1. **(c)** One vertex of square is (– 4, 5) and equation of one diagonal is $7x - y + 8 = 0$

Diagonal of a square are perpendicular and bisect each other

Let the equation of the other diagonal be $y = mx + c$ where m is the slope of the line and c is the y-intercept.

Since this line passes through (– 4, 5)

$$5 = -4m + c \qquad \ldots \text{(i)}$$

Since this line is at right angle to the line

$7x - y + 8 = 0$ or $y = 7x + 8$, having slope = 7,

$$7 \times m = -1 \quad \text{or} \quad m = \frac{-1}{7}$$

Putting this value of m in equation (i) we get

$$c = 5 - \frac{4}{7} = \frac{31}{7}$$

Hence equation of the other diagonal is

$$y = -\frac{1}{7}x + \frac{31}{7} \quad \text{or} \quad x + 7y = 31.$$

2. **(a)** The parametric equation of a line through A is

$$\frac{x+5}{\cos\theta} = \frac{y+4}{\sin\theta} = r$$

Let $AB = r_1$, $AC = r_2$ and $AD = r_3$

Then the coordinates of B, C, D are

$(-5, + r_i\cos\theta, -4 + r_i \sin\theta)$, $i = 1, 2, 3$

Now B lies on the line $x + 3y + 2 = 0$

$$\therefore -5 + r_1\cos\theta + 3(-4 + r_1 \sin\theta) + 2 = 0$$

$$\frac{15}{r_1} = \cos\theta + 3\sin\theta$$

C lies on $2x + y + 4 = 0$

$$\therefore 2(-5 + r_2\cos\theta) + (-4 + r_2\sin\theta) + 4 = 0$$

$$\Rightarrow \frac{10}{r_2} = 2\cos\theta + \sin\theta$$

D lies on x – y – 5 = 0

$$\therefore -5+r_3\cos\theta+4-r_3\sin\theta-5=0$$

$$\Rightarrow \frac{6}{r_3}=\cos\theta-\sin\theta \ .$$

From the given condition $\left(\frac{15}{r_1}\right)^2+\left(\frac{10}{r_2}\right)^2=\left(\frac{6}{r_3}\right)^2$

we get,

$$(\cos\theta+3\sin\theta)^2+(2\cos\theta+\sin\theta)^2=(\cos\theta-\sin\theta)^2$$

$$\Rightarrow \quad (2\cos\theta+3\sin\theta)^2=0 \quad \Rightarrow \quad \tan\theta=-\frac{2}{3}$$

∴ Equation of the line is

$$y+4=-\frac{2}{3}(x+5)\Rightarrow 2x+3y+22=0$$

3. **(b)** The equation of any line parallel to $2x + 6y + 7 = 0$ is $2x + 6y + k = 0$

This meets the axes at $A\left(-\frac{k}{2},0\right)$ and $B\left(0,-\frac{k}{6}\right)$

By hypothesis, AB = 10

$$\Rightarrow \sqrt{\frac{k^2}{4}+\frac{k^2}{36}}=10\Rightarrow\sqrt{\frac{10k^2}{36}}=10$$

$$\Rightarrow 10k^2 = 3600 \ \Rightarrow k = \pm6\sqrt{10}$$

Hence there are two lines given by $2x+6y\pm6\sqrt{10}=0$

4. **(c)** We have the equation

$$y^2 + xy + px^2 - x - 2y + p = 0$$

We know any general equation

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0 \qquad \text{... (1)}$$

represents two straight lines if

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 \qquad \text{... (2)}$$

On comparing given equation with (1), we get

$$a = p,\ b = 1,\ h=\frac{1}{2},\ g=-\frac{1}{2},\ f = -1,\ c = p$$

Put these value in equation (2)

$$p \times 1 \times p + 2 \times -1 \times -\frac{1}{2} \times \frac{1}{2} - p \times (-1)^2 - 1$$

