ALL INDIA TEST SERIES

TEST – 31

JEE (Advanced)

Time Allotted: 3 Hours **Maximum Marks: 198**

General Instructions:

- The test consists of total 54 questions.
- Each subject (PCM) has 18 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections**: **Section-A & Section-C.**

  Section-A (01 – 06, 19 – 24, 37– 42) contains **18 multiple** choice questions which have **ONLY ONE CORRECT ANSWER**. Each question carries **+3 marks** for correct answer and **–1 mark** for wrong answer.

  Section-A (07 – 12, 25 – 30, 43 – 48) *this section contains* ***18 multiple*** *choice questions.*
  Each question has FOUR options. ***ONE OR MORE THAN ONE*** *of these four option(s) is (are) correct answer(s).*
  For each question, choose the option(s) corresponding to (all) the correct answer(s)
  Answer to each question will be evaluated according to the following marking scheme:

  Full Marks **: +4** If only (all) the correct option(s) is (are) chosen:
  Partial Marks **: +3** If all the four options are correct but ONLY three options are chosen;
  Partial Marks **: +2** If three or more options are correct but ONLY two options are chosen and both of which are correct;
  Partial Marks **: +1** If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks **: 0** If none of the options is chosen (i. e. the question is unanswered);
  Negative Marks **: –2** In all other cases.

  Section-C (13 – 18, 31– 36, 49 – 54) *contains* ***18 Numerical*** *answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer and* ***0 marks*** *for wrong answer.*

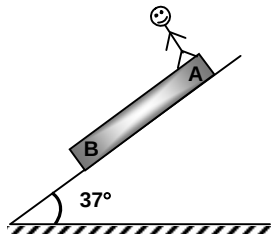
Physics

PART – I

SECTION – A
(One Options Correct Type)

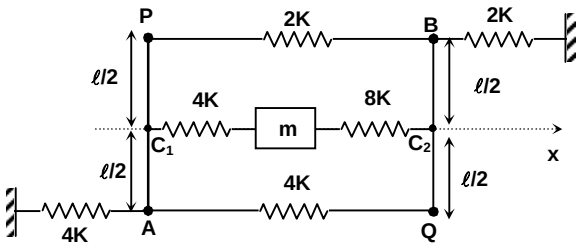
This section contains **06 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. A plank of mass 150 kg and length 16 m is placed on a rough inclined plane and a man of mass 50 kg walks down the plank. The coefficient of friction between plank and inclined plane is $\frac{1}{2}$, and the man starts moving downward with minimum acceleration such that plank does not slide. Find the time needed when the man moves from end A to end B of the plank. ($g = 10\ m/s^2$)



(A) 2 sec
(B) 4 sec
(C) 6 sec
(D) 8 sec

2. In the figure shown, arrangement is placed on horizontal frictionless plane. The mass of rod PA and QB are negligible and they are free to rotate about hinge points P and Q respectively in the horizontal plane. The block of mass 'm' is attached to springs. Springs are light and their stiffness are shown in the figure. The block is slightly displaced along x–axis then released. Then the time period of oscillation of the block is



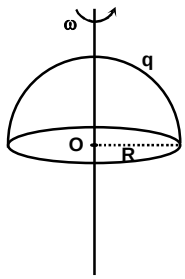
(A) $\frac{3\pi}{2}\sqrt{\frac{m}{5k}}$

(B) $\frac{3\pi}{4}\sqrt{\frac{m}{5k}}$

(C) $\frac{3\pi}{2}\sqrt{\frac{5m}{k}}$

(D) $\frac{3\pi}{4}\sqrt{\frac{5m}{k}}$

3. A hollow non conducting hemisphere of radius R has a positive charge q uniformly distributed on its surface. The hemisphere starts rotating with a constant angular velocity 'ω' about an axis passing through center of sphere, as shown in the figure. Then the net magnetic field at centre 'O' of the sphere is



(A) $\frac{\mu_0 q\omega}{2\pi R}$

(B) $\frac{\mu_0 q\omega}{3\pi R}$

(C) $\frac{\mu_0 q\omega}{6\pi R}$

(D) $\frac{\mu_0 q\omega}{12\pi R}$

4. Suppose that one of the slits in YDSE is wider than the other, so the amplitude of the light reaching on the screen from one slit, acting alone, is five times that from the other slit, acting alone. The intensity for central bright fringe is I_0, then find the intensity I on the screen for (angular position) $\theta = 37°$. (Take $\frac{d}{\lambda} = \frac{5}{18}$, where d is separation between the slits and λ is wavelength of the source)

(A) $\left(\frac{124}{225}\right)I_0$

(B) $\left(\frac{93}{225}\right)I_0$

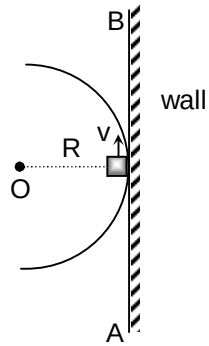
(C) $\left(\frac{62}{225}\right)I_0$

(D) $\left(\frac{31}{225}\right)I_0$

5. A cubical block 'A' of mass m_0 ($=a^3\rho$ = 3kg) of edge 'a' and density 'ρ' floats in a liquid of density '3ρ'. The lower surface of the cube just touches the free end of massless spring of spring constant k($=a^2\rho g$) fixed at the bottom of the vessel. Another block 'B' of mass 'm' placed over the block 'A', so that, the cube is immersed in liquid upto $\frac{2}{3}$ of cube height in equilibrium. Then the value of 'm' is

(A) 3 kg
(B) 6 kg
(C) 8 kg
(D) 4 kg

6. As shown in the figure AB represents an infinite wall tangential to horizontal semi-circular track. O is a point source of light on the ground at the centre of the circle. A block moves along the circular track with a constant speed v starting from the point where the wall touches the circular track. Then the acceleration of the shadow of the block along the length of wall is



(A) zero

(B) $\left(\frac{v^2}{R}\right)\sec^2\left(\frac{vt}{R}\right)\tan\left(\frac{vt}{R}\right)$

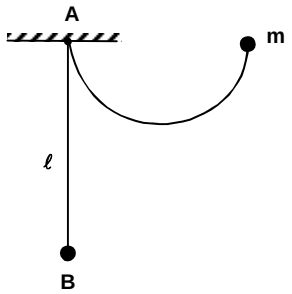
(C) $\left(\frac{v^2}{2R}\right)\sec^2\left(\frac{vt}{R}\right)\tan\left(\frac{vt}{R}\right)$

(D) $\left(\frac{2v^2}{R}\right)\sec^2\left(\frac{vt}{R}\right)\tan\left(\frac{vt}{R}\right)$

(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

7. A ball of mass m = 100 gm is suspended from a point A by an inextensible thread of length ℓ. Ball is drawn aside and held at same level as A but at a distance $\frac{\sqrt{3}}{2}\ell$ from A as shown in the figure. Now ball is released from rest. Then which of the following is/are correct. (Assume string applies only that much jerk which is required so that velocity along string becomes zero)



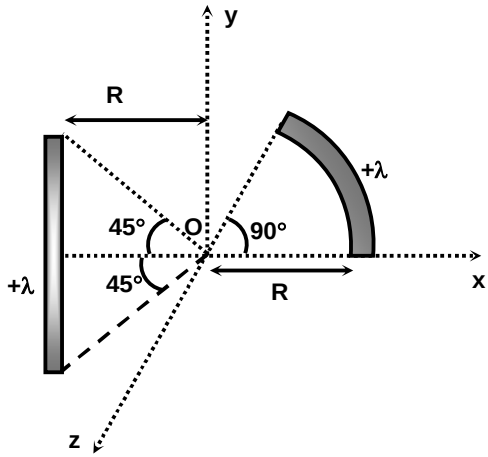
(A) Velocity of ball just before experiencing jerk is $\sqrt{g\ell}$

(B) Velocity of ball just after experiencing jerk is $\frac{\sqrt{3g\ell}}{2}$

(C) Impulse applied by string is $\frac{\sqrt{g\ell}}{20}$

(D) Velocity of ball when the ball comes at point B is $\frac{\sqrt{7g\ell}}{2}$

8. A rod having charge per unit length λ = 2 Coulomb/meter lies parallel to y-axis and a quarter ring having same charge density lies in x-z plane as shown in the figure (R = 2 m). Then



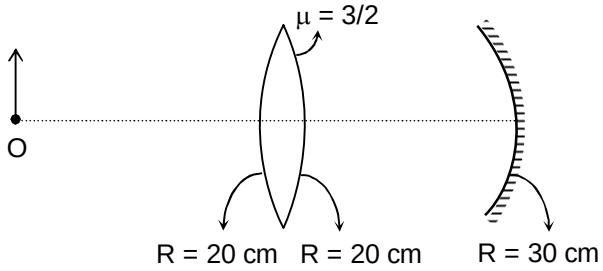
(A) The magnitude of electric field due to the rod is more than the electric field due to the quarter ring at the point 'O'

(B) The net electric field at point 'O' is zero.

