2 Polynomials

Topics Covered

1. Geometrical Meaning of the Zeroes of a Polynomial
2. Relationship between Zeroes and Coefficients of a Polynomial

Introduction

Polynomial: An expression of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0,$$

where n is a non-negative integer, $a_1, a_2, \dots, a_n$ are constants (real numbers) and $a_n \neq 0$, is called a polynomial in x of degree n.

Degree: If $p(x)$ is a polynomial in x, the highest power of x in $p(x)$ is called the degree of the polynomial $p(x)$.

Type of Polynomials: Polynomials of degree 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.

1. Geometrical Meaning of the Zeroes of a Polynomial

Zeroes of a Polynomial: A real number 'a' is said to be a **zeroes of a polynomial $p(x)$** if $p(a) = 0$.

Note: A polynomial may have no zero, one or more than one zeroes.

If $p(a) \neq 0$, then 'a' is not a zero of the polynomial $p(x)$.

Example 1. The value of k for which (–4) is a zero of the polynomial $x^2 - x - (2k + 2)$ is

(*a*) 2 (*b*) –6 (*c*) 9 (*d*) 8

Solution. Given that –4 is a zero of the polynomial $p(x) = x^2 - x - (2k + 2)$.

$\therefore \quad p(-4) = 0 \Rightarrow (-4)^2 + 4 - 2k - 2 = 0$

$\Rightarrow \quad 16 + 4 - 2k - 2 = 0 \Rightarrow 18 = 2k \Rightarrow k = 18 \div 2 = 9$

Thus, for $k = 9$, –4 is a zero of the given polynomial.

Hence, option (*c*) is the correct answer.

Example 2. The solution of $x^2 + 6x + 9 = 0$ is

(*a*) –1 (*b*) 3 (*c*) –3 (*d*) 1

Solution. Let $p(x) = x^2 + 6x + 9$

$\therefore \quad p(-3) = (-3)^2 + 6 \times (-3) + 9 = 9 + (-18) + 9 = 18 - 18 = 0$

$\therefore$ $x = -3$ is a solution of $x^2 + 6x + 9 = 0$.

Hence, option (*c*) is the correct answer.

Example 3. The number of polynomials having zeroes as –2 and 5 is

(*a*) 1 (*b*) 2 (*c*) 10 (*d*) infinite

Solution. As –2 and 5 are zeroes of the polynomial $p(x)$, so $(x + 2)$ and $(x - 5)$ are the factors of $p(x)$.

$\therefore \quad p(x) = k(x+2)(x-5) \Rightarrow p(x) = k(x^2 - 3x - 10)$, where k is a real number.

Now for different values of k, we get different polynomials.

Thus, the number of polynomials having zeroes as –2 and 5 is infinite.

Hence, option (*d*) is the correct answer.

Example 4. The graph of $y = f(x)$ is given in figure (i), for some polynomial $f(x)$. The number of zeroes of $f(x)$ is

(a) 1 (b) 2
(c) 3 (d) many

Solution. Polynomial $f(x)$ has 3 zeroes as graph intersects the x-axis at three distinct points.

Hence, option (c) is the correct answer.

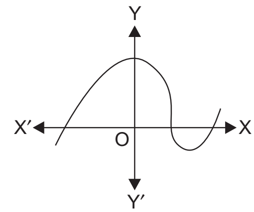


Figure (i)

Example 5. The graph of $y = f(x)$ is given in figure (ii). How many zeroes are there of $f(x)$?

(a) 0 (b) 1
(c) 2 (d) many

Solution. We observe that the graph of $y = f(x)$ intersects x-axis in one point only. Therefore, the number of zeroes of $f(x)$ is one.

Hence, option (b) is the correct answer.

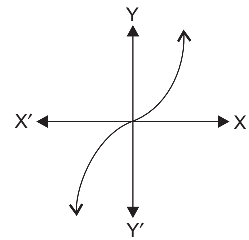


Figure (ii)

Example 6. The graph of $y = f(x)$ is given in figure (iii). How many zeroes are there of $f(x)$?

(a) 1 (b) 2
(c) 3 (d) None

Solution. The given graph $y = f(x)$ does not intersect x-axis. So, it has no zeroes.

Hence, option (d) is the correct answer.

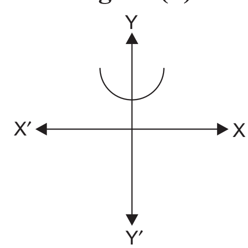


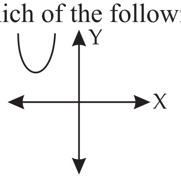
Figure (iii)

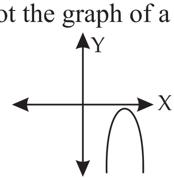
Exercise 2.1

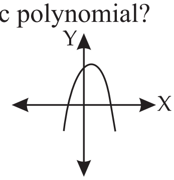
A. Multiple Choice Questions (MCQs)

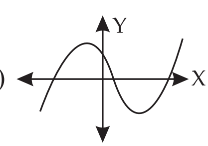
Choose the correct answer from the given options:

1. Which of the following is not the graph of a quadratic polynomial?

(a) 

(b) 

(c) 

(d) 

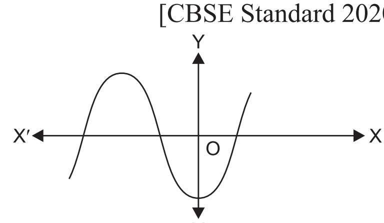
2. The zeroes of the polynomial $x^2 - 3x - m(m + 3)$ are

(a) $m, m + 3$ (b) $-m, m + 3$ (c) $m, -(m + 3)$ (d) $-m, -(m + 3)$

3. The graph of a polynomial is shown in figure, then the number of its zeroes is

[CBSE Standard 2020]

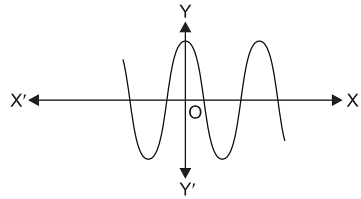
(a) 3 (b) 1
(c) 2 (d) 4



4. The value of p, for which (-4) is a zero of the polynomial $x^2 - 2x - (7p + 3)$ is

(a) 0 (b) 2 (c) 3 (d) None of these

5. The graph of $y = p(x)$, where $p(x)$ is a polynomial in variable x, is as follows:



The number of zeroes of $p(x)$ is

(*a*) 2 (*b*) 5 (*c*) 6 (*d*) None of these

6. If 2 is a zero of the polynomial $ax^2 - 2x$, then the value of 'a' is

(*a*) 3 (*b*) 1 (*c*) 2 (*d*) 5

7. A solution of $2x^2 + 5x - 3 = 0$ is

(*a*) –3 (*b*) 3 (*c*) –5 (*d*) 5

8. If 1 is a zero of polynomial $p(x) = ax^2 - 3(a - 1) - 1$, then the value of a is

(*a*) 0 (*b*) 1 (*c*) 3 (*d*) None of these

9. If the graph of a polynomial intersects the x-axis at exactly two points, is it necessarily a

(*a*) quadratic polynomial (*b*) cubic polynomial

(*c*) linear polynomial (*d*) None of these

10. What number should be added to the polynomial $x^2 - 5x + 4$, so that 3 is the zero of the polynomial?

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

11. For what value of k, (-4) is a zero of $p(x) = x^2 - x - (2k - 2)$?

(*a*) 8 (*b*) 11 (*c*) 13 (*d*) 15

12. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students:

$2x + 3$, $3x^2 + 7x + 2$, $4x^3 + 3x^2 + 2$, $x^3 + \sqrt{3x} + 7$, $7x + \sqrt{7}$, $5x^3 - 7x + 2$, $2x^2 + 3 - \frac{5}{x}$, $5x - \frac{1}{2}$, $ax^3 + bx^2 + cx + d$, $x + \frac{1}{x}$.