$$\times\left(-\frac{1}{2}\right)^2 - p\times\left(\frac{1}{2}\right)^2 = 0$$

$$\Rightarrow \quad p^2 + \frac{1}{2} - p - \frac{1}{4} - \frac{p}{4} = 0 \ \Rightarrow \ p^2 - \frac{5p}{4} + \frac{1}{4} = 0$$

$$\Rightarrow \quad 4p^2 - 5p + 1 = 0 \ \Rightarrow \ (4p - 1)(p - 1) = 0$$

$$p = 1, \frac{1}{4}$$

5. **(a)** Let the point (h, k) lie on a line $x + y = 4$

then $h + k = 4$ ...(i)

and $1 = \pm\frac{4h+3k-10}{\sqrt{4^2+3^2}} \Rightarrow 4h + 3k = 15$ ...(ii)

and $4h + 3k = 5$ ...(iii)

On solving (i) and (ii); and (i) and (iii), we get the required points (3, 1) and (–7, 11).

Trick : Check with options. Obviously, points (3, 1) and (–7, 11) lie on $x + y = 4$ and perpendicular distance of these points from $4x + 3y = 10$ is 1.

6. **(d)** Slope of the line in the new position is $\frac{b}{a}$, since it is $\perp$ to the line ax + by + c = 0 and it cuts the x-axis at (2,0). Hence, the required line passes through (2, 0) and its slope is $\frac{b}{a}$. Required eq. is

$$y - 0 = \frac{b}{a}(x-2) \ \Rightarrow ay = bx - 2b \ \Rightarrow ay - bx + 2b = 0$$

7. **(b)** We know that if m_1 and m_2 are the slopes of the lines represented by $ax^2 + 2hxy + by^2 = 0$,

then sum of slopes = $m_1 + m_2 = -\frac{2h}{b}$ and product of slopes = $m_1\, m_2 = \frac{a}{b}$.

Consider the given equation which is

$x^2 + 2hxy + 2y^2 = 0$

On comparing this equation with $ax^2 + 2hxy + by^2 = 0$,

we have $a = 1$, $2h = 2h$ and $b = 2$

Let the slopes be m_1 and m_2.

Given : $m_1 : m_2 = 1 : 2$

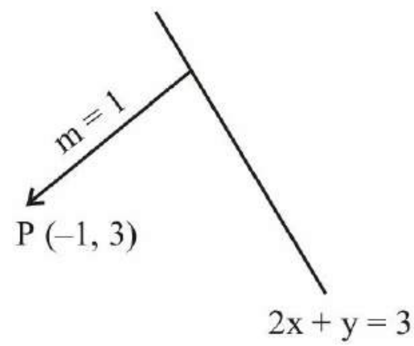
Let $m_1 = x$ and $m_2 = 2x$

$\therefore \quad m_1 + m_2 = -\frac{2h}{2} \Rightarrow x + 2x = -h \Rightarrow h = -3x$ ...(i)

and $m_1 m_2 = \frac{a}{b} \Rightarrow x \cdot 2x = \frac{1}{2} \Rightarrow x = \pm\frac{1}{2}$ ...(ii)

$\therefore$ From eqs. (i) and (ii), we have $h = \pm\frac{3}{2}$.

8. **(c)** The equation of the line through (–1, 3) and having the slope 1 is $\frac{x+1}{\cos\theta} = \frac{y-3}{\sin\theta} = r$.



