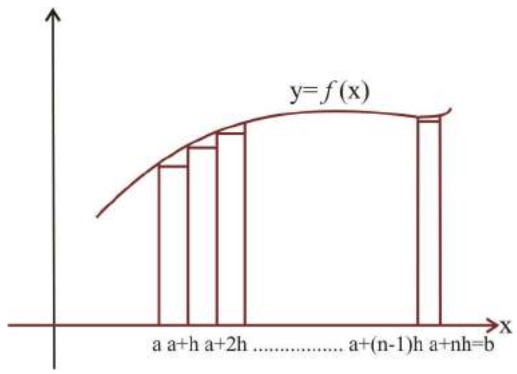
DEFINITE INTEGRATION, AREA UNDER CURVES

1. DEFINITION

Let $f(x)$ be a continuous real valued function defined on the closed interval [a, b] which is divided into n parts as shown in figure.



The point of division on x-axis are

$a, a+h, a+2h.........a+(n-1)h, a+nh$, where $\frac{b-a}{n} = h$.

Let S_n denotes the area of these n rectangles.

Then, $S_n = h f(a) + h f(a+h) + h f(a+2h) + ... + h f(a+(n-1)h)$

Clearly, S_n is area very close to the area of the region bounded by curve $y = f(x)$, x –axis and the ordinates x = a, x = b.

Hence $\int_a^b f(x)\,dx = \underset{n\to\infty}{\text{Lt}}\, S_n$

$$\int_a^b f(x)\,dx = \underset{n\to\infty}{\text{Lt}} \sum_{r=0}^{n-1} h\, f(a+rh)$$

$$= \underset{n\to\infty}{\text{Lt}} \sum_{r=0}^{n-1} \left(\frac{b-a}{n}\right) f\left(a + \frac{(b-a)\,r}{n}\right)$$

NOTES :

1. We can also write

   $S_n = hf(a+h) + hf(a+2h) + ... + hf(a+nh)$ and

   $$\int_a^b f(x)\,dx = \underset{n\to\infty}{\text{Lt}} \sum_{r=1}^{n} \left(\frac{b-a}{n}\right) f\left(a + \left(\frac{b-a}{n}\right) r\right)$$

2. If a = 0, b = 1, $\int_0^1 f(x)\,dx = \underset{n\to\infty}{\text{Lt}} \sum_{r=0}^{n-1} \frac{1}{n} f\left(\frac{r}{n}\right)$

2. DEFINITE INTEGRAL AS LIMIT OF SUM/SUM OF INFINITE SERIES USING DEFINITE INTEGRAL

Step 1. Replace $\frac{r}{n}$ by x, $\frac{1}{n}$ by dx and $\underset{n\to\infty}{\text{Lt}} \sum$ by $\int$

Step 2. Evaluate $\underset{n\to\infty}{\text{Lt}} \left(\frac{r}{n}\right)$ by putting least and greatest values of r as lower and upper limits respectively.

For example $\underset{n\to\infty}{\text{Lt}} \sum_{r=1}^{pn} \frac{1}{n} f\left(\frac{r}{n}\right) = \int_0^p f(x)\,dx$

$$\left[\underset{n\to\infty}{Lt}\left(\frac{r}{n}\right)\Bigg|_{r=1} = 0,\ \underset{n\to\infty}{Lt}\left(\frac{r}{n}\right)\Bigg|_{r=np} = p\right]$$

3. SECOND FUNDAMENTAL THEOREM OF CALCULUS

Let F (x) be any antiderivative of $f(x)$, then for any two values of the independent variable x, say a and b, the difference F (b) – F (a) is called the definite integral of $f(x)$ from a to b and is denoted by $\int_a^b f(x)\,dx$. Thus $\int_a^b f(x)\,dx = F(b) - F(a)$, The numbers a and b are called the limits of integration; a is the lower limit and b is the upper limit. Usually F (b) – F (a) is abbreviated by writing $F(x)\,|_a^b$.

4. GEOMETRICAL INTERPRETATION OF THE DEFINITE INTEGRAL

4.1

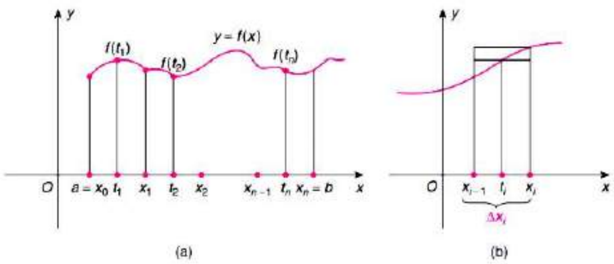


Figure 4.1

Let $f:[a,b] \to [0,\infty)$ be a function, $P = \{x_0, x_1, x_2, \ldots, x_n\}$ be a partition of [a, b] and $t_i \in [x_{i-1}, x_i]$ for i = 1, 2,...., n Then (see fig. 4.1) $f(t_i)\,\Delta x_i = f(t_i)\,(x_i - x_{i-1})$ = Area of the rectangle with width Δx_i and height $f(t_i)$

Hence

$$S(f,P) = \sum_{i=1}^{n} f(t_i)\Delta x_i$$

= Sum of the areas of the rectangles with width $\Delta x_i = x_i - x_{i-1}$ and height $f(t_i)$

Thus, the area A enclosed by the x-axis, the lines x = a, x=b and the curve y = f(x) is approximately equal to S (f,P). When the width of the rectangles becomes smaller, that is when Max $\{\Delta x_1, \Delta x_2, \ldots, \Delta x_n\}$ is small, the sum of the areas or S (f, P) is very nearly equal to A. If f is integrable, then

S(f, P) converges to $\int_a^h f(x)\,dx$ and hence

$$A = \int_a^b f(x)dx$$

Thus, definite integral of a non-negative function f, when integrable, may be interpreted over [a, b] as the area enclosed by the curve y = f(x), the lines x = a, x = b and the x-axis.

4.2 Geometrical Interpretation of the Definite Integral

If $y = f(x)$ is continuous and $\int_a^b f(x)\,dx = 0,$

then $f(x) = 0$ has at least one real root in (a, b).

5. PROPERTIES OF DEFINITE INTEGRALS

1. $\int_a^b f(x)\,dx = -\int_b^a f(x)$

2. $\int_a^b f(x)\,dx = \int_a^b f(y)\,dy$

3. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx,$ where c may or may not lie between a and b.

4. $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

5. $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

NOTES :

1. $\int_0^a \frac{f(x)}{f(x)+f(a-x)}dx = \frac{a}{2}$

2. $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx = \frac{b-a}{2}$

6. $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

$$= \begin{cases} 0 & \text{if } f(2a-x) = -f(x) \\ 2\int_0^a f(x)\,dx & \text{if } f(2a-x) = f(x) \end{cases}$$

7. $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(-x)=f(x) \text{ i.e. } f(x) \text{ is even} \\ 0 & \text{if } f(-x)=-f(x) \text{ i.e. } f(x) \text{ is odd} \end{cases}$

8. If $f(x)$ is a periodic function of period 'a', i.e. $f(a+x)=f(x)$, then

(a) $\int_0^{na} f(x)\,dx = n\int_0^a f(x)\,dx$

(b) $\int_a^{na} f(x)\,dx = (n-1)\int_0^a f(x)\,dx$

(c) $\int_{na}^{b+na} f(x)\,dx = \int_0^b f(x)\,dx$, where $b \in R$

(d) $\int_b^{b+a} f(x)\,dx$ independent of b.

(e) $\int_b^{b+na} f(x)\,dx = n\int_0^a f(x)\,dx$, where $n \in I$

9. If $f(x) \ge 0$ on the interval [a, b], then $\int_a^b f(x)\,dx \ge 0$.

10. If $f(x) \le g(x)$ on the interval [a, b], then

$$\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$$

11. $\left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx$

12. If $f(x)$ is continuous on [a, b], m is the least and M is the greatest value of $f(x)$ on [a, b], then

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

13. For any two functions $f(x)$ and $g(x)$, integrable on the interval [a, b], the Schwarz – Bunyakovsky inequality holds

$$\left|\int_a^b f(x)\,.\,g(x)\,dx\right| \le \sqrt{\int_a^b f^2(x)\,dx.\int_a^b g^2(x)\,dx}$$

14. If a function $f(x)$ is continuous on the interval [a, b], then there exists a point $c \in (a, b)$ such that

$$\int_a^b f(x)\,dx = f(c)(b-a), \text{ where } a < c < b.$$

6. DIFFERENTIATION UNDER INTEGRAL SIGN

NEWTON LEIBNITZ'S THEOREM :

If f is continuous on [a, b] and $g(x)$ & $h(x)$ are differentiable functions of x whose values lie in [a, b], then

$$\frac{d}{dx}\left[\int_{g(x)}^{h(x)} f(t)\,dt\right] = \frac{d}{dx}[h(x)]\,.\,f[h(x)] - \frac{d}{dx}[g(x)]\,.\,f[g(x)]$$

7. REDUCTION FORMULAE IN DEFINITE INTEGRALS

7.1 If $I_n = \int_0^{\frac{\pi}{2}} \sin^n x\,dx$, then show that $I_n = \left(\frac{n-1}{n}\right) I_{n-2}$

Proof: $I_n = \int_0^{\frac{\pi}{2}} \sin^n x\,dx$

$$I_n = \left[-\sin^{n-1} x \cos x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} (n-1)\sin^{n-2} x\,.\cos^2 x\,dx$$

$$= (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2} x\,.(1-\sin^2 x)\,dx$$

$$= (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2} x\,dx - (n-1)\int_0^{\frac{\pi}{2}} \sin^n x\,dx$$

$$I_n + (n-1)I_n = (n-1)I_{n-2}$$

$$I_n = \left(\frac{n-1}{n}\right) I_{n-2}$$

NOTES :

1. $\int_0^{\frac{\pi}{2}} \sin^n x\, dx = \int_0^{\frac{\pi}{2}} \cos^n x\, dx$

2. $I_n = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right) \ldots\ldots I_0 \text{ or } I_1$

according as n is even or odd. $I_0 = \frac{\pi}{2}, I_1 = 1$

$$\text{Hence } I_n = \begin{cases} \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right) \ldots\ldots \left(\frac{1}{2}\right) . \frac{\pi}{2} & \text{if } n \text{ is even} \\ \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right) \ldots\ldots \left(\frac{2}{3}\right) . 1 & \text{if } n \text{ is odd} \end{cases}$$

7.2 If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x\, dx$, then show that $I_n + I_{n-2} = \frac{1}{n-1}$

Proof. $I_n = \int_0^{\frac{\pi}{4}} (\tan x)^{n-2} . \tan^2 x\, dx$

$$= \int_0^{\frac{\pi}{4}} (\tan x)^{n-2} (\sec^2 x - 1)\, dx$$

$$= \int_0^{\frac{\pi}{4}} (\tan x)^{n-2} \sec^2 x\, dx - \int_0^{\frac{\pi}{4}} (\tan x)^{n-2}\, dx$$

$$= \left[\frac{(\tan x)^{n-1}}{n-1}\right]_0^{\frac{\pi}{4}} - I_{n-2}$$

$$I_n = \frac{1}{n-1} - I_{n-2}$$

$$\therefore \quad I_n + I_{n-2} = \frac{1}{n-1}$$

7.3 If $I_{m,n} = \int_0^{\frac{\pi}{2}} \sin^m x . \cos^n x\, dx$, then show that

$$I_{m,n} = \frac{m-1}{m+n} I_{m-2}, n$$

Proof. $I_{m,n} = \int_0^{\frac{\pi}{2}} \sin^{m-1} x\, (\sin x \cos^n x)\, dx$

$$= \left[-\frac{\sin^{m-1} x . \cos^{n+1} x}{n+1}\right]_0^{\frac{\pi}{2}} +$$

$$\int_0^{\frac{\pi}{2}} \frac{\cos^{n+1} x}{n+1} (m-1) \sin^{m-2} x \cos x\, dx$$

$$= \left(\frac{m-1}{n+1}\right) \int_0^{\frac{\pi}{2}} \sin^{m-2} x . \cos^n x . \cos^2 x\, dx$$

$$= \left(\frac{m-1}{n+1}\right) \int_0^{\frac{\pi}{2}} (\sin^{m-2} x . \cos^n x - \sin^m x . \cos^n x)\, dx$$

$$= \left(\frac{m-1}{n+1}\right) I_{m-2,n} - \left(\frac{m-1}{n+1}\right) I_{m,n}$$

$$\Rightarrow \left(1 + \frac{m-1}{n+1}\right) I_{m,n} = \left(\frac{m-1}{n+1}\right) I_{m-2,n}$$

$$I_{m,n} = \left(\frac{m-1}{m+n}\right) I_{m-2,n}$$

NOTES :

1. $I_{m,n} = \left(\frac{m-1}{m+n}\right)\left(\frac{m-3}{m+n-2}\right)\left(\frac{m-5}{m+n-4}\right) ....\, I_{0,n} \text{ or } I_{1,n}$

according as m is even or odd.

$$I_{0,n} = \int_0^{\frac{\pi}{2}} \cos^n x\, dx \text{ and } I_{1,n} = \int_0^{\frac{\pi}{2}} \sin x \,.\, \cos^n x\, dx = \frac{1}{n+1}$$

2. **Walli's Formula**

$$I_{m,n} = \begin{cases} \frac{(m-1)(m-3)(m-5)\ldots\ldots(n-1)(n-3)(n-5)\ldots\ldots}{(m+n)(m+n-2)(m+n-4)\ldots\ldots\ldots}\frac{\pi}{2} & \text{when both m, n are even} \\ \frac{(m-1)(m-3)(m-5)\ldots\ldots\ldots(n-1)(n-3)(n-5)\ldots\ldots\ldots}{(m+n)(m+n-2)(m+n-4)\ldots\ldots\ldots} & \text{otherwise} \end{cases}$$

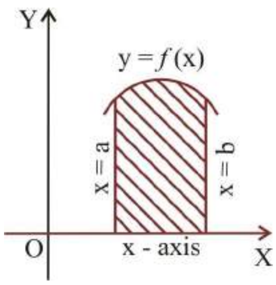
8. AREA OF PLANE REGIONS

1. The area bounded by the curve $y = f(x)$, x-axis and the ordinates x = a. and x = b (where b > a) is given by

$$A = \int_a^b |y|\, dx = \int_a^b |f(x)|\, dx$$

(i) If $f(x) > 0 \;\forall\; x \in [a, b]$

Then $A = \int_a^b f(x)\, dx$

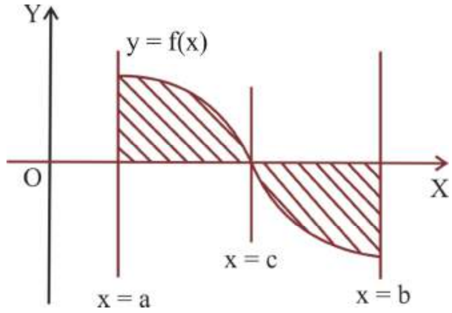


(ii) If $f(x) > 0 \;\forall\; x \in [a, c)$ &

$< 0 \;\forall\; x \in (c, b]$ Then

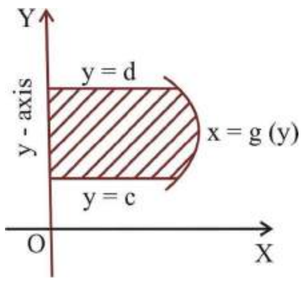
$$A = \left|\int_a^c y\, dx\right| + \left|\int_c^b y\, dx\right| = \int_a^c f(x)\, dx - \int_c^b f(x)\, dx$$

where c is a point in between a and b.



2. The area bounded by the curve $x = g(y)$, y – axis and the abscissae y = c and y = d (where d > c) is given by

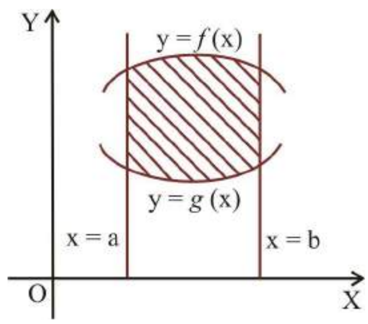
$$A = \int_c^d |x|\, dy = \int_c^d |g(y)|\, dy$$



3. If we have two curve $y = f(x)$ and $y = g(x)$, such that $y = f(x)$ lies above the curve $y = g(x)$ then the area bounded between them and the ordinates x = a and x = b (b > a), is given by

$$A = \int_a^b f(x)\, dx - \int_a^b g(x)\, dx$$

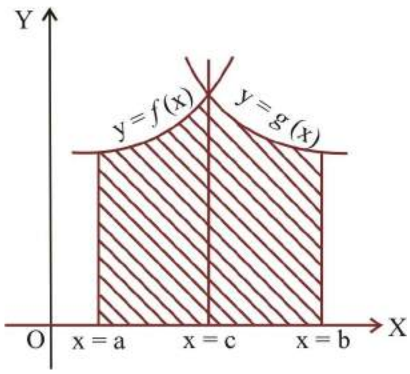
i.e. upper curve area – lower curve area.



4. The area bounded by the curves $y = f(x)$ and $y = g(x)$ between the ordinates $x = a$ and $x = b$ is given by

$$A = \int_a^c f(x)\,dx + \int_c^b g(x)\,dx,$$

where $x = c$ is the point of intersection of the two curves.



9. CURVE TRACING

In order to find the area bounded by several curves, it is important to have rough sketch of the required portion. The following steps are very useful in tracing a cartesian curve $f(x, y) = 0$.

Step 1 : Symmetry

(i) The curve is symmetrical about x-axis if all powers of y in the equation of the given curve are even.

(ii) The curve is symmetrical about y-axis if all powers of x in the equation of the given curve are even.

(iii) The curve is symmetrical about the line $y = x$, if the equation of the given curve remains unchanged on interchanging x and y.

(iv) The curve is symmetrical in opposite quadrants, if the equation of the given curve remains unchanged when x and y are replaced by – x and – y respectively.

Step 2 : Origin

If there is no constant term in the equation of the algebraic curve, then the curve passes through the origin.

In that case, the tangents at the origin are given by equating to zero the lowest degree terms in the equation of the given algebraic curve.

For example, the curve $y^3 = x^3 + axy$ passes through the origin and the tangents at the origin are given by $axy = 0$ i.e. $x = 0$ and $y = 0$.

Step 3 : Intersection with the Co-ordinate Axes

(i) To find the points of intersection of the curve with X-axis, put $y = 0$ in the equation of the given curve and get the corresponding values of x.

(ii) To find the points of intersection of the curve with Y-axis, put $x = 0$ in the equation of the given curve and get the corresponding values of y.

Step 4 : Asymptotes

Find out the asymptotes of the curve.

(i) The vertical asymptotes or the asymptotes parallel to y-axis of the given algebraic curve are obtained by equating to zero the coefficient of the highest power of y in the equation of the given curve.

(ii) The horizontal asymptotes or the asymptotes parallel to x-axis of the given algebraic curve are obtained by equating to zero the coefficient of the highest power of x in the equation of the given curve.

Step 5 : Region

Find out the regions of the plane in which no part of the curve lies. To determine such regions we solve the given equation for y in terms of x or vice-versa. Suppose that y becomes imaginary for $x > a$, the curve does not lie in the region $x > a$.

Step 6: Critical Points

Find out the values of x at which $\frac{dy}{dx} = 0$.

At such points y generally changes its character from an increasing function of x to a decreasing function of x or vice-versa.

Step 7: Trace the curve with the help of the above points.

SOLVED EXAMPLES

Example – 1

Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)\,g(x)\,dx$ is

(a) $e + \frac{e^2}{2} - \frac{2}{2}$ (b) $e - \frac{e^2}{2} - \frac{3}{2}$

(c) $e + \frac{e^2}{2} + \frac{5}{2}$ (d) $e - \frac{e^2}{2} - \frac{5}{2}$

Ans. (b)

Sol. As $f(x) = f'(x)$ and $f(0) = 1$

$$\Rightarrow \frac{f'(x)}{f(x)} = 1$$

$$\Rightarrow \log(f(x)) = x + c \Rightarrow f(x) = e^x + k$$

$$\Rightarrow f(x) = e^x \text{ as } f(0) = 1$$

Now $g(x) = x^2 - e^x$

$$\therefore \int_0^1 f(x)g(x)dx = \int_0^1 e^x\left(x^2 - e^x\right)dx$$

$$= \int_0^1 x^2 e^x dx - \int_0^1 e^{2x} dx$$

$$= \left[\left(x^2 - 2x + 2\right)e^x\right]_0^1 - \left(\frac{e^{2x}}{2}\right)_0^1$$

$$= (e-2) - \left(\frac{e^2-1}{2}\right) = e - \frac{e^2}{2} - \frac{3}{2}$$

Using $\int f^n(x)e^x dx = e^x\left[f^n(x) - f_1^n(x) + f_2^n(x) + \ldots + (-1)^n f_n(x)\right]$

Where $f_1, f_2, \ldots f_n$ are derivatives of first, second $\ldots n^{th}$ order.

Example – 2

Evaluate the following integrals :

(i) $\int_2^3 x^2\,dx$ (ii) $\int_1^3 \frac{x}{(x+1)(x+2)}\,dx$

Sol. (i) $\int_2^3 x^2 dx = \left[\frac{x^3}{3}\right]_2^3$

$$= \frac{27}{3} - \frac{8}{3} = \frac{19}{3}$$

(ii) $\frac{x}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{2}{x+2}$

$\therefore \int_1^3 \frac{x}{(x+1)(x+2)} dx = \left[-\log|x+1| + 2\log|x+2|\right]_1^3$

$= [-\log|4| + 2\log|5|] - [-\log|2| + 2\log|3|]$

$= [-\log 4 + 2\log 5] - [-\log 2 + 2\log 3]$

$= -2\log 2 + 2\log 5 + \log 2 - 2\log 3$

$= -\log 2 + \log 25 - \log 9 = \log 25 - \log 18$

$= \log\frac{25}{18}$

Example – 3

Evaluate : $\int_0^{\pi/4} \sec x \cdot \sqrt{\frac{1-\sin x}{1+\sin x}}\,dx$.

Sol. $I = \int_0^{\pi/4} \sec x \cdot \sqrt{\frac{1-\sin x}{1+\sin x}}\,dx$

$$= \int_0^{\pi/4} \sec x . \sqrt{\frac{1-\sin x}{1+\sin x}} \cdot \sqrt{\frac{1-\sin x}{1-\sin x}}\,dx$$

$$= \int_0^{\pi/4} \sec x \frac{1-\sin x}{\sqrt{1-\sin^2 x}}\,dx$$

$$= \int_0^{\pi/4} \sec x \frac{1-\sin x}{\cos x} dx$$

$$= \int_0^{\pi/4} (\sec^2 x - \sec x \tan x)\, dx$$

$$= \int_0^{\pi/4} \sec^2 x\, dx - \int_0^{\pi/4} \sec x \tan x\, dx$$

$$= [\tan x]_0^{\pi/4} - [\sec x]_0^{\pi/4}$$

$$= \left(\tan\frac{\pi}{4} - \tan 0\right) - \left(\sec\frac{\pi}{4} - \sec 0\right)$$

$$= (1-0) - (\sqrt{2}-1) = 2-\sqrt{2}.$$

Example – 4

Evaluate : $\int_{-1}^{1} 5x^4\sqrt{x^5+1}\, dx$.

Sol. Let $I = \int_{-1}^{1} 5x^4\sqrt{x^5+1}\, dx$

Put $x^5 = t$ so that $5x^4 dx = dt$.

When $x = -1, t = -1$. When $x = 1, t = 1$.

$\therefore$ $$I = \int_{-1}^{1} \sqrt{t+1}\, dt$$

$$= \left[\frac{(t+1)^{3/2}}{3/2}\right]_{-1}^{1} = \frac{2}{3}\left[(t+1)^{3/2}\right]_{-1}^{1}$$

$$= \frac{2}{3}[2^{3/2} - 0] = \frac{4\sqrt{2}}{3}.$$

Example – 5

Prove that $\int_0^{\pi/2} \sqrt{\sin\phi}\cos^5\phi\, d\phi = \frac{64}{231}$.

