Level-II

Chapter 10

Permutations and Combinations

Solutions

SECTION - A

Objective Type Questions (One option is correct)

1. If one quarter of all the subsets containing three elements of the integers 1, 2, 3, ..., m contain the integer 5, then m is equal to

   (1) 12 (2) 10 (3) 14 (4) 11

Sol. Answer (1)

Number of 3 element subsets = ${}^{m}C_3$

Number of 3 element subset which contains 5 = ${}^{m-1}C_2$

According to question ${}^{m}C_3 \times \frac{1}{4} = {}^{m-1}C_2$

$\Rightarrow \quad {}^{m}C_3 = 4.{}^{m-1}C_2$

$$\frac{m!}{3!(m-3)!} = 4.\frac{m-1!}{2!(m-3)!} = \frac{m}{6} = \frac{4}{2}$$

$\Rightarrow m = 12$

2. Let y be an element of the set $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and x_1, x_2, x_3 be integers such that $y = x_1x_2x_3$. Then the number of positive integral solutions of $x_1x_2x_3 = y$ is

   (1) 64 (2) 27 (3) 81 (4) 30

Sol. Answer (1)

When	$y = 1$	$x_1.x_2.x_3 = 1$	Only one solutions = $x_1 = x_2 = x_3 = 1$
	$y = 2$	$1 \times 1 \times 2$	no. of solutions = $\frac{3!}{2!} = 3$
	$y = 3$	$1 \times 1 \times 3$	no. of solutions = $\frac{3!}{2!} = 3$
	$y = 5$	$1 \times 1 \times 5$	no. of solutions = $\frac{3!}{2!} = 3$

$y = 6$ $1 \times 2 \times 3$ no. of solutions = $3! + \frac{3!}{2!} = 9$

or $1 \times 1 \times 6$

$y = 10$ $1 \times 2 \times 5$ no. of solutions = $3! + \frac{3!}{2!} = 9$

or $1 \times 1 \times 10$

$y = 15$ $1 \times 3 \times 5$ no. of solutions = $3! + \frac{3!}{2!} = 9$

or $1 \times 1 \times 15$

$y = 30$ $1 \times 5 \times 6$ no. of ways = $4 \times 3! + \frac{3!}{2!}$

or $2 \times 3 \times 5$ $= 24 + 3 = 27$

or $1 \times 3 \times 10$

or $1 \times 15 \times 2$

or $1 \times 1 \times 30$

$\therefore$ Total number of ways = 64.

3. The number of divisors of the form $4k + 2$, $k \geq 0$ of the integer 240 is

(1) 4 (2) 8 (3) 10 (4) 3

Sol. Answer (1)

$240 = 2^4 \times 3 \times 5$

Number of divisors of the form : $4k + 2$, $k \geq 0$ or $2(2k + 1)$

Such divisors are 2.1 or 2.3 or 2.5 or 2.15.

So number of such divisors = 4.

4. There are 3 apartments *A*, *B* and *C* for rent in a building. Each apartment will accept either 3 or 4 occupants. The number of ways of renting the apartments to 10 students

(1) 12600 (2) 10800 (3) 13500 (4) 15000

Sol. Answer (1)

This is clearly a case of "a four and two threes". Now *A* can be given 4 occupants, and *B* and *C*, 3 each in ${}^{10}C_4 \cdot {}^{6}C_3 \cdot {}^{3}C_3$ ways.

The total number will 3 times this

Hence, required number is = $3 \cdot {}^{10}C_4 \cdot {}^{6}C_3 \cdot {}^{3}C_3$

$= 12600$

5. A double-decker bus has 5 empty seats in the upper deck and 5 empty seats in the lower deck. 10 people board the bus of which 2 are old people and 3 are children. The children refuse to take seats lower deck while old people insist to stay lower deck. How many different sitting arrangements of the people are possible?

(1) 121000 (2) 81000 (3) 10000 (4) 144000

Sol. Answer (4)

We can select any 2 seats down stair and give seat to old person this can be done in = ${}^{5}C_2 \times 2!$ ways

Again we can select any 3 seats upstair and give it to children this can be done in = ${}^{5}C_3 \times 3!$ ways

Rest can be seated in 5! ways

$\therefore$ Required number of ways = ${}^{5}C_2 \times 2! \times {}^{5}C_3 \times 3! \times 5!$

$= 20 \times 60 \times 120 = 144000$

6. The number of ways, in which 10 identical objects of one kind, 10 of another kind and 10 of third kind can be divided between two persons so that each person has 15 objects, is

(1) 136 (2) 91 (3) 45 (4) 36

Sol. Answer (2)

$a + b + c = 15$

n = Coefficient of x^{15} in $(x^0 + x^1 + x^2 + ...x^{10})^3$

= Coefficient of x^{15} in $\dfrac{(1-x^{11})^3}{(1-x)^3}$

= Coefficient of x^{15} in $(1 - x^{11})^3 (1 - x)^{-3}$

= Coefficient of x^{15} in
$(1 - 3x^{11} + 3x^{22} - x^{33}) (1 - x)^{-3}$

$= {}^{15+3-1}C_{3-1} - 3\,{}^{4+3-1}C_{3-1}$

$= {}^{17}C_2 - 3\,{}^{6}C_2 = 17 \times 8 - 3 \times 3 \times 5$

$= 136 - 45 = 91$

7. There are 6 letters and 6 directed envelopes. The number of ways in which 4 letters are rightly placed and 2 letters are wrongly placed, is

(1) 45 (2) 30 (3) 15 (4) Zero

Sol. Answer (3)

$n = {}^6C_2 \times 1 = {}^6C_2 = 15$

8. Out of 6 consonants and 5 vowels, number of words, that can be made each containing 4 consonants and 2 vowels, is

(1) 75600 (2) 378000 (3) 151200 (4) 108000

Sol. Answer (4)

Number of such words formed

$= {}^6C_4 . {}^5C_3 \times \underline{|6}$

9. If the numbers of circular arrangements of 50 persons including A and B such that there are exactly 24 persons between A and B and there are 12 persons between A and B are m and n respectively, then $\dfrac{m}{n}$ is

(1) 1 (2) 2 (3) $\dfrac{1}{2}$ (4) $\dfrac{1}{3}$

Sol. Answer (3)

