COMPUTER BASED TEST (CBT)
Memory Based Questions & Solutions

Date: 26 June, 2022 (SHIFT-2) | TIME : (3.00 a.m. to 6.00 p.m)
Duration: 3 Hours | Max. Marks: 300

PART : MATHEMATICS

1. The value of $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4}$ is

(1) $\frac{1}{2}$ (2) $\frac{1}{6}$ (3) $\frac{1}{12}$ (4) $\frac{1}{3}$

Ans. **(2)**

Sol.. $\lim_{x\to 0} \frac{-2\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{\sin x - x}{2}\right)}{x^4}$

$\Rightarrow \lim_{x\to 0} \frac{-2\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{\sin x - x}{2}\right)}{x^4}$

$\Rightarrow -\frac{1}{2} \lim_{x\to 0} \frac{\left(\frac{\sin x}{x} + 1\right)(\sin x - x)}{x^3}$

$\Rightarrow -\frac{1}{2}(1+1) \lim_{x\to 0} \frac{\left[\left(x - \frac{x^3}{3!} + \frac{x^5}{5!}.........\right) - x\right]}{x^3}$

$\Rightarrow \frac{-1}{2}(2)\left(-\frac{1}{3!}\right) = \frac{1}{6}$

2. The value of 16(sin20º) (sin40º) (sin80º) is

(1) $2\sqrt{3}$ (2) $2\sqrt{5}$ (3) $-2\sqrt{5}$ (4) $-2\sqrt{3}$

Ans. **(1)**

Sol. $\because \sin x \cdot \sin(60^\circ - x)\cdot \sin(60^\circ + x) = \frac{\sin 3x}{4}$

$\Rightarrow 16(\sin 20^\circ)(\sin 40^\circ)(\sin 80^\circ)$

$= 16(\sin 20^\circ)(\sin(60^\circ - 20^\circ))(\sin 60^\circ + 20^\circ)$

$= 16 \times \frac{\sin 60^\circ}{4}$

$= 4 \times \frac{\sqrt{3}}{2}$

$= 2\sqrt{3}$

3. Let y(x) represents the solution of differential equation

$x\frac{dy}{dx} + 2y = xe^x$ and $z(x) = x^2\, y(x) - e^x$, and $y(1) = 0$, then the maximum value of z(x) is

(1) $\frac{4}{e} - e$ (2) $\frac{4}{e^2} - e$ (3) $\frac{2}{e} - e$ (4) $\frac{2}{e^2} - e$

Ans. (1)

Sol. $x\frac{dy}{dx} + 2y = xe^x$

$$\frac{dy}{dx}+\frac{2}{x}.y=e^x$$

I.F. = $e^{2\int\frac{1}{x}dx}=x^2$

final solution is

y.x^2 = $\int x^2.e^x\,dx$

$yx^2 = x^2e^x - 2xe^x + 2e^x + k$

given y(1) = 0

x = 1, y = 0 given

$0 = e - 2e + 2e + k \Rightarrow k = -e$

$y.x^2 - e^x = x^2e^x - 2xe^x + e^x - e$

$z(x) = (x - 1)^2\, e^x - e$

$z'(x) = 2(x - 1)e^x + (x - 1)^2\, e^x$

for maximum or minimum $z'(x) = 0$

$(x-1)e^x(2+(x-1))=0$

$(x-1)(x+1)e^x=0$

$z(x)\big|_{max.} = 4e^{-1} - e = \frac{4}{e}-e$

4. If $z^2 + z + 1 = 0$, ($z \in C$) then the value of $\left|\sum_{n=1}^{15}\left(z^n+\frac{(-1)^n}{z^n}\right)^2\right|$ is equal to

Ans. **(2)**

Sol. z = w, w^2

$\left(z^n+\frac{(-1)^n}{z^n}\right)^2 = z^{2n} + \frac{1}{z^{2n}}+(-1)^n.2$

exp. = $\left|\sum_{n=1}^{15}\left(z^n+\frac{(-1)^n}{z^n}\right)^2\right| = \left|\sum_{n=1}^{15}z^{2n}+2\sum_{n=1}^{15}(-1)^n+\sum\frac{1}{z^{2n}}\right|$

= 0 + 0 + 2 (1) = 2

5. Let p, q $\in$ R such that p + q = 3 and $p^4 + q^4 = 369$ then the value of $\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}$ is

