# 3 Predicate Logic

***Frege's... discovery of qualification, the deepest single technical advance ever made in logic.***

- ***Read the following argument.***

  All scientists are intelligent.

  All intelligents are creative.

  Therefore all scientists are creative.
- Is this argument valid?
- Test validity of this argument by using the method of truth table, shorter truth table, direct deductive proof. C. P and, I. P.
- What answer do you get?

## 3.1 Need for Predicate logic

The logic we have studied so far is known as propositional logic. The methods that we have studied in propositional logic like, Truth table, Shorter truth table, Direct deductive proof, C.P. and I.P. cannot decide or prove validity of all arguments. These methods can be used only for those arguments whose validity depends upon the ways in which simple statements are truth-functionally combined into compound statements. The branch of logic which deals with such type of arguments is called Propositional logic.

In Propositional logic a proposition is taken as one unit. It does not involve analysis of the proposition. It does not take into consideration how terms in the propositions are related. However there are certain types of arguments whose validity depends upon the inner logical structure of the non-compound statements it contains. Methods of propositional logic are not adequate in testing validity of such arguments. Let us take an example -

All singers are creative.

Mahesh is a singer.

Therefore, Mahesh is creative.

In propositional logic by using propositional constants one can symbolize the above argument as follows –

S

M / ∴C

It is obvious that the above given argument is valid but it cannot be proved to be valid by the methods of propositional logic. The method of truth table on the contrary shows that the argument is invalid. All the three statements involved in the argument are non-compound statements. The inner logical structure of these statements and the relation between the terms involved in the statements is important in deciding the validity of this argument. The relation between the class of singer and the class of creative people is stated in the first premise. It states that the class of singers is included in the class of creative people i.e. whoever is a singer is also creative. The second premise states that the individual Mahesh belongs to the class of singer and therefore in the conclusion it is validly inferred that Mahesh also belongs to the class of creative people. When the argument is symbolized in propositional logic as stated above the inner logical structure of the statements and the relation between the terms involved is not revealed. It is therefore necessary to symbolize the argument in such a way that the inner logical structure of the statements is revealed and then one can prove validity of such arguments. The branch of logic which deals with such types of arguments is known as Predicate logic or Predicate calculus.

Like propositional logic, in predicate logic a proposition is not taken as one unit. The propositions are analyzed and symbolized to reveal, how the terms in the propositions are related with each other. However, Predicate logic is not totally different from propositional logic. The methods and notations of propositional logic are used in predicate logic so far as they are applicable to the non-compound statements with which it deals. If a formula is valid in propositional logic, the corresponding formula in predicate logic will also be valid. Though predicate logic includes propositional logic and is based on it, predicate logic goes beyond propositional logic since it reveals the logical structure of the propositions and the relation between the different terms of the proposition.

*Can you recognize and state how the following non compound propositions differ from each other? How can we classify them?*

*Everything is beautiful.*

*Ashish is smart.*

*All birds have wings.*

*Some children are brilliant.*

*Nilesh is not tall.*

*No farmer is rich.*

*Nothing is permanent.*

*Some things change.*

*Some mobile phones are not expensive.*

*Some things are not attractive.*

## 3.2 Types of Propositions

The non compound propositions; whose inner logical structure is significant in proving validity of arguments in Predicate logic are of two types – (1) Singular propositions and (2) General propositions

### Singular Propositions :

Singular proposition makes an assertion about a particular/specific individual. **Singular Proposition states that an individual possesses or does not possess a certain property/ attribute (quality).** Thus we get two types of singular propositions, affirmative singular propositions and negative singular propositions. **Affirmative singular proposition states that an individual possesses a certain property,**

***For example :*** Sunita is a dancer.

Here 'Sunita' is a subject term and 'dancer' is a predicate term. **Negative singular proposition states that an individual does not possess a certain property,**

***For example :*** London is not an American city.

The word 'individual' here refers not only to persons but to anything like a city, a country, an animal or anything of which an attribute can be significantly predicated and the 'property'/ 'attribute' may be an adjective, a noun or even a verb. Following are some examples of singular propositions -

(1) Sahil is a good writer.

(2) This Dog is not a wild animal.

(3) Ashok is not a politician.

(4) Thames is not an Indian river.

(5) Nikita is an athlete.

### General Proposition :

**General propositions make an assertion about class/classes.** General propositions are broadly classified into two types – (1) General propositions making an assertion about one class and (2) General propositions making an assertion about two classes or giving relation between two classes. Each type is further classified into Universal and Particular (Existential) general proposition. Universal general proposition makes an assertion about all members of a class where as a particular general proposition makes an assertion about some members of a class. Universal general proposition can be either affirmative or negative. Similarly particular/existential general proposition can also be either affirmative or negative. Thus altogether we get eight types of general propositions as given below.

**General propositions**

| One class | Two classes |
|---|---|
| (1) Universal affirmative<br>e.g. Everything is interesting | (1) Universal affirmative ( A proposition)<br>e.g. All fruits are sweet |
| (2) Universal negative<br>e.g. Nothing is useless | (2) Universal negative ( E proposition)<br>e.g. No living being is immortal |
| (3) Existential affirmative<br>e.g. Some things are beautiful | (3) Particular affirmative ( I proposition)<br>e.g. Some children are creative |
| (4) Existential negative<br>e.g. Some things are not clean | (4) Particular negative ( O proposition)<br>e.g. Some cities are not crowed |

## 3.3 Symbolization of singular and general propositions

**Symbolizing singular propositions :**

The two important components of any singular propositions are – (1) Name of an individual (2) Property / Attribute. Two different symbols are used for symbolizing these components namely Individual constant and Predicate constant. **An Individual constant is a symbol which stand for the name of an individual.** Small letters of English alphabet 'a' to 'w' are used as individual constants. **Predicate constant is a symbol which stands for the particular property/attribute.** Capital letters of English alphabet 'A' to 'Z' are used as predicate constants. While symbolizing a singular proposition, the symbol for the property is written to the left of the symbol for the name of an individual

***For example :*** the singular proposition, 'Suraj is wise' is symbolized as 'Ws', here 'W' stands for the attribute 'wise' and 's' stands for the name of an individual i.e. Suraj. A negative singular proposition is symbolized by placing '∼' before the statement,

***For example :*** the statement 'Makarand is not cunning', is symbolized as '∼ Cm'.

While symbolizing it is necessary to follow the same two restrictions which we follow while symbolizing propositions in propositional logic namely:

(1) The same individual constant should be used for symbolizing the name of an individual if it occurs again in the same argument or proposition. Similarly the same predicate constant should be used for symbolizing the name of property if it occurs again in the same argument or proposition.