$m = \underline{|48}$, $n = 2 \times \underline{|48}$

$\dfrac{m}{n} = \dfrac{1}{2}$

10. Number of 5 digit numbers which have exactly three 5's is

(1) 774 (2) 486 (3) 288 (4) 144

Sol. Answer (1)

$n = {}^4C_2 \times 9^2 + {}^4C_3 \times 8 \times 9$

$= (6 \times 81) + (4 \times 72)$

$= 486 + 288 = 774$

11. Number of total numbers divisible by 3 and lying between 100 and 1000 using the digits 0, 1, 2, 3, 4, if the repetition of digits is not allowed, is

(1) 48 (2) 24 (3) 20 (4) 10

Sol. Answer (3)

0, 1, 2 or 0, 2, 4 or 1, 2, 3 or 2, 3, 4

Total numbers = (2 × 2 × 1) + (2 × 2 × 1) + (3 × 2 × 1) + (3 × 2 × 1) = 20

12. Number of outcomes, when 3 identical dice are thrown together, is

(1) 216 (2) 156 (3) 56 (4) 20

Sol. Answer (3)

n(numbers on all three dice are same) = n_1 = 6

n(numbers on exactly two dice are same)

$= n_2 = (6 \times 6 - 6) = 30$

n(numbers on all three dice are different)

$$= \frac{216 - 6 - 90}{\underline{|3}} = \frac{120}{6} = 20 \text{ or } {}^6C_3 = 20$$

n(outcomes) = 6 + 30 + 20 = 56

13. Number of ways in which 5 identical black balls, 6 identical red balls and 7 identical green balls can be arranged in a row so that all the balls of same colour are never togther

(1) $\frac{\underline{|18}}{\underline{|4}\,\underline{|5}\,\underline{|6}}$ (2) $\frac{\underline{|18}}{\underline{|5}\,\underline{|6}\,\underline{|7}}$ (3) $\frac{\underline{|18}}{\underline{|5}\,\underline{|6}\,\underline{|7}} - \underline{|3}$ (4) $\frac{\underline{|18}}{\underline{|5}\,\underline{|6}\,\underline{|7}} - \underline{|6}$

Sol. Answer (3)

Number of ways = $\frac{\underline{|18}}{\underline{|5}\,\underline{|6}\,\underline{|7}} - \underline{|3}$

14. A class has 15 students. The number of ways of forming a team of students including at least two students and also excluding at least two students, is

(1) 32768 (2) 32736 (3) 16384 (4) 16368

Sol. Answer (2)

Required number of ways

$= {}^{15}C_2 + {}^{15}C_3 + {}^{15}C_4 + \ldots + {}^{15}C_{13}$

$= 2^{15} - \left({}^{15}C_0 + {}^{15}C_1 + {}^{15}C_{14} + {}^{15}C_{15}\right)$

$= 2^{15} - 32 = 32736$

15. Five balls are to be placed in four boxes. Each can hold all the five balls. Number of ways in which we can place the balls so that no box remains empty, if the balls as well as boxes are identical, is

(1) 150 (2) 25 (3) 6 (4) 1

Sol. Answer (4)

When balls as well as boxes are identical the number of combinations and arrangements will be 1 each.

16. The total number of 5-digit numbers of different digits using the digits 0, 1, 2, ..., 7, in which the digit in the middle is the largest is

(1) 1130 (2) 1132 (3) 1134 (4) 1136

Sol. Answer (3)

Middle digit ≥ 4

No. of numbers with '4' in the middle = ${}^4P_4 - {}^3P_3$

No. of numbers with '5' in the middle = ${}^5P_4 - {}^4P_3$

and so on

$\therefore$ Required number of numbers = $({}^4P_4 - {}^3P_3) + ({}^5P_4 - {}^4P_3) + ({}^6P_4 - {}^5P_3) + ({}^7P_4 - {}^6P_3)$

= 1134

SECTION - B

Objective Type Questions (More than one options are correct)

1. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then the value of n is

(1) 14 (2) 11 (3) 7 (4) 8

Sol. Answer (1, 3)

$$2 \times {}^nC_5 = {}^nC_4 + {}^nC_6$$

$${}^nC_5 - {}^nC_4 = {}^nC_6 - {}^nC_5$$

On solving we get the answer.

2. The results of 11 chess matches (as win, lose or draw) are to be forecast. Out of all possible forecasts, the number of ways in which 8 correct and 3 incorrect results can be forecast is

(1) ${}^{11}C_8 \times 8$ (2) $2^3 \times {}^{11}C_3$ (3) 1320 (4) $8! \times 3!$

Sol. Answer (1, 2, 3)

Required number of ways = ${}^{11}C_8 \times 2^3 = {}^{11}C_3 \times 2^3 = 1320$

3. Ritu has to answer 10 out of 13 questions in an examination. The number of ways in which she can answer if she must answer at least 3 of the first five questions is

(1) 276 (2) ${}^{13}C_{10} - {}^5C_3$

(3) ${}^5C_3.{}^8C_7 + {}^5C_4.{}^8C_6 + {}^5C_5.{}^8C_5$ (4) ${}^{13}C_{10} - {}^5C_2$

Sol. Answer (1, 2, 3, 4)

Required number of ways = ${}^5C_3 \,.\, {}^8C_7 + {}^5C_4 \,.\, {}^8C_6 + {}^5C_5 \,.\, {}^8C_5$

$= {}^{13}C_{10} - {}^5C_2 = {}^{13}C_{10} - {}^5C_3 = 276$

4. Let N denote the number of ways in which n boys can be arranged in a line so that 3 particular boys are separated then, then N is equal to

(1) ${}^{n-2}P_3(n-3)!$ (2) $\dfrac{(n-2)!(n-3)!}{n-5!}$ (3) ${}^{n-2}C_3\, 3!(n-3)!$ (4) $(n-3)(n-4)(n-2)!$

Sol. Answer (1, 2, 3, 4)

First of all we arrange $(n-3)$ boys by $\lfloor n-3$ ways. Then between $(n-2)$ gaps between $(n-3)$ boys 3 particular boys can be arrange by ${}^{n-2}P_3$ ways

Total number of ways $= {}^{n-2}P_3 \times (n-3)!$

5. There are 25 students in a class. The teacher takes 5 of them at a time, to zoological garden as often as he can, without taking the same 5 students more than once. Then the number of visits, the teacher makes to the garden exceeds that of a student by

