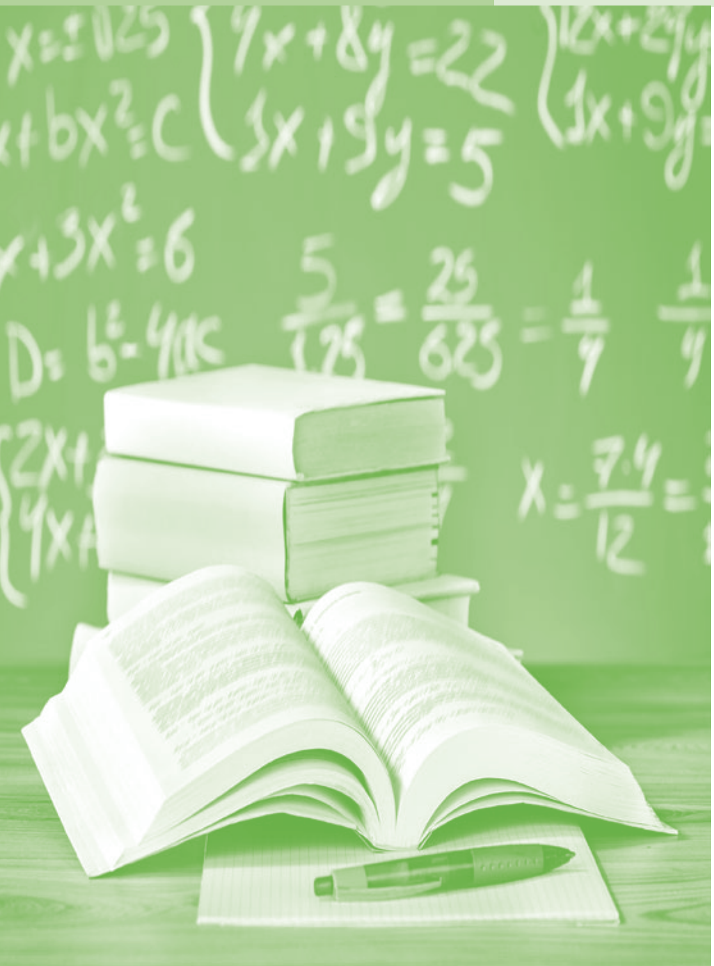
Chapter 16

Trigonometry



REMEMBER

Before beginning this chapter, you should be able to:

- Know the different types of angles and their values
- Understand the definitions of triangles and their types

KEY IDEAS

After completing this chapter, you should be able to:

- Know the systems of measurement of angle and their inter-relations
- Understand trigonometric ratios and identities
- Learn the trigonometric ratios of compound angles

INTRODUCTION

The word trigonometry is originated from the Greek word ‘tri’ means three, ‘gonia’ means angle and ‘metron’ means measure. Hence, the word trigonometry means three angle measure, i.e., it is the study of geometrical figures which have three angles, i.e., triangles.

The great Greek mathematician Hipapachus of 140 BCE gave relation between the angles and sides of a triangle. Further trigonometry is developed by Indian (Hindu) mathematicians. This was migrated to Europe via Arabs.

Trigonometry plays an important role in the study of Astronomy, Surveying, Navigation and Engineering. Now a days it is used to predict stock market trends.

ANGLE

A measure formed between two rays having a common initial point is called an angle. The two rays are called the arms or sides of the angle and the common initial point is called the vertex of the angle.

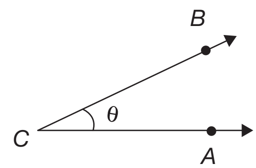


Figure 16.1

In the Fig. 16.1, OA is said to be the initial side and the other ray OB is said to be the terminal side of the angle.

The angle is taken positive when measured in anti-clockwise direction and is taken negative when measured in clockwise direction (see Fig. 16.2).

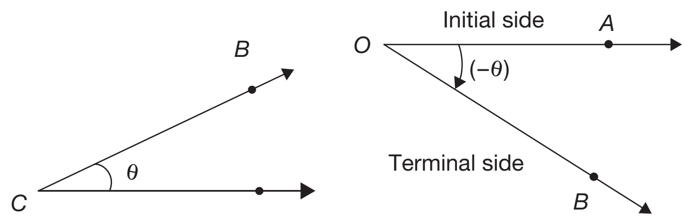


Figure 16.2

Systems of Measurement of Angle

We have the following systems of the measurement of angle.

Sexagesimal System

In this system, the angle is measured in degrees (°).

Degree When the initial ray is rotated through $\left(\frac{1}{360}\right)$ of one revolution, we say that an angle of one degree (1°) is formed at the initial point. A degree is divided into 60 equal parts and each part is called one minute (1 m).

Further, a minute is divided into 60 equal parts called seconds (″).
So, 1 right angle = 90°
$1^\circ = 60'$ (minutes) and
$1' = 60''$ (seconds)

Note This system is also called the British system.

Centesimal system

In this system, the angle is measured in grades.

Grade When the initial ray is rotated through $\left(\frac{1}{400}\right)$ of one revolution, an angle of one grade is said to be formed at the initial point. It is written as 1^g.

Further one grade is divided into 100 equal parts called minutes and one minute is further divided into 100 equal parts called seconds.

So, 1 right angle = 100^g
$1^g = 100'$ (minutes) and
$1' = 100''$ (seconds)

Note This system is also called the French system.

Circular System

In this system, the angle is measured in radians.

Radian The angle subtended by an arc of length equal to the radius of a circle at its centre is said to have a measure of one radian (see Fig. 16.3). It is written as 1^c.

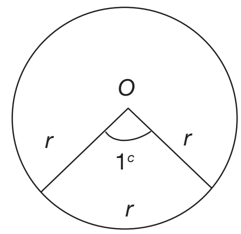


Figure 16.3

Note This measure is also known as radian measure.

Relation Between the Units of the Three Systems

When a rotating ray completes one revolution, the measure of angle formed about the vertex is $360°$ or 400^g or $2\pi^c$.

So,

$$360° = 400^g = 2\pi^c$$

(or)

$$90° = 100^g = \frac{\pi^c}{2}.$$

For convenience, the above relation can be written as,

$$\frac{D}{90} = \frac{G}{100} = \frac{R}{\frac{\pi}{2}}.$$

Where, D denotes degrees, G grades and R radians.

Remember

1. $1° = \frac{\pi}{180}$ radians = 0.0175 radians (approximately).
2. $1^c = \frac{180}{\pi}$ degrees = $57°17'44''$ (approximately).

Notes

1. The measure of an angle is a real number.
2. If no unit of measurement is indicated for any angle, it is considered as radian measure.

EXAMPLE 16.1

Convert 60° into circular measure.

SOLUTION

Given, $D = 60°$

We have, $\dfrac{D}{90} = \dfrac{R}{\frac{\pi}{2}}$

So, $\dfrac{60}{90} = \dfrac{R}{\frac{\pi}{2}}$

$$\frac{2}{3} \times \frac{\pi}{2} = R$$

$$R = \frac{\pi}{3}.$$

Hence, 60° in circular measure is $\dfrac{\pi}{3}$.

EXAMPLE 16.2

Convert 180^g into sexagesimal measure.

SOLUTION

Given, $G = 180^g$

We have, $\dfrac{D}{90} = \dfrac{G}{100}$

So, $\dfrac{D}{90} = \dfrac{180}{100}$

$$D = \frac{9}{5} \times 90 = 162.$$

Hence, sexagesimal measure of 180^g is 162°.

EXAMPLE 16.3

The angle measuring $\dfrac{\pi}{4}$ when expressed in sexagesimal measure is _________.

SOLUTION

Given, $R = \dfrac{\pi}{4}$.

We have, $\dfrac{D}{90} = \dfrac{R}{\frac{\pi}{2}}$.

So, $\dfrac{D}{90} = \dfrac{\frac{\pi}{4}}{\frac{\pi}{2}}$

$$D = \frac{2}{4} \times 90 = 45°$$

Hence, the sexagesimal measure of $\dfrac{\pi^c}{4}$ is 45°.

TRIGONOMETRIC RATIOS

Let AOB be a right triangle with $\angle AOB$ as 90°. Let $\angle OAB$ be θ. Notice that $0° < \theta < 90°$. That is, θ is an acute angle (see Fig. 16.4).

