FLUID
MECHANICS

Fluid property.

Ideal Fluid :-
- frictionless (noviscosity मतलब nonviscous)
- Incompressible
- no surface tension
- Bulk modulus → ∞ (∵ compressibility → 0)
- no such fluid exist practically.

note:- However fluid like air & water have very low value of viscosity and can be treated as ideal fluids for all practical purposes.

Viscosity / Dynamic viscosity (μ) :-

- It is measure of resistance of fluid to deformation
- It is due to internal frictional forces that develop between different layers of fluid when they are forced to move relative to each other.
- SI unit = [N-sec/m²] or [kg/m-sec]
- CGS unit = [Dyne-sec/cm²]
- 1 Poise = $\frac{1}{10}$ N-sec/m²
- unit (dimension unit) in MLT = $[ML^{-1}T^{-1}]$
- Viscosity
  - in liquid {(due to cohesion)}
  - in gas {due to molecular momentum transfer}
- Temp. effect: Viscosity
  - in liquid: Temp↑ μ↓
  - in gas: Temp↑ μ↑

- Pressure effect: viscosity
  - in liq: Practally indepent of pressure except at extremely high Pressure.
  - in gas: generally independent of pressure

note:- viscosity of water at 1°C ⇒ 1 centipoise $= \frac{1}{100} \times \frac{1}{10}$ NS/m2

Kinematic viscosity (ν) ⇒ $\frac{\text{dynamic viscosity } (\mu)}{\text{density } (\rho)}$

unit
- SI - m²/sec
- CGS. stokes or cm²/sec

1 stoke = 10^{-4} m²/sec

dimension = $[L^2T^{-1}]$

effect of Pressure :- Kinematic viscosity decreases (as density ∝ Pressure)

Bulk modulus (k) $= \frac{dp}{\left(\frac{-dV}{V}\right)} = \frac{dp}{\left(\frac{d\rho}{\rho}\right)}$

unit ⇒ N/m2

compressibility $= \frac{1}{\text{Bulk modulus}}$

↳ Ideal fluid ($k = \infty$) *

Isothermal Bulk modulus (K_T) = Pressure (P)

Adiabatic bulk modulus (K_a) = γP ↓ adiabatic Index

$\gamma = \frac{C_p}{C_V} = \frac{\text{Specific heat at constant pressure}}{\text{specific heat at constant volume}}$

note:- for Ideal gas $P = \rho RT$

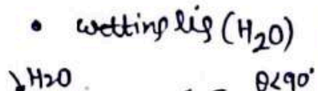
$P \propto \rho$ & $\rho \propto 1/T$

density depend on both temp. & Pressure.

Surface tension :- occures at interface of 2 liquid. (→ due to unbalanced cohesive force mainly.)

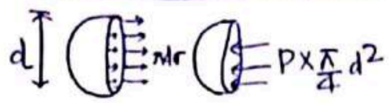
↳ elastic tendency of fluid surface which makes it aquire the least surface area possible

- magnitude of surface tension ⇒ force acting across imaginary short and straight elemental line divide by length of line.
- T↑ surface tension↓
- effect of Pressure on surface tension → Neglisible
- Phenomenon of surface tension arises due to 2 kind of intermolecular forces.
  ① cohesion force {attractive force b/w same kind of molecules}
  ⑪ Adhesive force {____ different ____}
- wetting liq (H_2O) | nonwetting liquid (Hg)



adhesive force > cohesive force (glass+water) (water+water) | cohesive force > Adhesive force (Hg-Hg) (Hg-glass)

① Pressure inside drop of liq :-



$$\boxed{P = \frac{4\sigma}{d}}$$

2) Pressure inside Bubble
(Soap bubble in air)
(There are two Interface b/w soap film and air) 1 inside the bubble & other outside. :

$$\boxed{P = \frac{8\sigma}{d}}$$

3) Pressure inside jet

$P(dl) = \sigma(2l)$

$$\boxed{P = 2\sigma/d}$$

note:- water air Interface at 20°C
surface tension = 0·0736 N/m

Imp.

Capillarity :- capillarity effect is a consequence of surface tension (cohesion) & adhesion.

- defined as rise/fall of liquid in a small-diameter tube inserted into liquid.

$$\boxed{\text{Capillary rise in tube } (h) = \frac{4T\cos\theta}{\rho g d}}$$

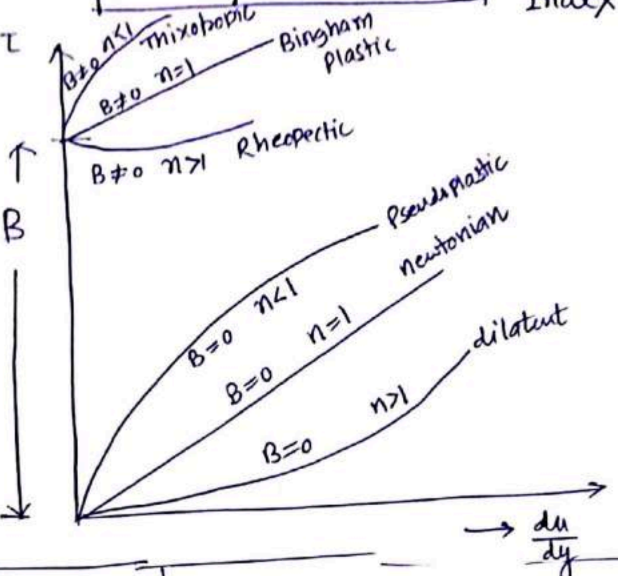
$\theta \Rightarrow$ angle of contact b/w liq. & material

$\theta = 0 \longrightarrow$ water + glass
$\theta = 128° \longrightarrow$ Hg + glass

General Relationship

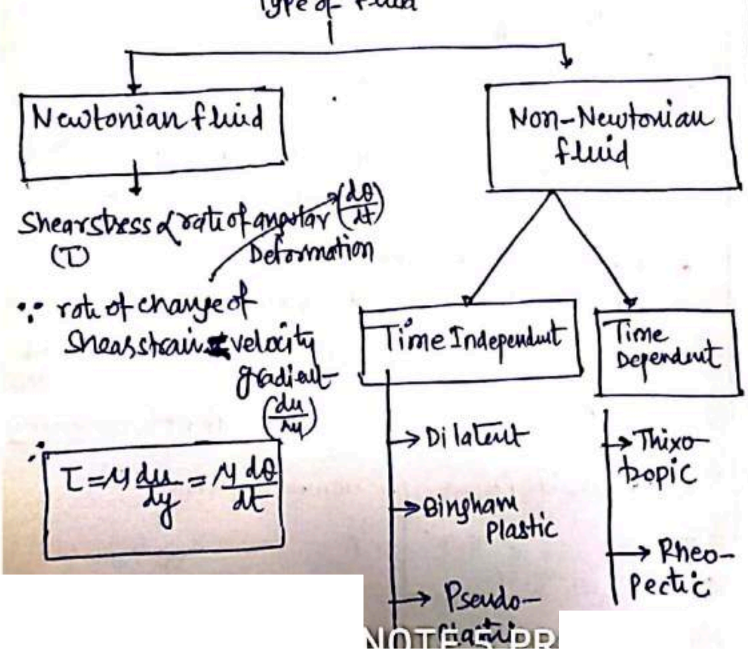
$$\boxed{\tau = A\left(\frac{du}{dy}\right)^n + B}$$

A → consistency Index
n → flow Behaviour Index



Fluid	Examples
Dilatent	Sugar solution, non colloidal solⁿ Butter, quick sand
Pseudoplastic	Blood, Paints,
Bingham plastic	toothpaste, sewage sludge, drilling mud
Rheopectic	Gypsum paste, Bentonite solⁿ lubricants
Thixotropic	Printer Ink, enamels

Type of fluid



Shear stress (τ) ∝ rate of angular Deformation $\left(\frac{d\theta}{dt}\right)$

∵ rate of change of shear strain = velocity gradient $\left(\frac{du}{dy}\right)$

$$\boxed{\tau = \mu\frac{du}{dy} = \mu\frac{d\theta}{dt}}$$

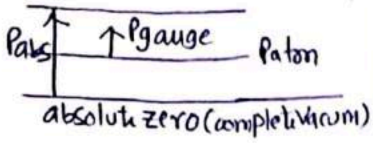
31/3/20

→ 760mm of Mercury

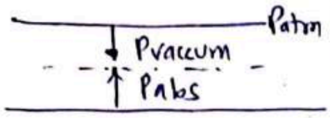
1 atm = 101.3 KPa = 14.696 psi = 10.3 m head of water

1 bar = 10^5 Pa = 10^5 N/m^2

atm. pressure → measure by <u>Barometer</u>.



$\boxed{P_{abs} = P_{gauge} + P_{atm}}$.



$\boxed{P_{abs} = P_{atm} - P_{vaccum}}$

<u>note:-</u> Pabsolute → actual pressure at a given position
- → measured wrt absolute zero / complete vacuum.
- → measurement by <u>Aneroid Barometer</u>.

Pgauge → measured wrt <u>local atm pressure as datum</u>
- → measurement by
  - manometer
  - Bourden gauge

measurement of pressure in fluid
- manometer
  - <u>Principle</u> → Balance column of fluid by same or other column of fluid.
    - Simple manometer (measure pressure at a point)
    - differential manometer (measure pressure difference b/w 2 points)
- mechanical gauge
  - Ex. Bourden gauge

- peizometer → for small (+)ve pressure measurement

1 torr = 1 mm of Hg   1 atm = 760 mm of Hg

- <u>Micromanometer</u> :- when higher precision / High sensitivity / measurement of small pressure difference rep. use it.

<u>Pascal law</u> → The pressure at a point <u>in static fluid</u> is same in all directions.

when fluid is at Rest, It exerts normal force on the surface of contact.

Pressure → Scalar quantity { It has magnitude But does not have definite direction }

<u>note:-</u> The atm. pressure with rise in altitude decreases first slow then steeply.

<u>note:-</u> Hg (mercury) used in manometer Reasons →
- (I) High density
- (II) <u>low vapour pressure</u> (so It does not evaporate easily)
- (III) freezing point is much lower than that of water.

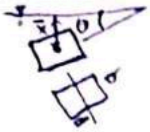
Hydrostatic force on submerged Body :-

$$\boxed{\begin{aligned} F &= \omega A\bar{x} \\ F &= \rho g A\bar{x} \\ F &= \gamma A\bar{x} \end{aligned}}$$

centre of Pressure ($h > \bar{x}$)

$$\boxed{\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}}\sin^2\theta}$$

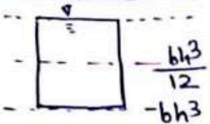
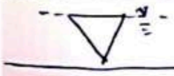
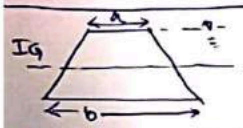
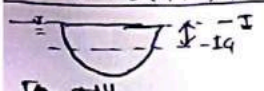
($I_G \rightarrow$ MoI about centroidal axis Parallel to free axis)



जैसे-2 depth of Immersion बढ़ेगी $h, \bar{x}$ में अन्तर कम होता ही चला जायेगा

θ = 90 (vertical position) $\bar{h} = \bar{x} + I_G/A\bar{x}$

θ = 0 (horizontal ") $\bar{h} = \bar{x}$

 $\frac{bh^3}{12}$ $\frac{-bh^3}{3}$	$\bar{x} = \frac{h}{2}$	$\bar{h} = \frac{2h}{3}$
 $\frac{bh^3}{36}$ $bh^3/12$	$\bar{x} = \frac{2h}{3}$	$\bar{h} = \frac{3h}{4}$
	$\bar{x} = h/3$	$\bar{h} = h/2$
 $\frac{\pi d^4}{64}$	$\bar{x} = d/2$	$\bar{h} = 5d/8$
 $I_G = \left(\frac{a^2+4ab+b^2}{36(a+b)}\right)h^3$	$\bar{x} = \left(\frac{a+2b}{a+b}\right)\frac{h}{3}$	$\bar{h} = \left(\frac{a+3b}{a+2b}\right)\frac{h}{2}$
 $I = \frac{\pi d^4}{2 \times 64}$ $I_G = I + \left(\frac{\pi d^2}{8}\right) \times \left(\frac{4r}{3\pi}\right)^2$	$\bar{x} = \frac{4r}{3\pi}$	$\bar{h} = \frac{6\pi r}{32}$

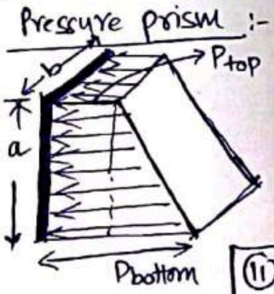
Pressure prism concept → valid only for plane surface.

