{2+x}\right)dx = |\ln(x+2)|_0^2$

$= \ln 4 - \ln 2 = \ln 2$