(2) In the same argument or proposition, different individual constants and predicate constants should be used for different names of individual and property respectively.

Before we learn symbolization of general propositions it is necessary to learn about two more important symbols used in predicate logic i.e. Individual variable and Predicate variable. **Individual variable is a symbol which stands for any individual whatsoever.**

Individual variable does not stand for any specific individual. It is only a **place marker** which marks the place of an individual. It can be replaced by a proper name of an individual or by an individual constant. The small letters 'x', 'y', 'z' of English alphabet are used as individual variables. ***For example,*** the proposition 'Mohini is beautiful' is about the specific individual. But in place of the name of a particular individual i.e. Mohini if we leave a blank space keeping the rest of the statement same, we shall get the expression – '---------------- is beautiful'. The blank space here is just a place marker that marks the place of an individual, so in place of blank space we can use individual variable 'x' and we will get the expression – 'x is beautiful' which can be symbolized as 'Bx'. Similarly **Predicate variable is a symbol which stands for any property/attribute whatsoever.** It can be replaced by any name of property or predicate constant. The Greek letters $\phi$ (phi) and $\psi$ (psi) are used as predicate variables. ***For example,*** in the expression Surekha is -----, blank space marks the place of some property, where we can use predicate variable say '$\phi$' and we will get an expression - 'Surekha is '$\phi$', which can be symbolized as '$\phi$s'. In predicate logic such expressions are called Propositional function. We shall learn in detail about the concept of propositional function later in the chapter.

**Symbolize the following singular propositions :**

(1) Nilesh is a singer.

(2) John is an engineer.

(3) Ramesh is not a science student.

(4) Hemangi is smart and Hemangi is creative.

(5) Zarin is beautiful.

(6) Amit is an actor but Amit is not a dancer.

(7) Neena is Indian or Neena is American.

(8) New york is not an Australian city.

## Symbolizing General propositions :

As stated earlier, general propositions are broadly classified into two types – (1) General propositions making an assertion about one class and (2) General propositions making an assertion about two classes or giving relation between two classes. Let us first learn to symbolize general propositions making an assertion about one class.

### (I) Symbolizing General propositions about one class

General propositions can either be universal or existential. These two types are further classified into affirmative and negative propositions. Thus we get four types of general propositions about one class and they are symbolized as stated below.

#### (1) Universal affirmative proposition :

The proposition 'Everything is perishable', for instance, is of this type. To symbolize this proposition let us first convert it into logical terminology. This proposition affirms the property 'perishable' of everything. In the logical terminology it can be expressed as follows -

Given anything, it is perishable

The expressions 'anything' and 'it' stand for any individual whatsoever. So we shall use individual variable in place of these words as follows –

Given any x, x is perishable.

In logic the expression 'Given any x' is customarily symbolized by the symbol '(x)'. This symbol is called 'Universal quantifier'. By using predicate constant 'P', 'x is perishable' can be symbolized as 'Px'. Accordingly the whole statement will be symbolized as –

(x) Px

The statement is to be read as, 'Given any x, x is perishable'. If we replace predicate constant 'P' by predicate variable then we get the form of such type of statements as given below –

(x) $\phi$x

### (2) Universal negative proposition :

The Proposition 'Nothing is everlasting' is of this type. The property 'everlasting' is denied of all things. In logical terminology the statement may be expressed as –

Given anything, it is not everlasting.

By using individual variables instead of the expressions 'thing' and 'it' we rewrite the statement as –

Given any x, x is not everlasting.

By using universal quantifier, predicate constant 'E' and the symbol for negation, we symbolize the whole statement as follows –

$(x) \sim Ex$

The form of such type of propositions is – $(x) \sim \phi x$

### (3) Existential affirmative proposition :

The below given statements are of this type.

(1) Something is beautiful.

(2) Dogs exist.

The first proposition affirms the property 'beautiful' of some things. In logic the expression 'some' means at least one. Accordingly the statement can be expressed in logical terminology as follows –

There is at least one thing such that, it is beautiful.

By using individual variable in place of 'thing' and 'it', the statement can be rewritten as –

There is at least one x such that, x is beautiful.

The symbol '$(\exists x)$' is used for the expression. 'there is at least one x such that'. The symbol is called 'Existential quantifier'. By using existential quantifier and predicate constant 'B' for the property 'beautiful' we symbolize the whole statement as given below –

$(\exists x)\ Bx$

This is to be read as –

'There is at least one x such that x is beautiful.' The form of such type of statement is – $(\exists x)\ \phi x$

The second statement, 'Dogs exist' affirms the existence of at least one dog. The statement can be expressed in logical terminology as follows –

There is at least one thing such that, it is a dog.

By using individual variable the statement can be rewritten as –

There is at least one x such that, x is a dog.

By using existential quantifier and predicate constant 'D' we symbolize the whole statement as given below –

$(\exists x)\ Dx$

This it to be read as –

'There is at least one x such that, x is a dog.' The form of such type of statement is - $(\exists x)\ \phi x$

### (4) Existential negative proposition :

The following statements are of this type.

(1) Something is not good.

(2) There are no giants.

The first proposition denies the property 'good' of some things. It states that there is at least one thing which is not good. The statement can be expressed in logical terminology as follows –

There is at least one thing such that it is not good.

By using individual variable the statement can be rewritten as –

There is at least one x such that, x is not good.

By using existential quantifier and predicate constant 'G' for the property 'good' we symbolize the whole statement as given below –

$(\exists x) \sim Gx$

This is to be read as –

'There is at least one x such that x is not good.' The form of such type of statement is - $(\exists x) \sim \phi x$

The second proposition 'There are no giants' denies existence of giants. 'Existence' is not a property/attribute. So the statement cannot be translated in logical terminology as the first statement. The proposition states that there is not even one giant. The correct translation of the statement in logical terminology is as given below –

It is not the case that, there is at least one x such that, x is a giant. This correctly expresses the statement's meaning that there is not even one giant.

