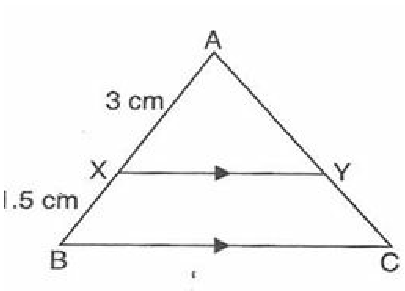
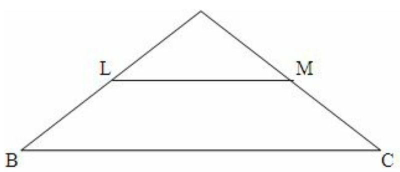
CBSE Test Paper 01

Chapter 6 Triangles

1. In an isosceles triangle ABC if AC = BC and $AB^2 = 2AC^2$ then the measure of $\angle C$ is **(1)**
   a. 90^0
   b. 45^0
   c. 60^0
   d. 30^0
2. In the given figure XY || BC. If AX = 3cm, XB = 1.5cm and BC = 6cm, then XY is equal to **(1)**



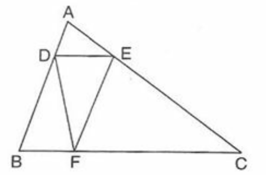
   a. 6 cm.
   b. 4.5 cm
   c. 3 cm.
   d. 4 cm.
3. What will be the length of the hypotenuse of an isosceles right triangle whose one side is $4\sqrt{2}\ cm$ **(1)**
   a. $12\sqrt{2}\ cm.$
   b. 12 cm.
   c. 8 cm.
   d. $8\sqrt{2}\, cm.$
4. In the given figure, if $\frac{ar(\Delta ALM)}{ar(trapezium\ LMCB)} = \frac{9}{16}$, and LM||BC, Then AL:LB is equal to **(1)**



   a. 3 : 5

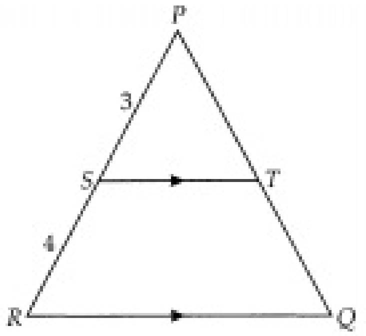
b. 4 : 1
c. 3 : 4
d. 2 : 3

5. In the follwoing figure AD : DB = 1 : 3, AE : EC = 1 : 3 and BF : FC = 1 : 4, then **(1)**

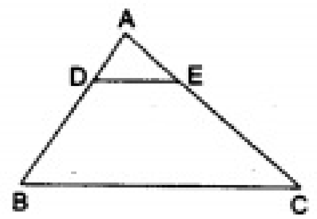


a. $AD||FC.$
b. $AD||FE.$
c. $DE||BC.$
d. $AE||DF.$

6. In the given figure, ST || RQ, PS = 3 cm and SR = 4 cm. Find the ratio of the area of △PST to the area of △PRQ. **(1)**



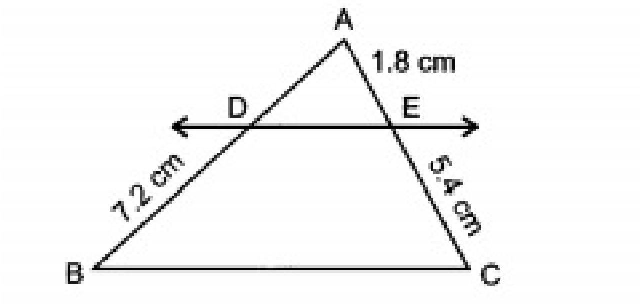
7. If D and E are points on the sides AB and AC respectively of $\triangle ABC$ such that AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm, show that DE ||BC. **(1)**



8. A ladder is placed in such a way that its foot is at a distance of 5 m from a wall and its tip reaches a window 12 m above the ground. Determine the length of the ladder. **(1)**

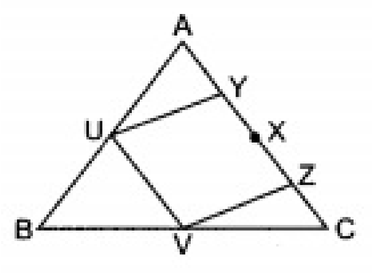
9. Triangles ABC and DEF are similar. If AC = 19 cm and DF = 8 cm, find the ratio of the area of two triangles. **(1)**

10. In the given figure, DE || BC.



Find AD. **(1)**

11. In $\triangle$ABC, X is any point on AC. If Y, Z, U and V are the middle points on AX, XC, AB and BC respectively, then prove that UY || VZ and UV || YZ.

