Chapter 6

GRAVITATION

A SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. A certain pendulum clock keeps good time on the earth.If the same clock were placed on the moon, where objects weigh only one sixth as much as on the earth,how many seconds will the clock tick out in an actual time of 1 minute?

(a) 24 sec (b) 24.5 sec
(c) 25 sec (d) 25.5 sec

2. A satellite of mass m_s revolving in a circular orbit of radius r_s round the earth of mass M has a total energy E. Then its angular momentum will be

(a) $(2Em_s r_s{}^2)^{1/2}$ (b) $(2Em_s r_s{}^2)$
(c) $(2Em_s r_s)^{1/2}$ (d) $(2Em_s r_s)$.

3. A shell is fired vertically from the earth with speed $\frac{V_{esc}}{N}$, where N is some number greater than one and V_{esc} is escape speed for the earth. Neglecting the rotation of the earth and air resistance, the maximum altitude attained by the shell will be (R_E is radius of the earth)

(a) $\frac{N^2 R_E}{N^2 - 1}$ (b) $\frac{NR_E}{N^2 - 1}$
(c) $\frac{R_E}{N^2 - 1}$ (d) $\frac{R_E}{N^2}$

4. The percentage change in the acceleration of the earth towards the sun from a total eclipse of the sun to the point where the moon is on a side of earth directly opposite to the sun is

(a) $\frac{M_s}{M_m}\frac{r_2}{r_1}\times 100$ (b) $\frac{M_s}{M_m}\left(\frac{r_2}{r_1}\right)^2\times 100$
(c) $2\left(\frac{r_1}{r_2}\right)^2\frac{M_m}{M_s}\times 100$ (d) $\left(\frac{r_1}{r_2}\right)^2\frac{M_s}{M_m}\times 100$

5. A straight rod of length L extends from x = a to $x = L + a$.Find the gravitational force it, exerts on a point mass m at $x = 0$ if the linear density of rod $\mu = A + Bx^2$

(a) $\text{G}m\left[\frac{A}{a} \quad BL\right]$ (b) $\text{G}m\left[A\left(\frac{1}{a} - \frac{1}{a \quad L}\right) \quad BL\right]$
(c) $Gm\left[BL \quad \frac{A}{a \quad L}\right]$ (d) $\text{G}m\left[BL - \frac{A}{a}\right]$

6. A geo-stationary satellite orbits around the earth in a circular orbit of radius 36,000km. Then, the time period of a spy satellite orbiting a few hundred km above the earth's surface (R_{earth} = 6,400km) will approximately be

(a) 1/2 hr (b) 1 hr (c) 2 hr (d) 4 hr

7. In a certain region of space, gravitational field is given by $I = -(K/r)$. Taking the reference point to be at $r = r_0$ with $V = V_0$, find the potential.

(a) $K\log\frac{r}{r_0} \quad V_0$ (b) $K\log\frac{r_0}{r} \quad V_0$
(c) $K\log\frac{r}{r_0} - V_0$ (d) $\log\frac{r_0}{r} - V_0 r$

8. A projectile leaves the earth's surface with a speed equal to $2\sqrt{gR_e}$, where R_e is the radius of earth. Its speed far away from the earth would be

(a) $2\sqrt{gR_e}$ (b) $\sqrt{gR_e}$
(c) $\sqrt{2gR_e}$ (d) $g\,R_e$

9. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of the escape velocity from the earth. Find the height (h) of the satellite above the earth's surface : (Take radius of earth as R_e)

(a) $h = R_e^2$ (b) $h = R_e$
(c) $h = 2R_e$ (d) $h = 4R_e$

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	

10. A planet is revolving around a star in an elliptic orbit. The ratio of the farthest distance to the closest distance of the planet from the star is 4. The ratio of kinetic energies of the planet at the farthest to the closest position is

(a) 1 : 16 (b) 16 : 1 (c) 1 : 4 (d) 4 : 1

11. A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is v. Due to the rotation of planet about its axis the acceleration due to gravity g at equator is 1/2 of g at poles. The escape velocity of a particle on the pole of planet in terms of v is

(a) $v_e = 2v$ (b) $v_e = v$

(c) $v_e = v/2$ (d) $v_e \quad \sqrt{3}v$

12. A point P lies on the axis of a fixed ring of mass M and radius R, at a distance $2R$ from its centre O. A small particle starts from P and reaches O under gravitational attraction only. Its speed at O will be

(a) zero (b) $\sqrt{\frac{2GM}{R}}$

(c) $\sqrt{\frac{2GM}{R}(\sqrt{5}-1)}$ (d) $\sqrt{\frac{2GM}{R}(1-\frac{1}{\sqrt{5}})}$

13. A planet of mass m is in an elliptical orbit about the sun ($m << M_{sun}$) with an orbit period T. If A be the area of orbit, then its angular momentum would be

(a) $2mA/T$ (b) mAT (c) $mA/2T$ (d) $2mAT$

14. If the radius of earth shrinks by one percent and its mass remains the same, then acceleration due to gravity on the earth surface will

(a) increase by 1% (b) decrease by 1%

(c) decrease by 2% (d) increase by 2%

15. The work done required to increase the separation distance from x_1 to $x_1 + d$ between two masses m_1 and m_2 is

(a) $\frac{G\,m_1 m_2 d}{x_1(x_1 \quad d)}$ (b) $\frac{G\,m_1 m_2 x_1}{(x_1 \quad d)d}$

(c) $\frac{-G\,m_1 m_2 x_1}{(x_1 \quad d)}$ (d) none

16. Two satellites of same mass are orbiting round the earth at the heights of R and $4R$ above the earth's surface; R being the radius of the earth. The kinetic energies are in the ratio

(a) 4 : 1 (b) 3 : 2 (c) 4 : 3 (d) 5 : 2

17. A body is projected vertically upward from the surface of the earth with a velocity equal to half the escape velocity. If R is the radius of the earth, the maximum height attained by the body is

(a) $\frac{R}{6}$ (b) $\frac{R}{3}$

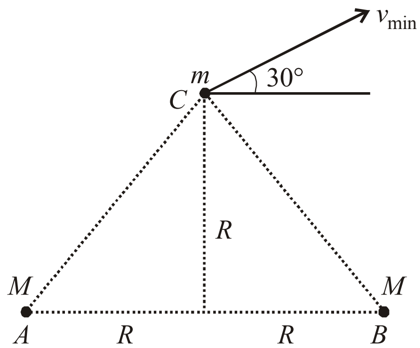
(c) $\frac{2R}{3}$ (d) R

18. A small satellite of mass m is revolving around earth in a circular orbit of radius r_0 with speed v_0. At certain point of its orbit, the direction of motion of satellite is suddenly changed by angle $\theta = \cos^{-1}(3/5)$ by turning its velocity vector, such that speed remains constant. The satellite consequently goes to elliptical orbit around earth. The ratio of speed at perigee to speed at apogee is

(a) 3 (b) 9

(c) 1/3 (d) 1/9

19. With what minimum speed should m be projected from point C in presence of two fixed masses M each at A and B as shown in the figure such that mass m should escape the gravitational attraction of A and B



(a) $\sqrt{\frac{2GM}{R}}$ (b) $\sqrt{\frac{2\sqrt{2}GM}{R}}$

(c) $2\sqrt{\frac{GM}{R}}$ (d) $2\sqrt{2}\sqrt{\frac{GM}{R}}$

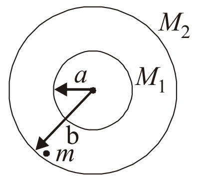
20. The radius of a planet is n times the radius of earth (R). A satellite revolves around it in a circle of radius $4nR$ with angular velocity ω. The acceleration due to gravity on planet's surface is

(a) $R\omega^2$ (b) $16\,R\omega^2$

(c) $32\,nR\omega^2$ (d) $64\,nR\omega^2$.

MARK YOUR RESPONSE					
	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ
	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ	19. ⓐⓑⓒⓓ
	20. ⓐⓑⓒⓓ				

21. Two concentric uniform shells of mass M_1 and M_2 are as shown in the figure. A particle of mass m is located just within the shell M_2 on its inner surface. Gravitational force on 'm' due to M_1 and M_2 will be



(a) zero (b) $\frac{GM_1 m}{b^2}$

(c) $\frac{G(M_1 \quad M_2)m}{b^2}$ (d) None

22. The minimum and maximum distances of a satellite from centre of earth are 2R and 4R respectively, where R is the radius of earth. The minimum and maximum speeds of the satellite will be

(a) $\sqrt{\frac{GM}{R}}, \sqrt{\frac{2GM}{R}}$ (b) $\sqrt{\frac{GM}{6R}}, \sqrt{\frac{2GM}{3R}}$

(c) $\sqrt{\frac{2GM}{3R}}, \sqrt{\frac{4GM}{3R}}$ (d) None

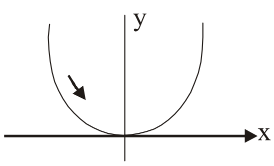
Note that M represents the mass of the earth.

23. An artificial satellite is first taken to a height equal to half the radius of earth. Assume that it is at rest on the earth's surface initially and that it is at rest at this height. Let E_1 be the energy required. It is then given the appropriate orbital speed such that it goes in a circular orbit at that height. Let E_2 be the energy required. The ratio $\frac{E_1}{E_2}$ is

(a) 4 : 1 (b) 3 : 1
(c) 1 : 1 (d) 1 : 2

24. In order to simulate different values of g, aspiring astronauts are put on a plane which dives in a parabola given by the equation :

$x^2 = 500\,y$

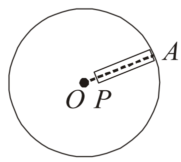


where x is horizontal, y is vertically upwards; both being measured in meter. The x–component of the velocity of the plane is constant throughout, and has the value of 360 km/h. The effective g ("g – force") experienced by an astronaut on the plane equals

(a) 4g (b) 3g

(c) $\frac{g}{5}$ (d) 5g

25. A planet of radius R has an acceleration due to gravity of g_s on its surface. A deep smooth tunnel is dug on this planet, radially inward, to reach a point P located at a distance of $\frac{R}{2}$ from the centre of the planet. Assume that the planet has uniform density. The kinetic energy required to be given to a small body of mass m, projected radially outward from P, so that it gains a maximum altitude equal to the thrice the radius of the planet from its surface, is equal to



(a) $\frac{63}{16} mg_s R$ (b) $\frac{3}{8} mg_s R$

(c) $\frac{9}{8} mg_s R$ (d) $\frac{21}{16} mg_s R$

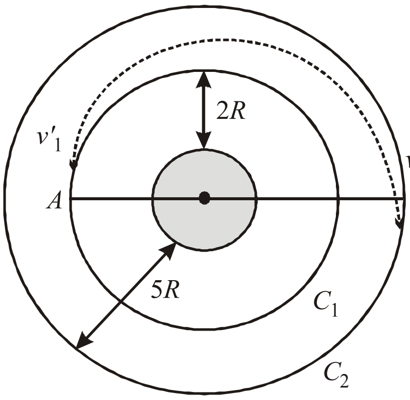
26. Six stars of equal mass are moving about the centre of mass of the system such that they are always on the vertices of a regular hexagon of side length a. Their common time period will be

(a) $4\pi\sqrt{\frac{a^3}{Gm}}$ (b) $2\pi\sqrt{\frac{4\sqrt{3}a^3}{Gm\; 5\sqrt{3} \quad 4}}$

(c) $4\pi\sqrt{\frac{3a^3}{Gm}}$ (d) None of these

MARK YOUR RESPONSE					
	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ
	26. ⓐⓑⓒⓓ				

27. An earth satellite of mass m orbits along a circular orbit C_1 at a height $2R$ from earth's surface. It is to be transferred to a circular orbit C_2, of bigger radius, at a height $5R$ from earth's surface. The transfer is affected by following an elliptical path as shown in figure. Calculate the change in the energies required at the transfer points A and B. [R = radius of earth]



(a) $\frac{mgR}{36}$ (b) $\frac{mgR}{18}$

(c) $\frac{mgR}{2}$ (d) $\frac{mgR}{9}$

28. If the radius of the earth were to shrink by one percent, its mass remaining the same, the acceleration due to gravity on the earth's surface would

(a) decrease (b) remain unchanged

(c) increase (d) be zero

29. The acceleration due to gravity on the surface of the moon is 1/6 that on the surface of earth and the diameter of the moon is one-fourth that of earth. The ratio of escape velocities on earth and moon will be

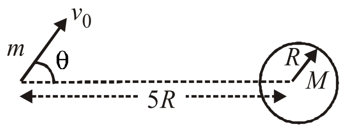
(a) $\frac{\sqrt{6}}{2}$ (b) $\sqrt{24}$ (c) 3 (d) $\frac{\sqrt{3}}{2}$

30. An artificial satellite of mass m of a planet of mass M, revolves in a circular orbit whose radius is n times the radius R of the planet. In the process of motion, the satellite experiences a slight resistance due to cosmic dust. Assuming resistance force on satellite depends on velocity as $F = av^2$ where 'a' is constant. Calculate how long the satellite will stay in orbit before it falls onto the planet's surface.

(a) $\frac{m\sqrt{R}\,(\sqrt{n}-1)}{a\sqrt{GM}}$ (b) $\frac{m\sqrt{R}\,(\sqrt{n}\quad 1)}{a\sqrt{GM}}$

(c) $\frac{2m\sqrt{R}\,(\sqrt{n}-1)}{a\sqrt{GM}}$ (d) $\frac{m\sqrt{R}\,(\sqrt{n}-1)}{a\sqrt{2GM}}$

31. A spaceship is sent to investigate a planet of mass M and radius R. While hanging motionless in space at a distance $5R$ from the centre of the planet, the spaceship fires an instrument package with speed v_0 as shown in the figure. The package has mass m, which is much smaller than the mass of the spaceship. For what angle θ will the package just graze the surface of the planet ?



