Chapter 6

FACTORS AND MULTIPLES

In the chapter on Integer, you have learnt that the operation on division doesn't follow the closure property. This means if an integer is divided by another integer, we don't always get an integer. Think about this and find out by which numbers 8 gets divided and doesn't give any remainder. And by which numbers does 7 get divided?

Factors

In $2 \times 5 = 10$, you have observed that 10 can be divided by 2 and 5 completely. The factors of a number are those numbers that divide the number completely.

$10 \div 2 = 5$

$10 \div 5 = 2$ thus, 2 and 5 are the factors of 10.

The factors of a number are all those numbers that can divide that number.

Every number at least gets compulsorily divided by 1 and itself.

For example: 12 get completely divided by 1.

12 get completely divided by 12.

Do you know any number that doesn't get completely divided by 1 or by the same number?

Divisible Numbers

Those numbers that get completely divided by numbers other than 1 and itself are known as *divisible numbers* or *composite numbers*.

Those numbers which get completely divided by any number are known as *factors* of that number.

Lets find out how many numbers can divide 12

$12 = 1 \times 12$

$12 = 2 \times 6$

$12 = 3 \times 4$

$12 = 3 \times 2 \times 2$

So, 12 can get divided by 1,2,3,4,6,12. Hence the number 12 is divisible by 1,2,3,4,6,12.

Indivisible or Prime Number

The numbers whose only factors are 1 and the number itself are called Prime numbers. For example: 13 can only get divided completely by 1 and 13. Other such numbers are 2, 3, 5, 7, 11 etc.

Numbers having more than 2 factors are called *Composite numbers*.

ACTIVITY 1

In the table below are given the factors of some numbers. Write down all the possible factors for the remaining numbers in the blank spaces and list out the numbers with one, two and more than two factors separately.

Number	All factors	Number	All factors
1	1	6	1, 6, 2, 3
2	1, 2	7	....
3	....	8	....
4	1, 4, 2	9	....
5	....	10	....

The table shows three types of numbers –

1. The numbers with only one factor. Such a number is 1. It is neither a (indivisible) prime nor a composite (divisible) number and therefore makes a separate category.
2. The numbers with two factors: 2, 3, 5, 7 etc. are numbers which have only two factors. These get divisible by that number and one only. Therefore these are prime numbers. Write such five more examples in your notebooks.
3. Numbers that have more than two factors e.g. 4, 6, 8, 9, 10 etc. which are known as composite numbers.

ACTIVITY 2

The Sieve of Eratosthenes

We can find prime numbers from 1 to 100 with the help of an easier method without actually checking the factors of a number. This method was given by a Greek mathematician Eratosthenes in 3rd century B.C. His method is known by the name the Sieve of Eratosthenes that helps us to separate Prime numbers and composite numbers.

The Sieve of Eratosthenes

Below, numbers from 1 to 100 are given in table. Now follow the instructions given:

Instructions:

1. Strike out 1, because 1 is not a prime number.
2. Encircle 2 and strike out all the numbers that get divided by 2, e.g. 4,6,8,10,12 etc.
3. Now take the next number that has not been struck out that is 3. Encircle 3 first and then strike out the numbers that get divided by 3.
4. Similarly, take the next number that has not been struk out, encircle it and strike off the remaining numbers in the list that get divided by the encircled number.

5. Repeat the process till all the numbers in the list upto 100 get crossed out.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	9	~~10~~
11	~~12~~	13	~~14~~	15	~~16~~	17	~~18~~	19	~~20~~
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

All the encircled numbers in the table are prime numbers. All the struck out numbers, other than 1 are composite numbers.

This method is known as the Sieve of Eratosthenes.

ACTIVITY 3

You know about the prime and composite numbers, now let us play the game of finding factors.

Draw some circles in you notebook. Beneath every circle write the numbers 1, 2, 3, 4, 5… etc. serially and then follow the instructions given below.

Instructions:

Write 1 in all those circles for the numbers that get divided by 1.

Write 2 in the circles for numbers that get divided by 2.

Write 3 in the circles for numbers that get divided by 3.

Now carry on this process for as many numbers you can and then answers the questions that follow:

1 2 3 4 5 6 7 8 9 10

11 12 13 14 15 16 17 18 19 20

Q 1. How many circles contain only one numbers? Write down the figures outside those circles.

