# 09 Differential Equations

9.1 Introduction
9.2 Basic Concepts
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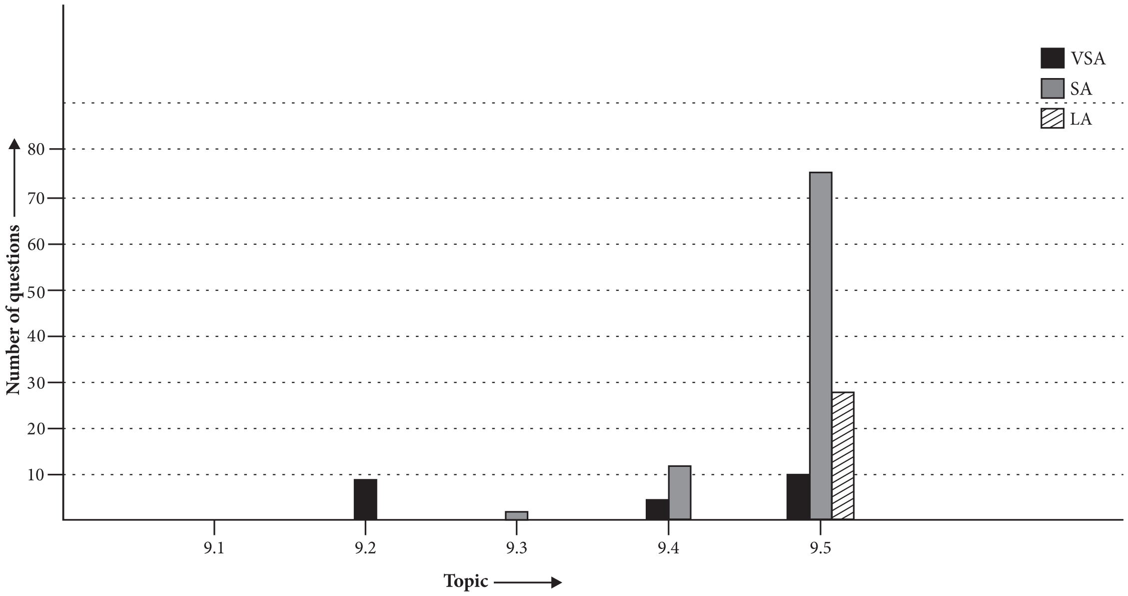
9.4 Formation of a Differential Equation whose General Solution is given
9.5 Methods of Solving First Order, First Degree Differential Equations

## Topicwise Analysis of Last 10 Years' CBSE Board Questions




- Maximum weightage is of *Methods of Solving First Order, First Degree Differential Equations*
- Maximum VSA, SA and LA type questions were asked from *Methods of Solving First Order, First Degree Differential Equations*
- No VBQ type questions were asked till now

## QUICK RECAP

### DIFFERENTIAL EQUATION

- An equation involving an independent variable, a dependent variable and the derivatives of the dependent variable is called differential equation.
  - A differential equation involving derivatives of the dependent variable with respect to only one independent variable is called an ordinary differential equation.
  - A differential equation involving derivatives with respect to more than one independent variables is called a partial differential equation.

- **Order and Degree of a Differential Equation**
  - The order of highest derivative appearing in a differential equation is called order of the differential equation.
  - The power of the highest order derivative appearing in a differential equation, after it is made free from radicals and fractions, is called degree of the differential equation.

  **Note :** Order and degree (if defined) of a differential equation are always positive integ rs.

## HOMOGENEOUS DIFFERENTIAL EQUATIONS

- A differential equation of the form

$$\frac{dy}{dx}=\frac{f(x, y)}{g(x, y)}$$

where, $f(x, y)$ and $g(x, y)$ are homogeneous functions of $x$ and $y$ of the same degree.

## LINEAR DIFFERENTIAL EQUATIONS

- An equation of the form $\frac{dy}{dx}+Py=Q$ where $P$ and $Q$ are functions of $x$ only (or constants) is called a linear differential equation of the first order.

## SOLUTION OF A DIFFERENTIAL EQUATION

- Solution of a differential equation is a function of the form $y=f(x)+C$ which satisfies the given differential equation.
- **General Solution :** The solution of a differential equation which contains a number of arbitrary constants equal to the order of the differential equation.
- **Particular Solution :** A solution obtained by giving particular values to arbitrary constants in the general solution.

## FORMATION OF A DIFFERENTIAL EQUATION

- Differential equation of the equation of a family of curves is obtained by eliminating arbitrary constants occurring in it with the help of equation of the curves and equations formed by its differentiation.