(1) 5 (2) $\frac{1}{4}$ (3) 4 (4) $\frac{1}{5}$

Ans. **(3)**

Sol. $p^4 + q^4 = (p^2 + q^2)^2 - 2p^2q^2$

$\Rightarrow$ 369 = $\left((p+q)^2-2pq\right)^2$ $-2\,(pq)^2$

$\Rightarrow$ 369 = $(9 - 2x)^2 - 2x^2$ (where x = pq)

$\Rightarrow$ 369 = $81 - 36x + 4x^2 - 2x^2$

$\Rightarrow$ $2x^2 - 36x - 288 = 0$

$\Rightarrow$ $x^2 - 18x - 144 = 0$

$\Rightarrow$ (x – 24) (x + 6) = 0

$\Rightarrow$ x = 24, – 6

$$\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}=\left(\frac{p+q}{pq}\right)^{-2}=\left(\frac{pq}{p+q}\right)^{2}=\left(\frac{-6}{3}\right)^{2} = 4$$

6. If the common tangent of $\frac{x^2}{16}+\frac{y^2}{9}=1$ and $x^2 + y^2 = 12$ is $y = mx + c$ then the value of $12m^2$ is

(1) 12 (2) 10 (3) 9 (4) 6

Ans. (3)

Sol. Slope of the common tangent is m

Then Equation of tangent to Ellipse is $y = mx \pm \sqrt{16m^2+9}$

Equation of tangent to Circle is $y = mx \pm 2\sqrt{3}\sqrt{1+m^2}$

Since both tangents are identical

$\Rightarrow \quad 16m^2 + 9 = 12 + 12m^2$

$\Rightarrow \quad 4m^2 = 3$

$\Rightarrow \quad 12m^2 = 9$

7. If $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x) + C$, then the value of $g\left(\frac{1}{2}\right)$ is

(1) $\frac{\pi}{3}+\ln\left(2+\sqrt{3}\right)$ (2) $\frac{\pi}{6}+\ln\left(2+\sqrt{3}\right)$ (3) $\frac{\pi}{3}-\ln\left(2+\sqrt{3}\right)$ (4) $\frac{\pi}{6}-\ln\left(2+\sqrt{3}\right)$

Ans. (3)

Sol. $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx$ Put $x = \frac{1}{t}$

$dx = -\frac{1}{t^2}\, dt$

$= \int \frac{-t}{t^2}\sqrt{\frac{t-1}{t+1}}dt$

$= -\int \frac{\sqrt{t-1}}{t\sqrt{t+1}}dt$

$= -\int \frac{t-1}{t\sqrt{t^2-1}}dt$

$= \int \frac{1}{t\sqrt{t^2-1}}dt - \int \frac{dt}{t^2-1}$

$\sec^{-1}t - \ell n\left(t+\sqrt{t^2-1}\right)+C$

$\sec^{-1}\frac{1}{x} - \ell n\left(\frac{1}{x}+\sqrt{\frac{1}{x^2}-1}\right)+C$

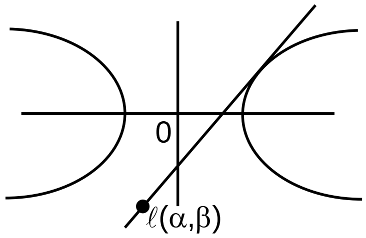
$g\left(\frac{1}{2}\right) = \sec^{-1}(2) - \ell n\left(2+\sqrt{2^2-1}\right)$

$= \frac{\pi}{3} - \ell n\left(2+\sqrt{3}\right)$

8. The locus of point of intersection of any tangent to the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ and a line perpendicular to the tangent and passing through (0,0) is $(x^2+y^2)^2 = \alpha x^2+\beta y^2$, then value of $(\alpha+\beta)$ is