(a) $\sin^{-1}\left(\frac{1}{2}\sqrt{1\quad\frac{8GM}{5v_0^2R}}\right)$ (b) $\sin^{-1}\left(\frac{1}{5}\sqrt{1\quad\frac{8GM}{5v_0^2R}}\right)$

(c) $\sin^{-1}\left(\frac{1}{5}\sqrt{1\quad\frac{2GM}{5v_0^2R}}\right)$ (d) $\sin^{-1}\left(\frac{1}{5}\sqrt{1\quad\frac{8GM}{3v_0^2R}}\right)$

32. In older times, people used to think that the earth was flat. Imagine that the earth is indeed not a sphere of radius R, but an infinite plate of thickness H. What value of H is needed to allow the same gravitational acceleration to be experienced as on the surface of the actual earth ? (Assume that the earth's density is uniform and equal in the two models

(a) $2R/3$ (b) $4R/3$

(c) $8R/3$ (d) $R/3$

33 A planet revolves about the sun is elliptical orbit. The areal velocity $\left(\frac{dA}{dt}\right)$ of the planet is 4.0×10^{16} m²/s. The least distance between planet and the sun is 2×10^{12} m. Then the maximum speed of the planet in km/s is

(a) 10 (b) 20

(c) 30 (d) 40

34. The mass M of a planet-earth is uniformly distributed over a spherical volume of radius R. Calculate the energy needed to disassemble the planet against the gravitational pull amongst its constituent particles.

Given : $MR = 2.5 \times 10^{31}$ kg-m and $g = 10$ m/s²

(a) 3.0×10^{32} J (b) 2.5×10^{32} J

(c) 0.5×10^{28} J (d) 1.5×10^{32} J

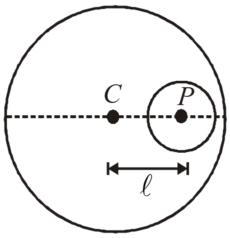
35. If gravitational forces alone prevent a spherical, rotating neutron star from disintegrating , the minimum mean density of a star that has a rotation period of one second will be

(a) 2.4×10^{10} kg/m³ (b) 1.4×10^{10} kg/m³

(c) 1.4×10^{11} kg/m³ (d) 3.4×10^{11} kg/m³

MARK YOUR RESPONSE					
	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ
	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ	

36. Inside a uniform sphere of density ρ there is a spherical cavity whose centre is at a distance ℓ from the centre of the sphere. Find the strength *F* of the gravitational field inside the cavity at the point *P*.



(a) $\frac{4}{3}G\pi\rho\vec{\ell}$ (b) $\frac{1}{3}G\pi\rho\vec{\ell}$

(c) $\frac{2}{3}G\pi\rho\vec{\ell}$ (d) $\frac{1}{2}G\pi\rho\vec{\ell}$

37. If g is the acceleration due to gravity on the earth's surface, the change in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius *R* of the earth is

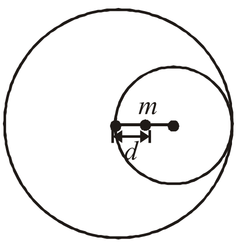
(a) $mgR/2$ (b) $2mgR$

(c) mgR (d) $-mgR$

38. If the distance between the earth and the sun were half its present value, the number of days in a year would have been

(a) 64.5 (b) 129 (c) 182.5 (d) 730

39. A spherical cavity is made in a lead sphere of radius *R* such that its surface touches the outside surface of the lead sphere and passes through its centre . The mass of the lead sphere before hollowing was *M*. What is the force of attraction that this sphere would exert on a particle of mass *m*, which lies at a distance *d* from the centre of the lead sphere on the straight line joining the centres of the sphere and the centre of cavity as shown in the figure.



(a) $\frac{3GMm}{2R^2}$ (b) $\frac{2GMm}{R^2}$

(c) $\frac{GMm}{R^2}$ (d) $\frac{GMm}{2R^2}$

40. The orbital speed of Jupiter is

(a) greater than the orbital speed of earth

(b) less than the orbital speed of earth

(c) equal to the orbital speed of earth

(d) zero

41. The eccentricity of the earth's orbit is 0.0167. The ratio of its maximum speed in its orbit to its minimum speed is

(a) 1.67 (b) 1.034

(c) 1 (d) 0.167

42. Two bodies of masses M_1 and M_2 are placed at a distance *d* apart. What is the potential at the position where the gravitational field due to them is zero ?

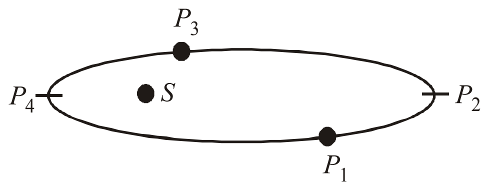
(a) $-\frac{G}{d}\ M_1\ \ M_2\ \ 2\sqrt{M_1}\sqrt{M_2}$

(b) $-\frac{G}{d}\ M_1+M_2-2\sqrt{M_1}\sqrt{M_2}$

(c) $-\frac{G}{d}\ 2M_1\ \ M_2\ \ 2\sqrt{M_1}\sqrt{M_2}$

(d) $-\frac{G}{2d}\ M_1\ \ M_2\ \ 2\sqrt{M_1}\sqrt{M_2}$

43. The figure shows a planet in elliptical orbit around the sun *S*. Where is the kinetic energy of the planet maximum ?



(a) P_1 (b) P_2

(c) P_3 (d) P_4

44. A missile, which missed its target when into orbit around the earth at a mean radius 3 times as great as the parking orbit of the satellite. The period of the missile is

(a) $\sqrt{2}$ day (b) 3 day

(c) $\sqrt{3}$ day (d) $3\sqrt{3}$ day

45. A satellite of mass m is orbiting the earth in a circular orbit of radius *R*. It starts losing energy due to small air resistance at the rate of *C* J/s. Find the time taken for the satellite to reach the earth.

(a) $\frac{GMm}{C}\left[\frac{1}{R}-\frac{1}{r}\right]$ (b) $\frac{GMm}{2C}\left[\frac{1}{R}\ \ \frac{1}{r}\right]$

(c) $\frac{GMm}{2C}\left[\frac{1}{R}-\frac{1}{r}\right]$ (d) $\frac{2GMm}{C}\left[\frac{1}{R}\ \ \frac{1}{r}\right]$

MARK YOUR RESPONSE					
	36. ⓐⓑⓒⓓ	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ	40. ⓐⓑⓒⓓ
	41. ⓐⓑⓒⓓ	42. ⓐⓑⓒⓓ	43. ⓐⓑⓒⓓ	44. ⓐⓑⓒⓓ	45. ⓐⓑⓒⓓ

46. The time taken by the earth to travel over half its orbit, remote from the sun, separated by the minor axis is about 2 days more than half the year, then the eccentricity of the orbit is

(a) 1/30 (b) 1/60
(c) 1/15 (d) 1/70

47. If a spherically symmetric star of radius R collapsed under its own weight, neglecting any forces other than gravitational ones, what is the time required for collapse ?

(a) $\frac{8\pi^2 R^3}{(GM)^{0.5}}$ (b) $\frac{2\pi^2 R^3}{(3GM)^{0.5}}$

(c) $\frac{\pi^2 R^3}{(8GM)^{0.5}}$ (d) None of these

48. A planet of mass m is in an elliptical orbit about the sun ($m << M_{sun}$), with an orbital period T. If A be the area of orbit then its angular momentum would be

(a) $\frac{2mA}{T}$ (b) mAT

(c) $\frac{mA}{2T}$ (d) $2mAT$

49. A binary star system consists of two stars A and B which have time period T_A and T_B, radius R_A and R_B and mass M_A and M_B. Then

(a) if $T_A > T_B$ then $R_A > R_B$ (b) if $T_A > T_B$ then $M_A > M_B$

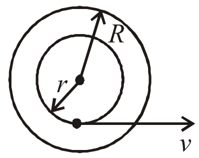
(c) $\left(\frac{T_A}{T_B}\right)^2 \quad \left(\frac{R_A}{R_B}\right)^3$ (d) $T_A = T_B$

50. Two particles of masses m_1 and m_2 moving in coplanar parabolas round the sun, collide at right angles and coalesce when their common distance from sun is R. The subsequent path of the combined particles is an ellipse of major axis

(a) $\frac{(m_1 \quad m_2)^2}{2m_1m_2}R$ (b) $\frac{(m_1 - m_2)^2}{2m_1m_2}R$

(c) $\frac{(m_1 \quad m_2)^2}{m_1m_2}R$ (d) $\frac{(m_1 \quad m_2)^2}{3m_1m_2}R$

51. Inside a large isolated shell of mass M having inner and outer radius r and R respectively, a particle is projected with a velocity v tangentially along the inner surface of the shell. Find the time taken for the particle to complete one revolution. The friction is absent.



(a) $\frac{2\pi r}{\sqrt{GM/R}}$

(b) $\frac{2\pi r^{3/2}}{\sqrt{GM}}$

(c) $\frac{2\pi r}{v}$

(d) The particle will not be able to complete a revolution, as it will fall off towards the centre

MARK YOUR RESPONSE					
	46. ⓐⓑⓒⓓ	47. ⓐⓑⓒⓓ	48. ⓐⓑⓒⓓ	49. ⓐⓑⓒⓓ	50. ⓐⓑⓒⓓ
	51. ⓐⓑⓒⓓ				

B COMPREHENSION TYPE

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1

It can be assumed that orbits of earth and mars are nearly circular around the sun. It is proposed to launch an artificial planet around the sun such that its apogee is at the orbit of mars while its perigee is at the orbit of earth. Let T_e and T_m be periods of revolution of earth and mars. Further the variables are assigned the meanings as follows.

$M_e \rightarrow$ Mass of earth
$M_m \rightarrow$ Mass of Mars.
$L_e \rightarrow$ Angular momentum of earth around the sun.
$L_m \rightarrow$ Angular momentum of mars around the sun.
$R_e \rightarrow$ Semi major axis of earth's orbit.
$R_m \rightarrow$ Semi major axis of mars orbit.
$M \rightarrow$ Mass of the artificial planet.
$E_e \rightarrow$ is total energy of earth.
$E_m \rightarrow$ is total energy of mars.

1. Time period of revolution of the artificial planet about sun will be (neglect gravitational effects of earth and mars)

(a) $\frac{T_e \quad T_m}{2}$ (b) $\sqrt{T_e T_m}$

(c) $\frac{2T_e T_m}{T_e \quad T_m}$ (d) $\left[\frac{T_e^{2/3} \quad T_m^{2/3}}{2}\right]^{3/2}$

2. The total energy of the artificial planet's orbit will be

(a) $\frac{2M}{M_e}\left(\frac{R_e E_e}{R_e \quad R_m}\right)$ (b) $\frac{2M}{M_m}\left(\frac{R_e E_e}{R_e \quad R_m}\right)$

(c) $\frac{2E_e M}{M_m}\left(\frac{R_e \quad R_m}{R_m}\right)$ (d) $\frac{2E_e M}{M_e}\left(\frac{R_e \quad R_m}{\sqrt{R_e^2 \quad R_m^2}}\right)$

3. Areal velocity of the artificial planet around the sun will be
(a) less than that of earth (b) more than that of mars
(c) more than that of earth (d) same as that of the earth

PASSAGE-2

If a tunnel is dug across the earth passing through centre and a particle is dropped from surface it perform SHM inside tunnel with time period, $T = 2\pi\sqrt{\frac{Radius}{g}}$

4. Assume that a tunnel is dug across the earth (radius = R) passing through its centre. Find the time, a particle takes to reach centre of the earth, if it is projected into the tunnel from surface of earth with speed needed for it to escape the gravitational field of earth.

(a) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\sqrt{\frac{R_e}{g}}$ (b) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\sqrt{\frac{2R_e}{g}}$

(c) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\sqrt{\frac{R_e}{2g}}$ (d) None of these

5. If a particle is drop from height h ($h << R$) into the tunnel then choose the correct option –
(a) Particle will perform SHM
(b) Particle will not perform SHM
(c) Particle will perform periodic motion
(d) Both (b) and (c)

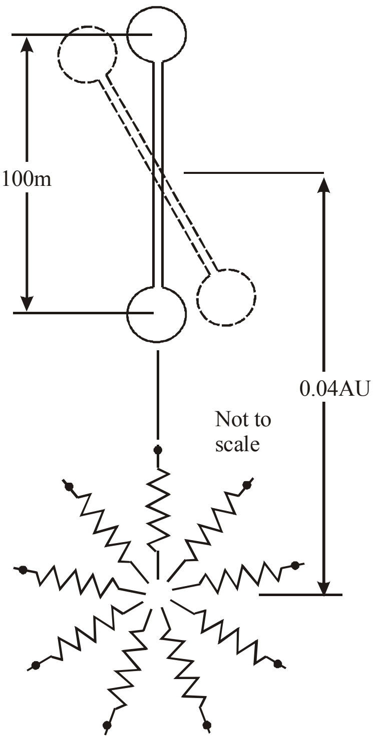
6. Find the time period for the motion in previous question

(a) $2\pi\sqrt{\frac{R}{g}} \quad 4\sqrt{\frac{2h}{g}}$ (b) $2\pi\sqrt{\frac{R}{g}} \quad 2\sqrt{\frac{2h}{g}}$

(c) $\pi\sqrt{\frac{R}{g}} \quad 2\sqrt{\frac{2h}{g}}$ (d) None of these

PASSAGE-3

An intergalactic observation station is in a circular orbit of radius 0.04 AU around a black hole of mass 10 solar masses. The observation station is constructed in the shape of a dumbbell with two spheres 20 m in diameter, connected by a hollow cylindrical beam of small cross section and 80 m long. [1 AU = 1.5×10^{11} m]



7. The orbital period of the station is
(a) 7.99×10^4 s (b) 7.99×10^2 s
(c) 3.99×10^4 s (d) 7.99×10^3 s

8. The tangential velocity of the station is
(a) 4.72×10^2 m/s (b) 2.72×10^5 m/s
(c) 4.72×10^5 m/s (d) 4.72×10^3 m/s

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ		

9. The station is aligned in orbit so that the long axis of the dumbbell is aligned radially with respect to the centre of the black hole. If each of the spheres weighs 100 tonnes, calculate the tension on the connecting beam.

(a) 1.124 N (b) 0.124 N (c) 3.114 N (d) 0.213 N

10. If the alignment of the station is stable configuration, any change in that alignment would have to result in forces which cause the station to return to its stable alignment. Calculate the magnitude of the gravitational forces which would occur if the dumbbell axis were rotated 30° out of position.