(C) The magnitude of electric field due to the rod and due to the quarter ring at the point 'O' are same.

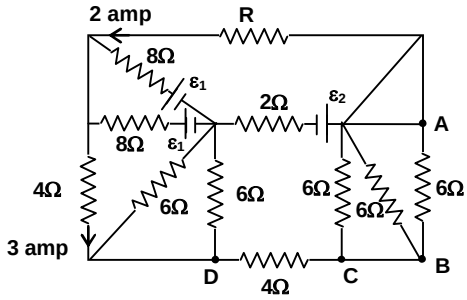
(D) The magnitude of net electric field at 'O' is $\left(\frac{\sqrt{4-2\sqrt{2}}}{4\pi\varepsilon_0}\right)\frac{\lambda}{R}$.

9. An object is placed in front of converging lens at a distance 40 cm from the lens. On the other side of the lens is concave mirror of focal length 15 cm separated from the lens by a distance 70 cm. Light from the object passes rightward through the lens and reflects from the mirror, passes leftward through the lens, and forms a final image of the object. Then which of the following is/are correct.



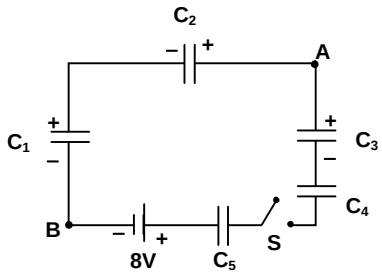
(A) The distance between the lens and the final image is equal to 40 cm.
(B) The distance between the mirror and the final image is equal to 110 cm.
(C) The final image lies in the same plane as the object.
(D) The final image is real, inverted and of the same size as that of the object.

10. In the circuit shown, ε_1 and ε_2 are two ideal sources of unknown emfs. Some currents are shown in some branches of the circuit. Potential difference appearing across resistance 6 Ω is $V_A - V_B = 10$ V. Then



(A) The current in the CD branch is 5 amp.
(B) The unknown emf ε_1 is 40 volts.
(C) The unknown emf ε_2 is 68 volts.
(D) The unknown resistance R is 9 Ω.

11. Three capacitors C_1 = 12F, C_2 = 6F and C_3 = 8F are separately charged with battery such that charge on capacitors are 48 coulomb, 18 coulomb and 56 coulomb respectively and then disconnected with the battery. After they are connected in a circuit along with two uncharged capacitors C_4 = 2F and C_5 = 8F and a battery of emf = 8 volts as shown in figure. Now the switch S is closed. Choose the correct option(s).



(A) Final charge on the capacitor C_4 is 8 coulomb.
(B) Final charge on the capacitor C_3 is 48 coulomb.
(C) Potential difference between A and B is 9 volt.
(D) Amount of charge flown in the circuit is 8 coulomb.

12. A solid sphere A moving with a speed v_0 and rotating with an angular velocity ω_0, makes a head on elastic collision with a hollow sphere (of same mass) B. There is no friction between the surface of A and B (Neglect the gravity). Then which of the following is/are correct.

(A) A will stop moving but continue to rotate with an angular speed ω_0.
(B) A will come to rest and stop rotating.
(C) B will move with same speed v_0 without rotating.
(D) B will move with a speed v_0 and rotate with an angular speed ω_0.

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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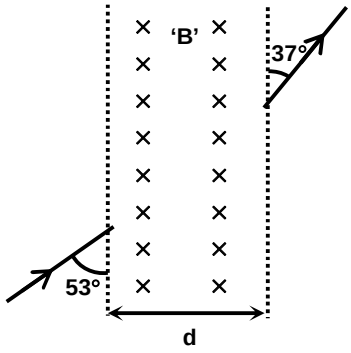
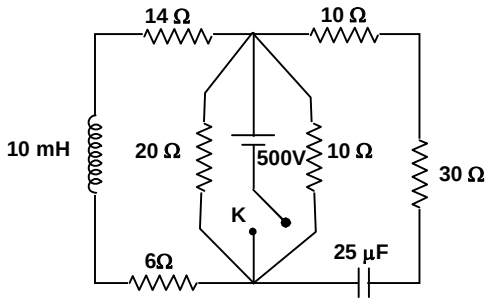
13. A cubical box of side 5 m contains helium gas at a pressure 320 N/m^2. During an observation time of 1 second, an atom travelling with root mean square speed parallel to one of the sides of the cube was found to make 1000 hits with a particular wall, without any collision with other atoms. (take $R = \frac{25}{3}$ J/mol-K). Then find the total mass (in gm) of helium gas in the box.

14. A positive charge particle enters and comes out from a uniform magnetic field which exists with direction inside the paper in a finite space as shown in the figure. If the radius of curvature of the charge particle is 4 m, when it is inside the magnetic field. Find the width 'd' (in cm) of the region in which magnetic field exists.

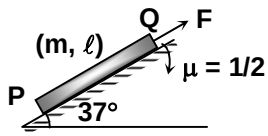


15. Two monochromatic (wavelength λ = a/5) and coherent sources of electromagnetic waves are placed on the x-axis at the point (2a, 0) and (–a, 0). A detector moves in a circle of radius R(R>>2a) whose centre is at origin. Find the number of maximas detected during one circular revolution by the detector.

16. In the circuit shown, the key(K) is closed at t = 0. Find the current (in ampere) through the key(K) at the instant t = $10^{-3}\ell n2$ sec.



17. A constant force F = 20N is applied on the rod of mass m = 8kg and length ℓ = 10m placed on a rough inclined plane $\left(\mu = \frac{1}{2}\right)$ of inclination θ = 37° as shown in the figure. A transverse wave pulse is created at the end point P. Find the time (in sec) needed to move the pulse from end P to end Q. (g = 10 m/s^2)



18. A target element A is bombarded with electrons and the wavelengths of the characteristic spectrum are measured. A second characteristic spectrum is also obtained, because of an impurity in the target. The wavelengths of the K_α lines are 784 pm (element A) and 1521 pm (impurity). Find the atomic number of impurity. (atomic number of element A is 40).

Chemistry

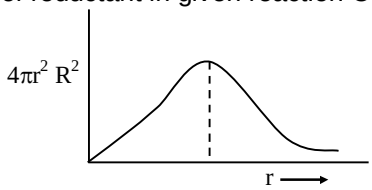
PART – II

SECTION – A
(One Options Correct Type)

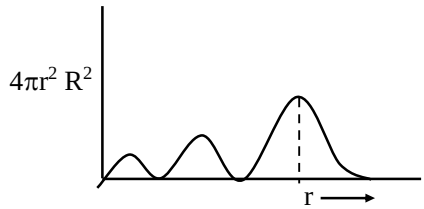
This section contains **06 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

19. Which of the following graphs represents the radial charge density for the electron of outer most sub shell of reductant in given reaction $Cu \rightarrow Cu^{+2} + 2e$?

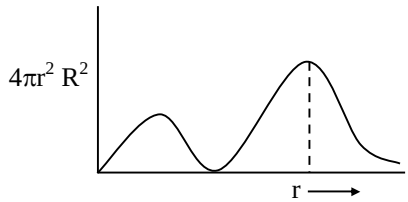
(A)



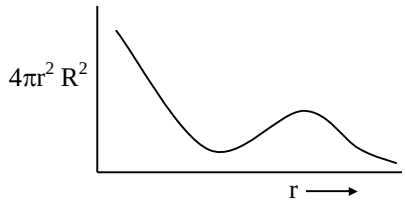
(B)



(C)



(D)



20. Which of the following represents the correct order for the wavelength of absorption in the visible region for the following?