Sol. $$I = \int_0^{\pi/2} \sqrt{\sin\phi}\cos^5\phi\, d\phi$$

$$= \int_0^{\pi/2} \sqrt{\sin\phi}\cos^4\phi\cos\phi\, d\phi$$

$$= \int_0^{\pi/2} \sqrt{\sin\phi}(1-\sin^2\phi)^2\cos\phi\, d\phi$$

Put $\sin\phi = t$ so that $\cos\phi\, d\phi = dt$.

When $\phi = 0, \sin 0 = t \Rightarrow t = 0$.

When $\phi = \frac{\pi}{2}, \sin\frac{\pi}{2} = t \Rightarrow t = 1$

$\therefore$ $$I = \int_0^1 \sqrt{t}(1-t^2)^2\, dt = \int_0^1 \sqrt{t}(1-2t^2+t^4)\, dt$$

$$= \int_0^1 (t^{1/2} - 2t^{5/2} + t^{9/2})\, dt$$

$$= \left[\frac{t^{3/2}}{3/2} - 2\frac{t^{7/2}}{7/2} + \frac{t^{11/2}}{11/2}\right]_0^1$$

$$= \left[\frac{2}{3}t^{3/2} - \frac{4}{7}t^{7/2} + \frac{2}{11}t^{11/2}\right]_0^1$$

$$= \left[\frac{2}{3}(1) - \frac{4}{7}(1) + \frac{2}{11}(1)\right] - [0-0+0]$$

$$= \frac{2}{3} - \frac{4}{7} + \frac{2}{11}$$

$$= \frac{154-132+42}{231} = \frac{64}{231}.$$

Example – 6

Evaluate : $\int_1^2 \left(\frac{x-1}{x^2}\right) e^x dx$

Or

$\int_1^2 e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

Sol. $\int \left(\frac{x-1}{x^2}\right) e^x dx = \int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

$= \int \frac{1}{x} . e^x dx - \int \frac{1}{x^2} . e^x dx$

$= \frac{1}{x} . e^x - \int \left(-\frac{1}{x^2}\right) e^x dx - \int \frac{1}{x^2} . e^x dx$

[Integrating first integral by parts]

$= \frac{1}{x} . e^x = F(x)$

$\therefore \quad \int_1^2 \left(\frac{x-1}{x^2}\right) e^x dx = \left[\frac{e^x}{x}\right]_1^2$

$= \frac{1}{2} . e^2 - \frac{1}{1} e^1 = \frac{1}{2} e^2 - e$.

Example – 7

Evaluate : $\int_a^b x^2 \, dx$ using limit of a sum formula.

Sol. Let $I = \int_a^b x^2 dx = \lim_{\substack{n \to \infty \\ h \to 0}} h \left[(a+h)^2 + (a+2h)^2 + ... + (a+nh)^2\right]$

$= \lim_{\substack{n \to \infty \\ h \to 0}} h \left[\left(a^2 + a^2 + ...\right) + \left(h^2 + 4h^2 + ... + n^2h^2\right)\right.$

$\left. + \left(2ah + 4ah + ... + 2anh\right)\right]$

$\Rightarrow \quad I = \lim_{\substack{n \to \infty \\ h \to 0}} \left[nha^2 + \frac{2ah^2 n(n+1)}{2} + \frac{h^3 n(n+1)(2n+1)}{6}\right]$

Using nh = b – a, we get :

$\Rightarrow \quad I = \lim_{n \to \infty} \left[a^2(b-a) + a(b-a)^2\left(1 + \frac{1}{n}\right)\right.$

$\left. + (b-a)^3 \frac{1}{6}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)\right]$

$\Rightarrow \quad I = a^2(b-a) + a(b-a)^2 + \frac{(b-a)^3}{6}(2)$

$\Rightarrow \quad I = (b-a)\left[a^2 + ab - a^2 + \frac{b^2 + a^2 - 2ab}{3}\right]$

$\Rightarrow \quad I = \frac{(b-a)}{3}\left[a^2 + b^2 + ab\right] = \frac{b^3 - a^3}{3}$

Example – 8

Evaluate : $\int_0^1 |3x-1| \, dx$

Sol. We know that $|3x - 1| = 3x - 1$,

when $x \ge \frac{1}{3}$ i.e. when $3x - 1 \ge 0$

and $|3x - 1| = -(3x - 1)$

when $x < \frac{1}{3}$ i.e. when $3x - 1 < 0$.

$\therefore \quad \int_0^1 |3x-1| dx = \int_0^{1/3} |3x-1| dx + \int_{1/3}^1 |3x-1| dx$

$= \int_0^{1/3} -(3x-1) \, dx + \int_{1/3}^1 (3x-1) \, dx$

$= \left[-\left(\frac{3}{2}x^2 - x\right)\right]_0^{1/3} + \left[\frac{3}{2}x^2 - x\right]_{1/3}^1$

$= -\left(\frac{3}{2} . \frac{1}{9} - \frac{1}{3}\right) + \left[\left(\frac{3}{2}(1) - 1\right) - \left(\frac{3}{2} . \frac{1}{9} - \frac{1}{3}\right)\right]$

$= -\left[\frac{1}{6} - \frac{1}{3}\right] + \left[\frac{3}{2} - 1\right] - \left[\frac{1}{6} - \frac{1}{3}\right]$

$= -2\left(\frac{1}{6} - \frac{1}{3}\right) + \frac{1}{2} = -2\left(-\frac{1}{6}\right) + \frac{1}{2} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$

Example – 9

Evaluate : $\int_{0.5}^{3.5} [x]\, dx$ where [.] is GIF

Sol. Here $f(x) = [x] = \begin{cases} 0, 0.5 \le x < 1 \\ 1, 1 \le x < 2 \\ 2, 2 \le x < 3 \\ 3, 3 \le x < 3.5 \end{cases}$

$$\therefore \quad \int_{0.5}^{3.5} [x]\, dx = \int_{0.5}^{1} [x]\, dx + \int_{1}^{2} [x]\, dx + \int_{2}^{3} [x]\, dx + \int_{3}^{3.5} [x]\, dx$$

$$= \int_{0.5}^{1} 0.dx + \int_{1}^{2} 1.dx + \int_{2}^{3} 2.dx + \int_{3}^{3.5} 3\, dx$$

$$= 0 + [x]_1^2 + 2[x]_2^3 + 3[x]_3^{3.5}$$

$$= (2-1) + 2(3-2) + 3(3.5-3)$$

$$= 1 + 2 + 1.5 = 4.5.$$

Example – 10

Evaluate : $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$

Sol. Let : $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$ ... (i)

Using property – 4, we have :

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2 - x)} + \sqrt{\cos(\pi/2 - x)}}\, dx$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx \quad \text{... (ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} dx = \frac{\pi}{2} \qquad \Rightarrow I = \frac{\pi}{4}$$

Example – 11

Prove that $\int_0^{\pi/2} \sin 2x \, \log \tan x \, dx = 0.$

Sol. Let $I = \int_0^{\pi/2} \sin 2x \log \tan x \, dx$ ... (1)

$$\therefore \quad I = \int_0^{\pi/2} \sin 2\left(\frac{\pi}{2} - x\right) \log \tan\left(\frac{\pi}{2} - x\right) dx$$

$$= \int_0^{\pi/2} \sin(\pi - 2x) \log \tan\left(\frac{\pi}{2} - x\right) dx$$

$$= \int_0^{\pi/2} \sin 2x \log \cot x \, dx \quad \text{... (2)}$$

Adding (1) and (2) :

$$2I = \int_0^{\pi/2} \sin 2x \, [\log \tan x + \log \cot x]\, dx$$

$$= \int_0^{\pi/2} \sin 2x \log(\tan x \cot x)\, dx$$

$$= \int_0^{\pi/2} \sin 2x \log 1 \, dx = 0$$

$[\because \log 1 = 0]$

Hence I = 0.

Example – 12

If $f(a+b-x)=f(x)$, then $\int_a^b x\,f(x)\,dx$ is equal to

(a) $\frac{a+b}{2}\int_a^b f(x)\,dx$

(b) $\frac{b-a}{2}\int_a^b f(x)\,dx$

(c) $\frac{a+b}{2}\int_a^b f(a+b-x)\,dx$

(d) $\frac{a+b}{2}\int_a^b f(b-x)\,dx$

Ans. (a,c)

Sol. Let $I=\int_a^b xf(x)dx$

$$I=\int_a^b (a+b-x)f(a+b-x)dx$$

$$I=\int_a^b (a+b)f(a+b-x)dx-\int_a^b xf(a+b-x)dx$$

$$I=\int_a^b (a+b)f(x)dx-\int_a^b xf(x)dx$$

$$\therefore I=\frac{a+b}{2}\int_a^b f(x)dx=\frac{a+b}{2}\int_a^b f(a+b-x)dx$$

Example – 13

Evaluate : $\int_{-2}^{2} f(x)\,dx$;

where $f(x)=\begin{cases}2x-1, -2\le x<1\\ 3x-2, 1\le x<2\end{cases}$

Sol. $\int_{-2}^{2} f(x)\,dx=\int_{-2}^{1} f(x)\,dx+\int_{1}^{2} f(x)\,dx$

$$=\int_{-2}^{1}(2x-1)\,dx+\int_{1}^{2}(3x-2)\,dx$$

$$=\left[2.\frac{x^2}{2}-x\right]_{-2}^{1}+\left[\frac{3x^2}{2}-2x\right]_{1}^{2}$$

$$=\left[x^2-x\right]_{-2}^{1}+\left[\frac{3}{2}x^2-2x\right]_{1}^{2}$$

$$=[(1-1)-(4+2)]+\left[\left(\frac{3}{2}(4)-2(2)\right)-\left(\frac{3}{2}(1)-2(1)\right)\right]$$

$$=(0-6)+\left(2+\frac{1}{2}\right)=-6+\frac{5}{2}=-\frac{7}{2}$$

Example – 14

Evaluate : $\int_0^{\pi/2}\sqrt{1-\sin 2x}\,dx.$

Sol. Let $I=\int_0^{\pi/2}\sqrt{1-\sin 2x}\,dx$

$$\Rightarrow\ I=\int_0^{\pi/2}\sqrt{(\sin x-\cos x)^2}\,dx$$

$$\Rightarrow\ I=\int_0^{\pi/2}|\sin x-\cos x|\,dx$$

$$\Rightarrow\ I=\int_0^{\pi/4}|\sin x-\cos x|\,dx+\int_{\pi/4}^{\pi/2}(\sin x-\cos x)\,dx$$

$$\Rightarrow\ I=\int_0^{\pi/4}(\cos x-\sin x)\,dx+\int_{\pi/4}^{\pi/2}(\sin x-\cos x)\,dx$$

$$\Rightarrow\ I=\left[\sin x+\cos x\right]_0^{\pi/4}+\left[-\cos x-\sin x\right]_{\pi/4}^{\pi/2}$$

$$\Rightarrow\ I=\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-1\right)+(-1)-\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow\ I=2\sqrt{2}-2$$

Example – 15

Evaluate : $\int_0^{\pi} \frac{x}{1+\cos^2 x} dx.$

Sol. Let $I = \int_0^{\pi} \frac{x}{1+\cos^2 x} dx$ ... (i)

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1+\cos^2(\pi - x)} dx$ [using property – 4]

... (ii)

Adding (i) and (ii), we get :

$\Rightarrow 2I = \int_0^{\pi} \frac{\pi}{1+\cos^2 x} dx$

$\Rightarrow I = \frac{\pi}{2} \int_0^{\pi} \frac{dx}{1+\cos^2 x} = \frac{2\pi}{2} \int_0^{\pi/2} \frac{dx}{1+\cos^2 x}$

[using property – 6]

Divide N^r and D^r by $\cos^2 x$ to get :

$\Rightarrow I = \pi \int_0^{\pi/2} \frac{\sec^2 x}{\sec^2 x + 1} dx$

Put $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$[\sec^2 x = 1 + \tan^2 x]$

For $x = \pi/2, t \to \infty$ and for $x = 0, t = 0$

$\Rightarrow I = \pi \int_0^{\infty} \frac{dt}{2+t^2}$

$\Rightarrow I = \frac{\pi}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} \Big|_0^{\infty} = \frac{\pi}{\sqrt{2}} \times \frac{\pi}{2} = \frac{\pi^2}{2\sqrt{2}}$

Example – 16

Evaluate : $\int_0^{\pi} \frac{x \sin(2x) \sin\left(\frac{\pi}{2}\cos x\right) dx}{2x - \pi}$

Sol. Let $I = \int_0^{\pi} \frac{x \sin(2x) \sin\left(\frac{\pi}{2}\cos x\right) dx}{2x - \pi}$ ... (i)

Apply property – 4 to get

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)\sin(2\pi - 2x) \sin\left(\frac{\pi}{2}\cos(\pi - x)\right) dx}{2(\pi - x) - \pi}$

$= \int_0^{\pi} - \frac{(\pi - x) \sin 2x \sin\left(\frac{\pi}{2}\cos x\right) dx}{2x - \pi}$ ... (ii)

Add (i) and (ii) to get

$2I = \int_0^{\pi} 2 \sin x \cos x \sin\left[\frac{\pi}{2}\cos x\right] dx$

Let $\frac{\pi}{2}\cos x = t \Rightarrow -\frac{\pi}{2} \sin x \, dx = dt$

$\Rightarrow I = \frac{4}{\pi^2} \int_{-\pi/2}^{\pi/2} t \sin t \, dt = \frac{8}{\pi^2} \int_0^{\pi/2} t \sin t \, dt$

$\Rightarrow I = \frac{8}{\pi^2}\left[(-t\cos t)_0^{\pi/2} + (\sin t)_0^{\pi/2}\right]$

$= \frac{8}{\pi^2}[0+1] = \frac{8}{\pi^2}$

Example – 17

Evaluate : $\int_0^{\pi/2} \log \sin x\, dx.$

Sol. Let $I = \int_0^{\pi/2} \log \sin x\, dx$ ... (i)

$\Rightarrow I = \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx$ [using property–4]

$\Rightarrow I = \int_0^{\pi/2} \log \cos x\, dx$ ... (ii)

Adding (i) and (ii) we get :

$$2I = \int_0^{\pi/2} \log(\sin x \cos x)\, dx = \int_0^{\pi/2} \log\left(\frac{\sin 2x}{2}\right) dx$$

$\Rightarrow 2I = \int_0^{\pi/2} \log \sin 2x\, dx - \int_0^{\pi/2} \log 2\, dx$

$\Rightarrow 2I = \int_0^{\pi/2} \log \sin 2x\, dx - \frac{\pi}{2} \log 2$ ... (iii)

Let $I_1 = \int_0^{\pi/2} \log \sin 2x\, dx$

Put $t = 2x \Rightarrow dt = 2dx$

For $x = \frac{\pi}{2}, t = \pi$ and for $x = 0, t = 0$

$\Rightarrow I_1 = \frac{1}{2}\int_0^{\pi} \log \sin t\, dt = \frac{2}{2}\int_0^{\pi/2} \log \sin t\, dt$

$\Rightarrow I_1 = \int_0^{\pi/2} \log \sin x\, dx$ [using property–2]

$\Rightarrow I_1 = I$

Substituting in (iii) we get :

$2I = I - \pi/2 \log 2$

$\Rightarrow I = -\pi/2 \log 2$

[learn this result so that you can directly apply it in other difficult problem]

Example – 18

Show that $\int_0^{n\pi+v} |\sin x|\, dx = 2n + 1 - \cos v$, where n is a +ve integer and $0 \le v \le \pi$.

Sol. Let $I = \int_0^{n\pi+v} |\sin x| dx = \int_0^{n\pi} |\sin x| dx + \int_{n\pi}^{n\pi+v} |\sin x|\, dx$

[using property – 3]

$\Rightarrow I = I_1 + I_2$ ... (i)

Consider I_1 :

$I_1 = \int_0^{n\pi} |\sin x| dx = n\int_0^{\pi} |\sin x|\, dx$

[using property and period of |sin x| is π]

$\Rightarrow I_1 = n\int_0^{\pi} \sin x\, dx$

[As $\sin x \ge 0$ in $[0, \pi]$, $|\sin x| = \sin x$]

$\Rightarrow I_1 = -n \cos x\big|_0^{\pi} = -n[-1-1] = 2n$

Consider I_2: $I_2 = \int_{n\pi}^{n\pi+v} |\sin x|\, dx = \int_0^{v} |\sin x|\, dx$

[as period of |sin x| = π]

$\Rightarrow I_2 = \int_0^{v} \sin x\, dx$

[as for $0 \le x \le \pi$, sin x is positive]

$= -\cos x\big|_0^{v} = 1 - \cos v$

On substituting the values of I_1 and I_2 in (i), we get

$I = 2n + (1 - \cos v) = 2n + 1 - \cos v.$

Hence proved.

Example – 19

Evaluate $\dfrac{d}{dx}\left(\int_{1/x}^{\sqrt{x}} \cos t^2 dt\right)$

Sol. Let, $f(x) = \int_{1/x}^{\sqrt{x}} \cos t^2 dt$

$$\therefore \quad \frac{d}{dx}(f(x)) = \cos(\sqrt{x})^2 \cdot \left\{\frac{d}{dx}(\sqrt{x})\right\} - \cos\left(\frac{1}{x}\right)^2 \left\{\frac{d}{dx}\left(\frac{1}{x}\right)\right\}$$

$$= \frac{1}{2\sqrt{x}}\cos x + \frac{1}{x^2}.\cos\left(\frac{1}{x^2}\right) \text{ (Using Leibnitz Rule)}$$

$$\Rightarrow \quad \frac{d}{dx}\left(\int_{1/x}^{\sqrt{x}} \cos t^2 dt\right) = \frac{1}{2\sqrt{x}}\cos x + \frac{1}{x^2}\cos\left(\frac{1}{x^2}\right).$$

Example – 20

Find the points of local minimum and local maximum of the function $\int_0^{x^2} \dfrac{t^2 - 5t + 4}{2 + e^t} dt.$

Sol. Let $y = \int_0^{x^2} \dfrac{t^2 - 5t + 4}{2 + e^t} dt = \int_0^{x^2} \dfrac{(t-1)(t-4)}{2 + e^t} dt$

For the points of Extremes,

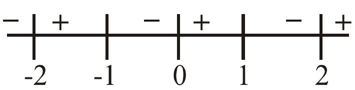
$$\frac{dy}{dx} = 0$$

$$\left[\frac{(x^2-1)(x^2-4)}{2 + e^{x^2}}\right](2x) = 0 \quad \text{[using property under point 3]}$$

$\Rightarrow \; x = 0$ or $x^4 - 5x^2 + 4 = 0$

$\Rightarrow \; x = 0$ or $(x-1)(x+1)(x-2)(x+2) = 0$

$\Rightarrow \; x = 0, x = \pm 1$ and $x = \pm 2$



$f'(x)$ changes sign from (–) to (+) at $x = -2, 0, 2$ where as $f'(x)$ changes sign from (+) to (–) at $x = -1, 1$

$\Rightarrow x = -2, 0, 2$ are points of local minimum and $x = -1, 1$ are points of local maximum.

Example – 21

Let $F : R \to R$ be a differentiable function having $f(2) = 6,\ f'(2) = \left(\dfrac{1}{48}\right)$. Then $\lim\limits_{x\to 2}\int_6^{f(x)} \dfrac{4t^3}{x-2} dt$ equals

(a) 36 (b) 24

(c) 18 (d) 12

Ans. (c)

Sol. $\lim\limits_{x\to 2}\int_6^{f(x)} \dfrac{4t^3}{x-2} dt$

$\because \left(\dfrac{0}{0}\right)$ form

$$= \lim_{x\to 2} \frac{f'(x)\times 4(f(x))^3}{1}$$

$$= 4f'(2)\times(f(2))^3 = \frac{1}{48}\times 4\times 6\times 6\times 6 = 18$$

Example – 22

If for a continuous function $\int_{-\pi}^{t}(f(x)+x)dx = \pi^2 - t^2$, for all $t \geq -\pi$, then $f\left(-\dfrac{\pi}{3}\right)$ is equal to:

(a) π (b) $\dfrac{\pi}{2}$

(c) $\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{6}$

Ans. (a)

Sol. $\int_{-\pi}^{t}(f(x)+x)dx = \pi^2 - t^2$

Applying Newton-Leibnitz theorem,

$f(t) + t = -2t$

$$f\left(-\frac{\pi}{3}\right) - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\Rightarrow f\left(-\frac{\pi}{3}\right) = \pi$$

Example – 23

Find the sum of the series :

$$\lim_{n\to\infty}\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+........+\frac{1}{6n}.$$

Sol. Let $S=\lim_{n\to\infty}\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+........+\frac{1}{n+5n}$

Take 1/n common from the series i.e.

$$S=\lim_{n\to\infty}\frac{1}{n}\left[1+\frac{1}{1+\frac{1}{n}}+\frac{1}{1+\frac{2}{n}}+.......+\frac{1}{1+\frac{5n}{n}}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{5n}\frac{1}{1+r/n}$$

For the definite integral,

$$\text{Lower limit}=a=\lim_{n\to\infty}\left(\frac{r}{n}\right)=\lim_{n\to\infty}\frac{1}{n}=0$$

$$\text{Upper limit}=b=\lim_{n\to\infty}\left(\frac{r}{n}\right)=\lim_{n\to\infty}\frac{5n}{n}=5$$

Therefore,

$$S=\frac{1}{n}\lim_{n\to\infty}\sum_{r=0}^{5n}\frac{1}{1+(r/n)}$$

$$=\int_0^5\frac{dx}{1+x}=\left[\ln|1+x|\right]_0^5=\ln 6-\ln 1=\ln 6$$

Example – 24

If $S_n=\frac{1}{1+\sqrt{n}}+\frac{1}{2+\sqrt{2n}}+\frac{1}{3+\sqrt{3n}}+...+\frac{1}{n+\sqrt{n^2}}$, then find $\lim_{n\to\infty}S_n$.

(a) log 2 (b) 2 log 2

(c) 3 log 2 (d) 4 log 2

Ans. (b)

Sol. $S_n=\frac{1}{1+\sqrt{n}}+\frac{1}{2+\sqrt{2n}}+\frac{1}{3+\sqrt{3n}}+...+\frac{1}{n+\sqrt{n^2}}$

$$S_n=\sum_{r=1}^{n}\frac{1}{n}\left(\frac{1}{\frac{r}{n}+\sqrt{\frac{r}{n}}}\right)$$

$$\therefore \lim_{n\to\infty}S_n=\int_0^1\frac{1}{x+\sqrt{x}}dx$$

$$=\int_0^1\frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)}dx$$

$$=2\ln\left(\sqrt{x}+1\right)\Big|_0^1$$

$$=2\ln 2$$

Example – 25

If $I_1=\int_0^1 2^{x^2}\,dx,\ I_2=\int_0^1 2^{x^3}\,dx,$

$I_3=\int_1^2 2^{x^2}\,dx$ and $I_4=\int_1^2 2^{x^3}\,dx$

then

(a) $I_1>I_2$ (b) $I_2>1_1$

(c) $I_3>I_4$ (d) $I_3=I_4$

Ans. (a)

Sol. For $0<x<1$,

$x^2>x^3$

$\therefore 2^{x^2}>2^{x^3}$

$\therefore \int_0^1 2^{x^2}dx>\int_0^1 2^{x^3}dx$

$\therefore I_1>I_2$

and for $1<x<2, x^3>x^2 \therefore 2^{x^3}>2^{x^2}$

i.e. $\int 2^{x^2}<\int 2^{x^3}\Rightarrow I_3<I_4$

Example – 26

$I_n = \int_0^{\pi/4} \tan^n x \, dx$, then $\lim_{n\to\infty} n\,[I_n + I_{n-2}]$ equals

(a) 1/2 (b) 1

(c) ∞ (d) 0

Ans. (b)

Sol. $I_n = \int_0^{\pi/4} \tan^n x dx$

$I_{n-2} = \int_0^{\pi/4} \tan^{n-2} x dx$

$\therefore I_n + I_{n-2} = \int_0^{\pi/4} \tan^n x dx + \int_0^{\pi/4} \tan^{n-2} x dx$

$= \int_0^{\pi/4} \tan^{n-2} x \times (\sec^2 x - 1) dx + \int_0^{\pi/4} \tan^{n-2} x dx$

$= \int_0^{\pi/4} \tan^{n-2} x \sec^2 x dx = \int_0^1 t^{n-2} dt$

$\therefore I_n + I_{n-2} = \dfrac{1}{n-1}$

$\therefore n(I_n + I_{n-2}) = \dfrac{1}{1-\frac{1}{n}}$

$\therefore \operatorname*{Lt}_{n\to\infty} n(I_n + I_{n-2}) = 1$

Example – 27

If for $n \geq 1$, $P_n = \int_1^e (\log x)^n dx$, then $P_{10} - 90P_8$ is equal to:

(a) - 9 (b) 10e

(c) -9e (d) 10

Ans. (c)

Sol. $P_n = \int_1^e \underbrace{(\log x)^n}_{I} \cdot \underbrace{1}_{II} \cdot dx$

$= x(\log x)^n \Big|_1^e - \int_1^e \underbrace{n(\log x)^{n-1}}_{I} \cdot \underbrace{1}_{II} \cdot dx$

$= e - n\left[x(\log x)^{n-1} \Big|_1^e - \int_1^e (n-1)(\log x)^{n-2} dx \right]$

$P_n = e - n[e - (n-1)P_{n-2}]$

Put $n = 10$,

$P_{10} = e - 10e + 90P_8$

$\Rightarrow P_{10} - 90P_8 = -9e$

Example – 28

Find the area bounded by the curve

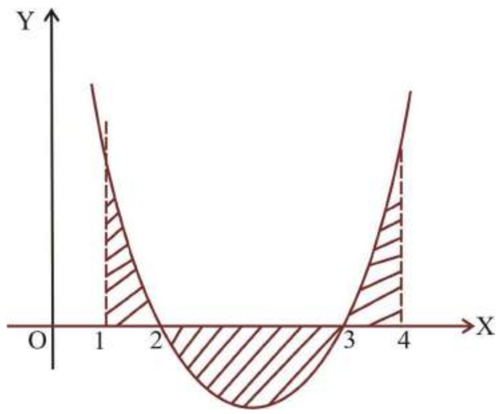
$y = x^2 - 5x + 6$, X–axis and the lines x = 1 and 4.