∵ In plane surface → direction of force is same at each point

↓ ↓ ↓ ↓

while in curved surface It is changing ↓↓↓↓

Pressure prism :-



(I) volume of Pressure prism

⇒ Net force on plate

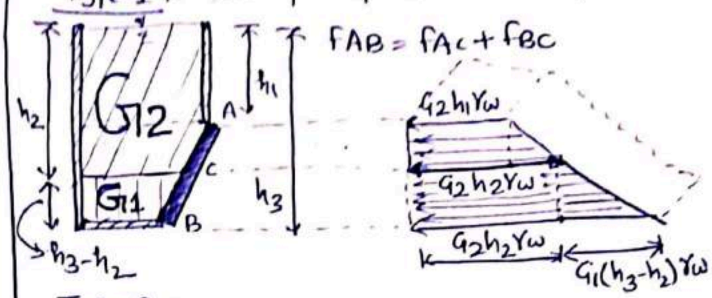
⇒ $\frac{1}{2}a(P_T + P_b) \times b$

(II) Point of application of resultant force

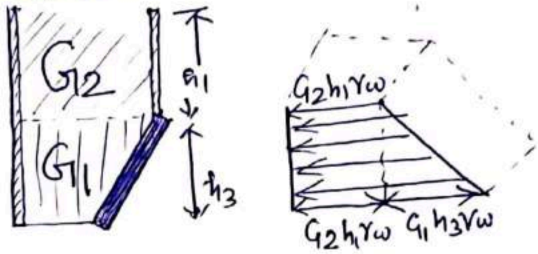
⇒ CG of Pressure prism Projected on plate.

Type-1 :- 2 liquid of different density (G_1, G_2)

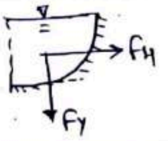
$F_{AB} = F_{AC} + F_{BC}$



Type-2 :-



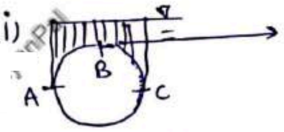
Hydrostatic force on curved surface :- $F_{net} = \sqrt{F_H^2 + F_V^2}$



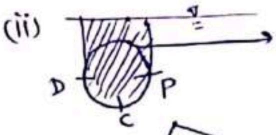
F_H ⇒ Project the surface on vertical Plane

F_H = Projected area × pressure at its ocon centre of area

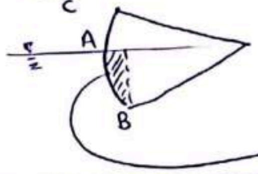
F_V ⇒ wt. of liquid block lying above curved surface up to free surface

(i) 

→ wt of shaded portion = $F_{V_{ABC}}$

(ii) 

→ wt of shaded portion = $F_{V_{DCP}}$

(iii) 

$F_{V_{AB}}$ = wt of shaded

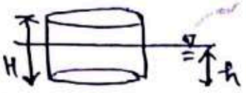
Buoyant force → $\boxed{\text{net upward force} \uparrow}$

→ ⇒ wt of liq displaced (Archimedes principle)

→ resultant force exert by static fluid on submerged/floating body

Centre of buoyance → The C.G of displaced fluid

Floatation: $\boxed{\underset{\text{(downward force)}}{\text{if wt of body} \downarrow} = \text{Buoyant force} \uparrow}$



$$(G\gamma_w)AH = (\gamma_w A h)$$

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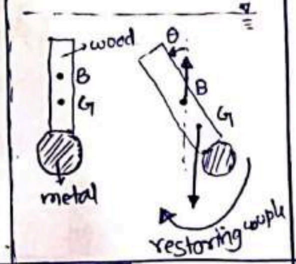
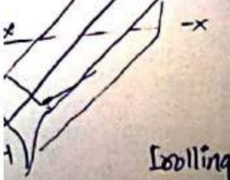
v.v.imp Rotational stability check

	completely submerged Body.	floating Body
stable eqb	$\boxed{G < B}$ G नीचे B ऊपर 	$\boxed{G < M}$ मतलब $\boxed{GM > 0}$ $GM = BM - BG$ $GM = \frac{I_{min}}{\forall} - BG$ $GM \uparrow \Rightarrow$ stability ↑
unstable eqb	$\boxed{G > B}$	$\boxed{G > m}$ $\boxed{Gm < 0}$
neutral eqb	$\boxed{G = B}$	$\boxed{Gm = 0} \Rightarrow (BM = BG)$

Time period of oscillation $\boxed{T = 2\pi\sqrt{\frac{k^2}{g \cdot GM}}}$ k → least radius of gyration

Cylinder stable condition $\boxed{\frac{L}{D} < \frac{1}{\sqrt{8S(1-S)}}}$ *

Cone stable condition $\boxed{\tan\alpha > \sqrt{\frac{1 - RD^{1/3}}{RD^{1/3}}}}$



rotation about longitudinal axis YY ⇒ rolling

rotation about transverse axis xx ⇒ pitching

$I_{rolling} < I_{pitching}$

∴ BM rolling < pitching

∴ GM rolling < GM pitching

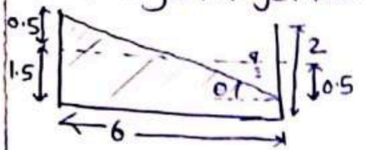
hence most critical condition of stability using GM rolling

अगर rolling मे safe तो pitching मे भी safe

Question →

a open container $L=6$ $B=2.5$ $H=2m$ having water depth $=1.5m$ find horizontal acceleration ?

① Spilling on verge then horizontal acceleration a_x?



$\forall spill = 0$

$\tan\theta = \frac{a_x}{g} = \frac{0.5+0.5}{6}$

$\boxed{a_x = g/6} = 1.635 m/s^2$

⇓ in above problem net force acting on wall ?

(M₁)

$\overleftarrow{f_2} = wA\bar{x}$
$= 9810 \times (\underset{h_2}{1} \times \underset{B}{2.5}) \times (\frac{1}{2})$

$\overrightarrow{f_1} = wA\bar{x}$
$= 9810 \times (\underset{h_1}{2} \times \underset{B}{2.5}) \times (\frac{2}{2})$

$\boxed{f_{net} = f_1 - f_2 = 36787.5 N}$

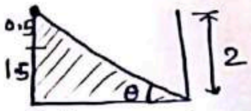
$f_1 =$

(M₂) $f_{net} = m \times a_x = \left\{ \underbrace{\left[\frac{1}{2}(2+1) \times 6\right] \times 2.5}_{volume} \times \underset{\rho}{1000} \right\} \times \underset{a_x}{1.635}$

$\boxed{f_{net} = 36787.5 N}$

② front Bottom corner just expose $a_x = ?$ $\forall spill = ?$

(0·5 नीचे आनेपर spilling शुरु हो जायेगी)

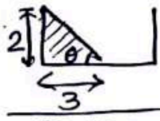


$\therefore \tan\theta = \frac{a_x}{g} = \frac{0.5+1.5}{g}$ $\boxed{a_x = g/3}$

$\forall spill = \forall original \left[6 \times 2.5 \times 1.5\right] - \forall exist \left[\frac{2 \times 6}{2} \times 2.5\right]$

$\forall spill = 7.5 m3$

③ Bottom of tank exposed to mid point $a_x = ?$ $\forall spill = ?$



$\tan\theta = \frac{a_x}{g} = 2/3$ $\boxed{a_x = 2/3 g}$ $\boxed{\forall spill = 15 m^3}$

④ if $a_x = g/5$ {or any value $a_x > g/6$} then $\forall spill = ?$

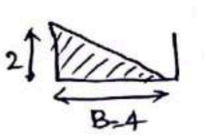
$\therefore \underset{\tan\theta}{\frac{a_x}{g}} = \frac{x}{6}$ $\therefore x = 1.20$

$\boxed{\therefore \forall spill = 4 m^3}$

⑤ if $a_x = g/2$ {$a_x > g/6$ case} then $\forall spill$?

$\tan\theta = \frac{a_x}{g} = x/6$ $x = 3$ which is not possible

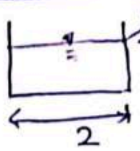
hence $\frac{a_x}{g} = \frac{g/2}{g} = \frac{x_{max}\ 2}{B}$ $\boxed{\therefore B = 4}$



$\boxed{\forall spill = 12.5 m^3}$

Conclusion :- $a_x =$	$g/6$	$g/5$	$g/3$	$g/2$
Spilled volume $\forall spill$	0 (verge)	4	7.5	12.5

Question :- lift type problems : ↑↓ {vertical motion}



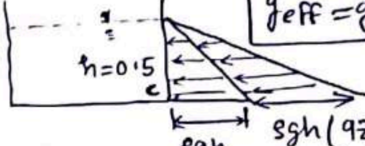
Case-1 :- $a_z = 2.45 m/s^2$ ↑

Case-2 $a_z = 2.45 m/s^2$ ↓

Case-3 $a_z = 0$ stationary

case-1 :-

$a_z = 2.45$↑ upward constant acceleration.



$\boxed{g_{eff} = g + a_z}$ *

Pressure Intensity at bottom (C) $= \rho gh + \rho gh(a_z/g) = 6130 N/m^2$

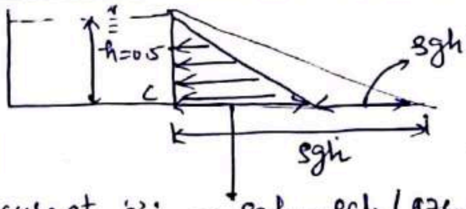
force exert on wall ⇒ volume of pressure diagram

$= \left[\frac{1}{2} \times 0.5 \times 6130\right] \times 2$ N

case-2 :-

$a_z = 2.45$↓

$\boxed{g_{eff} = g - a_z}$ *

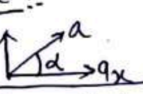


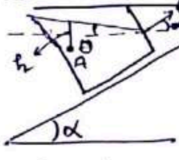
Pressure Intensity at 'c' $= \rho gh - \rho gh(a_z/g)$

case-3 :- $a_z = 0$ stationary

Types of Problem :- Inclined plane :-

① acceleration up the slope :-





$\boxed{a_x = a\cos\alpha \quad a_z = a\sin\alpha}$

$\tan\theta = \frac{a_x}{g + a_z}$

θ → from horizontal; $g + a_z$ → upward case $g_{eff} = g + a_z$

$P_a = \rho g_{eff} h$
$= \rho(g + \underset{a\sin\alpha}{a_z})h$

(II) acceleration down the slope :-

$\boxed{P_A = \rho(g - \overset{a\sin\alpha}{a_z})h}$

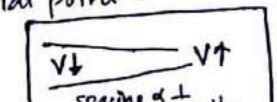
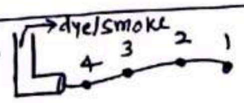
$\tan\theta = \frac{a_x}{g - \overset{a\sin\alpha}{a_z}}$

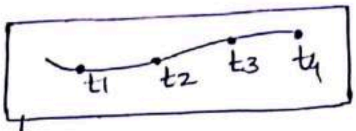
$g_{eff} = g - a_z$

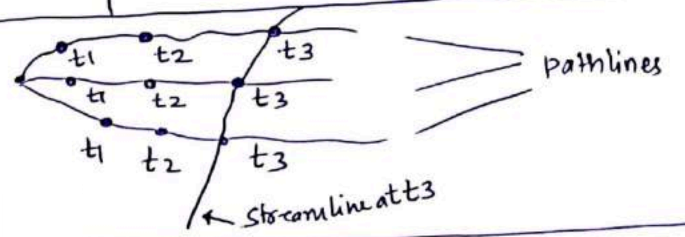
	Fluid Description by motion
lagrangian method	(लगे रहो) → related to single fluid Particle and its velocity
Euler method	take finite volume called controll volume

Type of fluids	Description
Steady flow	if flow and fluid properties at any given location does not change with time flow called steady flow otherwise unsteady. $\frac{\partial P}{\partial t}=0 \quad \frac{\partial V}{\partial t}=0 \quad \frac{\partial \rho}{\partial t}=0$ } for steady flow
uniform flow	when velocity does not change with location over a specified region, at a particular instant of time flow uniform otherwise nonuniform. $\frac{\partial V}{\partial s}=0 \rightarrow$ uniform flow
Rotational flow 	when fluid particle rotate about their mass centre during movement called rotational flow otherwise irrotational flow • Rotation of fluid particles ⟶ cause by viscosity. • forced vortex flow ⟶ rotational flow • free vortex flow ⟶ Irrotational flow.
Compressible flow	density of fluid changes from point to point मतलब $\left(\frac{\partial \rho}{\partial p} \neq 0\right)$ if $\frac{\partial \rho}{\partial p}=0 \Rightarrow$ Incompressible flow

Types of flow lines → to visualize the flow, these are drawn in flow space.