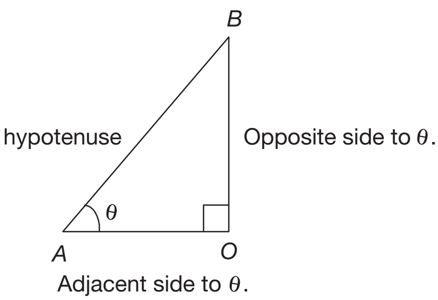


Figure 16.4

We can define six possible ratios among the three sides of the triangle AOB, known as trigonometric ratios. They are defined as follows.

1. Sine of the angle θ or simply $\sin\theta$:

$$\sin\theta = \frac{\text{Side opposite to angle } \theta}{\text{Hypotenuse}} = \frac{OB}{AB}.$$

2. Cosine of the angle θ or simply $\cos\theta$:

$$\cos\theta = \frac{\text{Side adjacent to angle } \theta}{\text{Hypotenuse}} = \frac{OA}{AB}.$$

3. Tangent of the angle θ or simply $\tan\theta$:

$$\tan\theta = \frac{\text{Side opposite to } \theta}{\text{Side adjacent to } \theta} = \frac{OB}{OA}.$$

4. Cotangent of the angle θ or simply $\cot\theta$:

$$\cot\theta = \frac{\text{Side adjacent to } \theta}{\text{Side opposite to } \theta} = \frac{OA}{OB}.$$

5. Cosecant of the angle θ or simply $\operatorname{cosec}\theta$:

$$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Side opposite to } \theta} = \frac{AB}{OB}.$$

6. Secant of the angle θ or simply $\sec\theta$:

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \theta} = \frac{AB}{OA}.$$

Observe that,

1. $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\sec\theta = \dfrac{1}{\cos\theta}$ and $\cot\theta = \dfrac{1}{\tan\theta}$.

2. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$.

EXAMPLE 16.4

If $\cos\theta = \frac{3}{5}$, then find the values of $\tan\theta$, $\operatorname{cosec}\theta$.

SOLUTION

Given, $\cos\theta = \frac{3}{5}$

Let PQR be the right triangle such that $\angle QPR = \theta$ (see Fig. 16.5)

Assume that $PQ = 3$ and $PR = 5$.

Then, $QR = \sqrt{PR^2 - PQ^2} = \sqrt{25 - 9} = 4$.

So, $\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{QR}{PQ} = \frac{4}{3}$ and $\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{PR}{QR} = \frac{5}{4}$.

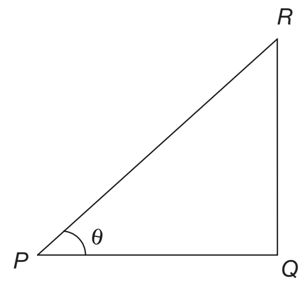


Figure 16.5

Pythagorean Triplets

1. 3, 4, 5
2. 5, 12, 13
3. 8, 15, 17
4. 7, 24, 25
5. 9, 40, 41

Trigonometric Identities

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\sec^2\theta - \tan^2\theta = 1$
3. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Values of Trigonometric Ratios for Specific Angles

Trigonometric Ratios	Angle 0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

From the above table, we observe that

1. $\sin\theta = \cos\theta$, $\tan\theta = \cot\theta$ and $\sec\theta = \operatorname{cosec}\theta$, if $\theta = 45°$.
2. $\sin\theta$ and $\tan\theta$ are increasing functions in $0° \le \theta \le 90°$.
3. $\cos\theta$ is a decreasing function in $0° \le \theta \le 90°$.

EXAMPLE 16.5

Find the value of $\sin 60° + 2\tan 45° - \cos 30°$.

SOLUTION

$$\sin 60° + 2\tan 45° - \cos 30°$$

$$= \frac{\sqrt{3}}{2} + 2(1) - \frac{\sqrt{3}}{2} = 2$$

$$\therefore\ \sin 60° + 2\tan 45° - \cos 30° = 2.$$

EXAMPLE 16.6

Using the trigonometric table and evaluate

(a) $\sin^2 45° + \cos^2 45°$

(b) $\sec^2 30° - \tan^2 30°$

SOLUTION

(a) $\sin^2 45° + \cos^2 45°$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} + \frac{1}{2} = 1.$$

Hence, $\sin^2 45° + \cos^2 45° = 1$.

(b) $\sec^2 30° - \tan^2 30°$

$$= \left(\frac{2}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= \frac{4}{3} - \frac{1}{3} = \frac{3}{3} = 1.$$

Hence, $\sec^2 30° - \tan^2 30° = 1$.

EXAMPLE 16.7

Find the values of $\dfrac{\tan 30° + \tan 60°}{1 - \tan 30° \tan 60°}$ and $\tan 90°$; what do you observe?

SOLUTION

$$\frac{\tan 30° + \tan 60°}{1 - \tan 30° \tan 60°}$$

$$= \frac{\frac{1}{\sqrt{3}} + \sqrt{3}}{1 - \frac{1}{\sqrt{3}} \times \sqrt{3}} = \frac{\frac{1+3}{\sqrt{3}}}{1-1} = \frac{4}{0}$$

= not defined or ∞.

$= \tan 90°$

Hence, $\dfrac{\tan 30° + \tan 60°}{1 - \tan 30° \tan 60°} = \tan 90°$.

Trigonometric Ratios of Compound Angles

1. $\sin(A + B) = \sin A \cos B + \cos A \sin B$ and
   $\sin(A - B) = \sin A \cos B - \cos A \sin B$.
2. $\cos(A + B) = \cos A \cos B - \sin A \sin B$ and
   $\cos(A - B) = \cos A \cos B + \sin A \sin B$.
3. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ and

   $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$.

Also, by taking $A = B$ in the above relations, we get,

1. $\sin 2A = 2\sin A \cos A$.
2. $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$.
3. $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$.

EXAMPLE 16.8

Find the value of $\sin 15°$.

SOLUTION

We have, $\sin 15° = \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$\therefore \sin 15° = \dfrac{\sqrt{3} - 1}{2\sqrt{2}}$.

EXAMPLE 16.9

Find the value of $\tan 75°$.

SOLUTION

We have, $\tan 75° = \tan(45° + 30°)$

$$\frac{\tan 45° + \tan 30°}{1 - \tan 45° \cdot \tan 30°}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}.$$

$$= \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right) = \frac{3+1+2\sqrt{3}}{2} = 2+\sqrt{3}$$

$$\therefore \tan 75° = \frac{\sqrt{3}+1}{\sqrt{3}-1} \text{ or } 2+\sqrt{3}.$$

EXAMPLE 16.10

Eliminate θ from the equations $a = x \sec\theta$ and $b = y \tan\theta$.

SOLUTION

We know that trigonometric ratios are meaningful when they are associated with some θ, i.e., we cannot imagine any trigonometric ratio without θ. Eliminate θ means, eliminating the trigonometric ratios by using suitable identity.

Given, $a = x \sec\theta$ and $b = y \tan\theta$

$$\frac{a}{x} = \sec\theta \text{ and } \frac{b}{y} = \tan\theta.$$

We know that, $\sec^2\theta - \tan^2\theta = 1$.

So, $$\left(\frac{a}{x}\right)^2 - \left(\frac{b}{y}\right)^2 = 1$$

$$\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1.$$

Hence, the required equation is $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

EXAMPLE 16.11

Find the relation obtained by eliminating θ from the equations $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$.

SOLUTION

Given, $x = a\cos\theta + b\sin\theta$

$$x^2 = (a\cos\theta + b\sin\theta)^2$$

$$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta.$$

Also $y = a \sin\theta - b \cos\theta$

$$y^2 = a^2\sin^2\theta + b^2\cos^2\theta - 2ab \sin\theta \cos\theta$$

$$x^2 + y^2 = a^2 (\sin^2\theta + \cos^2\theta) + b^2 (\cos^2\theta + \sin^2\theta)$$

$$= a^2 + b^2.$$

Hence, the required relation is $x^2 + y^2 = a^2 + b^2$.

EXAMPLE 16.12

Eliminate θ from the equations $P = a \operatorname{cosec}\theta$ and $Q = a \cot\theta$.