(1) ${}^{25}C_5 - {}^{24}C_5$ (2) ${}^{24}C_4$ (3) ${}^{24}C_5$ (4) ${}^{25}C_5 - {}^{24}C_4$

Sol. Answer (3, 4)

Number of visits the teacher makes to the garden = ${}^{25}C_5$

Number of visits the student makes to the garden = ${}^{24}C_4$

Required number of ways = ${}^{25}C_5 - {}^{24}C_4 = {}^{24}C_5$

6. The number of non-negative integral solutions of $x_1 + x_2 + x_3 + x_4 \le n$ (where n is a non-negative integer) is

(1) ${}^{n+3}C_3$ (2) ${}^{n+4}C_4$ (3) ${}^{n+3}C_n$ (4) ${}^{n+4}C_n$

Sol. Answer (2, 4)

$x_1 + x_2 + x_3 + x_4 \le n$

$\therefore$ Number of non-negative integral solutions

$= {}^{n+4-1}C_{4-1} + {}^{n-1+4-1}C_{4-1} + {}^{4-2+4-1}C_{4-1} + ... + {}^{0+4-1}C_{4-1}$

$= {}^{n+3}C_3 + {}^{n+2}C_3 + {}^{n+1}C_3 + ... + {}^{3}C_3 = {}^{n+4}C_4$

Alternate method

$x_1 + x_2 + x_3 + x_4 + x_5 = n,\ 0 \le x_5 \le n$

Number of solutions

$= {}^{n+5-1}C_{5-1} = {}^{n+4}C_4$

7. If N is the number of positive integral solutions of $x_1x_2x_3x_4 = 770$, then

(1) N is divisible by 4 distinct primes (2) N is a perfect square

(3) N is a perfect fourth power (4) N is a perfect 8th power

Sol. Answer (2, 3, 4)

$770 = 2 \times 5 \times 11 \times 7 = x_1\, x_2\, x_3\, x_4$

2 can be distributed in 4 ways, similarly 5, 11 and 7 can be distributed in 4 ways respectively.

Required number of ways = $4 . 4 . 4 = 4^4 = N$

8. There are n white and n red balls marked 1, 2, 3, ...n. The number of ways we can arrange these balls in a row so that neighbouring balls are of different colours is

(1) $2(n)!$ (2) $(2n)!$ (3) $2(n!)^2$ (4) $2.\dfrac{(2n)!}{{}^{2n}C_n}$

Sol. Answer (3, 4)

Required number of ways = 2 . $n!$ $n!$ = $2(n!)^2$ = 2 . $\frac{(2n!)}{^{2n}C_n}$.

(Either red will come first or white).

9. There are 15 bulbs in a room. Each one of them can be operated independently. The number of ways in which the room can be lighted is

(1) $8^5 - 1$ (2) $(32)^2 - 1$ (3) $(32)^3 - 1$ (4) $8^4 - 1$

Sol. Answer (1, 3)

Each bulb will be either 'on' or 'off'

Required number of ways = $2^{15} - 1 = (32)^3 - 1 = (8)^5 - 1$.

10. The number of six digit numbers that can be form from the digits 1, 2, 3, 4, 5, 6 and 7. So that the digits do not repeat and the terminal digits are even is

(1) $5.4! \cdot 3!$ (2) 72 (3) 720 (4) $5! \cdot 3!$

Sol. Answer (1, 3, 4)

Required number = $(^3C_2 \times 2!) \times (^5C_4 \times 4!) = 720$

11. The number of integers which lie between 1 and 10^6 and which have sum of the digits equal to 12 is

(1) $^{17}C_5 - 6 \times {}^7C_5$ (2) 5382 (3) 6062 (4) 8055

Sol. Answer (1, 3)

The number of ways in which the sum of digits will be equal to 12 is

= Coefficient of x^{12} in $(x^0 + x^1 + x^2 + \ldots\ldots + x^9)^6$

= Coefficient of x^{12} in $\left(\frac{1-x^{10}}{1-x}\right)^6$

= coefficient of x^{12} in $(1 - x^{10})^6 (1 - x)^{-6}$

= coefficient of x^{12} in $(1 - x)^{-6} (1 - {}^6C_1 \cdot x^{10} + \ldots\ldots)$

= coefficient of x^{12} in $(1 - x)^{-6} - {}^6C_1$ · coefficient of x^2 in $(1 - x)^{-6}$

= $^{12+6-1}C_{6-1} - {}^6C_1 \cdot {}^{2+6-1}C_{6-1}$

= $^{17}C_5 - {}^{6 \cdot 7}C_5 = 6062$

Hence answer is (1, 3)

12. $\sum_{r=0}^{m} {}^{n+r}C_n$ is equal to

(1) $^{n+m+1}C_{n+1}$ (2) $^{n+m+2}C_n$ (3) $^{n+m+1}C_m$ (4) $^{n+m-1}C_{m-1}$

Sol. Answer (1, 3)

$$\sum_{r=0}^{m} {}^{n+r}C_n = \sum_{r=0}^{m} {}^{n+r}C_r$$

$$= {}^nC_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + \ldots\ldots + {}^{n+m}C_m$$

$$= 1 + n + 1 + {}^{n+2}C_2 + \ldots\ldots + {}^{n+m}C_m$$

$$=\left(n+2+{}^{n+2}C_2\right)+{}^{n+2}C_3+........+{}^{n+m}C_m$$

$$=\left({}^{n+2}C_1+{}^{n+2}C_2\right)+{}^{n+3}C_3+........+{}^{n+m}C_m$$

$$=\left({}^{n+3}C_2+{}^{n+3}C_3\right)+{}^{n+4}C_4+........+{}^{n+m}C_m$$

$$={}^{n+m+1}C_m$$

13. Between two junction stations A and B there are 12 intermediate stations. The number of ways in which a train can be made to stop at 4 of these stations so that no two of these halting station are consecutive is

(1) ${}^{9}C_4$ (2) ${}^{12}C_4-4$ (3) ${}^{9}C_5$ (4) $\frac{9}{5}\cdot{}^{8}C_4$

Sol. Answer (1, 3, 4)

Number of remaining stations at which train do not halt consecutively = 8

$S_1 \times S_2 \times S_3 \times S_4 \times S_5 \times S_6 \times S_7 \times S_8$

Required number of ways = Number of ways of selection of 4 distinct places out of (8 + 1) places = ${}^{9}C_4$ = ${}^{9}C_5$

14. m parallel lines in a plane are intersected by family of n parallel lines. The total number of parallelogram so formed is

(1) ${}^{m}C_2\cdot{}^{n}C_2$ (2) $\frac{m(m-1)(n-1)}{4}$

(3) 150, for m = 5, n = 6 (4) 25, for m = 5, n = 6

Sol. Answer (1, 3)

A parallelogram is formed by choosing two straight lines from the set of m parallel lines and two straight lines from the set of n parallel lines.