Sol. For y = 0, we get $x^2 - 5x + 6 = 0$

$\Rightarrow x = 2, 3$

Hence the curve crosses X–axis at x = 2, 3 in the interval [1, 4].

Bounded Area $= \left|\int_1^2 y\,dx\right| + \left|\int_2^3 y\,dx\right| + \left|\int_3^4 y\,dx\right|$



$\Rightarrow A = \left|\int_1^2 (x^2 - 5x + 6)\,dx\right| + \left|\int_2^3 (x^2 - 5x + 6)\,dx\right|$

$+ \left|\int_3^4 (x^2 - 5x + 6)\,dx\right|$

$A_1 = \left[\dfrac{2^3 - 1^3}{3}\right] - 5\left(\dfrac{2^2 - 1^2}{2}\right) + [6(2-1)] = \dfrac{5}{6}$

$A_2 = \dfrac{3^3 - 2^3}{3} - 5\left(\dfrac{3^2 - 2^2}{2}\right) + 6(3-2) = -\dfrac{1}{6}$

$A_3 = \dfrac{4^3 - 3^3}{3} - 5\left(\dfrac{4^2 - 3^2}{2}\right) + 6(4-3) = \dfrac{5}{6}$

$\Rightarrow A = \dfrac{5}{6} + \left|-\dfrac{1}{6}\right| + \dfrac{5}{6} = \dfrac{11}{6}$ sq. units.

Example – 29

The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is

(a) 1 : 2 : 3 (b) 1 : 2 : 1

(c) 1 : 1 : 1 (d) 2 : 1 : 2

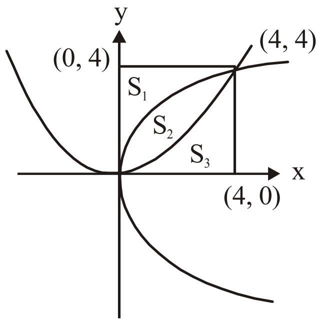
Ans. (c)

Sol. Total area $= 4 \times 4 = 16$ sq.units

Area of $S_3 = \int_0^4 \frac{x^2}{4} dx = \frac{16}{3} = S_1$

$\therefore S_2 = 16 - \frac{16}{3} \times 2 = \frac{16}{3}$.

$\therefore S_1 : S_2 : S_3$ is $1:1:1$



Example – 30

Find the area bounded by the circle $x^2 + y^2 = a^2$.

Sol. $x^2 + y^2 = a^2 \Rightarrow y = \pm \sqrt{a^2 - x^2}$

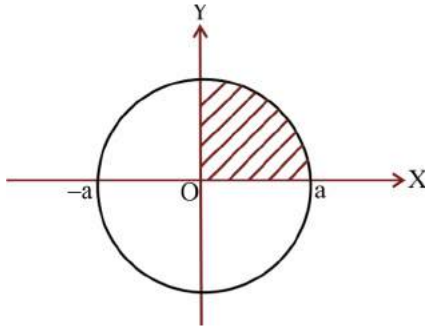
Equation of semicircle above X–axis is $y = + \sqrt{a^2 - x^2}$

Area of circle = 4 (shaded area)

$$= 4\int_0^a \sqrt{a^2 - x^2}\, dx$$

$$= 4\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

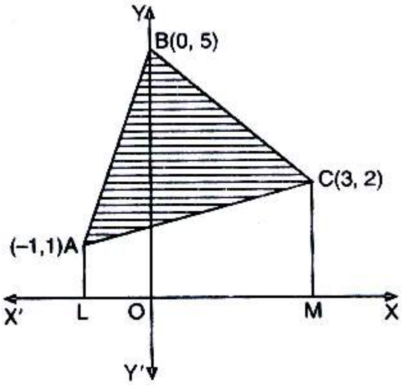
$$= 4\frac{a^2}{2}\left(\frac{\pi}{2}\right) = \pi a^2$$



Example – 31

Using integration, find the area of the region bounded by: (–1, 1), (0, 5) and (3, 2).

Sol. Let A (–1, 1), B (0, 5) and C (3, 2) be the vertices of the triangle as shown in the following figure :



Equation of AB is :

$$y - 1 = \frac{5-1}{0-(-1)}(x+1)$$

$$\left[\text{Using } y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

$\Rightarrow$ $y - 1 = 4(x + 1)$

$\Rightarrow$ $y = 4x + 5$ ... (1)

Equation of BC is :

$$y - 5 = \frac{2-5}{3-0}(x - 0)$$

$\Rightarrow$ $3y - 15 = -3x$

$\Rightarrow$ $3y = 15 - 3x$

$\Rightarrow$ $y = 5 - x$ ... (2)

Equation of AC is :

$$y-1=\frac{2-1}{3-(-1)}(x+1)$$

$\Rightarrow \quad y-1=\frac{1}{4}(x+1)$

$\Rightarrow \quad 4y-4=x+1$

$\Rightarrow \quad 4y=x+5$

$\Rightarrow \quad y=\frac{x}{4}+\frac{5}{4}$ ... (3)

Now ar (ΔABC) = ar (ALOB) + ar (OMCB) – ar (ALMC)

$$=\int_{-1}^{0}(4x+5)\,dx+\int_{0}^{3}(5-x)\,dx-\int_{-1}^{3}\left(\frac{1}{4}x+\frac{5}{4}\right)dx$$

$$=\left[\frac{4x^2}{2}+5x\right]_{-1}^{0}+\left[5x-\frac{x^2}{2}\right]_{0}^{3}-\left[\frac{x^2}{8}+\frac{5}{4}x\right]_{-1}^{3}$$

$$=\left[2x^2+5x\right]_{-1}^{0}+\left[5x-\frac{x^2}{2}\right]_{0}^{3}-\left[\frac{x^2}{8}+\frac{5}{4}x\right]_{-1}^{3}$$

$$=[(0+0)-(2-5)]+\left[\left(15-\frac{9}{2}\right)-(0-0)\right]$$

$$-\left[\left(\frac{9}{8}+\frac{15}{4}\right)-\left(\frac{1}{8}-\frac{5}{4}\right)\right]$$

$$=3+\frac{21}{2}-\left(\frac{9+30-1+10}{8}\right)=3+\frac{21}{2}-\frac{48}{8}$$

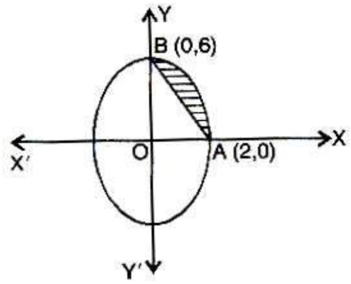
$$=3+\frac{21}{2}-6=\frac{21}{2}-3=\frac{15}{2}=7.5 \text{ sq. units}.$$

Example – 32

AOBA is the part of the ellipse $9x^2+y^2=36$ in the first quadrant such that OA = 2 and OB = 6. Find the area between the arc AB and the chord AB.

Sol. The given equation of the ellipse can be written as

$$\frac{x^2}{4}+\frac{y^2}{36}=1 \text{ i.e. } \frac{x^2}{2^2}+\frac{y^2}{6^2}=1$$



∴ A is (2, 0) and B is (0, 6).

∴ The equation of chord AB is :

$$y-0=\frac{6-0}{0-2}(x-2)$$

$\Rightarrow \quad y=-3x+6.$

∴ Reqd. area (shown shaded)

$$=\int_{0}^{2}3\sqrt{4-x^2}\,dx-\int_{0}^{2}(6-3x)\,dx$$

$$=3\left[\frac{x\sqrt{4-x^2}}{2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{0}^{2}-\left[6x-\frac{3x^2}{2}\right]_{0}^{2}$$

$$=3\left[\frac{2}{2}(0)+2\sin^{-1}(1)\right]-\left[6(2)-\frac{3(4)}{2}\right]$$

$$=3\left[2\times\frac{\pi}{2}\right]-[12-6]$$

$= (3\pi-6)$ sq. units.

Example – 33

Find the area bounded by the curves $y=x^2$ and $x^2+y^2=2$ above X–axis.

Sol. Let us first find the points of intersection of curves.

Solving $y=x^2$ and $x^2+y^2=2$ simultaneously, we get :

$x^2+x^4=2$

$\Rightarrow (x^2-1)(x^2+2)=0$

$\Rightarrow x^2=1$ and $x^2=-2$ [reject]

$\Rightarrow x=\pm 1$

$\Rightarrow A=(-1, 1)$ and $B=(1, 1)$

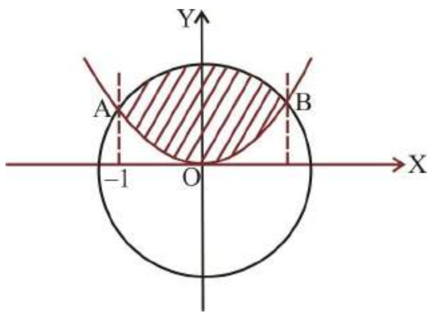
$$\text{Shaded Area}=\int_{-1}^{+1}\left(\sqrt{2-x^2}-x^2\right)dx$$

$$= \int_{-1}^{+1} \sqrt{2-x^2}\,dx - \int_{-1}^{+1} x^2\,dx$$

$$= 2\int_{0}^{1} \sqrt{2-x^2}\,dx - 2\int_{0}^{1} x^2\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{2-x^2} + \frac{2}{2}\sin^{-1}\frac{x}{\sqrt{2}}\right]_0^1 - 2\left(\frac{1}{3}\right)$$

$$= 2\left(\frac{1}{2}+\frac{\pi}{4}\right) - \frac{2}{3} = \frac{1}{3} + \frac{\pi}{2} \text{ sq. units.}$$



Example – 34

Find the area of the region included between :

The parabola $y = \frac{3}{4}x^2$ and the line $3x - 2y + 12 = 0$

Sol. The given parabola is $4y = 3x^2$.

i.e. $y = \frac{3}{4}x^2$ ... (1)

and the given line is $3x - 2y + 12 = 0$ ... (2)

Putting the value of y from (1) in (2), we get :

$$3x - 2\left(\frac{3}{4}x^2\right) + 12 = 0 \Rightarrow 3x - \frac{3}{2}x^2 + 12 = 0$$

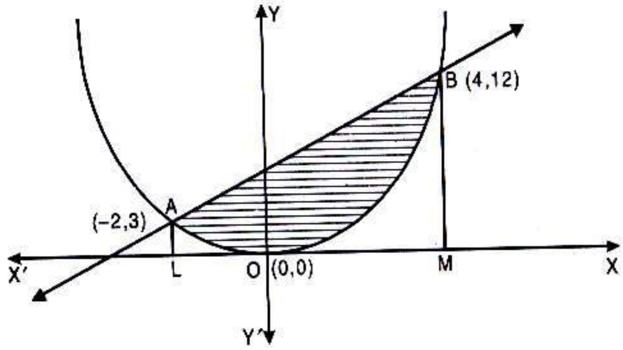
$\Rightarrow$ $6x - 3x^2 + 24 = 0$

$\Rightarrow$ $x^2 - 2x - 8 = 0$

$\Rightarrow$ $(x-4)(x+2) = 0$

$\Rightarrow$ $x = 4, -2.$

Putting in (1), $y = \frac{3}{4}(4)^2 = 12$ and $y = \frac{3}{4}(-2)^2 = 3.$



Hence the line (2) intersects parabola (1) in the points A (–2, 3) and B (4, 12).

$\therefore$ Reqd. area = area ALMB – (area ALO + area OMB)

$$= \int_{-2}^{4} \frac{3x+12}{2}\,dx - \left(\int_{-2}^{0} \frac{3}{4}x^2\,dx + \int_{0}^{4} \frac{3}{4}x^2 dx\right)$$

$$\left[\because \text{From (2), } y = \frac{3x+12}{2}\right]$$

$$= \frac{1}{2}\left[\frac{3x^2}{2} + 12x\right]_{-2}^{4} - \left[\frac{3}{4}\left\{\frac{x^3}{3}\right\}_{-2}^{0} + \frac{3}{4}\left\{\frac{x^3}{3}\right\}_{0}^{4}\right]$$

$$= \frac{1}{2}[(24+48)-(6-24)] - \left[\frac{3}{4}\left(0+\frac{8}{3}\right) + \frac{3}{4}\left(\frac{64}{3}-0\right)\right]$$

$$= \frac{1}{2}[72+18] - [2+16] = 45 - 18$$

= 27 sq. units.

Example – 35

Using integration, find the area of the region :

$\{(x, y) : |x-1| \le y \le \sqrt{5-x^2}\}$

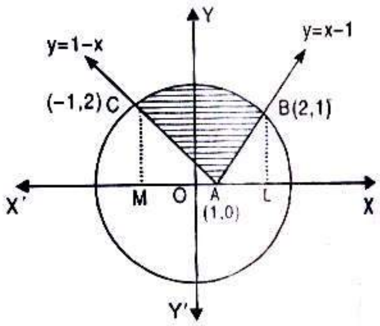
Sol. The given curves are :

$x^2 + y^2 = 5$

$[\because y = \sqrt{5-x^2} \Rightarrow y^2 = 5 - x^2 \Rightarrow x^2 + y^2 = 5]$

and $y = \begin{cases} 1-x, \text{ if } x < 1 \\ x-1, \text{ if } x \ge 1 \end{cases}$

The reqd. region is shown as shaded in the following figure :



$y = x - 1$ meets $x^2 + y^2 = 5$ at B (2, 1)

$y = 1 - x$ meets $x^2 + y^2 = 5$ at C (–1, 2)

$y = x - 1$ and $y = 1 - x$ meet at A (1, 0).

$\therefore$ Reqd. area = ar. (MCBLM) – ar (CMAC) – ar (ALBA)

$$= \int_{-1}^{2} \sqrt{5 - x^2}\, dx - \int_{-1}^{1} (1 - x)\, dx - \int_{1}^{2} (x - 1)\, dx$$

$$= \left[\frac{x\sqrt{5 - x^2}}{2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} - \left[x - \frac{x^2}{2}\right]_{-1}^{1} - \left[\frac{x^2}{2} - x\right]_{1}^{2}$$

$$= \left[\left(1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right) - \left(-\frac{1}{2}\times 2 + \frac{5}{2}\sin^{-1}\left(-\frac{1}{\sqrt{5}}\right)\right)\right]$$

$$-\left[\left(1 - \frac{1}{2}\right) - \left(-1 - \frac{1}{2}\right)\right] - \left[(2 - 2) - \left(\frac{1}{2} - 1\right)\right]$$

$$= 1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + 1 - \frac{5}{2}\sin^{-1}\left(-\frac{1}{\sqrt{5}}\right) - 2 - \frac{1}{2}$$

$$= -\frac{1}{2} + \frac{5}{2}\left[\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right]$$

$$= -\frac{1}{2} + \frac{5}{2}\times\frac{\pi}{2} = \frac{5\pi}{4} - \frac{1}{2} \text{ sq. units}$$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Definite Integration

Definite integration by methods of indefinite integration

1. Let $\frac{d}{dx}F(x)=\left(\frac{e^{\sin x}}{x}\right)$, $x>0$. If $\int_1^4 \frac{3}{x}e^{\sin x^3}\,dx = F(k)-F(1)$, then one of the possible values of k is

(a) 16 (b) 63

(c) 64 (d) 15

2. The solution for x of the equation $\int_{\sqrt{2}}^{x}\frac{dt}{t\sqrt{t^2-1}}=\frac{\pi}{12}$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{2}$

(c) 2 (d) π

3. If $\int_0^{\pi/2}\frac{d\theta}{9\sin^2\theta+4\cos^2\theta}=k\pi$, then the value of k is :

(a) $\frac{1}{16}$ (b) $\frac{1}{12}$

(c) $\frac{1}{8}$ (d) $\frac{1}{3}$

4. The value of $\int_0^{\infty}\frac{dx}{(1+x)^3}$ is:

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) 0 (d) ∞

5. $\int_0^{a/2}\frac{a\,dx}{(x-a)(x-2a)}$ equals to

(a) $\ln\frac{2}{3}$ (b) $\ln\frac{3}{2}$

(c) $\ln 6$ (d) none

6. $\int_0^{\pi/2}\cos^5\left(\frac{x}{2}\right).\sin x\,dx$ is equal to

(a) $\frac{2}{7}\left(1-\frac{1}{8\sqrt{2}}\right)$ (b) $\frac{-4}{7}\left(1-\frac{1}{8\sqrt{2}}\right)$

(c) $\frac{4}{7}\left(1-\frac{1}{8\sqrt{2}}\right)$ (d) None of these

7. For $0<x<\frac{\pi}{2}$, $\int_{1/\sqrt{2}}^{1/2}\cot x\,d(\cos x)$ equals to

(a) $\frac{\sqrt{3}-\sqrt{2}}{2}$ (b) $\frac{\sqrt{2}-\sqrt{3}}{2}$

(c) $\frac{1-\sqrt{3}}{2}$ (d) none

8. $\int_0^{\pi/4}\frac{x.\sin x}{\cos^3 x}dx$ equal to

(a) $\frac{\pi}{4}+\frac{1}{2}$ (b) $\frac{\pi}{4}-\frac{1}{2}$

(c) $\frac{\pi}{4}$ (d) none

9. The value of the integral $I=\int_0^1 x(1-x)^n\,dx$ is:

(a) $\frac{1}{n+2}$ (b) $\frac{1}{n+1}-\frac{1}{n+2}$

(c) $\frac{1}{n+1}+\frac{1}{n+2}$ (d) $\frac{1}{n+1}$

10. $\int_{-3\pi/2}^{-\pi/2}[(x+\pi)^3+\cos^2(x+3\pi)]\,dx$ is equal to

(a) $\frac{\pi^4}{32}$ (b) $\frac{\pi^4}{32}+\frac{\pi}{2}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{2}-1$

Properties of Definite Integrals

11. The integral $\int_0^{\frac{1}{2}} \frac{\ln(1+2x)}{1+4x^2} dx$, equals:

(a) $\frac{\pi}{4}\ln 2$ (b) $\frac{\pi}{8}\ln 2$

(c) $\frac{\pi}{16}\ln 2$ (d) $\frac{\pi}{32}\ln 2$

12. $\int_0^{\pi} x f(\sin x) dx$ is equal to :

(a) $\pi \int_0^{\pi} x f(\cos x) dx$ (b) $\pi \int_0^{\pi} f(\sin x) dx$

(c) $\frac{\pi}{2} \int_0^{\pi} f(\sin x) dx$ (d) $\pi \int_0^{\pi} f(\cos x) dx$

13. The value of the integral $\int_0^{\pi} x(\sin^4 x \cos^4 x) dx$ is

(a) $\frac{3\pi^2}{64}$ (b) $\frac{3\pi^2}{128}$

(c) $\frac{3\pi^2}{256}$ (d) none of these

14. The value of the integral $\int_{\log 1/3}^{\log 3} \log\left(x+\sqrt{1+x^2}\right) dx$ is

(a) log 3 (b) 2 log 3

(c) 0 (d) None

15. If [ ] denotes the greatest integer function, then the integral $\int_0^{\pi} [\cos x]\, dx$ is equal to:

(a) $\frac{\pi}{2}$ (b) 0

(c) -1 (d) $-\frac{\pi}{2}$

16. $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2 \sin^2 x} dx$ is equal to

(a) $\frac{\pi^2}{ab}$ (b) $\frac{\pi^2}{2ab}$

(c) $\frac{\pi^2}{4ab}$ (d) none of these

17. The value of $\int_{-1}^{1} \frac{1+\sin x}{1+x^2} dx$ is :

(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$

18. Let $f(x) = x - [x]$, for every real number x, where $[x]$ is the integral part of x. Then $\int_{-1}^{1} f(x) dx$ is:

(a) 1 (b) 2

(c) 0 (d) $\frac{1}{2}$

19. $\int_0^{100\pi} \sqrt{1-\cos 2x}\, dx$ is equal to

(a) $100\sqrt{2}$ (b) $50\sqrt{2}$

(c) $200\sqrt{2}$ (d) $400\sqrt{2}$

20. $\int_{\pi}^{10\pi} |\sin x| dx$ is equal to

(a) 18 (b) 20

(c) 40 (d) None

21. $\int_0^{10} e^{x-[x]} dx$ ([.] denotes GIF) is equal to

(a) $10(e-1)$ (b) $\frac{e-1}{10}$

(c) $\frac{e^{10}-1}{10}$ (d) $\frac{e^{10}-1}{e-1}$

22. $\int_0^{\sqrt{2}} [x^2]\, dx$ where [.] is GIF is :

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$

(c) $\sqrt{2}-1$ (d) $\sqrt{2}-2$

23. The integral $\int_0^{\pi} \sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\, dx$ equals :

(a) $4\sqrt{3}-4-\frac{\pi}{3}$ (b) $\pi-4$

(c) $\frac{2\pi}{3}-4-4\sqrt{3}$ (d) $4\sqrt{3}-4$

24. If $f(x)=|x|+|x-1|+|x-2|, x\in R$ then $\int_0^3 f(x)\, dx$ equals to

(a) $9/2$ (b) $15/2$

(c) $19/2$ (d) none

25. Let $y=\{x\}^{[x]}$ where $\{x\}$ denotes the fractional part of x & $[x]$ denotes greatest integer $\le x$ then $\int_0^3 y\, dx =$

(a) $5/6$ (b) $2/3$

(c) 1 (d) $11/6$

26. $\int_0^{\pi} |1+2\cos x|\, dx$ equals to

(a) $\frac{2\pi}{3}$ (b) π

(c) 2 (d) $\frac{\pi}{3}+2\sqrt{3}$

27. The value of $\int_1^a [x] f'(x)\, dx, a>1$, where [x] denotes the greatest integer not exceeding x, is

(a) $a f(a)-\{f(1)+f(2)+...+f([a])\}$

(b) $[a]f(a)-\{f(1)+f(2)+...+f([a])\}$

(c) $[a]f([a])-\{f(1)+f(2)+...+f(a)\}$

(d) $af([a])-\{f(1)+f(2)+...+f(a)\}$

28. $\int_{-1}^{3} \left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right) dx$ equals to

(a) 2π (b) π

(c) 4π (d) none

29. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is

(a) $\pi^2/4$ (b) π^2

(c) 5/4 (d) –3/4

30. Let function F be defined as $F(x)=\int_1^x \frac{e^t}{t} dt, x>0$ then the value of the integral $\int_1^x \frac{e^t}{t+a} dt$, where $a>0$, is:

(a) $e^a[F(x)-F(1+a)]$ (b) $e^{-a}[F(x+a)-F(a)]$

(c) $e^a[F(x+a)-F(1+a)]$ (d) $e^{-a}[F(x+a)-F(1+a)]$

Differentiation of Definite Integrals: Newton leibniz Theorem

31. If $f(x)=\int_0^x t\sin t\, dt$, then $f'(x)$ is

(a) $\cos x + x\sin x$ (b) $x\cos x+\sin x$

(c) $x\cos x$ (d) $x\sin x$

32. If $f(x)=\int_1^{x^3} \frac{dt}{1+t^4}$, then $f''(x)$ is equal to

(a) $\frac{6x(1-5x^{12})}{(1+x^{12})^2}$ (b) $\frac{6x(1+5x^{12})}{(1+x^{12})^2}$

(c) $-\frac{6x(1-5x^{12})}{(1+x^{12})^2}$ (d) none of these

33. If the variables x and y are connected by the relation $x=\int_1^y \frac{dz}{\sqrt{1+6z^3}}$, then $\frac{d^2y}{dx^2}$ is proportional to

(a) y (b) y^2

(c) y^3 (d) none of these

34. $\lim_{x\to 0^+} \frac{1}{x^3}\int_0^{x^2} \sin\sqrt{t}\, dt$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $-\frac{1}{3}$ (d) $-\frac{2}{3}$

35. The value of $\lim_{x\to 0} \frac{\int_0^{x^2} \sec^2 t\, dt}{x \sin x}$ is

(a) 2 (b) 1

(c) 0 (d) 3

36. Let $f: R \to R$ be a continuous function. Then

$\lim_{x\to\pi/4} \frac{\frac{\pi}{4}\int_2^{\sec^2 x} f(x)dx}{x^2 - \frac{\pi^2}{16}}$ is equal to

(a) $4f(2)$ (b) $f(2)$

(c) $2f(\sqrt{2})$ (d) $2f(2)$

Summation of series using integration

37. $\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + ... + \frac{1}{n}\sec^2 1\right]$ equals

(a) $\frac{1}{2}\operatorname{cosec} 1$ (b) $\frac{1}{2}\sec 1$

(c) $\frac{1}{2}\tan 1$ (d) tan 1

38. $\lim_{n\to\infty}\left[\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + ... + \frac{1}{8n}\right]$ is equal to

(a) $\frac{3}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) none of these

39. The value of the

$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2-1}} + \frac{1}{\sqrt{n^2-2^2}} + ... + \frac{1}{\sqrt{n^2-(n-1)^2}}\right]$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) none of these

40. If $S_n = \left[\frac{1}{2n} + \frac{1}{\sqrt{4n^2-1}} + \frac{1}{\sqrt{4n^2-4}} + ... + \frac{1}{\sqrt{3n^2+2n-1}}\right]$

then $\lim_{n\to\infty} S_n$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

41. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{\frac{r}{n}}$ is :

(a) e (b) $e-1$

(c) $1-e$ (d) $e+1$

Bounds of definite integrals

42. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}dx$. Then which one of the following is true ?