(a) 2.66×10^{-1} N (b) 1.66×10^{-2} N
(c) 2.66×10^{-4} N (d) 2.66×10^{-2} N

11. The cylindrical beam is made of a titanium alloy which has a yield strength of 830 MPa (mega Pascals). If the beam has a diameter of 2.00 m and a wall thickness of 5 cm, calculate how close the station can come to the black hole and not be torn apart by the differential gravitational forces.

(a) 4.72×10^2 m. (b) 4.72×10^6 m.
(c) 4.72×10^3 m. (d) 4.72×10^8 m.

MARK YOUR RESPONSE	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ		

C REASONING TYPE

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options :

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.
(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is true but Statement-2 is false.
(d) Statement-1 is false but Statement-2 is true.

1. **Statement - 1 :** A balloon filled with hydrogen will fall with acceleration $\frac{g}{6}$ of the moon.
Statement - 2 : Moon has no atmosphere.

2. **Statement - 1 :** The length of the day is slowly increasing
Statement - 2 : The dominant effect causing a slowdown in the rotation of the earth is the gravitational pull of other planets in the solar system.

3. **Statement - 1 :** Generally the path of a projectile from the earth is parabolic but it is elliptical for projectiles going to a very large height.
Statement - 2 : The path of a projectile is independent of the gravitational force of earth.

4. **Statement - 1 :** The time period of revolution of a satellite close to surface of earth is smaller than that revolving away from surface of earth.
Statement - 2 : The square of time period of revolution of a satellite is directly proportional to cube of its orbital radius.

5. **Statement - 1 :** We can not move even a finger without disturbing all the stars.
Statement - 2 : Every body in this universe attracts every other body with a force which is inversely proportional to the square of distance between them.

6. **Statement - 1 :** Orbital velocity of a satellite is greater than its escape velocity.
Statement - 2 : Orbit of a satellite is within the gravitational field of earth whereas escaping is beyond the gravitational field of earth.

7. **Statement - 1 :** If earth suddenly stops rotating about its axis, then the value of acceleration due to gravity will become same at all the places.
Statement - 2 : The value of acceleration due to gravity is independent of rotation of earth.

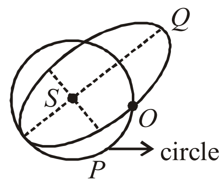
8. **Statement - 1 :** The square of the period of revolution of a planet is proportional to the cube of its mean distance from sun.
Statement - 2 : The intensity of the gravitational field of the sun acting on the planet is proportional to the mass of the sun and inversely proportional to square of its distance from the sun; it also acts towards the sun.

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ		

9. Statement - 1 : Gravitational potential due to earth at its own centre is negative.

Statement - 2 : Gravitational field due to earth is vector quantity.

10. Statement - 1 : A planet may orbit around a star either in orbit P or orbit Q. The speed of a planet at O is same for both orbits.



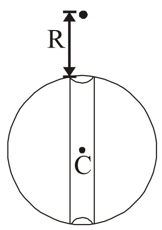
Statement - 2 : The radius vector of point O from the sun is same for both orbits P and Q.

MARK YOUR RESPONSE	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ			

D MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. A particle is dropped from a height equal to the radius of the earth into a tunnel dug through the earth along one of the diameters as shown in the figure.



R : Radius of earth
M : Mass of earth

(a) Particle will oscillate through the earth to a height R on both sides
(b) Particle will execute simple harmonic motion
(c) Motion of the particle is periodic
(d) Particle passes the centre of earth with a speed $\sqrt{\frac{2GM}{R}}$

2. The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform sphere of radius R and mass m are F_1 and F_2 respectively. Then:

(a) $\frac{F_1}{F_2} \quad \frac{r_1}{r_2}$ if $r_1 \quad R$ and $r_2 \quad R$

(b) $\frac{F_1}{F_2} \quad \frac{r_2^2}{r_1^2}$ if $r_1 \quad R$ and $r_2 \quad R$

(c) $\frac{F_1}{F_2} \quad \frac{r_1}{r_2}$ if $r_1 \quad R$ and $r_2 \quad R$

(d) $\frac{F_1}{F_2} \quad \frac{r_1^2}{r_2^2}$ if $r_1 \quad R$ and $r_2 \quad R$

3. Which of the following statements are true about acceleration due to gravity ?
(a) g decreases in moving away from the centre if $r > R$
(b) g decreases in moving away from the centre if $r < R$
(c) g is zero at the centre of earth
(d) g decreases if earth stops rotating on its axis

4. A satellite is launched and attains a velocity of 30400 km/hr relative to the centre of the earth at a height of 320km from the earth's surface. It has been guided into a path that is parallel to the earth's surface at burnout. Choose the correct options
(a) Satellite moves along an elliptical orbit
(b) Longest distance from the earth's surface is 3550 km.
(c) he period of revolution for the satellite is 2.09 hrs.
(d) The minimum escape velocity for this position of launching is 10930.08 m/s

5. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$
(a) T^2 is proportional to R^3
(b) T^2 is proportional to $R^{7/2}$
(c) T^2 is proportional to $R^{3/2}$
(d) T^2 is proportional to $R^{3/73}$

MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ

6. Which of the following statements is/are correct –

(a) The radius R of planet's orbit and period T bear the relation R^2/T^3 = constant for all planets

(b) The gravitational potential energy of a satellite earth system decreases as the period of the satellite decreases.

(c) The period of an earth-stationary satellite, as observed from earth, is infinite

(d) The mass of a body placed in a orbiting satellite becomes zero

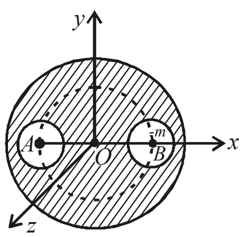
7. Consider an attractive central force of the form

$F(r) = -\dfrac{k}{r^n}$, k is a constant. For a stable circular orbit to exist

(a) $n = 2$ (b) $n < 3$

(c) $n > 3$ (d) $n = -1$

8. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at A (–2, 0 ,0) and B (2, 0, 0) respectively, are taken out of the solid leaving behind spherical cavities as shown in fig



Then :

(a) The gravitational force due to this object at the origin is zero.

(b) the gravitational force at the point B (2, 0 ,0) is zero.

(c) the gravitational potential is the same at all points of circle $y^2 + z^2 = 36$.

(d) the gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$.

9. For two satellites at distance R and $7R$ above the earth's surface, the ratio of their

(a) total energies is 4 and potential and kinetic energies is2

(b) potential energies is 4

(c) kinetic energies is 4

(d) total energies is 4

10. A rocket starts vertically upwards with speed v_0. Choose the correct option(s) (R is the radius of the earth).

(a) Speed v at a height is given by

$$v_0^2 - v^2 \quad \frac{2gh}{1 \quad \frac{h}{R}}$$

(b) The maximum height reached by the rocket fired with a speed of 90% of escape velocity is 2.26 R

(c) The maximum height reached by the rocket fired with a speed of 90% of escape velocity is 4.26 R

(d) Speed v at a height is given by

$$v_0^2 - v^2 \quad \frac{2gh}{1 \quad \frac{h}{2R}}$$

11. A communication earth satellite

(a) goes round the earth from east to west

(b) can be in the equatorial plane only

(c) can be vertically above any place on the earth

(d) goes round the earth from west to east

12. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth.

(a) The acceleration of S is always directed towards the centre of the earth.

(b) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant.

(c) The total mechanical energy of S varies periodically with time.

(d) The linear momentum of S remains constant in magnitude.

13. Titan, the largest satellite of Saturn has a radius of 2,575 km and a mass of 1.36×10^{23} kg. The NASA Cassini probe has measured its average surface temperature to be -178°C. Which of the following gases would you expect to remain in the atmosphere of Titan (A rule-of-thumb among astronomers is that if the average speed is below 1/6 of the escape velocity, the gas will remain for the age of the solar system)

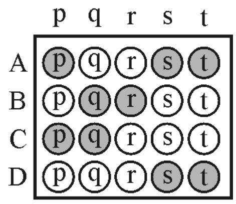
(a) ammonia (b) methane

(c) helium (d) hydrogen

MARK YOUR RESPONSE	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ
	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ		

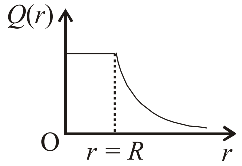
E MATRIX-MATCH TYPE

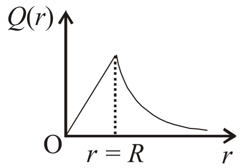
Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.

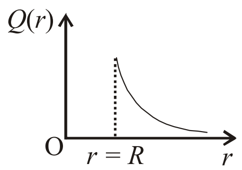


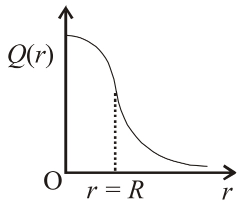
1. Match the following

Column-I

(A) 

(B) 

(C) 

(D) 

Column-II

(p) $Q(r)$ is variation of gravitational potential for a uniform volumetric spherical distribution of mass

(q) $Q(r)$ is variation of gravitational potential for a uniform spherical shell

(r) $Q(r)$ is variation of gravitational field intensity for a uniform volumetric spherical distribution of mass

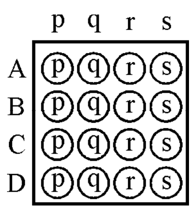
(s) $Q(r)$ is variation of gravitational field intensity for a uniform spherical shell

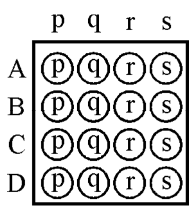
Where $Q(r)$ is magnitude of physical quantity as a function of r (distance from centre of spherical distribution of radius R)

2. Column I lists some parameter of the electron in the hydrogen atom. Column II lists their dependences with respect to the principle quantum number n. Match the entries in Column I with those in Column II.

Column I	**Column II**
(A) Radius of the orbit	(p) $\frac{1}{n}$
(B) Momentum of the electron	(q) $\frac{1}{n^2}$
(C) Time period of circular motion	(r) n^3
(D) Kinetic energy	(s) n^2

MARK YOUR RESPONSE

1. 

2. 

3. Match the columns I and II

Column I	Column II
(A) Elliptical orbit of planet	(p) Kinetic energy conservation
(B) Circular orbit of satellite	(q) Angular momentum conservation
(C) Escape velocity	(r) Independent of mass of particle/satellite
(D) Orbital velocity	(s) $\sqrt{\frac{GM}{R}}$
	(t) Areal velocity constant

4. Match the columns I and II

Column I	Column II
(A) Kinetic energy of a body projected from surface of earth, at large distance from surface of earth	(p) must be zero
(B) Gravitational potential energy of a bound system	(q) may be zero
(C) Change in potential energy of a point mass if left free to itself, with time	(r) positive
(D) Change in areal velocity of earth as earth moves from apogee towards perigee	(s) may be negative
	(t) must be negative

5. Considering earth to be a homogeneous sphere but keeping in mind its spin, match the columns I and II correctly.

Column I	Column II
(A) Acceleration due to gravity	(p) May change from point to point
(B) Orbital angular momentum of the earth as seen from a distant star	(q) Does not depend on direction of projection
(C) Escape velocity from the earth	(r) Remains constant
(D) Gravitational potential due to earth at particular point	(s) Changes with time
	(t) Depends on earth mass

MARK YOUR RESPONSE

3.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

4.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

5.

	p	q	r	s	t
A	p	q	r	s	t
B	p	q	r	s	t
C	p	q	r	s	t
D	p	q	r	s	t

NUMERIC/INTEGER ANSWER TYPE

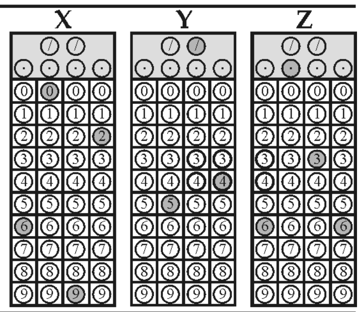
F

The answer to each of the questions is either numeric (eg. 304, 40, 3010, 3 etc.) or a fraction (2/3, 23/7) or a decimal (2.35, 0.546).

The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

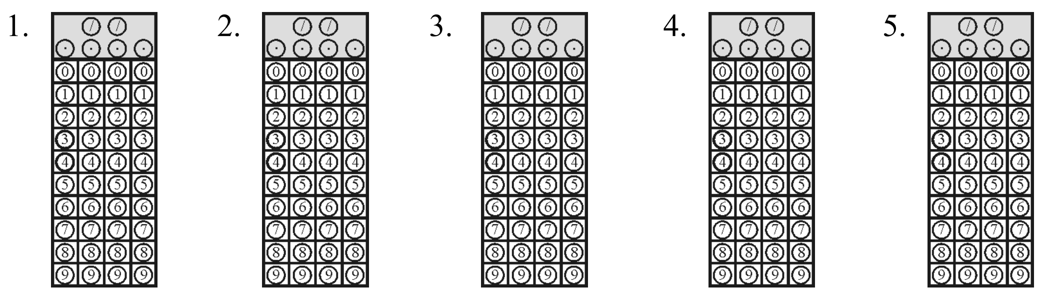
For example, if the correct answers to question X, Y & Z are 6092, 5/4 & 6.36 respectively then the correct darkening of bubbles will look like the following.

For single digit integer answer darken the extreme right bubble only.



1. If the radius of the earth were to shrink by two percent, its mass remaining the same, by how much percentage would the acceleration due to gravity on the earth's surface would increase ?
2. The period of revolution of planet A around the sun is 8 times that of B. The distance of A from the sun is how many times greater than that of B from the Sun?
3. Suppose earth's orbital motion around the sun is suddenly stopped. What time (in days) will the earth take to fall into the sun?
4. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed (in m/s) with which it hits the surface of the earth.
5. A body is projected vertically upwards from the bottom of a crater of moon of depth $\frac{R}{100}$ where R is the radius of moon with a velocity equal to the escape velocity on the surface of moon. How many times of the radius of the moon is the maximum height attained by the body from the surface of the moon ?