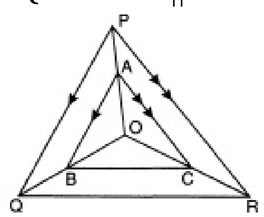


12. If the angles of one triangle are respectively equal to the angles of another triangle, Prove that the ratio of their corresponding sides is the same as the ratio of their corresponding angle bisectors. **(2)**

13. In a ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC. If AD = x, DB = x-2, AE = x + 2 and EC = x - 1, find the value of x. **(2)**

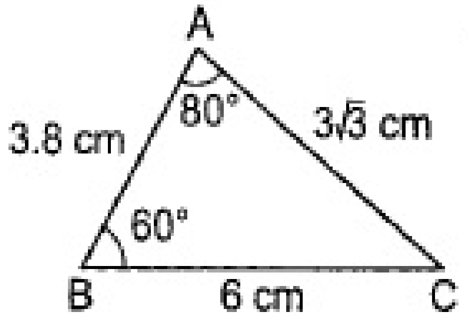
14. A man goes 10m due south and then 24m due west. How far is he from the starting point? **(3)**

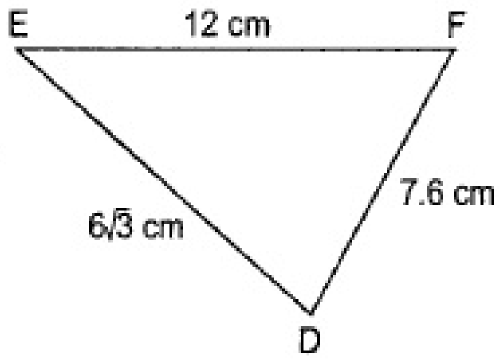
15. In the given figure A, B and C are points on OP, OQ and OR respectively such that AB $\|$ PQ and AC $\|$ PR. Prove that BC $\|$ QR.



16. In a ΔABC, D and E are points on the sides AB and AC respectively such that $DE \| BC.$ If AD = 8x - 7, DB = 5x - 3, AE = 4x - 3 and EC = (3x -1), find the value of x. **(3)**

17. In Fig. find $\angle$F. **(3)**

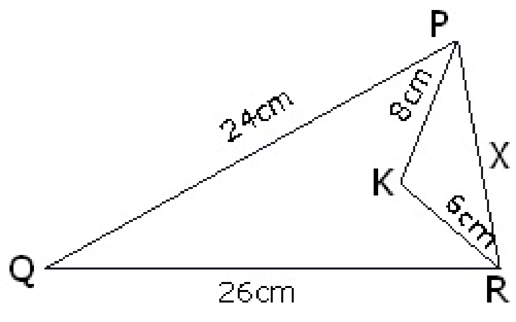




18. Prove that ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides. **(4)**

19. In a trapezium ABCD, AB | | DC and DC = 2AB. EF | | AB, where E and F lie on BC and AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$. Diagonal DB intersects EF at G. Prove that, 7EF = 11AB. **(4)**

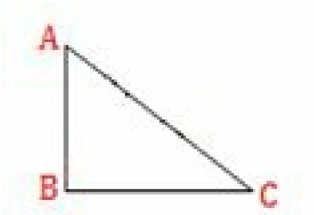
20. In a triangle, if the square of one side is equal to the sum of the squares on the other two sides. Prove that the angle opposite to the first side is a right angle. Use the above theorem to find the measure of $\angle$ PKR in the figure given below. **(4)**



