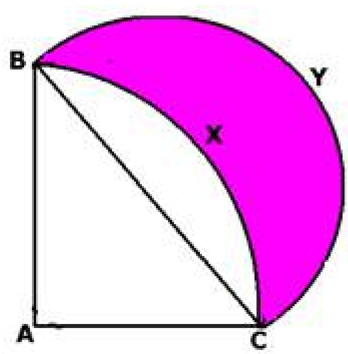
CBSE Board Class 10 Maths Chapter 12- Areas related to Circles
Objective Questions

Introduction

1. Find the area of the shaded region where ABC is a quadrant of radius 5cm and a semicircle is drawn with BC as diameter.



(A) 8.8 cm^2
(B) 7.14cm^2
(C) 12.5cm^2
(D) 19.64cm^2

Answer: (C)12.5 cm^2

Solution: Area of the shaded region = Area of semicircle-Area of segment of the sector BAC

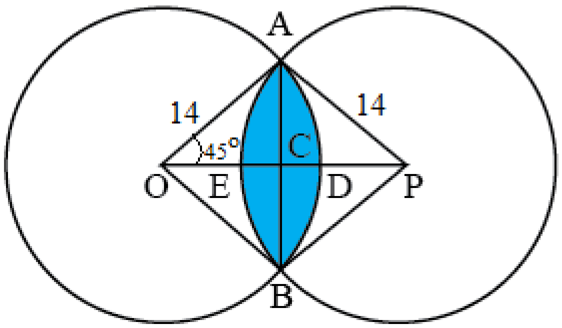
Area of the semicircle with BC as diameter = $\frac{1}{2} \times \frac{22}{7} \times \frac{5}{\sqrt{2}} \times \frac{5}{\sqrt{2}} =$ 19.64cm^2

Area of segment = Area of quadrant - Area of ΔABC

= 90/360 × 22/7 × 5^2- ½ × 5 × 5=19.64-12.5=7.14 cm^2..... (ii)

Area of the shaded region = (i) - (ii) = 19.64 - 7.14 = 12.5 cm^2

2. Area of the shaded portion in the following figure is equal to area of.

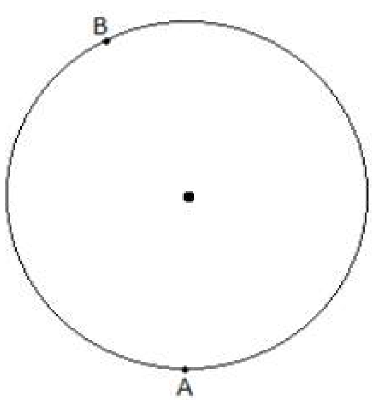


(A) sector OADBO - segment ADBA
(B) segment AEBA
(C) segment ADBA
(D) segments ADBA and AEBA

Answer: (D) segments ADBA and AEBA

Solution: The area of the shaded region will be the sum of areas of the segments of the circles that form the region. In this case, it is segments ADBA and AEBA.

3. Consider a point A on the circle of radius $7/\pi$ cm as shown in the figure. A ball on point A moves along the circumference until it reaches a point B. The tangent at B is parallel to the tangent at A. What is the distance travelled by the ball? (Consider the ball to be a point object)

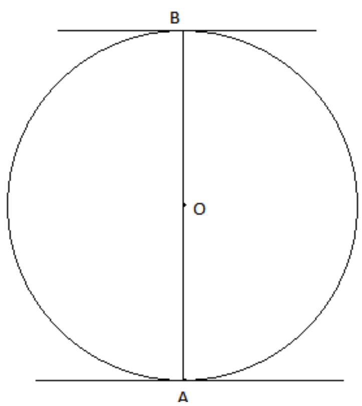


Note: The point B in the diagram may not represent its actual position.