Hence number of parallelogram such formed is

$= {}^{m}C_2\cdot{}^{n}C_2$

$$=\frac{m(m-1)}{2}\cdot\frac{n(n-1)}{2}$$

$$=\frac{mn(m-1)(n-1)}{4}$$

for m = 5, n = 6

Number of parallelogram = $\frac{5\times6\times4\times5}{4}$ = 150

Hence, answer is (1, 3)

15. There are n straight lines in a plane, no two of which are parallel and no three pass through the same point. Their point of intersection are joined. Then the number of fresh lines thus obtained is

(1) ${}^{n}C_2\cdot\frac{(n-2)(n-3)}{4}$ (2) $\frac{n(n-1)(n-2)(n-3)}{6}$

(3) $\frac{n(n-1)(n-2)(n-3)}{8}$ (4) $\frac{n(n-1)(n-2)}{8}$

Sol. Answer (1, 3)

Since no two lines are parallel and no three are concurrent, therefore n straight lines intersect at ${}^nC_2 = N$ (let) points. Since two points are required to determine a straight line, therefore, the total number of lines obtained by joining N points ${}^NC_2$. But in this each old line has been counted ${}^{n-1}C_2$ times, since on each old line there will be $n-1$ points of intersection made by remaining $(n-1)$ lines. Hence the required number of fresh lines.

$= {}^NC_2 - n \cdot {}^{n-1}C_2$

$$= \frac{N(N-1)}{2} - \frac{n(n-1)(n-2)}{2}$$

$$= \frac{{}^nC_2({}^nC_2 - 1)}{2} - \frac{n(n-1)(n-2)}{2}$$

$$= \frac{\frac{n(n-1)}{2} \cdot \left(\frac{n(n-1)}{2} - 2\right)}{2} - \frac{n(n-1)(n-2)}{2}$$

$$= \frac{n(n-1)(n-2)(n-3)}{8} = {}^nC_2 \cdot \frac{(n-2)(n-3)}{4}$$

Hence, answer is (1, 3)

16. On the modified chess board 10 × 10, Amit and Suresh two persons which start moving towards each other. Each person moving with same constant speed. Amit can move only to the right and upwards along the lines while Suresh can move only to the left or downwards along the lines of the chess board. The total number of ways in which Amit and Suresh can meet at same point during their trip.

(1) ${}^{20}C_{10}$

(2) $\left(\frac{11}{10}\right)\left(\frac{10}{9}\right)\left(\frac{9}{8}\right)\left(\frac{8}{7}\right)\left(\frac{7}{6}\right)\left(\frac{6}{5}\right)\left(\frac{5}{4}\right)\left(\frac{4}{3}\right)\left(\frac{3}{2}\right)\left(\frac{2}{1}\right)$

(3) $2^{10}\left(\frac{1}{1}\right)\left(\frac{3}{2}\right)\left(\frac{5}{3}\right)\left(\frac{7}{5}\right).......\left(\frac{19}{10}\right)$

(4) $\left(\frac{2}{1}\right)\left(\frac{6}{2}\right)\left(\frac{10}{3}\right)..........\left(\frac{38}{10}\right)$

Sol. Answer (1, 2, 3, 4)

Both persons are having same speed. According to instructions of movement of persons,

required number of ways = ${}^{20}C_{10}$

SECTION - C

Linked Comprehension Type Questions

Comprehension-I

If n things are arranged in a row, the number of ways in which they can be deranged so that none occupies its original position is

$$n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + ... + (-1)^n \frac{1}{n!}\right)$$

1. The number of ways of putting 6 letters into 6 addressed envelopes so that exactly 2 letters are in wrong envelops is

(1) ${}^6C_4$ (2) 30 (3) 45 (4) 28

Sol. Answer (1)

Required no. = ${}^6C_2 = {}^6C_4$

2. Five boys are to be seated in a row. The number of ways in which 3 boys are not seated in the place specified to them is

(1) ${}^5P_2$ (2) 10 (3) 40 (4) 25

Sol. Answer (1)

$${}^5C_2 \times 3!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right)$$

$= 10 \times 2 = 20 = {}^5P_2$.

3. The number of ways of putting at least 3 out of 6 letters in wrong envelops is

(1) 265 (2) 40 (3) 704 (4) 50

Sol. Answer (3)

$${}^6C_3 \times 3!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right)+{}^6C_4 \times 4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)+{}^6C_5 \times 5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$$

$$+{}^6C_6 \times 6!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}\right)=704$$

Comprehension-II

Five balls are to be placed in three boxes. Each can hold all the five balls. In how many different ways can we place the balls so that no box remains empty, if

1. Balls and boxes are all different

(1) 60 (2) 90 (3) 150 (4) 80

Sol. Answer (3)

As no box is empty, so balls can be of the group 1, 1, 3 or 1, 2, 2.

∴ Required number of arrangement

$$=\frac{5!}{1!1!3!}\times\frac{3!}{2!}+\frac{5!}{1!2!2!}\times\frac{3!}{2!} = \frac{3!}{2!}(20+30) = \frac{6\times 50}{2}=150$$

Alternate Method

Number of ways = $3^5 - {}^3C_1 2^5 + {}^3C_2 1^5 = 150$

2. Balls are different but boxes are identical

(1) 15 (2) 25 (3) ${}^5C_1 \times {}^4C_2 \times {}^2C_2$ (4) 30

Sol. Answer (2)

Required number of ways = $\dfrac{\frac{5!}{1!1!3!}}{2!}+\dfrac{\frac{5!}{2!2!1!}}{2!} = \dfrac{20}{2}+\dfrac{30}{2}$ = 10 + 15 = 25.

