GUIDED REVISION

PHYSICS **GR # ELASTICITY, THERMAL EXPANSION, CALORIMETRY, HEAT TRANSFER**

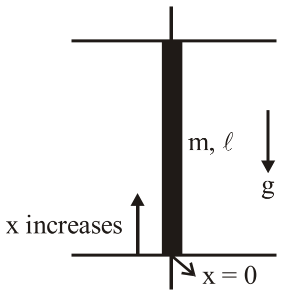
SECTION-I

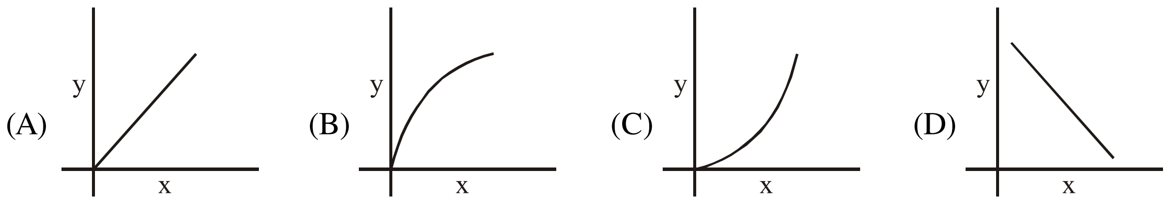
Single Correct Answer Type **11 Q. [3 M (–1)]**

1. A thermally insulated vessel contains some water at 0^0C. The vessel is connected to a vacuum pump to pump out water vapour. This results in some water getting frozen. It is given Latent heat of vaporization of water at 0°C = 21×10^5 J/kg and latent heat of freezing of water = 3.36×10^5 J/kg. The maximum percentage amount of water that will be solidified in this manner will be :-

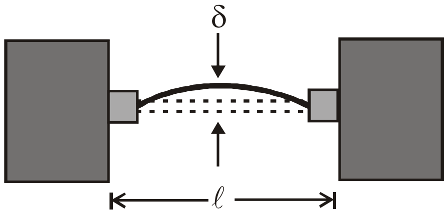
(A) 86.2% (B) 33.6% (C) 21% (D) 24.36%

2. A uniform dense rod with non uniform young's modulus is hanging from ceiling under gravity. If elastic energy density at every point is same then young's modulus with x will change as which of the shown graph :-





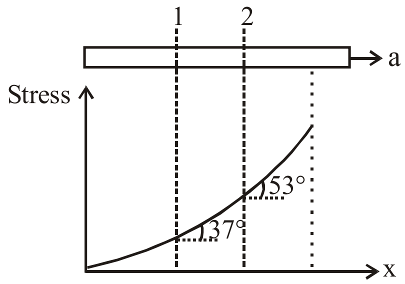
3. A long, thin metal bar of length ℓ is clamped rigidly at its ends at temperature t_0. When the temperature is increased to t, the expanding bar will bow out, as shown below. If the bowing is not too large, a fair first approximation to the shape of the bar is two equal straight segments in the form of a wide V. What is the arch δ of the bow as a function of t? (This is the distance between the corner of the V and the straight line that represents the form of the bar at t_0.)



(A) $\delta = (\ell/2)\sqrt{2\alpha(t-t_0)}$ (B) $\delta = (\ell)\sqrt{\alpha(t-t_0)}$

(C) $\delta = (\ell/2)\sqrt{\alpha(t-t_0)}$ (D) $\delta = (\ell)\sqrt{2\alpha(t-t_0)}$

4. Rod of constant cross-section moves towards right with constant acceleration. Graph of stress and distance from left end is given as in figure. If density of material of rod at cross section 1 is 9 gm/cm³. Find density at cross section 2.

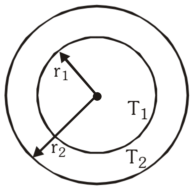


(A) 16 gm/cm³ (B) 20 gm/cm³ (C) 24 gm/cm³ (D) 12 gm/cm³

5. The volume of the bulb of a mercury thermometer at 0°C is V_0 and cross section of the capillary is A_0. The coefficient of linear expansion of glass is α_g per °C and the cubical expansion of mercury γ_m per °C. If the mercury just fills the bulb at 0°C, what is the length of mercury column in capillary at T°C.