____

Q 2. How many circles have the two numbers? Write down the figures outside those circles.

____

Q 3. How many circles contain more than two numbers? Write down the figures outside those circles.

____

The numbers inside the circles are the factors for the numbers outside the circles. **All those numbers which have only two factors are the prime numbers.**

Co Prime Number

Let us discuss on factors of 8 and 15

Factors of 8 = 1,2,4,8

Factors of 15 = 1,3,5,15

It is clear that there is no common factor for 8 and 15 except 1. In this condition 8 and 15 will know as Co prime number.

Like this, Let us think on factors 9, 10 and 49

Factors of 9 = 1,3,9

Factors of 10 = 1,2,5,10

Factors of 49 = 1,7,49

In the example given above only 1 is common factor for 9, 10 and 49. No other common factor is there for 9, 10 and 49. So 9, 10 and 49 are Co prime numbers.

Numbers, which do not have any common factor between them other than 1 are called Co prime numbers.

Practice 1

1. Write down all the prime numbers between 1 to 100.
2. Write the composite numbers between 75 to 100.
3. Which number between 70 to 80 has the maximum number of factors?
4. Whether the 12 and 25 are Co prime numbers?
5. Can two consecutive number be Co prime?

Some other Types of Numbers

1. **EVEN NUMBERS :** These numbers that are completely divided by 2 are called even numbers. Example: 2, 4, 6, 8, 10, 12 etc.

2. **ODD NUMBERS :** These numbers that are not completely divisible by 2 are called odd number. Example: 1, 3, 5, 7, 9, 11 etc.

Below are given some odd numbers, separate out the prime and composite numbers from the list.

41, 45, 47, 53, 55, 57, 63, 67, 69.

71, 73, 77, 81, 87, 89, 91, 93, 95, 97, 99.

COMPOSITE NUMBERS	PRIME NUMBERS

Are all odd numbers prime?

Prime Factors

Let us now find out what are the prime factors of 42.

$42 = 14 \times 3$, here 3 is a prime number.

Is 14 also a prime number?

Number 14 will be written as 2×7.

That is, $42 = 2 \times 7 \times 3$

Now, here 2, 7, and 3 are all prime numbers. These are known as prime factors of 42.

What are the prime factors of 6? Take some more numbers and find out the prime factors.

Determining Prime Factors

How should we find out the prime factor of a number? Shall we divide each number several times to do so? Generally, we use the method shown below to find out the prime factors of any number. First, divide the given number by 2. If the number is divisible by 2. Then the number and its quotients are continuously divided by 2 till they get divided by 2. Then, if the number is divisible by 3. Similarly if for numbers like 5, 7, 11……..etc this process is repeated until the quotient obtained is 1.

Example 1: Let us observe the prime factors of 24.

2	30
3	15
5	5
	1

$\therefore$ 24 = 2 × 2 × 2 × 3.

Now take another number.

Example 2: The prime factor of 30.

2	24
2	12
2	6
3	3
	1

$\therefore$ 30 = 2 × 3 × 5.

Practice 2

1. Find out the prime factors for the following numbers:
   (i) 16 (ii) 48 (iii) 60 (iv) 84

You have learnt how to find out prime factors. Now let us think about all the multiple factors of any number.

Example 3: Is 18, a factor of 108?

1st method: If 18, a factor of 108, then 108 must be completely divisible by 18.

18)108(6
108
―――
0

Remainder is zero. Therefore, 18 is a factor of 108.

2nd method: (i) Find out the prime factors of 18.
(ii) Find out the prime factors of 108.

Solution:

18 = 2 × 3 × 3
108 = 2 × 2 × 3 × 3 × 3.

Since all the numbers included in the prime factors of 18 are also the prime factors of 108 and they are repeated more times in 108, therefore 18 is a factor for 108.

By all factors of any number, we mean that 1, the number itself and all those numbers that completely divide the number for which we are finding factors.

Example 4: Write down all the factors of 18.

1st Method: As you already know that 18 gets divided by these numbers 1, 18, 2, 3, 6 and 9, remainder is zero.

Therefore, 1, 18, 2, 3, 6, 9 are the factors or dividends if 18.

2nd Method: Here all the dividends of 18 can be found out like this too.

18 = 1 × 18
18 = 2 × 9
18 = 3 × 6.