CBSE Test Paper 01
Chapter 6 Triangles

Solution

1. a. 90^{0}
   Explanation: Given: $AB^2 = 2AC^2$
   $\Rightarrow AB^2 = AC^2 + AC^2$
   $\Rightarrow AB^2 = AC^2 + BC^2$ [Given: AC = BC] $\therefore \Delta\mathrm{ABC}$is a right angled triangle, by converse of Pythagoras theorem
   Now, since $\Delta\mathrm{ABC}$ is an isosceles triangle also.
   Therefore, its two sides are equal i.e., AC = BC Therefore, AB is hypotenuse.
   $\therefore \angle$C is a right angle i.e., 90°
2. d. 4 cm.
   Explanation: Since $XY||BC$, then using Thales theorem,
   $\Rightarrow \frac{\mathrm{AX}}{\mathrm{AB}} = \frac{\mathrm{XY}}{\mathrm{BC}}$
   $\Rightarrow \frac{3}{4.5} = \frac{\mathrm{XY}}{6}$
   $\Rightarrow$XY = 4 cm
3. c. 8 cm.
   Explanation: Let AC be hypotenuse. Its equal sides are AB and BC and AB = BC = $4\sqrt{2}$cm.

   Using Pythagoras Theorem,
   $\mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2$
   $\Rightarrow \mathrm{AC}^2 = \left(4\sqrt{2}\right)^2 + \left(4\sqrt{2}\right)^2$ = 32 + 32 = 64 cm^2
   $\Rightarrow$AC = 8 cm
   

4. b. 4 : 1
   Explanation: In $\Delta\mathrm{ALM}$ and $\Delta\mathrm{ABC}$, $\angle\mathrm{A} = \angle\mathrm{A}$ [Common]
   $\angle\mathrm{ALM} = \angle\mathrm{ABC}$ [Corresponding angles as LM$||$BC]

$\therefore \Delta\text{ALM} \sim \Delta\text{ABC}$ [AA similarity]

$\therefore \frac{\text{ar}(\Delta\text{ALM})}{\text{ar}(\Delta\text{ABC})} = \frac{\text{AL}^2}{\text{AB}^2}$ Now, $\frac{\text{ar}(\text{trap.LMCB})}{\text{ar}(\Delta\text{ALM})} = \frac{9}{16}$

$\Rightarrow \frac{\text{ar}(\Delta\text{ABC}) - \text{ar}(\Delta\text{ALM})}{\text{ar}(\Delta\text{ALM})} = \frac{9}{16}$

$\Rightarrow \frac{\text{ar}(\Delta\text{ABC})}{\text{ar}(\Delta\text{ALM})} - 1 = \frac{9}{16}$

$\Rightarrow \frac{\text{ar}(\Delta\text{ABC})}{\text{ar}(\Delta\text{ALM})} = \frac{9}{16} + 1$

$\Rightarrow \frac{\text{ar}(\Delta\text{ABC})}{\text{ar}(\Delta\text{ALM})} = \frac{25}{16}$

$\Rightarrow \frac{\text{AB}^2}{\text{AL}^2} = \frac{25}{16}$

$\Rightarrow \frac{\text{AB}}{\text{AL}} = \frac{5}{4}$

Let AB = 5x and AL = 4x then LB = AB - AL = 5x - 4x = 1x

$\therefore \frac{\text{AL}}{\text{LB}} = \frac{4x}{1x} = \frac{4}{1}$

$\Rightarrow$ AL : LB = 4 : 1

5. c. $DE || BC$.

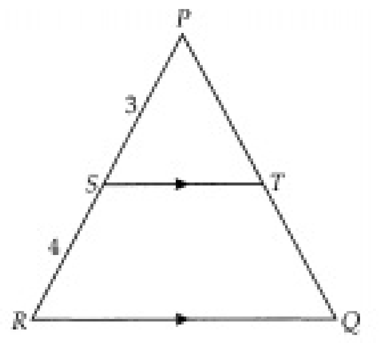
Explanation: Given: $\frac{\text{AD}}{\text{DB}} = \frac{1}{3}$ and $\frac{\text{AE}}{\text{EC}} = \frac{1}{3}$

Therefore, in $\triangle\text{ABC}, \frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$

$\therefore$ DE$||$BC [Using Thales Theorem]

Here we are not considering BF : FC =1 : 4.