(A) $\frac{V_0T(\gamma_m+3\alpha_g)}{A_0(1+2\alpha_gT)}$ (B) $\frac{V_0T(\gamma_m-3\alpha_g)}{A_0(1+2\alpha_gT)}$ (C) $\frac{V_0T(\gamma_m+2\alpha_g)}{A_0(1+3\alpha_gT)}$ (D) $\frac{V_0T(\gamma_m-2\alpha_g)}{A_0(1+3\alpha_gT)}$

6. The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2, respectively. The radial rate of flow of heat in a substance between the two concentric spheres, is proportional to- **[AIEEE - 2005]**



(A) $\frac{(r_2-r_1)}{(r_1r_2)}$ (B) $\ell n\left(\frac{r_2}{r_1}\right)$ (C) $\frac{r_1r_2}{(r_2-r_1)}$ (D) (r_2-r_1)

7. 'Gulab Jamuns' (assumed to be spherical) are to be heated in an oven. They are available in two sizes, one twice bigger (in radius) than the other. Pizzas (assumed to be discs) are also to be heated in oven. They are also in two sizes, one twice big (in radius) than the other. All four are put together to be heated to oven temperature. Choose the correct option from the following:

(A) Both size gulab jamuns will get heated in the same time.
(B) Smaller gulab jamuns are heated before bigger ones.
(C) Smaller pizzas are heated before bigger ones.
(D) Bigger pizzas are heated before smaller ones.

8. A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wien constant $b = 2.88 \times 10^6$ nm K. Then

(A) $U_1 = 0$ (B) $U_3 = 0$ (C) $U_1 > U_2$ (D) $U_2 > U_1$

9. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures 2T and 3T respectively. The temperature of the middle (i.e. second) plate under steady state condition is **[JEE 2012]**

(A) $\left(\frac{65}{2}\right)^{1/4} T$ (B) $\left(\frac{97}{4}\right)^{1/4} T$ (C) $\left(\frac{97}{2}\right)^{1/4} T$ (D) $(97)^{1/4} T$

10. Parallel rays of light of intensity $I = 912\ Wm^{-2}$ are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}\ Wm^{-2}\ K^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to :- **[JEE-Advance-2014]**

(A) 330 K (B) 660 K (C) 990 K (D) 1550 K

11. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400°C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is $1.2 \times 10^{-5}\ K^{-1}$, the change in length of the wire PQ is **[JEE-Advance-2016]**

(A) 0.78 mm (B) 0.90 mm (C) 1.56 mm (D) 2.34 mm

Linked Comprehension Type (1 Para × 3Q.) [3 M (-1)]
(Single Correct Answer Type)

Paragraph for Question no. 12 to 14

A calorimeter of mass m contains an equal mass of water in it. The temperature of the water and calorimeter is t_2. A block of ice of mass m and temperature $t_3 < 0°$ C is gently dropped into the calorimeter. Let C_1, C_2 and C_3 be the specific heats of calorimeter, water and ice respectively and L be the latent heat of ice.

12. The whole mixture in the calorimeter becomes ice if

(A) $C_1t_2 + C_2t_2 + L + C_3t_3 > 0$ (B) $C_1t_2 + C_2t_2 + L + C_3t_3 < 0$
(C) $C_1t_2 + C_2t_2 - L + C_3t_3 > 0$ (D) $C_1t_2 + C_2t_2 - L - C_3t_3 < 0$

13. The whole mixture in the calorimeter becomes water if

(A) $(C_1 + C_2)t_2 - C_3t_3 + L > 0$ (B) $(C_1 + C_2)t_2 + C_3t_3 + L > 0$
(C) $(C_1 + C_2)t_2 - C_3t_3 - L > 0$ (D) $(C_1 + C_2)t_2 + C_3t_3 - L > 0$

14. Water equivalent of calorimeter is:

(A) mC_1 (B) $\frac{mC_1}{C_2}$ (C) $\frac{mC_2}{C_1}$ (D) none

SECTION-II

Numerical Answer Type Question 4 Q. [3(0)]
(upto second decimal place)

1. One end of copper rod of uniform cross-section and of length 1.5 meters is in contact with melting ice and the other end with boiling water. At what point along its length should a temperature of 200°C be maintained, so that in steady state, the mass of ice melting is equal to that of steam produced in the same interval of time? Assume that the whole system is insulated from the surroundings.