How many of the above ten, are not polynomials?

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

13. How many of the above ten (in question 12), are quadratic polynomials?

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

14. The zeroes of the polynomial $p(y) = 5\sqrt{5}y^2 + 30y + 8\sqrt{5}$ are

(*a*) $\frac{-3}{\sqrt{5}}, \frac{-7}{\sqrt{11}}$ (*b*) $\frac{-2}{\sqrt{5}}, \frac{-4}{\sqrt{5}}$ (*c*) $\frac{3}{\sqrt{5}}, \frac{3}{\sqrt{11}}$ (*d*) $\frac{2}{\sqrt{5}}, \frac{4}{\sqrt{5}}$

15. If the zero of polynomials $3x^2 - px + 2$ and $4x^2 - qx - 10$ is 2, the value of $2p - 3q$ is

(*a*) 3 (*b*) 5 (*c*) 7 (*d*) –5

B. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement reason (R). Choose the correct choice as:

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(*c*) Assertion (A) is true but reason (R) is false.

(*d*) Assertion (A) is false but reason (R) is true.

1. **Assertion (A):** $x^2 + 4x + 5$ has two zeroes.

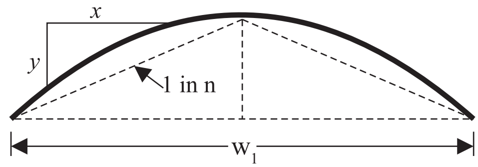
Reason (R): A quadratic polynomial can have at the most two zeroes.

2. **Assertion (A):** A quadratic polynomial whose zeroes are $5 + \sqrt{2}$ and $5 - \sqrt{2}$ is $x^2 - 10x + 23$.

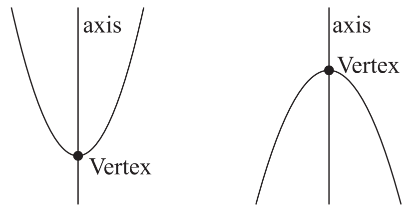
Reason (R): If α and β are zeroes of the quadratic polynomial $p(x)$, then $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$.

Case Study Based Questions

I. A highway underpass is parabolic in shape.



a. Parabolic Camber $y = 2x^2/n\text{w}$



Parabola: A parabola is the graph that results from $p(x) = ax^2 + bx + c$. Parabolas are symmetric about a vertical line known as the Axis of Symmetry. The Axis of Symmetry runs through the maximum or minimum point of the parabola which is called the vertex. [CBSE Standard SP 2020-21]

1. If the highway overpass is represented by $x^2 - 2x - 8$, then its zeroes are

(*a*) (2, –4) (*b*) (4, –2) (*c*) (–2, –2) (*d*) (–4, –4)

2. The highway overpass is represented graphically. Zeroes of a polynomial can be expressed graphically. Number of zeroes of polynomial is equal to number of points where the graph of polynomial

(*a*) intersects *x*-axis (*b*) intersects *y*-axis

(*c*) intersects *y*-axis or *x*-axis (*d*) None of these

3. Graph of a quadratic polynomial is a

(*a*) straight line (*b*) circle (*c*) parabola (*d*) ellipse

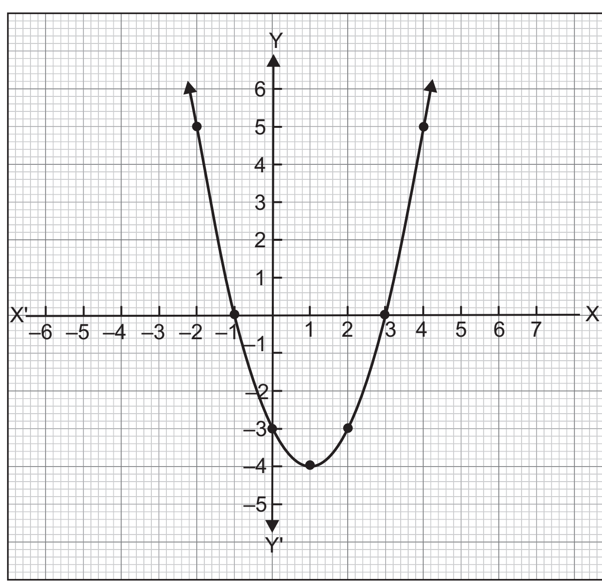
4. The representation of Highway Underpass whose one zero is 6 and sum of the zeroes is 0, is

(*a*) $x^2 - 6x + 2$ (*b*) $x^2 - 36$ (*c*) $x^2 - 6$ (*d*) $x^2 - 3$

5. The number of zeroes that polynomial $f(x) = (x - 2)^2 + 4$ can have is:

(*a*) 1 (*b*) 2 (*c*) 0 (*d*) 3

II. Due to heavy storm an electric wire got bent as shown in the figure. It followed a mathematical shape. Answer the following questions below:



1. Name the shape in which the wire is bent.

(*a*) spiral (*b*) ellipse

(*c*) linear (*d*) parabola

2. How many zeroes are there for the polynomial (shape of the wire)?

(*a*) 2 (*b*) 3

(*c*) 1 (*d*) 0

3. The zeroes of the polynomial are

(*a*) –1, 5 (*b*) –1, 3

(*c*) 3, 5 (*d*) –4, 2

4. What will be the expression of the polynomial?

(*a*) $x^2 + 2x - 3$ (*b*) $x^2 - 2x + 3$

(*c*) $x^2 - 2x - 3$ (*d*) $x^2 + 2x + 3$

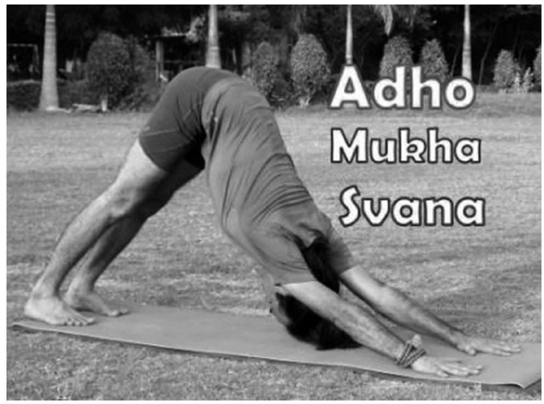
5. What is the value of the polynomial if $x = -1$?

(*a*) 6 (*b*) –18 (*c*) 18 (*d*) 0

III. An asana is a body posture, originally and still a general term for a sitting meditation pose, and later extended in hatha yoga and modern yoga as exercise, to any type of pose or position, adding reclining, standing, inverted, twisting, and balancing poses. In the figure, one can observe the poses that can be related to represent of a quadratic polynomial.