SOLUTION

Given, $P = a \operatorname{cosec}\theta$ and $Q = a \cot\theta$

$$\frac{P}{a} = \operatorname{cosec}\theta \text{ and } \frac{Q}{a} = \cot\theta.$$

We know that, $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$\left(\frac{P}{a}\right)^2 - \left(\frac{Q}{a}\right)^2 = 1$$

$$P^2 - Q^2 = a^2.$$

Hence, the required relation is $P^2 - Q^2 = a^2$.

EXAMPLE 16.13

Eliminate θ from the equations $s = \sin\theta + \operatorname{cosec}\theta$ and $r = \sin\theta - \operatorname{cosec}\theta$.

SOLUTION

Given, $s = \sin\theta + \operatorname{cosec}\theta$ (1)

$r = \sin\theta - \operatorname{cosec}\theta$ (2)

Adding Eqs. (1) and (2), we get

$$s + r = 2 \sin\theta$$

$$\sin\theta = \frac{s+r}{2}. \quad (3)$$

Subtracting Eq. (2) from Eq. (1), we get

$$s - r = 2 \operatorname{cosec}\theta$$

$$\operatorname{cosec}\theta = \frac{s-r}{2}. \quad (4)$$

Multiplying Eqs. (3) and (4), we get

$$\sin\theta \cdot \operatorname{cosec}\theta = \left(\frac{s+r}{2}\right)\left(\frac{s-r}{2}\right)$$

$$\sin\theta \cdot \frac{1}{\sin\theta} = \frac{s^2 - r^2}{4}$$

$$1 = \frac{s^2 - r^2}{4} \quad \text{(or)} \quad s^2 - r^2 = 4.$$

Hence, by eliminating θ, we obtain the relation $s^2 - r^2 = 4$.

EXAMPLE 16.14

If $\sin(A+B) = \dfrac{\sqrt{3}}{2}$ and cosec $A = 2$, then find A and B.

SOLUTION

Given, $\sin(A+B) = \dfrac{\sqrt{3}}{2}$

$$\sin(A+B) = \sin 60°$$

$$A + B = 60° \quad (1)$$

$$\text{cosec } A = 2 = \text{cosec } 30°$$

$$A = 30° \quad (2)$$

From Eqs. (1) and (2), we have

$A = 30°$ and $B = 30°$.

EXAMPLE 16.15

Find the length of the chord which subtends an angle of 90° at the centre 'O' and which is at a distance of 6 cm from the centre.

SOLUTION

Let the chord be PQ and OR be the distance of chord from the centre of circle (see Fig. 16.6).

Given $\angle POQ = 90°$ and $OR = 6$ cm.

Clearly, $\Delta POR \cong \Delta QOR$, by SSS axiom of congruency.

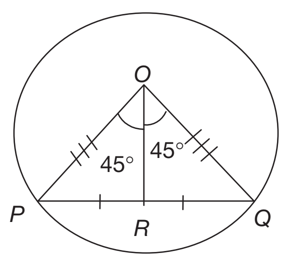


Figure 16.6

$$\angle POR = \angle QOR = \frac{1}{2}\angle POQ = 45°.$$

$$\text{In } \Delta POR, \tan 45° = \frac{PR}{OR}.$$

$$1 = \frac{PR}{6}$$

$$PR = 6 \text{ cm}.$$

$\therefore$ The length of the chord $PQ = 2\,PR = 12$ cm.

EXAMPLE 16.16

Evaluate $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$.

SOLUTION

Given $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

Rationalize the denominator, i.e.,

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}\cdot\frac{1-\cos\theta}{1-\cos\theta}}$$

$$=\sqrt{\frac{(1-\cos\theta)^2}{1+\cos^2\theta}}$$

$$=\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}} \quad (\because \sin^2\theta+\cos^2\theta=1)$$

$$=\frac{1-\cos\theta}{\sin\theta}$$

$$=\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}$$

$$=\text{cosec}\,\theta-\cot\theta\,.$$

$$\therefore \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\text{cosec}\,\theta-\cot\theta.$$

EXAMPLE 16.17

If $\sin\alpha$ and $\cos\alpha$ are the roots of the equation $ax^2 - bx - 1 = 0$, then find the relation between a and b.

SOLUTION

The given equation is $ax^2 - bx - 1 = 0$.

Here, $a = a$, $b = -b$ and $c = -1$.

$$\sin\alpha+\cos\alpha=\frac{-b}{a}=\frac{-(-b)}{a}=\frac{b}{a}$$

$$\sin\alpha\cdot\cos\alpha=\frac{c}{a}=\frac{-1}{a}.$$

Consider,

$$\sin\alpha+\cos\alpha=\frac{-b}{a}$$

Squaring on both sides,

$$(\sin\alpha+\cos\alpha)^2=\left(\frac{-b}{a}\right)^2$$

$$=1+2\left(\frac{-1}{a}\right)=\frac{b^2}{a^2}$$

$$=1-\frac{b^2}{a^2}=\frac{2}{a}$$

$$\therefore\ a^2-b^2=2a.$$

EXAMPLE 16.18

If $\cos\alpha = \frac{2}{3}$ and $\sin\beta = \frac{1}{4}$, then find $\cos(\alpha - \beta)$.

SOLUTION

Given,

$$\cos\alpha = \frac{2}{3} \Rightarrow \sin\alpha = \frac{\sqrt{5}}{3}$$

$$\sin\beta = \frac{1}{4} \Rightarrow \cos\beta = \frac{\sqrt{15}}{4}$$

$$\cos(\alpha - \beta) - \cos\,\alpha \cdot \cos\,\beta + \sin\,\alpha \cdot \sin\,\beta = \frac{2}{3} \times \frac{\sqrt{15}}{4} + \frac{\sqrt{5}}{3} \times \frac{1}{4}$$

$$\cos(\alpha - \beta) = \frac{2\sqrt{15} + \sqrt{5}}{12}.$$

EXAMPLE 16.19

Express the following as a single trigonometric ratio:

(a) $\sqrt{3}\cos\theta - \sin\theta$

(b) $\sin\theta - \cos\theta$

SOLUTION

(a) Given,

$$\sqrt{3}\cos\theta - \sin\theta$$
$$= 2\left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right)$$
$$= 2(\cos\theta \cdot \cos 30° - \sin\theta \cdot \sin 30°)$$
$$= 2(\cos(\theta + 30°))$$
$$\Rightarrow \sqrt{3}\cos\theta - \sin\theta = 2\cos(\theta + 30°).$$

(b) Here,

$$\sin\theta - \cos\theta$$
$$= \sqrt{2}\left(\frac{\sin\theta - \cos\theta}{\sqrt{2}}\right)$$
$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta\right)$$
$$= \sqrt{2}\left[\sin\theta\cos\left(\frac{\pi}{4}\right) - \cos\theta\sin\left(\frac{\pi}{4}\right)\right]$$
$$= \sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right). \ (\because \sin(A - B) = \sin A\cos B - \cos A\sin B)$$

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. The value of 144° in circular measure = _______.

2. The value of sin 30° · sin 45° · cosec 45° · cos 30° = _______.

3. If $\frac{1-\tan^2 60°}{1+\tan^2 60} = \cos X$, then the value of X is _______.

4. The value of log [(sec θ + tan θ) (sec θ − tan θ)] is _______.

5. If $\frac{\cos 13° + \sin 13°}{\cos 13° - \sin 13°} = \tan A$, then $A =$ _______.

6. If $\sin\theta = \frac{1}{2}$ and 0° < θ < 90°, then cos 2θ = _______.

7. If cosec θ − cot θ = x, then cosec θ + cot θ = _______.

8. The value of $\frac{\sin 20° \cos 70° + \cos 20° \sin 70°}{\sin 23° \text{coses} 23° + \cos 23° \sec 23°}$ is _______.