(a) $I > \frac{2}{3}$ and $J < 2$ (b) $I > \frac{2}{3}$ and $J > 2$

(c) $I < \frac{2}{3}$ and $J < 2$ (d) $I < \frac{2}{3}$ and $J > 2$

43. $I_1 = \int_1^2 e^x\, dx$ and $I_2 = \int_1^2 \log_e x\, dx$. Then

(a) $I_1 > I_2$ (b) $I_1 < I_2$

(c) $I_1 = I_2$ (d) None of these

44. $I=\int_1^2 \frac{e^x}{x}dx.$ Then

(a) $I < e$ (b) $I > \frac{e^2}{2}$

(c) $e < I < \frac{e^2}{2}$ (d) None of these

45. $I_1=\int_0^{\pi/4} \tan^3 x\,dx$ and $I_2=\int_0^{\pi/4} \tan^5 x\,dx$

$I_3=\int_0^{\pi/4} \tan^{1/2} x\,dx \quad I_4=\int_0^{\pi/4} \tan^{1/3} x\,dx$ then

(a) $I_1 < I_2$ (b) $I_1 > I_3$

(c) $I_3 > I_4$ (d) $I_1 > I_2$

46. $\int_0^2 \left(1+2^{-x^2}\right)dx=$

(a) 4 (b) $\frac{17}{2}$

(c) 2 (d) None of these

Reduction formula

47. If $I_n = \int_0^{\pi/4} \tan^n \theta\, d\theta$, where n is a positive integer, then $n\left(I_{n-1}+I_{n+1}\right)$ is equal to

(a) 1 (b) $n-1$

(c) $\frac{1}{n-1}$ (d) none of these

48. If $a_n = \int_{\pi/4}^{\pi/2} \cot^n x dx$, then $a_2+a_4, a_3+a_5, a_4+a_6$ are in

(a) G.P. (b) A.P.
(c) H.P. (d) None

49. Let $I_n = \int_0^{\pi/4} \tan^n x\, dx$,

then $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}, ...$ are in

(a) A.P. (b) G.P.
(c) H.P. (d) none

50. The value of the integral $\int_0^{\pi} \frac{\sin(2k\,x)}{\sin\,x} dx$, when $k \in I$, is

(a) $\frac{\pi}{2}$ (b) π

(c) 0 (d) none of these

Numerical Value Type Questions

51. $\int_0^{\pi}\left[\cos^2\left(\frac{3\pi}{8}-\frac{x}{4}\right)-\cos^2\left(\frac{11\pi}{8}+\frac{x}{4}\right)\right]dx$ equals $\sqrt{k}$.

Then the value of k is.

52. $\int_0^{\pi/4} \frac{\sqrt{\tan x}}{\sin x \cos x}dx$ equals to

53. $\int_0^{(\pi/2)^{1/3}} 3x^5.\sin x^3 dx$ equals to

54. If $\int_0^1 \cot^{-1}\left(1-x+x^2\right)dx = K.\int_0^1 \tan^{-1} x\, dx$, then K equals to

55. If the value of $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$ is $\frac{\pi}{k}$. Then the value of k is

56. The value of the integral, $2\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is :

57. If $f(x)$ and $\phi(x)$ are continuous functions on the interval $[0,4]$ satisfying

$f(x)=f(4-x), \phi(x)+\phi(4-x)=3$

and $\int_0^4 f(x)dx=2$, then $\int_0^4 f(x)\phi(x)\,dx=$

58. If $f(x)=\frac{x^7-3x^5+7x^3-x+1}{\cos^2 x}$ then, $\int_{-\pi/4}^{\pi/4} f(x)\,dx$ equals to

59. $\int_0^{100}\left(x-[x]\right)dx$ is equal to

60. $\int_0^{1.5} x[x^2]\,dx = 3/k$, where [ ] denotes greatest integer function. Then the value of k is.

61. Suppose for every integer n, $\int_n^{n+1} f(x)\,dx = n^2$. The value of $\int_{-2}^{4} f(x)\,dx$ is

62. If $\int_{-1}^{-4} f(x)\,dx = 4$ and $\int_{2}^{-4} (3 - f(x))\,dx = 7$ then the value of $\int_{-2}^{1} f(-x)\,dx$ is

63. $\lim_{x\to 0} \dfrac{\int_0^{x^2} \left(\tan^{-1} t\right)^2 dt}{\int_0^{x^4} \sin\sqrt{t}\,dt}$ is equal to $1/k$. Then the value of k is.

Area Under Curves

Plotting region and Area under curves

64. Area bounded by the curve $y = x^3$, the $x-$ axis and the ordinates $x = -2$ and $x = 1$ is :

(a) –9 (b) $-\frac{15}{4}$

(c) $\frac{15}{4}$ (d) $\frac{17}{4}$

65. The area of the region bounded by the curve $y = x - x^2, x-$ axis between $x = 0$ and $x = 1$ is :

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{5}{6}$

66. The area of the region bounded by the curve $y = \sqrt{1 - \cos x}$ between $x = 0$ and $x = \pi$ is :

(a) $\frac{\sqrt{2}}{2}$ (b) $2\sqrt{2}$

(c) $\sqrt{2}$ (d) 2

67. The area of the region bounded by the curve $y = x \sin x$ between $x = 0$ and $x = 2\pi$ is :

(a) π (b) 2π

(c) 3π (d) 4π

68. Area of the region bounded by the curve $y^2 = 4x, y-$ axis and the line $y = 3$ is :

(a) 2 (b) $\frac{9}{4}$

(c) $\frac{9}{3}$ (d) $\frac{9}{2}$

69. The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is

(a) 2 (b) 1

(c) 4 (d) 3

70. The area bounded by the curves y = *l*n x, y = *l*n |x|, y = |*l*n x| and y = |*l*n| x || is

(a) 4 sq. units (b) 6 sq. units

(c) 10 sq. units (d) none of these

71. The area of the region bounded by the curves y = |x–1| and y = 3 – |x| is

(a) 3 sq. units (b) 4 sq. units

(c) 6 sq. units (d) 2 sq. units

72. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d)

73. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line y = 2 is.

(a) $20\sqrt{2}$ (b) $\frac{10\sqrt{2}}{3}$

(c) $\frac{20\sqrt{2}}{3}$ (d) $10\sqrt{2}$

74. The area (in square units) bounded by the curves $y=\sqrt{x}$, $2y-x+3=0$, x-axis and lying in the first quadrant is

(a) 9 (b) 6

(c) 18 (d) $\frac{27}{4}$

75. The area of the region described by $A = \{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$ is

(a) $\frac{\pi}{2}+\frac{2}{3}$ (b) $\frac{\pi}{2}+\frac{4}{3}$

(c) $\frac{\pi}{2}-\frac{4}{3}$ (d) $\frac{\pi}{2}-\frac{2}{3}$

76. Let $A = \{(x, y) : y^2 \leq 4x, y - 2x \geq -4\}$. The area (in square units) of the region A is

(a) 8 (b) 9

(c) 10 (d) 11

77. The area of the circle $x^2+y^2=16$ exterior to the parabola $y^2=6x$ is :

(a) $\frac{4}{3}\left(4\pi-\sqrt{3}\right)$ (b) $\frac{4}{3}\left(4\pi+\sqrt{3}\right)$

(c) $\frac{4}{3}\left(8\pi-\sqrt{3}\right)$ (d) $\frac{4}{3}\left(8\pi+\sqrt{3}\right)$

78. The area between the curves $y=\cos x$ and the line $y=x+1$ in the second quadrant is –

(a) 1 (b) 2

(c) 3 / 2 (d) 1 / 2

79. The area bounded by the curves $y=\sin x, y=\cos x$ and $y-$ axis in first quadrant is –

(a) $\sqrt{2}-1$ (b) $\sqrt{2}$

(c) $\sqrt{2}+1$ (d) None of these

80. The area bounded by $y = x^2 - 4$ and $x + y = 2$ is

(a) $\frac{75}{6}$ (b) $\frac{100}{6}$

(c) $\frac{125}{6}$ (d) $\frac{150}{6}$

81. Smaller area enclosed by the circle $x^2+y^2=4$ and the line $x+y=2$ is :

(a) $2(\pi-2)$ (b) $\pi-2$

(c) $2\pi-1$ (d) $2(\pi+2)$

Numerical Value Type Questions

82. The area between the curves $y=\tan x$, $y=\cot x$ and $x-$ axis in the interval $[0,\pi/2]$ is $\log k$. Then the value of k is.

83. If $0 \leq x \leq \pi$; then the area bounded by the curve $y=x$ and $y=x+\sin x$ is –

84. The area bounded by the curves : $y^2 = 4a(x + a)$ and $y^2 = 4b\,(b - x)$ is $\frac{k}{3}(a+b)\sqrt{4ab}$. Then the value of k is.

85. If the area of the region $\{(x, y) : x^2 \leq y \leq |x|\}$ is 1/k sq. units, then the value of k is

86. If the line y = mx bisects the area enclosed by the lines $x = 0, y = 0, x = \frac{3}{2}$ and the curve $y = 1 + 4x - x^2$, and the value of m is equal to 13/k. then the value of k is

87. If the area enclosed between the parabolas $y^2 = 4ax$ and $x^2 = 4by$ is $\frac{16}{3}$, then value of ab is

88. AOB is the positive quadrant of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; where OA = a, OB = b. Area between arc AB and chord AB of ellipse is $\frac{(\pi-p)}{q}$ ab , then p + q equals

89. If the area bounded by the parabola $x^2 = 4y$, the x-axis and the line x = 4 is divided into two equal areas by the line $x = \alpha$, and the value of α is $(32)^{1/k}$, then the value of k is

90. The value of m for which the area included between the curves $y^2 = 4ax$ and $y = mx$ equals $\frac{a^2}{3}$ is

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

Definite Integration

1. The integral

$\int_2^4 \frac{\log x^2}{\log x^2 + \log(36-12x+x^2)} dx$ is equal to: **(2015)**

(a) 1 (b) 6

(c) 2 (d) 4

2. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$ is

(2015/Online Set–2)

(a) π (b) $a\pi$

(c) $\pi/2$ (d) 2π

3. If $2\int_0^1 \tan^{-1} x\, dx = \int_0^1 \cot^{-1}\left(1-x+x^2\right) dx$,

then $\int_0^1 \tan^{-1}\left(1-x+x^2\right) dx$ is equal to :

(2016/Online Set–1)

(a) log 4 (b) $\frac{\pi}{2} + \log 2$

(c) log 2 (d) $\frac{\pi}{2} - \log 4$

4. The value of the integral

$\int_4^{10} \frac{[x^2]\, dx}{[x^2 - 28x + 196] + [x^2]}$, where [x] denotes the greatest integer less than or equal to x, is :

(2016/Online Set–2)

(a) 6 (b) 3

(c) 7 (d) $\frac{1}{3}$

5. For $x \in R, x \neq 0$, if y(x) is a differentiable function such that $x\int_1^x y(t)\, dt = (x+1)\int_1^x t\, y(t)\, dt$, then y(x) equals :

(where C is a constant.) **(2016/Online Set–2)**

(a) $\frac{C}{x} e^{-\frac{1}{x}}$ (b) $\frac{C}{x^2} e^{-\frac{1}{x}}$

(c) $\frac{C}{x^3} e^{-\frac{1}{x}}$ (d) $Cx^3 e^{\frac{1}{x}}$

6. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to: **(2017)**

(a) – 2 (b) 2

(c) 4 (d) –1

7. The integral $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{8\cos 2x}{\left(\tan x + \cot x\right)^3} dx$ equals :

(2017/Online Set–1)

(a) $\frac{15}{128}$ (b) $\frac{15}{64}$

(c) $\frac{13}{32}$ (d) $\frac{13}{256}$

8. If $\int_1^2 \frac{dx}{(x^2-2x+4)^{\frac{3}{2}}} = \frac{k}{k+5}$, then k is equal to:

(2017/Online Set–2)

(a) 1 (b) 2

(c) 3 (d) 4

9. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is: **(2018)**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$

(c) $\frac{\pi}{2}$ (d) 4π

10. The value of the integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\left(1+\log\left(\frac{2+\sin x}{2-\sin x}\right)\right)dx \text{ is}$$

(2018/Online Set–1)

(a) 0 (b) $\frac{3}{4}$

(c) $\frac{3}{8}\pi$ (d) $\frac{3}{16}\pi$

11. The value of integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}dx$ is :

(2018/Online Set–2)

(a) $\pi\sqrt{2}$ (b) $\pi\left(\sqrt{2}-1\right)$

(c) $\frac{\pi}{2}\left(\sqrt{2}+1\right)$ (d) $2\pi\left(\sqrt{2}-1\right)$

12. If $I_1=\int_0^1 e^{-x}\cos^2 x\,dx$, $I_2=\int_0^1 e^{-x^2}\cos^2 x\,dx$ and

$I_3=\int_0^1 e^{-x^3}dx$; then: **(2018/Online Set–2)**

(a) $I_2>I_3>I_1$ (b) $I_2>I_1>I_3$

(c) $I_3>I_2>I_1$ (d) $I_3>I_1>I_2$

13. If $f(x)=\int_0^x t(\sin x-\sin t)dt$ then:

(2018/Online Set–3)

(a) $f'''(x)+f''(x)=\sin x$

(b) $f'''(x)+f''(x)-f'(x)=\cos x$

(c) $f'''(x)+f'(x)=\cos x-2x\sin x$

(d) $f'''(x)-f''(x)=\cos x-2x\sin x$

14. If $f(x)=\frac{2-x\cos x}{2+x\cos x}$ and $g(x)=\log_e x,(x>0)$ then the value of the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} g(f(x))dx$ is:

(8-04-2019/Shift-1)

(a) $\log_e 3$ (b) $\log_e e$

(c) $\log_e 2$ (d) $\log_e 1$

15. Let $f(x)=\int_0^x g(t)dt$, where g is a non-zero even function. If $f(x+5)=g(x)$, then $\int_0^x f(t)dt$ equals :

(8-04-2019/Shift-2)

(a) $\int_{x+5}^{5} g(t)dt$ (b) $\int_{5}^{x+5} g(t)dt$

(c) $2\int_{5}^{x+5} g(t)dt$ (d) $5\int_{5}^{x+5} g(t)dt$

16. The value of the integral $\int_0^1 x\cot^{-1}\left(1-x^2+x^4\right)dx$ is :

(9-04-2019/Shift-2)

(a) $\frac{\pi}{2}-\frac{1}{2}\log_e 2$ (b) $\frac{\pi}{4}-\log_e 2$

(c) $\frac{\pi}{2}-\log_e 2$ (d) $\frac{\pi}{4}-\frac{1}{2}\log_e 2$

17. If $f:R\to R$ is a differentiable function and $f(2)=6$, then $\lim_{x\to 2}\int_6^{f(x)}\frac{2t\,dt}{(x-2)}$ is : **(9-04-2019/Shift-2)**

(a) $24f'(2)$ (b) $2f'(2)$

(c) 0 (d) $12f'(2)$

18. The value of $\int_0^{2\pi}\left[\sin 2x\left(1+\cos 3x\right)\right]dx$, where [ ] denotes the greatest integer function, is: **(10-04-2019/Shift-1)**

(a) π (b) $-\pi$

(c) -2π (d) 2π

19. $\lim_{n\to\infty}\left(\frac{(n+1)^{1/3}}{n^{4/3}}+\frac{(n+2)^{1/3}}{n^{4/3}}+....+\frac{(2n)^{1/3}}{n^{4/3}}\right)$ is equal to:

(10-04-2019/Shift-1)

(a) $\frac{3}{4}(2)^{4/3}-\frac{3}{4}$ (b) $\frac{4}{3}(2)^{4/3}$

(c) $\frac{3}{2}(2)^{4/3}-\frac{4}{3}$ (d) $\frac{4}{3}(2)^{3/4}$

20. The integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\sec^{\frac{2}{3}}x\,\mathrm{cosec}^{\frac{4}{3}}x\,dx$ is equal to:

(10-4-2019/Shift-2)

(a) $3^{\frac{5}{6}}-3^{\frac{2}{3}}$ (b) $3^{\frac{4}{3}}-3^{\frac{1}{3}}$

(c) $3^{\frac{7}{6}}-3^{\frac{5}{6}}$ (d) $3^{\frac{5}{3}}-3^{\frac{1}{3}}$

21. If $\int_0^{\frac{\pi}{2}}\frac{\cot x}{\cot x+\mathrm{cosec}x}dx=m(\pi+n)$, then m. n is equal to

(12-04-2019/Shift-1)

(a) $-\frac{1}{2}$ (b) 1

(c) $\frac{1}{2}$ (d) -1

22. Let $f:R\to R$ be a continuously differentiable function such that $f(2)=6$ and $f'(2)=\frac{1}{48}$.

If $\int_6^{f(x)}4t^3dt=(x-2)g(x)$, then $\lim_{x\to2}g(x)$ is equal to

(12-04-2019/Shift-1)

23. A value of α such that

$\int_{\alpha}^{\alpha+1}\frac{dx}{(x+\alpha)(x+\alpha+1)}=\log_e\left(\frac{9}{8}\right)$ is ______.

(12-04-2019/Shift-2)

(a) -2 (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) 2

24. The value of $\int_0^{\pi}|\cos x|^3\,dx$ is: **(9-01-2019/Shift-1)**

(a) 0 (b) $\frac{4}{3}$

(c) $\frac{2}{3}$ (d) $\frac{-4}{3}$

25. If $\int_0^{\pi/3}\frac{\tan\theta}{\sqrt{2k\sec\theta}}d\theta=1-\frac{1}{\sqrt{2}},(k>0)$ then the value of k is: **(9-01-2019/Shift-2)**

(a) 4 (b) $\frac{1}{2}$

(c) 1 (d) 2

26. Let $I=\int_a^b(x^4-2x^2)dx$. If I is minimum then the ordered pair (a, b) is: **(10-1-2019/Shift-1)**

(a) $(0,\sqrt{2})$ (b) $(-\sqrt{2},0)$

(c) $(\sqrt{2},-\sqrt{2})$ (d) $(-\sqrt{2},\sqrt{2})$

27. If $\int_0^x f(t)dt=x^2+\int_x^1 t^2f(t)dt$, then $f'(1/2)$ is:

(10-01-2019/Shift-2)

(a) $\frac{24}{25}$ (b) $\frac{18}{25}$

(c) $\frac{4}{5}$ (d) $\frac{6}{25}$

28. The value of $\int_{-\pi/2}^{\pi/2}\frac{dx}{[x]+[\sin x]+4}$, where [t] denotes the greatest integer less than or equal to t, is:
(10-01-2019/Shift-2)

(a) $\frac{1}{12}(7\pi+5)$ (b) $\frac{1}{12}(7\pi-5)$

(c) $\frac{3}{20}(4\pi-3)$ (d) $\frac{3}{10}(4\pi-3)$

29. The value of the integral $\int_{-2}^{2}\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$ (where [x] denotes the greatest integer less than or equal to x) is : **(11-01-2019/Shift-1)**

(a) 0 (b) sin 4

(c) 4 (d) 4 – sin 4

30. The integral $\int_{\pi/6}^{\pi/4}\frac{dx}{\sin 2x\left(\tan^5 x+\cot^5 x\right)}$ equals : **(11-01-2019/Shift-2)**

(a) $\frac{1}{20}\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)$ (b) $\frac{1}{10}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{9\sqrt{3}}\right)\right)$

(c) $\frac{\pi}{40}$ (d) $\frac{1}{5}\left(\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{3\sqrt{3}}\right)\right)$

31. Let f and g be continuous functions on [0, a] such that $f(x)=f(a-x)$ and $g(x)+g(a-x)=4$ then $\int_0^a f(x)g(x)dx$ is equal to **(12-01-2019/Shift-1)**

(a) $4\int_0^a f(x)dx$ (b) $\int_0^a f(x)dx$

(c) $2\int_0^a f(x)dx$ (d) $-3\int_0^a f(x)dx$

32. The integral $\int_1^e\left\{\left(\frac{x}{e}\right)^{2x}-\left(\frac{e}{x}\right)^{x}\right\}\log_e x\, dx$ is equal to **(12-01-2019/Shift-2)**

(a) $\frac{1}{2}-e-\frac{1}{e^2}$ (b) $-\frac{1}{2}+\frac{1}{e}-\frac{1}{2e^2}$

(c) $\frac{3}{2}-\frac{1}{e}-\frac{1}{2e^2}$ (d) $\frac{3}{2}-e-\frac{1}{2e^2}$

33. $\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots+\frac{1}{5n}\right)$ is equal to **(12-01-2019/Shift-2)**

(a) $\frac{\pi}{4}$ (b) $\tan^{-1}(3)$

(c) $\frac{\pi}{2}$ (d) $\tan^{-1}(2)$

34. The integral $\int_0^2 ||x-1|-x|\,dx$ is equal to : **(2-9-2020/Shift-1)**

35. Let [t] denote the greatest integer less than or equal to t. Then the value of $\int_1^2 |2x-[3x]|\,dx$ is .... **(2-09-2020/Shift-2)**

36. $\int_{-\pi}^{\pi}|\pi-|x||\,dx$ is equal to : **(3-09-2020/Shift-1)**

(a) π^2 (b) $\frac{\pi^2}{2}$

(c) $\sqrt{2}\pi^2$ (d) $2\pi^2$

37. If the value of the integral $\int_0^{1/2}\frac{x^2}{(1-x^2)^{3/2}}dx$ is $\frac{k}{6}$, then k is equal to : **(03-09-2020/Shift-2)**