Stream-lines	• tangent at any point gives direction of velocity vector at that point. $\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$  V↓ V↑ spacing ∝ $\frac{1}{\text{velocity}}$ • no velocity component perpendicular (⊥) to Streamline (velocity vector)
streaklines	•  dye/smoke 4 3 2 1 • path followed by dye/smoke Ex. people coming from cinema hall.

Pathlines	• Based on lagrangian approach. • Curve traced by single fluid particle during its motion • Involve finite time period  t1 t2 t3 t4 ↳ Position of particle at different times, when all these positions are Joined ⟶ we get pathline.
stream tube	• Bundle of stream line. • at any instant mass flow rate constant  • flow within stream tube must remain there and Can not cross the boundry of stream tube



note:- Euler's eqn $\xrightarrow{\text{based on}}$ "momentum"
Bernoullis eqn ⟶ "Energy"

continuity eqn → Based on mass conservation.

$$\boxed{\frac{\partial}{\partial x}(\rho u) + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\rho z} + \frac{\partial \rho}{\partial t} = 0} \rightarrow 3D$$

(cartesian cordinate form)

if steady compressible flow $\left(\frac{\partial \rho}{\partial t}=0\right)$ $\rho \rightarrow$ const

①D

$$\boxed{\begin{gathered}\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \\ \rho_1 = \rho_2\end{gathered}}$$

②D

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

③D

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

polar 3D

$$\boxed{\frac{1}{r}\left[\frac{\partial(\rho r V_r)}{\partial r}\right] + \frac{\partial(\rho V_\theta)}{r\,\partial\theta} + \frac{\partial(\rho V_z)}{\partial z} + \frac{\partial \rho}{\partial t} = 0}$$

②D
Steady + Incompressible

$$\boxed{\frac{1}{r}\left[\frac{\partial(r V_r)}{\partial r} + \frac{\partial V_\theta}{\partial \theta}\right] = 0}$$

Imp

note:- $\boxed{\vec{V} = u(x,y,z,t)\hat{i} + v(x,y,z,t)\hat{j} + w(x,y,z,t)\hat{k}}$

acceleration of fluid :-

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \qquad \vec{a} = (\vec{\nabla}\cdot\vec{V})\vec{V} + \frac{\partial \vec{V}}{\partial t}$$

$$a_T = \underbrace{v\frac{dv}{ds}}_{\text{convective or adjective acc.}} + \underbrace{\frac{dv}{dt}}_{\text{local or temporal acceleration}}$$

$$\therefore\ a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \quad \left\{ \text{same } a_y, a_z \right.$$

① for steady flow $\frac{\partial v}{\partial t} = 0$ (local acc. $=0$)

$\therefore$ total acc(a_T) = convective acc.

② for uniform flow $\frac{\partial v}{\partial s} = 0$ (convective acc $=0$)

$\therefore a_T$ = local acc. or temporal acc.

③ for Steady + uniform $a_t = 0$

normal acceleration → due to change in direction of fluid moving in cusvepath.
(a_n)

$$a_n = \boxed{V_s}\underbrace{\overset{V_s/r}{\cancel{\frac{\partial V_n}{\partial s}}}}_{\text{convective normal acc.}} + \underbrace{\frac{\partial V_n}{\partial t}}_{\text{temporal normal acc.}}$$

Basically

$$a_n = \frac{dV_n}{dt} = \frac{\partial V_n}{\partial s}\times\overset{V_s}{\cancel{\frac{ds}{dt}}} + \frac{\partial V_n}{\partial n}\times\overset{V_n}{\cancel{\frac{dn}{dt}}} + \frac{\partial V_n}{\partial t}\frac{\times dt}{dt}$$

for streamline
$V_n = 0$ (no velocity vector perpendicular to flow)

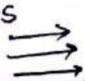
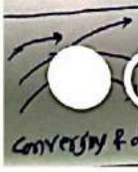
tangential acceleration (a_s) due to change in magnitude of velocity (if spacing b/w streamlines changes then tangential acceleration exist

$$a_s = \boxed{V_s}\frac{\partial V_s}{\partial s} + \frac{\partial V_s}{\partial t}$$

$$\therefore a_s = \frac{dV_s}{dt} = \frac{\partial V_s}{\partial s}\times\overset{V_s}{\cancel{\frac{ds}{dt}}} + \frac{\partial V_s}{\partial n}\times\overset{V_n}{\cancel{\frac{dn}{dt}}} + \frac{\partial V_s}{\partial t}\frac{\times dt}{dt}$$

for streamline $V_n = 0$

for steady flow $a = \sqrt{a_n^2 + a_s^2} = \sqrt{\left(\frac{V^2}{r}\right)^2 + \left(\frac{Vdv}{ds}\right)^2}$

cases 	$a_n=0$ (no cusvepath)	∵ magnitude → same $a_s=0$ (spacing same)
	$a_n=0$ (no cusvepath)	$a_s \neq 0$ (∵ spacing change)
	$a_n \neq 0$ (cusvepath)	$a_s = 0$ (spacing same)
 conversing & diversing streamlines.	$a_n \neq 0$ (curve path)	$a_s \neq 0$ (spacing changes)

Std. result :-

① Steady flow ⇒ discharge constant $\left(\frac{dQ}{dt} = 0 \ \& \ \frac{dv}{dt} = 0\right)$

$\therefore a = a_{convective}\left(v\frac{dv}{dx}\right)$

$$\boxed{a = \frac{32Q^2(d_2 - d_1)}{L\pi^2 d_x^5}}$$

(ii) if discharge changing with time { 20 − 40 lt in 30 min)

if $a_{15} = ?$ first Q_{15} पता करें, $\frac{dv}{dt} = \frac{d}{dt}\frac{Q}{\left(\frac{\pi}{4}d_x^2\right)}$

solve.

angular velocity (ω) ⇒ avg. of rotation rate of 2 initially ⊥ lines that intersect at that point.

$$\vec{\omega} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & \omega \end{vmatrix} \qquad \omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

flow Irrotational then $\omega = 0$ & $\Omega = 0$

vorticity $\Omega = 2\times\vec{\omega} \Rightarrow \vec{\nabla}\times\vec{V} \Rightarrow$ curl of velocity vector

$$\Omega = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

Imp

note :- Rapidly converging / accelerating flow ⇒ irrotational
large viscous flow ⇒ rotational.

circulation (Γ) = vorticity × area

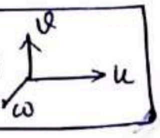
avg. deformation ⇒ $\frac{1}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$

Velocity potential / potential f^n (ϕ) :- $\boxed{\text{valid for 3D}}$ **

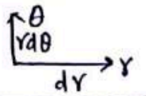
- scalar function of space & time
  (x, y, z) (t)

$$\phi = f^n(x, y, z, t)$$

$$\boxed{\frac{\partial\phi}{\partial x} = -u \quad \frac{\partial\phi}{\partial y} = -v \quad \frac{\partial\phi}{\partial z} = -w}$$



Polar (r, θ, z) :-



$$\boxed{\frac{\partial\phi}{\partial r} = -V_r \quad \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -V_\theta \quad \frac{\partial\phi}{\partial z} = -V_z}$$

note:- for steady Incompressible flow $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

Put $\frac{\partial\phi}{\partial x} = -u$ .... then

(laplaseeqn) $\longrightarrow$ $\boxed{\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0}$ ($\nabla^2\phi = 0$)

मतलब • v.v.imp — $\boxed{\text{if flow possible then velocity potential } (\phi) \text{ will satisfy laplaseeqn. \{ 1st it should satisfy continuity eqn) }}$

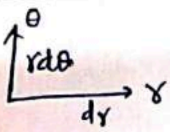
Imp point :- ① velocity potential f^n exist only for Irrotation flow & Ideal flow.
($w_z = 0$)

⑪ flow always occures in the decreasing potential

⑪ velocity potential concept used in Integration of euler eqn to get Bernauli eqn.

Streamline function (ψ) → $\boxed{\text{for 2D flow}}$ *

$$\boxed{\frac{\partial\psi}{\partial x} = v \quad \frac{\partial\psi}{\partial y} = -u}$$



$$\boxed{\frac{\partial\psi}{\partial r} = V_\theta \quad \frac{\partial\psi}{r\partial\theta} = -V_r}$$

- if ψ exist then it satisfy continuity eqn then flow can be < rotational / irrotational
- if stream fn satisfy laplase eqn then It is case of irrotational flow. ($\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$
- dischasge per unit length $= |\psi$ [illegible] $\psi_1|$

---

for Irrotational & Incompressible flow:-

$$\boxed{\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}} \quad \boxed{\frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x}} \Big] \rightarrow \text{Cauchy reiman eqn}$$

Flownet → (for 2D irrotational flow) only.

① for streamline $d\psi = 0$

$$\underset{v}{\cancel{\frac{\partial\psi}{\partial x}}}dx + \overset{-u}{\cancel{\frac{\partial\psi}{\partial y}}}dy = 0$$

$\therefore \frac{dy}{dx}$ = slope of streamline (m_1) $= \frac{v}{u}$

⑪ for equipotential line $d\phi = 0$

$$\underset{-u}{\cancel{\frac{\partial\phi}{\partial x}}}dx + \underset{-v}{\cancel{\frac{\partial\phi}{\partial y}}}dy = 0$$

$$\therefore \frac{dy}{dx} = m_2 = -u/v$$

$\because m_1 * m_2 = -1$
hence both are orthagonal.

---

methods to draw flownet :-

1- Analytical method (by φ, ψ , for simple & Ideal boundry condition)

2- Hydraulic model or seepage model
(streamlines can be traced by injecting a dye in this)

3- Graphical method (draw streamlines, equipotential lines & cut ⊥)

4- electrical analogy method
(flow of fluid & flow of electricity through a conductor are analogus)

Limitation of flownet :-

1- not apply near to boundry { ∵ viscous effect are dominant)

.2- not used to determine flow pattern past a solid boundry ond/s side due to separation of flow & eddies.

fluid flow various forces :-

Gravity force (fg), pressure force (fp), viscous force (fv), force due to turbulence (ft), force due to surface tension (fs), force due to compressibility (fc)

	force neglected	forces
Newton's Law	—	fg+fp+fv+fc+ft+fs
Reynold's Law	fc, fs	fg+fp+fv+(ft)
navier stokes imp	fc, fs, ft	fg + fp + fv (GPV)
Euler eqn of motion	fc, fs, ft, fv	fg + fp

note :- Euler eqn of motion :- → Based on momentum principle (basically newton's 2nd Law)

- flow homogeneous
- flow incompressible
- Euler's eqn for steady flow of an ideal fluid along streamline is a relation b/w the velocity pressure and density of moving fluid.

Bernauli eqn (Based on energy principle)

assumption ① steady ⑪ Incompressible ⑾ nonviscous (effect of friction → zero) (invisid) ⑽ flow along a stream line ⓥ Irrotation flow

note:- Integration of Euler's eqn of motion along a stream line under steady incompressible condition gives Bernaulli eqn.

note:- Bernauli eqn valid for Gases → if no transfer of kinetic or potential energy from the gas flow due to compression & expansion of gas. (means compressible gas should behave as an Incompressible fluid)

where Bernauli theorem is not applied →

① long narrow flow passage → ∵ friction exist

⑪ wake region d/s of an object → ∵ energy loss

⑾ diverging flow → ∵ chance of separation & wake formation

Ⓝ near solid Boundry → ∵ viscosity dominant

ⓥ Beyond M ≥ 0.3 → ∵ compressibility effect dominant.