3. Balls as well as boxes are identical

(1) ${}^5C_3$ (2) 2 (3) 1 (4) 6

Sol. Answer (2)

As boxes as well as balls are identical there will be only two arrangements (one in group of 1, 1, 3 and other 1, 2, 2).

Comprehension-III

One and only one straight line can be drawn passing through two given points and we can draw only one triangle through three non-collinear points. By integral coordinates (x, y) of a point we mean both x and y as integers.

1. The number of triangles whose vertices are the vertices of an octagon but none of whose sides happen to be come from the octagon is

(1) 16 (2) 28 (3) 56 (4) 70

Sol. Answer (1)

Total number of triangles = ${}^8C_3 = 56$.

Number of triangles whose one side is common to octagon = 4 × 8 = 32.

Number of triangle whose two sides are common to octagon = 8.

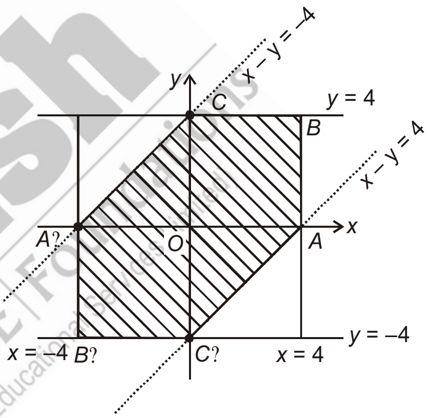
Required number of triangle = 56 – 32 – 8 = 16.

2. The number of points in the cartesian plane with integral coordinates satisfying the inequalities

$|x| \le 4$, $|y| \le 4$ and $|x - y| \le 4$ is

(1) 17 (2) 91 (3) 61 (4) 64

Sol. Answer (3)



$|x| \le 4 \Rightarrow -4 \le x \le 4$

$|y| \le 4 \Rightarrow -4 \le y \le 4$

$|x - y| \le 4 \Rightarrow -4 \le x - y \le 4$

There are 4 × 4 = 16 points inside square $OABC$.

Similarly then all 4 × 4 = 16 point middle square $OA'B'C'$

there are 6 points each in $\Delta OA'C$ and OAC'.

Also there are 17 points on line $A'A$ and CC'.

Number of solution = Number of integral co-ordinates in shaded region = 16 + 16 + 6 + 6 + 17 = 61

3. The sides AB, BC, CA of a triangle ABC have a, b and c interior points excluding vertices on them respectively. The number of triangles that can be contructed using these interior points as vertices is

(1) $\dfrac{ab+bc+ca}{2}$ (2) $\Sigma ab(a + b - 2)$

(3) $\Sigma ab(a + b)$ (4) ${}^{a+b+c}C_3 - ({}^aC_3 + {}^bC_3 + {}^cC_3)$

Sol. Answer (4)

Required number of triangle = ${}^{a+b+c}C_3 - {}^aC_3 - {}^bC_3 - {}^cC_3$.

Comprehension-IV

Let a_n denote the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0.

1. Which of the following is correct?

(1) $a_{17} = a_{16} + a_{15}$ (2) $c_{17} \ne c_{16} + c_{15}$ (3) $b_{17} \ne b_{16} + c_{16}$ (4) $a_{17} = c_{17} + b_{16}$

Sol. Answer (1)

Let k be the number of 1's and r be the number of zeros and the first digit obviously has to be "1". Clearly

$a_n = 1 + {}^{n-2}C_1 + {}^{n-3}C_2 + {}^{n-4}C_3 + ... + {}^{k}C_r \quad (k \geq r)$

$\Rightarrow \ a_{n-1} = 1 + {}^{n-3}C_1 + {}^{n-4}C_2 + {}^{n-5}C_3 + ...$

as $a_{n-2} = 1 + {}^{n-4}C_1 + {}^{n-5}C_2 + {}^{n-6}C_3 + ...$

Clearly, $\boxed{a_{n-2} + a_{n-1} = a_n}$

Hence, $a_{17} = a_{16} + a_{15}$

2. The value of b_6 is

(1) 7 (2) 8 (3) 9 (4) 11

Sol. Answer (2)

So, $b_n = 1 + {}^4C_1 + {}^3C_2 = 8$

SECTION - D

Matrix-Match Type Questions

1. How many seven letters words can be formed by using the letters of the word SUCCESS so that?

	Column-I		**Column-II**
(A)	The two C are together but no two S are together	(p)	60
(B)	Neither two C nor two S are together	(q)	96
(C)	All C are together and S are together and E always comes before U	(r)	24
(D)	U is the starting letter of the word	(s)	12

Sol. Answer A(r); B(q); C(s); D(p)

(A) SUCCESS

_ U _ CC _ E _

If 'S' is placed at the place indicated by _

Then no two 'S' will together

Assuming two 'C' as single unit,

Number of arrangement = 3! × ${}^4C_3$ = 6 × 4 = 24.

(B) _ U _ C _ C _ E _

Number of ways when no two 'S' is together

$= \dfrac{4!}{2!} \times {}^5C_3 = 4 \times 3 \times 10 = 120$

Number of arrangement two 'C' are together and no to 'S' are together = 24.

$\therefore$ Required number = 120 – 24 = 96.

(C) Assuming all C and S together.

Total number of arrangement = 4! = 24.

No. of ways when U will con before E = $\dfrac{24}{2} = 12$.

(D) $\boxed{\text{U}}$

Number of ways when U comes first = $\dfrac{6!}{3!.2!}$ = $\dfrac{6\times5\times4\times3\times2}{3\times2\times2}=60$

2. Match the following

Column-I	**Column-II**
(A) The number of ways in which 35 mangoes can be distributed among 3 boys so that each can have any number of mangoes	(p) 666
(B) The number of triangles whose vertices are at the vertices of a decagon with one side common with the decagon	(q) 60
(C) A box contains 2 white, 3 black and 4 red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be include in the draw?	(r) 64
(D) A child attempts to open a five disc-lock. He takes 5 seconds to dial a particular number on the disc. If he does so for 5 hours, every day, then the maximum number of days he would take to open the lock.	(s) 28

Sol. Answer A(p); B(q); C(r); D(s)

(A) Required number = ${}^{35+3-1}C_{3-1}$ = ${}^{37}C_2$ = 666.

(B) Total number of such triangles = 10 × 6 = 60.