MARK YOUR RESPONSE



Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(b)	**7**	(a)	**13**	(a)	**19**	(b)	**25**	(c)	**31**	(b)	**37**	(a)	**43**	(d)	**49**	(d)
2	(a)	**8**	(c)	**14**	(d)	**20**	(d)	**26**	(b)	**32**	(a)	**38**	(b)	**44**	(d)	**50**	(a)
3	(c)	**9**	(b)	**15**	(a)	**21**	(b)	**27**	(a)	**33**	(d)	**39**	(d)	**45**	(c)	**51**	(c)
4	(c)	**10**	(a)	**16**	(a)	**22**	(b)	**28**	(c)	**34**	(d)	**40**	(b)	**46**	(b)		
5	(b)	**11**	(a)	**17**	(b)	**23**	(c)	**29**	(b)	**35**	(c)	**41**	(b)	**47**	(d)		
6	(c)	**12**	(d)	**18**	(b)	**24**	(d)	**30**	(a)	**36**	(a)	**42**	(a)	**48**	(a)		

B COMPREHENSION TYPE

1	(d)	**3**	(c)	**5**	(d)	**7**	(a)	**9**	(b)	**11**	(b)
2	(a)	**4**	(a)	**6**	(a)	**8**	(c)	**10**	(d)		

C REASONING TYPE

1	(a)	**3**	(c)	**5**	(a)	**7**	(c)	**9**	(b)
2	(c)	**4**	(a)	**6**	(c)	**8**	(a)	**10**	(d)

D MULTIPLE CORRECT CHOICE TYPE

1	(a, c, d)	**4**	(a, b, c, d)	**7**	(a, b, d)	**10**	(a, c)	**13**	(a, b)
2	(a,b)	**5**	(b)	**8**	(a,c,d)	**11**	(b, d)		
3	(a, c)	**6**	(b, c)	**9**	(b, c, d)	**12**	(a)		

E MATRIX-MATCH TYPE

1. **A-q; B-r; C-s; D-p**

2. **A-s; B-p; C-r; D-q**

3. **A-q, t; B-p, q, t; C-r; D-r, s**

4. **A-q, r; B-t; C-q, s; D-p**

5. **A-p, t; B-r, t; C-p, q, t; D-r, t**

F NUMERIC/INTEGER ANSWER TYPE

1	4	**2**	4	**3**	65	**4**	7920	**5**	99.5

Solutions

A SINGLE CORRECT CHOICE TYPE

1. **(b)** On earth, $T = 2\pi\sqrt{\frac{l}{g}}$

On moon, $T' = 2\pi\sqrt{\frac{l}{g¢}}$

but $mg' = \frac{mg}{6}$

or $g' = \frac{g}{6}$

$\therefore T' = 2\pi\sqrt{\frac{6l}{g}} = \sqrt{6}\ T = \sqrt{6}\text{ min} \quad (\because T = 1\text{ min})$

So, in actual time of 1 minute on earth, on moon its ticks

out $= \frac{60}{\sqrt{6}}\text{sec} = 24.5\text{ sec}$

2. **(a)** The velocity v_s of the satellite is given by

$$\frac{GMm_s}{r_s{}^2} \quad \frac{m_s v_s{}^2}{r_s} \quad \therefore\ v_s \quad \sqrt{\left(\frac{GM}{r_s}\right)} \quad \ldots\text{(i)}$$

$$\text{K.E.} \quad \frac{1}{2}m_s v_s{}^2 \quad \frac{1}{2}m_s\left(\frac{GM}{r_s}\right);\ \text{P.E.} = -\frac{GMm_s}{r_s}$$

$$\therefore \text{Total energy } E = \text{K.E.} + \text{P.E.} = -\frac{GMm_s}{2r_s} \quad \ldots\text{(ii)}$$

The angular momentum L is given by

$$L \quad m_s v_s r_s \quad m_s\left(\frac{GM}{r_s}\right)^{1/2} r_s \quad (GMm_s{}^2 r_s)^{1/2} \ldots\text{(iii)}$$

From eqs. (ii) and (iii), we get

$$L \quad (2Em_s r_s{}^2)^{1/2}$$

3. **(c)** By conservation of energy

$$-\frac{GMm}{R_E} \quad \frac{1}{2}\frac{m}{N^2}\frac{GM}{2R_E} \quad -\frac{GMm}{H}$$

$$\Rightarrow\ H = \frac{N^2 R_E}{N^2 - 1}$$

$$\text{Altitude} = H - R \quad \frac{R_E}{N^2 - 1}$$

4. **(c)** During total eclipse :
Total attraction due to sun and moon,

$$F_1 \quad \frac{GM_sM_e}{r_1^2} \quad \frac{GM_mM_e}{r_2^2}$$

When moon goes on the opposite side of earth.
Effective force of attraction,

$$F_2 = \frac{GM_sM_e}{r_1^2} - \frac{GM_mM_e}{r_2^2}$$

Change in force, $\Delta F = F_1 - F_2 \quad \frac{2GM_mM_e}{r_2^2}$

Change in acceleration of earth

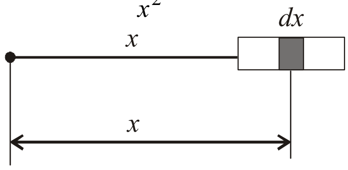
$$\Delta a = \frac{\Delta F}{M_e} \quad \frac{2GM_m}{r_2^2}$$

Average force on earth, $F_{av} = \frac{F_{av}}{M_e} \quad \frac{GM_s}{r_1^2}$ % age change in acceleration

$$= \frac{\Delta a}{a_{av}} \times 100 = \frac{2GM_m}{r_2^2} \times \frac{r_1^2}{GM_s} \times 100$$

$$= 2\left(\frac{r_1}{r_2}\right)^2 \frac{M_m}{M_s} \times 100$$

5. **(b)** $\because dF \quad \frac{Gm\ \mu dx}{x^2}$



$$F \quad Gm\int \ A \quad Bx \ \frac{dx}{x}$$

$$F = Gm\left[A\left(\frac{1}{a} - \frac{1}{a\quad L}\right) \quad BL\right]$$

6. **(c)** A satellite revolving near the earth's surface has a time period of 84.6 min.
We know that as the height increases, the time period increases. Thus the time period of the spy satellite should be slightly greater than 84.6 minutes.
$\therefore \quad T_s = 2\text{ hr}$

7. (a) We know that intensity is negative gradient of potential, i.e., $I = -(dV/dr)$ and as here $I = -(K/r)$, so

$$\frac{dV}{dr} = \frac{K}{r}, \qquad \text{i.e.,} \qquad \int dV = K$$

or $V - V_0 = \mathrm{K}\log\frac{r}{r_0}$

so $V = \mathrm{K}\log\frac{r}{r_0} + V$

8. (c) If a body is projected with ($2\sqrt{gR_e}$) greater than escape velocity $\sqrt{2gR_e}$ then by conservation of energy

$$\frac{1}{2}mv^2 - \frac{GMm}{R_e} = \frac{1}{2}m(v')^2 + 0$$

i.e., $(v')^2 = v^2 - \frac{2GM}{R_e} = v^2 - v_e^2$

i.e., $v' = \sqrt{v^2 - v_e^2} = \sqrt{4gR_e - 2gR_e} = \sqrt{2gR_e}$

9. (b) The escape velocity from earth is given by

$v_e = \sqrt{2g\,R_e}$ ...(i)

The orbital velocity of a satellite revolving around earth is given by

$$v_0 = \frac{\sqrt{GM_e}}{(R_e + h)}$$

where, M_e = mass of earth, R_e = radius of earth, h = height of satellite from surface of earth.

By the relation $GM_e = gR_e^2$

So, $v_0 = \frac{\sqrt{gR_e^2}}{(R_e + h)}$ ...(ii)

Dividing equation (i) by (ii), we get

$$\frac{v_e}{v_0} = \frac{\sqrt{2(R_e + h)}}{(R_e)}$$

Given, $v_0 = \frac{v_e}{2}$

$$\frac{2v_e}{v_e} = \frac{\sqrt{2(R_e + h)}}{R_e}$$

Squaring on both side, we get

$4 = \frac{2(R_e + h)}{R_e}$ or $R_e + h = 2R_e$ i.e., $h = R_e$

10. (a) Angular momentum remains conserved during the revolution of planet. Because gravitational force is a central force.

Now

$$\text{K.E.} = \frac{1}{2}mv^2 \quad \frac{m^2v^2r^2}{2mr^2} \quad \frac{L^2}{2I}$$

Since, L is constant, thererfore

$$\text{K.E.} \quad \frac{1}{I} \Rightarrow K.E. \quad \frac{1}{r^2}$$

$$\therefore \frac{E_2}{E_1} \quad \frac{r_1^2}{r_2^2} \quad \frac{1}{16}$$

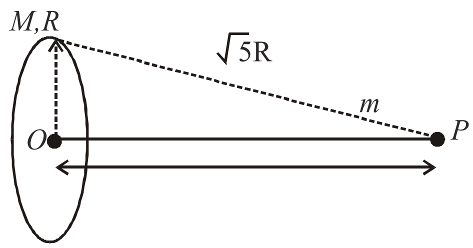
11. (a) $v = \omega R$

$g = g_0 - \omega^2 R$ [g = at equator, g_0 = at poles]

$$\frac{g_0}{2} = g_0 - \omega^2 R \;;\; \omega^2 R \quad \frac{g_0}{2} \;;\; v^2 \quad \frac{g_0 R}{2}$$

$$v_e \quad \sqrt{2g_0 R} \quad \sqrt{4v^2} \quad 2v$$

12. (d) Gravitational potential at P, $V_p \quad \frac{-GM}{\sqrt{5}R}$ Gravitational potential at O, $V_O \quad \frac{-GM}{R}$



By work energy theorem,

$$W = \Delta K \Rightarrow m[V_P - V_O] \quad \frac{1}{2}mv^2$$

$$m\left[\frac{GM}{R} - \frac{GM}{\sqrt{5}R}\right] \quad \frac{1}{2}mv^2 \quad \text{or} \quad \sqrt{\frac{2GM}{R}\left(1 - \frac{1}{\sqrt{5}}\right)}$$

13. (a) Using Kepler's 2nd law

$$\frac{dA}{dt} \quad \frac{J}{2m} \qquad [J = \text{angular momentum}]$$

Integrating, $2m\frac{A}{T} \quad J$

14. (d) Acceleration due to gravity $g = \frac{GM}{R^2}$

G = gravitational constant, M = mass and R = radius of earth

New radius $R' = R - \frac{R}{100} = \frac{99}{100}R$

$$\Rightarrow g' = \frac{GM}{\left(\left(\frac{99}{100}\right)R\right)^2} = \frac{GM}{R^2} \times \left(\frac{100}{99}\right)^2$$

$$\Rightarrow g' = g\left(\frac{100}{99}\right)^2$$

$\Rightarrow$ % change in g $= \dfrac{g'-g}{g} \times 100$

$= \left(\left(\dfrac{100}{99}\right)^2 - 1\right) \times 100 = \dfrac{(100)^2 - (99)^2}{(99)^2} \times 100$

$= \dfrac{199}{(99)^2} \times 100 \simeq 2\%$

15. (a) Work done, $W = U_f - U_i$

$= \dfrac{-G\, m_1 m_2}{r_f} - \left[\dfrac{-G\, m_1 m_2}{r_i}\right]$

$= Gm_1 m_2 \left[\dfrac{1}{r_i} - \dfrac{1}{r_f}\right] = Gm_1 m_2 \left[\dfrac{1}{x_1} - \dfrac{1}{x_1 \quad d}\right]$

16. (a) $\dfrac{v_1}{v_2} \quad \sqrt{\dfrac{r_2}{r_1}} \quad \sqrt{\dfrac{4R}{R}} \quad 2$

$\therefore \dfrac{K_1}{K_2} \quad \dfrac{\frac{1}{2} m v_1^2}{\frac{1}{2} m v_2^2} \quad \dfrac{v_1^2}{v_2^2} \quad 4$

17. (b) $v = \dfrac{v_e}{2} = \dfrac{\sqrt{\dfrac{2GM}{R}}}{2} = \sqrt{\dfrac{GM}{2R}}$

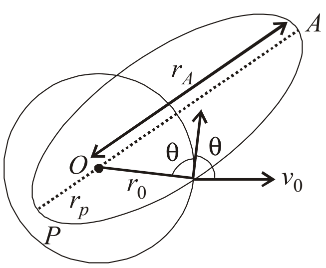
If h is the height reached by the body, then by conservation of mechanical energy

$\dfrac{-GMm}{R} \quad \dfrac{1}{2} m \left(\sqrt{\dfrac{GM}{2R}}\right)^2 \quad \dfrac{-GMm}{R \quad h} \quad 0$

or $\dfrac{3}{4R} \quad \dfrac{1}{R \quad h}$

$\therefore h = \dfrac{R}{3}$

18. (b) $TE \quad \dfrac{-GM_e m}{2r_0}$



Using conservation of angular momentum about O.

$mv_p r_p = mv_A r_A = mv_0 r_0 \cos\theta$

$v_A r_A = v_p r_p \quad \dfrac{3v_0 r_0}{5}$

Using conservation of energy

$\dfrac{1}{2} m v_p^2 \quad \dfrac{-GM_e m}{r_A} \quad \dfrac{-GM_e m}{r_0} \quad \dfrac{1}{2} m v_0^2$

$\Rightarrow \dfrac{9 v_0^2 r_0^2}{50 r_A^2} - \dfrac{v_0^2 r_0}{r_A} \quad \dfrac{v_0^2}{2} \quad \left[\text{Let} \dfrac{r_0}{r_A} \quad x\right]$

$\Rightarrow 9x^2 - 50x + 25 = 0$

$\Rightarrow x = 5$ or $(5/9)$

$\Rightarrow \dfrac{v_P}{v_A} \quad \dfrac{r_A}{r_p} \quad 9$

19. (b) $\dfrac{1}{2} m v_{\min}^2 - \dfrac{GMm}{R\sqrt{2}} \times 2 \quad 0$

$\Rightarrow \quad v_{\min} \quad \sqrt{\dfrac{2GM\sqrt{2}}{R}}$

20. (d) $mr\omega^2 = \dfrac{GMm}{r^2}$

$GM = r^3 \omega^2$ $\qquad (GM = gR^2)$

$g = \dfrac{r^3 . \omega^2}{R^2}$

$g' = \dfrac{(4nR)^3 . \omega^2}{n^2 . R^2}$

$g' = 64\, nR\omega^2$.