9. If $A + B = 45°$, then (1 + tan A)(1 + tan B) = _______.

10. If $\tan\theta = \frac{5}{6}$ and $\tan\phi = \frac{1}{11}$, then $\theta + \phi =$ _______.

11. If $A - B = 45°$ and $\tan A - \tan B = \sqrt{3}$, then $\tan A \cdot \tan B =$ _______.

12. If $\cos A + \sin A = \frac{1}{2(\cos A - \sin A)}$; (0° < A < 90°), then $\sin^2 A =$ _______.

13. The value of $(\sin A - \cos A)^2 + (\sin A + \cos A)^2$ is _______.

14. If $A + B = 60°$, then the value of $\sin A \cos B + \cos A \sin B =$ _______.

15. In ΔABC, the value of cos (A − B) − cos (C) is _______.

16. ABC is right triangle, right angled at A, then $\tan B \cdot \tan C =$ _______.

17. $\tan(A+B) = \sqrt{3}$ and $\sin A = \frac{1}{\sqrt{2}}$, then the value of B in radians is _______.

18. In ΔABC, the lengths of the three sides AB, BC and CA are 28 cm, 96 cm and 100 cm respectively. Find the value of cos C.

19. If sin A = cos B, where A and B are acute angles, then $A + B =$ _______.

20. ABC is a right isosceles triangle, right angled at B. Then $\sin^2 A + \cos^2 C =$ _______.

21. Evaluate: $\sin^2 60° \cos^2 45° \cos^2 60° \text{cosec}^2 90°$.

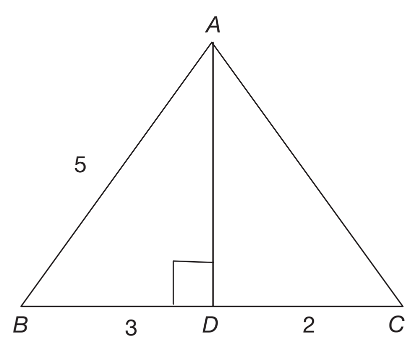
22. If $\sin\theta = \frac{3}{5}$ and θ is acute, then find the value of $\frac{\tan\theta - 2\cos\theta}{3\sin\theta + \sec\theta}$.

23. Find the values of the cos 15°.

24. Evaluate: $\text{cosec}^2 30° + \sec^2 60° + \tan^2 30°$.

25. Convert $\frac{\pi^c}{15}$ into the other two systems.

26. In the adjoining figure, find the values of tan B.



27. Simplify and express $\sec^4\alpha - \tan^4\alpha$ in their least exponents.

28. Convert $\left(\frac{200}{3}\right)^g$ into other two systems.

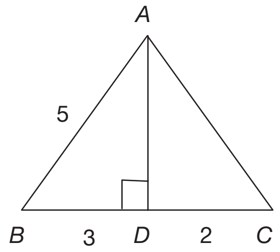
29. Convert 270° into other two systems.

30. Evaluate: $\cos 0° + \sqrt{2}\sec 45° - \sqrt{3}\tan 30°$.

PRACTICE QUESTIONS

Short Answer Type Questions

31. In the adjoining figure, find the value of sin C.



32. A wheel makes 200 revolutions in 2 minutes. Find the measure of the angle it describes at the centre in 24 seconds.

33. Find the length of the chord subtending an angle of 120° at the centre of the circle whose radius is 4 cm.

34. Find the value of tan 75°.

35. If $\sec^2\alpha + \cos^2\alpha = 2$, then find the value of $\sec\alpha + \cos\alpha$.

36. Eliminate θ from the equations, $a = x\sin\theta - y\cos\theta$ and $b = x\cos\theta + y\sin\theta$.

37. If $\cos\alpha = \frac{12}{13}$ and $\sin\beta = \frac{4}{5}$, then find $\sin(\alpha + \beta)$.

38. Express $\sin\theta$ in terms of $\cot\theta$.

39. If $\sin\alpha = \frac{4}{5}$, then find the value of $\sin 2\alpha$.

40. Find the value of $\sin 2\alpha$, if $\sin\alpha + \cos\alpha = \frac{1}{3}$.

41. The length of the minutes hand of a wall clock is 36 cm. Find the distance covered by its tip in 35 minutes.

42. If $\tan 2\alpha = \frac{3}{4}$, then find $\tan\alpha$.

43. If $A + B = 45°$, then find the value of $\tan A + \tan B + \tan A \tan B$.

44. If $\sin(A + B) = \frac{\sqrt{3}}{2}$ and $\cot(A - B) = 1$, then find A and B.

45. If $\sin\theta$ and $\cos\theta$ are the roots of the quadratic equation $lx^2 - mx - n = 0$, then find the relation between l, m and n.

Essay Type Questions

46. Show that $3(\sin x + \cos x)^4 - 6(\sin x + \cos x)^2 + 4(\sin 6x + \cos 6x) = 1$.

47. If $\cos^2\alpha + \cos\alpha = 1$, then find the value of $4\sin^2\alpha + 4\sin^4\alpha + 2$.

48. Obtain the relation by eliminating θ from the equations, $x = a + r\cos\theta$ and $y = b + r\sin\theta$.

49. One of the angles of a rhombus is 60° and the length of the diagonal opposite to it is 6 cm. Find the area of the rhombus (in sq.cm).

50. If $\alpha + \beta + \gamma = 90°$, then $\tan\alpha + \tan\beta + \tan\alpha\tan\beta\cot\gamma$ is _______.

CONCEPT APPLICATION

Level 1

1. The length of the minutes hand of a wall clock is 6 cm. Find the distance covered by the tip of the minutes hand in 25 minutes.

   (a) $\frac{270}{7}$ cm (b) 110 cm
   (c) $\frac{88}{7}$ cm (d) $\frac{110}{7}$ cm

2. The value of tan 10° · tan 20° · tan 45° · tan 70° · tan 80° =

   (a) 1 (b) $\frac{1}{\sqrt{3}}$
   (c) 0 (d) 8

3. The value of $\sin\theta$ in terms of $\tan\theta$ is _______.

   (a) $\frac{\tan\theta}{\sqrt{1-\tan^2\theta}}$ (b) $\frac{\tan^2\theta}{\sqrt{1+\tan^2\theta}}$
   (c) $\frac{\tan^2\theta}{\sqrt{1-\tan^2\theta}}$ (d) $\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$

4. The value of $\sin^2 60° + \cos^2 30° - \sin^2 45°$ is ______.

(a) 1 (b) sin 90°

(c) $\frac{1}{2}$ (d) Both (a) and (b)

5. The simplified value of $\text{cosec}^2\alpha\left(1+\frac{1}{\sec\alpha}\right)\left(1-\frac{1}{\sec\alpha}\right)$ is ________.

(a) 1 (b) 0

(c) 2 (d) −1

6. A wheel makes 240 revolutions in one minute. Find the measure of the angle it describes at the centre in 15 seconds.

(a) $60\,\pi^c$ (b) $120\,\pi^c$

(c) $8\,\pi^c$ (d) None of these

7. If $3\tan A = 4$, then find the value of $\frac{2\sin A - 7\cos A}{3\cos A + 4}$.

(a) $\frac{-13}{29}$ (b) $\frac{13}{11}$

(c) ∞ (d) $\frac{29}{13}$

8. If $\tan\alpha = 3$ and $\tan\beta = \frac{1}{2}$, then which of the following is true?

(a) $\alpha + \beta = \frac{\pi^c}{4}$ (b) $\alpha\beta = \frac{\pi^c}{4}$

(c) $\alpha - \beta = \frac{\pi^c}{4}$ (d) None of these

9. If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then which of the following is true?

(a) $A + B = \frac{\pi^c}{4}$ (b) $A - B = \frac{\pi^c}{4}$

(c) $2A + B = \frac{\pi^c}{4}$ (d) $A + 2 = \frac{\pi^c}{4}$

10. If $\sin A = \frac{1}{2}$ and $90° < A < 180°$, then the value of A in circular measure is ______.

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{5\pi}{6}$ (d) $\frac{7\pi}{6}$

11. The value of $\frac{3\pi^c}{5}$ in sexagesimal measure is ______.

(a) 216° (b) 144°

(c) 128° (d) 108°

12. The value of $\log_{\sec\theta}(1 - \sin^2\theta)$ is ______.

(a) 2 (b) −2 (c) 0 (d) 1

13. If $\sin A = \frac{\sqrt{3}}{2}$ and A is an acute angle, then find the value of $\frac{\tan A - \cot A}{\sqrt{3} + \text{cosec} A}$.