ⓥⓘ flow section involves temp change. → ∵ T↑↓ (change) ρ↑↓ (change) ∴ not incompressible

ⓥⓘⓘ flow section involve fan like turbine, pump. impeller, such device disrupts streamlines & constant energy interaction with fluid particle.

$$\frac{P}{\rho} + gZ + \frac{V^2}{2} = \text{constant}$$

$\frac{\text{Pressure energy}}{\text{mass}}$ $\frac{\text{Potential energy}}{\text{mass}}$ $\frac{\text{K.E}}{\text{mass}}$

$$\frac{P}{\rho g} + Z + \frac{V^2}{2g} = \text{constant}$$

$\frac{\text{Pressure energy}}{\text{weight}}$ $\frac{\text{Potential energy}}{\text{weight}}$ → KE/weight

→ Bernauli eqn

$$P + \frac{1}{2}\rho V^2 + \rho g Z = \text{constant}$$

P ↓ static pressure (actual pressure in fluid)

$\frac{1}{2}\rho V^2$ ↓ dynamic pressure ↳ Pressure rise when fluid motion is brought to rest.

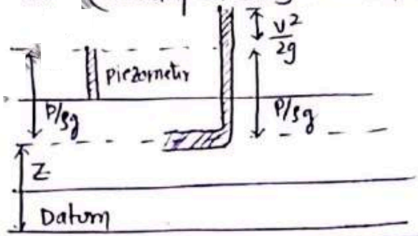
$\rho g Z$ → hydrostatic pressure

$$\underset{\text{Static pressure}}{P} + \underset{\text{dynamic pressure}}{\frac{1}{2}\rho V^2} \Rightarrow \text{Stagnation pressure}$$

$$\text{Stagnation pressure head} = \underset{\text{Pressure head}}{P/\rho g} + \underset{\text{velocity head}}{V^2/2g}$$

Pitot tube 'or' : ([redacted]) used to measure velocity at a point in a fluid flow. (basically it measures stagnation pressure)

मतलब (velocity at stagnation point)



$$\frac{P_{stag.}}{\rho g} = \frac{P}{\rho g} + \frac{V^2}{2g}$$

$$\therefore V = \sqrt{\frac{(P_{stag} - P)}{\rho g} \times 2g}$$

$$\boxed{V_{th} = \sqrt{2g(\text{stagnation head} - \text{static head}}}$$

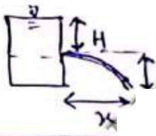
$$\boxed{V_{actual} = C_v\sqrt{2gh}}$$

C_v → 0.98 for pitot tube

note:- stagnation point → The point at which velocity reduces to zero.

Instrument	measure
Pitot tube	stagnation pressure, velocity at a point
U-tube manometer	absolute pressure at point
Venturimeter	discharge of liq flowing throgh pipe
• orifice meter • flow nozzle • Bend meter • rotatometer • elbowmeter • nozzle meter	discharge elbowmeter → principle:- when liq. moves along a pipebend its pressure increases with radius.
Current meter	velocity in open channel $aN_s + b = V$ (N_s → rps)
Hotwire anemometer	air & gas velocity
• Tensiometer • capillary tube	surface tension
• Viscometer or Rheometer	viscosity of fluid note:- piranigause → measurement of Pressure in vacum system.

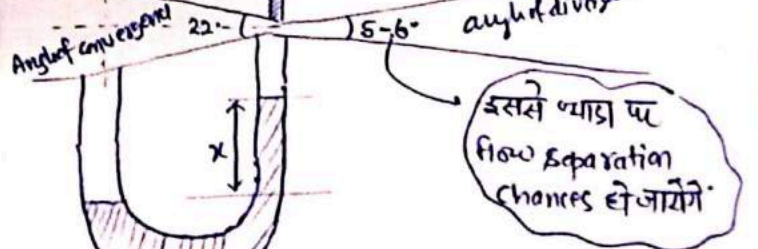
Hydraulic cofficients

cofficient of contraction (C_c)	$\frac{\text{area of jet at vena contracta}}{\text{area of orifice}}$
cofficient of velocity (C_v)	$\frac{\text{actual velocity of jet at vena contracta}}{\text{Theortical velocity}}$ $C_v = \frac{V}{\sqrt{2gH}} = \frac{x}{\sqrt{4YH}}$ 
Cofficient of discharge Cd	actual discharge / Theorticaldischarge $Cd = C_c \times C_v$

Application of Bernauli eqn :-

① venturimeter :- $\boxed{\text{used to find discharge throgh pipeline}}$

Principle :- reduction in area at throat result in increase in velocity in steady flow due to this Pressure decreases at throat. The decrease in pressure is noted and discharge found by Applying Bernauli eqn.



flow separated $\frac{\partial s}{\partial x}$ TO $\frac{du}{dy}$ LO

$$\boxed{h = x\left(\frac{S_m}{S} - 1\right) = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = \frac{v_2^2 - v_1^2}{2g}}$$

$$\boxed{Q_{actual} = \frac{Cd\, A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}}$$

$\frac{A_1}{A_2}$ → area ratio

$\boxed{Cd = \sqrt{\frac{h - h_L}{h}}}$ $Cd = 0.98$ (due to no flow separation)

advantage :- ① cheaper arrangement

Ⅱ Accuracy of venturimeter is quite good

Ⅲ less head loss ($Cd = 0.98$)

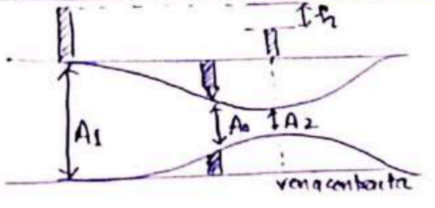
	Cd
venturimeter	0.95-0.98
orifice meter	0.60-0.65
nozzlemeter	0.96

② orificemeter :- • in this circular plate with concentric shape hole is installed in pipe such that the plate is ⊥ to the axis of pipe.

• In this case only small length of pipe is affected hence if there is space restriction orificemeter can be used in place of venturimeter.

• disadvantage → headloss due to flow separation more.

{The region where flow area is min. is called → vena contracta}



$C_c = \frac{A_2}{A_0}$

$A_2 = C_c A_0$

$$Q_{th} = \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} = \frac{C_c A_1 A_0 \sqrt{2gh}}{\sqrt{A_1^2 - C_c^2 A_0^2}}$$

$$\boxed{Q_{act} = \frac{Cd\, A_1 A_0 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}}$$

$Cd = 0.60 - 0.65$

$$\boxed{Cd = \frac{\sqrt{1 - \frac{A_0^2}{A_1^2}}}{\sqrt{\frac{1}{C_c^2} - \frac{A_0^2}{A_1^2}}}}$$

Vortex motion :

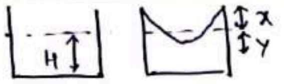
$$\boxed{dp = \left(\frac{\rho v^2}{r}\right) dr - \rho g dz}$$

freevortex motion	force vortex motion
$mvr = const.$ $V \propto 1/r$	$V = r\omega$ $V \propto r$
Bernauli valid	not valid
Ex. wash Basin flow, flow of liq. around a circular bend pipe, A whirl pool in river, centrifugal pump casing	Ex. container with liq. rotating about axis 'ω' • Impeller of centrifugal pump • Through runner of turbine

$\forall$ paraboloid = cylinder volume of same height / 2

$\forall$ cone = ———————————— /3

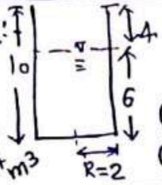
Proof (x=y)



Apply conservation of volume →

$\pi R^2 H = \pi R^2 (H+x) - \frac{\pi R^2 (x+y)}{2} \Rightarrow \boxed{x=y}$

Question :



$\forall i = \pi \times 2^2 \times 6 = 75.34\ m^3$

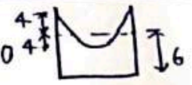
① $\omega_{max} = ?$ if $\forall spill = 0$

(ii) $\omega = ?$ so that axis depth = 0

(iii) $\omega \doteq 6.4$ rad/sec $\forall spill$?

(iv) $\omega = 8$ rad/sec $\forall spill$?

Case-1 :- ω_{max} if $\forall spill = 0$



here x = y = 4

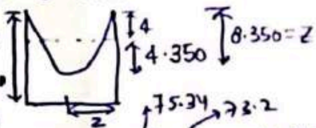
$\underset{(4+4)}{z} = \frac{\omega^2 r^2}{2g}$ $\therefore$ $\boxed{\omega = 6.2\text{ rad/s}}$

Case-2 :- axis (axial depth) = 0 ⇒ (z=10) ∴ $\omega = \sqrt{\frac{2gz}{r^2}}$

$\boxed{\omega = 7\text{ rad/sec}}$

Case-3 :- $\omega = 6.4$ ($> \omega_{max} = 6.26$ r/s)

$\therefore z = \frac{(6.4)^2 (2)^2}{2 \times 9.81} = 8.350 m$



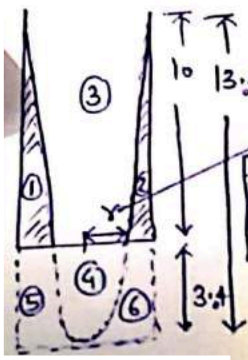
$\forall f = (\pi \times 2^2) \times 10 - \frac{\pi \times 2^2}{2} \times 8.350$

$Vf = 73.2$

$\therefore \forall spill = \overset{75.34}{\forall i} - \overset{73.2}{\forall f} = 2.138\ m^3$

case-4 :. $\omega = 8$ ($> \omega max = 6.26$) ————→ Important from conventional point of view.

$\therefore z = \frac{8^2 \times 2^2}{2 \times 9.81} = 13.04$ (> height of cylinder)



$\frac{\overset{8}{\omega^2} r^2}{2g} = z' = 3.04$ $\boxed{r = 0.965 m}$

we want (1+2) volume = ?

$$\boxed{\underbrace{\forall_{12}}_{\forall f} = \forall_{123456} - \forall_{34} - \underbrace{\forall_{56}}_{\{\forall_{456} - \forall_4\}}}$$

$\forall_{123456} = (\pi \times 2^2) \times 13.04$

$\forall_{34} = \left(\frac{\pi \times 2^2}{2}\right) \times 13.04$

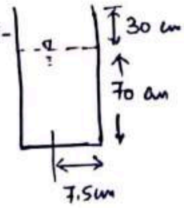
$\forall_{456} = (\pi \times 2^2) \times 3.04$

$\forall_4 = \left(\frac{\pi \times (0.965)^2}{2}\right) \times 3.04$

$\forall f = ✓$

$\forall spill = \forall i - \forall f = 27.23\ m^3$

Question :-



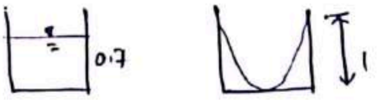
After rotation axial depth = 0

find total pressure force difference

① at side of cylinder

⑪ total Pressure force difference at bottom of cylinder.

case-1 :- at side of cylinder.



(axial depth = 0)

$fi = \omega A\bar{x}$

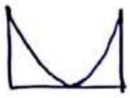
$fi = 9810(\pi \times 0.15 \times 0.70) \times \frac{0.70}{2}$ N

$Ff = 9810 \times (\pi \times 0.15 \times 1) * 1/2$ N

here $\boxed{f_{final} > f_{initial}}$

case-2 at bottom of cylinder

$fi = \underbrace{9810\left(\frac{\pi}{4} \times 0.15^2\right) \times 0.70}_{Just\ wt.} \Rightarrow$



⇒ as depth varies with radius

$\therefore Ff \Rightarrow \int_0^R P dA = \int_0^R \left[\frac{\omega^2 r^2}{2g} \times \rho g\right] \times 2\pi r\, dr$

$* \boxed{Ff = \frac{\pi \rho \omega^2 R^4}{4}}$ ✓ here $\boxed{f_{initial} > f_{final}}$

Special close container case :-

• Close cylinder (R,H) rotated about its vertical axis with 'ω' find total pressure force exerted by water.

on top of cylinder

$$\boxed{\frac{\pi \rho \omega^2 R^4}{4}}$$

on bottom of cylinder

$$\boxed{\frac{\pi \rho \omega^2 R^4}{4} + \underbrace{\rho g (\pi R^2 H)}_{wt.\ of\ water}}$$

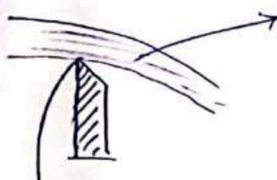
notch :- measure discharge through small channel/tank such that liq. surface in the tank/channel is below top edge of opening.

- used in lab / small size
- made of metallic plate.
- measure discharge of small stream or canal

weir :- concrete or masonry str. constructed across a river/canal over which flow occures

- used to measure discharge of rivers or big canals

Terminology :-



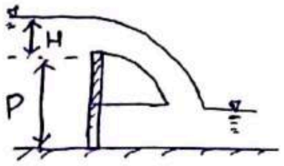
① Nappe or vein
↓
sheet of water flowing through a notch or weir

2- crest or sill
↓
bottom edge of notch or the top of weir

3- crest height :-
height above bottom of channel / tank

4- venacontracta → point in fluid stream where the diameter of stream is the last & fluid velocity is max.
(min)

① sharp crested Rectangular, suppressed weir :-



$$\boxed{Q_{actual} = \frac{2}{3} C_d L \sqrt{2g}\ H^{3/2} \quad Q_a \propto H^{3/2}}$$ → Rect.