TRIKONASANA

ADHOMUKHA SAVASANA



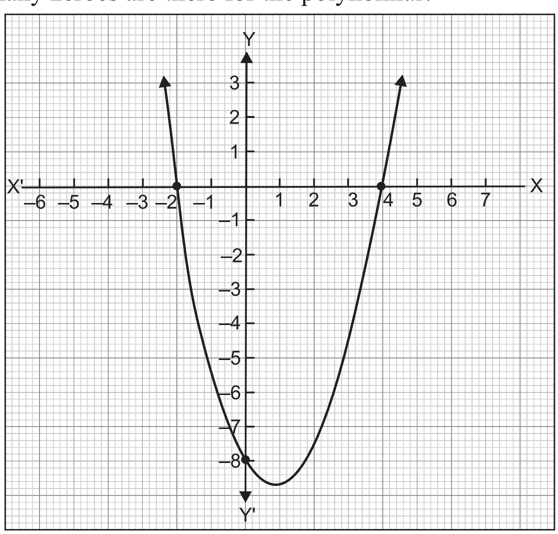
1. The shape of the poses shown is

(*a*) Spiral (*b*) Ellipse (*c*) Linear (*d*) Parabola

2. The graph of parabola opens downwards, if ________

(*a*) $a \geq 0$ (*b*) $a = 0$ (*c*) $a < 0$ (*d*) $a > 0$

3. In the graph, how many zeroes are there for the polynomial?



(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 2

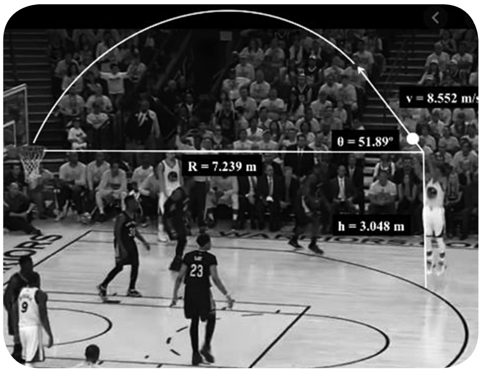
4. The two zeroes in the above shown graph are

(*a*) 2, 4 (*b*) –2, 4 (*c*) –8, 4 (*d*) 2, –8

5. The zeroes of the quadratic polynomial $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ are

(*a*) $\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$ (*b*) $-\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$ (*c*) $\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$ (*d*) $-\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$

IV. Basketball and soccer are played with a spherical ball. Even though an athlete dribbles the ball in both sports, a basketball player uses his hands and a soccer player uses his feet. Usually, soccer is played outdoors on a large field and basketball is played indoor on a court made out of wood. The projectile (path traced) of soccer ball and basketball are in the form of parabola representing quadratic polynomial.



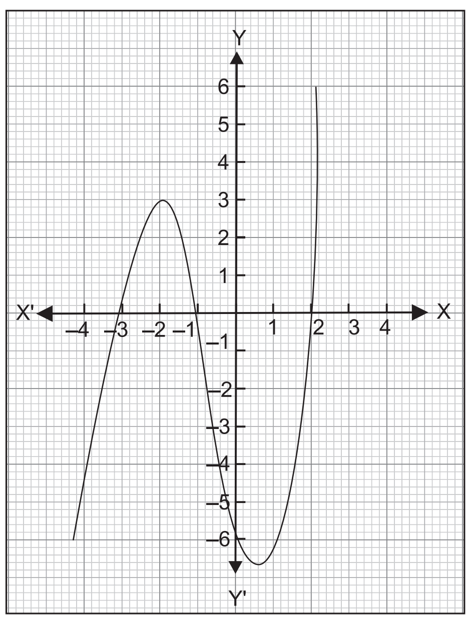
1. The shape of the path traced shown is

(*a*) Spiral (*b*) Ellipse (*c*) Linear (*d*) Parabola

2. The graph of parabola opens upwards, if ________

(*a*) $a = 0$ (*b*) $a < 0$ (*c*) $a > 0$ (*d*) $a \geq 0$

3. Observe the following graph and answer



In the above graph, how many zeroes are there for the polynomial?

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

4. The three zeroes in the above shown graph are

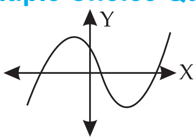
(*a*) 2, 3, –1 (*b*) –2, 3, 1 (*c*) –3, –1, 2 (*d*) –2, –3, –1

5. What will be the expression of the polynomial?

(*a*) $x^3 + 2x^2 - 5x - 6$ (*b*) $x^3 + 2x^2 - 5x + 6$ (*c*) $x^3 + 2x^2 + 5x - 6$ (*d*) $x^3 + 2x^2 + 5x + 6$

Answers and Hints

A. Multiple Choice Questions (MCQs)

1. (*d*) 

2. (*b*) $-m, m + 3$

3. (*a*) 3

4. (*c*) 3

5. (*b*) 5

6. (*b*) 1

7. (*a*) Given
$$p(x) = 2x^2 + 5x - 3$$
$$p(-3) = 2(-3)^2 + 5(-3) - 3$$
$$= 18 - 15 - 3$$
$$= 18 - 18 = 0$$
Since $p(-3) = 0$

$\therefore$ $x = -3$ is a solution of $2x^2 + 5x - 3 = 0$

8. (*b*) $\because$ 1 is a zero of $p(x)$

So, $p(1) = 0$

$\Rightarrow$ $a - 3a + 3 - 1 = 0$ $\Rightarrow$ $a = 1$

9. (*a*) Quadratic polynomial

10. (*c*) Let $f(x) = x^2 - 5x + 4$

Then $f(x) = 3^2 - 5 \times 3 + 4 = -2$

For $f(x) = 0$, 2 must be added to $f(x)$

11. (*b*) If -4 is a zero of $p(x) = x^2 - x - (2k - 2)$

then $p(-4) = 0$

$\Rightarrow 16 + 4 - (2k - 2) = 0$

$\Rightarrow$ $20 - 2k + 2$ 0

$\Rightarrow$ $-2k = -22$ $\Rightarrow$ $k = 11$

12. (*a*) 3

$x^3 + \sqrt{3x} + 7,\ 2x^2 + 3 - \dfrac{5}{x}$ and $x + \dfrac{1}{x}$

13. (*b*) 1

$3x^2 + 7x + 2$

14. (*b*) $\because$ $p(y) = 0$

$$\therefore \quad 5\sqrt{5}y^2 + 30y + 8\sqrt{5} = 0$$
$$\Rightarrow \quad 5\sqrt{5}y^2 + 20y + 10y + 8\sqrt{5} = 0$$
$$\Rightarrow \quad \sqrt{5}y(5y + 4\sqrt{5}) + 2(5y + 4\sqrt{5}) = 0$$
$$\Rightarrow \quad (\sqrt{5}y + 2)(5y + 4\sqrt{5}) = 0$$
$$\Rightarrow \quad y = \frac{-2}{\sqrt{5}},\ y = \frac{-4}{\sqrt{5}}$$
are the two zeroes of the polynomial.

15. (*b*) Given $p(x) = 3x^2 - px + 2$

and $q(x) = 4x^2 - qx - 10$ have their zero as 2.