2. A cylindrical block of length 0.4 m an area of cross-section $0.04 m^2$ is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross-section. The upper face of the cylinder is maintained at a constant temperature of 400K and the initial temperature of the disc is 300K. If the thermal conductivity of the material of the cylinder is 10 watt/m-K and the specific heat of the material of the disc in 600 J/kg-K, how long will it take for the temperature of the disc to increase to 350K? Assume, for purposes of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder.

3. A liquid takes 5 minutes to cool from 80°C to 50°C. How much time will it take to cool from 60°C to 30°C? The temperature of surrounding is 20°C. Use exact method.

4. In an insulated vessel, 0.05 kg steam at 373 K and 0.45 kg of ice at 253 K are mixed. Then, find the final temperature of the mixture.

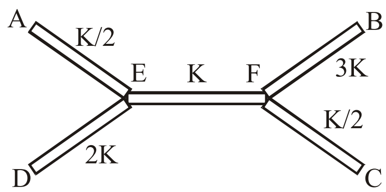
Given, $L_{fusion} = 80$ cal/g = 336 J/g, $L_{vaporization} = 540$ cal/g = 2268 J/g,

$S_{ice} = 2100$ J/kg K = 0.5 cal/gK and $S_{water} = 4200$ J/kg K = 1 cal /gK **[JEE 2006]**

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) 3 Q. [4 M (0)]

1. Five rods with identical geometries are arranged as shown. Their thermal conductivity are shown. Only A and C are maintained at 100°C and 0°C respectively. In other words, heat flows in from A and flows out of C. If temperature difference between ends D and B can be written as 10x °C where x is an integer. Then find x.



2. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487 °C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C ? **[JEE-Advance-2016]**

3. An object can be hanged to a 1.2m long wire of diameter 2 mm, and the wire does not break. However, if the object is slightly displaced and begins to slowly swing, the wire breaks. If another piece of wire having the same length and material but wider diameter is used, the speed of the same swinging object at lowest point can even be 6 m/s. Calculate the least diameter (in mm) of this wider wire.

Subjective Type (Ranging from 0 to 9) 7 Q. [4 M (0)]

1. If two rods of length L and 2 L having coefficients of linear expansion α and 2α respectively are connected so that total length becomes 3 L, determine the average coefficient of linear expansion of the composite rod.

2. A metal rod A of 25cm lengths expands by 0.050 cm, when its temperature is raised from 0°C to 100°C. Another rod B of a different metal of length 40cm expands by 0.040 cm for the same rise in temperature. A third rod C of 50cm length is made up of pieces of rods A and B placed end to end expands by 0.03 cm on heating from 0°C to 50°C. Find the lengths of each portion of the composite rod.

3. An isosceles triangle is formed with a rod of length l_1 and coefficient of linear expansion α_1 for the base and two thin rods each of length l_2 and coefficient of linear expansion α_2 for the two pieces, if the distance between the apex and the midpoint of the base remain unchanged as the temperatures varied show that

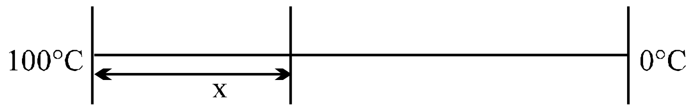
$$\frac{l_1}{l_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}.$$

4. A steel drill making 180 rpm is used to drill a hole in a block of steel. The mass of the steel block and the drill is 180 gm. If the entire mechanical work is used up in producing heat and the rate of raise in temperature of the block and the drill is 0.5 °C/s. Find

(a) the rate of working of the drill in watts, and (b) the torque required to drive the drill.