6. PS = 3 cm, SR = 4 cm and ST || RQ.



PR = PS + SR

= 3 + 4 = 7 cm

In $\triangle$PST and $\triangle$PRQ

$\angle$SPT $\cong$ $\angle$RPQ (common angle)

$\angle$PST $\cong$ $\angle$PRQ (Alternate angle)

$\triangle$PST $\sim$ $\triangle$PRQ (AA configuration)

$\frac{\text{ar } \Delta PST}{\text{ar } \Delta PQR} = \frac{PS^2}{PR^2} = \frac{3^2}{7^2} = \frac{9}{49}$

Hence required ratio = 9 : 49.

7. Given: AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm

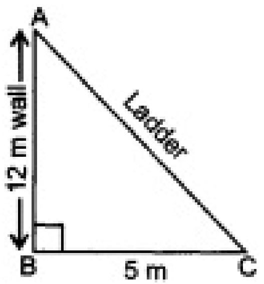
   $\therefore \quad \frac{AD}{AB} = \frac{1.4}{5.6} = \frac{1}{4}$ and $\frac{AE}{AC} = \frac{1.8}{7.2} = \frac{1}{4}$

   $\Rightarrow \quad \frac{AD}{AB} = \frac{AE}{AC}$

   Hence, by the converse of Thales' theorem, $DE \| BC$.

8. Let AC be the ladder, AB be the wall and BC be the distance of ladder from the foot of the wall.

   In right $\triangle$ABC,

   $AC^2 = AB^2 + BC^2$ { using Pythagoras theorm for right-angled triangle}

   $\Rightarrow AC^2 = (12)^2 + 5^2$

   $\Rightarrow AC^2 = 144 + 25$

   $\Rightarrow$ AC = 13 m

9. We have,

   ΔABC ~ ΔDEF

   AC = 19 cm and DF = 8 cm

   By area of similar triangle theorem

   $\frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{AC^2}{DF^2} = \frac{(19)^2}{8^2} = \frac{361}{64}$

10. $\because$ DE $\|$ BC

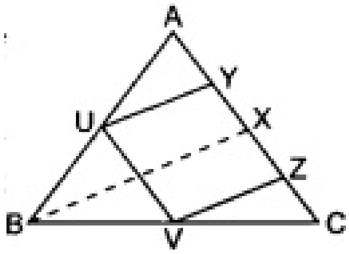
    $\therefore \quad \frac{\text{AD}}{\text{BD}} = \frac{\text{AE}}{\text{CE}}$ (from BPT)

    $\Rightarrow \quad \frac{\text{AD}}{7.2} = \frac{1.8}{5.4} \Rightarrow$ AD = 2.4 cm

11. Join BX

    In ABX, U is midpoint of AB and Y is mid-point AX (given)

$\therefore$ UY $\|$ BX (using mid-point theorem) .....(i)



In BCX, v is mid-point of BC and z is mid-point of XC

VZ $\|$ BX ..(ii)

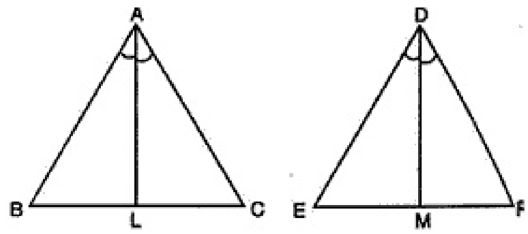
from (i) and (ii)

UY $\|$ VZ

In ABC, U is mid-point of AB and V is mid-point of BE.

$\therefore$ UV $\|$ AC

$\Rightarrow$ UV $\|$ YZ Hence proved.

12. 

Given: Two triangles ABC and DEF in which $\angle$ A = $\angle$D, $\angle$B = $\angle$E and $\angle$C = $\angle$F, AL and DM are angle bisectors of $\angle$A

and $\angle$D respectively

To prove: $\frac{BC}{EF} = \frac{AL}{DM}$

Proof: Triangle ABC and DEF are Similar.