(C) Required number of ways = ${}^{9}C_3 - {}^{6}C_3$ = 84 – 20 = 64.

(D) Number of 5 digit number on 5 disc-lock

= 10 × 10 × 10 × 10 × 10 = 10^5

Number of days = $\dfrac{10^5}{3600}=27.77=28$ days

3. Match the following

Column-I	**Column-II**
(A) The number of squares of any size in a chessboard is	(p) 1092
(B) The number of rectangles of any size which are not squares in a chessboard is	(q) 204
(C) The number of squares of size 3 × 3 in a chessboard is	(r) 36
(D) The number of rectangles of size 5 × 4 in a chessboard is	(s) 40

Sol. Answer A(q); B(p); C(r); D(s)

(A) Number of squares of any size = $\dfrac{8(8+1)(17)}{6}$ = 204

(B) Number of rectangles of any size which are not squares in a chess board = $\left(\dfrac{8(8+1)}{2}\right)^2-204=1092$

(C) Number of squares of size 3 × 3 in a chessboard

= (8 + 1 – 3) (8 + 1 – 3) = 6^2 = 36.

(D) Number of rectangles of size 5 × 4 in a chess board

= 2 × (5 × 4) = 40

4. Match the following

Column I	**Column II**
(A) There are 8 Hindi novels and 6 English novels. 4 Hindi novels and 3 English novels are selected and arranged in a row such that they are alternate then no. of ways is	(p) $\frac{5.(8!)}{56}$
(B) Number of arrangements of letters of the word CONCRETE such that no two vowels are together is	(q) 5(8!)
(C) There are 10 AC in a hall. In how many ways they can be operated such that atleast 4 AC is on?	(r) 848
(D) $A = \{1, 2, 3, ..., 10, 11\}$, Number of subsets of A having atleast 6 elements is equal to	(s) 1024
	(t) ${}^8P_4 \,.\, {}^6P_3$

Sol. Answer A(q, t); B(p); C(r); D(s)

(A) Books are in alternate order, then they must be in the order

$H_1\,E_1\,H_2\,E_2\,H_3\,E_3\,H_4$

Total no. of ways = ${}^8C_4 \times {}^6C_3 \times 4! \times 3! = 5.(8!)$

(B) CONCRETE

vowels are O, E, E

_ C _ N _ C _ R _ T _

If vowels are placed at the place indicated by _ then no two vowels will be together

Now required no. of ways = $\frac{5!}{2!} \times {}^6C_3 \times \frac{3!}{2!}$

$$\frac{5!}{2!} \times \frac{6 \times 5 \times 4}{3!} \times \frac{3!}{2!} = 5(6)! = \frac{5(8!)}{56}$$

(C) Required no. of ways = ${}^{10}C_4 \times 2^6 = \frac{8!}{3}$

(D) Required no. of subsets with the given condition $= {}^{11}C_6 + {}^{11}C_7 + {}^{11}C_8 + {}^{11}C_9 + {}^{11}C_{10} + {}^{11}C_{11}$

$$= \frac{1}{2}\,[{}^{11}C_0 + {}^{11}C_1 + \ldots\ldots\ldots + {}^{11}C_{11}]$$

$$= 2^{10} = 1024$$

5. Match the entries of Column-I with those of Column-II.

Column I	**Column II**
(A) The value of x satisfying the inequality ${}^{10}C_{x-1} > 2 \cdot {}^{10}C_x$ is	(p) 9
(B) A person wishes to make up as many different parties as he can out of his 20 friends such that each party consists of the same number of persons. The number of friends he should invite at a time is less than or equal to	(q) 7
(C) There are four letters and four directed envelopes. The number of ways in which all the letters can be put in the wrong envelopes is	(r) 8
(D) If a denotes the number of permutations of $x + 2$ things taken all at a time, b the number of permutations of x things taken 11 at a time and c the number of permutations of $x - 11$ things taken all a time such that $a = 182bc$, then the value of x is greater than or equal to	(s) 10
	(t) 12

Sol. Answer A(p, r, s); B(s, t); C(p); D(p, q, r, s, t)

(A) ${}^{10}C_{x-1} > 2 \cdot {}^{10}C_x$

$$\Rightarrow \frac{10!}{(x-1)!(10-x+1)!} > 2 \cdot \frac{10!}{(10-x)!\,x!}$$

$$\Rightarrow \frac{1}{11-x} > \frac{2}{x} \quad x \in I^+$$

$\Rightarrow x > 22 - 2x$

$3x > 22 \Rightarrow x > \frac{22}{3}$

$\therefore x = 8, 9, 10$

(B) Suppose he invites r friends at a time, then the total number of parties is ${}^{20}C_r$. We have to find maximum value of ${}^{20}C_r$, which is for $r = 10$

$\therefore$ He should invite 10 friends at a time in order to form the maximum number of parties.

(C) Required number of ways

$$= 4!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right]$$

$= 12 - 4 + 1 = 9$

(D) We have, $a = {}^{x+2}P_{x+2} = (x + 2)!$

$b = {}^{x}P_{11} = \frac{x!}{(x-11)!}$

$c = {}^{x-11}P_{x-11} = (x - 11)!$

Now, $a = 182bc$

$\Rightarrow (x + 2)! = 182 \cdot \frac{x!}{(x-11)!} \cdot (x-11)!$

$\Rightarrow (x + 2)! = 182 \times x!$

$\Rightarrow \boxed{x = 12}$

SECTION - E

Assertion-Reason Type Questions

1. STATEMENT-1 : The number of ways in which six different objects can be divided equally into 2 sets is 10.

and

STATEMENT-2 : The number of ways in which six different objects be distributed equally among two persons is 20.

Sol. Answer (2)

Number of ways of division of six different objects in two sets $= \dfrac{\frac{6!}{3!3!}}{2!} = 10$

Number of ways of division of six different objects among two persons $= \frac{6!}{3!3!} = 20$

Statement-1 is true, Statement-2 is true but is not a correct explanation.

2. STATEMENT-1 : The exponent of 7 in ${}^{100}C_{50}$ is 4.

and

STATEMENT-2 : The number of ways in which we can post 5 letters in 12 boxes is 12^5.

Sol. Answer (4)

Exponent of 7 is 100!