Specific heat of steel = 0.1 and J = 4.2 J/cal. Use : $P = \tau\,\omega$

5. A lagged stick of cross section area 1 cm^2 and length 1 m is initially at a temperature of 0°C. It is then kept between 2 reservoirs of temperature 100°C and 0°C. Specific heat capacity is 10 J/kg°C and linear mass density is 2 kg/m. Find



(a) temperature gradient along the rod in steady state.

(b) total heat absorbed by the rod to reach steady state.

6. There are two spheres of same radius and material at same temperature but one being solid while the other hollow. Which sphere will expand more if

(a) they are heated to the same temperature, (b) same heat is given to them ?

7. A thin uniform metallic rod of length 0.5 m and radius 0.1 m rotates with an angular velocity 400 rad/s in a horizontal plane about a vertical axis passing through one of its ends. Calculate (a) tension in the rod and (b) the elongation of the rod. The density of material of the rod is 10^4 kg/m^3 and the Young's modulus is 2×10^{11} N/m^2.

SECTION-I

Single Correct Answer Type **11 Q. [3 M (–1)]**

1. Ans. (A)	**2. Ans. (C)**	**3. Ans. (A)**	**4. Ans. (A)**
5. Ans. (B)	**6. Ans. (C)**	**7. Ans. (B)**	**8. Ans. (D)**
9. Ans. (C)	**10. Ans. (A)**	**11. Ans. (A)**	

Linked Comprehension Type **(1 Para × 3Q.) [3 M (-1)]**

(Single Correct Answer Type)

12. Ans. (B) **13. Ans. (D)** **14. Ans. (B)**

SECTION-II

Numerical Answer Type Question **4 Q. [3(0)]**

(upto second decimal place)

1. Ans. 10.34 cm **2. Ans.** 166.3 sec **3. Ans.** 10 min. **4. Ans.** 273 K

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) **3 Q. [4 M (0)]**

1. Ans. 2 **2. Ans. 9** **3. Ans. 4**

Subjective Type (Ranging from 0 to 9) **7 Q. [4 M (0)]**

1. Ans. $5\alpha/3$ **2. Ans.** 10cm , 40cm **4. Ans.** (a) 37.8 J/s (Watts), (b) 2.005 N-m

5. Ans. (a) – 100°C (b) 1000 J

6. Ans. (a) the expansion of both will be equal

(b) rise in temperature of hollow sphere will be more [as $\Delta\theta = \frac{Q}{mc}$] and hence its expansion will be more [as $\Delta V = V\gamma\Delta\theta$].

7. Ans. (a) $8\pi \times 10^{6}\left[\frac{1}{4} - r^{2}\right]$N (b) $\frac{1}{3} \times 10^{-3}$m

SOLUTIONS
SECTION-I

Single Correct Answer Type **11 Q. [3 M (–1)]**

1. Ans. (A)

Sol. Let the total amount be = 100 unit

Let m solidify

$m \times L_f = (100 - m)L_V$

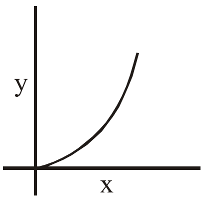
$m = 86.2$

2. Ans. (C)

Sol.

Elastic energy density at $A = \frac{1}{2}\frac{(\text{stress})^2}{Y} = \text{constant}$

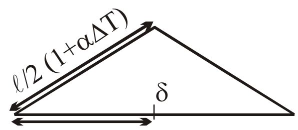
$$\frac{1}{2}\frac{\left(\frac{M}{L}\frac{x}{A}g\right)^2}{Y} = \text{constant}$$



$Y \propto x^2$

3. Ans. (A)

Sol. $\delta^2 = (\ell/2(1+\alpha\Delta T))^2 - (\ell/2)^2$

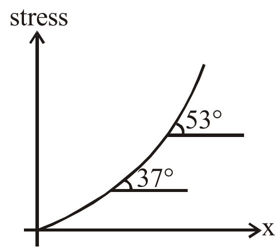


4. Ans. (A)

Sol. $\tan\theta = \frac{d(\text{stress})}{dx}$ (slope)