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$ ......(i)

In $\triangle$ ABL and $\triangle$ DEM, we have

$\angle$B= $\angle$E [Given]

$\angle$ BAL= $\angle$ EDM [$\because$ $\angle$ A= $\angle$ D $\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle D$]

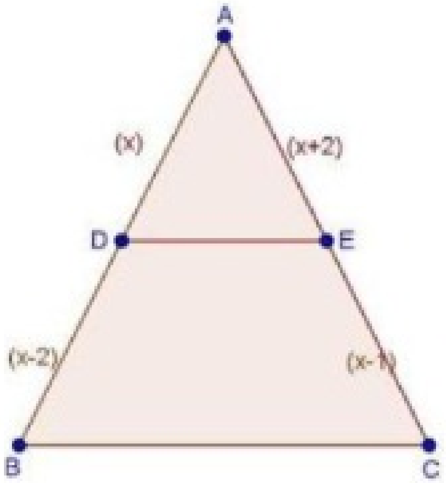
$\Rightarrow$ $\triangle$ ABL $\sim$ $\triangle$ DEM [AA similarity]

$\Rightarrow \frac{AB}{DE} = \frac{AL}{DM}$ ....(ii)

From (i) and (ii) we have

$\frac{BC}{EF} = \frac{AL}{DM}$

13. We have,



DE || BC

Therefore, by basic proportionality theorem,

We have,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

$\Rightarrow$ x(x - 1) = (x + 2)(x - 2)

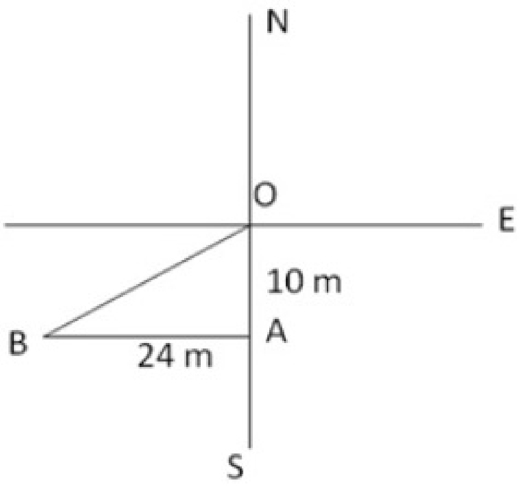
$\Rightarrow x^2 - x = x^2 - (2)^2$ [$\because (a - b)(a + b) = a^2 - b^2$]

$\Rightarrow$ -x = -4

$\Rightarrow$ x = 4 cm.

14. Starting from O, let the man goes from O to A and then A to B as shown in the figure. Then,

OA = 10m, AB = 24m and $\angle OAB = 90^0$



Using Pythagoras theorem:

$OB^2 = OA^2 + AB^2$

$\Rightarrow$ $OB^2 = 10^2 + 24^2$

$\Rightarrow$ $OB^2 = 100 + 576$

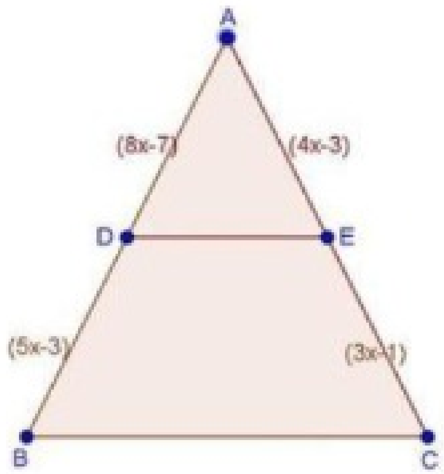
$\Rightarrow$ $OB^2 = 676$

$\Rightarrow$ OB = $\sqrt{676}$ = 26m

Hence, the man is 26m south-west from the starting position.