(a) $2\sqrt{3}+\pi$ (b) $3\sqrt{2}+\pi$

(c) $3\sqrt{2}-\pi$ (d) $2\sqrt{3}-\pi$

38. Let $f(x)=\int\frac{\sqrt{x}}{(1+x)^2}dx\,(x\ge 0)$ Then f(3) – f(1) is equal to : **(04-09-2020/Shift-1)**

(a) $-\frac{\pi}{6}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (b) $\frac{\pi}{6}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

(c) $-\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$ (d) $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

39. Let f (x) = |x – 2| and g (x) = f (f (x)), $x\in[0,4]$. Then $\int_0^3(g(x)-f(x))\,dx$ is equal to : **(4-09-2020/Shift-1)**

(a) $\frac{1}{2}$ (b) 0

(c) 1 (d) $\frac{3}{2}$

40. Let f be a twice differentiable function on (1, 6). If $f(2)=8, f'(2)=5, f'(x)\ge 1$ and $f''(x)\ge 4$ for all $x\in(1,6)$ $x\in(1,6)$ then : **(04-09-2020/Shift-1)**

(a) $f(5)+f'(5)\ge 28$ (b) $f'(5)+f''(5)\le 20$

(c) $f(5)\le 10$ (d) $f(5)+f'(5)\le 26$

41. The integral

$$\int_{\pi/6}^{\pi/3} \tan^3 x.\sin^2 3x\,(2\sec^2 x.\sin^2 3x+3\tan x.\sin 6x)\,dx$$

is equal to: **(4-9-2020/Shift-2)**

(a) $\frac{9}{2}$ (b) $-\frac{1}{18}$

(c) $-\frac{1}{9}$ (d) $\frac{7}{18}$

42. Let {x} and [x] denote the fractional part of x and the greatest integer $\le$ x respectively of real number x. If $\int_0^n \{x\}\,dx, \int_0^n [x]\,dx$ and $10(n^2-n)$, $(n\in N, n>1)$ are three consecutive terms of a G.P., then n is equal to .... **(4-9-2020/Shift-2)**

43. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{1+e^{\sin x}}dx$ is: **(5-09-2020/Shift-1)**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) π (d) $\frac{3\pi}{2}$

44. If $I_1=\int_0^1\left(1-x^{50}\right)^{100}dx$ and $I_2=\int_0^1\left(1-x^{50}\right)^{101}dx$ such that $I_2=\alpha I_1$ then α equal to : **(6-09-2020/Shift-1)**

(a) $\frac{5050}{5049}$ (b) $\frac{5050}{5051}$

(c) $\frac{5051}{5050}$ (d) $\frac{5049}{5050}$

45. The integral $\int_1^2 e^x.x^x\left(2+\log_e x\right)dx$ equal : **(6-09-2020/Shift-2)**

(a) $e(4e-1)$ (b) $e(4e+1)$

(c) $4e^2-1$ (d) $e(2e-1)$

46. If $f(a+b+1-x)=f(x)$ $\forall x$ where a and b are fixed positive real numbers, then $\frac{1}{(a+b)}\int_a^b x(f(x)+f(x+1))dx$ is equal to **(7-01-2020/Shift-1)**

(a) $\int_{a-1}^{b-a} f(x)dx$ (b) $\int_{a+1}^{b+1} f(x+1)dx$

(c) $\int_{a-1}^{b-1} f(x+1)dx$ (d) $\int_{a+1}^{b+1} f(x)dx$

47. The value of α for which $4\alpha\int_{-1}^{2} e^{-\alpha|x|}\,dx=5$, is: **(07-01-2020/Shift-2)**

(a) $\log_e 2$ (b) $\log_e \sqrt{2}$

(c) $\log_e\left(\frac{4}{3}\right)$ (d) $\log_e\left(\frac{3}{2}\right)$

48. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $[0,2\pi)-\{\pi\}$ which satisfy the equation, $2\cot^2\theta-\frac{5}{\sin\theta}+4=0$ then $\int_{\theta_1}^{\theta_2}\cos^2 3\theta\, d\theta$ is equal to **(07-01-2020/Shift-2)**

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{3}+\frac{1}{6}$ (d) $\frac{\pi}{9}$

49. If $I=\int_1^2\frac{dx}{\sqrt{2x^3-9x^2+12x+4}}$, then: **(8-01-2020/Shift-2)**

(a) $\frac{1}{6}<I^2<\frac{1}{2}$ (b) $\frac{1}{8}<I^2<\frac{1}{4}$

(c) $\frac{1}{9}<I^2<\frac{1}{8}$ (d) $\frac{1}{16}<I^2<\frac{1}{9}$

50. The value of $\int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x+\cos^8 x}dx$ is equal to:

(9-01-2020/Shift-1)

(a) 2π (b) 4π

(c) $2\pi^2$ (d) π^2

51. If for all real triplets (a,b,c), $f(x)=a+bx+cx^2$ then $\int_0^1 f(x)dx$ is equal to: **(9-01-2020/Shift-1)**

(a) $2\left(3f(1)+2f\left(\frac{1}{2}\right)\right)$

(b) $\frac{1}{3}\left(f(0)+f\left(\frac{1}{2}\right)\right)$

(c) $\frac{1}{2}\left(f(1)+3f\left(\frac{1}{2}\right)\right)$

(d) $\frac{1}{6}\left(f(0)+f(1)+4f\left(\frac{1}{2}\right)\right)$

52. The value of the integral $\int_{-1}^{1}\log_e\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$ is equal to: **(20-07-21/Shift-1)**

(a) $2\log_e 2+\frac{\pi}{4}-1$ (b) $\frac{1}{2}\log_e 2+\frac{\pi}{4}-\frac{3}{2}$

(c) $2\log_e 2+\frac{\pi}{2}-\frac{1}{2}$ (d) $\log_e 2+\frac{\pi}{2}-1$

53. Let a be a positive real number such that $\int_0^a e^{x-[x]}dx=10e-9$, where $[x]$ is the greatest integer less than or equal to x. Then a is equal to: **(20-07-21/Shift-1)**

(a) $10+\log_e 3$ (b) $10-\log_e(1+e)$

(c) $10+\log_e 2$ (d) $10+\log_e(1+e)$

54. If $[x]$ denotes the greater integer less than or equal to x, then the value of the integral $I=\int_{-\pi/2}^{\pi/2}\left[[x]-\sin x\right]dx$ is equal to ? **(20-07-21/Shift-2)**

(a) 0 (b) π

(c) 1 (d) $-\pi$

55. Let $g(t)=\int_{-\pi/2}^{\pi/2}\cos\left(\frac{\pi}{4}t+f(x)\right)dx$, where $f(x)=\log_e\left(x+\sqrt{x^2+1}\right), x\in R$. Then which one of the following is correct ? **(20-07-21/Shift-2)**

(a) $g(1)+g(0)=0$ (b) $g(1)=\sqrt{2}g(0)$

(c) $g(1)=g(0)$ (d) $\sqrt{2}g(1)=g(0)$

56. If $f:R\to R$ is given by $f(x)=x+1$, then the value of $\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+......+f\left(\frac{5(n-1)}{n}\right)\right]$, is:

(20-07-21/Shift-2)

(a) $\frac{3}{2}$ (b) $\frac{7}{2}$

(c) $\frac{5}{2}$ (d) $\frac{1}{2}$

57. If the real part of the complex number $(1-\cos\theta+2i\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta\in(0,\pi)$, then the value of the integral $\int_0^{\theta}\sin x\ dx$ is equal to ?

(20-07-21/Shift-2)

(a) 2 (b) –1

(c) 0 (d) 1

58. Let $f:[0,\infty)\to[0,\infty)$ be defined as $f(x)=\int_0^x [y]dy$

Where [x] is the greatest integer less than or equal to x. Which of the following is true ? **(25-07-21/Shift-1)**

(a) f is differentiable at every point in $[0,\infty)$

(b) f is continuous everywhere except at the integer points in $[0,\infty)$

(c) f is continuous at every point in $[0,\infty)$ and differentiable except at the integer points.

(d) f is both continuous and differentiable except at the integer points in $[0,\infty)$.

59. The value of the definite integral $\int_{\pi/24}^{5\pi/24} \frac{dx}{1+\sqrt[3]{\tan 2x}}$ is

(25-07-21/Shift-1)

(a) $\frac{\pi}{18}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

60. The value of the definite integral

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{(1+e^{x\cos x})(\sin^4 x+\cos^4 x)}$$ is equal to :

(27-07-21/Shift-1)

(a) $\frac{\pi}{\sqrt{2}}$ (b) $-\frac{\pi}{4}$

(c) $\frac{\pi}{2\sqrt{2}}$ (c) $-\frac{\pi}{2}$

61. The value of $\lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}\frac{(2j-1)+8n}{(2j-1)+4n}$ is equal to:

(27-07-21/Shift-1)

(a) $2-\log_e\left(\frac{2}{3}\right)$ (b) $3+2\log_e\left(\frac{2}{3}\right)$

(c) $1+2\log_e\left(\frac{3}{2}\right)$ (d) $5+\log_e\left(\frac{3}{2}\right)$

62. Let the domain of the function **(27-07-21/Shift-1)**

$f(x)=\log_4\left(\log_5\left(\log_3\left(18x-x^2-77\right)\right)\right)$ be (a,b).

Then the value of the integral

$$\int_a^b \frac{\sin^3 x}{\left(\sin^3 x+\sin^3(a+b-x)\right)}dx$$ is equal to.

63. If $\int_0^{\pi}(\sin^3 x)e^{-\sin^2 x}dx=\alpha-\frac{\beta}{e}\int_0^1\sqrt{t}\,e^t dt$, then $\alpha+\beta$ is equal to ________. **(27-07-21/Shift-2)**

64. If $\int_0^{100\pi}\frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}}dx=\frac{\alpha\pi^2}{1+4\pi^2}, \alpha\in R$, where $[x]$ is the greatest integer less than or equal to x, then the value of α is: **(22-07-21/Shift-2)**

(a) $100(1-e)$ (b) $200(1-e^{-1})$

(c) $150(e^{-1}-1)$ (d) $50(e-1)$

65. The value of the integral $\int_{-1}^{1}\log\left(x+\sqrt{x^2+1}\right)dx$ is:

(25-07-21/Shift-2)

(a) 1 (b) 0

(c) -1 (d) 2

66. If $f(x)=\begin{cases}\int_0^x (5+|1-t|)dt, & x>2\\ 5x+1, & x\le 2\end{cases}$ **(25-07-21/Shift-2)**

(a) $f(x)$ is not differentiable at $x=1$

(b) $f(x)$ is continuous but not differentiable at $x=2$

(c) $f(x)$ is not continuous at $x=2$

(d) $f(x)$ is everywhere differentiable

67. The function $f(x)$, that satisfies the condition

$f(x)=x+\int_0^{\pi/2}\sin x\cdot\cos y\, f(y)dy$, is **(01-09-21/Shift-2)**

(a) $x+(\pi-2)\sin x$ (b) $x+\frac{\pi}{2}\sin x$

(c) $x+\frac{2}{3}(\pi-2)\sin x$ (d) $x+(\pi+2)\sin x$

68. If the value of the integral $\int_0^5 \frac{x+[x]}{e^{x-[x]}}dx = \alpha e^{-1}+\beta$, where $\alpha, \beta \in R, 5\alpha+6\beta=0,$ and $[x]$ denotes the greatest integer less than to x, then the value of $(\alpha+\beta)^2$ is equal to: **(26-08-21/Shift-2)**

(a) 36 (b) 100

(c) 16 (d) 25

69. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right) dx$ is: **(26-08-21/Shift-2)**

(a) $\frac{3\pi}{2}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$ (d) $\frac{5\pi}{4}$

70. $\int_6^{16} \frac{\log_e x^2}{\log_e x^2+\log_e\left(x^2-44x+484\right)}dx$ is equal to: **(27-08-21/Shift-1)**

(a) 10 (b) 8

(c) 6 (d) 5

71. If $U_n = \left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2 \ldots \left(1+\frac{n^2}{n^2}\right)^n$, then $\lim_{n\to\infty}\left(U_n\right)^{\frac{-4}{n^2}}$ is equal to: **(27-08-21/Shift-1)**

(a) $\frac{4}{e^2}$ (b) $\frac{4}{e}$

(c) $\frac{16}{e^2}$ (d) $\frac{e^2}{16}$

72. The value of $\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{2n-1}\frac{n^2}{n^2+4r^2}$ is: **(26-08-21/Shift-1)**

(a) $\frac{1}{4}\tan^{-1}(4)$ (b) $\tan^{-1}(4)$

(c) $\frac{1}{2}\tan^{-1}(2)$ (d) $\frac{1}{2}\tan^{-1}(4)$

73. The value of $\int_{\frac{-1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \left(\left(\frac{x+1}{x-1}\right)^2+\left(\frac{x-1}{x+1}\right)^2-2\right)^{\frac{1}{2}} dx$ is: **(26-08-21/Shift-1)**

(a) $\log_e 4$ (b) $\log_e 16$

(c) $4\log_e\left(3+2\sqrt{2}\right)$ (d) $2\log_e 16$

74. The value of the integral $\int_0^1 \frac{\sqrt{x}dx}{(1+x)(1+3x)(3+x)}$ is: **(27-08-21/Shift-2)**

(a) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$ (b) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{6}\right)$

(c) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{2}\right)$ (d) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{6}\right)$

75. Let $[t]$ denote the greatest integer $\le t$. Then the value of $8\cdot\int_{-\frac{1}{2}}^{1}\left([2x]+|x|\right)dx$ is ________ ? **(31-08-21/Shift-1)**

76. If $x\phi(x)=\int_5^x\left(3t^2-2\phi'(t)\right)dt, x>-2,$ and $\phi(0)=4,$ then, $\phi(2)$ is __________ ? **(31-08-21/Shift-1)**

77. If $[x]$ is the greatest integer $\le x$, then $\pi^2\int_0^2\left(\sin\frac{\pi x}{2}\right)\left(x-[x]\right)^{[x]}dx$ is equal to: **(31-08-21/Shift-2)**

(a) $4(\pi-1)$ (b) $2(\pi-1)$

(c) $2(\pi+1)$ (d) $4(\pi+1)$

78. Let f be a non-negative function in $[0,1]$ and twice differentiable in $(0,1)$. If

$\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)dt, 0 \le x \le 1$ and

$f(0)=0$, then $\lim_{x\to 0}\frac{1}{x^2}\int_0^x f(t)dt$: **(31-08-21/Shift-1)**

(a) Equals 1 (b) Does not exist

(c) Equals $\frac{1}{2}$ (d) Equals 0

79. $\lim_{x\to 0}\frac{\int_0^{x^2}(\sin\sqrt{t})dt}{x^3}$ is equal to **(24-02-21/Shift-1)**

(a) $\frac{1}{15}$ (b) 0

(c) $\frac{2}{3}$ (d) $\frac{3}{2}$

80. If $\int_{-a}^{a}(|x|+|x-2|)dx = 22, (a>2)$ 40.and $[x]$ denotes the greatest integer $\le x$, then $\int_{a}^{-a}(x+[x])dx$ is equal to ________. **(24-02-21/Shift-1)**

81. Let $f(x)$ be a differentiable function defined on [0,2] such that $f'(x)=f'(2-x)$ for all $x\in(0,2), f(0)=1$ and $f(2)=e^2$. Then the value of $\int_0^2 f(x)dx$ is **(24-02-21/Shift-2)**

(a) $2(1-e^2)$ (b) $1-e^2$
(c) $2(1+e^2)$ (d) $1+e^2$

82. The value of the integral $\int_1^3[x^2-2x-2]dx$, where [x] denotes the greatest integer less than or equal to x, is **(24-02-21/Shift-2)**

(a) –5 (b) $-\sqrt{2}-\sqrt{3}-1$

(c) – 4 (d) $-\sqrt{2}-\sqrt{3}+1$

83. The value of $\int_{-1}^{1} x^2 e^{[x^3]}dx$, where $[t]$ denotes the greatest integer $\le t$, is: **(25-02-21/Shift-1)**

(a) $\frac{1}{3e}$ (b) $\frac{e+1}{3}$

(c) $\frac{e-1}{3e}$ (d) $\frac{e+1}{3e}$

84. If $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cot^n x\,dx$, then **(25-02-21/Shift-2)**

(a) $I_2+I_4, (I_3+I_5)^2, I_4+I_6$ are in G.P.

(b) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

(c) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in G.P.

(d) $I_2+I_4, I_3+I_5, I_4+I_6$ are in A.P.

85. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+....+\frac{n}{(2n-1)^2}\right]$ is equal to: **(25-02-21/Shift-2)**

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) 1

86. The value of $\int_{-2}^{2}|3x^2-3x-6|\,dx$ is: **(25-02-21/Shift-2)**

87. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos^2 x}{1+3^x}dx$ is **(26-02-21/Shift-1)**

(a) 2π (b) $\frac{\pi}{2}$

(c) 4π (d) $\frac{\pi}{4}$

88. The Value of $\sum_{n=1}^{100}\int_{n-1}^{n} e^{x-[x]}dx$ where [x] is the greatest integer $\leq x$, is **(26-02-21/Shift-1)**

(a) $100(e-1)$ (b) $100(1+e)$

(c) $100(1-e)$ (d) $100e$

89. The value of the integral $\int_0^{\pi}|\sin 2x|dx$ is _____. **(26-02-21/Shift-1)**

90. For $x>0$, if $f(x)=\int_1^x \frac{\log_e t}{(1+t)}dt$, then $f(e)+f\left(\frac{1}{e}\right)$ is equal to: **(26-02-21/Shift-2)**

(a) 1 (b) $\frac{1}{2}$

(c) 0 (d) -1

91. If $I_{m,n}=\int_0^1 x^{m-1}(1-x)^{n-1}dx$, for $m,n\geq 1$ and $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx=\alpha I_{m,n}, \alpha\in R$, then α equals _____. **(26-02-21/Shift-2)**

92. If the normal to the curve $y(x)=\int_0^x(2t^2-15t+10)dt$ at a point (a, b) is parallel to the line $x+3y=-5, a>1$, then the value of $|a+6b|$ is equal to ________. **(16-03-21/Shift-1)**

93. Let $f:(0, 2)\to R$ be defined as

$$f(x)=\log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right).$$

Then $\lim_{n\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+...+f(1)\right)$ is equal to _____. **(16-03-21/Shift-1)**

94. Let $f:R\to R$ be a continuous function such that $f(x)+f(x+1)=2$, for all $x\in R$. If $I_1=\int_0^8 f(x)dx$ and $I_2=\int_{-1}^3 f(x)dx$, then the value of I_1+2I_2 is equal to ________. **(16-03-21/Shift-1)**

95. Let $P(x)=x^2+bx+c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx=1$ and P(x) leaves remainder 5 when it is divided by $(x-2)$. Then the value of $9(b+c)$ is equal to **(16-03-21/Shift-2)**

(a) 11 (b) 9

(c) 7 (d) 15

96. Consider the integral $I=\int_0^{10}\frac{[x]e^{[x]}}{e^{x-1}}dx$ where [x] denotes the greatest integer less than or equal to x. Then the value of I is equal to : **(16-03-21/Shift-2)**

(a) $9(e-1)$ (b) $9(e+1)$

(c) $45(e+1)$ (d) $45(e-1)$

97. Which of the following statements is incorrect for the function $g(\alpha)$ for $\alpha\in R$ such that

$$g(\alpha)=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\sin^{\alpha}x}{\cos^{\alpha}x+\sin^{\alpha}x}dx$$ **(17-03-21/Shift-1)**

(a) $g(\alpha)$ has an inflection point at $\alpha=-\frac{1}{2}$

(b) $g(\alpha)$ is a strictly decreasing function

(c) $g(\alpha)$ is a strictly increasing function

(d) $g(\alpha)$ is an even function

98. If represents the greatest integer function, then the value of $\left|\int_0^{\sqrt{\frac{\pi}{2}}}\left[[x^2]-\cos x\right]dx\right|$ is …………… . **(17-03-21/Shift-1)**

99. Let $f: R \to R$ be defined as $f(x) = e^{-x} \sin x$. If $F:[0, 1] \to R$ is a differentiable function such that $F(x) = \int_0^x f(t)\, dt$, then the value of $\int_0^1 (F'(x) + f(x))\, e^x\, dx$ lies in the interval **(17-03-21/Shift-2)**

(a) $\left[\frac{330}{360}, \frac{331}{360}\right]$ (b) $\left[\frac{327}{360}, \frac{329}{360}\right]$

(c) $\left[\frac{331}{360}, \frac{334}{360}\right]$ (d) $\left[\frac{335}{360}, \frac{336}{360}\right]$

100. If the integral $\int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}}\, dx = \alpha e^{-1} + \beta e^{-\frac{1}{2}} + \gamma$, where α, β, γ are integers and $[x]$ denotes the greatest integer less than or equal to x, then the value of $\alpha + \beta + \gamma$ is equal to : **(17-03-21/Shift-2)**

(a) 0 (b) 20

(c) 10 (d) 25

101. Let $I_n = \int_1^e x^{19} (\log |x|)^n\, dx$, where $n \in N$. If $(20) I_{10} = \alpha I_9 + \beta I_8$, for natural numbers α and β, then $\alpha - \beta$ equals to .... **(17-03-21/Shift-2)**

102. Let $f(x)$ and $g(x)$ be two functions satisfying $f(x^2) + g(4-x) = 4x^3$ and $g(4-x) + g(x) = 0$, then the value of $\int_{-4}^{4} f(x^2)\, dx$ is .... **(18-03-21/Shift-1)**

103. Let $g(x) = \int_0^x f(t)\, dt$, where f is continuous function in [0, 3] such that $\frac{1}{3} \le f(t) \le 1$ for all $t \in [0, 1]$ and $0 \le f(t) \le \frac{1}{2}$ for all $t \in (1, 3]$. The largest possible interval in which $g(3)$ lies is: **(18-03-21/Shift-2)**

(a) $[1, 3]$ (b) $\left[-\frac{3}{2}, -1\right]$

(c) $\left[\frac{1}{3}, 2\right]$ (d) $\left[-1, -\frac{1}{2}\right]$

104. Let $P(x)$ be a real polynomial of degree 3 which vanishes at $x = -3$. Let $P(x)$ have Local minima at $x = 1$ local maxima at $x = -1$ and $\int_{-1}^{1} P(x)\, dx = 18$, then the sum of all the coefficients of the polynomial $P(x)$ is equal to .... **(18-03-21/Shift-2)**

Area Under Curves

105. The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is: **(2015)**

(a) $\frac{15}{64}$ (b) $\frac{9}{32}$

(c) $\frac{7}{32}$ (d) $\frac{5}{64}$

106. The area (in square units) of the region bounded by the curves $y + 2x^2 = 0$ and $y + 3x^2 = 1$, is equal to : **(2015/Online Set–1)**

(a) $\frac{1}{3}$ (b) $\frac{3}{4}$

(c) $\frac{3}{5}$ (d) $\frac{4}{3}$

107. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to **(2015/Online Set–2)**

(a) $\frac{4}{3}$ (b) $\frac{5}{3}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

108. The area (in sq. units) of the region **(2016)**

$\{(x, y) : y^2 \ge 2x \text{ and } x^2 + y^2 \le 4x, x \ge 0, y \ge 0\}$

is :

(a) $\pi - \frac{8}{3}$ (b) $\pi - \frac{4\sqrt{2}}{3}$

(c) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (d) $\pi - \frac{4}{3}$

109. The area (in sq. units) of the region described by $A=\{(x, y)|y\geq x^2-5x+4, x+y\geq 1, y\leq 0\}$ is:

(2016/Online Set–1)

(a) $\frac{7}{2}$ (b) $\frac{19}{6}$

(c) $\frac{13}{6}$ (d) $\frac{17}{6}$

110. The area (in sq. units) of the region $\{(x, y): x\geq 0, x+y\leq 3, x^2\leq 4y \text{ and } y\leq 1+\sqrt{x}\}$ is:

(2017)

(a) $\frac{59}{12}$ (b) $\frac{2}{3}$

(c) $\frac{7}{3}$ (d) $\frac{5}{2}$

111. The area (in sq. units) of the smaller portion enclosed between the curves, $x^2+y^2=4$ and $y^2=3x$, is :

(2017/Online Set–1)