(A) 3.5cm
(B) 7cm
(C) 14cm
(D) 28cm

Answer: (B) 7cm

Solution: Given that the tangent at point B is parallel to the tangent at point A. The points A and B would be as follows



The tangents at A and B are parallel, so the radii through A and B should be parallel to each other since the radius is perpendicular to the tangent at the point of contact. Since the radius passes through the center of the circle, the radii through A and B must lie on the same straight line. The line AB will be the diameter of the circle.
Now A and B are on two ends of the diameter of the circle. The distance travelled along the circle from A to B will be half the circumference of the circle.
Now, Circumference
$= 2\pi r = 2 \times \pi \times 7/\pi = 14cm$

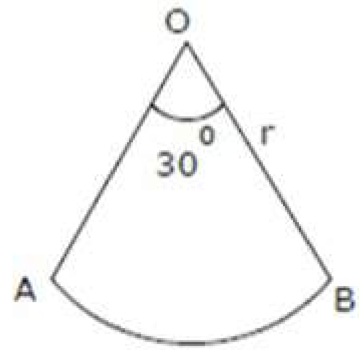
Distance
$= 1/2 \times circumference = 1/2 \times 14cm = 7cm$

4. A pendulum swings through on angle of 30∘ and describes an arc 8.8 cm in length. Find the length of pendulum in cm.

(A) 16.8
(B) 17.3
(C) 15.1
(D) 14.5

Answer: (A) 16.8

Solution:



Let r be the length of the pendulum

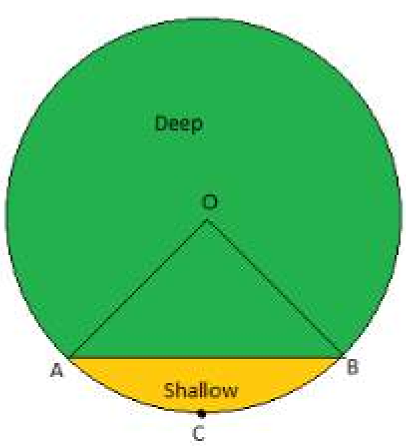
Length of arc = 8.8 cm

∠AOB = 30 °

Length of an arc of a sector of an angle θ = (θ/360) × 2πr

Length of arc = (θ/360) ×2πr
8.8= (30/360) × 2 × (22/7) × r
r= (8.8×21)/11=16.8 cm

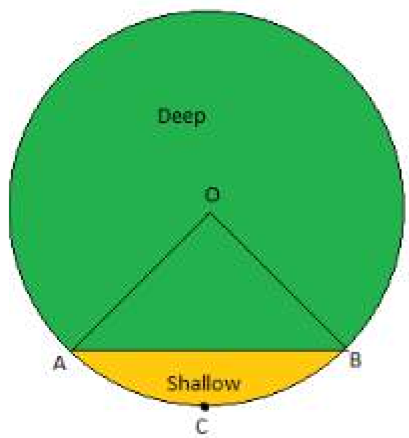
5. There is a circular swimming pool with center O. The radius of pool is 5 m. There are 2 points on the wall of the pool separated by distance of 7 m. These 2 points are named A and B. A rope is attached between A and B. This rope separates the shallow section of pool from deep section of pool. The shallow section is the smaller section. Which of following statements are true?



(A) The shallow section is an arc.
(B) The area of circle between OA and OB is an arc.
(C) The shallow section is a segment
(D) The shallow section is a sector

Answer: (C) The shallow section is a segment

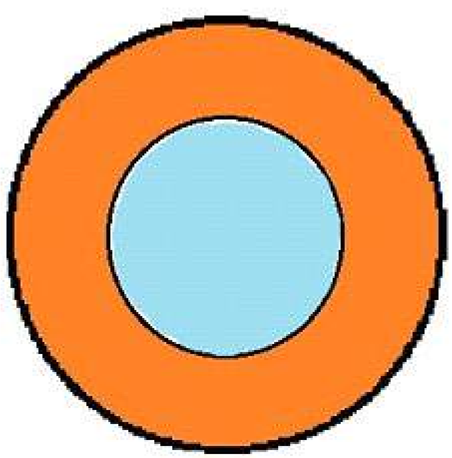
Solution:



Segment is that part of a circle which is made by a line and a connecting arc. Segments touches any two points in a circle. So here the shallow section is a segment.

