SAMPLE QUESTION PAPER - 02

Time: 3 Hrs 15 Min **Subject : Mathematics (35)** **Max Marks: 100**

Instructions: (1) The question paper has five Parts namely A, B, C, D and E. Answer all the parts.
(2) Use the graph sheet for the question on linear programming in PART-E.

PART-A

Answer any Ten of the following. (One Mark each) $\mathbf{10 \times 1 = 10}$

1. Examine whether the operation $* Z^+ \rightarrow Z^+$ defined by $a * b = |a - b|$ where Z^+ is the set of all positive integers, Is a binary operation or not?
2. Is the relation $R = \{(2,1), (1,2), (2,2)\}$ defined on the set $A = \{1,2,3\}$ transitive?
3. Find the domain of $sin^{-1}x$.
4. Find the principal value of $cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$
5. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{(i+j)^2}{2}$.
6. If A is a square matrix and $adjA = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ then find $|A|$.
7. Differentiate $cos\sqrt{x}$ with respect to x.
8. If $y = \log(\text{sinx})$ then find $\frac{dy}{dx}$
9. Evaluate: $\int \sqrt{ax + b}\, dx$
10. Evaluate $\int_0^1 \frac{1}{x^2+1} dx$
11. Find the vector component of the vector with initial point (2,1) and terminal point (-5,7) .
12. If $\hat{a} = \frac{1}{\sqrt{14}}(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$, then write the direction cosines of $\hat{a}$.
13. Find the distance of a plane $3x - 4y + 12z - 3 = 0$ from the origin.
14. Define the objective function in the linear programming problem.
15. If E is the event of a sample space S of an experiment then find $P(S/E)$

PART-B

Answer any Ten of the following.(Two Marks each) $\mathbf{10 \times 2 = 20}$

16. On R, * is defined by $a * b = \frac{a+b}{2}$, verify whether * is associative.
17. Evaluate: $cos^{-1}\left(\frac{1}{2}\right) + 2sin^{-1}\left(\frac{1}{2}\right)$.
18. Prove that $sin^{-1}\left(2x\sqrt{1 - x^2}\right) = 2sin^{-1}x$, $\frac{-1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$
19. Find the values of x, y, z and t If $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$
20. Find the equation of line joining $(1,2)$ and $(3,6)$ using determinants.
21. Find $\frac{dy}{dx}$ if $ax + by^2 = cosy$.
22. Differentiate $cos^{-1}(sinx)$ with respect to x.
23. If $y = x^{sinx}$, $x > 0$. $\frac{dy}{dx}$.
24. Find the local maximum and minimum value of the function $g(x) = x^3 - 3x$.
25. Evaluate $\int \frac{sin^2x - cos^2x}{sin^2xcos^2x} dx$.
26. Evaluate $\int log_e x\, dx$
27. Evaluate $\int \frac{(1+logx)^2}{x} dx$
28. Find the order and degree of differential equation $xy\left(\frac{d^2y}{dx^2}\right) + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

29. Find a vector in direction of the vector $\vec{a} = \hat{\imath} - 2\hat{\jmath}$ that has magnitude 7 units
30. Show that the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ is equally inclined to the positive direction of the axes.
31. Find the Cartesian equation of a line that passing through points (3,-2,-5) and (3,-2,6).
32. Find the Angle between line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and plane $10x + 2y - 11z = 3$
33. The random variable X has probability distribution P(X) of the following form where k some number $P(X) = \begin{cases} k & if\ x = 0 \\ 2k & if\ x = 1 \\ 3k & if\ x = 2 \\ 0 & otherwise \end{cases}$ determine the value of k and $P(X \le 2)$ and $P(X < 2)$.