21. (b) Since gravitational field intensity inside hollow sphere M_2 will be zero.

So, gravitational force will be due to M_1 only.

22. (b) By conservation of angular momentum

$mv_1 (2R) = mv_2 (4R)$

$v_1 = 2v_2$ ... (1)

By conservation of energy,

$\dfrac{1}{2} m v_1^2 - \dfrac{GMm}{2R} = \dfrac{1}{2} m v_2^2 - \dfrac{GMm}{4R}$ ... (2)

From (1) and (2), we get

$v_{min} = \sqrt{\dfrac{GM}{6R}}; v_{max} \quad \sqrt{\dfrac{2GM}{3R}}$

23. (c) $E_1 = -\dfrac{GM_e m}{R_e \quad \dfrac{R_e}{2}} - \left(-\dfrac{GM_e m}{R_e}\right) \quad \dfrac{GM_e m}{3R_e}$

$E_2 = \dfrac{1}{2} m v_0^2 \quad \dfrac{1}{2} m . \dfrac{GM_e}{R \quad \dfrac{R_e}{2}} \quad \dfrac{GM_e m}{3R_e}$

$\therefore \dfrac{E_1}{E_2} \quad 1:1$

24. **(d)** Differentiating the equation of the curve w.r.t. t we get,

$$2\left(\frac{dx}{dt}\right)^2 = 500\times\frac{d^2y}{dt^2}$$

or, $\frac{d^2y}{dt^2} \quad \frac{2\times10^4}{500} = 4g$. the effective 'g' = 5g.

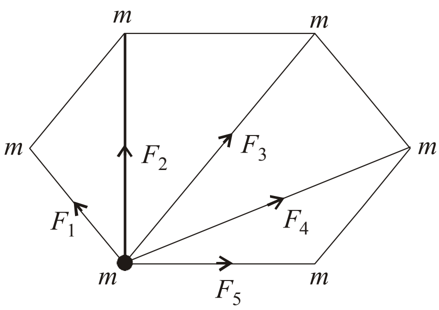
25. **(c)** The gravitational potential at a point Q ($OQ = x$) is given by

$$V(x) = \begin{Bmatrix} -g_s R\left(\frac{3}{2}-\frac{1}{2}\frac{x^2}{R^2}\right), \text{when } x \quad R \\ -g_s R\left(\frac{R}{x}\right), \quad \text{when } x \quad R \end{Bmatrix}$$

The energy required to project the body to a maximum altitude of $3R$ from its surface, is

$$m\left(V_B\Big|_{x \quad \frac{R}{2}} - V_P\big|_{x \quad 4R}\right) \quad \frac{9}{8}mg_s R.$$

26. **(b)** $\vec{F} \quad \vec{F}_1 \quad \vec{F}_2 \quad \vec{F}_3 \quad \vec{F}_4 \quad \vec{F}_5$



$$\left|\vec{F}_2\right| \quad \left|\vec{F}_5\right| \text{ and } \left|\vec{F}_2\right| \quad \left|\vec{F}_4\right|$$

$$\vec{F}_1 \quad \vec{F}_3 \quad 2F_2\cos 30 \quad\quad 2F_1\cos 60$$

$$F_3 \quad \frac{Gm^2}{4a^2}; F_2 \quad \frac{Gm^2}{3a^2}; F_1 \quad \frac{Gm^2}{a^2}$$

$$F \quad \frac{Gm^2}{a^2}\left(\frac{5}{4} \quad \frac{1}{\sqrt{3}}\right) = m\omega^2 a$$

$$\omega \quad \sqrt{\frac{Gm}{a^3}\left(\frac{5}{4} \quad \frac{1}{\sqrt{3}}\right)}$$

$$T = 2\pi\sqrt{\frac{4\sqrt{3}a^3}{Gm(5\sqrt{3} \quad 4)}}$$

27. **(a)** Let v_1 and v_2 be the velocity required for the satellite in order to maintain its motion along the circular orbits C_1 and C_2 respectively.

Then $v_1 \quad \sqrt{\frac{GM}{3R}}$ and $v_2 \quad \sqrt{\frac{GM}{6R}}$

Now, let v'_1 and v'_2 be the velocities of the satellite at A and B while the transfer is pursued from orbit C_1 to C_2. Evidently, the points A and B will be the positions of perigee and apogee for the satellite along its elliptical path.

Conserving angular momentum

$$v'_1(3R) = v'_2(6R)$$

$$\Rightarrow v'_2 \quad \frac{v'_1}{2}$$

Conserving energy between A and B, we have

$$\frac{1}{2}m(v'_1)^2 - \frac{GMm}{3R} = \frac{1}{2}m(v'_2)^2 - \frac{GMm}{6R}$$

$$\Rightarrow v_1'^2 - v_2'^2 \quad 2GM\left(\frac{1}{6R}\right)$$

$$\Rightarrow v_1'^2 - \frac{v_1'^2}{4} \quad \frac{GM}{3R}$$

$$\Rightarrow \frac{3v_1'^2}{4} = \frac{GM}{3R} \Rightarrow v'_1 \quad \frac{2}{3}\sqrt{\frac{GM}{R}}$$

So, $v'_2 = \frac{1}{3}\sqrt{\frac{GM}{R}}$

Now, change in energy required at point A

$$\Delta E_1 = \frac{1}{2}mv_1'^2 - \frac{1}{2}mv_1^2$$

$$\Rightarrow \frac{m}{2}\left[\frac{4}{9}(gR) - \frac{gR}{3}\right] \Rightarrow \frac{m}{2}\left(\frac{gR}{9}\right) \quad \frac{mgR}{18}$$

Similarly change in energy required at point B

$$\Delta E_2 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_2'^2 = \frac{m}{2}\left[\frac{gR}{6} - \frac{gR}{9}\right]$$

$$= \frac{m}{2}\left(\frac{gR}{18}\right) \quad \frac{mgR}{36}$$

28. **(c)** $$g = \frac{GM}{R^2}$$

$$g' = \frac{GM}{(0.99R)^2}$$

$$\therefore \quad \frac{g'}{g} \quad \left(\frac{R^2}{0.99R}\right)^2$$

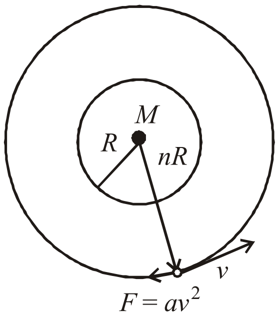
$$\Rightarrow g' > g$$

29. **(b)** $v_e \quad \sqrt{2g_e R_e}$; $v_m \quad \sqrt{2g_m R_m}$

$$\frac{v_e}{v_m} \quad \sqrt{\frac{g_e}{g_m}\frac{R_e}{R_m}} \quad \sqrt{\frac{g_e}{g_e/6}\frac{R_e}{R_e/4}} \quad \sqrt{24}$$

30. (a) $F \quad \dfrac{mdv}{dt} \quad av^2$

$$m\int_{\sqrt{\frac{GM}{nR}}}^{\sqrt{\frac{GM}{R}}}\frac{dv}{v^2} \quad a\int_0^t dt$$



$$\frac{\left[-\sqrt{\dfrac{R}{GM}} \quad \sqrt{\dfrac{nR}{GM}}\right]m}{a} \quad t$$

$$t \quad \frac{m\sqrt{R}\,(\sqrt{n}-1)}{a\sqrt{GM}}$$

Energy method : Total energy in circular orbit of radius r,

$$E=-\frac{GMm}{2r}$$

Rate of change of energy = $-\dfrac{dE}{dt}=-\dfrac{GMm}{2r^2}\dfrac{dr}{dt}$

$$-\frac{GMm}{2r^2}\frac{dr}{dt} \quad F.v \quad av^3 \qquad ....(1)$$

Centripetal force = gravitational force

$$\frac{mv^2}{r}=\frac{GMm}{r^2}\Rightarrow v^2 \quad \frac{GM}{r} \quad \therefore\ v^3 \quad \left(\frac{GM}{r}\right)^{3/2}$$

From eq. (1),

$$-\frac{GMm}{2r^2}\frac{dr}{dt} \quad a\frac{(GM)^{3/2}}{r^{3/2}}$$

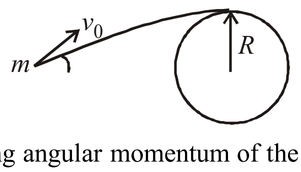
$$-\frac{GMm}{2r^{1/2}}dr \quad a(GM)^{3/2}dt$$

$$-\int_{nR}^{R}\frac{GMm}{2r^{1/2}}dr \quad \int_0^t a(GM)^{3/2}dt$$

$$t=\frac{GMm}{a(GM)^{3/2}}[\sqrt{nR}-\sqrt{R}]=\frac{m}{a\sqrt{GM}}(\sqrt{nR}-\sqrt{R})$$

$$=\frac{m\sqrt{R}\,(\sqrt{n}-1)}{a\sqrt{GM}}$$

31. (b) Let the speed of the instrument package is v when it grazes the surface of the planet.



Conserving angular momentum of the package about the centre of the planet

$mv_0 \times 5R\sin(\pi-\theta)=mvR\sin 90°$

$\Rightarrow v=5v_0\sin\theta$ ....(1)

Conserving mechanical energy

$$-\frac{GMm}{5R} \quad \frac{1}{2}mv_0^2 \quad -\frac{GMm}{R} \quad \frac{1}{2}mv^2$$

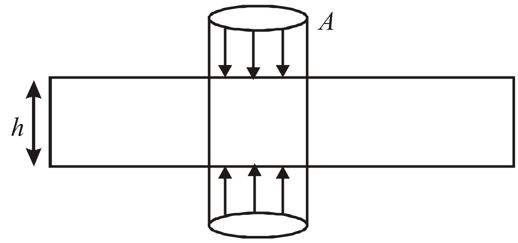
$$\Rightarrow \frac{1}{2}m(v^2-v_0^2)=\frac{4GMm}{5R}\Rightarrow v^2-v_0^2 \quad \frac{8GM}{5R} \quad .....(2)$$

Substituting the value of v from eq. (1) in equation (2)

$$25v_0^2\sin^2\theta-v_0^2 \quad \frac{8GM}{5R}$$

$$\theta \quad \sin^{-1}\left(\frac{1}{5}\sqrt{1 \quad \frac{8GM}{5v_0^2R}}\right)$$

32. (a)



Gauss law for gravitation

$$\int \vec{g}.d\vec{s}=-m_{in}.4\pi G \quad ; \quad g \quad \frac{GM}{R^2}$$

$$2\times\frac{GM}{R^2}\times A=\frac{M}{\frac{4}{3}\pi R^3}(h\times A)\times 4\pi G \ \Rightarrow\ h \quad \frac{2R}{3}$$

33. (d) $\dfrac{L}{2m} \quad \dfrac{dA}{dt}$ (L = angular momentum)

$$\frac{mv_{\max}r_{\min}}{2m} \quad \frac{dA}{dt} \ ;$$

$$v_{\max} \quad \frac{2\,dA/dt}{r_{\min}} \quad 40$$

34. (d) If M is the mass and R is the radius of earth, then the density $\rho \quad \dfrac{M}{\frac{4}{3}\pi R^3}$. The spherical volume may be supposed to be formed by a large number of their concentric spherical shells. Let the sphere be disassembled by removing such shells. When there is a spherical core of radius x the energy needed to disassemble a spherical shell of thickness dx is

$$dW \quad \frac{Gm_1m_2}{x} \ ; \ m_1=\frac{4}{3}\pi x^3\rho$$

m_2 = mass of spherical shell of radius x and thickness $dx = 4\pi x^2 dx\,\rho$.

$$\therefore\ dW=\frac{G\left(\frac{4}{3}\pi x^3\rho\right)(4\pi x^2dx\rho)}{x}=\frac{16}{3}\pi^2\rho^2Gx^4dx$$

$\therefore$ Total energy required

$$W = \int_0^R \frac{16}{3}\pi^2\rho^2 G x^3 dx = \frac{16}{3}\pi^2\rho^2 G\left[\frac{x^5}{5}\right]_0^R$$

$$= \frac{16}{3}\pi^2\rho^2 \frac{GR^5}{5} = \frac{16}{15}\pi^2\left(\frac{M}{\frac{4}{3}\pi R^3}\right) GR^5 \quad \frac{3}{5}\frac{GM^2}{R}$$

But $GM = gR^2$

$$\therefore \; W \quad \frac{3}{5}\frac{gR^2}{M}\frac{M^2}{R} \quad \frac{3}{5}gMR \quad \frac{3}{5}\times 10\times 2.5\times 10^{31}$$

$= 1.5 \times 10^{32}$ J

35. (c) For this (for a particle on its equator)

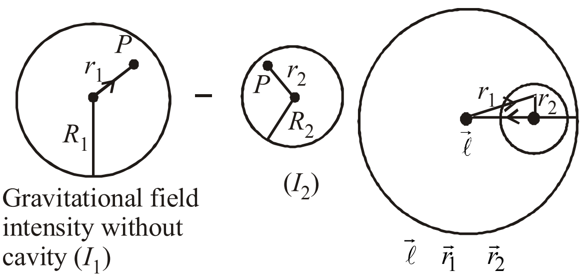
$$\frac{G\frac{4}{3}\pi r^3\rho}{r^2} > \omega^2 r$$

$$\therefore \; \rho \frac{3\omega^2}{4\pi G} \quad \frac{3\pi}{GT^2}$$

if $T = 1$ second

$\rho > 1.4 \times 10^{11}$ kg/m^3.