Therefore the divisors or factors of 18 would be 1, 2, 3, 6, 9, 18.

Example 5:

Write down all the factors of 60.

Solution:

$60 = 1 \times 60$

$60 = 2 \times 30$

$60 = 3 \times 20$

$60 = 4 \times 15$

$60 = 5 \times 12$

$60 = 6 \times 10$

Therefore the factors of 60 should be 1, 2, 3, 4, 5, 6, 10, 12, 20, 30, 60.

Practice 3

1. Write down all the dividends (factors) of the following numbers.

   (i) 28 (ii) 36 (iii) 45 (iv) 72

You might have found that determining factors takes time. But now you don't know the rules of verifying divisibility, because of which without equally dividing the number, you are not able to say whether a number can be divided by 3, 5, 7 etc. or not.

Let us now learn some techniques for verifying divisibility.

Verification Rule of Divisibility

1. Verification of Divisibility by 2

If the unit's place of any number has 0, 2, 4, 6 and 8, then this number is completely divisible by 2.

20, 62, 34, 26, 18, ... are divisible by 2.

21, 63, 33, 35, 17, ... are not divisible by 2.

here, 18 is divisible by 2. Let us verify this by division.

$$\begin{array}{r} 2)18(9 \\ -18 \\ \hline 0 \end{array}$$

Therefore, it is completely divisible by 2.

$$\begin{array}{r} 2)\,21(10 \\ -2 \\ \hline 01 \\ 00 \\ \hline 1 \end{array}$$

Remainder is 1

Therefore, it is not completely divisible by 2.

2. Verification of divisibility by 3

If the sum of all the digits of a number is divisible by 3, then the number is divisible by 3. For example: In 111111,the sum of all the digits would be 1 + 1 + 1 + 1 + 1 + 1 = 6, therefore the number is divisible by 3.

Similarly in 5112 the sum of all the numbers 5 + 1 + 1 + 2 = 9 and this number is divisible by 3.

The digits in 412 will give the sum 7, therefore this number will not be divisible by 3.

3. Verification of divisibility by 6

If any number is divisible by 2 and 3 separatly, then the number will be divisible by 6.
216, it is divisible by 2 (The digit in unit's place is 2)
It is divisible by 3 .(The sum of digits is 9)
So, it will be divisible by 6.
643212, is divisible by 2 (because the digit in unit's a place is 2)
is divisible by 3 (because the sum of digits is 18).

4. Verification of divisibility by 9

If the sum of its digits is divisible by 9, then the number will be divisible by 9.
The number 3663, is divisible by 9 (because the sum is 3 + 6 + 6 + 3 = 18, divisible by 9).
1827, is divisible by 9 (the of digits is 18, which is divisible by 9).
1227, is divisible by 9 (the of digits is 12, which is not divisible by 9).

5. Verification of divisibility by 5

If the digit in the unit's place is 0 or 5, the number will be divisble by 5.
e.g. 1045, is divisible by 5 (because the digit in the unit's place is 5).
940, is divisible by 5 (because the digit in the unit's place is 0).

6. Verification of divisibility by 10

If any number has 0 in its unit's place, then the number is divisible by 10.
Example:
1000, is divisible by 10 (digit in the unit's place is 0).
2130, is divisible by 10 (digit in the unit's place is 0).
5003, is not divisible by 10 (digit in the unit's place is 3).

7. Verification of divisibility by 4

If the number made by the ten's and unit's place digits of any number is divisible by 4 or the ten's and unit's place has zero, then the number is divisible by 4.

For example:

In 79412, the digits in ten's and unit's place are 1 and 2, so the number made by these two digits is 12. Since 12 is divisible by 4, therefore, 79412 will be divisible by 4.

1300 will be divisible by 4 (because the digits in ten's and unit's place are 0).

413 will not be divisible by 4 (because the digits 13 is not completely divisible by 4).

8. Verification of divisibility by 8

If the number made by unit's, ten's and hundredth places is divisible by 8, or the number contains 0 in all these three places, then the number would be divisible by 8.

31000	(divisible by 8)
1816	(divisible by 8, because 816 is divisible by 8)
12317	(not divisible by 8, because 317 is not divisible by 8)

9. Verification of divisibility by 7

Take a number and double its last digit. Now subtract this doubled number from the rest of the digits of the original number.