Visualizations

6. In the figure, the area of the portion in orange color is

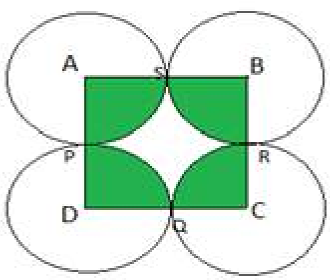


(A) Area of outer circle + Area of inner circle
(B) Area of outer circle – Area of inner circle
(C) Area of inner circle – Area of outer circle
(D) Area of outer circle

Answer: (B) Area of outer circle – Area of inner circle

Solution: (Area of outer circle – Area of inner circle) gives the area of the orange coloured region in the above figure.

7. Given below is a combination figure of square ABCD of side 26cm and four circles. Find the area of the shaded region.



(A) 530.64 cm^2
(B) 402.83cm^2
(C) 360cm^2
(D) 480.53cm^2

Answer: (A) 530.64 cm^2

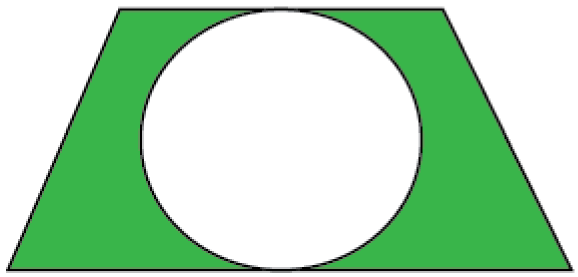
Solution: The given figure forms four sectors:

Area of a sector of angle $\Theta = \Theta/360^\circ \times \pi r^2$

Area of one sector APS = $(90^\circ/360^\circ) \times \pi \times 13^2 = 132.66$ cm^2

Total area of shaded region = Area of four sectors=4 × 132.66 cm^2 = 530.64cm^2

8. In the given figure, a circle is inscribed in a trapezium of height 14 cm and lengths of parallel sides are equal to 25 cm and 40 cm. What is the area of the shaded region?



(A) 455 sq cm
(B) 154 sq cm
(C) 509 sq cm
(D) 301 sq cm

Answer: (D) 301 sq cm

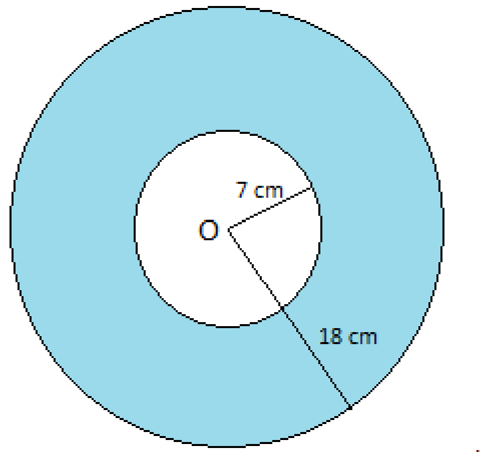
Solution: Area of the shaded region = Area of the trapezium - Area of the circle
Area of trapezium =1/2 × height × sum of parallel sides
=1/2×14× (25+40) =455 sq cm
Area of the circle = $\pi \times r^2$= (22/7) ×7×7=154 sq cm
Hence, area of the shaded region = 455 - 154 = 301 sq cm

9. Radius of the outer circle is 18 cm and the radius of the inner circle is 7 cm. What is the area of the region between the outer and the inner circles?