Ans. **(12.00)**

Sol. $\frac{x^2}{16}-\frac{y^2}{4}=1$



tangent $y = mx \pm \sqrt{16m^2-4}$

Given $y=\frac{-x}{m}$

$\Rightarrow m=\frac{-x}{y}$

then $y = \frac{-x^2}{y}+\frac{\sqrt{16.x^2-4y^2}}{y}$

$y^2 + x^2 = \sqrt{16x^2-4y^2}$

$(y^2 + x^2)^2 = 16x^2 - 4y^2$

$\alpha = 16,\ \beta = -4$

the value of $\alpha + \beta = 12$

9. Area bounded between the curve $y^2 = 8x$ and $y^2 = 16(3–x)$ is :

(1) 32 (2) 42 (3) 48 (4) 16

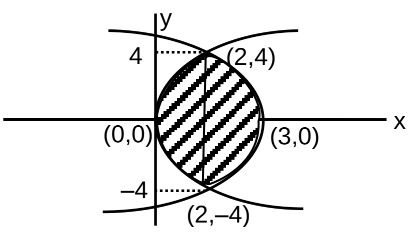
Ans. **(4)**

Sol. $y^2 = 8x$ & $y^2 = 16(3–x)$

Point of intersection $8x = 16 (3–x)$

$x = 6–2x$

$\Rightarrow x = 2 \therefore y = \pm 4$



$$A=2\left(\int_0^2 2\sqrt{2}\sqrt{x}\,dx+\int_2^3 4\sqrt{3-x}\,dx\right)$$

$$= 4\sqrt{2}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 + 8\left[-\frac{(3-x)^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^3$$

$$= \frac{8.\sqrt{2}}{3}.2\sqrt{2} - \frac{16}{3}(0-1)$$

$$= \frac{32}{3} + \frac{16}{3} = \frac{48}{3} = 16$$

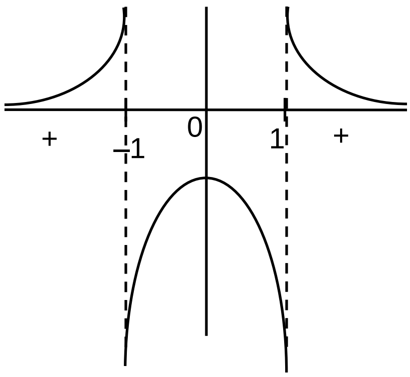
10. If f(x) = x–1 : R $\rightarrow$ R and g(x) = $\frac{x^2}{x^2-1}$: R – {–1,1} $\rightarrow$ R then f(g(x)) is

(1) one-one and onto function
(2) many-one and into function
(3) one-one and into function
(4) many-one and onto function

Ans. **(2)**

Sol. $f(g(x)) = f\left(\frac{x^2}{x^2-1}\right)$ $\quad x \neq \pm 1$

$= \frac{x^2}{x^2-1} - 1 = \frac{1}{x^2-1}$ $\quad x \neq \pm 1$



11. If $^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \ldots\ldots\ldots\ldots + {}^{60}C_{20} = \frac{m}{n}\ ({}^{60}C_{20})$ where m & n are co -prime then the value of m + n is

(1) 105 (2) 102 (3) 107 (4) 109

Ans. **(2)**

Sol. $^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \ldots\ldots\ldots\ldots + {}^{60}C_{20}$

$= {}^{40}C_{40} + {}^{41}C_{40} + {}^{42}C_{40} + \ldots\ldots\ldots\ldots + {}^{60}C_{40}$

$= {}^{41}C_{41} + {}^{41}C_{40} + {}^{42}C_{40} + \ldots\ldots\ldots\ldots$

$= {}^{42}C_{41} + {}^{42}C_{40} + \ldots\ldots\ldots\ldots$

$\vdots$

$= {}^{60}C_{41} + {}^{60}C_{40} = {}^{61}C_{41} = \frac{61}{41} \times {}^{60}C_{40}$

$= \left(\frac{61}{41}\right)\ {}^{60}C_{20}$

So, m = 61, n = 41

m + n = 102

12. The equation of normal to the hyperbola $\frac{x^2}{\alpha}+\frac{x^2}{9}$ = 1 at point $(8,3\sqrt{3})$

(1) $4x + \sqrt{3}y = 41$ (2) $4x - \sqrt{3}y = 41$ (3) $\sqrt{3}x + 4y = 41$ (4) $\sqrt{3}x - 4y = 41$

Ans. (1)

Sol. $\because$ $(8,3\sqrt{3})$ lies on Hyperbola $\frac{x^2}{\alpha}+\frac{x^2}{9} = 1$

$\Rightarrow \quad \frac{64}{\alpha}+\frac{27}{9} = 1 \Rightarrow \quad \alpha = -32$