$$= \frac{dF/A}{dx}$$

$$= \frac{(dm)a/A}{dx}$$



$$= \frac{\left(\rho \cdot d\left(\text{vol.}\right)\right)a}{A\ \ dx}$$

$\tan\theta = \rho a$

$$\frac{\tan 37°}{\tan 53°} = \frac{\rho_1}{\rho_2} = \frac{9}{\rho_2}$$

$\Rightarrow \rho_2 = 16\ \text{gm/cm}^3$

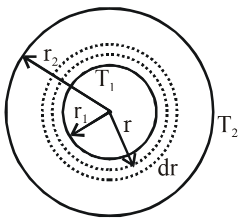
5. **Ans. (B)**

Sol. Volume exp. = $\Delta V = V_0(\gamma_m - 3\alpha_g)T$

$\Delta V = A_f \times L = A_0(1 + 2\alpha_g T)L$

$$\Rightarrow L = \frac{V_0 T\left(\gamma_m - 3\alpha_g\right)}{A_0\left(1 + 2\alpha_g T\right)}$$

6. **Ans. (C)**

Sol. 

Consider a shell of thickness (dr) and of radii (r) and the temp of inner and outer surfaces of this shell be T,(T – dT)

$\frac{dQ}{dt}$ = Rate of flow of heat through it

$$= \frac{kA\left[\left(T - dT\right) - T\right]}{dr} = -\frac{kAdT}{dr}$$

($\because A = 4\pi r^2$)

$$\frac{d\theta}{dt}\int_{r_1}^{r_2}\frac{1}{r^2}dr = -4\pi k\int_{T_1}^{T_2}dT$$

on solving

$$\frac{dQ}{dt} \propto \frac{r_1 r_2}{\left(r_2 - r_1\right)}$$

7. **Ans. (B)**

Sol. $\frac{dT}{dt} = \frac{\sigma e A}{ms}\left(T^4 - T_S^4\right)$

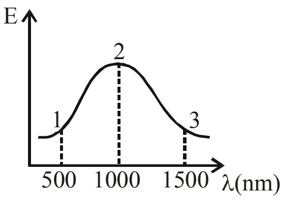
$\frac{dT}{dt} \propto \frac{A}{m}$ (m = ρV) for disc (pizzas) $m \propto A$ so $\frac{dT}{dt}$ is same for both

for $\frac{\text{gulab jamuns}}{\text{(sphere)}}$ — small (A, m) / big (4A, 8m)

$$\left(\frac{dT}{dt}\right)_{small} > \left(\frac{dT}{dt}\right)_{big} \quad \text{(for Gulab jamun)}$$

8. **Ans. (D)**

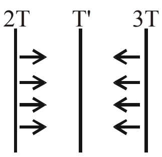
Sol. $\lambda_m T = b \Rightarrow \lambda = \frac{b}{T} = 1000nm$



$U_2 > U_1$, $U_2 > U_3$

9. **Ans. (C)**

Sol.



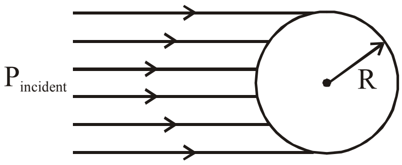
Let temperature of middle plate = T'

$k(2T)^4 + k(3T)^4 = 2k(T')^4$

$$T' = \left(\frac{97}{2}\right)^{1/4} T$$

10. **Ans. (A)**

Sol.



Energy incident = $I\pi R^2$

$= 912 \times \pi R^2$

Energy emitted, assuming temp of the sphere at steady state to be T,

$\sigma \times 4\pi R^2 (T^4 - 300^4)$

At quilibrium,

$\sigma \times 4\pi R^2 (T^4 - 300^4) = 912 \times \pi R^2$

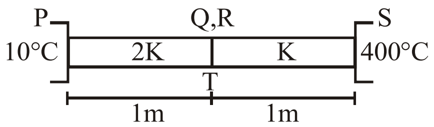
$\Rightarrow T^4 - 300^4 = \dfrac{912}{5.7\times10^{-8}\times4} = 40\times10^8$

$\therefore\ T^4 = (40 + 81)\times10^8$

$\therefore\ T = 331.66 \approx 330$ K

11. Ans. (A)

Sol.