(A) 361 πcm^2
(B) 133 cm^2
(C) 192.5 cm^2
(D) 275 πcm^2

Answer: (D) 275 πcm^2

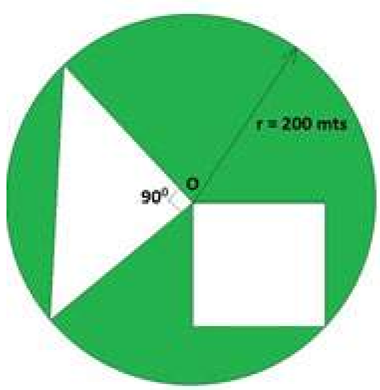
Solution:



Area of the region in between outer and inner circle = Area of outer circle – Area of inner circle

Area of the outer circle = $\pi\ (18)^2 = 324\ \pi cm^2$
Area of the inner circle = $\pi\ (7)^2 = 49\ \pi cm^2$
So, area of the required region = $324\ \pi - 49\pi = 275\ \pi cm^2$

10. A stadium is in circular shape. Within the stadium some areas have been allotted for a hockey court and a javelin range, as given in the figure. Assume the shape of the hockey court and the javelin range to be square and triangle, resp. The curators would like to accommodate a few more sports in the stadium. Help them by measuring the unallocated region within the stadium. (the radius of the stadium is 200 mts.)



(A) $40000\pi\ m^2$
(B) $40000(\pi-1)\ m^2$
(C) $20000(\pi-1)\ m^2$
(D) $20000\pi\ m^2$

Answer: (B) $40000(\pi-1)\ m^2$

Solution: We need to find the unallocated area within the stadium.

The unallocated area should be = The total circular area of the stadium – Area of the hockey court - Area of the Javelin Range.

The area of circular stadium = $\pi \times r^2$

= $\pi \times 200^2$

= 40000π

The area of hockey court (square), we know that the radius of the stadium forms the diagonal of the hockey court.

Therefore the sides of the hockey court will be $a = \frac{r}{\sqrt{2}}$, (applying **Pythagoras Theorem** in a square)

Then, the area of the square $= a^2$

$$= \left(\frac{r}{\sqrt{2}}\right)^2$$

$= r^2/2$

$= 200^2/2$

$= 20000\ m^2$

Javelin Range is a right angled triangle

area of the right triangle = $\frac{1}{2} \times r \times r$

$= \frac{1}{2} \times 200 \times 200$

$= 20000\ m^2$

Therefore the unallocated area in the stadium =Total area of stadium – Area of the hockey court – Area of the Javelin Range

$=40000\pi - 20000 - 20000$
$=40000(\pi - 1)\ m^2$

The unallocated area within the stadium is $40000(\pi - 1)\ m^2$**.**

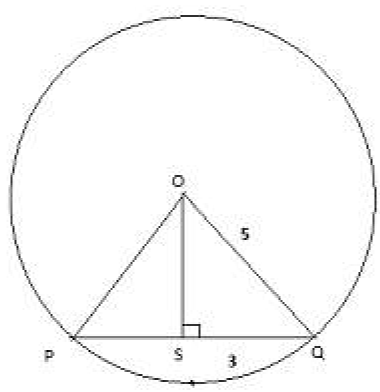
Areas of sector and Segments related to circles

11. There is a circle of diameter 10 cm. A chord of length 6 cm is drawn inside the circle. What is the distance between the centre and this chord in cm?

(A) 1.5
(B) 2
(C) 4
(D) 3

Answer: (C) 4

Solution:



Radius = 1/2 of diameter = 5 cm

OS is drawn from O and is perpendicular to chord PQ and bisects it.

Hence SQ = 3 cm

Hence length of OS is distance between O and PQ.

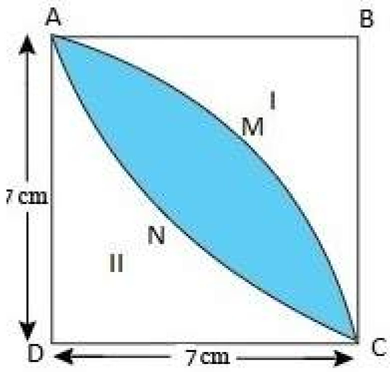
$OQ^2 = OS^2 + SQ^2$

$5^2 = OS^2 + 3^2$

$\Rightarrow OS = 4$ cm.

Hence distance between centre and chord is 4 cm.