$\underset{(I)}{[Co(NO_2)_6]^{4-}}$ $\underset{(II)}{[FeF_6]^{4-}}$ $\underset{(III)}{[Fe(H_2O)_6]^{2+}}$ $\underset{(IV)}{[Co(NH_3)_6]^{2+}}$

(A) I > IV > III > II
(B) II > IV > III > I
(C) II > III > IV > I
(D) I > III > II > IV

21. A + B → C + D is a stoichiometric ally balanced reaction. The initial rate of the reaction is doubled if the initial concentration of A is doubled, but is quadrupled if the initial concentration of B is doubled.
Select the incorrect statement

(A) The reaction is first order in B and second order in A
(B) The reaction is first order in A and second order in B
(C) The reaction cannot be a single-step reaction
(D) The overall order of the reaction is 3

22. Which of the following orders are/is correct ?
1) F > N > C > Si >Ca - Non-metallic character
2) F >Cl> O > N - oxidising nature
3) C > Si > P > N - electron affinity
4) O > N > F > C - second ionisation potential
5) $I_2 < F_2 < Br_2 < Cl_2$ -Bond energy
(A) 1,4 & 5
(B) 1,2 ,3 & 5
(C) 1 & 3 only
(D) 1 & 2 only

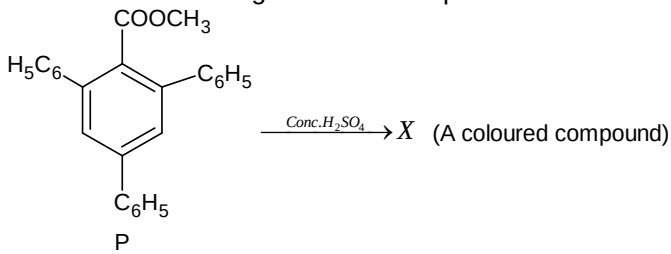
23. $Mg_3N_2 \xrightarrow{H_2O} A_{(g)} \xrightarrow[\Delta]{CuO} B + C_{(g)} + H_2O$
$C_{(g)}$ can be obtained by heating
(1) NH_4NO_2 (2) $(NH_4)_2SO_4$
(3) NH_4ClO_4 (4) $(NH_4)_2Cr_2O_7$
(A) 1,2 ,3 & 4
(B) 1,3 & 4
(C) 1 & 4 only
(D) 1 & 3 only

24. In thermodynamics, the P − V work done is given by
$w = -\int dV\, P_{ext}$.
For a system undergoing a particular process, the pressure is,
$$P_{ext} = \frac{RT}{V-b} - \frac{a}{V^2}$$
This equation is not applicable to a
(1) system that satisfies the van der Waals equation of state.
(2) process that is reversible and isothermal.
(3) process that is reversible and adiabatic.
(4) process that is reversible as well as irreversible adiabatic.
(5) process that is irreversible and at constant pressure.
(6) process that is irreversible and isobaric.
(A) 4,5 & 6
(B) 1,3 & 4
(C) 5 & 6 only
(D) 1,2 & 3

(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

25. In the following reaction, compound X is obtained from compound P via an ionic intermediate. Which of the following is true for compound of X?

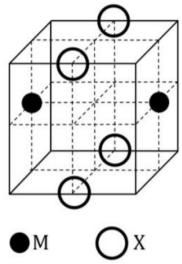


P

Select the correct statement(s)
(A) The degree of unsaturation of X is 18 and it can further react with 1 mol of RMgX.
(B) This reaction is an example of complex EAS reaction.

(C) The compound X has 5 cyclic 6 membered ring.
(D) The compound X is aromatic and it has a cyclic 5 membered ring.

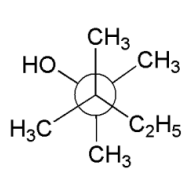
26. The cubic unit cell structure of a compound containing cation M and anion X is shown below. When compared to the anion, the cation has smaller ionic radius. Choose the correct statement(s).



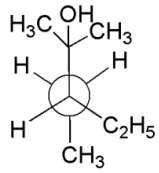
(A) The empirical formula of the compound is MX_2.
(B) The cation M and anion X have same coordination number which is 8.
(C) The ratio of M-X bond length to the cubic unit cell edge length is 0.866 and rank of unit cell is 2.
(D) The limiting ratio of the ionic radii of cation M to anion X is 0.732.

27. Choose the correct statement(s) among the following:
(A) Zeise's salt has tetrahedral geometry.
(B) $\left[Co(en)(NH_3)_2Cl_2\right]^+$ is inner orbital octahedral complex and has 4 stereo isomers.
(C) $[FeCl4]^-$ has higher spin-only magnetic moment than $\left[Co(en)(NH_3)_2Cl_2\right]^+$
(D) The cobalt ion in $\left[Co(en)_2(NH_3)_2\right]^{2+}$ has d^2sp^3 hybridization.

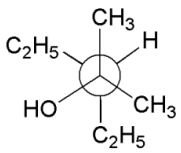
28. Newman projections P, Q, R and S are shown below:



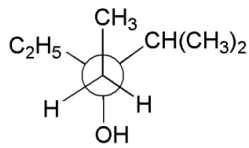
P



Q



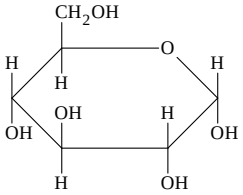
R



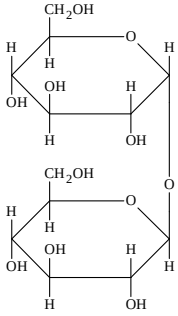
S

Choose the correct statement(s) for compound P ,Q, R and S
(A) (P, Q) and (R, S) Represents identical set of molecules.
(B) IUPAC name of Q and R are same which is 3-ethyl-2-methylpentan-2-ol.
(C) All the given molecules are Achiral.
(D) None of the molecule will be oxidized by acidic potassium dichromate solutions.

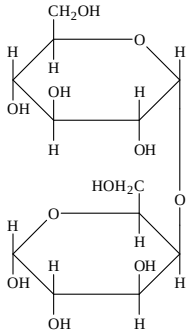
29. Some organic compounds are given below.



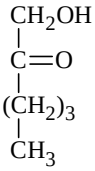
I



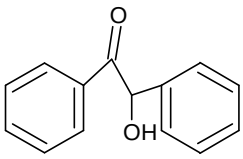
II



III



IV



VI

Select the incorrect statement(s) about the above compounds from the following

(A) Compounds I and III would reduce Fehling's solution and will also mutarotate

(B) Compound IV and VI will reduce Tollen's reagent .

(C) Compound II will not be reducing but mutarotating

(D) The product(s) of the acid catalysed hydrolysis of both II and III will be reducing but not mutarotating

30. A metallic salt (A) soluble in water, when heated with conc. H_2SO_4 produces a brown coloured gas. The metallic salt solution of (A) on treatment with H_2S/H^+ produces a black coloured ppt. insoluble in yellow ammonium sulfide. The black ppt on boiling with conc.H_2O_2 solution turns into (B) which is white in colour.

Select the correct statement(s) about the above compounds from the following

(A) The white pt. (B) finally formed is $PbSO_4$

(B) If the iodate of the same metal was initially taken and heated then Liberates both I_2 and O_2

(C) n-factor of Iodate salt while heating is 5

(D) The brown gas liberated is paramagnetic in nature.

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

---

31. A 2.0 mole mixture of $H_2(g)$, $O_2(g)$ and He(g) are placed together in a closed container at pressure equal to 50 atm. An electric spark is passed and pressure is noted as 12.5 atm after cooling the content to original temp. (room temp). Oxygen gas is introduced for pressure to change to 25 atm, keeping volume and temp constant. Again electrical spark is passed and pressure drops to 10 atm under original temp. conditions. The mole of $H_2(g)$ present in the initial mixture is:…….

32. A micro-organism, nano-turtle, is found autogenerated due to presence of preservatives in raw meat cane. On one day consumption of caned meat, it gets deposited in the inner surface of intestine and gets accumulated as a patch of 3.14 mm^2. Nano-turtles found to be hemispherical of diameter 10 $\overset{o}{A}$ each at the time of consumption and the gastric juice helps it grow slowly as per the equation mentioned below:

$$d_t = d_0 \times \exp.\left(2.732 \times 10^{-3} \times t\right)$$

Where, d_t : diameter after time t in days
d_0 : diameter on the day of consumption
Excessive consumption of caned meat may cause scalding inside the intestine and cause development of carcinogenic cells, which deters health.The number of nanoturtles deposited on the day of consumption of caned meat is 100billion. What is the base area (in $\overset{o}{A}$) of the nonoturtle, on his next birthday, if Mr. KHAN has consumed caned raw meat birthday this year of 7th January 2012?