$\therefore$ $p(2) = 0$

$\Rightarrow$ $12 - 2p + 2 = 0$

$\Rightarrow$ $-2p + 14 = 0$

$\Rightarrow$ $p = 7$

Also $q(2) = 0$

$\Rightarrow$ $16 - 2q - 10 = 0$

$\Rightarrow$ $-2q + 6 = 0$

$\Rightarrow$ $q = 3$

Now, $2p - 3q = 2(7) - 3(3) = 14 - 9 = 5$

B. Assertion-Reason Type Questions

1. (*d*) Assertion (A) is false but reason (R) is true.

2. (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Case Study Based Questions

I. 1. (*b*) (4, –2) **2.** (*a*) intersects *x*-axis **3.** (*c*) parabola **4.** (*b*) $x^2 - 36$ **5.** (*c*) 0

II. 1. (*d*) parabola **2.** (*a*) 2 **3.** (*b*) –1, 3 **4.** (*c*) $x^2 - 2x - 3$ **5.** (*d*) 0

III. 1. (*d*) Parabola **2.** (*c*) $a < 0$ **3.** (*c*) 2 **4.** (*b*) –2, 4 **5.** (*b*) $-\dfrac{2}{\sqrt{3}}, \dfrac{\sqrt{3}}{4}$

IV. 1. (*d*) Parabola **2.** (*c*) $a > 0$ **3.** (*d*) 3 **4.** (*c*) –3, –1, 2 **5.** (*a*) $x^3 + 2x^2 - 5x - 6$

2. Relationship between Zeroes and Coefficients of a Polynomial

- If α, β are zeroes of a quadratic polynomial $p(x) = ax^2 + bx + c$, where $a \neq 0$, then

  (*i*) Sum of zeroes $= \alpha + \beta = \dfrac{-b}{a}$ $\Rightarrow$ $\alpha + \beta = \dfrac{-\text{(Coefficient of } x)}{\text{Coefficient of } x^2}$

  (*ii*) Product of zeroes $= \alpha\beta = \dfrac{c}{a}$ $\Rightarrow$ $\alpha\beta = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

- If α, β are zeroes (or roots) of a quadratic polynomial $p(x)$, then $p(x) = x^2 - (\alpha + \beta)\,x + \alpha\beta$

  $\Rightarrow$ $p(x) = x^2 -$ (Sum of zeroes) x + (Product of zeroes)

Example 1. A quadratic equation $x^2 - 2x - 8$ is given. The zeroes of it are

(*a*) -2 and 4 (*b*) 3 and 5 (*c*) 1 and 6 (*d*) None of these

Solution. We have $p(x) = x^2 - 2x - 8$

Now, $x^2 - 2x - 8 = x^2 - 4x + 2x - 8 = x(x-4) + 2(x-4) = (x-4)(x+2)$

So, $p(x) = 0$

$\Rightarrow$ $(x-4)(x+2) = 0 \Rightarrow x = 4, x = -2$ are the zeroes of $p(x)$

Hence, option (*a*) is the correct answer.

Example 2. The sum and product of the zeroes of the quadratic equation given in example 1 are respectively

(*a*) 2, 4 (*b*) 5, -8 (*c*) 6, 8 (*d*) 2, -8

Solution. sum of its zeroes $= 4 + (-2) = 4 - 2 = 2 = \dfrac{-(-2)}{1} = \dfrac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

and product of its zeroes $= 4 \times (-2) = -8 = \dfrac{-8}{1} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, option (*d*) is the correct answer.

Example 3. The zeroes of the quadratic equation $4s^2 - 4s + 1$ are

(*a*) $\frac{1}{2}, \frac{1}{4}$ (*b*) $\frac{1}{2}, \frac{1}{2}$ (*c*) $\frac{1}{4}, \frac{1}{14}$ (*d*) $\frac{1}{3}, \frac{1}{4}$

Solution. We have $p(s) = 4s^2 - 4s + 1$

$$
\begin{aligned}
\text{Since} \quad 4s^2 - 4s + 1 &= 4s^2 - 2s - 2s + 1 = 2s(2s-1) - 1(2s-1) \\
&= (2s-1)(2s-1) \\
\text{So,} \quad p(s) &= 0 \\
\Rightarrow \quad (2s-1)(2s-1) &= 0 \Rightarrow s = \frac{1}{2}, \frac{1}{2} \text{ are the zeroes of } p(s)
\end{aligned}
$$

Hence, option (*b*) is the correct answer.

Example 4. The sum and product of zeroes of the quadratic equation given in example 3 are respectively

(*a*) 2, $\frac{3}{4}$ (*b*) 0, $\frac{1}{8}$ (*c*) 1, $\frac{1}{4}$ (*d*) None of these

Solution. Sum of its zeroes $= \dfrac{1}{2} + \dfrac{1}{2} = 1 = \dfrac{-(-4)}{4} = \dfrac{-(\text{Coefficient of } s)}{\text{Coefficient of } s^2}$

and product of its zeroes $= \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4} = \dfrac{\text{Constant term}}{\text{Coefficient of } s^2}$

Hence, option (*c*) is the correct answer.

Example 5. The set of the zeroes of the polynomial $x^2 - 25$, their sum and product is

(*a*) 4, 3; 7; 12 (*b*) -3, 3; 0; -9 (*c*) 5, -5; 0; -25 (*d*) None of these

Solution. We know that $a^2 - b^2 = (a-b)(a+b)$, so we can write

$$x^2 - 25 = (x-5)(x+5)$$

Thus value of $p(x) = x^2 - 25$ is zero, when either $x - 5 = 0$ or $x + 5 = 0 \Rightarrow x = 5$ or $x = -5$

So, sum of the zeroes $= 5 + (-5) = 0 = \dfrac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

and product of the zeroes $= (5)(-5) = -25 = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, option (*c*) is the correct answer.

Example 6. If the product of the zeroes of the polynomial $ax^2 - 6x - 6$ is 4, then value of a is

(*a*) $\frac{1}{8}$ (*b*) $-\frac{1}{4}$ (*c*) $\frac{-5}{3}$ (*d*) $\frac{-3}{2}$

Solution. Given polynomial is $ax^2 - 6x - 6$

Comparing it with $ax^2 + bx + c$, we get

$$\text{product of the zeroes} = \frac{c}{a} = \frac{-6}{a} = 4 \text{ (given)}$$

$$\therefore \quad \frac{-6}{a} = 4 \Rightarrow a = \frac{-6}{4} = \frac{-3}{2}$$

Hence, option (*d*) is the correct answer.

Example 7. The value of k, if the sum of the zeroes of the polynomial $x^2 - (k+6)x + 2(2k-1)$ is half of their product is

(*a*) 7 (*b*) 11 (*c*) 12 (*d*) None of these

Solution. Comparing $x^2 - (k+6)x + 2(2k-1)$ with $ax^2 + bx + c$, we get

$$\text{Sum of the zeroes of the quadratic polynomial} = \frac{-\{-(k+6)\}}{1} = k + 6$$

$$\text{Product of the zeroes of the quadratic polynomial} = \frac{c}{a} = \frac{2(2k-1)}{1} = 2(2k-1)$$

According to the question, we get

$$k + 6 = \frac{1}{2} \times 2(2k-1) \Rightarrow k + 6 = 2k - 1 \Rightarrow k = 7$$

Hence, option (*a*) is the correct answer.