Repeat the process till the result is a digit like 1 or 2. If the obtained number is divisible by 7, then the original number also be divisible by 7.

In 1729, the last digit is 9, twice of 9 is 18

172 - 18 = 154

In 154, the last digit is 4, twice of 4 is 8

15 - 8 = 7, 7 is the last digit.

Therefore the number will be divisible by 7.

Do you know that 1729 is also known as the Ramanujan Number?

When the great Indian mathematician, Ramanujan was in England, he once became very ill. Prof. Hardy met Ramanujan in the hospital and this was the dialogue that followed-

Ramanujan :	How did you come here, sir?
Prof. Hardy :	By a taxi.
Ramanujan :	What was the number of the taxi?
Prof. Hardy :	1729, not a very special number.
Ramanujan :	No, professor, you're mistaken.
	Its very interesting number that can be written as the sum of cubes of two numbers. In two different ways.
	i.e. $1729 = 1^3 + 12^3$
	$= 9^3 + 10^3$

10. Verification of divisibility by 11

For any number find out the sum of the digits in the odd places and the sum of the digits in the even places. If the difference between the sum of digits at odd places and the sum of digits in even places are 0, 11 or multiple of 11, then the number would be divisible by 11.

Example: In 856592,

the sum of digits in odd places

$= 8 + 6 + 9 = 23$

the sum of digits in even places

$= 5 + 5 + 2 = 12$

The difference between the two sums :

$23 - 12 = 11$

Therefore, the number is divisible by 11.

Example 6 :

Verify whether the number 805130425 is divisible by 11.

Solution :

In 805130425,

(1) The sum of digits in odd places $= 8 + 5 + 3 + 4 + 5 = 25$

(2) The sum of digits in even places $= 0 + 1 + 0 + 2 = 3$

The difference in the sums $= 25 - 3$

$= 22$

22 is divisible by 11, therefore, the number 805130425 is divisible by 11.

Practice 4

1. Put the ✓ mark on divisible number:

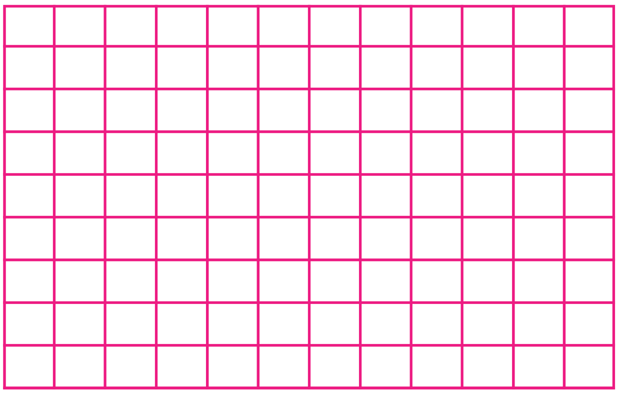
S.No.	Number	Number by which the number gets competely divided									
		1	2	3	4	5	6	7	8	9	10
1.	2550	✓	✓	✓		✓	✓				✓
2.	4914										
3.	9432										
4.	7332										
5.	13310										
6.	872										
7.	1210										

2. By which number is 27720 divisible?
3. Determine which of the statements are true or false.
   (1) 78 can be divided by 2.
   (2) 375 is completely divisible by 3, 5 and 10.
   (3) The number in which unit's place is 0, will be divisible by 5.
   (4) If the difference between sum of the digits in the even places and the sum of the digits in the odd places is zero, then the number would be divisible by 11.
   (5) The number 10080 is completely divisible by 2, 3, 4,5, 6, 7, 8, 9 respectively.

Highest Common Factor

ACTIVITY 4

Suppose you need to prepare a wooden scale to measure a 12 feet long and 9 feet wide room. What would be the maximum length of the long scale which can be used to measure both the length.



What lengths of scales can be used to measure this length of 12 feet. You will find that the length of 12 feet can be measured by scales that are 1, 2, 3, 4, 6 or 12 feet long. Note that all these numbers are multiple factors of 12.

Similarly, the measure of 9 feet can be taken by scales that are 1, 3 or 9 feet long. These are all the factors of 9. But we actually need a big scale that would measure both the lengths 12 feet and 9 feet. Note that the largest common factor for both 12 and 9 would be 3.