33. When two moles of an monoatomic ideal gas is expanded isoentropically and irreversibly at 27°C to 327°C then find out the final pressure $\ell n\left(P_2\right)$ (in atm) if initial pressure was 1 atm. (given $\ell n2 = 0.7$)

34. Standard electrode potential of $MnO_4^- \,|\, Mn^{2+} = 1.51\,V$ and $MnO_2 \,|\, Mn^{2+} = 1.18\,V$. Standard electrode potential (in V) of $MnO_4^- \,|\, MnO_2$ is ….

35. 100 ml, 0.1 M Na_3PO_4 solution is added in sequence [first Na_3PO_4 solution is mixed with (i) and the resulting mixture with (ii) and resulting mixture finally with (iii)] into the following reagents:
(K_{a_1}, K_{a_2} and K_{a_3} of H_3PO_4 are 10^{-4}, 10^{-8} and 10^{-11}) respectively. $\log_{10} 3 = 0.47$
(i) 100 ml, 0.1 M NaH_2PO_4 solution
(ii) 100 ml, 0.05 M $Ca(OH)_2$ solution
(iii) 100 ml, 0.125 M H_2SO_4 solution
So, the pH of the final mixture will be…..

36. Decomposition of non-volatile solute (A) into another non-volatile solute B and C. When dissolved in water follows first order kinetics as:
$A(s) \xrightarrow{H_2O} 2B(s) + C(s)$
If initially 2 moles of A is dissolved in 360 g of H_2O and left for decomposition at constant temperature (25°C). Then P_s in the given table is (assuming A, B, and C are miscible in water):

S.N.	Time	The vapour pressure of solution
1.	12 hr	20 mm Hg
2.	80 hr	P_s

Vapour pressure(mm Hg) of H_2O at 25°C is 24 mm Hg. (log2 = 0.30)

Mathematics

PART – III

SECTION – A
(One Options Correct Type)

This section contains **06 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

37. Let slope of tangent of any point (x, y) on the curve C is $\frac{1}{2}\left(\frac{3}{x^2}-y^2\right)$. If the curve passes through the point (1, –1), then
    (A) area of triangle formed by tangent at $(\alpha, y(\alpha))$ on C and coordinate axis increases $\forall\ \alpha > 0$
    (B) area of triangle formed by tangent at $(\alpha, y(\alpha))$ on C and coordinate axis decreases $\forall\ \alpha < 0$
    (C) area of triangle formed by tangent at $(\alpha, y(\alpha)$ on C and coordinate axis is constant $\forall\ \alpha > 0$
    (D) none of these

38. Line $x - 2y - 1 = 0$ and parabola $y^2 = 4x$ intersect at points A and B. Point C is on the parabola and $\angle ACB$ is 90°, then
    (A) number of possible points of C are 4
    (B) C may be (1, –2)
    (C) C may be (9, 6)
    (D) C may be (4, –4)

39. Find the sum $\sum_{k=1}^{64}[\log_2 k]$ (where [.] denote greatest integer function)
    (A) 246
    (B) 128
    (C) 526
    (D) 264

40. Suppose points F_1, F_2 are the foci of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$. P is a point on the ellipse, and $\frac{|PF_1|}{|PF_2|}=\frac{2}{1}$. Then the area of ΔPF_1F_2 is equal to
    (A) 4
    (B) 8
    (C) 12
    (D) 16

41. Let P be a variable plane satisfying $\left(\vec{r}-\left(\hat{i}+2\hat{j}+3\hat{k}\right)\right)\cdot\vec{n}=0$ and meet the positive x, y and z-axis at A, B and C respectively. If volume of tetrahedron OABC is minimum (O is origin), then the plane P contains the point
    (A) (2, 2, 1)
    (B) (1, 2, 2)
    (C) (2, 2, 0)
    (D) (2, 1, 3)

42. If $f(x) = x^3 + \log_2\left(x + \sqrt{x^2+1}\right)$, for any a, b $\in$ R to satisfy f(a) + f(b) $\geq$ 0, the condition a + b $\geq$ 0 is

(A) necessary and sufficient
(B) not necessary but sufficient
(C) necessary but not sufficient
(D) neither necessary nor sufficient

(One or More than one correct type)

This section contains **06** multiple choice questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

43. Let A = {a_1, a_2, a_3, a_4}. Suppose the set of sums of all the elements in ternary subset of A is B = {–1, 3, 5, 8}. Then which of the following is/are correct?

(A) The smallest element of A is –3
(B) The greatest element of A is 6
(C) The sum of all the elements of A is 5
(D) Product of all the elements of A is 54

44. Let a, b, c be real numbers with a > 0 such that the quadratic polynomial $p(x) = ax^2 + bcx + b^3 + c^3 – 4abc$ has non-real zeroes. Then which of the following statement is/are correct?

(A) Exactly one of the polynomial $T_1(x) = ax^2 + bx + c = 0$ and $T_2(x) = ax^2 + cx + b$ takes only positive values
(B) Both he polynomial $T_1(n) = ax^2 + bx + c$ and $T_2(x) = ax^2 + cx + b$ takes only positive values
(C) $(ax^2 + bx + c)(ax^2 + cx\ \ b) = 0$ has four real solution
(D) Only one of the equation $ax^2 + bx + c = 0$ and $ax^2 + cx + b = 0$ has real solution

45. Let y = f(x) satisfy the differential equation $e^x(1 + x)dx – (xe^x – ye^y)dy = 0$. If f(1) = 1, then the value of f(0) may be

(A) $\sqrt{3}$
(B) $-\sqrt{3}$
(C) 3
(D) –3

46. Sum of series ${}^nC_1 – {}^nC_2 2^r + {}^nC_3 3^r + ... + (–1)^{n–1} \cdot {}^nC_n n^r$ where $1 \leq r \leq n$ is

(A) 0 if r < n
(B) r! if r = n and r is odd
(C) –r! if r = n and r is even
(D) r! if r < n

47. If $\{a_n\}$ is a arithmetic sequence with non-zero common difference and $\{b_n\}$ is a geometric sequence, satisfying $a_1 = 3$, $b_1 = 1$, $a_2 = b_2$, $3a_5 = b_3$, further there are constants α, β such that for every positive integer n, we have $a_n = \log_\alpha(b_n) + \beta$, then

(A) $\alpha^3 – \beta = 2$
(B) $\alpha^3 + \beta = 3$
(C) $\alpha^3 + \beta = 6$
(D) $\alpha^3 – \beta = 0$

48. If $\cos^5\theta - \sin^5\theta < 7(\sin^3\theta - \cos^3\theta)$, $\theta \in [0, 2\pi]$, then the value of θ may belong to

(A) $\left(0, \frac{\pi}{4}\right)$

(B) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(C) $\left(\frac{\pi}{2}, \frac{5\pi}{4}\right)$

(D) $\left(\frac{5\pi}{4}, 2\pi\right)$

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

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49. Let $f(x) = 3 \sin x + 2 \cos x + 1$. If real numbers a, b and c are such that $af(x) + bf(x - c) = 1$ holds for any $x \in R$, then $\left|\frac{b\cos c}{a}\right|$ equals

50. It is known that $f(x) = ax^3 + bx^2 + cx + d$ $(a \neq 0)$ and $|f'(x)| \leq 1$ for $0 \leq x \leq 1$. Then the maximum value of a is $\frac{p}{q}$, where p and q are relatively prime, then p + q is

51. Suppose in ΔABC, we have $\sin A = 10 \sin B \sin C$, $\cos A = 10 \cos B \cos C$, then $\tan A$ is equal to

52. Let $\{a_n\}$ be an arithmetic progression with common difference $d(d \neq 0)$ and $\{b_n\}$ be a geometric progression with common ratio r, where r is a positive rational number less than 1. If $a_1 = d$, $b_1 = d^2$ and $\frac{a_1^2 + a_2^2 + a_3^2}{b_1 + b_2 + b_3}$ is a positive integer, then r equals

53. Take randomly five different numbers from 1, 2, 3, ….. 20. Then the probability that there are atleast two adjacent numbers among them is $\frac{k}{323}$. Then the value of k is

54. In a triangle ABC, the altitude, angle bisector and median from vertex C divide the angle C into four equal angles, then the value of $\sin^2 A + \sin^2 B + \sin^2 C$ is equal to

ALL INDIA TEST SERIES

TEST – 31

JEE (Advanced)

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. A

Sol. For plank

$f + 800 = 900$

$f = 100$

for man

$300 + f = 50a$

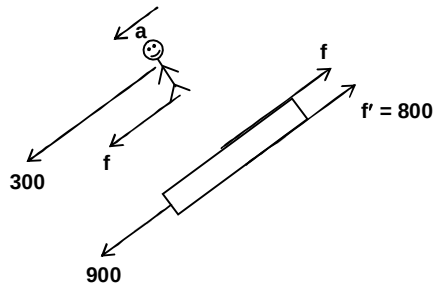
$400 = 50a$

$a = 8\ m/s^2$

$$S = ut + \frac{1}{2}at^2$$

$$16 = 0 + \frac{1}{2}\cdot 8 \cdot t^2$$

$t = 2$ sec



2. A

Sol. Block is displaced by 'x' along x–axis.