Any point on this line at a

distance r from P (–1, 3) is

$(-1 + r\cos\theta,\ 3 + r\sin\theta)$

This point is on the line 2x + y = 3 if

$2(-1 + r\cos\theta) + 3 + r\sin\theta = 3$

...(i)

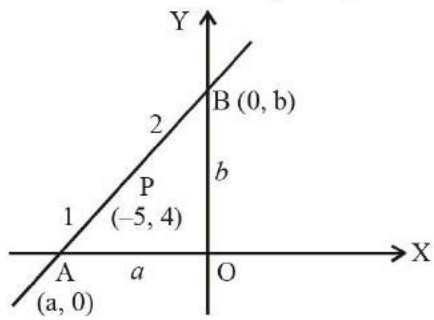
But tan θ = 1; $\Rightarrow \theta = 45°$

(i) becomes,

$$-2+2r.\frac{1}{\sqrt{2}}+3+r.\frac{1}{\sqrt{2}}=3 \Rightarrow \frac{3r}{\sqrt{2}}=2; \quad r=\frac{2\sqrt{2}}{3}$$

Hence the required distance = $\frac{2\sqrt{2}}{3}$.

9. **(c)** Let the equation of the line be $\frac{x}{a}+\frac{y}{b}=1$.



So, the coordinates of A and B are (a, 0) and (0, b) respectively. Since the point (–5, 4) divides AB in the ratio 1 : 2

$\therefore$ $-5 = \frac{1.0+2.a}{1+2}$

and $4 = \frac{1.b+2.0}{2+1}$ $\Rightarrow$ $a = -\frac{15}{2}$ and b = 12

So the line is $-\frac{2}{15}x+\frac{y}{12} = 1$, *i.e.* – 8x + 5y = 60

10. **(a)** Let Q(a, b) be the reflection of P(4, – 13) in the line $5x + y + 6 = 0$

Then the mid-point $R\left(\frac{a+4}{2},\frac{b-13}{2}\right)$ lies on $5x + y + 6 = 0$

$$5\left(\frac{a+4}{2}\right)+\frac{b-13}{2}+6=0 \Rightarrow 5a + b + 19 = 0 \text{...(i)}$$

Also PQ is perpendicular to $5x + y + 6 = 0$

Therefore $\frac{b+13}{a-4}\times\left(-\frac{5}{1}\right)=-1 \Rightarrow a - 5b - 69 = 0$..(ii)

Solving (i) and (ii), we get $a = -1$, $b = -14$.

11. **(b)** We have the equation $2x^2 - xy - y^2 = 0$

$\Rightarrow (2x + y)(x - y) = 0$

If (h, k) be the point then remaining pair is

$(2x + y + h)(x - y + k) = 0$

Where, $2x + y + h = 0$ and $x - y + k = 0$

It passes through the point (1, 0)

$\therefore 2 \times 1 + 0 + h = 0 \quad 2 + h = 0 \quad h = -2$

and $1 - 0 + k = 0 \quad 1 + k = 0 \quad k = -1$

Required pair is $(2x + y - 2)(x - y - 1) = 0$

$$2x^2 - 2xy - 2x + xy - y^2 - y - 2x + 2y + 2 = 0$$

$$2x^2 - xy - y^2 - 4x + y + 2 = 0$$

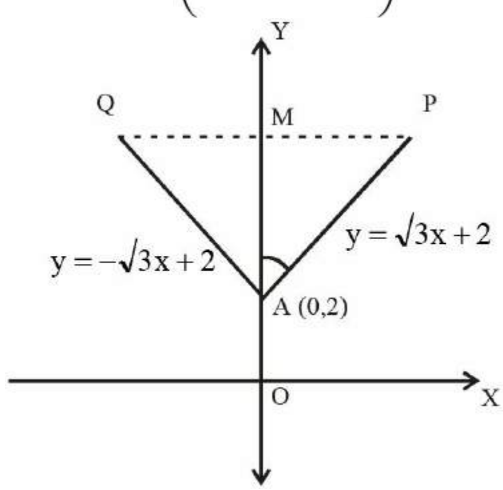
12. **(b)** Since $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$,

therefore the equations of two lines are

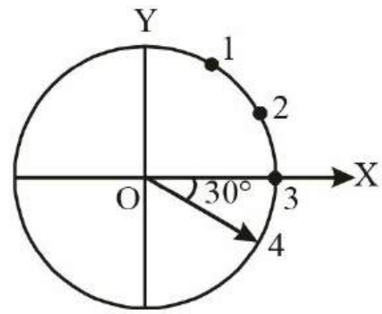
$y = \sqrt{3}x + 2, \ x \geq 0$ and $y = -\sqrt{3}x + 2, \ x < 0$

Clearly y-axis the only bisector of the angle between these two lines. There are two points P and Q on these lines at a distance of 5 units from A. Clearly M is the foot of the perpendicular from P and Q on y-axis (bisector). AM = AP cos 30° = $\frac{5\sqrt{3}}{2}$.