## METHODS OF SOLVING DIFFERENTIAL EQUATIONS

- **Equation in variable separable form :** If the differential equation is of the form $f(x)\,dx = g(y)\,dy$, then the variables are separable and such equations can be solved by integrating on both sides. The solution is given by $\int f(x)\,dx = \int g(y)\,dy + C$, where $C$ is an arbitrary constant.
- **Equation reducible to homogeneous form :** If the equation is of the form $\frac{dy}{dx}=\frac{f(x, y)}{g(x, y)}$, where $f(x, y)$ and $g(x, y)$ are homogeneous functions of the same degree in $x$ and $y$, then put $y = vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$ so that the dependent variable $y$ is changed to another variable $v$, then apply variable separable method.
- **Solution of Linear Differential Equation :** A differential equation of the form $\frac{dy}{dx}+Py=Q$, where $P$ and $Q$ are functions of $x$ (or constants) can be solved as :

1. Find Integrating Factor $(\text{I.F.})=e^{\int P\,dx}$
2. The solution of the differential equation is $y(\text{I.F.})=\int Q\,(\text{I.F.})\,dx+C$, wh re $C$ is constant of integ ation.

# Previous Years' CBSE Board Questions

## 9.2 Basic Concepts

**VSA (1 mark)**

**1.** Write the sum of the order and degree of the following differential equation

$$\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\}=0.$$ *(AI 2015)*

**2.** Write the sum of the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^3+x^4=0$$ *(Foreign 2015)*

**3.** Write the sum of the order and degree of the differential equation

$$1+\left(\frac{dy}{dx}\right)^4=7\left(\frac{d^2y}{dx^2}\right)^3.$$ *(Delhi 2015C)*

**4.** Write the degree of the differential equation

$$\left(\frac{dy}{dx}\right)^4+3x\frac{d^2y}{dx^2}=0.$$ *(Delhi 2013)*

**5.** Write the degree of differential equation

$$x^3\left(\frac{d^2y}{dx^2}\right)^2+x\left(\frac{dy}{dx}\right)^4=0.$$ *(Delhi 2013)*

**6.** Write the degree of the differential equation

$$x\left(\frac{d^2y}{dx^2}\right)^3+y\left(\frac{dy}{dx}\right)^4+x^3=0.$$ *(Delhi 2013)*

**7.** Write the degree of the differential equation :

$$\left(\frac{dy}{dx}\right)^4+3y\frac{d^2y}{dx^2}=0$$ *(Delhi 2013C)*

**8.** Write the degree of the differential equation

$$\left(\frac{d^2s}{dt^2}\right)^2+\left(\frac{ds}{dt}\right)^3+4=0.$$ *(AI 2013C)*

**9.** What is the degree of the following differential equation? $5x\left(\frac{dy}{dx}\right)^2-\frac{d^2y}{dx^2}-6y=\log x$

*(Delhi 2010)*

## 9.3 General and Particular Solutions of a Differential Equation

**SA (4 marks)**

**10.** Verify that $y = 3\cos(\log x) + 4\sin(\log x)$ is a solution of the differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0.$$ *(AI 2008 C)*

## 9.4 Formation of a Differential Equation whose General Solution is given

**VSA (1 mark)**

**11.** Find the differential equation representing the family of curves $v=\frac{A}{r}+B$, where $A$ and $B$ are arbitrary constants. *(Delhi 2015)*

**12.** Write the differential equation obtained by eliminating the arbitrary constant $C$ in the equation representing the family of curves $xy = C\cos x$. *(Delhi 2015C)*

**13.** Write the differential equation representing the family of curves $y = mx$, where $m$ is an arbitrary constant. *(AI 2013)*

**14.** Form the differential equation of the family of curves $y = a\cos(x + b)$, where $a$ and $b$ are arbitrary constants. *(Delhi 2007)*

**SA (4 marks)**

**15.** Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes. *(AI 2016, 2012)*

**16.** Form the differential equation of the family of parabolas having vertex at origin and axis along positive $y$-axis. *(Delhi 2011)*

**17.** Find the differential equation of the family of all circles touching the $y$-axis at the origin. *(Delhi 2010C, AI 2009)*

**18.** Form the differential equation of the family of circles touching the $x$-axis at origin. *(Delhi 2010C, AI 2009 C)*

**19.** Find the differential equation of all circles in the first quadrant which touch the coordinate axes. *(AI 2010 C)*

**20.** Form the differential equation representing the family of ellipses having foci on $x$-axis and centre at the origin. *(AI 2010C, Delhi 2009 C)*