Since three is the highest common factor for 12 and 9; this is also known as the **Highest Common Factor** or **H.C.F.**

Let us find out the H.C.F. of some more numbers with the help of divisions or factors.

Example 7:

All multiple factors of 48	1, 2, 3, 4, 6, 8, 12, 24, 48
Multiple factors of 64	1, 2, 4, 8, 16, 32, 64
Multiple factors of 72	1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

Encircle all the common factors and you'll see that the common (even) divisors are 1, 2, 4 and 8.

Therefore, the highest common factor for 48, 64 and 72 is 8.

This means 8 is the largest number that completely divides the numbers 48, 64 and 72. Hence the H.C.F. of 48, 64 and 72 is 8.

The highest or greatest common factor (divisor) out of the common factors for two or more than two number which completely divides the given number is known as the highest common factor for those numbers. This means,

H.C.F. = The largest equivalent or like factor

Method of Determining the H.C.F.

Example 8 : Find out the H.C.F. of 24, 36 and 60.

(1) The prime factor method :

24

2	24
2	12
2	6
3	3
	1

$24 = 2 \times 2 \times 2 \times 3$

36

2	36
2	18
3	9
3	3
	1

$36 = 2 \times 2 \times 3 \times 3$

60

2	60
2	30
3	15
5	5
	1

$60 = 2 \times 2 \times 3 \times 5$

Therefore the common multiple factor for 24, 36, 60 would be :

$$24 = 2 \times 2 \times 2 \times 3$$

$$36 = 2 \times 2 \times 2 \times 3$$

$$60 = 2 \times 2 \times 3 \times 5$$

$$= 2 \times 2 \times 3 = 12$$

(2) The factorisation method :

The factors of 24 — 1, 2, 3, 4, 6, 8, 12, 24

The factors of 36 — 1, 2, 3, 4, 6, 9, 12, 18, 36

The factors of 60 — 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Therefore, the common factors of 24, 36, 60 are 1, 2, 3, 4, 6, 12.

Highest common divisor = 12

Therefore, H.C.F. = 12

(3) The Division Method :

There are two methods in this process.

First Method :

Example 9 : Find out the H.C.F. for 16 and 36.

2	16, 36
2	8, 18
	4, 9

Steps:

(1) Divide 16 and 36 by the smallest prime number 2.

(2) Divide 8 and 18 by 2.

(3) It is not possible to divide 4 and 9 by any prime number.

(4) The highest common factor will be the product of these prime numbers which can act as divisor for all numbers.

Hence H.C.F. = $2 \times 2 = 4$

Example 10: Find out the highest common factor for 60,90 and 210.

2	60, 90, 210
3	30, 45, 105
5	10, 15, 35
	2, 3, 7

Steps:

(1) Divide 60, 90, 210 by 2.

(2) Divide 30, 45, 105 by 3.

(3) Divide 10, 15, 35 by 5.

Since 2, 3, 7 cannot be divided by any other prime number. Therefore,

H.C.F. = $2 \times 3 \times 5 = 30$

<u>Second Method</u> :

To find out the H.C.F. by this method, divide the bigger number by the smaller number till a remainder obtained is smaller than the divisor. Now consider the divisor as the dividend (number to be divided) and the remainder as the divisor and find the solution. Repeat this process till you get zero as the remainder. The divisor that gives a zero remainder will be the H.C.F. for the number. Let us work at these examples -

Example 11: Find out the H.C.F. of 15 and 63.

Solution :

```
15) 63 (4          Then,     3) 15 (5
   – 60                        – 15
   ----                        ----
      3                           0
```

Therefore, 3 is the highest commom factor for 15 and 63.

Example 12: Find out the largest common number which when used to divide 18 and 55, would give 2 and 3 as the remainder respectively.

Solution: Since we get remainder 2 when 18 is divided by the number to be found out. Therefore, the number is 18 - 2 = 16.

Similarly, when the remainder is 3, the other number would be 55 - 3 = 52.

This can be verified by finding out the H.C.F. of 16 and 52.

```
16) 52 (3
   – 48
   ----
      4) 16 (4
        – 16
        ----
           0
```

Thus, the H.C.F. of 16 and 52 is 4.

The Properties / Characteristics of Highest Common Factors

Let us understand the characteristics of H.C.F. through a few examples.

Example 13: Find out the H.C.F. of 15 and 60.