So equation of hyperbola is $-\frac{x^2}{32}+\frac{y^2}{9}=1$

Differentiating w.r.t. x $\Rightarrow -\frac{2x}{32}+\frac{2y}{9}\frac{dy}{dx}=0$

$\Rightarrow \quad \frac{dy}{dx} = \frac{9x}{32y}$

Slope of normal at $(8,3\sqrt{3}) \Rightarrow -\frac{32(3\sqrt{3})}{9(8)} = -\frac{4\sqrt{3}}{3}$

Equation of normal $\Rightarrow y - 3\sqrt{3} = -\frac{4}{\sqrt{3}}(x-8)$

$\Rightarrow \quad \sqrt{3}y - 9 = -4x + 32$

$\Rightarrow \quad 4x + \sqrt{3}y = 41$

13. The value of $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)+\frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$ is-

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Ans. **(4)**

Sol. $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right)+\frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$

$\cos^{-1}\left(\frac{3}{10}\cos\left(\cos^{-1}\frac{3}{5}\right)+\frac{2}{5}\sin\left(\sin^{-1}\frac{4}{5}\right)\right)$

$\cos^{-1}\left(\frac{3}{10}\times\frac{3}{5}+\frac{2}{5}\times\frac{4}{5}\right) = \cos^{-1}\left(\frac{9}{50}+\frac{8}{25}\right)$

$\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$

14. There are 50 observation $x_1,x_2,x_3 ....x_{49}$, a. The mean and standard deviation of these observation are 15 and 2 respectively. Now if 'a' is replaced by b, then mean of these 50 observations so obtained is 16 then the variance of new 50 observations is (it is given than a + b = 70)

Ans. (46)

Sol. Given $\frac{x_1+x_2..........+x_{49}+a}{50}=15$

$= \sum x_i + a = 750$

$\therefore \quad x_i = 750 - a$

$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$

$\Rightarrow \; 4 = \frac{\sum x_i^2 + a^2}{50} - 225$

$\Rightarrow \sum x_i^2 + a^2 = 11450$

New sum $\sum x_i + b$ = 750–a + b

$\therefore$ New mean $\frac{750 - a + b}{50} = 16$

–a + b = 800 – 750 = 50

a +b = 70

$\therefore$ 2b = 120 $\Rightarrow$ b = 60 & a = 10

$\therefore$ New $\Sigma x^2 = 11450 - a^2 + b^2$

= 14950

$\therefore$ new $\sigma^2 = \frac{\sum x^2}{n} - (16)^2 = \frac{14950}{50} - 256$

= 46

15. If the compound statement $(r \vee \sim p) \rightarrow (p \wedge q) \vee r$ is tautology then r is equivalent to

(1) ~ p (2) p (3) q (4) ~ q

Ans. (1)

Sol. $(r \vee \sim p) \rightarrow (p \wedge q) \vee r$

$\Rightarrow \sim (r \vee \sim p) \vee (p \wedge q) \vee r$

$\Rightarrow (\sim r \wedge p) \vee (p \wedge q) \vee r$

$\Rightarrow ((\sim r \wedge p) \vee r) \vee (p \wedge q)$

$\Rightarrow ((\sim r \vee r) \wedge (p \vee r)) \vee (p \wedge q)$

$\Rightarrow (t \wedge (p \vee r)) \vee (p \wedge q)$

$\Rightarrow (p \vee r) \vee (p \wedge q)$

$\Rightarrow r \vee (p \vee (p \wedge q))$

$\Rightarrow r \vee p$

will be a tautology if r = ~p.

16. If the probability that 6 digits number formed using digits 8 and 1. Which is divisible by 21 is P then the value of 96 P is

Ans. (33)

Sol. No. of numbers formed by 8 and 1 of 6 digits which is divisible by 3 as well as 7 will contain three 8 and three 1 like given below .

8	8	8	1	1	1

1	8	1	8	1	8

and any number of 6 digits using same number will be divisible by 3 and 7 so two cases will arise like

1	1	1	1	1	1

&

8	8	8	8	8	8

R. Prop = P = $\frac{\frac{\lfloor 6}{\lfloor 3 \lfloor 3} + 2}{2^6} = \frac{11}{32}$

96 P = 96 × $\frac{11}{32}$ = 33