($C_d = 0.62$)

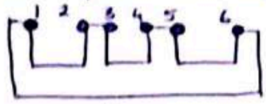
But if approach velocity consider

$$\boxed{Q_{actual} = \frac{2}{3} C_d \sqrt{2g}\ L \left[(H + h_a)^{3/2} - (h_a)^{3/2} \right]}$$

h_a → approach velocity head $= \frac{V_a^2}{2g}$

V_a → approach velocity $= Q/(P+H)L$

note:- if end contraction effect is taken



$$\boxed{L_{eff} = L - 0.1\, n H}$$

⇓

$\boxed{n = 6}$

$$\therefore Q_a = \frac{2}{3} C_d \sqrt{2g}\ L_{eff} H^{3/2}$$

② Triangular weir / v notch :-

$$\boxed{Q_a = \frac{8}{15} C_d \sqrt{2g}\ \tan\frac{\theta}{2}\ H^{5/2}} \quad \boxed{Q \propto H^{5/2}}$$ ($C_d = 0.52$)

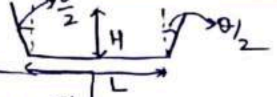
advantage of vnotch over rectangular notch →

* for small discharge, vnotch is more accurate

* in Triangular notch head is large even for small discharge

* cd is fairly constant with depth in triangular notch as it varies with depth in rectangular weir

③ Trapezoidal weir/notch :-



$$\boxed{Q = \frac{2}{3} C_{d_1} \sqrt{2g}\ L H^{3/2} + \frac{8}{15} C_{d_2} \sqrt{2g} \tan\theta/_2\ H^{5/2}}$$

Imp. note:- cipolletti weir :- → special type of trapezoidal weir in which $\boxed{1H : 4V}$ **

n=2

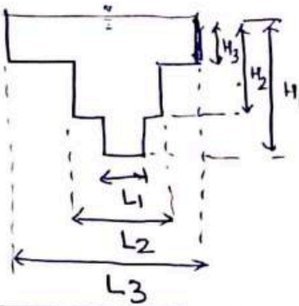
$\boxed{L_{eff} = L - 0.1 \times 2 \times H}$

or $\boxed{1V : 0.25 H \quad Z = 0.25}$

- discharge over cipolletti weir is calculated using suppressed rectangular weir formula.

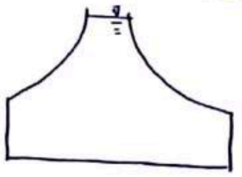
$$Q = 2/3\, C_d \sqrt{2g}\ L H^{3/2} \quad C_d = 0.63$$

④ Stepped notch :-



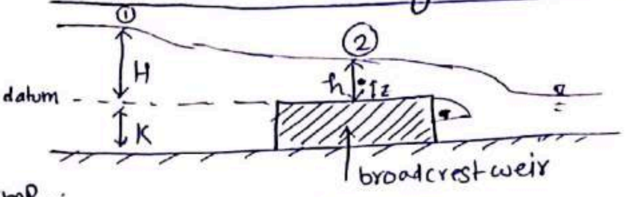
$$\boxed{Q = \frac{2}{3} C_d \sqrt{2g} \left[L_1 H_1^{3/2} + L_2 H_2^{3/2} + L_3 H_3^{3/2} \right]}$$

⑤ Sutro weir :- $\boxed{Q \propto H}$
or
Proportional weir :-



⑥ Broad crested weir :

* Broad crested weir supports nappe so that Pressure variation is hydrostatic at section-2.



Imp.

① $\boxed{q_{actual} = 1.7 C_d L H^{3/2}}$

② discharge over a Broad Crested weir is max.

v.imp. $\boxed{\text{when depth of flow } (h) = \frac{2}{3} H}$

Proof. Apply Bernauli's b/w 1 & 2

$$0 + 0 + H = (h - z) + \frac{V_2^2}{2g} + z$$

$$V_2 = \sqrt{2g(H - h)}$$

$$Q_{th} = L \times h \times V_2 \Rightarrow Q_a = C_d L h \sqrt{2g(H-h)}$$

- in broad crested weir flow adjust itself to have max. discharge for the available head H, The Critical depth is achieved by itself.

$\frac{dQ}{dh} = 0$ $\boxed{h = \frac{2}{3} H}$ for max. discharge

$h = \frac{2}{3} H \longrightarrow$ critical depth

Put in eqn ∴ $\boxed{Q = 1.7 \underset{(0.85-1)}{C_d} L H^{3/2}}$

v. Imp

effect on discharge due to error in head measurement

$Q = KH^n$
- $n = 1$ ⏢ Sutro/proportional
- $n = 3/2$ ⊔ rect.
- $n = 5/2$ ∨ triangular

$$\boxed{\frac{dQ}{Q} \times 100 = n \frac{dH}{H} \times 100}$$

Special :- for V notch :- if error in 'θ' :-

$Q = K \tan\theta/2$ $\quad dQ = K \sec^2\theta/2 \times \frac{1}{2} \times d\theta$

$$\frac{dQ}{Q} = \frac{d\theta}{\theta} \times \theta/\sin\theta$$

v. Imp

$$\boxed{\%\text{ error in } Q = \left(\frac{\theta}{\sin\theta}\right) \times \%\text{ error in } \theta}$$

flow through pipe	laminar Re< 2000	Turbulent Re>4000
open channel flow	Re<500	Re>2000
flow thro soil	Re<1	Re>2

Head loss Through pipe

- major loss (frictional loss) 80-90% of total loss
- minor losses (due to pipe fitting) 10-20% of total loss.

major loss → due to friction, given by

- Darcy's weisback eqn

$$h_f = \frac{fLV^2}{2gd} = \frac{fLQ^2}{12.1 d^5}$$

- chezy's eqn

$$V = C\sqrt{RS}$$

$$C = \sqrt{8g/f}$$ $$S = \frac{h_f}{L}$$

friction factor $\leftarrow f = 4f' \rightarrow$ friction coefficient(f')

- laminar Flow (Re<2000)

$$f = \frac{64}{Re}$$

- Turbulent flow Re>4000
  - Smooth pipe
    - Re<105

$$f = \frac{0.316}{(Re)^{1/4}}$$

    - Re>105
  - Rough pipe

$$\frac{1}{\sqrt{f}} = 1.74 + 2\log\frac{R}{K}$$

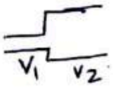
note:- friction cofficient

$$f' = C_{f_x} = \frac{\tau_w}{\frac{1}{2}\rho U_0^2} = \frac{\tau}{\frac{1}{2}\rho V^2}$$

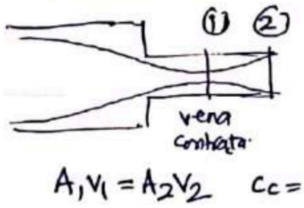
minor losses due to:-

① sudden expansion

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$



Ⓘ sudden contraction → loss due to contraction is not for the contraction itself, but due to expansion followed by contraction.



$$h_L = \frac{kV_2^2}{2g} = \frac{0.5V_2^2}{2g}$$

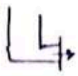
$A_1V_1 = A_2V_2$ $C_c = A_1/A_2$

$$k = \left(\frac{1}{C_c} - 1\right)^2$$

(iii) entry loss

$$h_L = \frac{0.5V^2}{2g} \text{ or } \frac{kV^2}{2g}$$

(iv) exit loss



$$h_L = \frac{V^2}{2g} = \frac{kV^2}{2g}$$

K=1 Turbulent flow

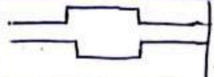
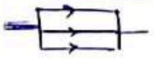
k=2 laminar flow

(v) head loss due to fitting and bends

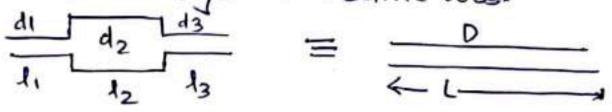
$$h_L = \frac{kV^2}{2g}$$

45° elbow K=0.4

90° bend K=1.2

Series pipe		Q → same $h_L = h_{f1} + h_{f2} + h_{f3}$
Parallel Pipe		hf → same $Q = Q_1 + Q_2 + Q_3$

Hydraulically equivalent pipe :- carry same discharge under same loss.



neglect minor loss, assume f → same

$$\frac{fl_1Q^2}{12.1 d_1^5} + \frac{fl_2Q^2}{12.1 d_2^5} + \frac{fl_3Q^2}{12.1 d_3^5} = \frac{fLQ^2}{12.1 D^5}$$

$$\frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{D_3^5} = \frac{L}{D^5}$$ **

Syphon :- long bent pipe which is used to transfer liquid from reservoir at higher elevation to another reservoir at lower level when 2 reservoir are separated by hill or high level ground.

Imp.

note:- for no vaporisation

P_{summit} (absolute) $\nless$ P_{vapour} (abs)

2.5m @ 20°C

HGL
(hydraulicgradient-Line) $\longrightarrow \left\{ \frac{p}{\rho g} + z \right\}$

$\hookrightarrow$ HGL may rise or fall.

Total energy line TEL $\longrightarrow \left(\frac{p}{\rho g} + z \right) + \frac{v^2}{2g}$

$\hookrightarrow$ always fall

note:- sudden rise in TEL/HGL → duetopump
∵ ME added

sudden fall in TEL/HGL → duetoturbine
∵ Mechanical energy extracted.

for Ideal flow ⇒ (loss=0) ∴ TEL → horizontal

Inopenchannel ⇒ HGL coincide with free surface

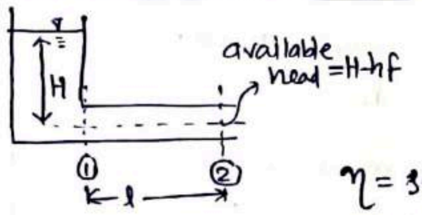
at pipe exit ⇒ HGL coincide with pipe surface
∵ $P/\rho g = 0$

Pressure in flow section (aboveHGL) =(−)ve
(belowHGL) =(+)ve

HGL not defined at upstream of pipe (∵ dueto entranceloss

gap b/w TEL & HGL ⟶ always $\frac{V^2}{2g}$
if velocity ↑ (sizeofpipe↓) ⇒ gap↑

Power transmission throgh pipe :-



$\eta_{\text{power transmission}} = \frac{\text{Power available at 2}}{\text{'' , '' ①}}$

$\eta = \rho g q(H-hf)/\rho q g H$

$\eta = H-hf/H$

$P_2 = \rho g Q\left(H - \frac{f l Q^2}{12.1 d^5}\right)$

$\frac{dp}{dQ} = 0 \rightarrow$ for max. power transmission $\boxed{hf = H/3}$ $\boxed{\therefore \eta_{max} = 66.6\%}$

Power available from nozzle :-

$\boxed{av = AV}$

neglect entry loss

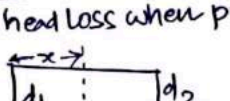
$H = hf + \frac{v^2}{2g}$

$\downarrow$

$\frac{flV^2}{2gD}$

for max. power transmission dia of nozzle $\boxed{d = \left(\frac{D^5}{2fl}\right)^{1/4}}$ **

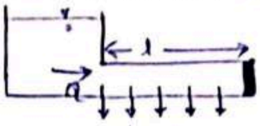
head loss when pipe diameter varies →



$\boxed{d_x = d_1 - \frac{(d_1-d_2)x}{l}}$

$hf = \int_0^l \frac{fQ^2 dx}{12.1 d_x^5} \rightarrow$ solve

water discharge from side at uniform rate at regular interval and end is closed.