Since 60 gets completely divided by 15, therefore 15 will be the H.C.F. of 15 and 60.

Example 14: Determine the H.C.F. of 12, 36.

Since 36 is completely divisible by 12, therefore 12 will be the H.C.F. for 12, 36.

Example 15: Find out H.C.F. for 20, 40.

Since 40 is completely divisible by 20, therefore, the H.C.F of 20 and 40 is 20.

Example 16: Find the H.C.F.for the number 15, 16 and 13, 17.

Except 1 there is no common multiple factor for 15 and 16 except 1. Therefore, the H.C.F. for 15, 16 will be 1.

Similarly except 1 there is no common multiple factors for 13 and 17. Therefore the H.C.F. for 13 and 17 will be 1.

Properties :

(1) For two given numbers, if the bigger number is completely divisible by the smaller number, then the smaller number is the H.C.F. for the two numbers.

(2) The H.C.F. for those numbers which do not have any other common factor except 1, is 1.

EXERCISE 6.1

(1) Find out H.C.F. for the following with the help of prime factorisation method.

(i)	120, 104	(ii)	144, 198
(iii)	150, 140, 210	(iv)	108, 135, 162

(2) Find out H.C.F. by the division method.

(i)	252, 576	(ii)	300, 450
(iii)	72, 96, 144	(iv)	120, 300, 105

(3) What would be the H.C.F. for two consecutive numbers?

(4) Divide the numerator and denominators of the given fractions by there highest common factors and write the fractions in simplified forms.

(i)	1444/256	(ii)	2211/3025

(5) Two small tankers contain 85 and 68 liters of petrol respectively. Find out the maximum capacity of the measuring vessel with which the total quantity of petrol can be measured.

(6) Find out the largest divisor for the numbers 389, 436 and 542 that would give 4, 7 and 3 respectively as remainders.

(7) In classes (VI, VII and VIII) 6, 7, and 8 of a school, there are 220, 176 and 132 students respectively. Find the maximum number of students that can be included in a group so that the groups have equal number of students and can be made in each class.

(8) Hamida has 527 apples, 646 chickoos and 748 oranges. These fruits have to be distributed in equal heaps. What would be the number of fruits in the largest heap? How many such heaps will be formed?

(9) A rectangular room is 122 m long and 92 m wide from outside. If its walls are 1 m thick then find out the maximum length of a scale that can be used to measure the inner length and width of that room.

Hints : Inner length = 122 - 2 = 120 m.
Inner width = 92 -2 = 90 m.
(Determine the H.C.F. of 120 and 90)

Multiples

You must have learnt multiplication tables in your previous classes. you have also used tables while doing multiplications and divisions.

You know that $7 \times 3 = 21$, which means that the product 7 and 3 is 21, i.e. both 7 and 3 have a multiple 21.

All the number that occur in the multiplication table of 2 are the multiples of 2. Similarly 13, 26, 39, 52, 65, 78 etc. are all multiples of 13.

ACTIVITY 5

Below are given some numbers, write down their first five multiples in the boxes provided.

Multiple of	4	4	8	12	16	20
Multiple of	7					
Multiple of	10					
Multiple of	12					
Multiple of	15					
Multiple of	16					
Multiple of	20					

Lowest Common Multiple

We find several examples of lowest common multiples in our everyday life. Let us discuss some such situations.

Example 17 :

Ram went to the market to buy fruits. The shopkeeper showed him two types of bananas. The first type of bananas were Rs.10 for 6 bananas and the second kind were Rs. 10 for 8 bananas. Ram wanted to buy equal number of both the kinds of bananas using some 10 rupee note without getting any money as change. How many bananas of both kinds can he buy?

Solution : To find this out, we shall write down the multiples of 6 and 8 first.

Multiples of 6 : 6, 12, 18,24 30, 36, 42, 48, 54, 60, 66, 72, 78.

Multiples of 8 : 8, 16, 24, 32, 40, 48, 56, 64, 72

This means that Ram can have 6, 12, 18, 24,................. numbers of bananas of the first kind without getting any change for one, two, three and four 10-10 rupee notes and so on.

Similarly he can buy 8, 16, 24,.........etc. numbers of bananas of the second kind with 10-10 rupee notes without getting any change. To buy the same number of both type of bananas, he will have to buy the number of bananas that are common to both groups.