(a) $\frac{-2}{5}$ (b) $\frac{2}{5}$

(c) $\frac{2}{3+2\sqrt{3}}$ (d) −2

14. The length of an arc, which subtends an angle of 30° at the centre of the circle of radius 42 cm is ______.

(a) 22 cm (b) 44 cm

(c) 11 cm (d) $\frac{22}{7}$ cm

15. If $\sin^4\theta + \cos^4\theta = \frac{1}{2}$, then find $\sin\theta\cos\theta$.

(a) $\pm\frac{1}{8}$ (b) $\pm\frac{1}{4}$

(c) ±1 (d) $\pm\frac{1}{2}$

16. If $\sin\theta = \frac{a}{b}$, then $\cos\theta$ and $\tan\theta$ in terms of a and b are

(a) $\frac{\sqrt{b^2 - a^2}}{b}$ and $\frac{b}{\sqrt{b^2 - a^2}}$

(b) $\frac{b}{\sqrt{b^2 - a^2}}$ and $\frac{a}{\sqrt{b^2 - a^2}}$

(c) $\frac{\sqrt{a^2 - b^2}}{a}$ and $\frac{b}{\sqrt{a^2 - b^2}}$

(d) $\frac{\sqrt{b^2 - a^2}}{b}$ and $\frac{a}{\sqrt{b^2 - a^2}}$

PRACTICE QUESTIONS

17. The distance covered by the tip of a minute hand in 35 minutes is 33 cm. What is the length of the minute hand?

(a) 6 cm (b) 9 cm

(c) 10 cm (d) 12 cm

18. If $\sin\alpha = \frac{4}{5}$, where $(0° \le \alpha \le 90°)$, then find $\sin 2\alpha$.

(a) $\frac{12}{25}$ (b) $-\frac{24}{25}$

(c) $\frac{25}{24}$ (d) $\frac{24}{25}$

19. In a ΔABC, $\cos\left(\frac{A+B}{2}\right) =$ _____.

(a) $\cos\frac{C}{2}$ (b) $-\sin\frac{C}{2}$

(c) $\cos\left(\frac{A-B}{2}\right)$ (d) $\sin\frac{C}{2}$

20. $\sin^4\theta - \cos^4\theta =$

(a) -1 (b) $\cos 2\theta$

(c) $2\sin^2\theta - 1$ (d) $\sin 2\theta$

21. If $P : Q = \tan 2A : \cos A$ and $Q : R = \cos 2A : \sin 2A$, then $P : R$ is _________.

(a) $\tan 2A$ (b) $2\sin A$

(c) 1 (d) $\sec A$

22. If $A = \sin\theta + \cos\theta$ and $B = \sin\theta - \cos\theta$, then which of the following is true?

(a) $A^2 + B^2 = 1$

(b) $A^2 - B^2 = 2$

(c) $A^2 + B^2 = 2$

(d) $2A^2 + B^2 = 4$

23. If $\cot(A - B) = 1$ and $\cos(A + B) = \frac{1}{2}$, then find B.

(a) $42\frac{1}{2}°$

(b) $7\frac{1}{2}°$

(c) $15\frac{1}{2}°$

(d) 60°

24. Find the measure of angle A, if $(2\sin A+1)(2\sin A-1) = 0$.

(a) 75° (b) 60°

(c) 45° (d) 30°

25. The value of $\cos 2\theta$ in terms of $\cot\theta$ is _______.

(a) $\frac{\cot^2\theta+1}{\cot^2\theta-1}$

(b) $-\frac{1+\cot^2\theta}{\cot^2\theta-1}$

(c) $\frac{\cot^2\theta-1}{\cot^2\theta+1}$

(d) $\frac{1-\cot^2\theta}{1+\cot^2\theta}$

26. The simplified value of $\sin^4\alpha + \cos^4\alpha + \frac{1}{2}\sin^2 2\alpha$ is

(a) -1 (b) $\sin\alpha + \cos\alpha$

(c) 0 (d) 1

27. $\sqrt{\frac{(1+\sin 2\theta)}{1-\cos^2\theta}}\left[\text{where } \theta \in \left[0, \frac{\pi}{4}\right]\right] =$

(a) $\operatorname{cosec}^2\theta$ (b) 1

(c) $1 + \cot\theta$ (d) $1 + \tan\theta$

28. If A and B are complementary angles, then the value of $\frac{\sin^2 A + \sin^2 B}{\operatorname{cosec}^2 A - \tan^2 B}$ is _______.

(a) 0 (b) 1

(c) -1 (d) 2

29. Find the value of $4(\sin^4 30° + \cos^4 30°) - 3(\cos^2 45° + \sin^2 90°)$.

(a) $-\frac{1}{2}$ (b) -2

(c) 2 (d) $\frac{1}{2}$

30. If $\sec\theta + \tan\theta = \frac{4}{3}$, then $\sec\theta\tan\theta =$

(a) $\frac{175}{24}$ (b) $\frac{25}{576}$

(c) $\frac{27}{576}$ (d) $\frac{175}{576}$

PRACTICE QUESTIONS

Level 2

31. If $\sin\alpha + \cos\alpha = n$, then $\sin^6\alpha + \cos^6\alpha$ in terms of n is _______.
    (a) $4 + 3(n - 1)^2$ (b) $\frac{4+3(n^2-1)}{4}$
    (c) $\frac{4-3(n^2-1)^2}{4}$ (d) None of these

32. Find the value of $\frac{\sin 25°}{\cos 35°} - \frac{\cos 25°}{\sin 35°}$.
    (a) cosec 70° (b) sin 70°
    (c) –sin 70° (d) –cosec 70°

33. The value of $\frac{3\tan 30° - \tan^3 30°}{1 - 3\tan^2 30°}$ is _______.
    (a) tan 90° (b) tan 60°
    (c) tan 45° (d) tan 30°

34. If $\cot^4 x - \cot^2 x = 1$, then the value of $\cos^4 x + \cos^2 x$ is _______.
    (a) –1 (b) 0
    (c) 2 (d) 1

35. If $\frac{\cos(A-B)}{\cos(A+B)} = \frac{8}{3}$, then $\tan A \cdot \tan B$ is _______.
    (a) $\frac{5}{11}$ (b) $\frac{7}{13}$
    (c) $\frac{8}{5}$ (d) $\frac{11}{5}$

36. If $A + B + C = 45°$, then the value of $\sum(\tan A + \tan A \tan B)$ is _______.
    (a) $1 - \pi\tan A$ (b) 1
    (c) $1 + \pi\tan A$ (d) $1 + \Sigma\tan A$

37. If $\cot\theta + \tan\theta = 2$, then the value of $\tan^2\theta - \cot^2\theta$ is _______.
    (a) 1 (b) 0
    (c) –1 (d) 2

38. The value of cot 5° · cot 15° · cot 25° · cot 35° · cot 45° · cot 55° · cot 65° · cot 75° · cot 85° is _______.
    (a) 0 (b) –1
    (c) –2 (d) 1

39. The simplified form of $\sqrt{1+\sin\left(\frac{x}{8}\right)}$ is _______.
    (a) $\sin\left(\frac{x}{8}\right) + \cos\left(\frac{x}{8}\right)$
    (b) $\sin\left(\frac{x}{16}\right) + \cos\left(\frac{x}{16}\right)$
    (c) $\sin\left(\frac{x}{4}\right) + \cos\left(\frac{x}{4}\right)$
    (d) None of these

40. $\sin^4\theta + \cos^4\theta$ in terms of $\sin\theta$ is _______.
    (a) $2\sin^4\theta - 2\sin^2\theta - 1$
    (b) $2\sin^4\theta - 2\sin^2\theta + 1$
    (c) $2\sin^4\theta + 2\sin^2\theta - 1$
    (d) $2\sin^4\theta - 2\sin^2\theta$

41. If $\tan(A - B) = 1$ and $\sin(A+B) = \frac{\sqrt{3}}{2}$, then find B.
    (a) $42\frac{1°}{2}$ (b) $7\frac{1°}{2}$
    (c) $15\frac{1°}{2}$ (d) 60°

42. If $\sin\beta + \cos\beta = \frac{5}{4}$, then find the value of $\sin\beta \cdot \cos\beta$.
    (a) $\frac{1}{4}$ (b) $\frac{9}{32}$
    (c) $\frac{5}{16}$ (d) $\frac{11}{32}$

43. If $\sin^2\alpha + \sin\alpha = 1$, then the value of $\cos^4\alpha + \cos^2\alpha$ is _______.
    (a) 0 (b) –1
    (c) 1 (d) 2

44. If $\tan P + \cot P = 2$, then the value of $\tan^n P + \cot^n P$ is _______.
    (a) 2 (b) 2^n
    (c) 2^{n-1} (d) 2^{n+1}

45. The value of $\sqrt{3}\tan 10° + \sqrt{3}\tan 20° + \tan 10° \cdot \tan 20°$ is _______.
    (a) –1 (b) 0
    (c) 1 (d) 2

PRACTICE QUESTIONS

46. If tan $(A+B)=1$ and $(A-B)=\frac{1}{\sqrt{2}}$, then find A and B.

The following are the steps involved in solving the above problem. Arrange them in sequential order.