By using the symbol for negation, existential quantifier and predicate constant 'G' we can symbolize the whole statement as –

$\sim(\exists x)\ Gx$

This is to be read as –

'It is not the case that, there is at least one x such that, x is a giant'. The form of such type of statement is - $\sim(\exists x)\ \phi x$

## (II) Symbolizing General propositions about two classes

General propositions about two classes are also of four types namely –

(1) Universal affirmative or 'A' proposition.

(2) Universal negative or 'E' proposition.

(3) Particular affirmative or 'I' proposition.

(4) Particular negative or 'O' proposition.

Let's symbolize such types of proposition.

### (1) Universal affirmative or 'A' proposition:

The proposition 'All women are attractive', for example is of this kind. This proposition states the relation between two classes namely – the class of 'women' and the class of 'attractive'. It is a universal affirmative proposition because in this proposition the property 'attractive' is affirmed of all women. This statement is expressed in logical terminology as given below -

Given anything, if it is a woman then it is attractive.

The terms 'thing' and 'it' stand for any individual whatsoever. So we can replace them by individual variable say 'x'. Accordingly the statement can be rewritten as –

Given any x, if x is a woman then x is attractive. By using the symbol universal quantifier for the expression 'Given any x', predicate constant 'W' for 'woman', 'A' for 'attractive' and the connective '$\supset$' we symbolize the whole proposition as follows –

$(x)\ (Wx \supset Ax)$

By replacing predicate constants by predicate variables we can get the form of such type of propositions as --- $(x)\ (\phi x \supset \psi x)$

### (2) Universal negative or 'E' proposition :

The proposition 'No child is wicked' is an example of Universal negative or 'E' proposition. This proposition states the relation between two classes namely – the class of 'children' and the class of 'wicked'. It is a Universal negative proposition because here the property 'wicked' is denied of all children. In logical terminology this statement may be expressed as –

Given anything, if it is a child then it is not wicked.

By using individual variable instead of 'thing' and 'it', we express this statement as –

Given any x, if x is a child then x is not wicked.

By using universal quantifier, predicate constants and the connective '⊃', the whole statement is symbolized as follows –

$(x)\,(Cx \supset \sim Wx)$

The form of 'E' proposition is – $(x)\,(\phi x \supset \sim \psi x)$

### (3) Particular affirmative or 'I' proposition:

In particular affirmative or 'I' proposition a property is affirmed of some members of a class. The proposition 'Some men are rich', for example, is a particular affirmative or 'I' proposition. This proposition states the relation between two classes namely – the class of 'men' and the class of 'rich'. It is a particular affirmative proposition as the property 'rich' is affirmed of some members of the class of 'men'. This proposition can be stated in logical terminology as –

There is at least one thing such that, It is a man and it is rich.

The statement can be expressed by using individual variables as follows –

There is at least one x such that, x is a man and x is rich.

The whole statement is symbolized as follows by using existential quantifier, predicate constants and the symbol for connective 'and'.

$(\exists x)\,(Mx \cdot Rx)$

The form of 'I' proposition is $(\exists x)\,(\phi x \cdot \psi x)$

### (4) Particular negative or 'O' proposition :

The proposition 'Some animal are not wild', for instance is an 'O' proposition. This proposition states the relation between two classes namely – the class of 'animals' and the class of 'wild'. It is a particular negative proposition as the property 'wild' is denied of some members of the class of 'animals'. This proposition can be translated in logical terminology by using individual variable as follows :

There is at least one x such that, x is an animal and x is not wild

The whole statement is symbolized as follows by using existential quantifier, predicate constants and the symbols for connective 'and' and 'not'

$(\exists x)\,(Ax \cdot \sim Wx)$

The form of 'O' proposition is -- $(\exists x)\,(\phi x \cdot \sim \psi x)$

General propositions do not always use the expressions – 'All', 'No' and 'Some'. Apart from these words there are many other words in English language which express these propositions. Some common expressions in English language which indicate these types of propositions are given in the following table.

**'A' proposition :** Affirmative sentences with words 'all', 'every', 'each', 'any', 'always', 'whatever', 'invariable', 'necessarily', 'absolutely'

**'E' proposition :** Statements with words 'no', 'never', 'not at all', 'not a single', 'not even one', 'none'

**'I' proposition :** Affirmative statements with words 'most', 'many', 'a few', 'certain', 'all most all', 'several', 'mostly', 'generally', 'frequently', 'often', 'perhaps', 'nearly always', 'sometimes', 'occasional'

Negative statements with 'few', 'seldom', 'hardly', 'scarcely', 'rarely'

**'O' proposition :** When affirmative statements which contain words indicating 'I' proposition are denied we get 'O' proposition.

Affirmative statements with the word 'few', 'seldom', 'hardly', 'scarcely', 'rarely'

When 'A' proposition is denied we get 'O' proposition.

*Give examples of affirmative and negative singular proposition and symbolize them.*

*Give examples of all eight types of general propositions and symbolize them.*

### Propositional Function

Propositional function is an important concept in predicate logic. 'Deepa is an artist' and 'Suresh is a sportsman', are propositions. They are either true or false. However the expressions, 'x is an artist' or 'Ax' and 'Suresh is $\phi$' or '$\phi$s'are not propositions as they are neither true nor false. Such expressions are called Propositional functions. **A propositional function is defined as an expression which contains at least one (free/real) variable and becomes a proposition when the variable is replaced by a suitable constant.**

**Free variable is one which falls beyond the scope of a quantifier. It is neither a part of a quantifier nor preceded by an appropriate quantifier.**

**Bound variable is one which is a part of a quantifier or preceded by an appropriate quantifier.** For example, 'Everything is expensive' is symbolized as – (x) (Ex). This is a proposition and not a propositional function as both the variables occurring in the expression are not free but bound. In '(x)' variable 'x' is a part of the quantifier and in 'Ex'; 'x' is preceded by an appropriate quantifier. The expression, '(y) (Dx)' however is a propositional function because though the 'y' being part of the quantifier is a bound variable, 'x' in the expression is free variable as it is neither a part of a quantifier nor preceded by an appropriate quantifier. Similarly following expressions are also propositional functions – 'Bx', Mx, $\psi$x or '$\phi$x' here both the variables 'x' and '$\phi$' are free/real.