**21.** Form the differential equation representing the family of curves given by $(x - a)^2 + 2y^2 = a^2$, where $a$ is an arbitrary constant. *(AI 2009)*

**22.** Form the differential equation of the family of curves $y = A\cos 2x + B\sin 2x$, where $A$ and $B$ are constants. *(Delhi 2007)*

## 9.5 Methods of Solving First Order, First Degree Differential Equations

**VSA (1 mark)**

**23.** Find the integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1.$ *(Delhi 2015, AI 2015C)*

**24.** Write the integrating factor of the following differential equation :
$\left(1 + y^2\right) + \left(2xy - \cot y\right)\frac{dy}{dx} = 0$ *(AI 2015)*

**25.** Write the solution of the differential equation
$\frac{dy}{dx} = 2^{-y}$ *(Foreign 2015)*

**26.** Find the solution of the differential equation $\frac{dy}{dx} = x^3 e^{-2y}.$ *(AI 2015C)*

**27.** Solve the following differential equation :
$x\cos y\, dy = (xe^x \log x + e^x)\, dx.$ *(Delhi 2007)*

**28.** Solve the following differential equation : $\tan y\, dx + \sec^2 y \tan x\, dy = 0.$ *(Delhi 2007, AI 2007)*

**29.** Solve the following differential equation :
$\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0.$ *(AI 2007)*

**30.** Solve the following differential equation :
$y(1 - x^2)\frac{dy}{dx} = x(1 + y^2).$ *(AI 2007)*

**SA (4 marks)**

**31.** Find the general solution of the following differential equation :
$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$ *(Delhi 2016)*

**32.** Find the particular solution of the differential equation $(1 - y^2)(1 + \log x)dx + 2xy\, dy = 0$, given that $y = 0$ when $x = 1$. *(Delhi 2016)*

**33.** Solve the differential equation :
$y + x\frac{dy}{dx} = x - y\frac{dy}{dx}$ *(AI 2016)*

**34.** Solve the following differential equation
$y^2 dx + (x^2 - xy + y^2)dy = 0$ *(Foreign 2016)*

**35.** Solve the following differential equation
$(\cot^{-1} y + x)dy = (1 + y^2)dx$ *(Foreign 2016)*

**36.** Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$, given that $y = \frac{\pi}{2}$, when $x = 1$. *(Delhi 2014)*

**37.** Solve the following differential equation :
$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{2}{x^2 - 1}, |x| \neq 1$ *(Delhi 2014)*

**38.** Find the particular solution of the differential equation $e^x\sqrt{1 - y^2}\,dx + \frac{y}{x}dy = 0$ given that $y = 1$ when $x = 0$. *(Delhi 2014)*

**39.** Solve the following differential equation :
$\operatorname{cosec} x \log y \frac{dy}{dx} + x^2 y^2 = 0.$ *(Delhi 2014)*

**40.** Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that $y = 0$ when $x = 1$ *(AI 2014)*

**41.** Solve the differential equation
$\left(1 + x^2\right)\frac{dy}{dx} + y = e^{\tan^{-1} x}$ *(AI 2014)*

**42.** Find the particular solution of the differential equation $x(1+y^2)\,dx - y(1+x^2)dy = 0$, given that $y = 1$ when $x = 0$. *(AI 2014)*

**43.** Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$, given that $y = 0$ when $x = 0$. *(AI 2014)*

**44.** Solve the differential equation $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$, given that $y = 1$ when $x = 1$. *(Foreign 2014)*

**45.** Solve the differential equation $\frac{dy}{dx} + y\cot x = 2\cos x$, given that $y = 0$ when $x = \frac{\pi}{2}$. *(Foreign 2014)*

**46.** Find a particular solution of the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0$, when $x = \frac{\pi}{3}$. *(Foreign 2014)*

**47.** Solve the differential equation $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$. *(Foreign 2014, Delhi 2010)*

**48.** If $y(x)$ is a solution of the differential equation $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$ and $y(0) = 1$, then find the value of $y\left(\frac{\pi}{2}\right)$. *(Delhi 2014C)*

**49.** Find the general solution of the differential equation $(x - y)\frac{dy}{dx} = x + 2y$. *(Delhi 2014C, AI 2010)*

**50.** Find the particular solution of the differential equation $x\frac{dy}{dx} - y + x\,\text{cosec}\left(\frac{y}{x}\right) = 0$; given that $y = 0$ when $x = 1$. *(AI 2014C, 2011C, Delhi 2009)*

**51.** Solve the differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x$, given $y\left(\frac{\pi}{2}\right) = 1$. *(AI 2014C)*