36. (a) For calculation of gravitational field intensity inside the cavity.



$$\vec{I}_1 \quad \frac{G\left(\frac{4}{3}\pi R_1^3\right)\rho\,(-\vec{r}_1)}{R_1^3}, \quad \vec{I}_2 \quad \frac{G\left(\frac{4}{3}\pi R_2^3\right)\rho\,(-\vec{r}_2)}{R_2^3}$$

$\vec{I} = \vec{I}_1 - \vec{I}_2$ ($\vec{I}$ – intensity inside the cavity)

$$= \frac{4}{3}G\pi\rho\,[-\vec{r}_1 \quad \vec{r}_2] \quad \frac{4}{3}G\pi\rho\vec{\ell}$$

37. (a) $U_g = -\frac{GM_1M_2}{R}$

$$\Rightarrow U_f - U_i = -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right)$$

$$\Delta U \quad \frac{GMm}{2R} \quad \frac{mgR}{2} \quad \left\{g \quad \frac{GM}{R^2}\right\}$$

38. (b) According to Kepler's law

$$\frac{T_1^2}{T_2^2} \quad \frac{R_1^3}{R_2^3}$$

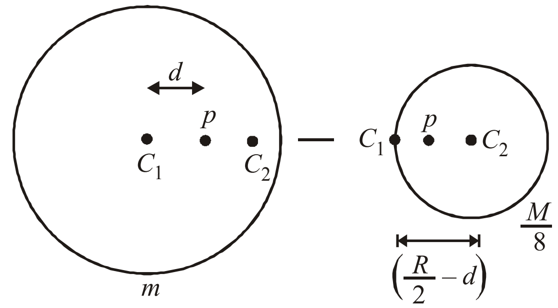
Here $T_1 = 365$ days ; $T_2 = ?$

$R_1 = R$; $R_2 = \frac{R}{2}$

$$\Rightarrow T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2}$$

$$= 365\left[\frac{R/2}{R}\right]^{3/2} \quad 129\,\text{days}$$

39. (d)

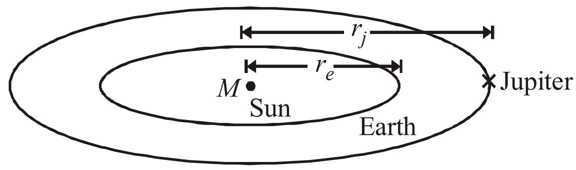


$$(E_p) = \frac{GMd}{R^3} \quad \frac{G\left(\frac{M}{8}\right)\left(\frac{R}{2}-d\right)}{\left(\frac{R}{2}\right)^3}$$

$$= \frac{GMd}{R^3} + \frac{GM\left(\frac{R}{2}\right)}{R^3} - \frac{GMd}{R^3} \quad \frac{GM}{2R^2}$$

$$\therefore \text{ Force} = \frac{GMm}{2R^2}$$

40. (b)

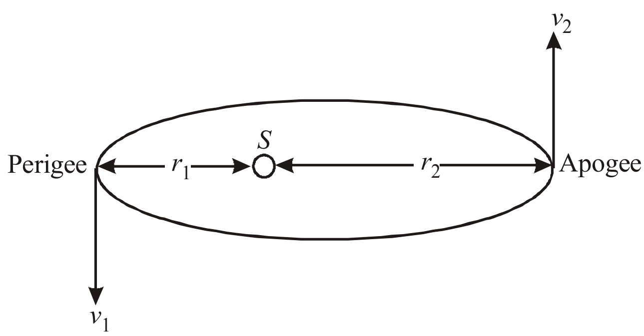


$$v \quad \sqrt{\frac{GM}{r}}$$

Distance of Jupiter from Sun is more than Earth.

i.e., $r_j > r_e$. $\therefore \; v_j < v_e$

41. (b)



According to the law of conservation of angular momentum, $mv_1r_1 = mv_2r_2$

$$\frac{v_1}{v_2} \quad \frac{r_2}{r_1} \quad \frac{a\,(1 \quad e)}{a\,(1-e)},$$

where e is eccentricity of the earth's orbit

$$= \frac{(1 \quad 0.0167)}{(1-0.0167)} \quad 1.034$$

42. (a) Let the gravitational field be zero at a point distant x from M_1.

$$\frac{GM_1}{x^2} \quad \frac{GM_2}{(d-x)^2} \;;\; \frac{x}{d-x} \quad \sqrt{\frac{M_1}{M_2}}$$

$$x\sqrt{M_2} = \sqrt{M_1}d - x\sqrt{M_1}$$

$$x\left[\sqrt{M_1} \quad \sqrt{M_2}\right] \quad \sqrt{M_1}d$$

$$x \quad \frac{d\sqrt{M_1}}{\sqrt{M_1} \quad \sqrt{M_2}},\; d-x \quad \frac{d\sqrt{M_2}}{\sqrt{M_1} \quad \sqrt{M_2}}$$

Potential at this point due to both the masses will be

$$-\frac{GM_1}{x} - \frac{GM_2}{(d-x)}$$

$$= -G\left[\frac{M_1 \quad \sqrt{M_1} \quad \sqrt{M_2}}{d\,\sqrt{M_1}} \quad \frac{M_2 \quad \sqrt{M_1} \quad \sqrt{M_2}}{d\,\sqrt{M_2}}\right]$$

$$= -\frac{G}{d} \quad \sqrt{M_1} \quad \sqrt{M_2} \quad ^2$$

$$= -\frac{G}{d} \quad M_1 \quad M_2 \quad 2\sqrt{M_1}\sqrt{M_2}$$

43. (d) Angular momentum of the planet about S is conserved.
So, mvr = constant.
v is maximum when r is minimum.
So, v is maximum at point P_4. Hence K.E. is maximum at P_4.

44. (d) From Kepler's law, $T^2 \quad R^3$
Parking orbit, $1^2 \quad R^3$
After missing, $T^2 \quad (3R)^3$

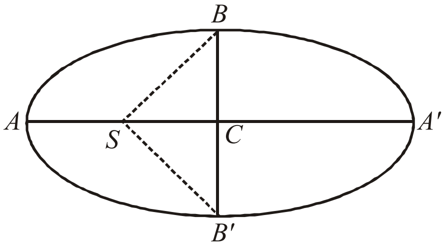
$$\frac{T^2}{1} \quad \frac{27R^3}{R^3} \quad 27 \;;\; T \quad \sqrt{27} \quad 3\sqrt{3} \text{ day}$$

45. (c) $E = -\dfrac{GMm}{2r}$

$$-\frac{dE}{dt} \quad \frac{GMm}{2r}\frac{1}{r^2}\frac{dr}{dt}$$

$$\int_0^t dt = -\frac{GMm}{2C}\int_r^R \frac{dr}{r^2} \;;\; t = \frac{GMm}{2C}\left[\frac{1}{R} - \frac{1}{r}\right]$$

46. (b) Remote half from the sun is the arc $B'A'B$.



Let t be the time of description of the arc $B'A'B$, therefore rate of description of the sectorial area $SB'A'BS$

$$= \frac{\text{area } SB'A'BS}{t}$$

Since the whole area of the ellipse is described in a year, therefore rate of description of the sectorial area is

$$= \frac{\text{area of the ellipse}}{\text{a year}}$$

Hence, $\dfrac{\text{area } SB'A'BS}{t} \quad \dfrac{\text{area of the ellipse}}{\text{a year}}$

or $\dfrac{t}{\text{year}} \quad \dfrac{\text{area } SB'A'BS}{\text{area of the ellipse}}$

$$= \frac{\frac{1}{2}\text{area of the ellipse} + \Delta\; SBB'}{\pi ab}$$

$$= \frac{\frac{1}{2}\pi ab \quad \frac{1}{2}.2bae}{\pi ab} \quad \frac{1}{2} \quad \frac{e}{\pi}$$

$$\therefore\; t \quad \left(\frac{1}{2} \quad \frac{e}{\pi}\right)\text{year} \quad \frac{1}{2}\text{year} \quad \frac{e}{\pi}\text{year}$$

$$= \frac{1}{2} \text{ year} + 2 \text{ days nearly}$$

Solving $e = 1/60$

47. (d) $4\pi r^2 dr$ = volume of an element of thickness dr.
The number of particles, $n_i = 4\pi r^2 dr.\,\rho$, assuming ρ is the number of particles per unit volume. The acceleration due to gravity $= \dfrac{g}{R} \times r$ for each particle.

$$n_i g = 4\pi r^2 dr\rho.\frac{g}{R}r = 4\pi\frac{g}{R}\rho r^3 dr$$

Mean acceleration of the particles due to gravity,

$$\bar{g} \quad \frac{\int_0^R \left(\frac{4\pi g}{R}r^3 dr\right)\rho}{(4/3)\,\pi R^3\rho} \quad \frac{4\pi g}{R}.\frac{R^4}{4}\times\frac{1}{(4/3)\pi R^3}$$

$$= 4\pi g.\frac{R^3}{4}\times\frac{1}{(4/3)\pi R^3} \quad \frac{3}{4}g$$

This is the mean acceleration due to gravity.

Average distance travelled by the particle is = $\dfrac{\sum n_i r}{\sum n_i}$

$$\bar{r} = \frac{\int_0^R 4\pi r^2 dr\, \rho . r}{\frac{4}{3}\pi R^3 \rho} \Rightarrow \bar{r} = 4\pi \frac{R^4}{4}\rho \frac{1}{\frac{4}{3}\pi R^3 \rho}$$

$$\Rightarrow \bar{r} \quad \frac{3}{4}R$$

$$\bar{r} \quad \frac{1}{2}\bar{a}t^2 \text{ where } \bar{a} = \text{average acceleration}$$

$$\Rightarrow \frac{3}{4}R = \frac{1}{2}.\frac{3}{4}g.t^2 \Rightarrow t \quad \sqrt{\frac{2R}{g}}$$

$$g = \frac{GM}{R^2} \Rightarrow t \quad \left(\frac{2R.R^2}{GM}\right)^{1/2} \quad \left(\frac{2R^3}{GM}\right)^{0.5}$$

Collision between particles is neglected. It is as if every particle is moving unhindered with no other interaction towards the centre. The density of particles P is very small. Otherwise it will be making collisions like the electrons hitting each other but moving under the electric potential. Here it will be gravitational potential. The model used is a simplified one.

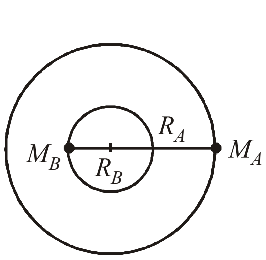
48. (a) By Kepler's law

$$\frac{dA}{dt} \quad \frac{A}{T} \quad \frac{L}{2m}$$

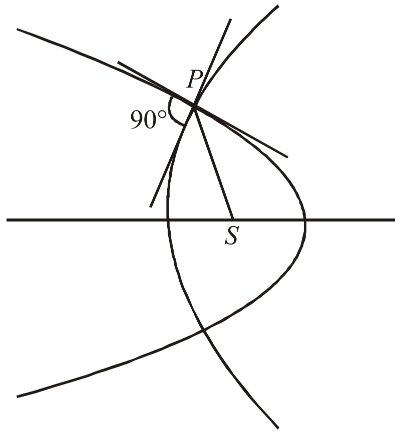
$$\therefore \; L \quad \frac{2mA}{T}$$

49. (d) The gravitational force of attraction between the stars will provide the necessary centripetal forces.
In this case angular velocity of both stars is the same.

Therefore time period remains the same. $\left(\omega \quad \dfrac{2\pi}{T}\right)$.



50. (a) Let P be the common point of the two parabolas where the two particles collide. Let v_1, v_2 be the velocities of m_1 and m_2 at the time of collision.



Since the two paths are parabolic, therefore

$$v_1^2 \quad \frac{2\mu}{R} \quad v_2^2 \qquad \text{(at the point } P) \qquad ....(1)$$

$$\left[\text{acc.} \quad \frac{\mu}{(\text{distance})^2}\right]$$

After collision the two masses combine into one, let V be the velocity after impact of the single mass $(m_1 + m_2)$.
By the principle of conservation of momentum the resultant momentum must be the same after and before the impact :

$$\therefore \; (m_1 \quad m_2)^2 V^2 \quad m_1^2 v_1^2 \quad m_2^2 v_2^2 \qquad ....(2)$$

because v_1 and v_2 are at right angles.
From (1) it is seen that $v_2 = v_1$ and then from (2), we have

$$(m_1 \quad m_2)^2 V^2 \quad (m_1^2 \quad m_2^2) v_1^2$$

$$\text{i.e. } V^2 \quad \frac{m_1^2 \quad m_2^2}{(m_1 \quad m_2)^2} v_1^2 \qquad ....(3)$$

It is seen from (3) that after the collision the $(\text{velocity})^2$ becomes less than v_1^2, that is less than $\dfrac{2\mu}{R}$, which is the condition for describing an ellipse.
So after the collision the path will be an ellipse. Let $2a'$ be its major axis.
For the path to be an ellipse,

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a'}\right)$$

At the point P, just after the collision $v = V$ and $r = R$.

$$\text{So, } V^2 = \mu\left(\frac{2}{R} - \frac{1}{a'}\right)$$

Substituting for V^2 from (3), we have

$$\frac{m_1^2 \quad m_2^2}{(m_1 \quad m_2)^2} v_1^2 = \mu\left(\frac{2}{R} - \frac{1}{a'}\right)$$

Now substituting for v_1^2 from (1), we have

$$\frac{m_1^2 \quad m_2^2}{(m_1 \quad m_2)^2}\frac{2\mu}{R} = \mu\left(\frac{2}{R} - \frac{1}{a'}\right)$$

$$\therefore \frac{1}{a'} = \left\{1 - \frac{m_1^2 \quad m_2^2}{(m_1 \quad m_2)^2}\right\}.\frac{2}{R} \quad \frac{2m_1m_2}{(m_1 \quad m_2)^2}.\frac{2}{R}$$

i.e. $2a' \quad \frac{(m_1 \quad m_2)^2}{2m_1m_2}.R$

that is, major axis $= \frac{(m_1 \quad m_2)^2}{2m_1m_2}.R$

51. (c) Inside the shell there will be no gravity. Hence, $T \quad \frac{2\pi r}{v}$

B COMPREHENSION TYPE

1. (d) $T^2 \quad R^3$

$T_e^2 \quad KR_e^3; T_m^2 \quad kR_m^3; T^2 \quad kR^3$

$R \quad \frac{R_e \quad R_m}{2}$

$$\Rightarrow T^2 \quad k\left[\frac{T_e^{2/3}}{k^{1/3}} \quad \frac{T_m^{2/3}}{k^{1/3}} \times \frac{1}{2}\right]^3$$

$$\Rightarrow T \quad \left[\frac{T_e^{2/3} \quad T_m^{2/3}}{2}\right]^{3/2}$$

2. (a) $E_e = -\frac{GM_sM_e}{2R_e} \quad -\frac{GM_sM}{2R}$

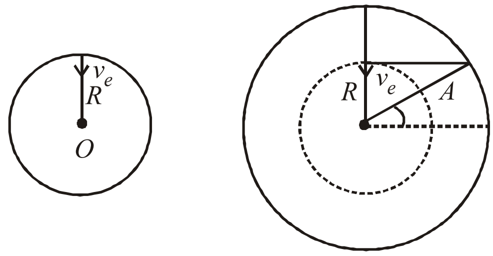
$$= \frac{2R_eE_e}{M_e} \times \frac{M}{2\left(\frac{R_e \quad R_m}{2}\right)}$$

$$\frac{2M}{M_e}\left(\frac{R_e}{R_e \quad R_m}\right)E_e$$

3. (c) Areal velocity of the artificial planet around the sun will be more than that of earth.