Propositional function may be either simple or complex. Simple propositional function is one which does not contain propositional connectives. For example –

(1) x is big. (Bx)

(2) y is smart (Sy)

(3) Mukund is $\phi$ ($\phi$m)

Propositional functions which contain propositional connectives are called complex propositional functions. For example –

(1) x is not a philosopher. – ($\sim$ Px)

(2) x is a doctor and x is a social worker. (Dx $\cdot$ Sx)

(3) x is either an actor or x is a dancer. (Ax $\vee$ Dx)

(4) If x is a man then x is rational. (Mx $\supset$ Rx)

**Distinction between Proposition and propositional function**

| Proposition | Propositional function |
|---|---|
| (1) A proposition does not contain any free variable. | (1) A propositional function contains at least one free variable. |
| (2) A proposition has a definite truth value it is either true or false. | (2) It is neither true nor false. |
| (3) A proposition can be interpreted. | (3) A propositional function cannot be interpreted. |
| (4) e.g. Akash is handsome - Ha | (4) e.g. x is handsome - Hx |

*Can you recognize which of the following expressions are propositions and which are propositional function?*

| | | | |
|---|---|---|---|
| *(1)* | $Cx$ | *(7)* | $Ta \cdot Fa$ |
| *(2)* | $Ma \supset Sa$ | *(8)* | $\phi s$ |
| *(3)* | $(x)\ (Fx \supset Ny)$ | *(9)* | $(x)\ (Gx \supset \sim Kx)$ |
| *(4)* | $(z)\ (Az \supset \sim Tz)$ | *(10)* | $(x)\ (Rx \supset Px)$ |
| *(5)* | $(x)\ (Ay \supset \sim Wx)$ | *(11)* | $Rx \supset Px$ |
| *(6)* | $By \cdot \sim Hx$ | *(12)* | $Ms \vee Kd$ |

## 3.4 Methods of obtaining propositions from propositional function –

In the last section we learned that a propositional function is an expression which contains at least one (free/real) variable and becomes a proposition when the variable is replaced by a suitable constant. Thus one can obtain propositions from propositional functions by replacing variables by suitable constants. As there are two types of propositions namely singular and general propositions, there are two ways of obtaining propositions from propositional functions. (1) Instantiation (2) Quantification

### (1) Instantiation

**The process of obtaining singular propositions from a propositional function by substituting a constant for a variable is called Instantiation.** For instance, 'x is a logician'/ 'Lx', is a propositional function. From this propositional function by replacing an individual variable 'x' with the proper name of an individual eg 'Aristotle' or with a symbol for the proper name(i.e. an individual constant) say 'a', we can obtain a singular proposition as follows- 'Aristotle is a logician'/ 'La'.

Individual variable 'x' can be replaced by any name of an individual or by an individual constant. By replacing 'x' by 'Newton' / 'n', we shall get a singular proposition as—'Newton is a logician'/ 'Ln'. Each singular proposition obtained from a propositional function in this manner is a substitution instance of that propositional function. A propositional function is neither true nor false; however every substitution instance of it is either true or false. The first singular proposition, 'Aristotle is a logician;, is true whereas the second proposition; 'Newton is a logician', is false.

A propositional function is either simple or complex. In case of a complex propositional function, the substitution instances obtained are truth- functions of singular propositions. For example 'x is a dancer and x is an engineer'/ ($Dx \cdot Ex$ )is a complex propositional function. By replacing 'x' by proper name eg Ketan or individual constant 'k' we get a substitution instance which is a truth- function of a singular propositions as follows –

'Ketan is a dancer and Ketan is an engineer' / ( $Dk \cdot Ek$ )

### (2) Quantification or Generalization

The process used to obtain general propositions from a propositional function is called Quantification or Generalization. **Quantification or Generalization is a process of obtaining a general proposition from a propositional function by placing an Universal or Existential quantifier before the propositional function.** As there are two types of general propositions, quantification is of two types. (1) Universal Quantification/ generalization. (2) Existential Quantification/ generalization.

The Process of universal quantification / generalisation is used to obtain a universal general proposition from a propositional function whereas existential general propositions are obtained by the process of Existential Quantification/ generalization from a propositional function.

**(1) Universal Quantification / generalization :**

**The process of Universal Quantification consists in obtaining an universal general proposition by placing an universal quantifier before the propositional function.** For example the expression 'x is 'gorgeous' or 'Gx' is a propositional function. Here the property 'gorgeous' is asserted of an individual variable 'x'. If we assert this property of all x then we shall get an universal general proposition as follows –

'Given any x, x is 'gorgeous'

(x) Gx

Universal general proposition thus obtained may be either true or false. **The universal quantification of a propositional function is true if and only if all its substitution instances are true.**

**(2) Existential Quantification / generalization :**

**The process of Existential Quantification consists in obtaining an existential general proposition by placing an existential quantifier before the propositional function.** For example in propositional function – 'x is noble' or 'Nx', the property 'noble' is asserted of an individual variable 'x'. by asserting this property of some 'x' we can obtain existential general proposition as given below –

'There is at least one x such that, x is noble'

(∃x) Nx

Existential general propositions obtained by the process of Existential Quantification may be true or false. The existential quantification of a propositional function is true even if one of its substitution instance is true.

## 3.5 Quantificational Deduction

After having learned how to symbolize non compound propositions i.e. singular and general propositions, one can symbolize the arguments which contain such non compound propositions and prove their validity. The method used to prove validity of such arguments is called Quantificational Deduction.

Like Deductive Proof, the Quantificational Deduction consists in deducing the conclusion of an argument with the help of certain rules. The difference between the two is that in case of the Quantificational Deduction, along with 19 rules of inference we require four more rules of quantificational deduction. This is because symbolization of arguments containing non compound propositions involves use of propositional functions and quantifiers; hence their validity cannot be proved by 19 rules of inference only.