Hence, coordinates of M are $\left(0, 2 + \frac{5\sqrt{3}}{2}\right)$



13. **(c)** Since the hour, minute and second hand always pass through origin because one end of these hands is always at origin. Now at 4 O' clock, the hour hand makes 30° angle in fourth quadrant. So the equation of hour hand is



$$y = mx \Rightarrow y = -\frac{1}{\sqrt{3}}x$$

$$\Rightarrow x + \sqrt{3}y = 0$$

14. **(d)** Mid-point of P(2, 3) and Q(4, 5) = (3, 4)

Slope of PQ = 1, Slope of the line L = – 1

Mid-point (3, 4) lies on the line L.

Equation of line L,

$$y - 4 = -1(x - 3) \Rightarrow x + y - 7 = 0 \quad \text{...(i)}$$

Let image of point R(0, 0) be S(x_1, y_1)

Mid-point of RS = $\left(\frac{x_1}{2}, \frac{y_1}{2}\right)$

Mid-point $\left(\frac{x_1}{2}, \frac{y_1}{2}\right)$ lies on the line (i)

$$\therefore x_1 + y_1 = 14 \quad \text{...(ii)}$$

Slope of RS = $\frac{y_1}{x_1}$; Since RS ⊥ line L

$$\therefore \frac{y_1}{x_1} \times (-1) = -1 \quad \therefore x_1 = y_1 \quad \text{...(iii)}$$

From (ii) and (iii), $x_1 = y_1 = 7$

Hence the image of R = (7, 7)

15. **(a)** Clearly the point (3, 0) does not lie on the diagonal x = 2y. Let m be the slope of a side passing through (3, 0). Then its equation is

y – 0 = m (x – 3) ....(i)

Since the angle between a diagonal and a side of a square is $\frac{\pi}{4}$.

Therefore angle between

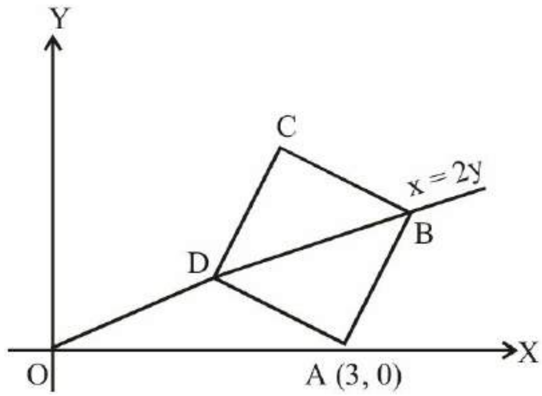
x = 2y & y – 0 = $m(x-3)$ is also $\frac{\pi}{4}$

Consequently, $\tan\frac{\pi}{4} = \pm\frac{m-\frac{1}{2}}{1+\frac{m}{2}} \Rightarrow m = 3, -\frac{1}{3}$

∴ from (i), we get the required equations

$y = 3(x-3) \Rightarrow y - 3x + 9 = 0$ or

$y = -\frac{1}{3}(x-3) \Rightarrow 3y + x - 3 = 0$



16. **(a)** The line passing through the intersection of lines $ax + 2by = 3b = 0$ and $bx - 2ay - 3a = 0$ is

$ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$

$(a + b\lambda)\, x + (2b - 2a\lambda)y + 3b - 3\lambda a = 0$

As this line is parallel to x-axis.