12. Find the area of the shaded region

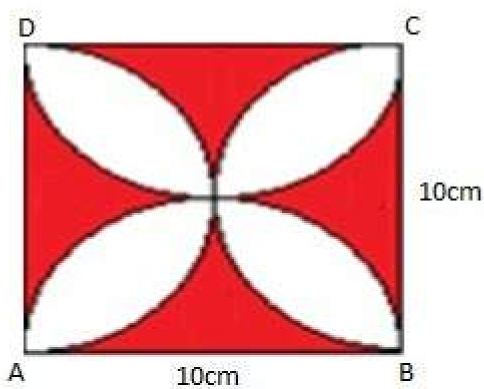


(A) 24 cm^2
(B) 25cm^2
(C) 28cm^2
(D) 21cm^2

Answer: (C) 28cm^2

Solution: Area of square ABCD = 7×7 =49cm^2
Area of AMCDA = ¼ × π× 7^2 = 1/4×22/7×7^2 = 38.5cm^2
Area of ADCNA = Area of AMCDA = 38.5cm^2
Area of shaded region = Area of ADCNA + Area of AMCDA - Area of square ABCD
→2× Area of ADCNA - Area of square ABCD
= 2×38.5 - 49 =28cm^2

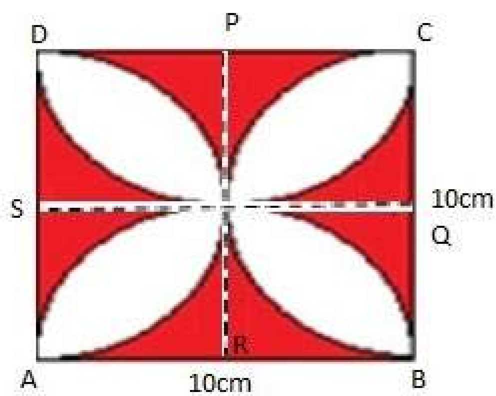
13. Find the area of the shaded region



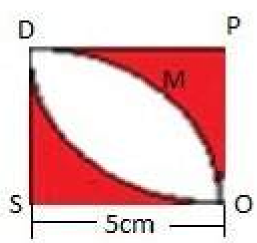
(A) $38cm^2$
(B) $57cm^2$
(C)$43cm^2$
(D) $62cm^2$

Answer: (C) $43cm^2$

Solution: Divide the square ABCD into equal four parts



and take one part which is given by



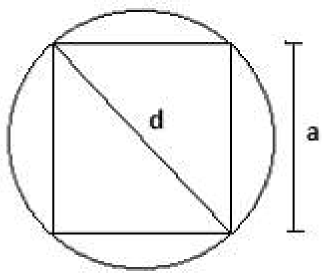
Area of shaded region in this part = Area of square DPOS - Area of the unshaded region and area of PDMO = $5^2 - ¼ \times \pi \times r^2$ = 25− ¼ ×3.14×25 = 5.375cm2

Therefore area of the shaded region in square DPOS = 5.375×2 = 10.75 cm^2

As DPOS is a quarter of square ABCD,
Total area of the shaded region = 4×10.75= $43cm^2$

Areas of combination of plane figures

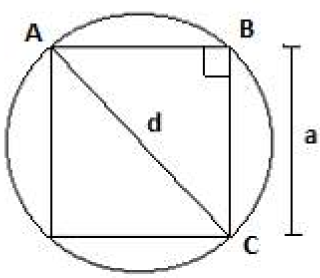
14. If a square with side ‘a’ is inserted within a circle such that the corners coincide with the circumference of the circle with diameter‘d’. Find the relation between ‘a’ and‘d’.



(A) $a = \frac{d}{\sqrt{2}}$
(B) a=d/2
(C) a=2d
(D) a=d

Answer: (A) $a = \frac{d}{\sqrt{2}}$

Solution: We are asked to find the relation between the diameter of the circle and the side of the square.



Let us consider the triangle ABC, $\angle ABC = 90^\circ$

Therefore we can consider triangle ABC as a right angled triangle with sides = ‘a’, and hypotenuse =‘d’.