The four rules of quantificational deduction are :

(1) Universal Instantiation (UI)

(2) Universal Generalization (UG)

(3) Existential Generalization (EG)

(4) Existential Instantiation (EI)

These rules are necessary since quantifiers are used while symbolizing general propositions. The rules of UI and EI are used to infer truth functional compound statements from general propositions. Once they are changed into truth functional compound statements, we can apply 19 rules of inference to derive the conclusion. The rules of UG and EG are used for inferring general propositions from truth functional compound statements.

**Rules of Quantification (Primary version)**

**(1) The rule of Universal Instantiation (UI)**

The rule of Universal Instantiation (UI) enables us to obtain truth functional compound statement from universal general proposition. This rule is based on the nature of universal general proposition. As the universal quantification of a propositional function is true if and only if all its substitution instances are true, the rule of UI states that, any substitution instance of a propositional function can be validly inferred from its universal quantification. In simple words it means, what is true of all members of a class is true of each member of that class. The symbolic representation of the rule is -

$(x)\ (\phi x)$

$\therefore \phi \nu$

(Where '$\nu$' is any individual symbol)

The rule of UI allows us to derive two types of inferences. The Greek letter '$\nu$' (nu) in rule, may stand for either a specific / particular individual (individual constant) or an arbitrarily selected individual. From the fact that what is true of all members of a class is true of each member of that class, it follows that this member can either be a specific member or an arbitrarily selected individual. ***For example,*** from the universal general proposition, 'everything is beautiful', one can infer a proposition about specific individual eg, 'Rita is beautiful' or may infer that any arbitrarily selected individual is beautiful. The symbol 'y' is used for an arbitrarily selected individual and a particular individual is symbolizes by individual constant. Accordingly symbolic representations of these two inferences are as given below –

(1) $(x)\ (\phi x)$ (2) $(x)\ (\phi x)$

$\therefore$ Br $\therefore$ By

Let us now take the argument, we had taken in the beginning of the chapter and construct formal proof of validity for it by using the rule UI

All singers are creative.

Mahesh is a singer.

Therefore, Mahesh is creative.

We first symbolize the argument as follows:

(1) $(x)\ (Sx \supset Cx)$

(2) Sm / $\therefore$ Cm

Now we can apply the rule of U I to the first premise –

(1) $(x)\ (Sx \supset Cx)$

(2) Sm / $\therefore$ Cm

(3) $Sm \supset Cm$ 1, U I

After inferring truth functional compound statement from general statement, by rule of UI rules of inference can be applied. By applying the rule of M.P. to the statement 3 and 2 we can infer the conclusion. Thus the validity of the argument is proved.

(1) $(x)\ (Sx \supset Cx)$

(2) Sm / $\therefore$ Cm

(3) $Sm \supset Cm$ 1, U I

(4) Cm 3,2 M.P.

While applying the rule of UI one has an option of taking any individual constant or arbitrarily selected individual – 'y'. From the nature of premises and the conclusion one can decide whether to take an individual constant or 'y'. in the above example the conclusion and the second premise is about specific individual Mahesh (m) so we used the same individual constant, which enabled us to apply rule of M.P. to derive the conclusion, which would not have been possible if we had used 'y' or any other constant other than 'm'.

### (2) Universal Generalization (UG)

The rule of Universal Generalization (UG) allows us to derive a universal general proposition from a truth functional compound statement. One can validly infer that what is true of all members of a class is true of each member of that class but one cannot in the same fashion say that what is true of a specific individual of a class is true of all the members of that class. For instance, we cannot say that Aurobindo is a philosopher therefore all men are philosophers. However one can say that, what is true of a man in general (i.e. without considering any specific qualities) is true of all men. To take an example, one can validly infer that a man is rational therefore all men are rational. From this it follows that, from statement which is about an arbitrarily selected individual one can infer a universal general statement. So the rule of UG is stated as follows –

**Universal quantification of a propositional function can be validly inferred from its substitution instance which is an arbitrarily selected individual.** The symbolic representation of the rule is –

$\phi y$

$\therefore$ (x) ($\phi$x)

(where 'y' denotes any arbitrarily selected individual.)

Let us now construct formal proof of validity for the following argument by using both the rules of UI and UG.

All men are honest.

All honest people are good.

Therefore, all men are good.

Let us first symbolize the argument as follows –

(1) (x) (Mx ⊃ Hx)

(2) (x) (Hx ⊃ Gx) /∴ (x) (Mx ⊃ Gx)

Next step is to apply the rule of UI to step no.1 and 2 then derive the conclusion by the rule of H.S and apply the rule of UG to step 5 to get the conclusion as shown below. While applying UI it is necessary to take 'y' in the place of 'x' because the conclusion is a universal general proposition and to get conclusion we will have to use the rule of UG at the end, which is possible only if we take 'y'

| | | |
|---|---|---|
| (1) | (x) (Mx ⊃ Hx) | |
| (2) | (x) (Hx ⊃ Gx) | /∴ (x) (Mx ⊃ Gx) |
| (3) | My ⊃ Hy | 1, UI |
| (4) | Hy ⊃ Gy | 2, UI |
| (5) | My ⊃ Gy | 3, 4, H.S |
| (6) | (x) (Mx ⊃ Gx) | 5, UG |

## (3) Existential Generalization (EG)

The rule of EG is used to get an existential general proposition from a truth functional compound statement. Existential general proposition makes an assertion about some members of a class. The term 'some', means 'at least one' in logic. So unlike the rule of UG, in case of the rule of EG one can validly infer that, what is true of a specific individual of a class is true of some individuals of that class. One can also infer existential general proposition from a statement about an arbitrarily selected individual. The rule of EG is stated as follows –

**The existential quantification of a propositional function can be validly inferred from any of its substitution instance.** The symbolic form of the rule is –

$\phi \nu$

$\therefore$ (∃x) ($\phi$x)

(Where '$\nu$' is any individual symbol)

To take an example we can infer a proposition, 'some men are handsome' from a statement about specific individual eg, 'Anil is handsome' or about an arbitrarily selected individual. These may be symbolically expressed as follows –

(1) Ha (2) Hy

∴ (∃x) (Hx) ∴ (∃x) (Hx)