Torque about P

$$8k(2x_1)\ell = 4k(x - x_1)\frac{\ell}{2}$$

$$x_1 = \frac{x}{9} \quad ...(i)$$

Torque about Q

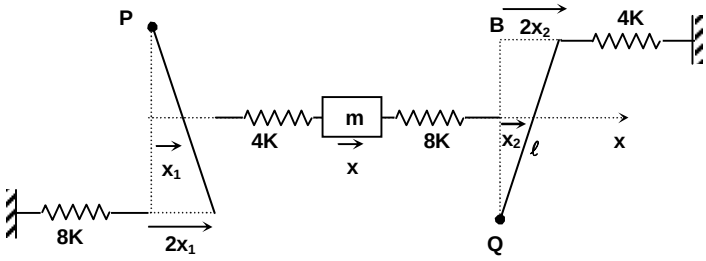
$$4k.2x_2.\ell = 8k(x - x_2)\frac{\ell}{2}$$

$$x_2 = \frac{x}{3}$$

$$ma = -\frac{80}{9}kx$$

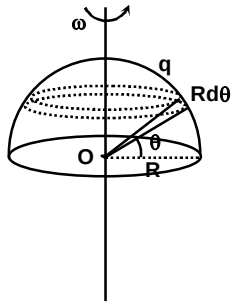
$$a = -\left(\frac{80k}{9m}\right)x$$

$$T = 2\pi\sqrt{\frac{9m}{80k}} = \frac{3\pi}{2}\sqrt{\frac{m}{5k}}$$



3. C

Sol. Magnetic field due to the ring on its axis is



$$B = \frac{\mu_0 \pi r^2 i}{2\pi(r^2 + x^2)^{3/2}} \text{ (due to ring)}$$

$$B = \int_0^{+\pi/2} \frac{\mu_0 \pi (R\cos\theta)^2 \left(\frac{2\pi R\cos\theta R d\theta \sigma}{2\pi/\omega}\right)}{2\pi\left[(R\cos\theta)^2 + (R\sin\theta)^2\right]^{3/2}}$$

Where $\sigma = \frac{q}{2\pi R^2}$

$$B = \frac{\mu_0 q\omega}{6\pi R}$$

4. A

Sol. $I_{CBF} = (K)\left(A_1^2 + A_2^2 + 2A_1A_2\cos\theta\right) = I_0$

$A_1 = A,\ A_2 = 5A,\ \theta = 0°$

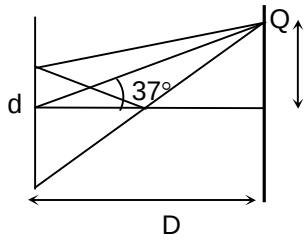
$$kA^2 = \frac{I_0}{36}$$

$$I_Q = I = \frac{16}{25}K\left(A_1^2 + A_2^2 + 2A_1A_2\cos\phi\right)$$

$$\phi = \frac{2\pi}{\lambda}\Delta x$$

$$\phi = \frac{2\pi}{\lambda}\cdot\frac{3}{5}d = \frac{\pi}{3}$$

$$I_Q = \left(\frac{124}{225}\right)I_0$$



$\Delta x = d\sin 37°$

$\Delta x = (3/5)d$

5. D

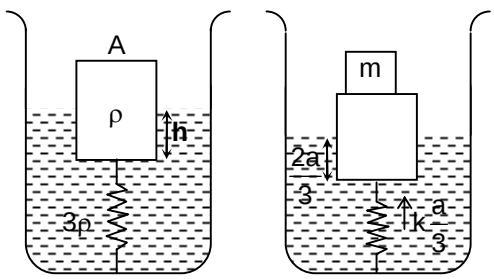
Sol. $(Ah)3\rho g = (Aa)\rho g$

$h = a/3$

$$mg + (a^3\rho g) = k\left(\frac{a}{3}\right) + \left(\frac{2}{3}a^3\right)(3\rho)g$$

$$mg = \frac{4}{3}a^3\rho g$$

$m = 4$ kg

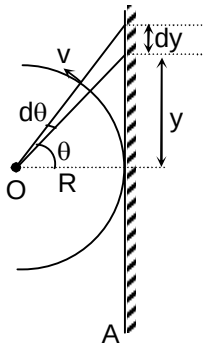


6. D

Sol. $y = R \tan\theta$

$$v_{shadow} = R\sec^2\theta\left(\frac{d\theta}{dt}\right) = v\sec^2\left(\frac{vt}{R}\right)$$

$$a_{shadow} = \frac{dv_{shadow}}{dt} = \frac{2v^2}{R}\sec^2\left(\frac{vt}{R}\right)\tan\left(\frac{vt}{R}\right)$$



7. A, B, C, D

Sol. By conservation of energy

$$\frac{1}{2}mv^2 = mg\frac{\ell}{2} \Rightarrow v = \sqrt{g\ell}$$

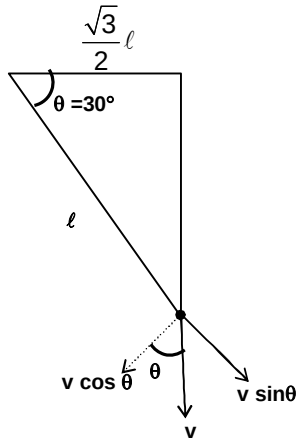
Just after jerk v sin θ becomes zero

Impulse applied by string = mv sin θ = $(0.1)\sqrt{g\ell}\,\frac{1}{2} = \frac{\sqrt{g\ell}}{20}$

Velocity of ball after experiencing jerk = $v\cos\theta = \frac{\sqrt{3g\ell}}{2}$

Velocity at B $v_B^2 = \frac{3g\ell}{4} + 2\cdot g\frac{\ell}{2} = \frac{7}{4}g\ell$

$$v_B = \left(\frac{\sqrt{7g\ell}}{2}\right)$$



8. C, D

Sol. Electric field due to rod

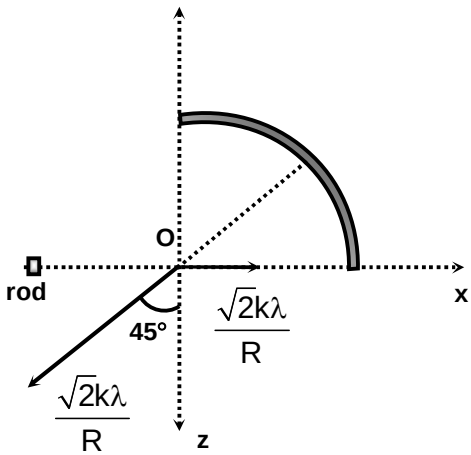
$$E = \frac{k\lambda}{R}(\sin\theta_1 + \sin\theta_2)$$

Electric field due to circular arc

$$E = \frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right)$$

$$E_0 = \left[\sqrt{(\sqrt{2}-1)^2 + (1)^2}\right]\frac{k\lambda}{R}$$

$$E_0 = \left(\frac{\sqrt{4-2\sqrt{2}}}{4\pi\varepsilon_0}\right)\frac{\lambda}{R}$$



9. A, B, C, D

Sol. Focal length of lens

$$\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{2}{R}\right) = \left(\frac{3}{2}-1\right)\left(\frac{2}{20}\right)$$

Position of Image from lens

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$\frac{1}{20} = \frac{1}{v} + \frac{1}{40} \Rightarrow v = 40$ cm

So, position of objet from the concave mirror is 30 which is centre of concave mirror. Hence final image will form at the position of object.