(a) $\frac{1}{2\sqrt{3}}+\frac{\pi}{3}$ (b) $\frac{1}{\sqrt{3}}+\frac{2\pi}{3}$

(c) $\frac{1}{2\sqrt{3}}+\frac{2\pi}{3}$ (d) $\frac{1}{\sqrt{3}}+\frac{4\pi}{3}$

112. Let $g(x)=\cos x^2$, $f(x)=\sqrt{x}$, and $\alpha,\beta(\alpha<\beta)$ be the roots of the quadratic equation $18x^2-9\pi x+\pi^2=0$. Then the area (in sq. units) bounded by the curve $y=(gof)(x)$ and the lines $x=\alpha, x=\beta$ and $y=0$, is :

(2018)

(a) $\frac{1}{2}\left(\sqrt{2}-1\right)$ (b) $\frac{1}{2}\left(\sqrt{3}-1\right)$

(c) $\frac{1}{2}\left(\sqrt{3}+1\right)$ (d) $\frac{1}{2}\left(\sqrt{3}-\sqrt{2}\right)$

113. The area (in sq. units) of the region $\left\{x\in R: x\geq 0, y\geq 0, y\geq x-2 \text{ and } y\leq\sqrt{x}\right\}$ is :

(2018/Online Set–1)

(a) $\frac{13}{3}$ (b) $\frac{8}{3}$

(c) $\frac{10}{3}$ (d) $\frac{5}{3}$

114. If the area of the region bounded by the curves, $y=x^2$, $y=\frac{1}{x}$ and the lines $y=0$ and $x=t$ $(t>1)$ is 1 sq. unit, then t is equal to : **(2018/Online Set–3)**

(a) $e^{\frac{3}{2}}$ (b) $\frac{4}{3}$

(c) $\frac{3}{2}$ (d) $e^{\frac{2}{3}}$

115. The area (in sq. units) of the region $A=\left\{(x,y)\in R\times R \mid 0\leq x\leq 3, 0\leq y\leq 4, y\leq x^2+3x\right\}$ is

(8-04-2019/Shift-1)

(a) $\frac{53}{6}$ (b) 8

(c) $\frac{59}{6}$ (d) $\frac{26}{3}$

116. Let $S(\alpha)=\{(x, y): y^2\leq x, 0\leq x\leq\alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ, $0<\lambda<4$, $A(\lambda): A(4)=2:5$, then λ equals : **(8-04-2019/Shift-2)**

(a) $2\left(\frac{4}{5}\right)^{\frac{1}{3}}$ (b) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$

(c) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (d) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

117. The area (in sq. units) of the region $A=\left\{(x,y): x^2\leq y\leq x+2\right\}$ is: **(09-04-2019/Shift-1)**

(a) $\frac{10}{3}$ (b) $\frac{9}{2}$

(c) $\frac{31}{6}$ (d) $\frac{13}{6}$

118. The area (in sq. units) of the region

$A = \left\{(x, y) : \frac{y^2}{2} \le x \le y+4\right\}$ is: **(9-04-2019/Shift-2)**

(a) $\frac{53}{3}$ (b) 30

(c) 16 (d) 18

119. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x+1|$, in the first quadrant is: **(10-4-2019/Shift-2)**

(a) $\ln 2 + \frac{3}{2}$ (b) $\frac{3}{2}$

(c) $\frac{1}{2}$ (d) $\frac{3}{2} - \frac{1}{\ln 2}$

120. If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x, x + y \le 1, x \ge 0, y \ge 0\}$ is $a\sqrt{2} + b$, then a – b is equal to : **(12-04-2019/Shift-1)**

(a) $\frac{10}{3}$ (b) 6

(c) $\frac{8}{3}$ (d) $-\frac{2}{3}$

121. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x, \lambda > 0$ is $\frac{1}{9}$ then λ is equal to ______. **(12-04-2019/Shift-2)**

(a) $2\sqrt{6}$ (b) 48

(c) 24 (d) $4\sqrt{3}$

122. The area of the region

$A = \{(x, y) : 0 \le y \le x|x| + 1 \text{ and } -1 \le x \le 1\}$ in sq. units is: **(09-01-2019/Shift-2)**

(a) $\frac{2}{3}$ (b) 2

(c) $\frac{4}{3}$ (d) $\frac{1}{3}$

123. If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, $(k > 0)$, is 1 square unit. Then k is: **(10-1-2019/Shift-1)**

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $\frac{2}{\sqrt{3}}$

124. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is **(11-01-2019/Shift-1)**

(a) $\frac{5}{4}$ (b) $\frac{9}{8}$

(c) $\frac{7}{8}$ (d) $\frac{3}{4}$

125. The area (in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is : **(11-01-2019/Shift-2)**

(a) $\frac{8}{3}$ (b) $\frac{37}{24}$

(c) $\frac{187}{24}$ (d) $\frac{14}{3}$

126. The maximum area (in sq. units) of a rectangle having its base on the X-axis and its other two vertices on the parabola, $y = 12 - x^2$ such that the rectangle lies inside the parabola, is ______. **(12-01-2019/Shift-1)**

(a) 36 (b) $20\sqrt{2}$

(c) 32 (d) $18\sqrt{3}$

127. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is : **(12-01-2019/Shift-1)**

(a) $\frac{15}{4}$ (b) $\frac{21}{2}$

(c) $\frac{17}{4}$ (d) $\frac{15}{2}$

128. Area (in sq. units) of the region outside $\frac{|x|}{2}+\frac{|y|}{3}=1$ and inside the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is :

(2-9-2020/Shift-1)

(a) $3(\pi-2)$ (b) $6(\pi-2)$

(c) $6(4-\pi)$ (d) $3(4-\pi)$

129. Consider a region $R=\{(x, y)\in R^2 : x^2\le y\le 2x\}$. If a line y = α divides the area of region R into two equal parts, then which of the following is true ?

(2-09-2020/Shift-2)

(a) $\alpha^3-6\alpha^2+16=0$ (b) $3\alpha^2-8\alpha^{3/2}+8=0$

(c) $\alpha^3-6\alpha^{3/2}-16$ (d) $3\alpha^2-8\alpha+8=0$

130. The area (in sq. units) of the region

$\left\{(x,y): 0\le y\le x^2+1, 0\le y\le x+1, \frac{1}{2}\le x\le 2\right\}$ is

(3-09-2020/Shift-1)

(a $\frac{23}{16}$ (b) $\frac{79}{16}$

(c) $\frac{23}{6}$ (d) $\frac{79}{24}$

131. The area (in sq. units) of the region

$A=\{(x, y):(x-1)[x]\le y\le 2\sqrt{x}, 0\le x\le 2\}$

where[t] denotes the greatest integer function, is:

(5-09-2020/Shift-2)

(a) $\frac{4}{3}\sqrt{2}-\frac{1}{2}$ (b) $\frac{8}{3}\sqrt{2}-\frac{1}{2}$

(c) $\frac{8}{3}\sqrt{2}-1$ (d) $\frac{4}{3}\sqrt{2}+1$

132. The area (in sq. units) of the region

$A=\{(x,y):|x|+|y|\le 1, 2y^2\ge|x|\}$ **(6-09-2020/Shift-1)**

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$

(c) $\frac{1}{3}$ (d) $\frac{7}{6}$

133. The area (in sq. units) of the region enclosed by the curves $y=x^2-1$ and $y=1-x^2$ is equal to :

(6-09-2020/Shift-2)

(a) $\frac{4}{3}$ (b) $\frac{7}{2}$

(c) $\frac{16}{3}$ (d) $\frac{8}{3}$

134. The area of the region, enclosed by the circle $x^2+y^2=2$ which is not common to the region bounded by the parabola $y^2=x$ and the straight line $y=x$

(7-01-2020/Shift-1)

(a) $\frac{1}{3}(12\pi-1)$ (b) $\frac{1}{3}(6\pi-1)$

(c) $\frac{1}{6}(12\pi-1)$ (d) $\frac{1}{6}(24\pi-1)$

135. The area (in sq. units) of the region $\{(x,y)\in R \mid 4x^2\le y\le 8x+12\}$ is: **(07-01-2020/Shift-2)**

(a) $\frac{125}{3}$ (b) $\frac{128}{3}$

(c) $\frac{124}{3}$ (d) $\frac{127}{3}$

136. For a > 0, let the curves $C_1 : y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a point P. let the line $x = b(0 < b < a)$ intersect the chord OP and the x-axis at points Q and R, respectively. If the line x = b bisects the area bounded by the curves, C_1 and C_2, and the area of $\Delta OQR=\frac{1}{2}$, then 'a' satisfies the equation:

(8-01-2020/Shift-1)

(a) $x^6-12x^3+4=0$ (b) $x^6-12x^3-4=0$

(c) $x^6+6x^3-4=0$ (d) $x^6-6x^3+4=0$

137. The area (in sq. units) of the region $\{(x,y)\in R^2 : x^2 \le y \le 3-2x\}$, is:

(8-01-2020/Shift-2)

(a) $\frac{31}{3}$ (b) $\frac{32}{3}$

(c) $\frac{29}{3}$ (d) $\frac{34}{3}$

138. Given:

$$f(x)=\begin{cases} x, & 0\le x<\frac{1}{2} \\ \frac{1}{2}, & x=\frac{1}{2} \\ 1-x, & \frac{1}{2}<x\le 1 \end{cases}$$

and $g(x)=\left(x-\frac{1}{2}\right)^2, x\in R$. Then the area (in sq. units) of the region bounded by the curves $y=f(x)$ and $y=g(x)$ between the lines $2x=1$ to $2x=\sqrt{3}$ is

(9-1-2020/Shift-2)

(a) $\frac{\sqrt{3}}{4}-\frac{1}{3}$ (b) $\frac{1}{3}+\frac{\sqrt{3}}{4}$

(c) $\frac{1}{2}+\frac{\sqrt{3}}{4}$ (d) $\frac{1}{2}-\frac{\sqrt{3}}{4}$

139. Let T be the tangent to the ellipse $E: x^2+4y^2=5$ at the point $P(1,1)$. If the area of the region bounded by the tangent T, ellipse E, lines $x=1$ and $x=\sqrt{5}$ is $\alpha\sqrt{5}+\beta+\gamma\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $\left|\frac{4}{5}(\alpha+\beta+\gamma)\right|$ is equal to

(20-07-21/Shift-1)

140. The area (in sq. units) of the region, given by the set $\{(x,y)\in R\times R \,|\, x\ge 0, 2x^2\le y\le 4-2x\}$

(25-07-21/Shift-1)

(a) $\frac{7}{3}$ (b) $\frac{13}{3}$

(c) $\frac{17}{3}$ (d) $\frac{8}{3}$

141. If the area of the bounded region

$R=\left\{(x,y): \max\{0, \log_e x\}\le y\le 2^x, \frac{1}{2}\le x\le 2\right\}$ is

$\alpha(\log_e 2)^{-1}+\beta(\log_e 2)+\gamma$, then the value of $(\alpha+\beta-2\gamma)^2$ is equal to : (27-07-21/Shift-1)

(a) 4 (b) 1

(c) 8 (d) 2

142. The area of the region bounded by $y-x=2$ and $x^2=y$ is equal to (27-07-21/Shift-2)

(a) $\frac{4}{3}$ (b) $\frac{9}{2}$

(c) $\frac{16}{3}$ (d) $\frac{2}{3}$

143. The area (in sq. units) of the region bounded by the curves $x^2+2y-1=0, y^2+4x-4=0$ and $y^2-4x-4=0$, in the upper half plane is ____. (22-07-21/Shift-2)

144. The area, enclosed by curves $y=\sin x+\cos x$ and $y=|\cos x-\sin x|$ and the lines $x=0, x=\frac{\pi}{2}$, is:

(01-09-21/Shift-2)

(a) $2\sqrt{2}(\sqrt{2}+1)$ (b) $4(\sqrt{2}-1)$

(c) $2(\sqrt{2}+1)$ (d) $2\sqrt{2}(\sqrt{2}-1)$

145. Let a and b respectively be the points of local maximum and local minimum of the function $f(x) = 2x^3 - 3x^2 - 12x$. If A is the total area of the region bounded by $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$, then 4A is equal to _______. **(26-08-21/Shift-2)**

146. The area of the region $S = \{(x, y) : 3x^2 \le 4y \le 6x + 24\}$ is ________. **(26-08-21/Shift-1)**

147. The area of the region bounded by the parabola $(y-2)^2 = (x-1)$, the tangent to it at the point whose ordinate is 3 and the x-axis is : **(27-08-21/Shift-2)**

(a) 6 (b) 4

(c) 10 (d) 9

148. If the line $y = mx$ bisects the area enclosed by the lines $x = 0, y = 0, x = \frac{3}{2}$ and the curve $y = 1 + 4x - x^2$ then $12m$ is equal to ____ **(31-08-21/Shift-2)**

149. The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is **(24-02-21/Shift-1)**

(a) $24\pi + 3\sqrt{3}$ (b) $12\pi - 3\sqrt{3}$

(c) $12\pi + 3\sqrt{3}$ (d) $24\pi - 3\sqrt{3}$

150. The area of the region $R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$ is **(24-02-21-Shift/2)**

(a) $11\sqrt{3}$ square units (b) $12\sqrt{3}$ square units

(c) $6\sqrt{3}$ square units (d) $9\sqrt{3}$ square units

151. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to _______. **(25-02-21/Shift-1)**

152. The area bounded by the lines $y = \left||x-1| - 2\right|$ is ______ **(26-02-21/Shift-1)**

153. Let A_1 be the area of the region bounded by the curves $y = \sin x$, $y = \cos x$ and y-axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves $y = \sin x$, $y = \cos x$ x, x-axis and $x = \frac{\pi}{2}$ in the first quadrant. Then **(26-02-21/Shift-2)**

(a) $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$

(b) $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

(c) $2A_1 = A_2$ and $A_1 + A_2 = 1 + \sqrt{2}$

(d) $A_1 : A_2 = 1 : 2$ and $A_1 + A_2 = 1$

154. Let $f : [-3, 1] \to R$ be given as

$$f(x) = \begin{cases} \min\{(x+6), x^2\}, & -3 \le x \le 0 \\ \max\{\sqrt{x}, x^2\}, & 0 \le x \le 1. \end{cases}$$

If the area bounded by $y = f(x)$ and x-axis is A, then the value of 6A is equal to .... **(17-03-21/Shift-2)**

155. The area bounded by the curve $4y^2 = x^2(4-x)(x-2)$ is equal to: **(18-03-21/Shift-2)**

(a) $\frac{\pi}{8}$ (b) $\frac{3\pi}{2}$

(c) $\frac{3\pi}{8}$ (d) $\frac{\pi}{16}$

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Definite Integration

Objective Questions I [Only one correct option]

1. If $f(x) = A \sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)\,dx = \frac{2A}{\pi}$, then constants A and B are :

(a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ (b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$

(c) 0 and $-\frac{4}{\pi}$ (d) $\frac{4}{\pi}$ and 0

2. The value of $\int_{\pi}^{2\pi} [2\sin x]\,dx$ where [.] represents the greatest integral functions, is :

(a) $-\frac{5\pi}{3}$ (b) $-\pi$

(c) $\frac{5\pi}{3}$ (d) -2π

3. $\int_3^{29} \frac{\sqrt[3]{(x-2)^2}}{3+\sqrt[3]{(x-2)^2}}\,dx =$

(a) $4+\frac{3\sqrt{3}}{2}\pi$ (b) $2+\frac{3\sqrt{3}}{2}\pi$

(c) $4+\frac{\sqrt{3}}{2}\pi$ (d) $8+\frac{3\sqrt{3}}{2}\pi$

4. $\int_0^{1/\sqrt{3}} \frac{dx}{(2x^2+1)\sqrt{x^2+1}} =$

(a) $\frac{\pi}{2}$ (b) $\tan^{-1} 2$

(c) $\tan^{-1} 1/2$ (d) π

5. If $a \neq b$ and $af(x)+bf\left(\frac{1}{x}\right)=\frac{1}{x}-5$ for all $x \neq 0$, then

$$\int_1^2 f(x)\,dx = \frac{1}{a^2-b^2}\left[a(\log 2-\alpha)+\beta\left(\frac{b}{2}\right)\right]$$

where $\beta-\alpha$ is equal to

(a) 12 (b) 5

(c) 7 (d) 2

6. $\int_{-\pi}^{\pi} (\cos px - \sin qx)^2\,dx$ is equal to (where p, q $\in$ Z)

(a) 0 (b) $\frac{\pi}{2}$

(c) π (d) 2π

7. $\int_1^e \frac{1+\log_{10} x}{x}\,dx =$

(a) $\frac{1}{2}\log_{10} e$ (b) $\frac{1+\log_{10} e}{2}$

(c) $\frac{1}{2}\log_{10} e+1$ (d) $2\log_{10} e$

8. $\int_{1/e}^{\tan x} \frac{t}{1+t^2}\,dt + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)} =$

(a) $2(\tan e-1)$ (b) $2\tan e$

(c) 1 (d) $\tan e+\cot e$

9. $\int_0^{\log_e 5} \frac{e^x\sqrt{e^x-1}}{e^x+3}\,dx =$

(a) $3+\pi$ (b) $3-\pi$

(c) $4+\pi$ (d) $4-\pi$

10. $\int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx = \pi \frac{\cos \alpha}{1+\sin^2 \alpha}$

(a) for no value of α

(b) for exactly two values of α in $(0, \pi)$

(c) for at least one α in $\left(\frac{\pi}{2}, \pi\right)$

(d) for exactly one α in $\left(0, \frac{\pi}{2}\right)$

11. $\int_{-2}^{2} |x(x-1)| dx =$

(a) $\frac{17}{3}$ (b) $\frac{11}{3}$

(c) $\frac{13}{3}$ (d) $\frac{16}{3}$

12. If $[t]$ stands for the integral part of t, then

$\int_0^{5\pi/12} [\tan x] dx =$

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{\pi}{4}$ (d) 2π

13. If $[t]$ denotes the integeral part of t, then

$\int_0^1 \cos(\pi x)\cos([2x]\pi) dx =$

(a) 1 (b) -1

(c) $-\frac{2}{\pi}$ (d) $\frac{2}{\pi}$

14. $\int_0^1 \frac{dx}{(5+2x-2x^2)(1-e^{2-4x})} =$

(a) $\frac{1}{\sqrt{11}} \log_e \left(\frac{\sqrt{11}+2}{\sqrt{11}}\right)$ (b) $\frac{1}{\sqrt{11}} \log_e \left(\frac{\sqrt{11}+1}{\sqrt{10}}\right)$

(c) $\frac{1}{\sqrt{10}} \log_e \left(\frac{\sqrt{10}+2}{\sqrt{11}}\right)$ (d) $\frac{1}{\sqrt{10}} \log_e \left(\frac{\sqrt{10}+1}{\sqrt{11}}\right)$

15. Let $f: R \to R$ and $g: R \to R$ be continuous functions. Then the value of the integral

$\int_{-\pi/2}^{\pi/2} [f(x)+f(-x)][g(x)-g(-x)] dx$ is :

(a) π (b) 1

(c) -1 (d) 0

16. $\int_0^{2\pi} \frac{e^{|\sin x|} \cos x}{1+e^{\tan x}} dx =$

(a) e^{π} (b) 1

(c) $e^{\pi}-1$ (d) 0

17. If $m = \int_{-2}^{0} \frac{|\sin x|}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx$ and $n = \int_0^2 \frac{|\sin x|}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} dx$, where $[.]$ represents greatest integer function, then

(a) $m = n$ (b) $m = -n$

(c) $m = 2n$ (d) $m = -2n$

18. The maximum value of $\int_{a-1}^{a+1} e^{-(x-1)^2} dx$ is attained (a is real) at

(a) $a = 2$ (b) $a = 1$

(c) $a = -1$ (d) $a = 0$

19. If $f(x)$ is differentiable & defined on R^+ such that $\int_0^{t^2} x f(x) dx = \frac{2}{3} t^5$ then $f(4/25) =$

(a) $\frac{2}{3}$ (b) $-\frac{3}{2}$

(c) 1 (d) $\frac{3}{2}$

20. The function $F(x)=\int_{\pi/6}^{x}(4\sin t+3\cos t)\,dt$ attains least value on [π/4, 3π/4] at x equals.

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{3}$

(c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{4}$

21. If $f(x)=\int_{\pi^2/16}^{x^2}\frac{\cos x\cos\sqrt{t}}{1+\sin^2\sqrt{t}}dt$ then $f'(\pi)$ is equal to

(a) 0 (b) π

(c) 2π (d) $\frac{\pi}{2}$

22. If $f(t)=\begin{cases}at-1 & t<1\\ t^2+b & t\geq 1\end{cases}$ then possible set of values of (a,b) so that $\int_0^x f(x)\,dx$ is differentiable for all $x\geq 0$ is

(a) $(5,1)$ (b) $(1,3)$

(c) $(4,2)$ (d) none of these

23. If $f(x)=e^{g(x)}$ and $g(x)=\int_2^x\frac{t}{1+t^4}dt$ then $f'(2)$ has the value equal to

(a) $2/17$ (b) 0

(c) 1 (d) cannot be determined

24. The value of the function

$f(x)=1+x+\int_1^x(\ln^2 t+2\ln t)\,dt$ where $f'(x)$ vanishes is

(a) e^{-1} (b) 0

(c) $2e^{-1}$ (d) $1+2e^{-1}$

25. $\lim_{h\to 0}\frac{\int_a^{x+h}\ln^2 t\,dt-\int_a^x\ln^2 t\,dt}{h}$ equals to

(a) 0 (b) $\ln^2 x$

(c) $\frac{2\ln x}{x}$ (d) does not exist

26. If $\int_0^x f(t)\,dt=x+\int_x^1 tf(t)\,dt$ then $f(1)$ is

(a) $\frac{1}{2}$ (b) 0

(c) 1 (d) $\frac{-1}{2}$

27. $\int_0^{1/2}e^x\left(\sin^{-1}x-\frac{x}{(1-x^2)^{3/2}}\right)dx=$

(a) $\sqrt{e}\left(\frac{\pi}{6}-\frac{1}{\sqrt{3}}\right)+1$ (b) $\sqrt{e}\left(\frac{\pi}{6}+\frac{2}{\sqrt{3}}\right)+1$

(c) $\sqrt{e}\left(\frac{\pi}{6}-\frac{2}{\sqrt{3}}\right)+1$ (d) $\sqrt{e}\left(\frac{\pi}{6}+\frac{1}{\sqrt{3}}\right)+1$

28. Let $f(x)=\begin{cases}\int_0^x\{5+|1-y|\}dy & if\ x>2\\ 5x+1 & if\ x\leq 2\end{cases}$

Then

(a) $f(x)$ is continuous but not differentiable at $x=2$

(b) $f(x)$ is not continuous at $x=2$

(c) $f(x)$ is differentiable everywhere

(d) The right derivative of $f(x)$ at $x=2$ does not exist

29. Consider the function $f(x)=\int_0^x [t]dt$

where $x>0$ and $[t]$ is the integral part of t. Then

(a) $f(x)$ is not defined for $x=1,2,3,...$

(b) $f(x)$ is defined for all $x>0$ but is not continuous at $x=1,2,3,...$

(c) $f(x)$ is continuous for all $x>0$

(d) $f(x)$ is differentiable for all $x>0$

30. $\int_0^{\pi} \frac{x^2 \sin 2x \sin\left[(\pi/2)\cos x\right]}{2x-\pi}dx=$

(a) $\frac{4}{\pi^2}$ (b) $\frac{\pi^2}{4}$

(c) $\frac{\pi^2}{8}$ (d) $\frac{8}{\pi}$

31. Let $f(x)=\frac{e^x}{1+e^x}$

$I_1=\int_{f(-a)}^{f(a)} x\,g(x(1-x))dx$

$I_2=\int_{f(-a)}^{f(a)} g(x(1-x))dx$

then I_2/I_1 is

(a) 1 (b) -3

(c) -1 (d) 2

32. $\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{\cos^{-1}\left(\frac{2x}{1+x^2}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)}{e^x+1}dx=$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{4\sqrt{3}}$ (d) $\frac{\pi}{2\sqrt{3}}$

33. For any integer n, the integral $\int_0^{\pi} e^{\cos^2 x}\cos^3(2n+1)x\,dx$ has the value :

(a) π (b) 1

(c) 0 (d) none of these

34. If $I_1=\int_{-4}^{-5} e^{(x+5)^2}dx$ and $I_2=3\int_{1/3}^{2/3} e^{9\left[x-(2/3)\right]^2}dx$ then the value of I_1+I_2 is