Heat flow from P to Q

$$\frac{dQ}{dt} = \frac{2KA(T-10)}{1}$$

Heat flow from Q to S

$$\frac{dQ}{dt} = \frac{KA(400-T)}{1}$$

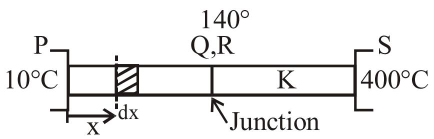
At steady state heat flow is same in whole combination

$$\frac{2KA(T-10)}{1} = KA(400\text{-}T)$$

$2T - 20 = 400 - T$

$3T = 420$

$T = 140°$



Temp of junction is 140°C

Temp at a distance x from end P

is $T_x = (130x + 10°)$

Change in length dx is dy

$dy = \alpha dx(T_x - 10)$

$$\int_0^{\Delta y} dy = \int_0^1 \alpha dx(130x + 10 - 10)$$

$$\Delta y = \left[\frac{\alpha x^2}{2}\times130\right]_0^1$$

$\Delta y = 1.2\times10^{-5}\times65$

$\Delta y = 78.0\times10^{-5}$ m $= 0.78$ mm

Linked Comprehension Type **(1 Para × 3Q.) [3 M (-1)]**

(Single Correct Answer Type)

12. **Ans. (B)**

Sol. Heat taken by ice > heat given by water + calorimeter

$C_1t_2 + C_2t_2 + L < (-C_3t_3)$

13. **Ans. (D)**

Sol. Heat given by water + calorimeter > Heat required by ice to metl completely.

$mC_2t_2 + mC_1t_2 > -mC_3t_3 + mL$

$(C_1 + C_2)\ t_2 + C_3t_3 - L > 0$

14. **Ans. (B)**

Sol. Water equivalent is given as

$m_WS_W = m_CS_C$

$$m_W = \frac{m_CS_C}{S_W}$$

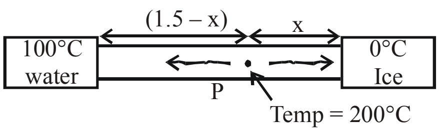
$$= \frac{mC_1}{C_2}$$

SECTION-II

Numerical Answer Type Question **4 Q. [3(0)]**

(upto second decimal place)

1. **Ans.** 10.34 cm

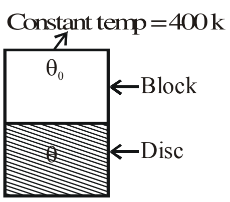
Sol. 

$$\frac{kA}{x}\left(200° - 0°\right) = 80\left(\frac{dm}{dt}\right)_{ice} \quad ....(i)$$

$$\frac{kA}{(1.5 - x)}\left(200° - 100°\right) = 540\left(\frac{dm}{dt}\right)_{vapour} \quad ....(ii)$$

$$\left(\frac{dm}{dt}\right)_{ice} = \left(\frac{dm}{dt}\right)_{vapour}$$

2. **Ans.** 166.3 sec

Sol. 

θ = temp of disc

θ_0 = constant temp

Heat input to disc = $\frac{kA(\theta_0 - \theta)}{L}$

Heat utilised by disc = $ms\frac{d\theta}{dt}$

S = Specific heat of disc

$$ms\frac{d\theta}{dt} = \frac{kA(\theta_0 - \theta)}{L}$$

$$\Rightarrow \int_{300}^{350}\frac{d\theta}{(\theta_0 - \theta)} = \frac{kA}{msL}\int_0^t dt$$

$$\Rightarrow t = \frac{msL}{kA}\ell n\left(\frac{\theta_0 - 300}{\theta_0 - 350}\right)$$

$\Rightarrow$ t = 166.32 sec

3. **Ans.** 10 min.