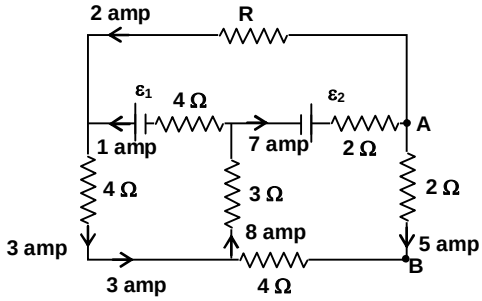
10. A, B, C, D

Sol. By applying Kirchhoff's law

$\varepsilon_1 = 40$ V

$\varepsilon_2 = 68$ V

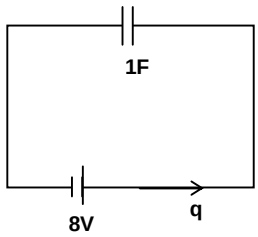
R = 9 Ω and current through 4Ω = 5 amp.



11. A, B, C, D

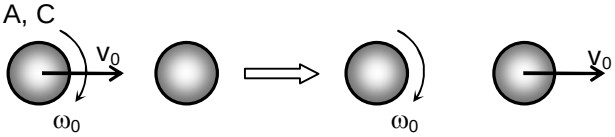
Sol. Equivalent circuit

q is the amount of charge flown in the circuit. q = 8 coulomb.



12. A, C

Sol.



SECTION – C

13. 00001.20

Sol. Time between two consecutive collisions = $\frac{1}{1000}$ sec

So, $\frac{2\ell}{v_{rms}} = \frac{1}{1000}$

$v_{rms} = 2000 \times 5 = 10000$ m/s

$v_{rms} = \sqrt{\frac{3RT}{M}} = 10000$

So, $T = \frac{10^8 \times 4 \times 10^{-3}}{3 \times \left(\frac{25}{3}\right)} = 16 \times 10^3 K$

Mass of helium gas $\Rightarrow$ PV = nRT

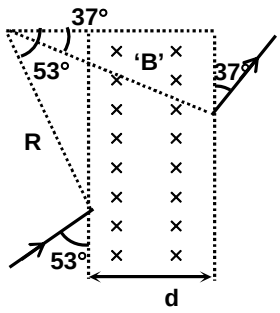
$PV = \frac{m}{M}RT$

$m = \frac{PVM}{RT} = \frac{320 \times 125 \times (4 \times 10^{-3})}{\left(\frac{25}{3}\right) \times 16 \times 10^3} = 1.2$ gm

14. 00080.00

Sol. $d = R\cos 37° - R\cos 53°$

$d = 80$ cm



15. 00060.00

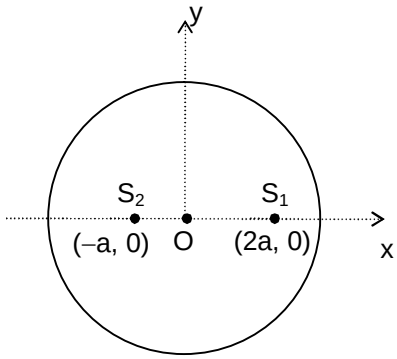
Sol. $\Delta r_{max} = 3a$

$n\lambda = 3a$

$n = \dfrac{3a}{\lambda}$

$n = \dfrac{3a}{(a/5)} = 15$

So, total number of maxima detected during one circular revolution = $15 \times 4 = 60$



16. 00100.00

Sol. i_1 is current in the first loop

$i_1 = \dfrac{500}{20}\left(1 - e^{-\frac{t}{5\times10^{-4}}}\right) = \dfrac{75}{4}$ amp

i_2 is current in the second loop

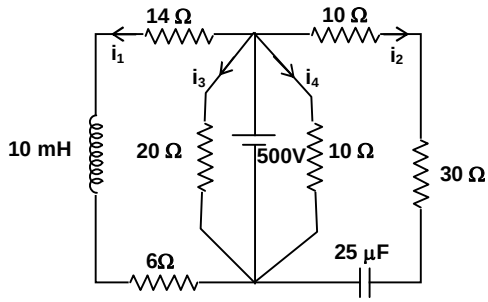
$i_2 = \dfrac{500}{40}e^{-\frac{t}{10^{-3}}} = \dfrac{25}{4}$ amp

$i_3 = \dfrac{500}{20}$ amp

$i_4 = \dfrac{500}{10}$ amp

So, total current through key (K)

$i = i_1 + i_2 + i_3 + i_4 = 100$ amp



17. 00004.00

Sol. $T = \dfrac{F}{\ell}x$

$v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{F}{\ell}\dfrac{x}{m/\ell}} = \sqrt{\dfrac{Fx}{m}}$

$\dfrac{dx}{dt} = \sqrt{\dfrac{Fx}{m}}$

$\int_0^{\ell} \dfrac{dx}{\sqrt{x}} = \sqrt{\dfrac{F}{m}}\int_0^{t} dt$

$$t = 2\sqrt{\frac{m\ell}{F}}$$

18. 00029.00

Sol. Using Moseley's law

$\sqrt{\nu}$ = a(Z - 1) , we have

$\sqrt{\frac{c}{\lambda_A}}$ = a(Z_A - 1)

and $\sqrt{\frac{c}{\lambda_x}} = a(Z_x - 1)$

Dividing yields $\sqrt{\frac{\lambda_A}{\lambda_x}} = \frac{Z_x - 1}{Z_A - 1}$

$\Rightarrow$ Z_x = 29.

Chemistry

PART – II

SECTION – A

19. A

Sol. The number of maxima in the radial charge density curve is $(n - l)$
For 3d electron, n = 3 and d = 2
Hence, $n - l = 3 - 2 = 1$
There will be only one maximum in the curve for 3 d electron.

20. C

Sol. The order of ligand strength in the spectrochemical series $F^- < H_2O < NH_3 < NO_2^-$.
And CFSE for octahedral complex is $Fe^{+2} < Co^{+2}$ due to high ENC . A strong ligand causes a larger degree of splitting resulting in high value of E (energy). Therefore, corresponding low value of $\lambda \left[E = \frac{hc}{\lambda} \right]$

21. A

Sol. **Rate =** $k[A]^{\alpha}[B]^{\beta}$ Using the given data α=1,β = 2 overall order is 3 and reaction may or may not be elementary.

22. B

Sol. Fact based

23. B

Sol. $Mg_3N_2 + H_2O \longrightarrow \underset{(A)}{Mg(OH)_2} + NH_3$

$NH_3 + CuO \longrightarrow Cu + \underset{(C)}{N_2} + H_2O$

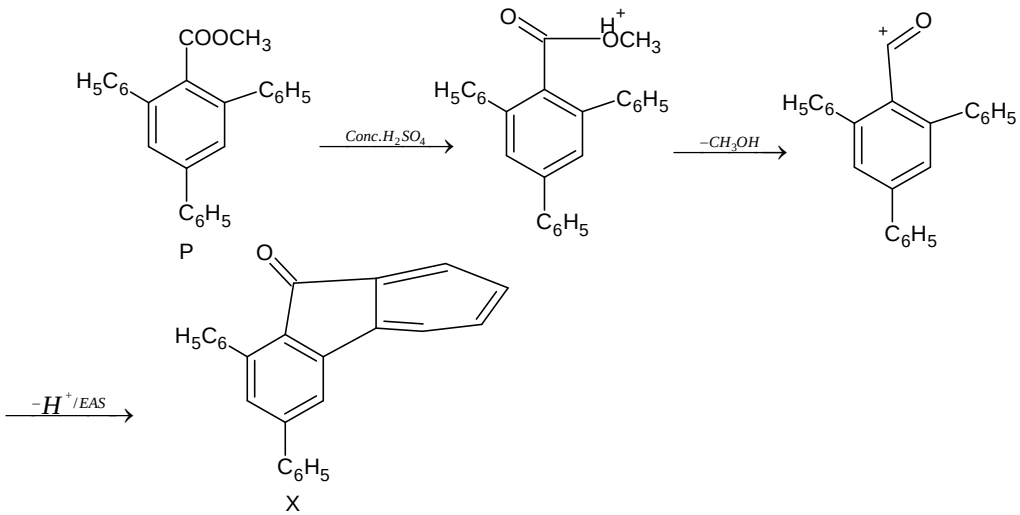
Ammonium salts containing anions which are oxidizing in nature produces nitrogen on heating ammonium

24. A

Sol. The given equation is for one mol van der waal's gas.
If P ext =P means process is reversible and this expression is not valid for any irreversible process.

25. A, B, D

Sol.



Compound X is aromatic in nature and it has 5 cyclic ring, 4 are 6-membered and 1 is 5-membered ring and it can react with 1mol Grignard reagent.