**52.** Solve the following differential equation : $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x;\ x \neq 0$. *(AI 2014C, Delhi 2012C)*

**53.** Find the particular solution of the following differential equation : $xy\frac{dy}{dx} = (x+2)(y+2);\ y = -1$ when $x = 1$. *(Delhi 2012)*

**54.** Solve the following differential equation : $2x^2\frac{dy}{dx} - 2xy + y^2 = 0$ *(Delhi 2012)*

**55.** Find the particular solution of the following differential equation; $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$. *(Delhi 2012)*

**56.** Find the particular solution of the following differential equation : $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$; $y = 0$ when $x = 0$. *(Delhi 2012)*

**57.** Find the particular solution of the following differential equation : $x(x^2 - 1)\frac{dy}{dx} = 1$; $y = 0$ when $x = 2$ *(AI 2012)*

**58.** Solve the following differential equation : $(1 + x^2)\,dy + 2xy\,dx = \cot x\,dx;\ x \neq 0$ *(AI 2012, 2012C, Delhi 2011C)*

**59.** Find the particular solution of the differential equation : $\frac{dy}{dx} + y\cot x = 4x\,\text{cosec}\,x$, $(x \neq 0)$, given that $y = 0$ when $x = \frac{\pi}{2}$. *(AI 2012)*

**60.** Find the particular solution of the following differential equation : $x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$, given that when $x = 2$, $y = \pi$. *(AI 2012)*

**61.** Solve the following differential equation : $\frac{dy}{dx} - y = \cos x$, given that if $x = 0$, $y = 1$. *(Delhi 2012C)*

**62.** Find the particular solution of the following differential equation, given that $x = 2$, $y = 1$ : $x\frac{dy}{dx} + 2y = x^2, (x \neq 0)$ *(Delhi 2012C)*

**63.** Find the particular solution of the differential equation :
$\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x, x \neq 0$, given that $y = 0$, when $x = \frac{\pi}{2}$. *(Delhi 2012C)*

**64.** Solve the following differential equation :
$\frac{dy}{dx} + \sec x \cdot y = \tan x, \left(0 \le x < \frac{\pi}{2}\right)$.
*(AI 2012C, Delhi 2008C)*

**65.** Solve the following differential equation :
$x\frac{dy}{dx} + y - x + xy\cot x = 0, x \neq 0$.
*(AI 2012C, Delhi 2011C)*

**66.** Solve the following differential equation :
$\cos^2 x \frac{dy}{dx} + y = \tan x$.
*(Delhi 2011, 2008, 2008C, AI 2009, 2008)*

**67.** Solve the following differential equation :
$(1 + y^2)(1 + \log x)\,dx + x\,dy = 0$. *(Delhi 2011)*

**68.** Solve the following differential equation :
$e^x \tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0$. *(Delhi 2011)*

**69.** Solve the differential equation :
$x\,dy + (y - x^3)\,dx = 0$. *(AI 2011)*

**70.** Solve the differential equation :
$x\,dy - (y + 2x^2)\,dx = 0$. *(AI 2011)*

**71.** Solve the differential equation :
$(y + 3x^2)\frac{dx}{dy} = x$. *(AI 2011)*

**72.** Solve the following differential equation :
$x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$ *(AI 2011)*

**73.** Solve the following differential equation :
$\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + x\,dy = 0$ *(Delhi 2011C)*

**74.** Find the particular solution of the differential equation satisfying the given conditions :
$\frac{dy}{dx} = y\tan x$, given that $y = 1$, when $x = 0$.
*(Delhi 2010)*

**75.** Solve the following differential equation :
$(x^2 + 1)\frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$.
*(AI 2010, 2008, Delhi 2008)*

**76.** Solve the following differential equation :
$(x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x$. *(AI 2010)*

**77.** Show that the following differential equation is homogeneous and then solve it.
$y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0$. *(AI 2010)*

**78.** Solve the following differential equation:
$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}; |x| \neq 1$ *(AI 2010)*

**79.** Solve the following differential equation :
$xy\log\left(\frac{y}{x}\right)dx + \left[y^2 - x^2\log\left(\frac{y}{x}\right)\right]dy = 0$.
*(Delhi 2010 C)*

**80.** Solve the following differential equation :
$x\log x\frac{dy}{dx} + y = 2\log x$. *(Delhi 2009)*

**81.** Solve the following differential equation :
$\frac{dy}{dx} + y = \cos x - \sin x$. *(Delhi 2009)*

**82.** Solve the following differential equation :
$(1 + x^2)\frac{dy}{dx} + y = \tan^{-1} x$. *(Delhi 2009)*

**83.** Solve : $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$. *(AI 2009)*

**84.** Solve the following differential equation :
$(x^2 - y^2)\,dx + 2xy\,dy = 0$, given that $y = 1$, when $x = 1$. *(Delhi 2008)*