Sol. $-\frac{d\theta}{dt} = k(\theta - \theta_0)$

$$\int_{80}^{50}\frac{d\theta}{\theta - \theta_0} = -k\int_0^5 dt$$

$$\ell n\frac{50-20}{80-20} = -5k$$

$$k = -\frac{\ell n\left(\frac{1}{2}\right)}{5} \quad ....(i)$$

then $\int_{60}^{30}\frac{d\theta}{(\theta - 20)} = \frac{\ell n\left(\frac{1}{2}\right)}{5}\times t$

$$\ell n\frac{30-20}{60-20} = \frac{\ell n\left(\frac{1}{2}\right)}{5}t$$

$$5\ell n\left(\frac{1}{4}\right) = \ell n\left(\frac{1}{2}\right)t$$

$$5\ell n\left(\frac{1}{2}\right)^2 = \ell n\left(\frac{1}{2}\right)t$$

$\Rightarrow$ t = 10 min

4. **Ans.** 273 K

Sol. (i) Heat lost by steam at 100°C to change to 100°C water = $Q_1 = mL_{vap} = 50 \times 540 = 27{,}000$ cal

(ii) Heat lost by 100°C water to change to 0°C water = $Q_2 = ms_{water}\Delta T = 50 \times 1 \times 100 = 5000$ cal

(iii) Heat requird by 0.45 kg of ice to change is temperature from 253 k to 273 k

$= Q_3 = m \times S_{ice} \times \Delta T = 450 \times \frac{1}{2} \times 20 = 4500$ cal

(iv) Heat required by 0.45 kg ice at 273 K to convert into 0.45 kg water at 273 K is

$= Q_4 = mL_f = 450 \times 80 = 36000$ cal

as $Q_4 + Q_3 > Q_1 + Q_2$

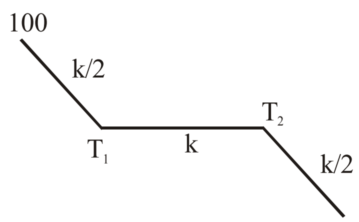
whole ice do not melt, so final temp will be 273 k or 0°C

SECTION-III

Numerical Grid Type (Ranging from 0 to 9) 3 Q. [4 M (0)]

1. **Ans. 2**

Sol. $R_{TH} = \frac{2L}{kA} + \frac{L}{kA} + \frac{2L}{kA}$



$= \frac{5L}{kA}$

$\frac{dH}{dt} = \frac{100}{5L}kA = 20\frac{kA}{L}$

$100 - T_1 = 40$

$\frac{dH}{dt} = \frac{T_2 - 0}{\frac{2L}{kA}} = 20\frac{kA}{L} \Rightarrow T_2 = 40$

$T_B = T_2,\ T_D = T_1 \Rightarrow \Delta T = 20$

2. **Ans. 9**

Sol. $P = eA\sigma T^4$ where T is in kelvin

$$\log_2 \frac{eA\sigma(487+273)^4}{P_0} = 1 \quad \text{...(i)}$$

$$\log_2 \frac{eA\sigma(2767+273)^4}{P_0} = x \quad \text{...(ii)}$$

(ii) – (i)

$$\log_2\left(\frac{3040}{760}\right)^4 = x - 1$$

$\therefore\ x = 9$

3. **Ans. 4**

Sol. $mg = \sigma\left(\pi\left(\frac{d_1}{2}\right)^2\right)$

$$mg+\frac{mv^2}{\ell}=\sigma\left(\pi\left(\frac{d_2}{2}\right)^2\right)$$

So, $1+\frac{v^2}{\ell g}=\left(\frac{d_2}{d_1}\right)^2 \Rightarrow 1+\frac{36}{12}=\left(\frac{d_2}{2mm}\right)^2 \Rightarrow d_2=4mm$

Subjective Type (Ranging from 0 to 9) 7 Q. [4 M (0)]

1. Ans. $5\alpha/3$

Sol. $L\,\alpha\,\Delta T + (2L)(2\alpha)\Delta T = 3L\alpha'\Delta T$

$$\alpha' = \frac{5}{3}\alpha$$

2. Ans. 10cm , 40cm

Sol. $25\alpha_1 \times 100 = 0.05$

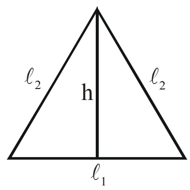
$40\alpha_2 \times 100 = 0.04$

$(\ell_1\alpha_1 + \ell_2\alpha_2) \times 50 = 0.03$

$\ell_1 + \ell_2 = 50$

3.