PART-C

Answer any Ten of the following. (Three Marks each) **$10 \times 3 = 30$**

34. Show that the relation R in the set of all integers Z defined by $R = \{(a,b): 2\ divides\ a - b\}$ is an equivalence relation
35. Write the simplest form of $tan^{-1}\left(\frac{acosx - bsinx}{bcosx + asinx}\right)$, if $\frac{a}{b}tan^{-1}x > -1$
36. If A and B are the symmetric matrices of same order then show that AB is symmetric if and only if A and B are commute. that is $AB = BA$
37. Verify that the value of the determinant remains unchanged if its row and columns are interchanged by considering third order determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$
38. Find $\frac{dy}{dx}$, if $x = a(\theta + sin\theta)\ and\ y = a(1 - cos\theta)$.
39. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for $-1 < x < 1$ and $x \ne y$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.
40. Verify mean value theorem for the function $f(x) = x^2 - 4x - 3$ in the interval $[1,4]$
41. Find the point, at which the tangent to the curve $y = \sqrt{4x-3} - 1$ has its slope $\frac{2}{3}$.
42. Evaluate $\int e^x\left(\frac{1+sinx}{1+cosx}\right)dx$.
43. Evaluate: $\int \frac{x}{(x+1)(x+2)}dx$.
44. Evaluate: $\int_0^{\pi/4} sin2x\, dx$.
45. Find the Area of region bounded by the curve $y = x^2$ and the lines $y = 2$.
46. Form the differential equation representing family of curves $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are arbitrary constants
47. Find the general solution of differential equation $ylog_e ydx - xdy = 0$
48. For three vectors $\vec{a}, \vec{b}\ and\ \vec{c}$ Prove that $[\vec{a} + \vec{b}, \vec{b} + \vec{c},\ \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$
49. Find unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}\ and\ \vec{a} - \vec{b}$ where $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$
50. Find the distance between two parallel lines $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and lines $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$
51. Bag I contains 3 red and 4 black balls and while another bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from bag II

PART-D

Answer any Six of the following. (Five Marks each) $\mathbf{6 \times 5 = 30}$

52. Verify whether the function $f: N \to N$ defined by $f(x) = x^2$ is one-one, onto and bijective.
53. Prove that the function f : R → R defined by $f(x) = 4x + 3$ is invertible function and find the inverse of f
54. If $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$ then verify that $(AB)' = B'A'$.
55. Solve the following system of linear equation by matrix method

$$4x + 3y + 2z = 60$$
$$2x + 4y + 6z = 90$$
$$6x + 2y + 3z = 70\ .$$

56. If $y = e^{a\cos^{-1}x}$, $-1 \le x \le 1$ then prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$.
57. Sand is pouring from a pipe at the rate of 12cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm.
58. Find the integral of $\frac{1}{x^2 - a^2}$ w.r.t x and evaluate $\int \frac{1}{4x^2 - 9}dx$
59. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $(a > b)$ by the method of integration .
60. Find the general solution of differential equation $ydx - (x + 2y^2)dy = 0$.
61. Derive the Equation of plane in normal form both in the vector and Cartesian form.
62. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that

    (i) All the five cards are spades? (ii) Only 3 cards are spades? (iii) None is a spade?
63. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that (i) both balls are red (ii) first ball is black and second is red

    (iii) one of them is black and other is red

PART-E

Answer any One of the following. (Ten Marks) $\mathbf{1 \times 10 = 10}$

64. a) Prove that $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$ and hence find the value of $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{tanx}}$

    b) Find the value of a and b if $f(x) = \begin{cases} 5 & if\ x \le 2 \\ ax + b & if\ 2 < x < 10 \\ 21 & if\ x \ge 10 \end{cases}$ is continuous function
65. a) Minimise and Maximise $Z = x + 2y$

    subject to $x + 2y \ge 100$

    $$2x - y \le 0$$
    $$2x + y \le 200$$
    $$x, y \ge 0$$

    b) Prove that $\begin{vmatrix} a - b - c & 2a & 2a \\ 2b & b - c - a & 2b \\ 2c & 2c & c - a - b \end{vmatrix} = (a + b + c)^3$
66. a) Show that the right circular cylinder of given surface and maximum volume is such that is heights is equal to the diameter of the base.

    b) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, satisfies the equation $A^2 - 4A + I = O$,

    where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. Find A^{-1}