$= \left[\frac{100}{7}\right]+\left[\frac{100}{7^2}\right]+\left[\frac{100}{7^3}\right]$

= 14 + 2 + 0 = 16

Exponent of 7 is 50!

$= \left[\frac{50}{7}\right]+\left[\frac{50}{7^2}\right]+\left[\frac{50}{73}\right]$

= 7 + 1 + 0 = 8

So, exponent of 7 is ${}^{100}C_{50}$ is = 16 – 8 – 8

= 0

Hence, Statement-1 is false.

3. STATEMENT-1 : The tens digit of 1! + 2! + 3! + 4! + 5! + ... + 50! is 1.

and

STATEMENT-2 : The sum of divisors of $2^4 3^5 5^2 7^3$ is $2^5 \cdot 3^6 \cdot 5^3 \cdot 7^4 - 2 \cdot 3 \cdot 5 \cdot 7$.

Sol. Answer (3)

The sum of divisor of $2^4\ 3^5\ 5^2\ 7^3$ is

$= (2^0 + 2^1 + 2^2 + \ldots\ldots + 2^4)\ (3^0 + 3^1 + \ldots\ldots + 3^5)\ (5^0 + 5^1 + 5^2)\ (7^0 + 7^1 + 7^2 + 7^3)$

Hence, Statement-2 is false.

4. STATEMENT-1 : The range of the function $f(x) = {}^{7-x}C_{x-3}$ is {3, 4, 5}.

and

STATEMENT-2 : The number of squares which can be formed on a chessboard (8 × 8) will be 204.

Sol. Answer (4)

{3, 4, 5} is the domain of $f(x)$ but not the range

Hence, Statement-1 is false.

5. STATEMENT-1 : The number of ways of choosing 15 couples out of 15 women and 15 men is 1240.

and

STATEMENT-2 : The number of ordered pairs of integers (x, y) which satisfies the equation $y^2 + 6y + x^2 = 4$ is 8.

Sol. Answer (2)

In Statement-1 the required number of ways is

$$\frac{1}{15!}\left({}^{15}C_1 \cdot {}^{15}C_1\right)\left(\left({}^{14}C_1 \cdot {}^{14}C_1\right)\left({}^{13}C_1 \cdot {}^{13}C_1\right)\ldots\ldots\left({}^{1}C_1 \cdot {}^{1}C_1\right)\right) = \frac{1}{15!}(15!)^2 = 15!$$

In Statement-2

$y^2 + 6y + 9 = 13 - x^2$

$\Rightarrow (y + 3)^2 = 13 - x^2$

When $x = 2$	$y = 0, -6$
$x = -2$	$y = 0, -6$
$x = 3$	$y = -1, -5$
$x = -3$	$y = -1, -5$

$\therefore$ Then number of ordered pair of integers is 8.

SECTION - F

Integer Answer Type Questions

1. An eight digit number is formed from 1, 2, 3, 4 such that product of all digits is always 3072, the total number of ways is (23. ${}^8C_k$), where the value of k is _______.

Sol. Answer (3) or (5)

$3072 = 2^{10} \times 3$

2	2	2	2	4	4	4	3

No. of ways = $\dfrac{8!}{4!3!}$

2	2	2	2	4	4	4	3

No. of ways = $\dfrac{8!}{4!2!}$

1	1	4	4	4	4	4	3

No. of ways = $\dfrac{8!}{5!2!}$

Total no. of ways = $\dfrac{8!}{4!3!} + \dfrac{8!}{4!2!} + \dfrac{8!}{5!2!} = \dfrac{23.8!}{5!\,3!} = 23.\,{}^8C_3 = 23.{}^8C_5$

2. The number of ways of distributing six identical mathematics books and six identical physics books among three students such that each student gets atleast one mathematics book and atleast one physics book is $\dfrac{5.5!}{k}$, then k is _______.

Sol. Answer (6)

Required no. of ways ${}^{6-1}C_{3-1} . {}^{6-1}C_{3-1}$

$= 10 \times 10 = 100 = \dfrac{5! \times 5}{6}$

$\therefore\ k = 6$

3. There are red, green and white identical balls, each being 10 in number. The number of selections of 10 balls in which the number of red balls is double the number of green balls is ________.

Sol. Answer (4)

Since all the balls are identical

Let x green balls be selected, y white ball is selected

So, red balls will be $2x$.

Now, $x + 2x + y = 10$

$\Rightarrow\ 3x + y = 10$

Clearly $x = 0, 1, 2, 3$ *i.e.*, 4 possible selections.

Corresponding values of y are 10, 7, 4, 1

4. If the number of ways in which a lawn-tennis mixed double be made from seven married couples if no husband and wife play in the same set is k, then greatest prime divisor of k is ________ .

Sol. Answer (7)

Required no. of ways $k = {}^7C_2 \cdot {}^5C_2 \cdot 2 = 420$

Greatest prime divisor of k = 7

5. The number of ways can 14 identical toys distributed among three boys so that each one gets atleast one toy and no two boys get equal number of toys is n, then $\frac{n}{10}$ is equal to _____ .

Sol. Answer (6)

Number of ways to distribute at least one toy to each $= {}^{14-1}C_{3-1} = {}^{13}C_2 = 78$

If toys are distributed in the following way then two will get equal number of toys

	No. of ways
1 1 12	$\frac{3!}{2!} = 3$ ways
2 2 10	3 ways
3 3 8	3 ways
4 4 6	3 ways
5 5 4	3 ways
6 6 2	3 ways

$\therefore$ Required number of ways = 78 – 18 = 60.

6. The number of ordered triplets of positive integers which satisfy the inequality $20 \le x + y + z \le 50$ is p, then $\frac{p-631}{2000}$ is equal to ________.

Sol. Answer (9)

$x + y + z = 19$

$20 \le x + y + z \le 50$

$P = {}^{20-1}C_{3-1} + {}^{21-1}C_{3-1} + \ldots + {}^{50-1}C_{3-1}$

$= {}^{19}C_2 + {}^{20}C_2 + \ldots. + {}^{49}C_2$

$= {}^{19}C_3 + {}^{20}C_2 + \ldots. + {}^{49}C_2 - {}^{19}C_3$

$= {}^{50}C_3 - {}^{19}C_3 = 19600 - 969 = 18631.$

Now, $\frac{p-631}{100} = \frac{18000}{100} = 180$

7. The number of 6 digit numbers that contains 6 exactly once is $7^{k_1} \cdot 3^{k_2}$, then $|k_2 - k_1|$ is equal to _______ .