Sol. $\ell_2^2 = h^2 + \frac{\ell_1^2}{4}$



$2\ell_2 d\ell_2 = \frac{\ell_1}{2}d\ell_1$ .....(i)

Put $d\ell_2 = \ell_2\alpha_2\Delta\theta$

$d\ell_1 = \ell_1\alpha_1\Delta\theta$

in equation (i)

4. Ans. (a) 37.8 J/s (Watts), (b) 2.005 N-m

Sol. (a) $P = ms\frac{dT}{dt} = 180 \times 0.1 \times 4.2 \times 0.5 = 37.8$ J/s

(b) 1 min $\rightarrow$ 180 revolutions

1 sec $\rightarrow$ 3 revolution

$\omega = 2\pi(3) = 6\pi$ rad/sec

$\tau = P/\omega$

5. Ans. (a) – 100°C (b) 1000 J

Sol. (i) Temperature gradient $=\frac{(T_2-T_1)}{1}=\frac{(0-100)}{1}=-100°C/m$

(ii) Steady state temp of element dx is = $T = 100\,(1 - x)$

Heat absorbed by the element to reach steady state is dQ

$dQ = (dm)s\Delta T = (\lambda dx)s(T - 0)$

$\Rightarrow dQ = 20[100\,(1 - x)]dx$

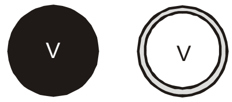
Total heat absorbed by the red

$Q = \int dQ = 2000\int_0^1 (1-x)\,dx = 1000J$

6. **Ans.** (a) the expansion of both will be equal

(b) rise in temperature of hollow sphere will be more [as $\Delta\theta = \frac{Q}{mc}$] and hence its expansion will be more [as $\Delta V = V\gamma\Delta\theta$].

Sol. (a) As thermal expansion of isotropic solids is similar to true photographic enlargement,



expansion of a cavity is same as if it had been a solid body of the same material

i.e. $\Delta V = V\gamma\,\Delta\theta$

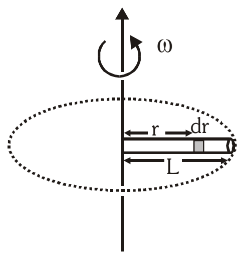
As here V, γ and $\Delta\theta$ are same for both solid and hollow spheres treated (cavity) ;

so the expansion of both will be equal.

(b) If same heat is given to the two spheres due to lesser mass, rise in temperature of hollow sphere will be more [as $\Delta\theta = \frac{Q}{mc}$] and hence its expansion will be more [as $\Delta V = V\gamma\Delta\theta$].

7. Ans. (a) $8\pi \times 10^6\left[\frac{1}{4} - r^2\right]N$ (b) $\frac{1}{3}\times 10^{-3}m$

Sol. (a) Consider an element of length dr at a distance r from the axis of rotation as shown in figure. The centripetal force acting on this element will be $dT = dmr\omega^2 = (\rho A dr)r\omega^2$. As this force is provided by tension in the rod (due to elasticity), so the tension in the rod at a distance r from the axis of rotation will be due to the centripetal force due to all elements between x = r to x = L



i.e.,$T = \int_r^L \rho A\omega^2 r dr = \frac{1}{2}\rho A\omega^2\left[L^2 - r^2\right]$...(i)

So here $T = \frac{1}{2} \times 10^4 \times \pi \times 10^{-2} \times (400)^2\left[\left(\frac{1}{2}\right)^2 - r^2\right] = 8\pi \times 10^6\left[\frac{1}{4} - r^2\right]N$

(b) Now if dy is the elongation in the element of length dr at position r then strain

$\frac{dy}{dr} = \frac{stress}{Y} = \frac{T}{AY} = \frac{1}{2}\frac{\rho\omega^2}{Y}[L^2 - r^2]\,dr$

So the elongation of the whole rod

$\Delta L = \frac{\rho\omega^2}{2Y}\int_0^L (L^2 - r^2)\,dr = \frac{1}{3}\frac{\rho\omega^2 L^3}{Y} = \frac{1}{3}\times\frac{10^4 \times (400)^2 (0.5)^3}{2\times 10^{11}} = \frac{1}{3}\times 10^{-3}m$