26. B,C,D

Sol. Its BCC unit cell having rank 2 can be compared with CsCl structure so radius of M to X is 0.732:1.00 .CN for both is 8 nearest distance between M and X is $a\sqrt{3}/2=0.866a$.

27. B, C, D

Sol. Zeise's salt $K[PtCl_3C_2H_4]$ has Square planar geometry. $[Co(en)(NH_3)2Cl_2]^+$ is inner orbital octahedral d^2sp^3 complex and has 3 G.I and 2 optical isomers. $[FeCl_4]^-$ has higher spin-only magnetic moment as it has 5 unpaired electron than $[Co(en)(NH_3)2Cl_2]^+$ which has only 1 unpaired electron. In option (D) The cobalt ion in $[Co(en)_2(NH3)_2]^{2+}$ has d^2sp^3 hybridization as both act as SFL for Co^{+2}.

28. B,C,D

Sol. Draw the bond line structure of each molecule you will get to know (P, Q) and (R, S) Represents different set of molecules while Q and R are same molecules. All the given molecules has no chiral centre so all are achiral. None of the molecule will be oxidized by acidic potassium dichromate solutions as all are tertiary alcohols.

29. C, D

Sol. All reducing sugars are mutarotating. Although IV and VI are an α-hydroxy ketone and hence reducing but it can't mutarorate as it is not a carbohydrate/can't form ring. In II the glycosidic bond is in between two anomeric carbons and hence ring opening can't occur. Thus non-reducing as well as non-mutarotating.

30. A. B, D

Sol. The white pt. (B) finally formed is $PbSO_4$. The lead iodate on heated t Liberates both I_2 and O_2 n-factor of Iodate while heating is 10 and The brown gas liberated is NO2 which is paramagnetic in nature.

$$\underset{\text{Black}}{PbS\downarrow} \xrightarrow{H_2O_2} \underset{\text{White}}{PbSO_4\downarrow}$$

$$Pb(IO_3)_2 \xrightarrow{\Delta} PbO + I_2 + O_2$$

SECTION – C

31. 00001.40

Sol. Let the no. of moles of H_2, O_2 and He are x, y and z respectively

$\therefore x + y + z = 2.0$

Pressure exerted by 2 moles of gaseous mixture is 50 atm.

After the first electric spark decrease in pressure = 50 – 12.5 = 37.5 atm

$\therefore$ decrease in no. of moles of gaseous mixture is 1.5

But from the given information limiting reagent in first step is O_2

$\therefore$ from $H_2(g)+\frac{1}{2}O_2(g)\longrightarrow H_2O(\ell)$

$\therefore$ H_2 and O_2 reacted are in the ratio of 2 : 1

$\therefore$ 0.5 mole of O_2 should be reacted with 1.0 mole of H_2

$\therefore$ No. of mole of O_2 in the mixture (y) = 0.5

When again O_2 is passed, pressure increased from 12.5 to 25.0 atm

$\therefore$ Change in pressure = 12.5 atm

$\therefore$ No. of moles of O_2 added = $\frac{12.5\times 2}{50}=0.5$ mole

Now H_2 will be completely reacted

No. of moles H_2 actually left after first electric spark = x – 1

$H_2 \quad + \frac{1}{2}O_2 \longrightarrow H_2O$

$(x-1) \quad \left(\frac{x-1}{2}\right)$

Now change in pressure in 25 – 10 = 15 atm

$\therefore$ change in no. of moles is $\frac{15\times 2}{50} = 0.6$

$\therefore (x-1)+\left(\frac{x-1}{2}\right) = 0.6$

$\Rightarrow x = 1.4$

$\therefore$ no. of moles of H_2 = 1.4

32. 00027.20

Sol. We need to calculate on the same day, one leap year later, so, number of days, nanoturtle got to grow is 366.

So, $d_{366} = d_0 \times e^{+2.732\times10^{-3}\times t}$

$= 10\,\mathring{A}\times e^{+2.732\times10^{-3}\times 366}$

$= \left(10\,\mathring{A}\right)\times e^1 = 10\times 2.720 = 27.20$

33. 00001.75

Sol. For isoentropic process $\Delta_{system} = 0$

$\therefore nC_P, m\ell n\left(\frac{T_2}{T_1}\right)+nR\ell n\frac{P_1}{P_2} = 0$

$\Rightarrow \ell n(P_2) = \left(\frac{5}{2}\right)\times \ell n\left(\frac{600}{300}\right) = 1.75$ atm

34. 00001.73

Sol. (i) $MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O \quad \left(\Delta G_1^0\right)$

(ii) $MnO_2 + 4H^+ + 2e^- \longrightarrow Mn^{2+} + 2H_2O \quad \left(\Delta G_2^0\right)$

Equation (i) and (ii) give the required equation

$MnO_4^- + 4H^+ + 3e^- \longrightarrow MnO_2 + 2H_2O \quad \left(\Delta G_3^0\right)$

$\Delta G_3^0 = \Delta G_1^0 - \Delta G_2^0$

$-3FE_3^0 = -5FE_1^0 - \left(-2FE_2^0\right)$

$E_3^0 = \frac{5E_1^0 - 2E_2^0}{3}$

$E_3^0 = \frac{(5\times 1.51)-(2\times 1.18)}{3} = 1.73$ V.

35. 00007.53

Sol. (a) on mixing into (i)

	PO_4^{3-}	$+ H_2PO_4^-$	$\rightleftharpoons 2HPO_4^{2-}$
	10	10	0
at eqbm	0	0	20

(b) on mixing this into (ii)

$$HPO_4^{2-} + OH^- \rightleftharpoons PO_4^{3-}$$

	20	10	0
at eqbm	10	0	10

(c) Now, on mixing this into (iii)

$$PO_4^{3-} + H_2PO_4^- \rightleftharpoons 2HPO_4^{2-}$$

	10	10	0
at eqbm	0	0	10

$$HPO_4^{2-} + H^+ \rightleftharpoons H_2PO_4^-$$

	20	15	0
at eqbm	5	0	15

So, final mixture is a buffer.

Hence, $pH = pK_{a_2} - \log_{10} 3 = 7.53$

36. 00018.75

Sol. $A \longrightarrow 2B + C$

$2 - x \quad 2x \quad x$

$$P_s = \frac{20}{22 + 2x} \times 24 \qquad (\text{at } t = 12 \text{ hr})$$

$x = 1$ mole which is half of the initial value.

Hence, 12 hour is half-life for the above first order reaction.

The above reaction will be completed 99% in 80 hrs.

$$P_s = \frac{20}{22 + 2 \times 1.8} \times 24 \qquad (\text{at } t = 80 \text{ hr})$$

$P_s = 18.75$ mm Hg

SECTION – A

37. C

Sol. Given $2\frac{dy}{dx} = \frac{3}{x^2} - y^2 = \frac{3-(xy)^2}{x^2}$

Let $t = xy \Rightarrow x\frac{dy}{dx} + y = \frac{dt}{dx}$

$\Rightarrow x\left(\frac{1}{2}\left(\frac{3-t^2}{x^2}\right)\right) + \frac{t}{x} = \frac{dt}{dx} \Rightarrow \frac{3-t^2}{2x} + \frac{t}{x} = \frac{dt}{dx}$

$\Rightarrow \int \frac{-dt}{t^2-2t-3} = \int \frac{dx}{2x} \Rightarrow \frac{1}{2}\left(\ln\left|\frac{t+1}{t-3}\right|\right) = \ln|x| + \ln C$

$\Rightarrow \sqrt{\left|\frac{t+1}{t-3}\right|} = C|x| \Rightarrow \frac{xy+1}{xy-3} = kx^2$ passes through (–1)

$\Rightarrow k = 0 \Rightarrow$ curve C is $y = -\frac{1}{x}$

38. B

Sol. Let $A(x_1, y_1)$, $B(x_1, y_2)$, $C(t^2, 2t)$ from $x - 2y - 1 = 0$ and $y^2 = 4x$, we get $y^2 - 8y - 4 = 0$
$y_1 + y_2 = 8$ and $y_1y_2 = -4$
$x_1 = 2y_1 + 1$, $x_2 = 2y_2 + 1$
$x_1 + x_2 = 2(y_1 + y_2) + 2 = 18$
$x_1 \cdot x_2 = 2(y_1 + y_2) + 4y_1y_2 + 1 = 1$
$\angle ACB = 90° \Rightarrow m_{AC} \cdot m_{BC} = -1 \Rightarrow (t^2 - x_1)(t^2 - x_2) + (2t - y_1)(2t - y_2) = 0$
$\Rightarrow t^4 - 14t^2 - 16t - 3 = 0 \Rightarrow t^2 + 4t + 3 = 0$ or $t^2 - 4t - 1 = 0$
$t = -1, -3$
Coordinate of C is (1, –2) or (9, –6)
$t^2 - 4t - 1 = 0$ not possible as C. Coincides with A or B