This means, he will have to take the number of bananas that is common for multiples of both 6 and 8.

These number are 24, 48, 72 etc.(These number have been encircled)

To buy the same number of bananas of both kinds 24, 48 or 72 bananas can be purchased with the help of 10 rupee notes.

Let us took at another example.

Example 18 :

A shopkeeper wants to purchase pens for his shop from the wholesale market. He chooses two kinds of pen. The first type of pen is available in 12 pieces per packet and the second type of pen as 15 pieces per packet. The wholesale dealer doesn't sell the loose pens, that is, he doesn't open the packets. Can you find out the minimum number of packets the shopkeeper will need to buy so that the number of each type of pen bought is equal?

Let us solve this problem with the help of a table.

No. of pens in the packets	Total number of pens				
	In 1 packet	In 2 packets	In 3 packets	In 4 packets	In 5 packets
12	12	24	36	48	60
15	15	30	45	60	75

Now you can see that if 5 packets containing 12 pens each are bought, the shopkeeper will have 60 pens and if 4 packets containing 15 pens each are taken, then again they'll be 60 pens.

To solve the abo ve problem, you have found the multiple of 12 and 15 and the common multiple which is the smallest among these, is the expected answer.

Smallest Common Multiple is called the Lowest Common Multiple or L.C.M.

Can you find the lowest common multiple of 10, 12 and 15.
Write down the multiples of 10 = ...
Write down the multiples of 12 = ...
Write down the multiples of 15 = ...
The smallest common multiple or the lowest common multiple = ...
Find out the L.C.M. of the numbers in table given below.

S. No.	Numbers	The multiples of the numbers	L.C.M.
1.	3, 5, 6	Multiples of 3 = 3, 6, 9, 12, 15, 18, 21 24, 27, 30 Multiples of 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50 Multiples of 6 = 6, 12, 18, 24, 30, 36, 42, 48, 54, 60	30
2.	4, 6, 9		
3.	4, 9, 12		
4.	6, 15, 18		

So, we can say that

(1) The Lowest Common Multiple (LCM) of two or more given numbers is the lowest (or smallest or least) of their common multiples.

(2) L.C.M. of two or more than two numbers is that smallest number which can be divided by each of the given numbers.

Method of Determining L.C.M.

(1) Prime Factorisation Method

Example 19 : (i) Find out the prime factors of the given numbers : 16, 24.

16

$$\begin{array}{c|l} 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$16 = 2 \times 2 \times 2 \times 2$

24

$$\begin{array}{c|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$24 = 2 \times 2 \times 2 \times 3$

(ii) Take the smallest prime factors out of all these numbers. Write down the factors occuring maximum number of times in any number.

(iii) Choose the next greater prime factor and write down the factor occurs maximum number of times in any number.

(iv) Similarly, write all prime factors. Their multiplication will give L.C.M.

Thus $16 = 2 \times 2 \times 2 \times 2$

$24 = 2 \times 2 \times 2 \times 3$

The lowest prime factor 2 occures 4 times in 16. The lowest prime factor 3 occures only once in 24.

L.C.M. = $2 \times 2 \times 2 \times 2 \times 3$

$= 48$

(2) The Division Method

Example 20 : Find out the L.C.M. of 12, 16, 24.

2	12, 16, 24	All numbers are divisible by 2.
2	6, 8, 12	All numbers are divisible by 2.
2	3, 4, 6	Two numbers are divisible by 2.
2	3, 2, 3	One numbers are divisible by 2.
3	3, 1, 3	Two numbers is divisible by 3.
	1, 1, 1	

The product of all the divisors is the L.C.M.

$2 \times 2 \times 2 \times 2 \times 3 = 48$

Relationship Between L.C.M. and H.C.F. and the Product of two Numbers

Example 21: Consider two numbers 12 and 16.

Let us multiply the two numbers, where the first number is 12 and the second number is 16.

Multiplication of the two numbers $= 1^{st}$ number $\times$ 2^{nd} number

$= 12 \times 16$

$= 192$

Now we shall find the H.C.F. and L.C.M. of the two numbers also.

H.C.F.

2	12, 16
2	6, 8
	3, 4

H.C.F. $= 2 \times 2 = 4$

L.C.M.