$\boxed{Q_x = Q - q'x = Q - \frac{Qx}{l}}$

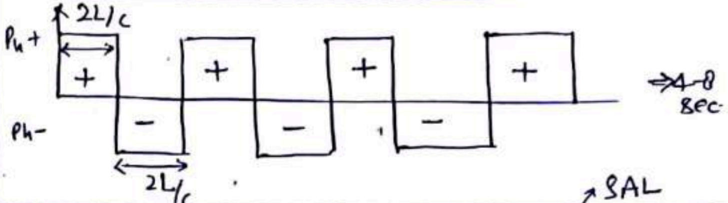
$q' = Q/L$

$hf = \int_0^l \frac{f Q_x^2 dx}{12.1 d^5} = \frac{1}{3}\frac{f l Q^2}{12.1 d^5} = \frac{1}{3}$rd of headloss in constant discharge

water hammer analysis ⟶ By Joukowsky

$\boxed{T_0 = \frac{2L}{C}} \rightarrow$ critical time < gradual closure $\boxed{t > \frac{2L}{C}}$; suddenclosure $\boxed{t < 2L/C}$

$\boxed{C \rightarrow \text{velocity of pressure wave} = \sqrt{K/\rho} \quad \text{order } 900 - 1200 \text{ m/s}}$



① Gradual closure :-

force duetopressure = retarding force → mass × a

(P·A) ; ρAL ; $\left(\frac{V}{T}\right)$

$\boxed{P = \rho VL/T}$ *

Ⓘⓘ suddenclosure

ⓐ elasticpipe ⟶ (elasticity of pipe reduces pressure increase)

loss of KE = gain of S.E in water + gain of SE in pipematerial

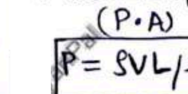
$\frac{1}{2}mv^2$; $\left(\frac{P^2}{2K}\right) AL$; $\frac{1}{2E}\{\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2\}$ [pipe volume]

(ρAL) ; $\sigma_1 = \frac{pd}{2t}$; $\sigma_2 = \frac{pd}{4t}$; $\mu = 0.25$; $\pi d t l$

$$\therefore \boxed{P = V\sqrt{\frac{\rho}{1/K + D/Et}}} \quad **$$

ⓑ rigid pipe :- gain of SE in pipematerial ⇒ zero
(Pressure more than elastic case) & $E \rightarrow \infty$ ∴ $\boxed{P = \rho VC}$ $\boxed{C = \sqrt{K/\rho}}$

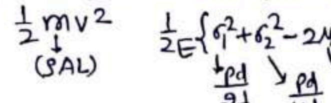
controll water Hammer ① Zerovelocity valve Ⓘⓘ Aircusion valve
ⓘⓘⓘ opposite poppet valve



$\boxed{D = \sqrt{2}d}$ *

varying pressure is max. or Pressure loss is max for given discharge

$\frac{d\Delta P}{dD} = 0$ $\quad \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \frac{(V_2-V_1)^2}{2g}$



↑$\left(\frac{K}{D}\right)$↑ (moody's & stanton diagram)

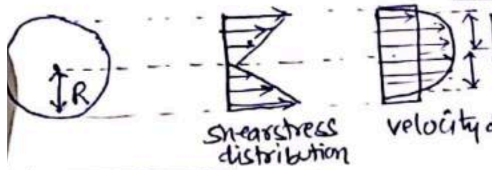
moody diagram → 'f' value for turbulent flow based on Cole-brook white data on commercial pipe.

In laminar flow $\boxed{\frac{\mu du}{dy} = \tau}$

∵ in laminar flow → turbulent shear = 0, eddy's viscosity = 0

Entrance length where flow attains fully developed flow profile
- laminar flow $\frac{Le}{D} = 0{\cdot}05 \text{ or } 0{\cdot}07$
- Turbulent flow $\frac{Le}{D} = 25\text{–}40D$ or $50D$

laminar flow thr. circular pipe (Hagenpoiseulle flow)



$\boxed{r = 0{\cdot}707R}$ $\boxed{u = u_{mean}}$

1- $\boxed{\tau = \left(\frac{-\partial p}{\partial x}\right)\frac{r}{2}}$ (–) sign ∵ pressure decrease in direction of flow [to compensate resistance of flow]

2- $\boxed{u = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)[R^2 - r^2]}$ or $\boxed{u = u_{max}\left(1 - \frac{r^2}{R^2}\right)}$

3- $\boxed{u = u_{mean} @ r = \frac{R}{\sqrt{2}} = 0{\cdot}707R}$ 4- $\boxed{u_{mean} = \frac{u_{max}}{2}}$

5- $\boxed{u_{max} @ r = 0 \Rightarrow -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)R^2}$ & $\boxed{u_{mean} = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right)R^2}$

6- How to get u_{mean} ⇒ $\boxed{Q = Au_{mean} = \int_0^R (2\pi r)dr\,u}$

7- Pressure drop in circular pipe / h_f :- Imp

$$\boxed{h_f = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{p_1 - p_2}{\rho g} = \frac{32\mu VL}{\rho g d^2} \propto \frac{1}{d^4}}$$

$$\boxed{h_f = \frac{32\mu VL}{\rho g d^2} = \frac{fLV^2}{2gd} \quad \therefore f = \frac{64}{Re} \text{ laminar flow}}$$

8- loss in laminar flow:

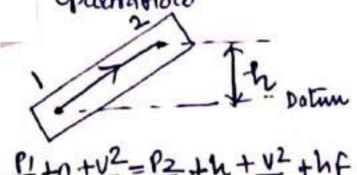
$\frac{-\partial p}{\partial x}$ = force/volume

Powerloss (ΔP) = force × velocity

$$\Delta P = \left(\frac{-\partial p}{\partial x}\right)[A \times L] \times V$$

$$\boxed{\Delta P = (P_1 - P_2)\,Q}$$

upward flow

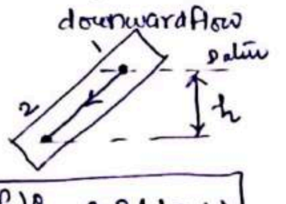


$$\frac{P_1}{\rho g} + 0 + \frac{V^2}{2g} = \frac{P_2}{\rho g} + h + \frac{V^2}{2g} + h_f$$

$$\underbrace{(P_1 - P_2)Q}_{\Delta P} = \boxed{\rho g Q(h + h_f)}$$

Powerloss बढ़ेगा

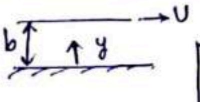
downward flow



$$\boxed{\underbrace{(P_1 - P_2)Q}_{\Delta P} = \underbrace{\rho g Q(h_f - h)}_{कम}}$$

∵ downward motion gravity will also contribute towards power, ∴ less power loss.

Laminar flow b/w 2 parallel plates → (upper plate) moving {Couette flow} → stationary {plane Poiseuille flow}



$$\boxed{u = \frac{Uy}{b} + \frac{1}{2\mu}\left(\frac{-\partial p}{\partial x}\right)(by - y^2)}$$

if upper plate fix or stationary ($U = 0$)

$$\therefore \boxed{u = \frac{1}{2\mu}\left(\frac{-\partial p}{\partial x}\right)(by - y^2)}$$

$$\boxed{\text{for } u_{max}:\ \frac{\partial u}{\partial y} = 0 \quad y = b/2 \qquad u_{max} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right)b^2}$$

How to get u_{mean} : $(b \times 1)$

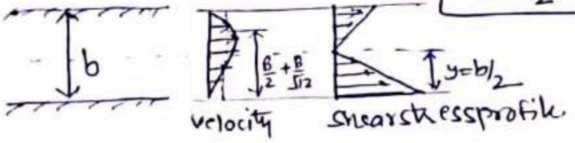
$$\int_0^b u\,dy = Q = Au_{mean}$$

∴ ?

$$\boxed{u_{mean} = u_{max} \times 2/3} \ *$$

$$\boxed{u = u_{mean} @ y = B/2 \pm \frac{B}{\sqrt{12}}}$$

$$\boxed{\tau = \mu\frac{du}{dy} = \mu\left[\frac{U}{b} - \frac{\partial p}{\partial x}\left(\frac{b}{2} - y\right)\right] \quad \text{at } y = b/2 \quad \tau = 0 \ \{\text{if } U = 0\}}$$



Pressure drop h_f :- Imp

$$\boxed{h_f = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = \frac{P_1 - P_2}{\rho g} = \frac{12\mu V l}{\rho g b^2} \propto \frac{1}{b^3}}$$

B/b

* circular pipe : KE correction factor (α) $= \int u^3 dA / V^3 A$

momentum correction factor $\beta = \int u^2 dA / V^2 A$

Ideal flow profile $\alpha = 1$ $\beta = 1$ uniform flow profile

	α	β
laminar	2	1.33
Turbulent	1.33 power law	1.015
	logarithm variation 1.03–1.06	
Parallel plate	1.54	1.2

measurement of viscosity

1- Rotating cylinder method

2- capillary tube method

3- orifice type viscometers

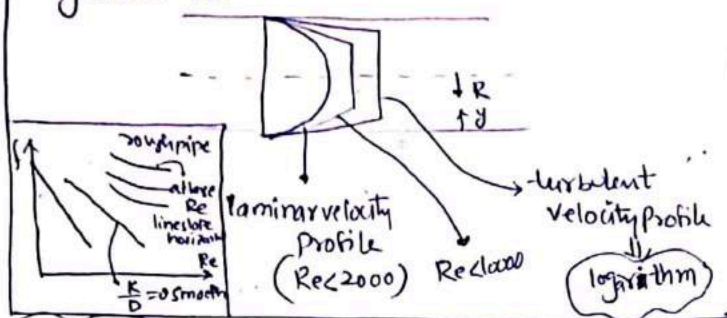
Saybolt, Redwood, Engler viscometer.

$\boxed{u/u_\infty = (y/\delta)^{1/7}}$

turbulent flow:

- velocity profile ⟶ much flatter

The profile becomes more flat at higher reynold's no.



moody's diagram used ⟵ friction factor(f) (turbulent flow Re>4000)

Smooth pipe $f = f^n(Re)$

- $Re < 10^5$: $\dfrac{0.316}{(Re)^{1/4}}$
- $10^5 < Re < 4\times10^7$: $0.0032 + \dfrac{0.221}{(Re)^{0.237}}$

rough pipe $f = f^n(Re, k)$

$$\frac{1}{\sqrt{f}} = 1.74 + 2\log\left(\frac{R}{k}\right)$$

Smooth + rough pipe

$$\frac{u_{max}}{u_{avg}} = 1 + 1.33\sqrt{f}$$

note - As pipe becomes older roughness increases $k = k_o + \alpha t$

Colebrook eq^n :- $\dfrac{1}{\sqrt{f}} = 1.14 - 2\log\left[\dfrac{k_s}{D} + \dfrac{9.35}{Re\sqrt{f}}\right]$

$$\tau_{total} = \tau_{viscous} + \tau_{turbulent} = \mu\frac{du}{dy} + \underset{\downarrow}{\eta}\frac{d\bar{u}}{dy}$$

eddy viscosity or turbulent mixing coff.

$\mu \to$ fluid property
$\eta \to$ flow charateristic

- eddy viscosity (η) decreases towards wall of pipe & becomes zero at wall. (eddy viscosity not constant)
- In turbulent flow, molecular viscosity is insignificant compared with eddy viscosity. (laminar flow eddy viscosity = 0)

for smooth + rough pipe :- **

$$\boxed{\frac{u - \bar{u}}{V_*} = 3.75 + 5.75\log\left(y/R\right)}$$

put $y = R$, $u \to u_{max}$

$$\boxed{\frac{u_{max} - \bar{u}}{V_*} = 3.75}$$

Smooth pipe velocity distribution	rough pipe velocity distribution
$\dfrac{u}{V_*} = 5.5 + 5.75\log\dfrac{u_* y}{\nu}$	$\dfrac{u}{V_*} = 8.5 + 5.75\log\dfrac{y}{k}$
$\dfrac{\bar{u}}{V_*} = 1.75 + 5.75\log\dfrac{u_* R}{\nu}$	$\dfrac{\bar{u}}{V_*} = 4.75 + 5.75\log\dfrac{R}{k}$

- effect of viscosity is max. near boundry hence laminar sublayer exist near boundary.

Thickness of this boundry $\propto \underset{\substack{\downarrow \\ \text{Kinematic viscosity}}}{\nu} \propto \dfrac{1}{V(\text{flow velocity})}$

$$\delta' = \frac{11.6\nu}{V_*}$$

$V_* \Rightarrow$ shear velocity $= V_{avg}\sqrt{f/8} = \sqrt{\tau_w/\rho}$

Hydrodynamically smooth surface

(thickness of laminar sublayers' $\delta' \uparrow$ ⇒ eddies will not penetrate till boundry.