4. (a) We know if a particle is released (initial speed zero) from earth surface time period $= 2\pi\sqrt{\frac{Radius}{g}}$, so we imagine a earth of radius A from where particle is released and when it reaches at distance R from earth centre its speed is v_e. Mass of assumed earth of radius is A.

$$M' \quad \left(\frac{A}{R}\right)^3 M$$



According to energy conservation

$$-\frac{GM'm}{A} \quad 0 \quad -\frac{GM'}{2A^3}(3A^2 - R^2)\,m \quad \frac{1}{2}m\left(\frac{2GM}{R}\right)$$

$$-\frac{A^3}{R^3}\frac{M}{A} = -\frac{A^3}{R^3}\frac{M}{2A^3}(3A^2 - R^2) \quad \frac{M}{R}$$

$$-A^2 = -\frac{1}{2}(3A^2 - R^2) \quad R^2$$

$$-A^2 = -\frac{3}{2}A^2 \quad \frac{R^2}{2} \quad R^2 \;;\; A \quad \sqrt{3}R$$

$$\sin\phi \quad \frac{1}{\sqrt{3}} \;;\; \phi \quad \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$T \quad \left(\frac{\phi}{2\pi}\right)T \quad \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\sqrt{\frac{A}{g_A}} \quad \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\sqrt{\frac{R_e}{g}}$$

$$T \quad \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\sqrt{\frac{R_e}{g}}$$

5. (d) At a height h from earth surface acceleration due to gravity is not proportional to distance from centre hence particle will not perform SHM but the motion will be periodic.

6. (a) $h = \frac{1}{2}gt^2 \Rightarrow t \quad \sqrt{\frac{2h}{g}}$

Time period $= 2\pi\sqrt{\frac{R}{g}} \quad 4t = 2\pi\sqrt{\frac{R}{g}} \quad 4\sqrt{\frac{2h}{g}}$

For a circular orbit, with the central mass much greater than the satellite mass:

$F_{central} = F_{gravitational}$

$m\,\omega^2 r = GMm / r^2$

$\omega = [GM/r^3]^{1/2}$

$= [6.67 \times 10^{-11} \times 20 \times 10^{30} / (0.04 \times 1.5 \times 10^{11})^3]^{1/2}$

$\omega = 7.86 \times 10^{-5}$

Period $= 1/f = 2\pi/\omega = 7.99 \times 10^4$ s

Tangential Velocity $= \omega r = 4.72 \times 10^5$ m / s

The tension in the beam is due to the difference in gravitational force between the inner and outer sphere.

$$T = \frac{GMm_1}{(R-\Delta R)^2} - \frac{GMm_2}{(R+\Delta R)^2}$$

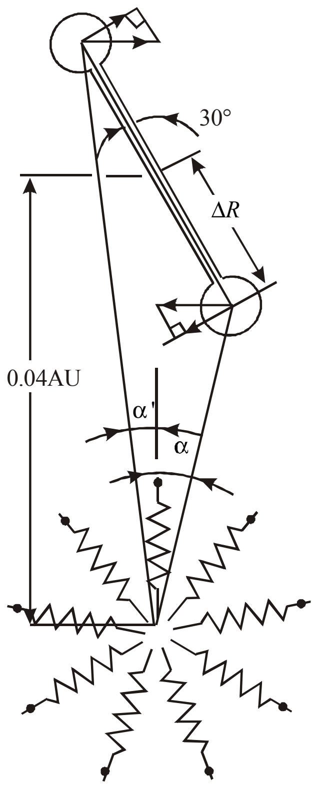
$$= \frac{GMm}{R^2}\left[\frac{1}{(1-\Delta R/R)^2} - \frac{1}{(1+\Delta R/R)^2}\right]$$

As $\Delta R / R << 1$, we can use the binomial expansion on both fractions.

$$= -\frac{4GMm\Delta R}{R^3}$$

$$= \frac{4\times 6.67\times 10^{-11}\times 20\times 10^{30}\times 100\times 10^3\times 50}{(0.04\times 1.5\times 10^{11})^3}$$

$= -0.124$ N



To calculate the displacement angle between one of the spheres, the black hole and the radial axis, the distance from the sphere to the radial axis $D = \Delta R \sin(30°) = 25$ m.

Then, $\sin(\alpha) = \dfrac{D}{R - \Delta R\cos(30\ )} \approx \dfrac{D}{R}$

The restoring force perpendicular to the radial axis, $F_{\perp RA}$ (a component of the force of the black hole on one sphere) is

$$F_{\perp RA} = \frac{GMm}{R^2}\sin(\alpha) \approx \frac{GMmD}{R^3} = 1.54\times 10^{-2}\,N$$

The restoring force F_R is a component of $F_{\perp RA}$, perpendicular to the axis of the observation station is given by

$F_{jR} = F_{\perp RA}\cos(30°) = 1.33 \times 10^{-2}$ N.

A similar restoring force acts on the other sphere.

As $\alpha' \approx \alpha$, this force is equal to the force on the first sphere, so the total restoring force is 2.66×10^{-2} N.

The maximum tension force F_{max} that can be applied to the beam is given by

F_{max} = Cross sectional area of beam x yield strength

$= \pi(1 - 0.95^2) \times 830 \times 10^6 = 2.54 \times 10^8$ N

The distance of closest approach is given by :

$$F_{\max} \quad \frac{4GMm\Delta R}{R^3}$$

or $$R^3 \quad \frac{4GMm\Delta R}{F_{\max}}$$

$R_{min} = 4.72 \times 10^6$ m.

C REASONING TYPE

1. **(a)** A balloon will not experience any buoyant force on the moon because it has no atmosphere, so it will have free fall under gravitational pull of the moon with acceleration equal to $\frac{g}{6}$.

2. **(c)** The length of the day is slowly increasing not due to gravitational pull of other planets in the solar system but due to viscous force between the earth and the atmosphere around it. So Statement-1 is true but Statement-2 is false.

3. **(c)** Upto ordinary heights, the change in the distance of a projectle from the centre of earth is negligible compared to the radius of earth. Hence the projectile moved under a nearly uniform gravitational force and the path is parabolic. But for the projection moving to a large height the gravitional forced decreases quite rapidly $\left(as\ F \quad \frac{1}{r^2}\right)$. Under such a rapidly decreasing variable force, the path of projectile become elliptical.

4. **(a)** The time period of satellite, $T \quad r^{3/2}$

   or $T \quad (R_e \quad h)^{3/2}$

   For a satellite revolving close to surface of earth $h = 0$

   $\therefore T \quad R_e^{3/2}$. It is evident that the period of revolution of a satellite depends upon its height above the earth's surface. Greater is the height of a satellite above the earth's surface, greater is its period of revolution.

5. **(a)** According to Newton's law of gravitation, every body in this universe attracts every other body with a force which is inversely proportional to the square of the distance between them. When we move our finger, the distance of the object with respect to finger changes, hence the force of attraction changes, disturbing the entire universe, including stars.

6. **(c)** The orbital velocity, if a satellite close to earth is $v_0 \quad \sqrt{gR_e}$, While the escape velocity for a body thrown from the earth's surface is $v_e \quad \sqrt{2gR_e}$.

   Thus $\frac{v_0}{v_e} \quad \frac{\sqrt{gR_e}}{\sqrt{2gR_e}} = \frac{1}{\sqrt{2}}$

   or $v_e \quad \sqrt{2}v_0$

   i.e., if the orbital velocity of a satellite revolving close to the earth happens to increase to $\sqrt{2}$ times, the satellite would escape.

7. **(c)** The value of g at any place is given by the relation,

   $g' = g - R_e\omega^2\cos^2\lambda$

   Where λ is angle of latitude and ω is the angular velocity of earth. If earth suddenly stops rotating, then

   $\omega = 0 \quad \therefore g' \quad g$

   i.e., the value of g will be same at all places.

8. **(a)** Statement-1 is true and Statement-2 is true and Statement - 2 is correct explanation of Statement - 1

9. **(b)** Statement – 1 is True, Statement – 2 is True; Statement – 2 **is NOT** a correct explanation for Statement – 1.

10. **(d)** Speed changes in elliptical orbit, angular momentum remain same.

D MULTIPLE CORRECT CHOICE TYPE

1. **(a, c, d)** The force outside the earth varies as inverse square of the distance.

   $\therefore$ Motion is not simple harmonic.

   However, from symmetry of motion, the motion will be periodic.

   From COE

   $$\frac{1}{2}mv^2 = -\frac{GMm}{2R} - \left(\frac{3GMm}{2R}\right)$$

   $$\Rightarrow v = \sqrt{\frac{2GM}{R}}$$

2. **(a,b)** For $r > R$, the gravitational field is $F = \frac{GM}{r^2}$

   $\therefore F_1 \quad \frac{GM}{r_1^2}$

   and $F_2 = \frac{GM}{r_2^2} \Rightarrow \frac{F_1}{F_2} \quad \frac{r_2^2}{r_1^2}$

   For $r < R$, the gravitational field is $F = \frac{GM}{R^3} \times r$

   $\therefore F_1 = \frac{GM}{R^3} \times r_1$ and $F_2 = \frac{GM}{R^3} \times r_2$

   $\Rightarrow \frac{F_1}{F_2} \quad \frac{r_1}{r_2}$

3. **(a, c)** If $r > R$, $g' \quad g\dfrac{R^2}{(R \quad h)^2} \quad g\dfrac{R^2}{r^2}$

If $r < R$, $g' \quad g\left[\dfrac{R-d}{R}\right]$

where d is the depth below the surface of earth.
If $d = R, g = 0$.
Further, due to rotation
$g' = g - R\omega^2 \cos^2 \lambda$
If $\omega = 0$, then g increases.

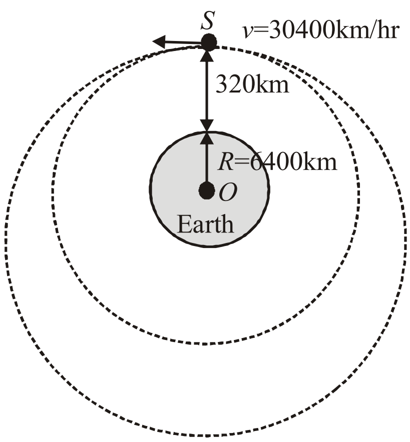
4. **(a, b, c, d)** Given that the velocity of the satellite is

$v = 30400 \times \dfrac{5}{18}$ m/s $= 8444.44$ m/s

Let us calculate the orbital velocity v_0 for the satellite to move along a circular orbit of height 320 km.

$\because \; v_0 \quad \sqrt{\dfrac{GM}{r}} \quad \sqrt{\dfrac{gR^2}{r}}$

$\sqrt{\dfrac{9.8\times(6.4\times10^6)^2}{672\times10^4}} \quad 7728.73 \text{ m/s}$



Also, the escape velocity for the satellite in this position is given by $v_e = \sqrt{2}\, v_0 = 10930.08$ m/s
Evidently, $7728.73 < 8444.44 < 10930.08$
i.e., $v_0 < v < v_e$
So, the satellite moves along an elliptical orbit, with the position of launch as the perigee. (figure).

(b) Let r_{max} be the maximum distance of the satellite from the earth's centre, and v' the corresponding velocity.
From, conservation law of angular momentum and energy between perigee and apogee, we have
$8444.44\,(672 \times 10^4) = v'(r_{max})$

and $\dfrac{1}{2}(8444.44)^2 - \dfrac{gR^2}{672\times10^4} = \dfrac{1}{2}v'^2 - \dfrac{gR^2}{r_{\max}}$

Eliminating v' from the above two equations, we have

$$3565.43\times10^4\left[1-\left(\frac{672}{r'}\right)^2\right]$$

$$= \frac{9.8\times(640\times10^4)^2}{10^4}\left[\frac{1}{672}-\frac{1}{r'}\right]$$

where $r' \quad \dfrac{r_{\max}}{10^4}\, m$

$$\Rightarrow 3565.43\times10^4\left(\frac{r'^2-(672)^2}{r'^2}\right)$$

$$= 4014.08\times10^7\left[\frac{r'-672}{672r'}\right]$$

$$\Rightarrow r' + 672 = 1.675\, r' \Rightarrow r' \quad \frac{672}{0.675} \quad 995\text{m}$$

$\therefore r_{max} = 995 \times 10^4$m
$\Rightarrow$ The maximum height of the satellite
$= r_{max} - R = 9950 - 6400 = 3550$ km.

(c) The semi-major axis of the elliptical path will be

$$a \quad \frac{r_{\max} \quad r_{\min}}{2} \quad \left(\frac{995 \quad 672}{2}\right)\times10^4\text{ m}$$

$= 833.5\times10^4$ m
From Kepler's III[rd] law, of planetary motion,

$$T = 2\pi\sqrt{\frac{a^3}{GM}} \Rightarrow T = 2\pi\sqrt{\frac{a^3}{gR^2}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{(833.5\times10^4)^3}{9.8\times(64\times10^5)^2}}$$

$= 2\times3.14\times1201.06 = 7546.5$ sec. $= 2.09$ hrs.

(d) The escape velocity is given by

$v_e \quad \sqrt{2}v_0 = 10930.08$ m/s

5. **(b)** The centripetal force is provided by the gravitational force of attraction
$mR\omega^2 = GMmR^{-5/2}$

$$\Rightarrow \frac{mR\times4\pi^2}{T^2} \quad \frac{GMm}{R^{5/2}}$$

$\Rightarrow \; T^2 \quad R^{7/2}$

6. **(b, c)** As per Kepler's law, $T^2 \quad R^3$ as T decreases radius decreases, $GPE = -\dfrac{GMm}{R}$ decreases.