(A) $\tan(A+B)=1 \Rightarrow \tan(A+B)=\tan 45°$ and $\sin(A-B)=\frac{1}{\sqrt{2}} \Rightarrow \sin(A-B)=\sin 45°$.

(B) $2A=90° \Rightarrow A=45°$.

(C) $A+B=45°$ and $A-B=45°$.

(D) $\therefore A=45°$ and $B=0°$.

(a) DBCA (b) CABD

(c) ACDB (d) ACBD

47. If $\cos(A-B)=\frac{1}{2}$ and $\sin(A+B)=\frac{\sqrt{3}}{2}$, then find A and B.

The following are the steps involved in solving the following problem. Arrange them in sequential order.

(A) $2A=120° \Rightarrow A=60°$.

(B) $\therefore A=60°, B=0°$.

(C) $\cos(A-B)=\frac{1}{2} \Rightarrow \cos(A-B)=\cos 60°$ and $\sin(A+B)=\frac{\sqrt{3}}{2} \Rightarrow \sin(A+B)=\sin 60°$.

(D) $A+B=60°$ and $A-B=60°$.

(a) DCAB (b) CADB

(c) DCBA (d) CDAB

48. If $\sin\alpha+\sin\beta+\sin\gamma=3$, then $\sin^3\alpha+\sin^3\beta+\sin^3\gamma=$ _______.

(a) 0 (b) 2

(c) 3 (d) 1

49. $\sec^4\theta-\sec^2\theta=$ _______.

(a) $\tan^2\theta\sec^2\theta$ (b) $\frac{\tan^2\theta}{\sec^2\theta}$

(c) $\text{cosec}^2\theta\cot^2\theta$ (d) $\frac{\cot^2\theta}{\text{cosec}^2\theta}$

50. $\sin\theta+\cos\theta=\sqrt{2}$, then $\sin^{16}\theta=$ _______.

(a) $\frac{\cos^{16}\theta}{2^{16}}$ (b) $\frac{\sec^{16}\theta}{2^8}$

(c) $\frac{1}{2\sec^{16}\theta}$ (d) $\frac{1}{2^{16}\cos^{16}\theta}$

Level 3

51. If $\frac{\sin(x-y)}{\sin(x+y)}=\frac{3}{5}$, then $\tan x\cdot\cot y$ is _______.

(a) 1 (b) 2

(c) 3 (d) 4

52. $\sqrt{\sqrt{16\sin^4\theta+\text{cosec}^4\theta+8}-4}=$

(a) $2\sin\theta-\text{cosec}\,\theta$

(b) $2\sin\theta+\text{cosec}\,\theta$

(c) $2\text{cosec}\,\theta+\cos\theta$

(d) $2\text{cosec}\,\theta+\sin\theta$

53. If $\sin\theta$ and $\cos\theta$ are the roots of the quadratic equation $px^2+qx+r=0$ $(p\neq 0)$, then which of the following relation holds good?

(a) $q^2-p^2=2pr$

(b) $p^2-q^2=2pr$

(c) $p^2+q^2+2pr=0$

(d) $(p-q)^2=2pr$

54. If $\sin\alpha-\cos\alpha=m$, then the value of $\sin^6\alpha+\cos^6\alpha$ in terms of m is _______.

(a) $1+\frac{3}{4}(1+m^2)^2$ (b) $1-\frac{4}{3}(m^2-1)^2$

(c) $1-\frac{3}{4}(1-m^2)^2$ (d) $1-\frac{3}{4}(1+m^2)^2$

55. The value of $\frac{8\sec^4\theta-8\tan^4\theta}{4+8\tan^2\theta}-\frac{2\cos^6\theta+2\sin^6\theta}{1-3\sin^2\theta\cos^2\theta}$ is _______.

(a) 0 (b) 1

(c) −1 (d) 3

56. If $\frac{1+\tan\theta}{1-\tan\theta}=\sqrt{3}$, then find the value of θ.

(a) 30° (b) 25°

(c) 15° (d) 45°

PRACTICE QUESTIONS

57. If $A \times B = 1$, $A + B = \operatorname{cosec}\theta \cdot \sec\theta$ then $\dfrac{A}{B}$ can be _______.

(a) $\tan^2\theta$ (b) $\sec^2\theta$

(c) $\sin^2\theta\cos^2\theta$ (d) $\operatorname{cosec}^2\theta\sec^2\theta$

58. If $7\sin^2\theta + 3\cos^2\theta = 4$, then find $\tan\theta$.

(a) $\dfrac{1}{\sqrt{3}}$ (b) $\dfrac{2}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) 1

59. If $\sin(A+B) = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$ and $\sec A = 2$, then the value of B in circular measure is _______.

(a) $\dfrac{\pi}{12}$ (b) $\dfrac{3\pi}{5}$

(c) $\dfrac{7\pi}{5}$ (d) $\dfrac{5\pi}{12}$

60. If $\tan\theta - \cot\theta = 7$, then the value of $\tan^3\theta - \cot^3\theta$ is

(a) 250 (b) 354

(c) 343 (d) 364

61. If $x^n = a^m\cos^4\theta$ and $y^n = b^m\sin^4\theta$, then

(a) $\dfrac{x^{n/2}}{a^{m/2}} + \dfrac{y^{n/2}}{b^{m/2}} = 1$ (b) $\dfrac{x^n}{a^m} + \dfrac{y^n}{b^m} = 1$

(c) $\dfrac{x^{n/2}}{y^{n/2}} + \dfrac{a^{m/2}}{b^{m/2}} = 1$ (d) None of these

62. If $\sin^2\theta + 2\cos^2\theta + 3\sin^2\theta + 4\cos^2\theta + \cdots +$ 40 terms = 405 where θ is acute, then find the value of $\tan\theta$.

(a) $\dfrac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) 1 (d) ∞

63. If $\sin\alpha + \sin\beta = 2$, then find the value of $\cos^2\alpha + \cos^2\beta$.

(a) 0 (b) 1

(c) 2 (d) 3

64. If $x = a^2\cos 3\theta$ and $y = b^2\sin 3\theta$, then

(a) $\dfrac{x^2}{a} + \dfrac{y^2}{b} = 1$

(b) $\left(\dfrac{x}{a^2}\right)^{1/3} + \left(\dfrac{y}{b^2}\right)^{1/3} = 1$

(c) $\left(\dfrac{x^2}{a^2}\right)^{2/3} + \left(\dfrac{y^2}{b^2}\right)^{2/3} = 1$

(d) $\left(\dfrac{x}{a^2}\right)^{2/3} + \left(\dfrac{y}{b^2}\right)^{2/3} = 1$

65. If $\cos^2\theta + 2\sin^2\theta + 3\cos^2\theta + 4\sin^2\theta + \cdots$ (200 terms) = 10025, where θ is acute, then the value of $\sin\theta - \cos\theta$ is

(a) $\dfrac{1-\sqrt{3}}{2}$

(b) $\dfrac{1+\sqrt{3}}{2}$

(c) $\dfrac{\sqrt{3}-1}{2}$

(d) 0

PRACTICE QUESTIONS

TEST YOUR CONCEPTS

Very Short Answer Type Questions

1. $\frac{4\pi}{5}$
2. $\frac{\sqrt{3}}{4}$
3. $120°$
4. 0
5. $58°$
6. $\frac{1}{2}$
7. $\frac{1}{x}$
8. $\frac{1}{2}$
9. 2
10. $45°$
11. $\sqrt{3}-1$
12. $\frac{1}{4}$
13. 2
14. $\frac{\sqrt{3}}{2}$
15. $2\cos A \cos B$
16. 1(B and C are complementary)
17. $\frac{\pi}{12}$
18. $\cos C = \frac{24}{25}$
19. $A + B = 90°$
20. $1(A = 45°,\ C = 45°)$