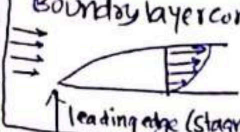
$\dfrac{K}{\delta'} < 0.25$ ⟶ Nikurdse experiment ⟶ $K/\delta' > 6$

$\dfrac{u_* K}{\nu} < 5$ ⟶ Reynold's roughness no. ⟶ $\dfrac{u_* K}{\nu} > 70$

Hydrodynamically rough surface

∵ $\delta' \downarrow$ ∴ penetration of eddies will be at boundry.

Boundry layer concept → for real fluid only



∴ Bernauli Theorem/eqⁿ is not Applicable inside IL.

K/δ → relative roughness.

Boundry layer Separation

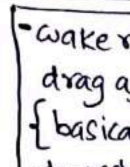
- onverge of separation: $\boxed{\frac{\partial u}{\partial y}=0}$ P_{min}
- separated: $\boxed{\frac{\partial P}{\partial x}>0}$ adverse pressure gradient $\left(\frac{\partial u}{\partial y}<0\right)$
- not separated: $\frac{\partial P}{\partial x}<0$ negative Pressure gradient 'or' favorable Pressure gradient

Separated flow Ex. diffuser, pumps, fans, turbine blades, airfoil, open channel transition.

- wake region is a low Pressure which creates a drag against flow of fluid
{basically large turbulent eddies are formed at downstream of the point of separation, This is called turbulent wake.

effect of separation → increase in pressure drag $\left(\frac{dP}{dx}>0\right)$
→ increase in flow loss.

method of controlling separation :- ① stream lining of Body
② accelerate the fluid in Boundry layer by injecting fluid.
③ Suction of fluid from BL (reduce skin friction)
④ supply additional energy from blower
⑤ rotation of cylinder on body.



$x=0$ (leading edge)	$\delta=0$
$y=0$	$u=0$
$y=\delta$	$u=U_\infty$ constant $\frac{\partial u}{\partial y}=0\ \frac{\partial^2 u}{\partial y^2}=0$

Boundary layer thickness (δ)	$u=0.99U_\infty$ (99% of freestream vel.)
Displacement thickness (δ^*) (mass flow rate)	$\int_0^\delta\left(1-\frac{u}{U_\infty}\right)dy$
momentum thickness (θ) (momentum)	$\int_0^\delta\frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right)dy$
energy Thickness (δ_e) (energy)	$\int_0^\delta\frac{u}{U_\infty}\left[1-\frac{u^2}{U_\infty^2}\right]dy$
Shape factor	δ^*/θ {Disp. Thic / Mom. Thick}

* flat plate < laminar $Re<5\times10^5$
Turbulent $Re>5\times10^5$

changes of Boundry layer from laminar to turbulent :-
1- Roughness of plate 2- Plate curvature
3- Pressure gradient 4- Intensity & scale of turbulence

vonKarman momentum eqⁿ :- $\boxed{\frac{\partial\theta}{\partial x}=\frac{\tau_w}{\rho U_\infty^2}}$
{laminar BL, Steady, 2D flow, Incompressible, $\partial P/\partial x=0$ }
(approx true for T.BL}

std. result :- laminar flow general velocity profile

$$\boxed{\frac{u}{U_\infty}=\frac{3}{2}(y/\delta)-\frac{1}{2}(y/\delta)^3}$$

→ $V_{avg}=5/8\,U_\infty$
→ $\delta^*=3/8\,\delta$ $\alpha=1.67$
→ $\theta=\frac{39}{280}\delta$

when velocity profile not given (Blassius Experiment result}

laminar	Turbulent
$\delta=\frac{5x}{\sqrt{Re_x}}$	$\delta=\frac{0.376x}{(Re_x)^{1/5}}$
$C_{fx}=f'=\frac{0.664}{\sqrt{Re_x}}$	$C_{fx}=f'=\frac{0.059}{(Re_x)^{1/5}}$
$C_D=\frac{1.328}{\sqrt{Re_L}}$	$C_D=\frac{0.072}{(Re_L)^{1/5}}$ or $\frac{0.074}{(Re_L)^{1/5}}$

$C_{fx}=\frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}$

note:- $\boxed{C_{fx}=f'=\frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}}$ $\boxed{C_D=\frac{F_D}{\frac{1}{2}\rho A U_\infty^2}}$

Imp
1- Bluff body → Plate perpendicular to flow
2- Streamline body → Thin plate parallel to flow

$F_D=3\pi\mu DV$ < form drag (deformation drag) ⇒ $\pi\mu DV$
surface drag (friction drag) ⇒ $2\pi\mu DV$

$Re<0.5$	$C_D=\frac{24}{Re}$
$0.5<Re<10^4$	$C_D=\frac{24}{Re}+\frac{3}{\sqrt{Re}}+0.34$
$Re>10^4$	$C_D=0.40$

*

1 metric horse power = 736 watt
1 HP = 746 watt

modal analysis & dimensionalanalysis :-

fundamental quantity	mass	length	time	force
M-L-T system	M	L	T	MLT-2
F-L-T system	M	L	T	F

Geometric similarity ⓛ	similarity of shape	area (L^2), volume ($L3$) moment of Inertia (L^4)
kinematic similarity Ⓣ	similarity of motion	velocity (LT^{-1}) acceleration (LT^{-2}) $\omega(T^{-1})$ $\alpha(T^{-2})$ discharge (L^3T^{-1}) $\nu(L^2T^{-1})$ stream function (L^2T^{-1}) velocity potential (L^2T^{-1}) vorticity (T^{-1})
Dynamic similarity Ⓜ	similarity of force	Force (MLT^{-2}) density (ML^{-3}) $\mu(ML^{-1}T^{-1})$, $\sigma(MT^{-2})$ $P(ML^{-1}T^{-2})$, $\tau(ML^{-1}T^{-2})$ $E(ML^{-1}T^{-2})$

method of dimensional analysis < Rayleigh method / Bukingham π method

Buckingham π method :- if all n variable are described by m fundamental dimension, they may be grouped into (n-m) dimensionless π terms

$$\text{no. of } \pi \text{ terms} = (\underset{\text{total}}{n} - \underset{\substack{\text{primary variable} \\ \text{fundamental dimension} \\ \text{or repeating variable}}}{m})$$

note:- Dimensional Homogenity based on Principle of Homogenity

note:- motion of ship in deep sea (ship model) < froude no. / Reynold's no.

v.Imp Important dimensionless numbers.	Applications
Reynold's no. ⇒ $\frac{\text{Inertia force}}{\text{viscous force}}$ ⇒ $\frac{\rho v^2 L^2}{\mu (V/L) L^2}$ ⇒ $\boxed{\frac{\rho VL}{\mu}}$ *	• pipe flow (where viscosity → dominant Ex. laminar flow) (chl-pimdut p(x)) • flow around completely submerged bodies (submarine, airoplane, Automobil • Parachute when there is drag (when not falling in vaccum)
froude no. $\sqrt{\frac{\text{Inertia force}}{\text{gravity force}}}$ $\sqrt{\frac{\rho v^2 L^2}{(\rho L^3) g}}$ * $\boxed{\frac{V}{\sqrt{gL}}}$ Trial :- $V \propto \sqrt{L}$ $T \propto \sqrt{L}$ $a \propto 1$ $Q \propto L^{2.5}$ $P \propto L^{3.5}$	• free surface flow (spillway, weir, Jump) (ocf) • Flow of liq. jet from orifice • weir, notches • motion of ship in rough & turbulent sea (motion resistance is due to waves formed in sea due to gravity. Ex. harbour designing, ship model design (only for surface of sea motion of ship), canal designing.
Euler no. $\sqrt{\frac{\text{Inertia force}}{\text{Pressure force}}}$ $\sqrt{\frac{\rho v^2 L^2}{(P) L^2}}$ * $\boxed{\frac{V}{\sqrt{(P/\rho)}}}$	• pipe flow at high pressure.. (in which viscous effect is negligible like in Turbulent flow) • Cavitation study • Pressure due to sudden closure of valve
Weber no. $\sqrt{\frac{\text{Inertia force}}{\text{Surface tension force}}}$ $\sqrt{\frac{\rho v^2 L^2}{\sigma L}}$ * $\boxed{\frac{V}{\sqrt{\left(\frac{\sigma/L}{\rho}\right)}}}$	• capillary rise in narrow passage • flow over weir for small head • Rising Bubble, seepage through soil. • flow of blood in vein & arteries.
mach no. $\sqrt{\frac{\text{Inertial force}}{\text{elastic force}}}$ $\boxed{\frac{V}{\sqrt{K/\rho}} = \frac{V}{C}}$ * $= M$	• where compressibility force are dominating ($M \geq 0.3$) • Aerodynamic Testing { launching of rocket, missile) • underwater Testing of torpedo's • water hammer pressure • flow of gases through pipe at high velocity

note:- if $M < 0.3$ ⇒ effect of compresibility neglected.

$M < 1$ → subsonic flow
$M = 1$ → sonic "
$1 < M < 5$ → supersonic "
$M > 5$ → hypersonic "

Rivermodel → froude's Law Applicable

↓

here distorted model used

- → not geometrical similarity.
- → different scale ratio for horizontal & vertical direction.

note:- in river model, if undistorted model used then depth → very small.

→ difficult in measurement.

→ depth becomes so small that flow in model may not remain turbulent as is there in case of original river.

$$\boxed{L_{rH} = \frac{L_p}{L_m} = \frac{B_p}{B_m}}$$

$$\boxed{L_{rV} = \frac{h_p}{h_m}}$$

① velocity scale ratio	$V_r = \sqrt{(L_r)_V}$
(II) area ratio scale	$A_r = (L_rH)(L_rV)$
(III) discharge scale ratio	$Q_r = (L_rH)(L_rV)^{3/2}$

for Rotodynamic machine :-

$Q \propto D^3 N$

$$\boxed{\frac{Q_r}{D_r^3 N_r} = 1} \quad \text{———} ①$$

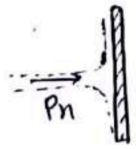
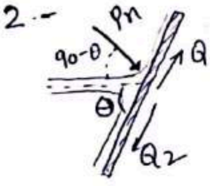
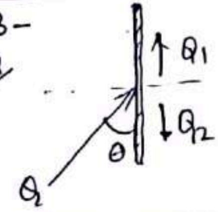
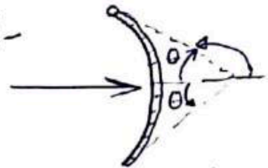
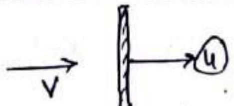
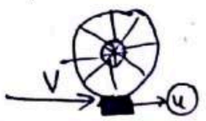
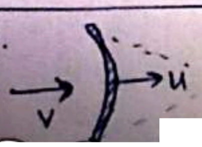
$P \propto \rho D^5 N^3$

$$\boxed{\frac{P_r}{\rho_r D_r^5 N_r^3} = 1} \quad \text{———} (II)$$

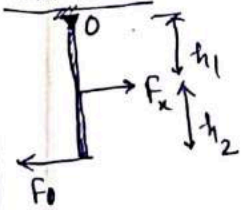
$DN \propto \sqrt{H}$

$$\boxed{\frac{\sqrt{H_r}}{D_r N_r} = 1} \quad \text{———} (III)$$

Plate stationary Jet case :- resultant (P_n) will always strike normal to plate.

1- Imp 	force exerted by water jet normal to plate $\boxed{P_n = \rho a v^2}$
2.-  Plate inclined θ from jet	$\boxed{P_n = \rho a v^2 \sin\theta}$ P_x ⇓ $P_n \cos(90-\theta)$ P_y ⇓ $P_n \sin(90-\theta)$
3- Imp 	$\boxed{Q_1 = \frac{Q}{2}(1+\cos\theta)}$ $\boxed{Q_2 = \frac{Q}{2}(1-\cos\theta)}$
4-  Curve plate	angle of deflection (180-θ) $\boxed{P_n = \rho a v^2(1+\cos\theta)}$
5.  Plate moving	$\boxed{P_n = \rho a (v-u)^2}$ $\boxed{\text{Work done per sec} = P_n \times u}$
6. Plate mounted on periferi of wheel 	$\boxed{P_n = \rho a V(v-u)}$ $\boxed{WDPS = P_n \times u}$ $\eta = \frac{\text{output of jet}}{\text{Input of jet}}$ $\eta = \frac{\rho a V(v-u)}{\frac{1}{2}\rho Q V2} = \frac{2(v-u)u}{V^2}$ $\boxed{\eta_{max} = 50\% \text{ @ } u = \frac{v}{2}}$
7. 	$\boxed{P_n = \rho a (v-u)^2(1+\cos\theta)}$ $\boxed{WDPS = P_n \times u}$

Std result :-

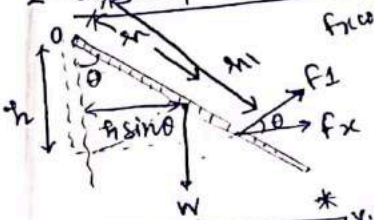


1- horizontal force at bottom edge of plate to keep it vertical

sol^n: $\Sigma M_o = 0$ $\quad \cancel{F_x} \times h_1 = F_o(h_1+h_2)$

$\rho a v^2$

2- angle of deflection where it stay in eqb.