$a + b\lambda = 0 \qquad \lambda = -a/b$

$ax + 2by + 3b - \frac{a}{b}(bx - 2ay - 3a) = 0$

$ax + 2by + 3b - ax + \frac{2a^2}{b}y + \frac{3a^2}{b} = 0$

$y\left(2b + \frac{2a^2}{b}\right) + 3b + \frac{3a^2}{b} = 0$

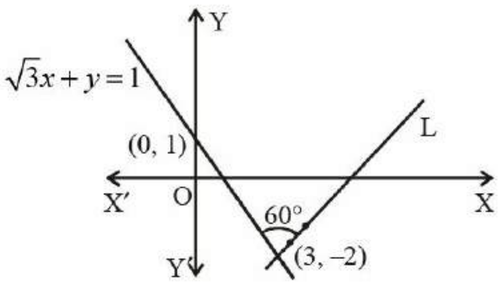
$y\left(\frac{2b^2 + 2a^2}{b}\right) = -\left(\frac{3b^2 + 3a^2}{b}\right)$

$y = \frac{-3(a^2 + b^2)}{2(b^2 + a^2)} = \frac{-3}{2}$

So it is 3/2 units below x-axis.

17. **(b)** Let the slope of line L be m.

Then $\left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$



$\Rightarrow m + \sqrt{3} = \pm(\sqrt{3} - 3m)$

$\Rightarrow 4m = 0$ or $2m = 2\sqrt{3}$

$\Rightarrow m = 0$ or $m = \sqrt{3}$

L intersects x-axis, $\therefore m = \sqrt{3}$

$\therefore$ Equation of L is $y + 2 = \sqrt{3}(x - 3)$

or $\sqrt{3}x - y - (2 + 3\sqrt{3}) = 0$

18. **(c)** Equation of line passing through (a, 0) is $y = m(x - a)$

$\Rightarrow mx - y - ma = 0$ ....(i)

Its distance from the point (2a, 2a) is

$$\left|\frac{2am - 2a - ma}{\sqrt{m^2+1}}\right| = a \quad \text{(given)}$$

$$\Rightarrow (m-2)^2 = (m^2+1) \quad \Rightarrow m^2 - 4m + 4 = m^2 + 1$$

$$\Rightarrow 0m^2 - 4m + 3 = 0 \Rightarrow m = \frac{3}{4}, \infty$$

The required equation of lines are, from (i)

$3x - 4y - 3a = 0$ and $x - a = 0$.

19. **(a)** Let (1,3) and (5,1) represent vertices A and C. The middle point G(3,2) must lie on the diagonal BD, whose equation is $y = 2x + c$

$\therefore 2 = 2.3 + c \quad \Rightarrow c = -4$

$\therefore$ equation of BD is $y = 2x - 4$

Also GA = GB = GC = GD $= \frac{1}{2}AC = \sqrt{5}$

We have to find two points along BD at distances $\pm\sqrt{5}$ from G. For this we convert equation of BD into distance form.

Slope of line BD = $\tan\theta = 2$

$\therefore\ \sin\theta = \frac{2}{\sqrt{5}}$ and $\cos\theta = \frac{1}{\sqrt{5}}$

$\therefore$ Distance form of the line BD through G is

$$\frac{x-3}{\frac{1}{\sqrt{5}}} = \frac{y-2}{\frac{2}{\sqrt{5}}} = r$$

Put $r = \pm\sqrt{5}$ to get the vertices of B and D as (4,4) and (2,0)

20. (d) As (sinθ, cosθ) and (3, 2) lie on the same side of $x + y - 1 = 0$, they should be of same sign.