Example 8. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, the values of p and q are respectively

(*a*) 5, 6 (*b*) 4, 7 (*c*) –5, –6 (*d*) –4, –7

Solution. Let α and β be the zeroes of $2x^2 - 5x - 3$.

$$\therefore \quad \alpha + \beta = \frac{5}{2}, \quad \alpha\beta = \frac{-3}{2}$$

As per the question,

It is given that 2α, 2β are the zeroes of $x^2 + px + q$

$$\therefore \quad 2\alpha + 2\beta = -p$$

$$\Rightarrow \quad 2(\alpha + \beta) = -p \Rightarrow 2 \times \frac{5}{2} = -p \Rightarrow p = -5$$

Also, $(2\alpha)(2\beta) = q$

$$\Rightarrow \quad 4\alpha\beta = q$$

$$\Rightarrow \quad \alpha\beta = \frac{q}{4} \Rightarrow \frac{-3}{2} = \frac{q}{4} \Rightarrow q = -6$$

Hence, option (*c*) is the correct answer.

Example 9. A quadratic polynomial whose zeroes are 1 and –3 is

(*a*) $x^2 + 3x - 2$ (*b*) $x^2 + 5x - 5$ (*c*) $x^2 + 2x - 3$ (*d*) None of these

Solution. Given that zeroes of the polynomial are 1 and –3.

Thus sum of the zeroes $= 1 + (-3) = -2$

and product of the zeroes $= 1 \times (-3) = -3$

Required quadratic polynomial is

$p(x) = x^2 -$ (Sum of the zeroes) x + Product of the zeroes $= x^2 + 2x - 3$

Hence, option (*c*) is the correct answer.

Example 10. The quadratic polynomial, sum of whose zeroes is 8 and their product is 12, is given by

(*a*) $x^2 - 8x + 12$ (*b*) $x^2 + 8x - 12$ (*c*) $x^2 - 5x + 7$ (*d*) $x^2 + 5x - 7$

Solution. Sum of the zeroes is 8 and product of zeroes is 12.

So, the required polynomial $p(x) = x^2 -$ (Sum of zeroes) x + Product of zeroes

$= x^2 - 8x + 12$

Hence, option (*a*) is the correct answer.

Example 11. If α, β are the zeroes of the polynomial $2x^2 - 5x + 7$, then a polynomial whose zeroes are $2\alpha + 3\beta$, $3\alpha + 2\beta$ is

(*a*) $k\left(x^2 - \frac{3}{5}x + 21\right)$ (*b*) $k\left(x^2 - \frac{25}{2}x + 41\right)$ (*c*) $k\left(x^2 + \frac{9}{2}x - 45\right)$ (*d*) None of these

Solution. Since α, β are the zeroes of $2x^2 - 5x + 7$

$$\therefore \quad \alpha + \beta = \frac{-(-5)}{2} = \frac{5}{2} \text{ and } \alpha\beta = \frac{7}{2} \qquad \left[\begin{array}{l} \text{sum of the zeroes} = \frac{-b}{a} \\ \text{product of the zeroes} = \frac{c}{a} \end{array}\right]$$

The given zeroes of required polynomial are $2\alpha + 3\beta$ and $3\alpha + 2\beta$

Sum of the zeroes $= 2\alpha + 3\beta + 3\alpha + 2\beta = 5\alpha + 5\beta = 5(\alpha + \beta) = 5 \times \frac{5}{2} = \frac{25}{2}$

Again, product of the zeroes$=(2\alpha + 3\beta)(3\alpha + 2\beta) = 6(\alpha^2 + \beta^2) + 13\alpha\beta$

$$= 6[(\alpha + \beta)^2 - 2\alpha\beta)] + 13\alpha\beta = 6(\alpha + \beta)^2 + \alpha\beta$$

$$= 6\left(\frac{5}{2}\right)^2 + \frac{7}{2} = \frac{75}{2} + \frac{7}{2} = \frac{82}{2} = 41$$

Now, required polynomial is given by

$k[x^2 -$ (Sum of the zeroes) x + Product of the zeroes$] = k\left[x^2 - \frac{25}{2}x + 41\right]$,

where k is any non-zero real number.

Hence, option (*b*) is the correct answer.

Example 12. A quadratic polynomial whose product and sum of zeroes are $-\frac{13}{5}$ and $\frac{3}{5}$, respectively.

(*a*) $k(x^2 + 12x + 5)$ (*b*) $k[x^2 - (8x) + (-9))$

(*c*) $k\left[x^2 - \left(\frac{1}{2}x\right) + \left(-\frac{7}{5}\right)\right]$ (*d*) $k\left[x^2 - \left(\frac{3}{5}x\right) + \left(-\frac{13}{5}\right)\right]$

Solution. Product of zeroes of a quadratic polynomial $= -\frac{13}{5}$

and sum of zeroes of quadratic polynomial $= \frac{3}{5}$.

Thus, required quadratic polynomial $= k\left[x^2 - \left(\frac{3}{5}x\right) + \left(\frac{-13}{5}\right)\right]$, where k is a non-zero real number.

Hence, option (*d*) is the correct answer.

Exercise 2.2

A. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

1. If one of the zeroes of quadratic polynomial $(k - 1)x^2 + kx + 1$ is –3, then $k =$
   (*a*) $\frac{4}{3}$ (*b*) $-\frac{4}{3}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{3}{2}$
2. Sum of the zeroes of the polynomial $x^2 + 7x + 10$ are
   (*a*) 7 (*b*) – 7 (*c*) 10 (*d*) – 10
3. The quadratic polynomial, the sum of whose zeroes is –5 and their product is 6, is
   (*a*) $x^2 + 5x + 6$ (*b*) $x^2 - 5x + 6$ (*c*) $x^2 - 5x - 6$ (*d*) $-x^2 + 5x + 6$
4. Quadratic polynomial having zeroes α and β is
   (*a*) $x^2 - (\alpha\beta)x + (\alpha + \beta)$ (*b*) $x^2 - (\alpha + \beta)x + \alpha\beta$ (*c*) $x^2 - \frac{\alpha}{\beta}x + \alpha\beta$ (*d*) None of these
5. If one zero of the quadratic polynomial $x^2 - 5x - 6$ is 6, then other zero is
   (*a*) 0 (*b*) 1 (*c*) –1 (*d*) 2
6. If α, β are the zeroes of the polynomial $2y^2 + 7y + 5$, the value of $\alpha + \beta + \alpha\beta$ is
   (*a*) 0 (*b*) 1 (*c*) –1 (*d*) 2
7. A quadratic polynomial, the sum and product of whose zeroes and (–3) and 2 respectively is
   (*a*) $x^2 + 3x + 2$ (*b*) $x^2 - 3x + 2$ (*c*) $x^2 + 3x - 2$ (*d*) None of these
8. If the sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then the value of k is
   (*a*) 3 (*b*) 6 (*c*) 9 (*d*) 0
9. A quadratic polynomial, whose sum of zeroes is 2 and product is –8 is
   (*a*) $x^2 - 3x - 3$ (*b*) $x^2 + 2x + 8$ (*c*) $x^2 + 3x + 3$ (*d*) $x^2 - 2x - 8$
10. The other zero of the polynomial $2x^3 + x^2 - 6x - 3$, if two of its zeroes are $-\sqrt{3}$ and $\sqrt{3}$ is
    (*a*) $\frac{1}{2}$ (*b*) $-\frac{1}{2}$ (*c*) $\frac{1}{3}$ (*d*) $-\frac{1}{3}$
11. If two zeroes of the polynomial $p(x) = x^3 - 4x^2 - 3x + 12$ are $\sqrt{3}$ and $-\sqrt{3}$, then its third zero is
    (*a*) $\sqrt{2}$ (*b*) $\sqrt{5}$ (*c*) 5 (*d*) 4
12. If 2 and –3 are the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$; then the values of a and b respectively are
    (*a*) 0, –6 (*b*) 0, 0 (*c*) 6, –6 (*d*) 2, –3
13. If α and β are the zeroes of the polynomial $ax^2 + bx + c$; then the value of $\alpha^2 + \beta^2$ is
    (*a*) $\frac{b^2 - ca}{a^2}$ (*b*) $\frac{b^2 - 2ca}{a^2}$ (*c*) $\frac{b^2 + ca}{a^2}$ (*d*) $\frac{b^2 + 2ca}{a^2}$
14. A quadratic polynomial whose zeroes are $5 - 3\sqrt{2}$ and $5 + 3\sqrt{2}$ is
    (*a*) $x^2 - 10x + 7$ (*b*) $x^2 + 10x + 7$ (*c*) $x^2 - 5x + 9$ (*d*) $x^2 + 5x - 9$
15. The zeores of the quadratic polynomial $6x^2 - 3 - 7x$ are
    (*a*) $\frac{3}{2}, \frac{-1}{3}$ (*b*) $\frac{2}{3}, -3$ (*c*) $\frac{3}{5}, \frac{-3}{7}$ (*d*) None of these
16. The zeroes of the quadratic polynomial $4x^2 - 4x - 3$ are
    (*a*) $\frac{3}{2}, \frac{-1}{3}$ (*b*) $\frac{3}{2}, \frac{-1}{2}$ (*c*) $\frac{2}{5}, \frac{-2}{5}$ (*d*) None of these