(a) 0 (b) 1

(c) e^{-1} (d) e

35. If $I=\int_0^1 \frac{e^t}{t+1}dt$, then $\int_{a-1}^{a} \frac{e^{-t}}{t-a-1}dt=$

(a) Ie^a (b) $(-I)e^a$

(c) $(-I)e^{-a}$ (d) Ie^{-a}

36. Let $\lambda=\int_0^1 \frac{dx}{1+x^3}$, $p=\lim_{n\to\infty}\left[\frac{\prod_{r=1}^{n}(n^3+r^3)}{n^{3n}}\right]^{1/n}$, then ln p is equal to

(a) $\ln 2-1+\lambda$ (b) $\ln 2-3+3\lambda$

(c) $2\ln 2-\lambda$ (d) $\ln 4-3+3\lambda$

37. Consider the integrals

$I_1=\int_0^1 e^{-x}\cos^2 x\,dx,\ I_2=\int_0^1 e^{-x^2}\cos^2 x\,dx,$

$I_3=\int_0^1 e^{-\frac{x^2}{2}}\cos^2 x\,dx,\ I_4=\int_0^1 e^{-\frac{x^2}{2}}dx$ Then

(a) $I_2>I_4>I_1>I_3$ (b) $I_2<I_4<I_1<I_3$

(c) $I_1<I_2<I_3<I_4$ (d) $I_1>I_2>I_3>I_4$

38. If $I_n=\int_1^e (\log_e x)^n dx$ (n is a positive integer), then $I_{2012}+(2012)I_{2011}=$

(a) $I_{2011}+(2010)I_{2010}$ (b) $I_{2013}+(2013)I_{2012}$

(c) $I_{2011}+(2010)I_{2009}$ (d) $I_{2012}-(2012)I_{2011}$

39. A function $f(x)$ which satisfies the relation

$f(x)=e^x+\int_0^1 e^x f(t)\,dt$, then $f(x)$ is

(a) $\frac{e^x}{2-e}$ (b) $(e-2)e^x$

(c) $2e^x$ (d) $\frac{e^x}{2}$

40. If $\int_0^{\pi}\frac{x^2}{(1+\sin x)^2}dx=A$ then $\int_0^{\pi}\frac{2x^2\cos^2(x/2)}{(1+\sin x)^2}dx=$

(a) $A+\pi-\pi^2$ (b) $A-\pi+\pi^2$

(c) $A-\pi-\pi^2$ (d) $A+2\pi-\pi^2$

41. $\int_0^{\sin^2 x}\left(\sin^{-1}\sqrt{t}\right)dt+\int_0^{\cos^2 x}\left(\cos^{-1}\sqrt{t}\right)dt=$

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2\sqrt{2}}$

42. For each positive integer n, define

$f_n(x)=Min\left(\frac{x^n}{n!},\frac{(1-x)^n}{n!}\right)$ for $0\le x\le 1$. Let

$I_n=\int_0^1 f_n(x)dx$ for $n\ge 1$. Then the value of $\sum_{n=1}^{\infty}I_n$ is

(a) $2\sqrt{e}-3$ (b) $2\sqrt{e}-2$

(c) $2\sqrt{e}-1$ (d) $2\sqrt{e}$

43. $\int_0^{\pi/2} f(\sin 2x)\sin x\,dx=k\int_0^{\pi/4} f(\cos 2x)\cos x\,dx$ where k equals

(a) 2 (b) 4

(c) $\sqrt{2}$ (d) $2\sqrt{2}$

44. The value of the definite integral $\int_0^1(1+e^{-x^2})\,dx$ is :

(a) -1 (b) 2

(c) $1+e^{-1}$ (d) none of these

45. If $\int_0^x e^{zx}.e^{-z^2}\,dz=f(x)\int_0^x e^{-z^2/4}dz$

then $\int e^x\left(\log_e(f(x))+\frac{x}{2}\right)dx=$

(a) $\frac{xe^x}{2}+c$ (b) $\frac{x^2e^x}{4}+c$

(c) $\frac{x^2e^x}{2}+c$ (d) $\frac{xe^x}{4}+c$

Objective Questions II [One or more than one correct option]

46. $\int_0^{\infty}\frac{x}{(1+x)(1+x^2)}dx$

(a) $\pi/4$

(b) $\pi/2$

(c) is same as $\int_0^{\infty}\frac{dx}{(1+x)(1+x^2)}$

(d) cannot be evaluated

47. If $f(2-x)=f(2+x)$ and $f(4-x)=f(4+x)$ and $f(x)$ is a function for which $\int_0^2 f(x)\,dx=5$, then $\int_0^{50} f(x)\,dx$ is equal to

(a) 125 (b) $\int_{-4}^{46} f(x)\,dx$

(c) $\int_1^{51} f(x)\,dx$ (d) $\int_2^{52} f(x)\,dx$

48. If $F(x)=\frac{1}{x^2}\int_4^x (4t^2-2F'(t))\,dt$, then $F'(4)$ equals –

(a) $\frac{32}{9}$ (b) $\frac{64}{9}$

(c) $\frac{(2F(4))^5}{9}$ (d) $\frac{11F(8)}{28}$

49. If $f(x)$ is integrable over $[1,2]$, then $\int_1^2 f(x)\,dx$ is equal to

(a) $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)$ (b) $\lim_{n\to\infty}\frac{1}{n}\sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$

(c) $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r+n}{n}\right)$ (d) $\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

50. If $I=\int_0^{\pi/2} e^{-\alpha \sin x}dx$, where $\alpha\in(0,\infty)$, then

(a) $I<\frac{\pi}{2}$ (b) $I>\frac{\pi}{2}\left(e^{-\alpha}+1\right)$

(c) $I>\frac{\pi}{2}e^{-\alpha}$ (d) $I>0$

51. If $I_n=\int_0^1\frac{dx}{(1+x^2)^n}$, $n\in N$, then which of the following statements hold good ?

(a) $2nI_{n+1}=2^{-n}+(2n-1)I_n$

(b) $I_2=\frac{\pi}{8}+\frac{1}{4}$

(c) $I_2=\frac{\pi}{8}-\frac{1}{4}$

(d) $I_3=\frac{\pi}{16}-\frac{5}{48}$

Numerical Value Type Questions

52. Determine a positive integer $n\le 5$, such that

$$\int_0^1 e^x(x-1)^n\,dx=16-6e$$

Assertion & Reason

(A) **If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.**

(B) **If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.**

(C) **If ASSERTION is true, REASON is false.**

(D) **If ASSERTION is false, REASON is true.**

53. **Assertion :** $\int_0^{\pi} x\sin x\cos^2 x\,dx=\frac{\pi}{2}\int_0^{\pi}\sin x\cos^2 x\,dx$

Reason : $\int_a^b x\,f(x)\,dx=\frac{a+b}{2}\int_a^b f(x)\,dx$

(a) A (b) B

(c) C (d) D

54. **Assertion :**

$$\int e^{\sin^{-1}x}\left(1-\frac{x}{\sqrt{1-x^2}}\right)dx=e^{\sin^{-1}x}.\sqrt{1-x^2}+c$$

Reason : $\int e^{g(x)}\left(g'(x)f(x)+f'(x)\right)dx=e^{g(x)}f(x)+c$

(a) A (b) B

(c) C (d) D

55. **Assertion :** $1\le\int_0^{\pi/2}\frac{\sin x}{x}\le\frac{\pi}{2}$

Reason : If $f(x)$ is continuous in $[a,b]$ and m and l are greatest and least value of $f(x)$ in $[a,b]$, then

$$l(b-a)\le\int_a^b f(x)\,dx\le m(b-a)$$

(a) A (b) B

(c) C (d) D

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

56.

Column - I	Column - II
(A) If $\int_0^1 \frac{\cos x}{1+x}\,dx = k$ and $\int_{6\pi-3}^{6\pi} \frac{\cos(x/3)}{6\pi+3-x}dx = mk$, then m is	**(P)** 4
(B) $\int_{-1}^{1}\left(\sin^{-1}\left[x+\frac{3}{4}\right]\right)dx = \frac{\pi}{k}$, then k is {where [.] denotes greatest integer function}	**(Q)** 1
(C) If $f(x) = \max\left(x-\lvert x\rvert, x+[x]\right)$ and $\int_{-3}^{3} f(x)\,dx = -\frac{3}{k}$, then k is {where [.] denotes greatest integer function}	**(R)** 3
(D) If $\int_0^{20} \sqrt{1-\cos \pi x}\;dx = \frac{10k\sqrt{2}}{\pi}$, then k is	**(S)** 2

The correct matching is :

(a) A-Q; B-P; C-S; D-P

(b) A-Q; B-P; C-S; D-R

(c) A-Q; B-R; C-Q; D-P

(d) A-S; B-P; C-S; D-P

Using the following passage, solve Q.57 to Q.59

Passage

Using integral $\int_0^{\pi/2} \ln(\sin x)dx$

$$= -\int_0^{\pi/2} \ln(\sec x)\,dx = -\frac{\pi}{2}\ln 2,$$

$$\int_0^{\pi/2} \ln(\tan x)\,dx = 0 \text{ and } \int_0^{\pi/4} \ln(1+\tan x)\,dx = \frac{\pi}{8}\ln 2.$$

57. Evaluate $\int_0^{\pi/4} \ln(\sin 2x)\,dx =$

(a) $\frac{-\pi \ln 2}{2}$ (b) $\pi \ln 2$

(c) $\frac{\pi \ln 2}{4}$ (d) none of these

58. Evaluate $\int_{-\pi/4}^{\pi/4} \ln\left(\frac{\sin x + \cos x}{\cos x - \sin x}\right)dx =$

(a) $\pi \ln 2$ (b) $\frac{\pi \ln 2}{2}$

(c) 0 (d) $-\pi \ln 2$

59. Evaluate $\int_{-\pi/4}^{\pi/4} \ln(\sin x + \cos x)\,dx =$

(a) $\frac{\pi \ln 2}{2}$ (b) $\frac{-\pi \ln 2}{4}$

(c) $\pi \ln 2$ (d) 0

Text

60. Determine the value of $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx$

61. Evaluate $\int_2^3 \frac{2x^5+x^4-2x^3+2x^2+1}{(x^2+1)(x^4-1)}dx.$

Area Under Curves

Objective Questions I [Only one correct option]

62. The area bounded by the curve $y=3+2x-x^2, y=0$ & the ordinate at $x=1$ & $x=4$ is

(a) $25/3$ (b) $23/3$

(c) $19/3$ (d) none

63. The area bounded by the curve $y=x(1-\ln x)$ and positive $x-$ axis between $x=e^{-1}$ and $x=e$ is

(a) $\left(\frac{e^2-4e^{-2}}{5}\right)$ (b) $\left(\frac{e^2-5e^{-2}}{4}\right)$

(c) $\left(\frac{4e^2-e^{-2}}{5}\right)$ (d) $\left(\frac{5e^2-e^{-2}}{4}\right)$

64. The area enclosed by $y=x^3$, its normal at $(1,1)$ and $x-$ axis is equal to

(a) $\frac{7}{4}$ (b) $\frac{9}{4}$

(c) $\frac{5}{4}$ (d) $\frac{8}{4}$

65. Area of the region bounded by $x=0, y=0, x=2, y=2,$ $y \le e^x$ and $y \ge \ln x$, is

(a) $6-4\ln 2$ (b) $4\ln 2-2$

(c) $2\ln 2-4$ (d) $6-2\ln 2$

66. The area bounded by the curve $y=e^x$ and the lines $y=|x-1|, x=2$ is given by

(a) e^2+1 (b) e^2-1

(c) e^2-2 (d) none

67. The area of the closed figure bounded by the curves $y=\sqrt{x}, y=\sqrt{4-3x}$ & $y=0$ is

(a) $4/9$ (b) $8/9$

(c) $16/9$ (d) none

68. The area of the closed figure bounded by the curves $y=\cos x; y=1+\frac{2}{\pi}x$ & $x=\frac{\pi}{2}$ is

(a) $\frac{\pi+4}{4}$ (b) $\frac{3\pi}{4}$

(c) $\frac{3\pi+4}{4}$ (d) $\frac{3\pi-4}{4}$

69. The ratio in which the curve $y=x^2$ divides the region bounded by the curve; $y=\sin\left(\frac{\pi x}{2}\right)$ & the $x-$ axis as x varies from 0 to 1, is

(a) $2:\pi$ (b) $1:3$

(c) $3:\pi$ (d) $(6-\pi):\pi$

70. The area bounded by $y=2-|2-x|$ and $y=\frac{3}{|x|}$ is

(a) $\frac{4+3\ell n3}{2}$ (b) $\frac{4-3\ell n3}{2}$

(c) $\frac{3}{2}+\ell n3$ (d) $\frac{1}{2}+\ell n3$

71. The area of the region enclosed between the curves $7x^2+9y+9=0$ and $5x^2+9y+27=0$ is

(a) 2 (b) 4

(c) 8 (d) 16

72. Value of the parameter a such that the area bounded by $y=a^2x^2+ax+1$, co-ordinate axes and the line $x=1$, attains it's least value, is equal to

(a) $-\frac{1}{4}$ (b) $-\frac{1}{2}$

(c) $-\frac{3}{4}$ (d) -1

73. The area enclosed by the curves $y=\sqrt{4-x^2}, y\ge\sqrt{2}\sin\left(\frac{x\pi}{2\sqrt{2}}\right)$ and $x-$ axis is divided by $y-$ axis in the ratio

(a) $\frac{\pi^2-8}{\pi^2+8}$ (b) $\frac{\pi^2-4}{\pi^2+4}$

(c) $\frac{\pi-4}{\pi+4}$ (d) $\frac{2\pi^2}{2\pi+\pi^2-8}$

74. The area of the plane figure bounded in first quadrant by $y = x^{1/3}; y = -x^2 + 2x + 3; y = 2x - 1$ and the axis of ordinates is

(a) $12/55$ (b) $55/12$

(c) $32/55$ (d) none

75. If $f(x) = \sin x \forall\, x \in \left[0, \frac{\pi}{2}\right], f(x) + f(\pi - x) =$ $2 \forall\, x \in \left(\frac{\pi}{2}, \pi\right]$ and $f(x) = f(2\pi - x) \forall x \in (\pi, 2\pi]$, then the area enclosed by $y = f(x)$ and $x-$ axis is

(a) π (b) 2π

(c) 2 (d) 4

76. The area bounded by curve $y = ex \log x$ and $y = \frac{\log x}{ex}$ is–

(a) $\frac{e^2 - 5}{4e}$ (b) $\frac{e^2 + 5}{4e}$

(c) $\frac{e^2}{4} - \frac{5}{e}$ (d) None of these

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

77.

	Column - I		Column - II
(A)	The area bounded by the curve $y = x + \sin x$ and its inverse function between the ordinates $x = 0$ to $x = 2\pi$ is $4s$ Then the value of s is	**(P)**	2
(B)	The area bounded by $y = xe^{\lvert x \rvert}$ and lines $\lvert x \rvert = 1, y = 0$ is	**(Q)**	1
(C)	The area bounded by the curves $y^2 = x^3$ and $\lvert y \rvert = 2x$ is	**(R)**	$\frac{32}{5}$
(D)	The smaller area included between the curves $\sqrt{x} + \sqrt{y} = 1$ and $\lvert x \rvert + \lvert y \rvert = 1$ is	**(S)**	$\frac{1}{3}$

The correct matching is :

(a) A-P; B-Q; C-R; D-S

(b) A-Q; B-P; C-R; D-S

(c) A-P; B-R; C-S; D-P

(d) A-P; B-P; C-R; D-S

Using the following passage, solve Q.78 to Q.80

Passage

Consider the curve defined implicity by the equation $y^2 - 2ye^{\sin^{-1} x} + x^2 - 1 + [x] + e^{2\sin^{-1} x} = 0$, Where $[x]$ denotes the greatest integer function

78. The area of the region bounded by the curve between the lines $x = -1$ and $x = 0$ is

(a) $\frac{\pi}{2} + 1$ (b) $\pi - 1$

(c) $\pi + 1$ (d) $\frac{\pi}{2} - 1$

79. The area of the region bounded by the curve between the lines $x = 0$ & $x = 1$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{2} - 1$

(c) $\frac{\pi}{2} + 1$ (d) $\pi + 1$

80. The Area of the region bounded by the curve between the lines $x = 0$ & $x = \frac{1}{2}$ is

(a) $\frac{\sqrt{3}}{4} + \frac{\pi}{6}$ (b) $\frac{\sqrt{3}}{2} + \frac{\pi}{6}$

(c) $\frac{\sqrt{3}}{4} - \frac{\pi}{6}$ (d) $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Definite Integration

Objective Questions I [Only one correct option]

1. Let $g(x) = \int_0^x f(t)\,dt$, where f is such that $\frac{1}{2} \le f(t) \le 1$ for $t \in [0, 1]$ and $0 \le f(t) \le \frac{1}{2}$ for $t \in [1, 2]$. Then $g(2)$ satisfies the inequality. **(2000)**

(a) $-\frac{3}{2} \le g(2) < \frac{1}{2}$ (b) $0 \le g(2) < 2$

(c) $\frac{3}{2} < g(2) \le 5/2$ (d) $2 < g(2) < 4$

2. The value of the integral $\int_{e^{-1}}^{e^2} \left|\frac{\log_e x}{x}\right| dx$ is : **(2000)**

(a) 3/2 (b) 5/2

(c) 3 (d) 5

3. If $f(x) = \begin{cases} e^{\cos x} \sin x, \text{ for } |x| \le 2, \\ = 2, \text{ otherwise} \end{cases}$

then $\int_{-2}^{3} f(x)dx$, is equal to **(2000)**

(a) 0 (b) 1

(c) 2 (d) 3

4. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$ is : **(2001)**

(a) π (b) $a\pi$

(c) $\pi/2$ (d) 2π

5. Let $f: (0, \infty) \to R$ and $F(x) = \int_0^x f(t)\,dt$. If $F(x^2) = x^2(1+x)$, then $f(4)$ equals : **(2001)**

(a) 5/4 (b) 7

(c) 4 (d) 2

6. Let $f(x) = \int_1^x \sqrt{2-t^2}\,dt$. Then the real roots of the equation $x^2 - f'(x) = 0$ are : **(2002)**

(a) ± 1 (b) $\pm\frac{1}{\sqrt{2}}$

(c) $\pm\frac{1}{2}$ (d) 0 and 1

7. Let $T > 0$ be a fixed real number, Suppose f is a continous function such that for all $x \in R$. $f(x + T) = f(x)$. If $I = \int_0^T f(x)\,dx$, then the value of $\int_3^{3+3T} f(2x)\,dx$ is : **(2002)**

(a) 3/2 I (b) I

(c) 3 I (d) 6 I

8. The integral $\int_{-1/2}^{1/2} \left([x] + \ln\left(\frac{1+x}{1-x}\right)\right) dx$ equals **(2002)**

(a) $-\frac{1}{2}$ (b) 0

(c) 1 (d) $2\ln\left(\frac{1}{2}\right)$

9. If $I(m, n) = \int_0^1 t^m (1+t)^n\,dt$, then the expression for $I(m, n)$ in terms of $I(m+1, n-1)$ is : **(2003)**

(a) $\frac{2^n}{m+1} - \frac{n}{m+1} I(m+1, n-1)$

(b) $\frac{n}{m+1} I(m+1, n-1)$

(c) $\frac{2^n}{m+1} + \frac{n}{m+1} I(m+1, n-1)$

(d) $\frac{m}{m+1} I(m+1, n-1)$

10. If $f(x) = \int_{x^2}^{x^2+1} e^{-t^2}\, dt$, then $f(x)$ increases in : **(2003)**

(a) (2, 2) (b) no value of x

(c) $(0, \infty)$ (d) $(-\infty, 0)$

11. The value of the integral $\int_0^1 \sqrt{\frac{1-x}{1+x}}\, dx$ is **(2004)**

(a) $\frac{\pi}{2}+1$ (b) $\frac{\pi}{2}-1$

(c) – 1 (d) 1

12. If $f(x)$ is differentiable and $\int_0^{t^2} x\, f(x)\, dx = \frac{2}{5}t^5$, then $f(4/25)$ equals : **(2004)**

(a) 2/5 (b) – 5/2

(c) 1 (d) 5/2

13. The value of $\int_{-2}^{0} [x^3+3x^2+3x+3+(x+1)\cos(x+1)]\, dx$ is: **(2005)**

(a) 0 (b) 3

(c) 4 (d) 1

14. If $\int_{\sin x}^{1} t^2 f(t)\, dt = 1-\sin x\ \forall\ x \in (0, \pi/2)$ then $f\left(\frac{1}{\sqrt{3}}\right)$ is : **(2005)**

(a) 3 (b) $\sqrt{3}$

(c) 1/3 (d) none of these

15. Let f be a non-negative function defined on the interval [0, 1]. If $\int_0^x \sqrt{1-(f'(t))^2}\, dt = \int_0^x f(t)\, dt$, $0 \le x \le 1$ and $f(0) = 0$, then **(2009)**

(a) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(b) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(c) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

(d) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

16. The value of the integral

$\int_{-\pi/2}^{\pi/2} \left(x^2 + \log\frac{\pi-x}{\pi+x}\right)\cos x\, dx$ is **(2012)**

(a) 0 (b) $\frac{\pi^2}{2}-4$

(c) $\frac{\pi^2}{2}+4$ (d) $\frac{\pi^2}{2}$

17. Let $f:\left[\frac{1}{2},1\right] \to R$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$. Then, the value of $\int_{1/2}^{1} f(x)\, dx$ lies in the interval **(2013)**

(a) (2e – 1, 2e) (b) (e – 1, 2e – 1)

(c) $\left(\frac{e-1}{2}, e-1\right)$ (d) $\left(0, \frac{e-1}{2}\right)$

18. The following integral $\int_{\pi/4}^{\pi/2} (2\operatorname{cosec} x)^{17}\, dx$ is equal to **(2014)**

(a) $\int_0^{\log(1+\sqrt{2})} 2(e^u+e^{-u})^{16}\, du$

(b) $\int_0^{\log(1+\sqrt{2})} (e^u+e^{-u})^{17}\, du$

(c) $\int_0^{\log(1+\sqrt{2})} (e^u-e^{-u})^{17}\, du$

(d) $\int_0^{\log(1+\sqrt{2})} 2(e^u-e^{-u})^{16}\, du$

19. Let $f'(x) \frac{192x^3}{2+\sin^4 \pi x}$ for all $x \in R$ with $f\left(\frac{1}{2}\right) = 0$. If $m \le \int_{1/2}^{1} f(x)\, dx \le M$, then the possible values of m and M are **(2015)**

(a) m = 13, M = 24 (b) $m = \frac{1}{4}$, $M = \frac{1}{2}$

(c) m = –11, M = 0 (d) m = 1, M = 12

20. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+e^x} dx$ is equal to **(2016)**

(a) $\frac{\pi^2}{4}-2$ (b) $\frac{\pi^2}{4}+2$

(c) $\pi^2 - e^{\frac{\pi}{2}}$ (d) $\pi^2 + e^{\frac{\pi}{2}}$

Objective Questions II [One or more than one correct option]

21. Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2+kn+k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2+kn+k^2}$, for $n = 1, 2, 3, \ldots$, then **(2008)**

(a) $S_n < \frac{\pi}{3\sqrt{3}}$ (b) $S_n > \frac{\pi}{3\sqrt{3}}$

(c) $T_n < \frac{\pi}{3\sqrt{3}}$ (d) $T_n > \frac{\pi}{3\sqrt{3}}$

22. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx$, $n = 0, 1, 2, \ldots$, then **(2009)**

(a) $I_n = I_{n+2}$ (b) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

(c) $\sum_{m=1}^{10} I_{2m} = 0$ (d) $I_n = I_{n+1}$

23. For $a \in R$ (the set of all real numbers), $a \neq -1$,

$$\lim_{n\to\infty} \frac{(1^a+2^a+\ldots+n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+\ldots+(na+n)]} = \frac{1}{60}$$

Then, a is equal to **(2013)**

(a) 5 (b) 7

(c) $\frac{-15}{2}$ (d) $\frac{-17}{2}$

24. Let $f: (0, \infty) \to R$ be given by

$f(x) = \int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$. Then **(2014)**