$\Sigma M_o = 0$

$F_x\cos\theta$

$\boxed{F_1 \times h_1 = W(h\sin\theta)}$

put $\cos\theta = h/h_1$

$\therefore h_1 = h\sec\theta$

* v.imp

$\therefore \boxed{\sin\theta = \frac{\rho a v^2}{W l}}$

Draft tube :- only used in reaction turbine

- is always submerged below the water level in tail race
- not prefer straight cylindrical

- draft tube connect the outlet of runner to tail race
- Imp • draft tube converts large proportions of KE (rejected at the outlet of turbine) to useful pressure energy {due to this efficiency of turbine increased} Imp

$\eta = \frac{\text{actual conversion of kinetic head into pressure head}}{\text{kinetic head at inlet of draft tube}}$

$$\boxed{\eta = \frac{\left(\overset{\text{Inlet}}{\frac{V_1^2}{2g}} - \overset{\text{outlet}}{\frac{V_2^2}{2g}}\right) - h_L}{\underset{\text{Inlet}}{\left(\frac{V_1^2}{2g}\right)}} \times 100}$$

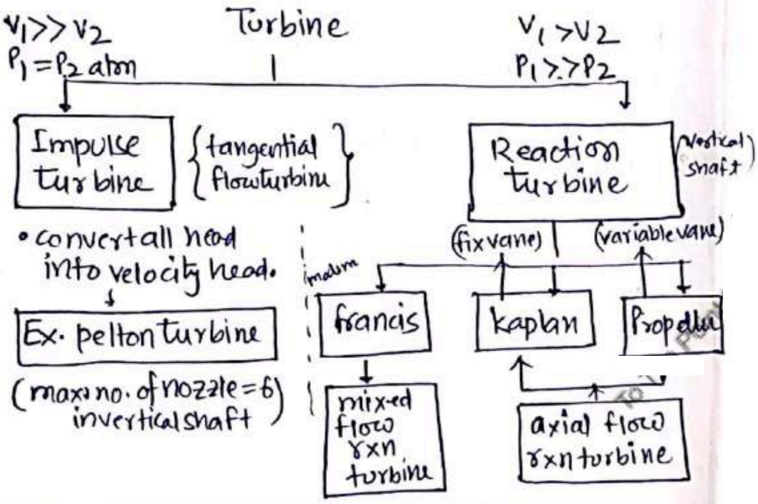
$$\boxed{\eta = \left(1 - \overset{\text{loss}}{k}\right)\left(\frac{V_1^2 - V_2^2}{V_1^2}\right)}$$

Turbine → fn $\boxed{\text{Hydraulic energy} \longrightarrow \text{mechanical energy}}$

- The mechanical so developed is supplied to generator coupled to the runner which then generated electrical energy.

Component of Hydroelectric plant :-

Reservoir → Sluicegate (Gate opening) → canal (or tunnel) → forebay / surgetank → intake str. → penstock → powerhouse → tailrace → River



Impulse turbine (pelton wheel)	High head (>300)	Low discharge
francis (mixed flow rxn)	(50-150) medium head	medium discharge
Kaplan & propeller	Low head (25-50)	High discharge

* $\boxed{H\downarrow Q\uparrow}$

Specific speed के basis पर classification :-

	Pelton (single jet) <	Pelton (double jet) <	francis <	Kaplan
Ns	10 – 35	35 – 60	60 – 300	300 – 1000

→ H↓

note :- old francis → radial turbine

Pump ⓠ

$$\boxed{N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}} \quad N \to \text{rpm}, \quad Q \to \frac{m^3}{s}$$

$[L^{3/4}\, T^{-3/2}]$

$\underbrace{\text{if } N_{s_1} = N_{s2}}_{\text{(homologus pump)}}$

- dimensionless specific speed

$$N_s = \frac{N\sqrt{Q}}{(gH_m)^{3/4}}$$

- specific speed used to compare performance of different pumps

$\boxed{u = \pi DN/60 = \sqrt{2gH}}$ *

Turbine Ⓟ

$$\boxed{N_s = \frac{N\sqrt{P}}{H^{5/4}}} \quad N \to \text{rpm}, \quad P \to \text{KW}$$

→ It has dimension $F^{1/2} L^{-3/4} T^{-3/2}$

- dimensionless specific speed or shape no.

$$N_s = \frac{N\sqrt{P/\rho}}{(gH_m)^{5/4}}$$

① $u \propto DN$	① $u \propto \sqrt{H}$
② $Q \propto D^3N$ {AV}	(II) $Q \propto \sqrt{H}$
③ $P \propto D^5N^3$ {QH}	(III) $P \propto H^{3/2}$

note :-

multistage centrifugal pump

- in series
  - Q → same
  - ∴ for High head
  - $H = H_1 + H_2 - - -$
- in parallel
  - H → same
  - for High discharge
  - $Q = Q_1 + Q_2 - -$

$$Q_{used} = \frac{Q\ total}{\text{no of pump used}}$$

$$\boxed{\text{Speed ratio} = \frac{u}{\sqrt{2gH}}}$$

multijet pelton turbine :-

(I) Power = n × Power single jet

(II) discharge = n × Q single

(III) $N_s = \sqrt{n} \times N_s$ single

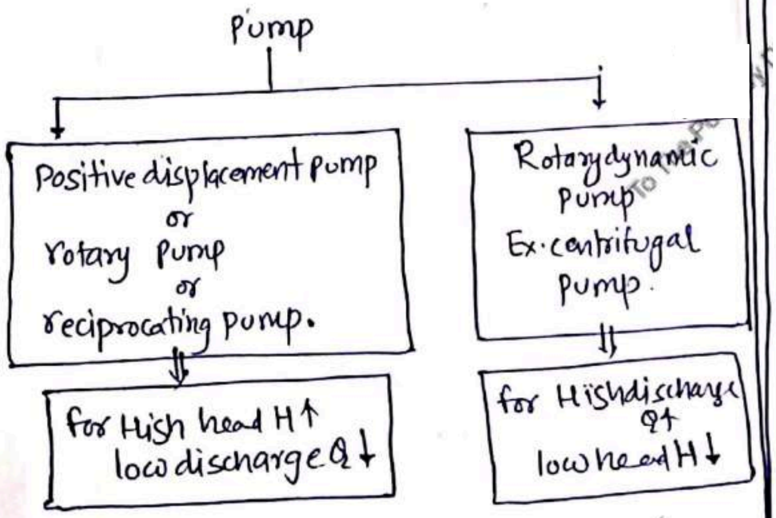
(IV) Higher specific speed turbines are more reliable to cavitation

(V) Ns↑ ⇒ smaller size and nessisate proper design of draft tube so that losses due to whirled component at exit is less.

Pump :- • mechanical device to increase the pressure energy of liquid.

- pumps are used to raising fluid form a lower to higher level

Imp
- not a main component of hydroelectric plant.



Reciprocating Pump :-

$$\boxed{Slip = Q_{th} - Q}$$

$$Q_{th} = \frac{A\,L\,N}{60}$$

L → stroke length, N → rpm, A → $\frac{\pi}{4}d^2$, d → piston dia

$$\boxed{\% slip = \frac{Q_{th} - Q}{Q_{th}} * 100}$$

'or'

$$\boxed{\% slip = \left(1 - Q/Q_{th}\right) \times 100}$$

Slip is negative :-

1- delievery pipe small & suction pipe long

2- pump running at very High speed.

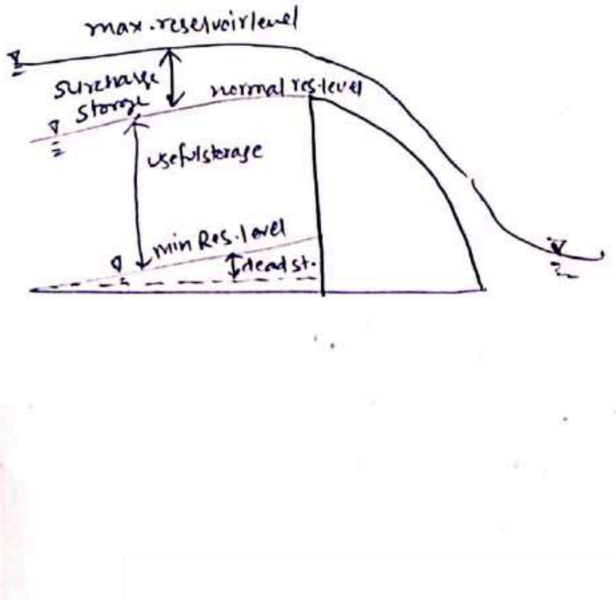
---

forebay → is used at Jn of [Powerchannel & penstock] *

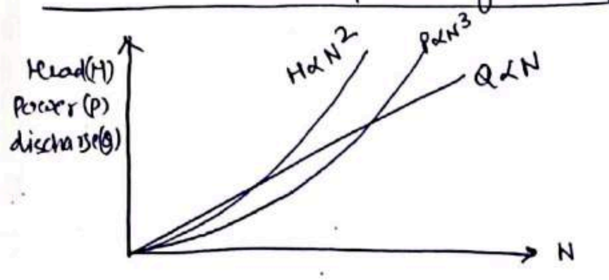
→ It stores water temporarily when reject by plant (when electric load is reduced) and also to meet the instantaneous increased demand of water due to sudden increase in load.

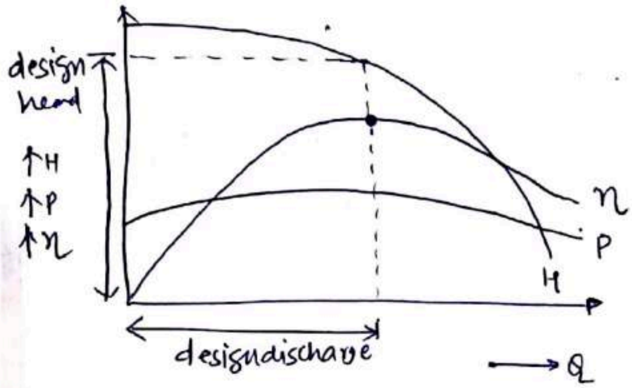
Surcharge storage :- volume of water stored b/w max. reservoir level and normal reservoir level

Useful storage :- volume of water stored b/w normal reservoir level and min. reservoir level.



Charactrstic curve of centrifugal pump :-





- Discharge increase with speed.
- Head ———
- manometric head decreases with discharge

Turbine head :-

(#) $\boxed{\text{net head} = \text{Gross head} - \text{frictional loss}}$

(H) → effective head used to calculate power production.

Gross head → difference in elevation b/w head race level of intake and the tail race level at discharge side.

Pump head :-

① Static head (Hs) :- $\boxed{\overset{\text{suction head}}{h_s} + \overset{\text{delievery head}}{h_d}}$

→ difference b/w water level in sump and top storage reservoir where water stored after pumping.

② manometric head (Hm) :- total head must produced by pump to satisfy external requirement.

$$\boxed{H_m = \underbrace{(h_s + h_d)}_{H_s} + \underbrace{h_{fs}}_{\downarrow \text{ frictionloss in suctionpipe}} + \underbrace{h_{fd}}_{\downarrow \text{ frictionloss in delievery pipe}}}$$

NPSH :- when pump operates at high speed {∵ P↓} that's why chance of cavitation increases.

→ total energy available at inlet of pump above vapour pressure of water

→ head available to push liquid into pump to replace liq discharged by pump.

$$\boxed{NPSH_A = \left[\frac{P_{atm} - P_v}{\rho g}\right] - h_s - h_{fs}}$$

$\frac{P_{atm}}{\rho g}$ ⇒ atm. pressure head = 10.3 m at MSL

$\frac{P_{vap}}{\rho g}$ ⇒ vapour pressure head

2.5m at 20°c

10.3m at 100°C

$\boxed{\text{for no cavitation } (NPSH)_A > (NPSH)_{Required}}$

$\boxed{\text{Thoma's cavitation no. } \sigma = \frac{NPSH}{H_m}}$

3/4/2020