17. If α and β are the zeroes of the polynomial $4x^2 + 3x + 7$, the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is

(*a*) $-\frac{1}{2}$ (*b*) $-\frac{5}{2}$ (*c*) $-\frac{3}{7}$ (*d*) $\frac{3}{7}$

18. If α, β are the zeroes of the polynomial $x^2 - 4x + 3$, a quadratic polynomial whose zeroes are 3α and 3β is

(*a*) $x^2 + 8x + 17$ (*b*) $x^2 + 12x + 27$ (*c*) $x^2 - 12x + 27$ (*d*) $x^2 - 8x + 17$

19. If α and β are the zeroes of the polynomial $5x^2 - 7x - 2$, the sum of the reciprocals of zeroes is

(*a*) $\frac{7}{2}$ (*b*) $-\frac{7}{2}$ (*c*) $\frac{5}{9}$ (*d*) $-\frac{5}{9}$

20. If α and β are zeroes of a polynomial $x^2 + 6x + 9$, then a polynomial whose zeroes are –α and –β is

(*a*) $x^2 - 6x + 9$ (*b*) $x^2 + 6x - 9$ (*c*) $x^2 + 5x + 4$ (*d*) None of these

21. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. A quadratic polynomial whose zeroes are 3α and 3β is

(*a*) $\frac{k}{2}(3x^2 + 5x - 5)$ (*b*) $\frac{k}{2}(3x^2 - 5x + 5)$ (*c*) $\frac{k}{2}(2x^2 + 9x + 9)$ (*d*) $\frac{k}{2}(2x^2 - 9x + 9)$

22. The value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeros equal to half of their product is

(*a*) 3 (*b*) 5 (*c*) 7 (*d*) None of these

23. The zeroes of the quadratic polynomial $x^2 + 7x + 10$ are

(*a*) –2, –5 (*b*) 2, 5 (*c*) –3, –8 (*d*) 3, 8

24. A quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial :

$f(x) = ax^2 + bx + c,\ a \neq 0,\ c \neq 0$ is

(*a*) $\frac{1}{c}(cx^2 + bx + a)$ (*b*) $\frac{1}{c}(cx^2 - bx + a)$ (*c*) $\frac{1}{c}(cx^2 - bx - a)$ (*d*) $\frac{1}{c}(-cx^2 + bx + a)$

25. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other, then 'a' is

(*a*) 1 (*b*) 3 (*c*) 12 (*d*) 19

B. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement reason (R). Choose the correct choice as:

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(*c*) Assertion (A) is true but reason (R) is false.

(*d*) Assertion (A) is false but reason (R) is true.

1. **Assertion (A):** If one zero of polynomial $p(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of each other, then $k = 2$.

Reason (R): If $(x - \alpha)$ is a factor of $p(x)$, then $p(\alpha) = 0$, i.e. α is a zero of $p(x)$.

2. **Assertion (A):** If both zeroes of the quadratic polynomial $x^2 - 2kx + 2$ are equal in magnitude but opposite in sign, then value of k is $\frac{1}{2}$.

Reason (R): Sum of zeroes of a quadratic polynomial $ax^2 + bx + c$ is $-\frac{b}{a}$.

Case Study Based Questions

I. Skipping Rope

Skipping rope is a good exercise. It burns calories, makes bones strong and improves heart health. During skipping, when rope goes up and down it makes the shape of parabolas (graphs of quadratic polynomials). Observe the following skipping pictures.

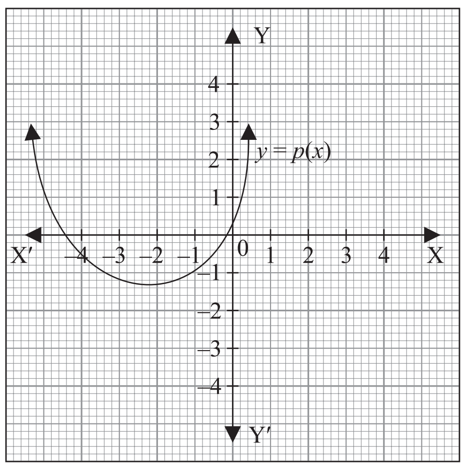

Picture 1


Picture 2


Picture 3

Refer Picture 1

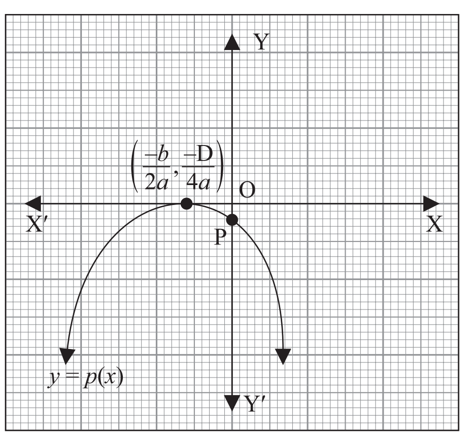
1. The graph of polynomial $p(x)$ represented by Picture 1 is shown below. The number of zeroes of the polynomial is



(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 4

Refer Picture 2

2. The graph of polynomial $p(x)$ represented by Picture 2 is shown below. Which of the following has negative (–) sign?



(*a*) *a* (*b*) *b* (*c*) *c* (*d*) All of these.