Shot Answer Type Questions

21. $\frac{3}{32}$
22. $\frac{-17}{61}$
23. $\frac{\sqrt{3}+1}{2\sqrt{2}}$
24. $8\frac{1}{3}$
25. $\frac{40^g}{3}$
26. $\frac{4}{3}$
27. $\sec^2\alpha + \tan^2\alpha$
28. $60°, \frac{\pi}{3}$
29. $300^g, \frac{3\pi^c}{2}$
30. 2
31. $\frac{2\sqrt{5}}{5}$
32. $80\pi^c$
33. $4\sqrt{3}$ cm
34. $\frac{\sqrt{3}+1}{\sqrt{3}-1}$
35. ± 2
36. $x^2 + y^2 = a^2 + b^2$
37. $\frac{63}{65}$
38. $\sqrt{\frac{1}{1+\cot^2\theta}}$
39. $\frac{24}{25}$
40. $-\frac{1}{X}$
41. 132 cm
42. $\frac{1}{3}$ or -3
43. 1
44. $A = 52\frac{1}{2}°$ and $B = 7\frac{1}{2}°$
45. $l^2 - m^2 = 2nl$

ANSWER KEYS

Essay Type Questions

47. 6

48. $(x - a)^2 + (y - b)^2 = r^2$

49. $18\sqrt{3}$

50. cot r

CONCEPT APPLICATION

Level 1

1. (d)	**2.** (a)	**3.** (d)	**4.** (d)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (c)	**9.** (a)	**10.** (c)
11. (d)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (d)	**16.** (d)	**17.** (b)	**18.** (d)	**19.** (d)	**20.** (c)
21. (d)	**22.** (c)	**23.** (b)	**24.** (d)	**25.** (c)	**26.** (d)	**27.** (c)	**28.** (b)	**29.** (b)	**30.** (d)

Level 2

31. (c)	**32.** (d)	**33** (a)	**34.** (d)	**35.** (a)	**36.** (c)	**37.** (b)	**38.** (d)	**39.** (b)	**40.** (b)
41. (b)	**42.** (b)	**43.** (c)	**44.** (a)	**45.** (c)	**46.** (d)	**47.** (d)	**48.** (c)	**49.** (a)	**50.** (d)

Level 3

51. (d)	**52.** (c)	**53.** (a)	**54.** (c)	**55.** (a)	**56.** (c)	**57.** (a)	**58.** (a)	**59.** (a)	**60.** (d)
61. (a)	**62.** (b)	**63.** (a)	**64.** (d)	**65.** (a)					

CONCEPT APPLICATION

Level 1

1. (i) The minutes hand covers an angle of 6° per minute.
   (ii) Use $l = r \times \theta$.
2. Use $\tan\theta \cdot \tan(90 - \theta) = 1$.
3. Use $\sec^2\theta - \tan^2\theta = 1$.
4. Take the values of trigonometric ratios from the table.
5. Use $(a + b)(a - b) = a^2 - b^2$ and $\cos^2\theta + \sin^2\theta = 1$.
6. Use, 1 revolution $= 2\pi$.
7. Find $\sin A$, $\cos A$ using triangle ABC.
8. Use the identity $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$.
9. Apply $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$.
10. Find the value of A and then multiply it by $\dfrac{\pi}{180}$.
11. Replace π^c by 180°.
12. Use $\cos^2\theta + \sin^2\theta = 1$.
13. Observe the values of trigonometric ratios from table.
14. Use $l = r \times \theta$, where θ is in radians.
15. Use the identity $a^2 + b^2 = (a + b)^2 - 2ab$.
16. Apply $\cos\theta = \dfrac{\text{Side adjacent to } \theta}{\text{Hypotenuse}}$,
    $\tan\theta = \dfrac{\text{Side opposite to } \theta}{\text{Side adjacent to } \theta}$.
17. (i) Minutes hand moves 6° in one minute.
    (ii) Use $l = r \times \theta$.
18. Use $\sin 2\alpha = 2\sin\alpha\cos\alpha$.
19. In triangle ABC, $\dfrac{A + B}{2} = \dfrac{180 - C}{2}$.
20. Apply $a^2 - b^2 = (a + b)(a - b)$.
22. Apply $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$ and $\sin^2\theta + \cos^2\theta = 1$.
23. Use $\tan 45° = 1$ and $\sin 60° = \dfrac{\sqrt{3}}{2}$.
24. Use $(a + b)(a - b) = a^2 - b^2$ and solve.
25. Use $\cos 2\theta = \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$.
26. Use $a^2 + b^2 = (a + b)^2 - 2ab$.
27. $1 + \sin 2\theta = (\sin\theta + \cos\theta)2$, when $\theta \in \left[0, \dfrac{\pi}{4}\right]$.
28. (i) $\sin^2 A = \sin^2(90 - B)$. If A and B complementary angles.
    (ii) If $A + B = 90°$, then $\sin A = \cos B$ and $\tan B = \cot A$.
29. Use trigonometric ratios values from table.
30. Squaring both sides of given equation.

Level 2

31. (i) $\sin^6\alpha + \cos^6\alpha = (\sin^2\alpha)^3 + (\cos^2\alpha)^3$. Use the formula $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$.
    (ii) Find $\sin\alpha \cdot \cos\alpha$ by substituting $\sin\alpha + \cos\alpha = n$ in the above equation.
32. (i) Use $\cos A \cos B - \sin A \sin B = \cos(A + B)$ and $\sin 2A = 2\sin A \cos A$.
    (ii) Find the LCM of denominators and simplify.
    (iii) Use the formula $\cos(A + B) = \cos A \cos B - \sin A \sin B$.
33. (i) Use $\tan 30° = \dfrac{1}{\sqrt{3}}$.
    (ii) Substitute the value of tan 30° and simplify.
    (iii) Then check from the options.
34. Use $\mathrm{cosec}^2 x = 1 + \cot^2 x$ and $\sin^2 x + \cos^2 x = 1$.
35. (i) Use componendo and dividendo theorem, i.e., $\dfrac{a}{b} = \dfrac{a + b}{a - b}$.
    (ii) Use the formula $\cos(A - B)$ and $\cos(A + B)$.
    (iii) Take cross multiplication and find $\tan A + \tan B$.

36. Use $\tan(A + B) = \tan(45° - C)$ and proceed.

37. (i) Put $\theta = 45°$.

(ii) If $\tan\theta + \cot\theta = 2$, then $\theta = 45°$.

38. Use $\cot A \cdot \cot(90 - A) = 1$.

39. (i) Apply $\sqrt{1 + \sin 2A} = \sin A + \cos A$.

(ii) $\sin\frac{x}{8} = \sin 2\left(\frac{x}{16}\right)$ and $\sin 2\theta = 2\sin\theta\cos\theta$.

(iii) Use the identity $1 = \sin^2\theta + \cos^2\theta$.

40. (i) Use $(a + b)^2 = a^2 + b^2 + 2ab$ and $\cos^2\theta = 1 - \sin^2\theta$.

(ii) Take $\cos 4\theta$ as $(1 - \sin^2\theta)^2$ and simplify.

41. (i) Use $\tan 45° = 1$ and $\sin 60° = \frac{\sqrt{3}}{2}$.

(ii) If $\tan(A - B) = 1$, then $A - B = 45°$.

(iii) If $\sin(A + B) = \frac{\sqrt{3}}{2}$, then $A + B = 60°$.

(iv) Subtract the above two equations and find B.

42. (i) By squaring on both the sides of the given equation we can obtain.

(ii) Square on both sides of $\sin\beta + \cos\beta = \frac{5}{4}$.

43. (i) Use $\sin^2 x + \cot^2 x = 1$.

(ii) $\cos^4\alpha = (1 - \sin^2\alpha)^2$.

(iii) $\cos^2\alpha = (1 - \sin^2\alpha)$.