Applying the Pythagoras Equation, we get $d^2=a^2+a^2$

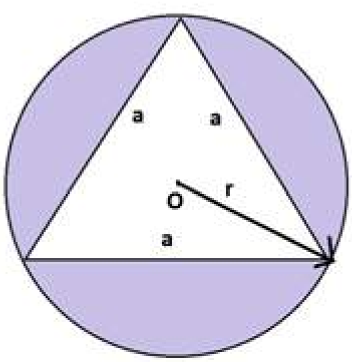
$\Rightarrow a^2 = \frac{d^2}{\sqrt{2}}$

$\Rightarrow$ a = $\sqrt{(\frac{d^2}{2})}$

$\Rightarrow$a= $\frac{d}{\sqrt{2}}$

Therefore the relation between the sides of the square and the diameter of the circle is, a= $\frac{d}{\sqrt{2}}$

15. If an equilateral triangle is drawn inside a circle such that the circle is the circum-circle of the triangle, find the relation between the length of the triangle and the radius of the circle.



(A) $\sqrt{3} \times r$

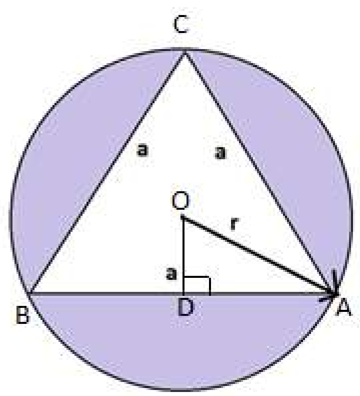
(B) $\frac{r}{\sqrt{3}}$

(C) 3r
(D) r/3

Answer: (A) $\sqrt{3} \times r$

Solution: Here we need to find the relation between the circum radius and the side length of the equilateral triangle.

Let us consider the radius of the circle is ‘r’ and the side length of the triangle is ‘a’.



Construction: Join OA. Draw OD perpendicular to AB. Since OD is perpendicular to AB it bisects the chord. Therefore, AD = 1/2×AB

In the triangle AOD,

∠AOD=1/2∠AOB (triangle AOB is isosceles triangle)

Because, angle subtended by a chord A the centre O is double the angle subtended by the chord at any point on the circumference.

∠AOB=2×∠ACB ∠ACB is internal angle of the equilateral triangle)

= 2×60°

=120°

Therefore, ∠AOD=12×120°

∠AOD=60°

∠ADO=90° (OD perpendicular to AB)

Applying trigonometric properties in rt triangle AOD,

AD/AO = sin 60

AD = AO Sin 60

$$AD = \frac{\sqrt{3}}{2} \times AO$$

$$AD = \frac{\sqrt{3}r}{2}$$

AB = 2 x AD

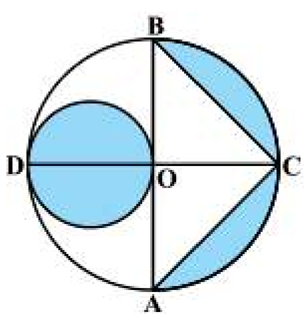
$$= 2 \times \frac{\sqrt{3}r}{2}$$

$$a = \sqrt{3} \times r$$

The relation between the side of the equilateral triangle and the radius of circumcircle is,

$$a = \sqrt{3} \times r$$

16. In the figure below, AB and CD are two diameters of a circle (with center O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.



(A) 65.5 cm^2
(B) 66.5 cm^2
(C) 67.5 cm^2
(D) 68.5 cm^2

Answer: (B) 66.5 cm^2

Solution: Radius of larger circle, R = 7 cm
Radius of smaller circle, r=7/2cm
Height of ΔBCA=OC=7 cm
Base of ΔBCA=AB=14 cm
Area of $\Delta BCA=1/2\times AB\times OC=1/2\times7\times14=49\ cm^2$
Area of larger circle $=\pi\times R^2=22/7\times7^2=154cm^2$
Area of larger semicircle $=154/2cm^2=77\ cm^2$
Area of smaller circle $=\pi\times R^2=22/7\times7/2\times7/2=77/2cm^2$
Area of the shaded region = Area of larger circle - Area of triangle - Area of larger semicircle + Area of smaller circle
Area of the shaded region $=(154-49-77+77/2)cm^2$
$=133/2cm^2=66.5\ cm^2$