Let us construct formal proof of validity for the following argument.

| | | |
|---|---|---|
| (1) | (x) (Dx ⊃ Ax) | |
| (2) | (x) (Dx) | / ∴ (∃x) (Ax) |
| (3) | Da ⊃ Aa | 1, UI |
| (4) | Da | 2, UI |
| (5) | Aa | 3, 4, M.P. |
| (6) | (∃x) (Ax) | 5, EG |

We can also construct a formal proof of validity for this argument by using 'y' in place 'a' as follows -

| | | | |
|---|---|---|---|
| (1) | (x) (Dx ⊃ Ax) | | |
| (2) | (x) (Dx) | / ∴ (∃x) (Ax) | |
| (3) | Dy ⊃ Ay | 1, UI | |
| (4) | Dy | 2, UI | |
| (5) | Ay | 3, 4, M.P. | |
| (6) | (∃x) (Ax) | 5, EG | |

### (4) Existential Instantiation (EI)

**The rule of Existential Instantiation states that from the existential quantification of a proposition function we may infer the truth of its substitution instance.** The rule enables us to infer a truth functional compound statement from an existential general proposition.

Existential quantification of a propositional function is true only if it has at least one true substitution instance. As what is true of some members of a class cannot be true of any arbitrarily selected individual of that class, the substitution instance cannot be an arbitrarily selected individual. From the statement 'some men are caring', one cannot infer that any arbitrarily selected man is caring. The truth functional statement that we drive can be about a particular individual only, but we may not know anything else about that person. So while applying the rule of EI one must take that individual constant which has not occurred earlier in the context. The symbolic form of this rule is as given below –

(∃x) ($\phi$x)

∴ $\phi \nu$

(Where '$\nu$' is an individual constant, other than 'y', that has not occurred earlier in the context.)

Let us take an example –

| | | |
|---|---|---|
| (1) | (x) (Bx ⊃ ∼ Px) | |
| (2) | (∃x) (Px · Tx) | / ∴ (∃x) (∼Bx) |
| (3) | Pa · Ta | 2, EI |
| (4) | Ba⊃ ∼ Pa | 1, UI |
| (5) | Pa | 3, Simp. |
| (6) | ∼ ∼ Pa | 5, D.N. |
| (7) | ∼ Ba | 4, 6, M.T. |
| (8) | (∃x) (∼ Bx) | 7, EG |

**The important point one needs to remember here is that, when in an argument, one has to use both rule of UI and EI, the rule of EI should be used first.** This is because for use of EI there is a restriction that, only that individual constant should be used which has not occurred earlier in the context. In the above argument if UI was used first, then while applying EI we could not have taken the same individual constant and with different constants we could not have arrived at the conclusion.

Let us take some more examples –

| | | | |
|---|---|---|---|
| (I) | (1) | (x) (Mx ⊃ Px) | |
| | (2) | (x) (Px ⊃ Tx) | |
| | (3) | Md | / ∴ (∃x) (Tx) |
| | (4) | Md ⊃ Pd | 1, UI |
| | (5) | Pd ⊃ Td | 2,UI |
| | (6) | Md ⊃ Td | 4,5, H.S. |
| | (7) | Td | 6,3, M.P. |
| | (8) | (∃x) (Tx) | 7, EG |

(II)

| | | |
|---|---|---|
| (1) | $(x)(Bx \supset Px)$ | |
| (2) | $(\exists x)(Bx \cdot Tx)$ | |
| (3) | $Bd$ | $/ \therefore (\exists x)(Px \cdot Tx)$ |
| (4) | $Ba \cdot Ta$ | 2, EI |
| (5) | $Ba \supset Pa$ | 1, UI |
| (6) | $Ba$ | 4, Simp. |
| (7) | $Pa$ | 5, 6, M.P. |
| (8) | $Ta \cdot Ba$ | 4, Com. |
| (9) | $Ta$ | 8, Simp. |
| (10) | $Pa \cdot Ta$ | 7, 9, Conj. |
| (11) | $(\exists x)(Px \cdot Tx)$ | 10, EG |

(III)

| | | |
|---|---|---|
| (1) | $(x)(Tx \supset Nx)$ | |
| (2) | $(x)(Nx \supset Bx)$ | |
| (3) | $(x)(Bx \supset \sim Ax)$ | |
| (4) | $(\exists x)(Px \cdot Tx)$ | $/ \therefore (\exists x)(Px \cdot \sim Ax)$ |
| (5) | $Pa \cdot Ta$ | 4, EI |
| (6) | $Ta \supset Na$ | 1,UI |
| (7) | $Na \supset Ba$ | 2,UI |
| (8) | $Ba \supset \sim Aa$ | 3, UI |
| (9) | $Ta \supset Ba$ | 6, 7 H.S. |
| (10) | $Ta \supset \sim Aa$ | 9, 8, H.S. |
| (11) | $Pa$ | 5, Simp. |
| (12) | $Ta \cdot Pa$ | 5, Com. |
| (13) | $Ta$ | 12, Simp. |
| (14) | $\sim Aa$ | 10, 13, M.P. |
| (15) | $Pa \cdot \sim Aa$ | 11, 14, Conj. |
| (16) | $(\exists x)(Px \cdot \sim Ax)$ | 15, EG |

**Summary**

- In Propositional logic a proposition is taken as one unit. It does not involve analysis of proposition.
- Predicate logic involves analysis of proposition. It deals with certain types of arguments whose validity depends upon the inner logical structure of the non-compound statements it contains.
- The non compound statements in Predicate logic are of two types – Singular propositions and General propositions.
- Singular propositions states that an individual possesses or does not possess a certain property/ attribute (quality).
- Singular propositions are of two types – affirmative singular propositions and negative singular propositions
- General propositions make an assertion about class.
- General propositions are classified into two types – (1) General propositions about one class and (2) General propositions about two classes.
- Each type is further classified in to Universal affirmative, Universal Negative, Particular (Existential) affirmative, Particular (Existential) Negative.