(iv) Substitute the above values in the given expression.

44. If $x + \frac{1}{x} = 2$, then $x^n + \frac{1}{x^n} = 2$.

45. (i) Use $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$.

(ii) Take $\tan(10° + 20°) = \tan 30°$, i.e.,

$$\frac{\tan 10° + \tan 20°}{1 - \tan 10° \cdot \tan 20°} = \frac{1}{\sqrt{3}}$$ and simplify.

46. ACBD is the required sequential order.

47. CDAB is the required sequential order.

48. Given, $\sin\alpha + \sin\beta + \sin\gamma = 3$.

This is possible only if $\sin\alpha = \sin\beta = \sin\gamma = 1$, i.e., if $\alpha = \beta = \gamma = 90°$.

$\therefore \sin^3\alpha + \sin^3\beta + \sin^3\gamma = 1^3 + 1^3 + 1^3 = 3$.

49. $\sec^4\theta - \sec^2\theta = \sec^2\theta(\sec^2\theta - 1) = \sec^2\theta\tan^2\theta$.

50. $\sin\theta + \cos\theta = \sqrt{2}$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$

$\Rightarrow 2\sin\theta\cos\theta = 1$

$\sin\theta\cos\theta = \frac{1}{2}$

$\sin^{16}\theta = \frac{1}{2^{16}\cos^{16}\theta}$.

Level 3

51. (i) Apply componendo and dividendo rule, i.e., $\frac{a}{b} = \frac{a + b}{a - b}$.

(ii) Use the formula $\sin(x - y)$ and $\sin(x + y)$.

(iii) And then apply cross multiplication.

52. Use $a^2 + b^2 + 2ab = (a + b)^2$ and $\sin\theta \cdot \operatorname{cosec}\theta = 1$.

53. (i) For the equation $ax^2 + bx + c = 0$, the sum of roots is $\frac{-b}{a}$, and product of the roots is $\frac{c}{a}$.

(ii) Sum of the roots, $\sin\theta + \cos\theta = \frac{-q}{p}$.

(iii) Product of the roots $\sin\theta \cdot \cos\theta = \frac{r}{p}$.

(iv) Eliminate θ, by using the formula $(a + b)^2 = a^2 + b^2 + 2ab$ from the above equations.

54. (i) Use the identity $a^3 + b^3 = (a + b)^3 = 3ab(a + b)$.

(ii) Find the value of $\sin\alpha\ \cos\alpha$ by squaring $\sin\alpha - \cos\alpha = m$.

55. (i) Put $\theta = 0$ and simplify.

(ii) $\sec^4\theta - \tan^4\theta = \sec^2\theta + \tan^2\theta$.

(iii) $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$.

56. $\frac{1 + \tan\theta}{1 - \tan\theta} = \sqrt{3}$

$\frac{\tan 45 + \tan\theta}{1 - \tan 45 \cdot \tan\theta} = \sqrt{3}$

$\tan(45° + \theta) = \tan 60°$

$\Rightarrow 45° + \theta = 60°$

$\theta = 60° - 45° = 15°$.

HINTS AND EXPLANATION

57. $AB = 1 \Rightarrow A = \dfrac{1}{B}$

$$A + B = \frac{1}{\sin\theta}\ \frac{1}{\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$A + B = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$A + B = \tan\theta + \cot\theta.$

Since, $\tan\theta \times \cot\theta = 1 = AB.$

$\therefore\ A = \tan\theta,\ B = \cot\theta.$

$$\frac{A}{B} = \frac{\tan\theta}{\cot\theta} = \tan\theta \cdot \tan\theta = \tan^2\theta.$$

$$A = \cot\theta,\ B = \tan\theta,\ \frac{A}{B} = \cot^2\theta.$$

58. $3\sin^2\theta + 4\sin^2\theta + 3\cos^2\theta = 4$

$3 + 4\sin^2\theta = 4$

$4\sin^2\theta = 1$

$$\sin\theta = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

$\theta = 30°.$

$$\therefore \tan\theta = \frac{1}{\sqrt{3}}.$$

59. $\sin(A + B) = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

$\sin(A + B) = \sin 75°$

$A + B = 75°.$ (1)

$\sec A = 2$

$\sec A = \sec 60°$

$A = 60°.$

Substitute the value of A in Eq. (1),

$60° + B = 75°$

$B = 15°.$

In circular measure,

$$B = 150° \times \frac{\pi}{180°} = \frac{\pi^c}{12}.$$

60. Given, $\tan\theta - \cot\theta = 7.$

We know that,

$a^3 - b^3 = (a - b)^3 + 3ab\,(a - b)$

$\tan^3\theta - \cot^3\theta = (\tan\theta - \cot\theta)^3 + 3\tan\theta\cot\theta(\tan\theta - \cot\theta)$

$= 7^3 + 3(7) = 343 + 21 = 364.$

61. $x^n = a^m\cos^4\theta$, and $y^n = b^m\sin^4\theta$

$$\Rightarrow \cos^4\theta = \frac{x^n}{a^m} \text{ and } \sin^4\theta = \frac{y^n}{b^m}$$

$$\Rightarrow \cos^2\theta = \frac{x^{n/2}}{a^{m/2}},\ \sin^2\theta = \frac{y^{n/2}}{b^{m/2}}.$$

But, $\sin^2\theta + \cos^2\theta = 1$

$$\therefore\ \frac{x^{n/2}}{a^{m/2}} + \frac{y^{n/2}}{b^{m/2}} = 1.$$

62. Given,

$\sin^2\theta + 2\cos^2\theta + 3\sin^2\theta + \ldots$ 40 terms $= 405$

$\Rightarrow (\sin^2\theta + 3\sin^2\theta + \ldots$ 20 terms$) + (2\cos^2\theta + 4\cos^2\theta + \ldots$ 20 terms$) = 405$

$\Rightarrow 20^2\sin^2\theta + (20^2 + 20)\cos^2\theta = 405$

$\Rightarrow 400(\sin^2\theta + \cos^2\theta) + 20\cos^2\theta = 405$

$\Rightarrow 400 + 20\cos^2\theta = 405 = 20\cos^2\theta = 5$

$$\Rightarrow \cos^2\theta = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{2}\quad (\because \theta \text{ is acute})$$

$\Rightarrow \theta = 60°.$

$\therefore\ \tan 60° = \sqrt{3}.$

63. Given, $\sin\alpha + \sin\beta = 2$

$\alpha = \beta = 90°.$

$\therefore\ \cos^2\alpha + \cos^2\beta = \cos^2 90° + \cos^2 90° = 0.$

64. Given, $x = a^2\cos^3\theta$ and $y = b^2\sin^3\theta.$

$$\Rightarrow \frac{x}{a^2} = \cos^3\theta \text{ and } \frac{y}{b^2} = \sin^3\theta$$

$$\Rightarrow \left(\frac{x}{a^2}\right)^{2/3} = \cos^2\theta;\ \left(\frac{y}{b^2}\right)^{2/3} = \sin^2\theta.$$

$$\sin^2\theta + \cos^2\theta = \left(\frac{x}{a^2}\right)^{2/3} + \left(\frac{y}{b^2}\right)^{2/3}$$

$$\therefore\ \left(\frac{x}{a^2}\right)^{2/3} + \left(\frac{y}{b^2}\right)^{2/3} = 1.$$

65. Given,

$\cos^2\theta + 2\sin^2\theta + 3\cos^2\theta + 4\sin^2\theta + \cdots + 200$ terms $= 10025$

$\Rightarrow (\cos^2\theta + 3\cos^2\theta + 5\cos^2\theta + \cdots 100 \text{ terms}) + (\sin^2\theta + 2\sin^2\theta + \cdots + 100 \text{ terms}) = 10025$

$\Rightarrow 100^2\cos^2\theta + (100^2 + 100)\sin^2\theta = 10025$

$\Rightarrow 10000(\cos^2\theta + \sin^2\theta) + 100\sin^2\theta = 10025$

$\Rightarrow 100\sin^2\theta = 25$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \frac{1}{2}$ ($\because \theta$ is acute)

$\Rightarrow \theta = 30°$.

$\therefore \sin\theta - \cos\theta = \sin 30° - \cos 30°$

$= \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$.