Sol. Answer (6)

If repetition of digits is not allowed.

When 6 is at lakh's place, remaining five places can be filled by remaining digits 0, 1, 2, 3, 4, 5, 7, 8, 9 in ${}^9P_5$ ways. When 6 is at any other place, the lakh's place can be filled by 8 different ways and remaining five places in ${}^9P_3$ ways.

Hence total number of six digit numbers that contains 6 exactly one

$= {}^9P_5 + 8.{}^9P_5 = 9.{}^9P_5$

$= 9 \times 9 \times 8 \times 7 \times 6 = 27,216.$

If repetition of digits is permissible, then total number of 6 digit numbers with the given condition is $9^5 + 5 \times 8 \times 9^4$.

8. The number of five digits can be made with the digits 1, 2, 3 each of which can be used atmost thrice in a number is K, then $\frac{K}{30}$ is equal to ________ .

Sol. Answer (7)

We have the digits 1, 1, 1, 2, 2, 2, 3, 3, 3 to make numbers of five digits. The digits will be as follows

(i) **Case I :** Three identical, one pair

The number of selections of three identical digits, one pair $= {}^3C_1 \times {}^2C_1$

Corresponding to each selection, the number of numbers that can be made $= \frac{5!}{3!2!}$

$\therefore$ The total number of numbers of three identical digits and one pair

$= {}^3C_1 \cdot {}^2C_1 \cdot \frac{5!}{3!2!} = 3 \times 2 \times \frac{5 \times 4}{2} = 60$

(ii) **Case II :** Three identical, two different

The number of selections of three identical digits and two different digits $= {}^3C_1 \times {}^2C_2$

$\therefore$ The total number of numbers of three identical digits and two different digits

$= {}^3C_1 \cdot {}^2C_2 \cdot \frac{5!}{3!} = 60$

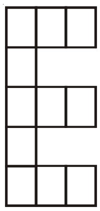
(iii) **Case III :** The number of selections of two pairs and one different digit $= {}^3C_2 \times {}^1C_1$

$\therefore$ The total number of numbers of two pairs and one different digit

$= {}^3C_2 \cdot {}^1C_1 \cdot \frac{5!}{2!2!} = 3 \times 30 = 90$

$\therefore$ The required number of numbers = 60 + 60 + 90 = 210

9. The symbols +, +, ×, ×, ✯, •, are placed in the squares of the adjoining figure. The number of ways of placing symbols so that no row remains empty is k, then the ten's digit of k is _______ .



Sol. Answer (8)

Required number of ways $= {}^3C_1 \times {}^3C_2 \times {}^3C_1 \times {}^3C_1 \times {}^1C_1 \times {}^1C_1 \times \frac{6!}{2!2!} = 14580$

10. The total number of non-similar triangles which can be formed such that all the angles of the triangle are integers is n, then $\frac{n}{900}$ is equal to ______ .

Sol. Answer (3)

Let the angles be x, y and z

$\Rightarrow x + y + z = 180$

Now $x, y, z \geq 1$

So, total number of solutions = ${}^{179}C_2 = 15931$

Now, total number of triangles having two or all the angles same

$\Rightarrow 2x_1 + x_3 = 180$

$x_3 = 180 - 2x_{-1}$

$1 \leq x\text{-}_1 \leq 89$

So, there are 89, such triangles.

So, the total number of such triangles

$= \dfrac{15931 - 89 \times 3 + 2}{3!} + 89 = 2700$

11. If total number of integral solutions of $xyz = 42$ is n then $|n - 103|$ is equal to

Sol. Answer (2)

$xyz = 2 \cdot 3 \cdot 7$

Total positive integral solutions = 27

Total integral solutions = $27 + 27 \times {}^3C_2 - 3$

$= 105$

so $|n - 103| = 2$

12. If the number of integral solutions of $x + y + z = 13$ such that $x \geq 1$, $y > 2$ and $z \geq 3$ is m, then the number of divisors of m is

Sol. Answer (6)

m = Coeff. of x^{13} in $(x + x^2 + ... + x^7)(x^3 + x^4 + ... + x^9)^2$

= Coeff. of x^6 in $(1 - x^7)^3(1 - x)^{-3}$

$= 28$

$= 2^{2} \cdot 7$

So number of divisiors of m is 6.

13. A six faced ordinary cubical die marked with alphabets A, B, C, D, E, F is thrown 'n' times and the total number of ways in which among the alphabets A, B, C, D, E and F only three of them appear in the list is 10800. Then n is equal to

Sol. Answer (6)

3 faces can be selected in ${}^6C_3$ ways.

Required number of ways = ${}^6C_3\,[3^4 - {}^3C_2\,(2^4 - 2) - 3] = 10800$

$\Rightarrow n = 6$

14. The number of seven digit numbers in which every digit is either greater than or equal to immediately preceeding one is n, then the unit's digit of n is

Sol. Answer (5)

Out of infinite one's, two's, … and nine's, we have to choose 7 numbers and to be arranged in increasing order in one way only which equals

Coefficient of x^7 in $(1 + x + x^2 + \ldots)^9$ *i.e.* in $(1 - x)^{-9}$

$= {}^{15}C_8 = 6435$

15. In a certain test there are n questions. In this test 2^{n-k} students gave wrong answers to at least k questions, where k = 1, 2, ..., n. If the total number of wrong answers given is 2047, then sum of digits of n equals

Sol. Answer (2)

The number of answering exactly r questions wrongly is $2^{n-r} - 2^{n-r-1}$

The number of students answering r questions wrongly is r

$\therefore$ Total number of wrong answers

$$= 1(2^{n-1} - 2^{n-2}) + 2(2^{n-2} - 2^{n-3}) + ... + 2^0.n$$

Total number of wrong answers = $2^{n-1} + 2^{n-2} + 2^{n-3} + ... + 2^0$

$$= \frac{2^n - 1}{2-1} = 2047$$

$\Rightarrow$ $2^n - 1 = 2047$ $\Rightarrow$ $2^n = 2048 = 2^{11}$

$\Rightarrow$ $n = 11$

$\Rightarrow$ Sum of digits = 1 + 1 = 2

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