- A propositional function is defined as an expression which contains at least one (free/real) variable and becomes a proposition when the variable is replaced by a suitable constant.
- The process of obtaining a singular proposition from a propositional function by substituting a constant for a variable is called Instantiation.
- Quantification and Generalization is a process of obtaining a general proposition from a propositional function by placing a universal or Existential quantifier before the propositional function.
- Quantification is of two types. (1) Universal Quantification/ generalization. (2) Existential Quantification/generalization
- The Quantificational Deduction consists in deducing the conclusion of an argument from its premises with the help of certain rules.
- Rules of quantificational deduction are – (1) Universal Instantiation (U I), (2) Universal Generalization (U G), (3) Existential Generalization (E G), (4) Existential Instantiation (E I)
- The rules of UI and EI are used to infer truth functional compound statements from general propositions.
- The rules of UG and EG are used for inferring general propositions from truth functional compound statements.

## Exercises

**Q. 1. Fill in the blanks with suitable words from those given in the brackets :**

(1) ……….is an individual variable. *($\psi$, x)*

(2) ……….is a predicate variable. *(A, $\phi$)*

(3) Individual ………. stands for a specific individual. *(Constant, Variable)*

(4) The process of ………. helps to derive singular proposition. *(Quantification, Instantiation)*

(5) General propositions are obtained by the process of ………. . *(Instantiation, Generalization)*

(6) A ………. is neither true nor false. *(Propositional function, Proposition)*

(7) A predicate constant stands for ………. property. *(any, specific)*

(8) An individual variable stands for ………. proposition. *(specific, any)*

(9) ………. proposition is Universal Negative proposition. *(E, O)*

(10) ………. is a Universal Quantifier.

*[(x), ($\exists$x)]*

(11) ………. is either true or false. *(Proposition/ propositional function)*

(12) The expression 'Given anything' is an ………. Quantifier. *(Existential/ Universal)*

(13) In ………. logic proposition is taken as one unit. *(propositional/predicate)*

(14) Propositions are analyzed in ………. logic. *(propositional/predicate)*

(15) ………. Propositional states that an individual possesses or does not possess a certain property/ attribute. *(singular/ general)*

**Q. 2. State whether the following statements are true or false.**

(1) The expression 'Given anything' is an Existential Quantifier.

(2) A singular proposition can be obtained from a propositional function by the process of Instantiation.

(3) A general proposition can be obtained from a propositional function by the process of Quantification.

(4) The rule of UG says that what is true of the whole class is true of each member of the class.

(5) The rule of EG says that what is true of an arbitrary object is true of all the members of a class.

(6) The rule of EG says that an Existential Quantification of a propositional function can be validly inferred from its substitution instance.

(7) ($\phi$) is a universal Quantifier.

(8) In the formal proof of validity by quantificational deduction, if both the rule UI and EI are to be used then E.I. should be used first.

(9) The rules of UI and EI are used to drop quantifiers from general propositions.

(10) The rules of UG and EG are used for inferring general propositions from truth functional compound propositions.

(11) In predicate logic proposition is taken as one unit.

(12) Singular propositions make an assertion about class.

(13) Proportional function contains at least one bound variable.

(14) Singular proposition states that an individual possesses or does not possess a certain property/attribute.

**Q. 3. Match the columns :**

| | (A) | (B) |
|---|---|---|
| (1) | Proposition | (a) a |
| (2) | Propositional function | (b) (x) Sx |
| (3) | Individual variable | (c) B |
| (4) | Predicate constant | (d) x |
| (5) | Universal quantifier | (e) Hx |
| (6) | Individual constant | (f) (x) |

**Q. 4. Give logical terms :**

(1) Branch of logic in which proposition is taken as one unit.

(2) Branch of logic that involves analysis of proportion.

(3) Proposition which states that an individual possesses or does not possess a certain property/attribute.

(4) Proposition which makes an assertion about class.

(5) An expression which contains at least one (free/real) variable and becomes a proposition when the variable is replaced by a suitable constant.

(6) The process of obtaining a singular proposition from a propositional function by substituting a constant for a variable.

(7) The process of obtaining a general proposition from a propositional function by placing a universal or Existential quantifier before the propositional function.

(8) The symbol which stand for the name of an individual.

(9) The symbol which stands for a particular property/attribute.

(10) The symbol which stands for any individual whatsoever.

(11) The symbol which stands for name of any property/attribute whatsoever.

(12) The variable which is neither a part of a quantifier nor preceded by an appropriate quantifier.

(13) The variable which is either a part of a quantifier or preceded by an appropriate quantifier.

**Q. 5. Give reasons for the following.**

(1) When both U.I. and E.I. are used in a proof, E.I. should be used first.

(2) The rule of U.G. allows us to infer universal general proposition only from an arbitrarily selected individual.

(3) One cannot derive a statement about an arbitrarily selected individual from an existential general proposition while using the rule of E.I.

(4) Rules of inference and replacement along with C.P. and I.P. are not sufficient to prove validity of all argument.

(5) Propositional function is neither true nor false.

(6) Quantifiers are not used while symbolizing singular propositions.

**Q. 6. Explain the following.**

(1) The Rule of UI.

(2) The Rule of UG.

(3) The Rule of EG.

(4) The Rule of EI.

(5) Method of Instantiation.

(6) Method of Quantification.

(7) The difference between Propositional logic and Predicate logic.

(8) Distinction between Singular proposition and General proposition.

(9) Distinction between Proposition and propositional function.

(10) The nature of Quantificational Deduction.

(11) Singular Proposition in modern logic.

(12) Propositional function.

**Q. 7. Symbolize the following propositions using appropriate quantifiers and propositional functions.**

(1) No animals lay eggs.

(2) Everything is valuable.

(3) Some shopkeepers are not straightforward.

(4) A few homes are beautiful.

(5) Hardly any enterprise in the city is bankrupt.

(6) There are elephants.

(7) Unicorns do not exist.

(8) Few bureaucrats are honest.

(9) A few teenagers like swimming.

(10) Not a single pupil in the class passed the test.

(11) All singers are not rich.

(12) Every child is innocent.

(13) Few men are not strong.

(14) Dodos do not exist.

(15) Nothing is enduring.

(16) Some things are elegant.

(17) All men are sensible.

(18) Not all actors are good dancers.

(19) Rarely business men are scientists.