2	12, 16
2	6, 8
2	3, 4
2	3, 2
3	3, 1
	1, 1

L.C.M. $= 2 \times 2 \times 2 \times 2 \times 3 = 48$

Therefore, H.C.F. × L.C.M.

= 4 × 48 = 192

The product obtained in both the situations for the numbers 12 and 16 is the same (192).

So, we can now say that

First number × second number = H.C.F. × L.C.M.

or **Product of two numbers = H.C.F. × L.C.M.**

ACTIVITY 6

Verify the above stated relationship by finding H.C.F. and L.C.M. of the numbers given in the table.

First Number	Second Number	H.C.F.	L.C.M.	H.C.F. × L.C.M.	1^{st} no. × 2^{nd} no.
6	8	2	24	2 × 24 = 48	6 × 8 = 48
4	9				
30	36				
42	48				
108	18				
21	105				

EXERCISE 6.2

Oral Questions :

(1) The H.C.F. of two numbers is 2 and their L.C.M. is 12. If one number is 6. What will be the other number?

(2) If the product of two numbers are 338, what is the product of their L.C.M. and H.C.F.?

(3) What would be the L.C.M. of 2, 6, 8?

(4) Is the L.C.M. of 7 and 14 greater than 7 or smaller than 7?

(5) Can the L.C.M. of 15 and 30 be lesser than 30?

Written Questions :

(1) Find the L.C.M. of the following by factorisation method?

(i) 14, 28 (ii) 108, 162

(iii) 12, 15, 45 (iv) 40, 36, 126

(2) Find the L.C.M. of the following by division method?

(i) 28, 56 (ii) 112, 168

(iii) 36, 45, 72 (iv) 180, 184, 144

(3) On a field 55m long and 22m wide square shaped matresses have to be spread out. Find out the minimum number of same size matresses, that would be needed to cover field.

(4) If six bells starts ringing together and if they ring at intervals of 1, 2, 4, 6, 8, 10 and 12 seconds in a sequence, then how many times would they ring together in 30 minutes?

(5) A trader goes to Raipur every fourth day while another trader goes to Raipur every tenth day. If both the traders are goning to Raipur together on 3 January, what will be the next date on which they would go to Raipur together?

(6) If two numbers 24 and 36 have the H.C.F. 12, find out their L.C.M.

(7) The H.C.F. of two numbers is 13 and their L.C.M. is 1989. If one of the number is 117, find out the other number

(8) Shashank saves 4.65 rupees everyday. In atleast how many days, will he be able to save the complete amount of rupee value?

[**Hint** - In Rs. 4.65, Rs. 4 is in whole number. Now find the L.C.M. of 65 paise and 100 paise. Then number of whole days = $\frac{\text{L.C.M}}{65}$, which represent total amount in rupees]

(9) Can two numbers have a H.C.F. of 14 and L.C.M. of 204? Give reasons for your answer.

(10) On a particular day buses from Ratanpur to Raipur run at intevals of every 40 minutes and buses from Raipur to Ratanpur run every 45 minutes. If buses from two opposite directions cross each other on a bridge at 10.15 a.m., then at what time at the earliest next buses from opposite directions will cross each other at the bridge.

[**Hints :** L.C.M. of 40 and 45 = 360 minutes

$$\frac{360}{60} = 6 \text{ hours}$$

$$10.15 + 6.00 = 16.15$$

$$16.15 - 12.00 = 4.15 \text{ p.m.}]$$

What Have We Learnt ?

(1) A number is totally divisible by its factor.

(2) The multiples of a number is completely divisible by that number.

(3) Every number is a multiple and factor of itself.

(4) 1 is the factor for all numbers and it is neither prime nor composite.

(5) Only 2 is even prime number.

(6) The H.C.F. of two or more numbers is its greatest common factor.

(7) The L.C.M. of two or more given numbers is their lowest or smallest of their common multiples.

(8) The product of two numbers is equal to the product of their H.C.F. and L.C.M.

(9) All the multiple of 2 are even numbers.

(10) Those numbers which are not multiples of 2, are odd numbers.

(11) The H.C.F. of two numbers is one factor of their L.C.M.

(12) The H.C.F. of numbers, cannot be greater than the numbers themselves.

(13) The L.C.M. of numbers cannot be smaller than the numbers themselves.

(14) The numbers which have only one common factor (1) are called Co prime numbers