39. D

Sol. For $2^i \le k \le 2^{i+1} - 1$; $\left[\log_2 k\right] = i$

So, $\sum_{k=1}^{2^n}\left[\log_2 k\right] = \left[\log_2 1\right] + \left[\log_2 2\right] ..... \left[\log_2 2^n\right]$

$= 0 + 1 \times 2 + 2 \times 2^2 + 3 \times 2^3 ..... (n-1) \cdot 2^{n-1} + n$
$= (n-2) \cdot 2^n + n + 2$

40. D

Sol. $|PF_1| + |PF_2| = 2a = 12$ by definition of an ellipse, since $\frac{|PF_1|}{|PF_2|} = \frac{2}{1}$

Then $|PF_1| = 8$; and $|PF_2| = 4$ and $|F_1F_2| = 2ae = 12\sqrt{1 - \frac{16}{36}} = 4\sqrt{5}$

and $|PF_1|^2 + |PF_2|^2 = 8^2 + 4^2 = 80$
Then ΔPF_1F_2 is a right = $|F_1F_2|^2$ triangle
So, $PF_1F_2 = \frac{1}{2}|PF_1||PF_2| = \frac{1}{2} \times 8 \times 4 = 16$

41. C

Sol. Equation of plane P is $\left(\vec{r} - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)\right) \cdot \left(6\hat{i} + 3\hat{j} + 2\hat{k}\right) = 0$

42. A

Sol. Obviously $f(x) = x^3 + \log_2\left(x + \sqrt{x^2+1}\right)$ is odd function and is monotonically increasing

So if $a + b \geq 0$, i.e. $a \geq -b$, we get $f(a) \geq f(-b) > f(a) \geq -f(b)$

$\Rightarrow f(a) + f(b) \geq 0$

On other hand if $f(a) + f(b) \geq 0$, then $f(a) \geq -f(b)$

So, $a \geq -b$, $a + b \geq 0$

43. A, B, C

Sol. Every element of A appears 3 times. Then we have

$3(a_1 + a_2 + a_3 + a_4) = -1 + 3 + 5 + 8 = 5 \Rightarrow a_1 + a_2 + a_3 + a_4 = 5$

$A = \{-3, 0, 2, 6\}$

44. A, D

Sol. $\Delta = b^2c^2 - 4a\left(b^3 + c^3 - 4abc\right) = \left(b^2 - 4ac\right)\left(c^2 - 4ab\right) < 0$

So, any one of $b^2 - 4ac$ and $c^2 - 4ab$ is less than zero and other is greater than zero

45. A, B

Sol. $e^x\,dx + xe^x\,dx - xe^x\,dy + ye^y\,dy = 0$

$\Rightarrow e^x\,dx + xe^x(dx - dy) + ye^y\,dy = 0$

$\Rightarrow e^{x-y}\,dx + xe^{x-y}(dx - dy) + y\,dy = 0$

$\Rightarrow d(xe^{x-y}) + d\left(\frac{y^2}{2}\right) = 0$

$\Rightarrow xe^{x-y} + \frac{y^2}{2} = c$ as $f(1) = 1 \Rightarrow c = \frac{3}{2}$

$xe^{x-y} + \frac{y^2}{2} = \frac{3}{2}$. So, value of f(0) is $\pm\sqrt{3}$

46. A, B, C

Sol. Let r < n, then total number of ways of distributing r differents things to n has as such that each should get atleast one things is

$n^r - {}^nC_1(n-1)^r + {}^nC_2(n-2)^r + ...(-1)^{n-1} \cdot {}^nC_{n-1} = 0$

Let r = n

$n^n - {}^nC_1(n-1)^n + {}^nC_2(n-2)^n - .....(-1)^{n-1} \cdot {}^nC_{n-1} = n!$

$\Rightarrow {}^nC_1 - {}^nC_2 2^n + {}^nC_3 3^n - .... = (-1)^{n-1} \cdot n!$

47. C, D

Sol. Let common difference of $\{a_n\}$ is d, common ratio of $\{b_n\}$ is r, then

$3 + d = r$ ..... (1)

$3(3 + 4d) = r^2$ ..... (2)

$\Rightarrow r = 9, d = 6$

$\Rightarrow a_n = \log_\alpha^{b_n} + \beta$

$\Rightarrow 3 + (n-1)6 = \log_\alpha^{9^{n-1}} + \beta$

$\Rightarrow 3 + (n-1)6 = 2(n-1)\log_\alpha^3 + \beta$ hold for every positive integer n

$\Rightarrow \alpha = (3)^{\frac{1}{3}}, \beta = 3$

48. B, C

Sol. From the given inequality $\cos^5\theta - \sin^5\theta < 7\left(\sin^3\theta - \cos^3\theta\right)$

$\sin^3\theta + \frac{1}{7}\sin^5\theta > \cos^3\theta + \frac{1}{7}\cos^5\theta$

$\therefore$ $f(x) = x^3 + \frac{1}{7}x^5$ is increasing over R, then $\sin\theta > \cos\theta$

and that mean $2k\pi + \frac{\pi}{4} < \theta < 2k\pi + \frac{5\pi}{4}$ $(k \in I)$

So, range of $\theta \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

SECTION – C

49. 00001.00

Sol. $af(x) + bf(x - c) = a(3\sin x + 2\cos x + 1) + b(3\sin(x - c) + 2\cos(x - c) + 1) = 1$

So, $c = (2n+1)\overline{x}$ $x \in Z$ and $a = b = \frac{1}{2}$; $\frac{b\cos c}{a} = -1$

50. 00011.00

Sol. $f'(x) = 3ax^2 + 2bx + c$; $f'(0) = c$, $f'\left(\frac{1}{2}\right) = \frac{3}{4}a + b + c$; $f'(1) = 3a + 2b + c$, then

$3a = 2f'(0) + 2f'(1) - 4f'\left(\frac{1}{2}\right)$, we get $3|a| = \left|2f'(0) + 2f'(1) - 4f'\left(\frac{1}{2}\right)\right| \le 2|f'(0)| + 2|f'(1)| + 4\left|f'\left(\frac{1}{2}\right)\right|$

$3|a| \le 8$, $|a| \le \frac{8}{3}$, $a \in \left[-\frac{8}{3}, \frac{8}{3}\right]$. So, p + q = 11

51. 00011.00

Sol. $\sin A - \cos A = 10[\sin B \sin C - \cos B \cos C] = -10\cos(B + C) = 10\cos A$

$\Rightarrow \tan A = 11$

52. 00000.50

Sol. $\frac{a_1^2 + a_2^2 + a_3^2}{b_1 + b_2 + b_3} = \frac{14}{1 + r + r^2}$

Possible if $r = \frac{1}{2}$

53. 00232.00

Sol. Suppose $a_1 < a_2 < a_3 < a_4 < a_5$ taken from 1, 2, ….., 20.

If a_1, a_2, a_3, a_4, a_5 are not adjacent to each other then, we have

$1 \le a_1 < a_2 - 1 < a_3 - 2 < a_4 - 3 < a_5 - 4 \le 16$

From which we known that number of ways to select five numbers not adjacent to each other from 1 to 20 is the same as selecting five different numbers from 1 to 16

So, required probability = $\frac{{}^{16}C_5}{{}^{20}C_5} = \frac{232}{323}$

54. 00002.00

Sol. Let altitude, angle bisector and median from vertex C meet the side AB at D, E and F respectively

Now, $\frac{AE}{EB} = \frac{AC}{CB} = \frac{b}{a} \Rightarrow AE = \frac{bc}{a+b}$

Also, $\frac{FB}{FE} = \frac{BC}{CE} = \frac{a}{b} \Rightarrow FB = \frac{c}{2}$ and $FE = \frac{c(a-b)}{2(a+b)}$

$\Rightarrow \left(\frac{b}{a}\right)^2 + 2\left(\frac{b}{a}\right) - a = 0 \Rightarrow \frac{b}{a} = \sqrt{2} - 1 \Rightarrow \angle C = 90°$