15. Proof : In $\triangle POQ, AB||PQ$,(Given)
 $\frac{AO}{AP} = \frac{OB}{BQ}$.......(i) (BPT)
 In $\triangle$ OPR $AC\|PR$
 $\frac{OA}{AP} = \frac{OC}{CR} \ldots\ldots (ii)$
 From eqn (I) and (ii)
 $\frac{OB}{BQ} = \frac{OC}{CR}$
 Hence $BC\|QR$ (By converse of BPT)

16. We have,



We are given that, DE || BC

Therefore, by thales theorem,

We have,
 $\frac{AD}{DB} = \frac{AE}{EC}$
 $\Rightarrow \frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$
 $\Rightarrow$ (8x - 7)(3x - 1) = (4x - 3)(5x - 3)

$\Rightarrow$ $24x^2 - 8x - 21x + 7 = 20x^2 - 12x - 15x + 9$

$\Rightarrow$ $24x^2 - 20x^2 - 29x + 27x + 7 - 9 = 0$

$\Rightarrow$ $4x^2 - 2x - 2 = 0$

$\Rightarrow$ 2[2x^2 - x - 1] = 0

$\Rightarrow$ 2x^2 - x - 1 = 0

$\Rightarrow$ 2x^2 - 2x + 1x - 1 = 0

$\Rightarrow$ 2x(x - 1) + 1(x - 1) = 0

$\Rightarrow$ (2x + 1)(x - 1) = 0

$\Rightarrow$2x + 1 = 0 or x - 1 = 0

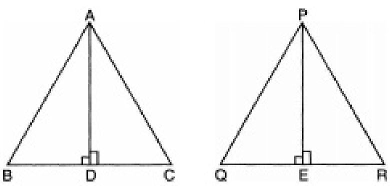
$\Rightarrow x = -\frac{1}{2}$ or x = 1

$x = -\frac{1}{2}$ is not possible.

$\therefore$ x = 1.

17. In triangles ABC and DEF, we have
$\frac{AB}{DF} = \frac{BC}{FE} = \frac{CA}{ED} = \frac{1}{2}$
Therefore, by SSS-criterion of similarity, we have
$\Delta ABC \sim \Delta DFE$
$\Rightarrow$ ∠A = ∠D, ∠B = ∠F and ∠C = ∠E
$\Rightarrow$ ∠D = 80°, ∠F = 60°
Hence, ∠F = 60°.

18. Given : $\Delta ABC \sim \Delta PQR$
To Prove : $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$
Construction: Draw AD ⊥BC and PE ⊥ QR
Proof :



$\Delta ABC \sim \Delta PQR$
$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ (Ratio of corresponding sides of similar triangles are equal) ...(i)
$\angle B = \angle Q$ (Corresponding angles of similar triangles)......... (ii)
In ΔADB and ΔPEQ
$\angle B = \angle Q$ (From (ii))
$\angle ADB = \angle PEQ$ [each 90°]
$\therefore \Delta ADB \sim \Delta PEQ$ [By AA criteria]

$\Rightarrow \frac{AD}{PE} = \frac{AB}{PQ}$ (Corresponding sides of similar triangles) ...(iii)

From equation (i) and equation (iii)

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PE}$ ...(iv)

$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PE}$

$= \left(\frac{BC}{QR}\right) \times \left(\frac{AD}{PE}\right)$

$(\frac{AD}{PE} = \frac{BC}{QR})$

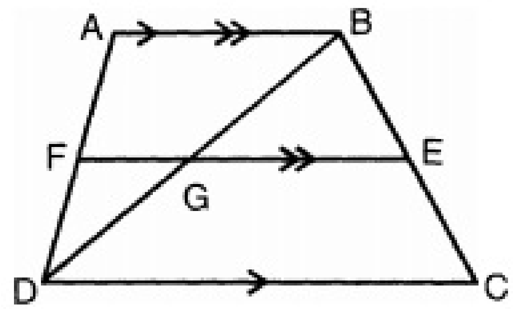
$= \frac{BC}{QR} \times \frac{BC}{QR}$

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$ ....(v) [from eq. (iv)]

From equation (iv) and equation (v),

$\therefore \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

$\therefore$ Ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

19. 