(a) $f(x)$ is monotonically increasing on $[1, \infty)$

(b) $f(x)$ is monotonically decreasing on $(0, 1)$

(c) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0, \infty)$

(d) $f(2^x)$ is an odd function of x on R

25. The option(s) with the value of a and L that satisfy the following equation is(are)

$$\frac{\int_0^{4\pi} e^t(\sin^6 at + \cos^4 at\, dt)}{\int_0^{\pi} e^t(\sin^6 at + \cos^4 at\, dt)} = L\ ?$$ **(2015)**

(a) $a = 2, L = \frac{e^{4\pi}-1}{e^{\pi}-1}$ (b) $a = 2, L = \frac{e^{4\pi}+1}{e^{\pi}+1}$

(c) $a = 4, L = \frac{e^{4\pi}-1}{e^{\pi}-1}$ (d) $a = 4, L = \frac{e^{4\pi}+1}{e^{\pi}+1}$

26. Let $f(x) = \lim_{n\to\infty} \left(\frac{n^n (x+n)\left(x+\frac{n}{2}\right)\ldots\left(x+\frac{n}{n}\right)}{n!(x^2+n^2)\left(x^2+\frac{n^2}{4}\right)\ldots\left(x^2+\frac{n^2}{n^2}\right)} \right)^{\frac{x}{n}}$,

for all $x > 0$. Then **(2016)**

(a) $f\left(\frac{1}{2}\right) \geq f(1)$ (b) $f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right)$

(c) $f'(2) \leq 0$ (d) $\frac{f'(3)}{f(3)} \geq \frac{f'(2)}{f(2)}$

27. If $I = \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)} dx$, then **(2017)**

(a) $I > \frac{49}{50}$ (b) $I < \frac{49}{50}$

(c) $I < \log_e 99$ (d) $I > \log_e 99$

28. For $\lim_{n\to\infty} \frac{1+\sqrt[3]{2}+......+\sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(na+1)^2}+\frac{1}{(na+2)^2}+....+\frac{1}{(na+n)^2}\right)}=54$

$a \in R$, $|a| > 1$, Then possible value(s) of a is/are – **(2019)**

(a) 8 (b) -9

(c) 7 (d) -6

29. Which of the following inequalities is/are TRUE ? **(2020)**

(a) $\int_0^1 x\cos x\,dx \geq \frac{3}{8}$ (b) $\int_0^1 x\sin x\,dx \geq \frac{3}{10}$

(c) $\int_0^1 x^2\cos x\,dx \geq \frac{1}{2}$ (d) $\int_0^1 x^2\sin x\,dx \geq \frac{2}{9}$

30. Let $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to R$ be a continuous function such that $f(0) = 1$ and $\int_0^{\frac{\pi}{3}} f(t)dt = 0$ Then which of the following statements is (are) TRUE ? **(2021)**

(a) The equation $f(x) - 3\cos 3x = 0$ has a least one solution in $\left(0,\frac{\pi}{3}\right)$

(b) The equation $f(x)-3\sin 3x=-\frac{6}{\pi}$ has at least one solution in $\left(0,\frac{\pi}{3}\right)$

(c) $\lim_{x\to 0}\frac{x\int_0^x f(t)dt}{1-e^{x^2}}=-1$

(d) $\lim_{x\to 0}\frac{\sin x\int_0^x f(t)dt}{x^2}=-1$

Numerical Value Type Questions

31. The value of $\frac{(5050)\int_0^1 (1-x^{50})^{100}\,dx}{\int_0^1 (1-x^{50})^{101}\,dx}$ is **(2006)**

32. The value of $\int_0^1 4x^3\left\{\frac{d^2}{dx^2}\left(1-x^2\right)^5\right\}dx$ is **(2014)**

33. Let $f: R \to R$ be a function defined by $f(x)=\begin{cases}[x], & x\leq 2\\ 0, & x>2\end{cases}$ where [x] is the greatest integer less than or equal to x, if $I=\int_{-1}^{2}\frac{xf(x^2)}{2+f(x+1)}dx$, then the value of (4I – 1) is **(2015)**

34. If $\alpha=\int_0^1\left(e^{9x+3\tan^{-1}x}\right)\left(\frac{12+9x^2}{1+x^2}\right)dx$ where $\tan^{-1}x$ takes only principal values, then the value of $\left(\log_e|1+\alpha|-\frac{3\pi}{4}\right)$ is **(2015)**

35. Let $f: R\to R$ be a continuous odd function, which vanishes exactly at one point and $f(1)=\frac{1}{2}$. Suppose that $F(x)=\int_{-1}^{x} f(t)dt$ for all $x \in [-1, 2]$ and $G(x)=\int_{-1}^{x} t\left|f(f(t))\right|dt$ for all $x \in [-1, 2]$. If $\lim_{x\to 1}\frac{F(x)}{G(x)}=\frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is **(2015)**

36. The total number of distinct $x \in [0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4}dt = 2x-1$ is **(2016)**

37. The value of the integral $\int_0^{\frac{1}{2}}\frac{1+\sqrt{3}}{\left((x+1)^2(1-x)^6\right)^{\frac{1}{4}}}dx$ is__. **(2018)**

38. $I=\frac{2}{\pi}\int_{-\pi/4}^{\pi/4}\frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$ then find $27I^2$ equals _____ **(2019)**

39. The value of the integral

$\int_0^{\pi/2}\frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5}d\theta$ equals **(2019)**

40. Let $f:R\to R$ be a differentiable function such that its derivative f' is continuous and $f(\pi)=-6$.

If F : [0, π] → R is defined by $F(x)=\int_0^x f(t)\,dt$, and if

$\int_0^{\pi}(f'(x)+F(x))\cos x\,dx=2$ then the value of $f(0)$ is _____. **(2020)**

41. For any real number x, let [x] denote the largest integer less than or equal to x. If $I=\int_0^{10}\left[\sqrt{\frac{10x}{x+1}}\right]dx$, then the value of 9I is ___ **(2021)**

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

42. Match the conditions/expressions in **Column I** with statement in **Column II**. **(2007)**

Column–I	Column–II
(A) $\int_{-1}^{1}\frac{dx}{1+x^2}$	(P) $\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B) $\int_0^1\frac{dx}{\sqrt{1-x^2}}$	(Q) $2\log\left(\frac{2}{3}\right)$
(C) $\int_2^3\frac{dx}{1-x^2}$	(R) $\frac{\pi}{3}$
(D) $\int_1^2\frac{dx}{x\sqrt{x^2-1}}$	(S) $\frac{\pi}{2}$

The correct matching is:

(a) (A–S; B–S; C–P; D–R)

(b) (A–Q; B–S; C–P; D–R)

(c) (A–S; B–Q; C–P; D–R)

(d) (A–S; B–R; C–P; D–S)

43.

List I		List II	
P.	The number of polynomials f(x) with non-negative integer coefficients of degree ≤ 2, satisfying f(0) = 0 and $\int_0^1 f(x)\,dx=1$, is	**1.**	8
Q.	The number of points in the interval $\left[-\sqrt{13},\sqrt{13}\right]$ at which $f(x)=\sin(x^2)+\cos(x^2)$ attains its maximum value is	**2.**	2
R.	$\int_{-2}^{2}\frac{3x^2}{(1+e^x)}dx$ equals	**3.**	4
S.	$\frac{\left(\int_{-\frac{1}{2}}^{\frac{1}{2}}\cos 2x\log\left(\frac{1+x}{1-x}\right)dx\right)}{\left(\int_0^{\frac{1}{2}}\cos 2x\log\left(\frac{1+x}{1-x}\right)dx\right)}$ equals	**4.**	0

(2014)

	P	Q	R	S
(a)	3	2	4	1
(b)	2	3	4	1
(c)	3	2	1	4
(d)	2	3	1	4

Using the following passage, solve Q.44 and 45

Passage – 1

Read the following passage and answer the questions.

Suppose we define definite integral using formula

$$\int_a^b f(x)\,dx = \left(\frac{b-a}{2}\right)\{f(a)+f(b)\},$$

for more acurate results for $c \in (a, b)$,

$$F(c) = \frac{c-a}{2}[f(a)-f(c)] + \frac{b-c}{2}[f(b)-f(c)]$$

When $c = \frac{a+b}{2}$

$$\int_a^b f(x)dx = \frac{b-a}{4}\{f(a)+f(b)+2f(c)\}\,dx$$ **(2006)**

44. Good approximation of $\int_0^{\pi/2} \sin x\,dx$, is

(a) $\pi/4$ (b) $\pi(\sqrt{2}+1)/4$

(c) $\pi(\sqrt{2}+1)/8$ (d) $\pi/8$

45. If $\lim_{t\to a} \frac{\int_a^t f(x)\,dx - \frac{(t-a)}{2}\{f(t)+f(a)\}}{(t-a)^3} = 0,$

then degree of polynomial function $f(x)$ at–most is

(a) 0 (b) 1

(c) 3 (d) 2

Using the following passage, solve Q.46 and 47

Passage – 2

Given that for each $a \in (0, 1)$, $\lim_{h\to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1}dt$

exists. Let this limit be g(a). In addition, it is given that the function g(a) is differentiable on (0,1). **(2014)**

46. The value of $g\left(\frac{1}{2}\right)$ is

(a) π (b) 2π

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

47. The value of $g'\left(\frac{1}{2}\right)$ is

(a) $\frac{\pi}{2}$ (b) π

(c) $-\frac{\pi}{2}$ (d) 0

Using the following passage, solve Q.48 and 49

Passage – 3

Let $F: R \to R$ be a thrice differentiable function. Suppose that F(1) = 0, F(3) = –4 and F′(x) < 0 for all $x \in \left(\frac{1}{2}, 3\right)$. Let $f(x) = xF(x)$ for all $x \in R$. **(2015)**

48. The correct statement(s) is (are).

(a) $f'(1) < 0$

(b) $f(2) < 0$

(c) $f'(x) \neq 0$ for any $x \in (1, 3)$

(d) $f'(x) = 0$ for some $x \in (1, 3)$

49. If $\int_1^3 x^2 F'(x)dx = -12$ and $\int_1^3 x^3 F''(x)dx = 40$, then the correct expression(s) is (are)

(a) $9f'(3) + f'(1) - 32 = 0$

(b) $\int_1^3 f(x)dx = 12$

(c) $9f'(3) - f'(1) + 32 = 0$

(d) $\int_1^3 f(x)dx = -12$

Using the following passage, solve Q.50 and 51

Passage – 4

Let $g_i : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to R$, $i = 1, 2$, and $f : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to R$ be functions such that $g_1(x) = 1$, $g_2(x) = |4x - \pi|$ and $f(x) = \sin^2 x$, for all $x \in \left[\frac{\pi}{8}, \frac{3\pi}{8}\right]$

Define $S_i = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} f(x).g_i(x)dx, \quad i = 1, 2$ **(2021)**

50. The value of $\frac{16S_1}{\pi}$ is --------.

51. The value of $\frac{48S_2}{\pi^2}$ is --------.

Using the following passage, solve Q.52 and 53

Passage – 5

Let $\psi_1 : [0, \infty) \to R, \psi_2 : [0, \infty) \to R, f : [0, \infty) \to R$, and $g : [0, \infty) \to R$ be functions such that $f(0) = g(0) = 0$, $\psi_1(x) = e^{-x} + x, x \ge 0$, $\psi_2(x) = x^2 - 2x - 2e^{-x} + 2$, $x \ge 0$, $f(x) = \int_{-x}^{x} \left(|t| - t^2\right)e^{-t^2}dt, x > 0$ and

$g(x) = \int_0^{x^2} \sqrt{t}e^{-t}dt, x > 0$ **(2021)**

52. Which of the following statements are TRUE?

(a) $f\left(\sqrt{\ln 3}\right) + g\left(\sqrt{\ln 3}\right) = \frac{1}{3}$

(b) For every $x > 1$, there exists an $\alpha \in (1, x)$ such that $\psi_1(x) = 1 + \alpha x$

(c) For every $x > 0$, there exists a $\beta \in (0, x)$ such that $\psi_2(x) = 2x(\psi_1(\beta) - 1)$

(d) f is an increasing function on the interval $\left[0, \frac{3}{2}\right]$

53. Which of the following statements is TRUE?

(a) $\psi_1(x) \le 1$, for all $x > 0$

(b) $\psi_2(x) \le 0$, for all $x > 0$

(c) $f(x) \ge 1 - e^{-x^2} - \frac{2}{3}x^3 + \frac{2}{5}x^5$, for all $x \in \left(0, \frac{1}{2}\right)$

(d) $g(x) \le \frac{2}{3}x^3 - \frac{2}{5}x^5 + \frac{1}{7}x^7$, for all $x \in \left(0, \frac{1}{2}\right)$

Text

54. For $x > 0$, let $f(x) = \int_1^x \frac{\ell nt}{1+t}dt$. Find the function $f(x) + f(1/x)$ and show that $f(e) + f(1/e) = 1/2$. Here, $ln\, t = \log_e t$ **(2000)**

55. Evaluate $\int_{-\pi/3}^{\pi/3} \frac{\pi + 4x^3}{2 - \cos\left(|x| + \frac{\pi}{3}\right)}dx.$ **(2004)**

56. Evaluate

$$\int_0^{\pi} e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right) + 3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x\, dx$$

(2005)

Area Under Curves

Objective Questions I [Only one correct option]

57. The area enclosed by the cuves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is **(2013)**

(a) $4\left(\sqrt{2} - 1\right)$ (b) $2\sqrt{2}\left(\sqrt{2} - 1\right)$

(c) $4\left(\sqrt{2} + 1\right)$ (d) $2\sqrt{2}\left(\sqrt{2} + 1\right)$

58. Area of the region

$\left\{(x, y) \in R^2 : y \ge \sqrt{|x + 3|}, 5y \le x + 9 \le 15\right\}$ is equal to **(2016)**

(a) $\frac{1}{6}$ (b) $\frac{4}{3}$

(c) $\frac{3}{2}$ (d) $\frac{5}{3}$

59. The area of the region $\{(x, y) : xy \le 8, 1 \le y \le x^2\}$ is **(2019)**

(a) $16\log_2 2 - 6$ (b) $8\log_e 2 - \frac{7}{3}$

(c) $16\log_e 2 - \frac{14}{3}$ (d) $8\log_e 2 - \frac{14}{3}$

60. Let the functions $f : R \to R$ and $g : R \to R$ be defined by

$$f(x) = e^{x-1} - e^{-|x-1|} \text{ and } g(x) = \frac{1}{2}(e^{x-1} + e^{1-x}).$$

Then the area of the region in the first quadrant bounded by the curves $y = f(x)$, $y = g(x)$ and $x = 0$ is **(2020)**

(a) $\left(2-\sqrt{3}\right)+\frac{1}{2}\left(e-e^{-1}\right)$ (b) $\left(2+\sqrt{3}\right)+\frac{1}{2}\left(e-e^{-1}\right)$

(c) $\left(2-\sqrt{3}\right)+\frac{1}{2}\left(e+e^{-1}\right)$ (d) $\left(2+\sqrt{3}\right)+\frac{1}{2}\left(e+e^{-1}\right)$

61. The area of the region

$$\left\{(x, y) : 0 \le x \le \frac{9}{4},\ 0 \le y \le 1,\ x \ge 3y,\ x + y \ge 2\right\} \text{ is}$$

(2021)

(a) $\frac{11}{32}$ (b) $\frac{35}{96}$

(c) $\frac{37}{96}$ (d) $\frac{13}{32}$

Objective Questions II [One or more than one correct option]

62. If S be the area of the region enclosed by $y = e^{-x^2}$, $y = 0$, $x = 0$ and $x = 1$. Then, **(2012)**

(a) $S \ge \frac{1}{e}$ (b) $S \ge 1 - \frac{1}{e}$

(c) $S \le \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$ (d) $S \le \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

63. If the line $x = \alpha$ divided the area of region

$$R = \left\{(x, y) \in R^2 : x^3 \le y \le x, 0 \le x \le 1\right\}$$

into two equal parts, then **(2017)**

(a) $0 < \alpha \le \frac{1}{2}$ (b) $2\alpha^4 - 4\alpha^2 + 1 = 0$

(c) $\alpha^4 + 4\alpha^2 - 1 = 0$ (d) $\frac{1}{2} < \alpha < 1$

Numerical Value Type Questions

64. Let $F(x) = \int_x^{x^2+\frac{\pi}{6}} 2\cos^2 t\,(dt)$ for all $x \in R$ and $f : \left[0, \frac{1}{2}\right] \to [0, \infty)$ be a continuous function. For $a \in \left[0, \frac{1}{2}\right]$, if $F'(a) + 2$ is the area of the region bounded by $x = 0$, $y = 0$, $y = f(x)$ and $x = a$, then $f(0)$ is **(2015)**

65. A farmer F_1 has a land in the shape of a triangle with vertices at P(0, 0), Q(1, 1) and R(2, 0). From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n\ (n > 1)$. If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR, then the value of n is **(2018)**

Answer Key

CHAPTER - 6 | DEFINITE INTEGRATION, AREA UNDER CURVES

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (c)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (b)
6. (c)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (c)
11. (c)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (d)
16. (b)	**17.** (c)	**18.** (a)	**19.** (c)	**20.** (a)
21. (a)	**22.** (c)	**23.** (a)	**24.** (c)	**25.** (d)
26. (d)	**27.** (b)	**28.** (b)	**29.** (b)	**30.** (d)
31. (d)	**32.** (a)	**33.** (b)	**34.** (b)	**35.** (b)
36. (d)	**37.** (c)	**38.** (a)	**39.** (c)	**40.** (b)
41. (b)	**42.** (c)	**43.** (a)	**44.** (c)	**45.** (d)
46. (d)	**47.** (a)	**48.** (c)	**49.** (a)	**50.** (c)
51. (2)	**52.** (2)	**53.** (1)	**54.** (2)	**55.** (4)
56. (3)	**57.** (3)	**58.** (2)	**59.** (50)	**60.** (4)
61. (19)	**62.** (29)	**63.** (2)	**64.** (d)	**65.** (a)
66. (b)	**67.** (d)	**68.** (b)	**69.** (b)	**70.** (a)
71. (b)	**72.** (a)	**73.** (c)	**74.** (a)	**75.** (b)
76. (b)	**77.** (c)	**78.** (d)	**79.** (a)	**80.** (c)
81. (b)	**82.** (2)	**83.** (2)	**84.** (4)	**85.** (3)
86. (6)	**87.** (1)	**88.** (6)	**89.** (3)	**90.** (2)

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. (a)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (c)
6. (b)	**7.** (a)	**8.** (a)	**9.** (a)	**10.** (c)
11. (b)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (a)
16. (d)	**17.** (d)	**18.** (b)	**19.** (a)	**20.** (c)
21. (d)	**22.** (18)	**23.** (a)	**24.** (b)	**25.** (d)
26. (d)	**27.** (a)	**28.** (c)	**29.** (a)	**30.** (b)
31. (c)	**32.** (d)	**33.** (d)	**34.** (1.50)	**35.** (1)
36. (a)	**37.** (d)	**38.** (d)	**39.** (c)	**40.** (a)
41. (b)	**42.** (21)	**43.** (a)	**44.** (b)	**45.** (a)
46. (c)	**47.** (a)	**48.** (b)	**49.** (c)	**50.** (d)
51. (d)	**52.** (d)	**53.** (c)	**54.** (d)	**55.** (d)
56. (b)	**57.** (d)	**58.** (c)	**59.** (d)	**60.** (c)
61. (c)	**62.** (1)	**63.** (5)	**64.** (b)	**65.** (b)
66. (b)	**67.** (a)	**68.** (d)	**69.** (c)	**70.** (d)
71. (d)	**72.** (d)	**73.** (b)	**74.** (a)	**75.** (5)
76. (4)	**77.** (a)	**78.** (c)	**79.** (c)	**80.** (3)
81. (d)	**82.** (b)	**83.** (d)	**84.** (b)	**85.** (a)
86. (19)	**87.** (d)	**88.** (a)	**89.** (2)	**90.** (b)
91. (1)	**92.** (406)	**93.** (1)	**94.** (16)	**95.** (c)
96. (d)	**97.** (d)	**98.** (1)	**99.** (a)	**100.** (a)
101. (1)	**102.** (512)	**103.** (c)	**104.** (8)	**105.** (b)
106. (d)	**107.** (a)	**108.** (a)	**109.** (b)	**110.** (d)
111. (d)	**112.** (b)	**113.** (c)	**114.** (d)	**115.** (c)
116. (d)	**117.** (b)	**118.** (d)	**119.** (d)	**120.** (b)
121. (c)	**122.** (b)	**123.** (b)	**124.** (b)	**125.** (b)
126. (c)	**127.** (d)	**128.** (b)	**129.** (b)	**130.** (d)
131. (b)	**132.** (b)	**133.** (d)	**134.** (c)	**135.** (b)
136. (a)	**137.** (b)	**138.** (a)	**139.** (1)	**140.** (a)
141. (d)	**142.** (b)	**143.** (2)	**144.** (d)	**145.** (114)
146. (27)	**147.** (d)	**148.** (26)	**149.** (d)	**150.** (b)
151. (64)	**152.** (4)	**153.** (b)	**154.** (41)	**155.** (b)

CHAPTER -6 | DEFINITE INTEGRATION, AREA UNDER CURVES

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (d)	**2.** (a)	**3.** (d)	**4.** (c)	**5.** (d)
6. (d)	**7.** (c)	**8.** (c)	**9.** (d)	**10.** (d)
11. (a)	**12.** (c)	**13.** (d)	**14.** (b)	**15.** (d)
16. (d)	**17.** (b)	**18.** (b)	**19.** (a)	**20.** (d)
21. (c)	**22.** (c)	**23.** (a)	**24.** (d)	**25.** (b)
26. (a)	**27.** (c)	**28.** (a)	**29.** (c)	**30.** (d)
31. (d)	**32.** (d)	**33.** (c)	**34.** (a)	**35.** (c)
36. (b)	**37.** (c)	**38.** (b)	**39.** (a)	**40.** (d)
41. (c)	**42.** (a)	**43.** (c)	**44.** (d)	**45.** (b)
46. (a,c)	**47.** (a,b,d)		**48.** (a,d)	**49.** (b, c)
50. (a,c,d)		**51.** (a,b)	**52.** (3)	**53.** (c)
54. (a)	**55.** (a)	**56.** (a)	**57.** (d)	**58.** (c)
59. (b)	**60.** π^2	**61.** $\frac{1}{2}\log 6-\frac{1}{10}$		**62.** (b)
63. (b)	**64.** (a)	**65.** (a)	**66.** (c)	**67.** (b)
68. (d)	**69.** (d)	**70.** (b)	**71.** (c)	**72.** (c)
73. (d)	**74.** (b)	**75.** (b)	**76.** (a)	**77.** (d)
78. (a)	**79.** (a)	**80.** (a)		

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (b)	**2.** (b)	**3.** (c)	**4.** (c)	**5.** (c)
6. (a)	**7.** (c)	**8.** (a)	**9.** (a)	**10.** (d)
11. (b)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (c)
16. (b)	**17.** (d)	**18.** (a)	**19.** (d)	**20.** (a)
21. (a,d)	**22.** (a,b,c)	**23.** (b,d)	**24.** (a,c,d)	
25. (a,c)	**26.** (b,c)	**27.** (a,c)	**28.** (a,b)	**29.** (a,b,d)
30. (a,b,c)		**31.** (5051)	**32.** (2)	**33.** (0)
34. (9)	**35.** (7)	**36.** (1)	**37.** (2)	**38.** (4)
39. (0.50)	**40.** (4)	**41.** (182)	**42.** (a)	**43.** (d)
44. (c)	**45.** (b)	**46.** (a)	**47.** (d)	**48.** (a,b,c)
49. (c,d)	**50.** (2)	**51.** (1.50)	**52.** (c)	**53.** (d)
54. $\frac{1}{2}(\ln x)^2$		**55.** $\frac{4\pi}{\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\right)$		
56. $\frac{24}{5}\left(e\cos\left(\frac{1}{2}\right)+\frac{e}{2}\sin\left(\frac{1}{2}\right)-1\right)$			**57.** (b)	**58.** (c)
59. (c)	**60.** (a)	**61.** (a)	**62.** (a,b,d)	**63.** (b,d)
64. (3)	**65.** (4)			