7. **(a, b, d)** $F(r) = -\dfrac{k}{r^n}$

$$\Rightarrow U(r) = -\int F(r)\,dr = -\frac{k}{(n-1)}.\frac{1}{r^{n-1}}$$

If L is the angular momentum of the particle of mass m in an orbit of radius r, then

Kinetic energy $= \dfrac{L^2}{2I} \quad \dfrac{L^2}{2mr^2} \quad K(r)$

Since total energy $E(r) = U(r) + K(r)$

$$\Rightarrow E(r) = -\frac{k}{(n-1)}.\frac{1}{r^{n-1}} \quad \frac{L^2}{2mr^2}$$

The criterion that a circular orbit of radius r_0 be stable is that $E(r)$ is minimum.

For $E(r)$ to be minimum, 2 conditions must be fulfilled.

$$\Rightarrow \left.\frac{\partial E}{\partial r}\right|_{r \quad r_0} \quad 0 \text{ and } \left.\frac{\partial^2 E}{\partial r^2}\right|_{r \quad r_0} \quad 0,$$

using both conditions $(3-n)\dfrac{L^2}{m} \quad 0$

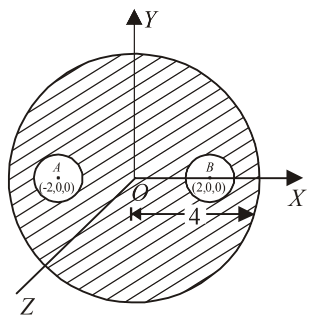
This is possible only when $n < 3$.

We also note that inverse square law belongs to this category $n = -1$ also gives stable circular orbits [Law of direct distance]. But $n = 3$ gives circular orbits which are unstable [Inverse cube law]

8. **(a,c,d)** The gravitational field intensity at the point O is zero (as the cavities are symmetrical with respect to O). Now the force acting on a test mass m_0 placed at O is given by

$F = m_0 E = m_0 \times 0 = 0$

Now, $y^2 + z^2 = 36$ represents the equation of a circle with centre (0, 0, 0) and radius 6 units the plane of the circle is perpendicular to x-axis.



Since the spherical mass distribution behaves as if the whole mass is at its centre (for a point outside on the sphere) and since all the points on the circle is equidistant from the centre of the sphere, the circle is a gravitational equipotential.

The same logic holds good for option (d).

9. **(b, c, d)** Distances of the two satellites from the centre of earth are $r_1 = 2R$ and $r_2 = 8R$ respectively. (R = earth's radius). Their potential energies are :

$$V_1 = -\frac{GmM}{r_1} \text{ and } V_2 = -\frac{GmM}{r_2}$$

Their ratio is : $\dfrac{V_1}{V_2} \quad \dfrac{r_2}{r_1} \quad \dfrac{8R}{2R} \quad 4$

The kinetic energy of a satellite can be obtained from relation

$$\frac{mv^2}{r} \quad \frac{GmM}{r^2} \text{ or } K \quad \frac{1}{2}mv^2 \quad \frac{GmM}{2r}$$

Thus, $K_1 \quad \dfrac{GmM}{2r_1}$ and $K_2 \quad \dfrac{GmM}{2r_2}$

The ratio of their kinetic energies is

$$\frac{K_1}{K_2} \quad \frac{r_2}{r_1} \quad \frac{8R}{2R} \quad 4$$

Their total energies are

$$E_1 = -\frac{GmM}{r_1} \quad \frac{GmM}{2r_1} \quad -\frac{GmM}{2r_1}$$

and $E_2 = -\dfrac{GmM}{r_2} \quad \dfrac{GmM}{2r_2} \quad -\dfrac{GmM}{2r_2}$

Their ratio is

$$\frac{K_1}{K_2} \quad \frac{r_2}{r_1} \quad \frac{8R}{2R} \quad 4$$

$$\frac{E_1}{E_2} \quad \frac{r_2}{r_1} \quad \frac{8R}{2R} \quad 4$$

10. **(a, c)** Kinetic energy on the surface of earth $= \dfrac{1}{2}mv_0^2$

Potential energy on the surface of earth $= \dfrac{-GMm}{R}$

Total energy $= \dfrac{1}{2}mv_0^2 - \dfrac{GMm}{R}$

Kinetic energy at a height $h = \dfrac{1}{2}mv^2$

Potential energy at this height $= \dfrac{-GMm}{(R \quad h)}$

Total energy $= \dfrac{1}{2}mv^2 - \dfrac{GMm}{R \quad h}$

By the principle of conservation of energy,

$$\frac{1}{2}mv^2 - \frac{GMm}{R \quad h} = \frac{1}{2}mv_0^2 - \frac{GMm}{R}$$

$$\frac{1}{2}(v_0^2 - v^2) = \frac{GM}{R} - \frac{GM}{R \quad h}$$

But $GM = gR^2$

$$\therefore \frac{1}{2}(v_0^2 - v^2) \quad \frac{gR^2 h}{R(R \quad h)}$$

$$v_0^2 - v^2 \quad \frac{2gRh}{R \quad h} \quad \frac{2gh}{1 \quad \frac{h}{R}}$$

At maximum height $v = 0$.

The initial velocity $v_0 = (90\%)v_{escape} = 0.9\sqrt{2gR}$ then

$$(0.9\sqrt{2gR})^2 - 0 \quad \frac{2gh_{max}}{1 \quad \frac{h_{max}}{R}} \quad ;$$

$$0.81\ \mathrm{R} = \frac{h_{max}}{1 \quad \frac{h_{max}}{R}}$$

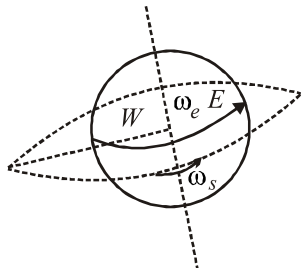
$0.81\ \mathrm{R} + 0.81\ h_{max} = h_{max}$

$0.19\ h_{max} = 0.81\ \mathrm{R}$

Maximum height reached by the rocket

$$h_{max} = \frac{0.81R}{0.19} \quad 4.26\ \mathrm{R}$$

11. (b, d) Satellite revolve in equatorial plane with same angular velocity as that of earth in magnitude and direction.



12. (a) Force on satellite is always towards earth, therefore, acceleration of satellite S is always directed towards centre of the earth. Net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about centre of earth is constant throughout. Since the force F is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.

13. (a, b) The escape velocity v_e of an object from a planet of radius R is obtained by setting the kinetic energy of the object near the surface equal to the gravitational potential energy difference between the surface and infinity. As the mass of the atoms and molecules m in this case is much smaller than the mass of the satellite, we don't need to use the reduced mass of the system in the calculation.

$$\frac{mv_e^2}{2} - \frac{GMm}{R} \quad 0$$

$$v_e \quad \left[\frac{2GM}{R}\right]^{1/2} \quad 2655\ \mathrm{m/s}$$

The speed of a gas molecule or atom follows the Maxwell-Boltzmann distribution. Their average speed, as a function of temperature is given by:

$$\frac{mv^2}{2} \quad \frac{3kT}{2}$$

where k is Boltzmann's constant and T the temperature in Kelvin. The following table gives the masses and average velocity for the gases in the question.

Gas	Mas (molar)	Mass (kg)	Average speed
Ammonia (NH_3)	17.031	2.83×10^{-26}	364 m/s
Methane (CH_4)	16.043	2.66×10^{-26}	385 m/s
Helium (He)	4.0026	6.65×10^{-27}	770 m/s
Hydrogen (H_2)	2.016	3.35×10^{-27}	1085 m/s
Hydrogen (H)	1.008	1.67×10^{-27}	1534 m/s

As the Maxwell-Boltzmann distribution has a long tail in the direction of higher speeds, even though the average speed is below the escape velocity, some molecules (atoms) of gas will escape. A rule-of-thumb among astronomers is that if the average speed is below 1/6 of the escape velocity, the gas will remain for the age of the solar system.

So ammonia and methane will remain on Titan, helium would for a shorter time but hydrogen (molecular or atomic) will escape more quickly.

E MATRIX-MATCH TYPE

1. **A-q; B-r; C-s; D-p**
2. **A-s; B-p; C-r; D-q**
3. **A-q, t; B-p, q, t; C-r; D-r, s**

Gravitational force due to planet or satellite is a central force, hence, torque = 0 ⇒ Angular momentum conservation

Kepler's 2nd law ⇒ Constant areal velocity.

In elliptical orbit as distance from focus changes, speed changes, hence KE, while in circular orbit speed remains same hence KE.

Escape velocity $= \sqrt{\frac{2\mathrm{GM}}{\mathrm{R}}}$,

where M = Mass of planet

Orbital velocity $= \sqrt{\frac{GM}{R}}$

4. A-q, r; B-t; C-q, s; D-p

(A) Kinetic energy of a body projected from the surface of earth at large distance may be zero (body momentarily comes to rest) or positive.

(B) Gravitational potential energy of a bound system must be negative.

(C) Change in potential energy of a point mass if left free to itself, with time may be zero (point mass on horizontal surface) or negative (point mass falling freely vertically).

(D) Change in areal velocity of earth as earth moves from apogee towards perigee is zero as areal velocity remains constant according to Kepler's second law.

5. A-p, t; B-r, t; C-p, q, t; D-r, t

Due to rotation

$g_{eff} = g_0 - \omega^2 R \cos^2\lambda$

$\therefore$ g changes with λ (altitude angle)

g_0 depends on earth mass

τ on earth = 0 $\Rightarrow$ Angular momentum constant

Escape velocity is independent on direction of projection but depends on earth mass and distance from earth centre.

Gravitational potential, $V = -\frac{GM}{r}$.

At particular point it is constant but depends on mass.

F NUMERIC/INTEGER ANSWER TYPE

1. 4

$g \quad \frac{GM}{R^2}$; If R decreases then g increases. Taking logarithm of both the sides;

$\log g = \log G + \log M - 2 \log R$

Differentiating it we get $\frac{dg}{g} = 0 + 0 - \frac{2\,dR}{R}$;

$\therefore \frac{dg}{g} \quad -2\left(\frac{-2}{100}\right) \quad \frac{4}{100}$

$\therefore$ % increase in $g = \frac{dg}{g} \times 100 = \frac{4}{100} \times 100 \quad$ **4%**

2. 4

$\left(\frac{T_A}{T_B}\right)^2 \quad \left(\frac{R_A}{R_B}\right)^3$ (Kepler law);

$\therefore \left(\frac{8T_B}{T_B}\right)^2 \quad \left(\frac{R_A}{R_B}\right)^3$

or $64 = \left(\frac{R_A}{R_B}\right)^3$

or $(4)^3 \quad \left(\frac{R_A}{R_B}\right)^3$

or $4 \quad \frac{R_A}{R_B}$

$\therefore$ $\mathbf{R_A = 4\,R_B}$

3. 65

When the earth's motion is suddenly stopped, it would fall into the sun and (suppose) it comes back. If the effect of temperature of sun is ignored, we can say that the earth would continue to move along a strongly extended flat ellipse whose extreme points are located at the earth's orbit and at the centre of the sun.

The semi major axis of such ellipse is $R/2$.

Now $\frac{T'^2}{T^2} \quad \left[\frac{R}{2}\right]^3 \left[\frac{1}{R^3}\right]$

Where T is the time period of normal orbit of earth.

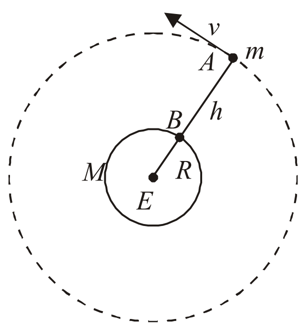
or $T'^2 \quad \frac{T^2}{8}$

or $T' \quad \frac{T}{2\sqrt{2}}$

Now, time required to fall into the sun,

$t \quad \frac{T'}{2} \quad \frac{T}{4\sqrt{2}} \quad \frac{365}{4\sqrt{2}} \approx$ **65** days

4. 7920



For the satellite in the circular orbit, we have

$$\frac{mv_s^{\,2}}{R \quad h} \qquad \frac{GMm}{R \quad h^{\,2}}$$

$$\Rightarrow \quad v_s^{\,2} \quad \frac{GM}{R \quad h}$$

$$\Rightarrow \quad \left(\frac{1}{2}\sqrt{\frac{2GM}{R}}\right)^2 \quad \frac{GM}{R \quad h} \quad \left(\because \; v_s \quad \frac{1}{2}v_e\right)$$

$$\Rightarrow \quad \frac{GM}{2R} \quad \frac{GM}{R \quad h}$$

$$\Rightarrow \quad R \quad h \quad 2R$$

$$\Rightarrow \quad h = R$$

When the satellite is stopped, its kinetic energy is zero. When it falls freely on to the Earth, its potential energy decreases and converts into kinetic energy.

$\therefore \quad (P.E.)_A - (P.E.)_B = K.E.$

$$\Rightarrow \quad \frac{-GMm}{2R} - \left(\frac{-GMm}{R}\right) \quad \frac{1}{2}mv^2$$

$$\Rightarrow \quad \text{v} = \sqrt{\frac{GM}{R}} \quad \sqrt{gR} \quad \sqrt{9.8 \times 6.4 \times 10^6}$$

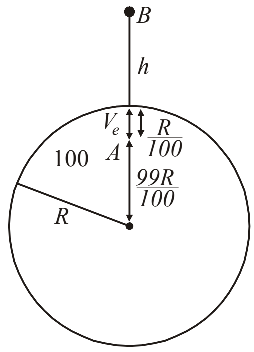
= **7920 m/s**

5. 99.5

Total energy at A = Total energy at B

$$(K.E.)_A + (P.E.)_A = (P.E.)_B$$

$$= \frac{1}{2}m \times \frac{2GM}{R} + \left[\frac{-GMm}{2R^3}\left\{3R^2 - \left(\frac{99R}{100}\right)^2\right\}\right] = -\frac{GMm}{R \quad h}$$



On solving, we get h = **99.5 *R*.**

$\therefore$ The maximum height attained by the body from the surface of the moon is 99.5 times the radius of the moon.

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