3. If $\frac{1}{4}$ and –1 are the sum and product of zeroes of a polynomial whose graph is represented by Picture (3), the quadratic polynomial is

(*a*) $k\left(x^2 - \frac{1}{4}x - 1\right)$ (*b*) $k\left(\frac{1}{4}x^2 - x - 1\right)$ (*c*) $k\left(x^2 + \frac{1}{4}x + 1\right)$ (*d*) $k\left(\frac{1}{4}x^2 + x + 1\right)$

4. Let the Picture (1) represents the quadratic polynomial $f(x) = x^2 - 8x + k$ whose sum of the squares of zeroes is 40. The value of k is

(*a*) 8 (*b*) 10 (*c*) 12 (*d*) 20

5. Let the Picture (3) represents the quadratic polynomial $f(x) = x^2 + 7x + 10$. Then its zeroes are

(*a*) –1, –5 (*b*) –2, –5 (*c*) 1, 5 (*d*) 2, 5

II. Rainbow is an arch of colours that is visible in the sky, caused by the refraction and dispersion of the sun's light after rain or other water droplets in the atmosphere. The colours of the rainbow are generally said to be red, orange, yellow, green, blue, indigo and violet.

Each colour of rainbow makes a parabola. We know that for any quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, the graph of the corresponding equation $y = ax^2 + bx + c$ has one of the two shapes either open upwards like $\cup$ or open downwards like $\cap$ depending on whether $a > 0$ or $a < 0$. These curves are called parabolas.

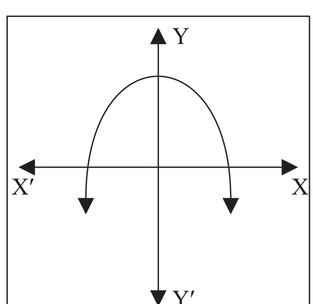
1. A rainbow is represented by the quadratic polynomial $x^2 + (a + 1)x + b$ whose zeroes are 2 and –3. Then

(*a*) $a = -7, b = -1$ (*b*) $a = 5, b = -1$ (*c*) $a = 2, b = -6$ (*d*) $a = 0, b = -6$

2. The polynomial $x^2 - 2x - (7p + 3)$ represents a rainbow. If –4 is zero of it, then the value of p is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

3. The graph of a rainbow $y = f(x)$ is shown below.



The number of zeroes of $f(x)$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

4. If graph of a rainbow does not intersect the x-axis but intersects y-axis in one point, then number of zeroes of the polynomial is equal to

(*a*) 0 (*b*) 1 (*c*) 0 or 1 (*d*) none of these

5. The representation of a rainbow is a quadratic polynomial. The sum and the product of its zeroes are 3 and –2 respectively. The polynomial is

(*a*) $k(x^2 - 2x - 3)$, for some real k. (*b*) $k(x^2 - 5x - 9)$, for some real k.

(*c*) $k(x^2 - 3x - 2)$, for some real k. (*d*) $k(x^2 - 8x + 2)$, for some real k.

III. The below picture are few natural examples of parabolic shape which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms.

1. In the standard form of quadratic polynomial, $ax^2 + bx + c$; a, b and c are

(*a*) All are real numbers. (*b*) All are rational numbers.

(*c*) 'a' is a non-zero real number and b and c are any real numbers.

(*d*) All are integers.

2. If the roots of the quadratic polynomial are equal, where the discriminant $D = b^2 - 4ac$, then

(*a*) $D > 0$ (*b*) $D < 0$ (*c*) $D \geq 0$ (*d*) $D = 0$

3. If α and $\frac{1}{\alpha}$ are the zeroes of the quadratic polynomial $2x^2 - x + 8k$, then k is

(*a*) 4 (*b*) $\frac{1}{4}$ (*c*) $\frac{-1}{4}$ (*d*) 2

4. The graph of $x^2 + 1 = 0$

(*a*) Intersects x-axis at two distinct points (*b*) Touches x-axis at a point

(*c*) Neither touches nor intersects x-axis (*d*) Either touches or intersects x-axis

5. If the sum of the roots is $-p$ and product of the roots is $-\frac{1}{p}$, then the quadratic polynomial is

(a) $k\left(-px^2 + \frac{x}{p} + 1\right)$ (b) $k\left(px^2 - \frac{x}{p} - 1\right)$ (c) $k\left(x^2 + px - \frac{1}{p}\right)$ (d) $k\left(x^2 - px + \frac{1}{p}\right)$

Answers and Hints

A. Multiple Choice Questions (MCQs)

1. (a) $\frac{4}{3}$

2. (b) -7

3. (a) $x^2 + 5x + 6$

4. (b) $x^2 - (\alpha + \beta)x + \alpha\beta$

5. (c) -1

6. (c) $p(y) = 2y^2 + 7y + 5$

$\therefore \quad \alpha + \beta = \frac{-7}{2}$

$\alpha\beta = \frac{5}{2}$

So, $\alpha + \beta + \alpha\beta = \frac{-7}{2} + \frac{5}{2} = -1$

7. (a) Let α and β be the zeroes of the required polynomial $f(x)$.

Here, $\alpha + \beta = -3$ and $\alpha\beta = 2$

$\therefore \quad f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (-3)x + 2$

$= x^2 + 3x + 2$

8. (c) $\alpha + \beta = \frac{k}{3}$

$3 = \frac{k}{3}$

$\therefore \quad k = 9$

9. (d) Quadratic polynomial is given by

$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 2x - 8$

10. (b) Let $p(x) = 2x^3 + x^2 - 6x - 3$

$= x^2(2x + 1) - 3(2x + 1)$

$= (x^2 - 3)(2x + 1)$

$= (x - \sqrt{3})(x + \sqrt{3})(2x + 1)$

Since $-\sqrt{3}, \sqrt{3}$ are two zeroes of $p(x)$.

$\therefore$ All zeroes of given polynomial are $-\sqrt{3}, \sqrt{3}$ and $-\frac{1}{2}$.

11. (d) As $\sqrt{3}$ and $-\sqrt{3}$ are the zeroes of the polynomial $p(x)$.

$\therefore \quad (x + \sqrt{3})(x - \sqrt{3})$ are the factors of $p(x)$.

Now, $p(x) = x^3 - 4x^2 - 3x + 12$

$= x^2(x - 4) - 3(x - 4)$

$= (x^2 - 3)(x - 4)$

$= (x - \sqrt{3})(x + \sqrt{3})(x - 4)$

So third zero = 4.

12. (a) Since 2 and -3 are zeroes of $p(x) = x^2 + (a + 1)x + b$

$\therefore \quad 2 + (-3) = -(a + 1) \Rightarrow a = 0$

and $2(-3) = b \Rightarrow b = -6.$

13. (b) $\frac{b^2 - 2ca}{a^2}$

14. (a) Sum of zeroes $= 5 - 3\sqrt{2} + 5 + 3\sqrt{2} = 10$

Product of zeroes $= \left(5 - 3\sqrt{2}\right)\left(5 + 3\sqrt{2}\right) = 7$

Therefore $P(x) = x^2 - 10x + 7$

15. (a) $p(x) = 6x^2 - 7x - 3 \quad ...(i)$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

$= (2x - 3)(3x + 1)$

The zeroes of $p(x)$ is given by $p(x) = 0$.