Arc length

17. A wire is bent to form a circle of radius 7 cm. From the resulting shape, a chunk of the wire is cut off, and the wire cut off measures 4 cm in length. The length of the remaining wire is

(A) 45cm
(B) 50cm
(C) 40cm
(D) 42cm

Answer: (C) 40cm

Solution: The length of the wire = circumference of the circle with radius 7cm
$=2\times\pi\times r=2\times22/7\times7=44$ cm.
When a wire of length 4 cm is cut off, the length of the wire remaining = circumference - length of arc cut off = 44 - 4 = 40 cm.

Perimeter and areas related to circles

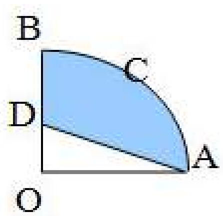
18. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(A) 3 units
(B) 4 units
(C) π units
(D) 2 units

Answer: (D) 2 units

Solution: Let the radius of the circle be r.
∴ Perimeter of the circle = Circumference of the circle $=2\pi r$
∴ Area of the circle $=\pi r^2$
Given: Area = Perimeter
$\Rightarrow 2\pi r=\pi r^2$
$\Rightarrow 2=r$
Thus, the radius of the circle is 2 units.

19. In the given figure below, OACB is a quadrant of a circle. The radius OA = 3.5 cm, OD = 2 cm. Calculate the area of the shaded region.



(A) 5.125cm2
(B) 6.5cm2
(C) 7cm2
(D) 6.125cm2

Answer: (D) $6.125 cm^2$

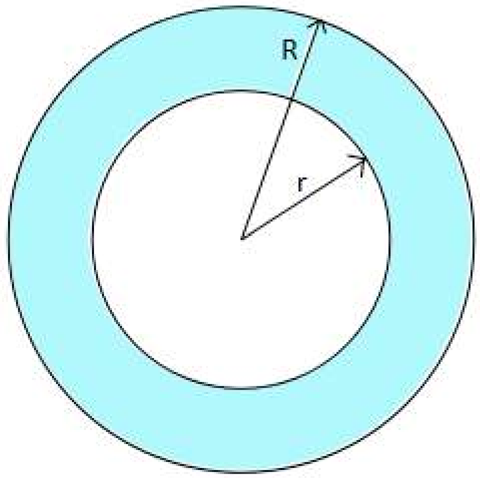
Solution: Area of a quadrant of a circle = 1/4× area of the circle.
∴ Area of the shaded region = 1/4× Area of the circle - Area of the triangle.
⟹ Area of the shaded region = $1/4\times\pi r^2 \; -1/2 base\times height$
⟹ Area of the shaded region = $1/4\times22/7\times3.5\times3.5 -1/2\times3.5\times2$
⟹ Area of the shaded region = 9.625 - 3.5 = 6.125 cm^2

20. The shaded area in the adjoining figure, between the circumferences of two concentric circles is 346.5 cm^2. The circumference of the inner circle is 88 cm. Calculate the radius of the outer circle. [Take $\pi=22/7$]

(A) 35 cm
(B) 32 cm
(C) 17.5cm
(D) 16.5cm

Answer: (C) 17.5cm

Solution:



Let the radii of inner circle and outer circle be r and R respectively.
Then, $2 \times 22/7 \times r = 88$
$\therefore r = 14$ cm.

Area of the inner circle = $22/7 \times 14 \times 14 cm^2 = 616 cm^2$
$\therefore$ Area of the outer circle = $(616 + 346)\ cm^2 = 962.5\ cm^2$
Then, $22/7 \times R^2 = 962.5$
$\Longrightarrow R^2 = 962.5 \times 7/22 = 6.25 \times 7 \times 7$
$\Longrightarrow R = 2.5 \times 7 = 17.5$ cm