$\sin\theta + \cos\theta - 1 > 0$ as $3 + 2 - 1 > 0$

$$\Rightarrow \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) > 1$$

$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}} \Rightarrow 0 < \theta < \frac{\pi}{4}$$

21. (0.2) The slope of the line $3x - y = 7$ is $\tan\theta = 3$.

or $\frac{P}{B} = \frac{3}{1} \Rightarrow H = \sqrt{9+1} = \sqrt{10}$

$$\therefore\ \sin\theta = \frac{3}{\sqrt{10}}, \cos\theta = \frac{1}{\sqrt{10}}$$

The eqn of line passing through (1, 2) and parallel to $y = 3x - 7$ is

$$\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} \qquad \text{...(i)}$$

Let r be the required distance.

$\therefore$ $(1 + r\cos\theta, 2 + r\sin\theta)$ lies on $x + y + 5 = 0$

$\Rightarrow 1 + r\cos\theta + 2 + r\sin\theta + 5 = 0$

$$\Rightarrow 1 + r\frac{1}{\sqrt{10}} + 2 + r\frac{3}{\sqrt{10}} + 5 = 0 \Rightarrow r = 2\sqrt{10}$$

22. (5) $p_1^2 = 4a^2\cos^2 4\theta$

$$p_2^2 = \frac{16a^2\cos^2 2\theta}{\sec^2\theta + \mathrm{cosec}^2\theta} = 16a^2\cos^2 2\theta\cos^2\theta\sin^2\theta$$

$= a^2 \sin^2 4\theta$

$\therefore p_1^2 + 4p_2^2 = 4a^2$

$\Rightarrow m + n = 1 + 4 = 5$

23. **(3)** Let line be $y - 3 = m(x - 2)$

y intercept is $(3 - 2m)$, x intercept is $\left(2 - \frac{3}{m}\right)$

Area = 12

$$12 = \frac{1}{2}\left|2 - \frac{3}{m}\right| |3 - 2m|$$

$$\Rightarrow 12 - \frac{9}{m} - 4m = +24$$

$4m^2 + 12m + 9 = 0 \Rightarrow m = -3/2$

or $12 - \frac{9}{m} - 4m = -24 \Rightarrow 4m^2 - 36m + 9 = 0$; $D > 0$

$\Rightarrow$ There are two values of m. Hence total 3 values of m.

24. **(1)** The length of perpendicular from P (2, –3) on the given family of lines

$$= \frac{a(4 - 3 + 4) + b(2 + 6 - 3)}{\sqrt{(2a + b)^2 + (a - 2b)^2}} = \pm\sqrt{10} \text{ (given)}$$

$$\Rightarrow 5a + 5b = \pm\sqrt{10(5a^2 + 5b^2)}$$

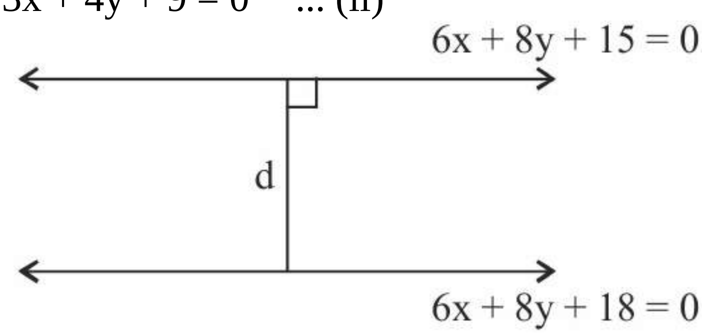
$$\Rightarrow 25(a + b)^2 = 50(a^2 + b^2)$$

$$\Rightarrow 25(a - b)^2 = 0 \Rightarrow a = b$$

For which we get only line $3x - y + 1 = 0$

25. **(0.3)** $6x + 8y + 15 = 0$ ... (i)

and $3x + 4y + 9 = 0$ ... (ii)



Multiply equation (ii) by 2, we get

6x + 8y + 18 = 0

Distance between the straight lines

$$\frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}} = \frac{18 - 15}{\sqrt{(6)^2 + (8)^2}} = \frac{3}{10} \text{ unit}$$