In a trapezium ABCD, AB|| DC ,. EF || AB and CD=2AB

and also $\frac{BE}{EC} = \frac{4}{3}$ -------------(1)

AB || CD and AB || EF

$\therefore \frac{AF}{FD} = \frac{BE}{EC} = \frac{4}{3}$

In ΔBGE and ΔBDC

$\angle BEG = \angle BCD$ ($\because$ corresponding angles)

$\angle GBE = \angle DBC$ (Common)

$\therefore \Delta BGE \sim \Delta BDC$ [By AA similarity]

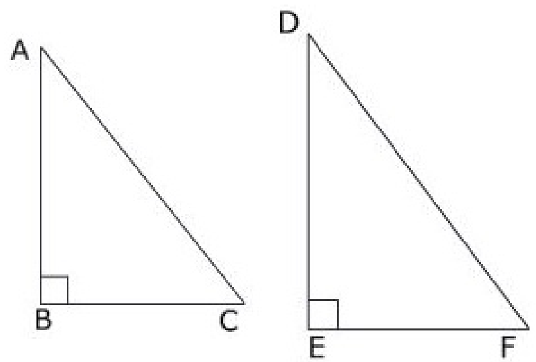
$\Rightarrow \frac{EG}{CD} = \frac{BE}{BC}$ ....(2)

Now, from (1) $\frac{BE}{EC} = \frac{4}{3}$

$\Rightarrow \frac{EC}{BE} = \frac{3}{4}$

$\Rightarrow \frac{EC}{BE} + 1 = \frac{3}{4} + 1$

$\Rightarrow \frac{EC+BE}{BE} = \frac{7}{4}$
$\Rightarrow \frac{BC}{BE} = \frac{7}{4}$ or $\frac{BE}{BC} = \frac{4}{7}$
from equation (2), $\frac{EG}{CD} = \frac{4}{7}$
So $EG = \frac{4}{7}CD$ ......(3)
Similarly, $\Delta DGF \sim \Delta DBA$ (by AA similarity)
$\Rightarrow \frac{DF}{DA} = \frac{FG}{AB}$
$\Rightarrow \frac{FG}{AB} = \frac{3}{7}$
$\Rightarrow FG = \frac{3}{7}AB$ ...(4)
$\left[\begin{array}{l} \because \frac{AF}{AD} = \frac{4}{7} = \frac{BE}{BC} \\ \Rightarrow \frac{EC}{BC} = \frac{3}{7} = \frac{DE}{DA} \end{array}\right]$
Adding equations (3) and (4),we get,
$EG + FG = \frac{4}{7}CD + \frac{3}{7}AB$
$\Rightarrow EF = \frac{4}{7} \times (2AB) + \frac{3}{7}AB$
$= \frac{8}{7}AB + \frac{3}{7}AB = \frac{11}{7}AB$
$\therefore \quad 7EF = 11AB$

20. 

i. Given: In $\triangle$ABC such that

   $AC^2 = AB^2 + BC^2$

   To prove: Triangle ABC is right angled at B

   Construction: Construct a triangle DEF such that

   DE = AB, EF = BC and $\angle E = 90^\circ$

   Proof: $\because \triangle$ DEF is a right angled triangle right angled at E [construction]

   $\therefore$ By Pythagoras theorem, we have

   $DF^2 = DE^2 + EF^2$

   $\Rightarrow DF^2 = AB^2 + BC^2$ [$\because$ DE = AB and EF = BC]

$\Rightarrow DF^2 = AC^2 [\because AB^2 + BC^2 = AC^2]$

$\Rightarrow DF = AC$

Thus,in $\triangle$ ABC and $\triangle$DEF, we have

AB = DE

BC = EF

and AC = DF [By Construction and (i)]

$\therefore \triangle ABC \cong \triangle DEF$ (SSS)

$\Rightarrow \angle B = \angle E = 90^o$

Hence, $\triangle$ ABC is a right triangle.

ii. In $\triangle$QPR , $\angle$ QPR = 90^o

$\Rightarrow 24^2 + x^2 = 26^2$

$\Rightarrow x = 10$

$\Rightarrow$ PR = 10 cm

Now in $\triangle$PKR, $PR^2 = PK^2 + KR^2$[as $10^2 = 8^2 + 6^2$]

$\therefore$ PKR is right angled at K

$\Rightarrow \angle PKR = 90^o$