(20) Not a single story from the book is fascinating.

(21) All tigers are carnivorous animals.

(22) No book is covered.

(23) Some shops are open.

(24) Some shares are not equity.

(25) Air Tickets are always costly.

(26) Cunning people are never caring.

(27) Several banks are nationalized.

(28) Hardly children are interested in studies.

(29) Whatever is durable is worth buying.

(30) Not a single ladder is long.

**Q. 8. Construct formal proofs of validity for the following arguments.**

(1) (1) $(x)(Ax \supset \sim Px)$

(2) $(\exists x)(Ox \cdot Px)$ $/ \therefore (\exists x)(Ox \cdot \sim Ax)$

(2) (1) $(x)(Cx \supset \sim Kx)$

(2) $(x)(\sim Yx \supset Ax)$

(3) $(x)(\sim Kx \supset \sim Yx)$ $/ \therefore (x)(Cx \supset Ax)$

(3) (1) $(x)(\sim Ax \supset \sim Sx)$

(2) $(x)(Jx \supset \sim Ax)$

(3) $Ja$ $/ \therefore \sim Sa$

(4) (1) $(x)(Dx \supset Sx)$

(2) $Dc$

(3) $Wc$ $/ \therefore Sc \cdot Wc$

(5) (1) $(x)(Tx \supset Ax)$

(2) $(\exists x)(Mx)$

(3) $(x)(Ax \supset \sim Mx)$

$/ \therefore (\exists x)(\sim Ax \cdot \sim Tx)$

(6) (1) $(x)(Mx \supset Sx)$

(2) $(x)(Nx \supset Lx)$

(3) $\sim Sa \cdot Na$ $/ \therefore \sim Ma \cdot La$

(7) (1) $(x)(Px \supset Sx)$

(2) $(\exists x)(Px \cdot Lx)$

(3) $Pa$ $/ \therefore (\exists x)(Sx \cdot Lx)$

(8) (1) $(x)(Tx \supset Nx)$

(2) $(x)(Nx \supset Mx)$

(3) $Td$ $/ \therefore Ad \lor Md$

(9) (1) $(x)(Tx \supset Rx)$

(2) $(\exists x)(Tx \cdot Nx)$

(3) $(x)(Rx \supset Kx)$ $/ \therefore (\exists x)(Rx \cdot Kx)$

(10) (1) $(x)(Nx \supset Hx)$

(2) $\sim Hm \cdot Cm$ $/ \therefore (\exists x)(Cx \cdot \sim Nx)$

(11) (1) $(x)[(Qx \lor Rx) \supset Tx]$

(2) $(x)\, Qx$ $/ \therefore (x)\, Tx$

(12) (1) $(x)[(Jx \lor Kx) \supset Lx]$

(2) $Ka$

(3) $(\exists x) \sim Lx$ $/ \therefore (\exists x) \sim Jx$

(13) (1) $(x)[Dx \supset (Hx \cdot \sim Kx)]$

(2) $(x)(Hx \supset Px)$

(3) $Dg$ $/ \therefore (\exists x)(Px \cdot \sim Kx)$

(14) (1) $(x)(Hx \supset Gx)$

(2) $(\exists x)(Hx \cdot Lx)$ $/ \therefore (\exists x)(Lx \cdot Gx)$

(15) (1) $(x)(Ux \supset Wx)$

(2) $(x)\, Ux$

(3) $(\exists x)\, Zx$ $/ \therefore (\exists x)(Wx \cdot Zx)$

(16) (1) $(x)[Px \supset (Qx \supset Rx)]$

(2) $(x)(Rx \supset Tx)$

(3) $(x)\, Px$ $/ \therefore (x)(Qx \supset Tx)$

(17) (1) $(x)[Ix \supset (Px \cdot \sim Lx)]$

(2) $(x)(Px \supset Qx)$

(3) $Pd$

(4) $(\exists x) Ix$ $/ \therefore (\exists x)(Qx \cdot \sim Lx)$

(18) (1) $(x)[Ax \supset (Rx \lor Tx)]$

(2) $(x)\, Ax$

(3) $(\exists x)(Sx \cdot \sim Tx)$ $/ \therefore (\exists x)(Sx \cdot Rx)$

(19) (1) $(x)[Ax \supset (Bx \supset Fx)]$

(2) $(\exists x)(Ax \cdot Bx)$ $/ \therefore (\exists x)\, Fx$

(20) (1) $(x)(Dx \supset \sim Gx)$

(2) $Db$

(3) $(\exists x)[Dx \cdot (Gx \lor Kx)]$ $/ \therefore (\exists x)\, Kx$

(21) (1) $(x)(Fx \supset Gx)$

(2) $(x)(Gx \supset Hx)$ $/ \therefore (x)(Fx \supset Hx)$

(22) (1) $(x)(Ax \supset Bx)$

(2) $\sim Bx$ $/ \therefore (x) \sim Ax$

(23) (1) $(x)(Hx \supset Px)$

(2) $(x)(Px \supset Tx)$ $/ \therefore Hy \supset Ty$

(24) (1) $(x)(Bx \supset Kx)$

(2) $(\exists x) \sim Kx$ $/ \therefore \sim Bt$

(25) (1) (x) (Nx ⊃ Rx)

(2) (∃x) (Qx · ∼ Rx) / ∴ (∃x) (Qx · ∼ Nx)

(26) (1) (x) [Fx ⊃ (Lx · Ox)]

(2) (x) Fx / ∴ (∃x) Ox

(27) (1) (x) (Mx ⊃ Nx)

(2) (x) (Nx⊃ Rx) / ∴ (x) (Mx ⊃ Rx)

(28) (1) (x) (Ax ⊃ Bx)

(2) (x) (Bx ⊃ Cx)

(3) (x) (Cx ⊃ Dx) / ∴(x) (Ax ⊃ Dx)

(29) (1) (x) [Cx ⊃ (Fx ⊃ Gx)]

(2) Cp / ∴∼ Gp ⊃ ∼ Fp

(30) (1) (x) (Dx ⊃ ∼ Gx)

(2) (∃x) [(Dx · (Gx ∨ Kx)] / ∴ (∃x) Kx

❖ ❖ ❖