So the two zeroes are $x = \frac{3}{2}, \frac{-1}{3}$

16. (b) $\frac{3}{2}, \frac{-1}{2}$

17. (c) $\frac{-3}{7}$

18. (c) $x^2 - 12x + 27$

19. (b) $\frac{-7}{2}$

20. (a) $p(x) = x^2 + 6x + 9$

Since α, β are zeroes of $p(x)$

$\therefore \quad \alpha + \beta = -6, \alpha\beta = 9$

Now, $-\alpha - \beta = -(\alpha + \beta)$
$= 6$

and $(-\alpha)(-\beta) = \alpha\beta$
$= 9$

$\therefore$ Required quadratic polynomial is given by

$q(x) = x^2 - (-\alpha - \beta)x + (-\alpha)(-\beta)$
$= x^2 - 6x + 9$

21. (*d*) α and β are zeroes of the polynomial $2x^2 - 3x + 1$.

$\therefore \ \alpha + \beta = -\frac{b}{a} = \frac{3}{2}$ and $\alpha\beta = \frac{c}{a} = \frac{1}{2}$

Now zeroes of the required polynomial are 3α, 3β.

So, $3\alpha + 3\beta = 3(\alpha + \beta) = \frac{9}{2}$

and $(3\alpha)(3\beta) = 9(\alpha\beta) = \frac{9}{2}$

$\therefore$ Required polynomial

$= k[x^2 - (3\alpha + 3\beta)x + (3\alpha)(3\beta)]$

$= k\left(x^2 - \frac{9}{2}x + \frac{9}{2}\right) = \frac{k}{2}(2x^2 - 9x + 9)$

where k is a constant.

22. (*c*) Given polynomial : $x^2 - (k + 6)x + 2(2k - 1)$

Sum of zeroes $= \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{k+6}{1} = k + 6$

$\Rightarrow \quad \alpha + \beta = k + 6$

Product of zeroes $= \frac{c}{a} = \frac{2(2k-1)}{1}$

$\Rightarrow \quad \alpha\beta = 4k - 2$

ATQ, $\quad \alpha + \beta = \frac{1}{2}(\alpha\beta)$

$\Rightarrow \quad k + 6 = \frac{1}{2}(4k - 2) = \frac{1}{2} \times 2(2k - 1)$

$\Rightarrow \quad k + 6 = 2k - 1 \quad \Rightarrow \quad k = 7$

23. (*a*) We have $\quad p(x) = x^2 + 7x + 10$

$p(x) = x^2 + 2x + 5x + 10$
$= x(x + 2) + 5(x + 2)$
$= (x + 2)(x + 5)$

The zeroes of polynomial $p(x)$ is given by

$p(x) = 0$

$\Rightarrow \quad (x + 2)(x + 5) = 0$

Either $\quad x + 2 = 0$ or $x + 5 = 0$

$\Rightarrow \quad x = -2, x = -5$

$\Rightarrow \quad x = -2, -5$

Thus, the zeroes of $x^2 + 7x + 10$ are $\alpha = -2$ and $\beta = -5$

24. (*a*) Let α and β are the zeroes of polynomial, $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$

Then, $\quad \alpha + \beta = \frac{-b}{a}$

$\alpha \cdot \beta = \frac{c}{a}$

Now, $\quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

$= \frac{-\frac{b}{a}}{\frac{c}{a}} = \frac{-b}{a} \times \frac{a}{c} = \frac{-b}{c}$

$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{\frac{c}{a}} = \frac{a}{c}$

$\therefore$ The required polynomial

$= x^2 - \left(\frac{-b}{c}\right)x + \frac{a}{c} = x^2 + \frac{b}{c}x + \frac{a}{c}$

$= \frac{cx^2 + bx + a}{c} = \frac{1}{c}(cx^2 + bx + a)$

25. (*b*) 3

B. Assertion-Reason Type Questions

1. (*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

2. (*d*) Assertion (A) is false but reason (R) is true.

Case Study Based Questions

I. 1. (*c*) 2 **2.** (*d*) All of these.

3. (*a*) $k\left(x^2 - \frac{1}{4}x - 1\right)$ **4.** (*c*) 12

5. (*b*) $-2, -5$

II. 1. (*d*) $a = 0, b = -6$ **2.** (*c*) 3

3. (*c*) 2 **4.** (*a*) 0

5. (*c*) $k(x^2 - 3x - 2)$, for some real k.

III. 1. (*c*) '*a*' is a non zero real number and b and c are any real numbers.

2. (*d*) D = 0 **3.** (*b*) $\frac{1}{4}$

4. (*c*) Neither touches nor intersects x-axis.

5. (*c*) $k\left(x^2 + px - \frac{1}{p}\right)$

Experts' Opinion

Questions based on following types are very important for exams. So, students are advised to revise them thoroughly.

1. Finding zeroes of the quadratic polynomial and the relation between the zeroes and the coefficients.
2. Forming a quadratic polynomial with given sum of zeroes and product of zeroes.
3. Finding all the zeroes of a polynomial if its some zeroes are given.

IMPORTANT FORMULAE

- If α, β are zeroes of the quadratic polynomial $ax^2 + bx + c$, then

  (*i*) $\alpha + \beta = \frac{-b}{a}$ (*ii*) $\alpha\beta = \frac{c}{a}$

- If α, β are zeroes of the quadratic polynomial $p(x)$, then $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

QUICK REVISION NOTES

- A polynomial $p(x)$ in one variable x is an algebraic expression in x of the form

  $p(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_2x^2 + a_1x + a_0$ where $a_n, a_{n-1}, ..., a_0$ are real numbers with $a_n \neq 0$.
- The highest power of x in $p(x)$ is called the degree of the polynomial $p(x)$.
- A quadratic polynomial in x with real coefficients is of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.
- Let $p(x)$ be a polynomial in x and k be any real number, then the value obtained by replacing x by k in $p(x)$ is called the value of $p(x)$ at $x = k$ and is denoted by $p(k)$.
- A real number k is said to be a zero of a polynomial $p(x)$ if $p(k) = 0$.
- A polynomial may have no zero, one or more than one zeroes.
- A polynomial $p(x)$ of degree n has atmost n zeroes.
- The zeroes of a polynomial $p(x)$ are precisely the x-coordinates of the points where the graph of $y = p(x)$ intersects the x-axis.
- If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then $\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$.

COMMON ERRORS

Errors	Corrections
(*i*) Finding incorrectly $p(x)$ at $x = k$.	(*i*) To find $p(x)$ at $x = k$, replace x by k in $p(x)$ carefully and get the answer.
(*ii*) Doing mistakes in finding zeroes of a polynomial from graph.	(*ii*) Remember that x-coordinate(s) of point(s), where the graph of given polynomial intersects the x-axis are zeroes of the polynomial.
(*iii*) Splitting the middle term of a polynomial incorrectly to find the zeroes.	(*iii*) To split b of the polynomial $ax^2 + bx + c$, write b as the sum of two numbers whose product is ac.
(*iv*) Formating incorrectly a quadratic polynomial when zeroes are α, β as $(x - \alpha)(x - \beta)$.	(*iv*) Form a quadratic polynomial with zeroes α, β as $k(x - \alpha)(x - \beta)$, where k is real.
(*v*) Finding the incorrect value of y for various values of x.	(*v*) Transpose y term to either side so as to get positive coefficient.
(*vi*) Finding the incorrect value of y for various values of x.	(*vi*) Transpose y term to either side so as to get positive coefficient.