**85.** Solve the following differential equation :
$\frac{dy}{dx} = \frac{x(2y - x)}{x(2y + x)}$, if $y = 1$, when $x = 1$.
*(Delhi 2008)*

**86.** Solve : $\frac{dy}{dx} + 2y\tan x = \sin x$. *(AI 2008)*

**87.** Solve the following differential equation :
$x^2\frac{dy}{dx} = 2xy + y^2$.
Given that $y = 1$, when $x = 1$ *(AI 2008)*

**88.** Solve the following differential equation :
$(1 + e^{2x})\,dy + (1 + y^2)e^x\,dx = 0$. *(AI 2008 C)*

**89.** Solve the differential equation :
$\frac{dy}{dx} + 2y = 6e^x$. *(Delhi 2007)*

**90.** Solve the following differential equation :
$x\cos y\,dy = (xe^x\log x + e^x)\,dx$. *(Delhi 2007)*

**91.** Solve the following differential equation :
$$4\frac{dy}{dx}+8y=5e^{-3x}.$$ *(AI 2007)*

## LA (6 marks)

**92.** Solve the differential equation : $(\tan^{-1}y - x)dy = (1 + y^2)dx$. *(Delhi 2015)*

**93.** Find the particular solution of the differential equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$ given that $y = 1$, when $x = 0$. *(Delhi 2015)*

**94.** Show that the differential equation $\frac{dy}{dx}=\frac{y^2}{xy-x^2}$ is homogeneous and also solve it. *(AI 2015)*

**95.** Find the particular solution of the differential equation $(\tan^{-1}y - x)\,dy = (1+y^2)dx$, given that $x = 1$ when $y = 0$. *(AI 2015)*

**96.** Solve the following differential equation :
$$\left[y-x\cos\left(\frac{y}{x}\right)\right]dy+\left[y\cos\left(\frac{y}{x}\right)-2x\sin\left(\frac{y}{x}\right)\right]dx=0$$ *(Foreign 2015)*

**97.** Solve the following differential equation :
$$\left(\sqrt{1+x^2+y^2+x^2y^2}\right)dx+xy\,dy=0$$ *(Foreign 2015, AI 2010)*

**98.** Find the particular solution of the differential equation $x\frac{dy}{dx}+y-x+xy\cot x=0;\ x\neq 0$, given that when $x=\frac{\pi}{2}, y=0$. *(Delhi 2015C)*

**99.** Solve the differential equation $x^2\,dy + (xy+y^2)dx = 0$ given $y = 1$, when $x = 1$ *(Delhi 2015C, 2013C, 2010)*

**100.** Solve the differential equation
$$\left(x\sin^2\left(\frac{y}{x}\right)-y\right)dx+xdy=0 \quad \text{given} \quad y=\frac{\pi}{4}$$
when $x = 1$ *(AI 2015C, 2014C, 2013C, 2013)*

**101.** Solve the differential equation $\frac{dy}{dx}-3y\cot x=\sin 2x$ given $y = 2$ when $x=\frac{\pi}{2}$. *(AI 2015C)*

**102.** Show that the differential equation
$$x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)+x-y\sin\left(\frac{y}{x}\right)=0$$
is homogeneous. Find the particular solution of this differential equation, given that $x = 1$ when $y=\frac{\pi}{2}$. *(Delhi 2013)*

**103.** Show that the differential equation $2ye^{x/y}\,dx + (y - 2x\,e^{x/y})\,dy = 0$ is homogeneous. Find the particular solution of this differential equation, given that $x = 0$ when $y = 1$ *(Delhi 2013, AI 2012C)*

**104.** Show that the differential equation $(x\,e^{x/y} + y)\,dx = xdy$ is homogeneous. Find the particular solution of this differential equation, given that $x = 1$ when $y = 1$ *(Delhi 2013)*

**105.** Find the particular solution of the differential equation $(\tan^{-1} y - x)\,dy = (1 + y^2)\,dx$, given that when $x = 0, y = 0$ *(AI 2013)*

**106.** Find the particular solution of the differential equation $\frac{dx}{dy}+x\cot y=2y+y^2\cot y, (y\neq 0)$, given that $x=0$ when $y=\frac{\pi}{2}$. *(AI 2013)*

**107.** Find the particular solution of the differential equation $(3xy + y^2)\,dx + (x^2 + xy)\,dy = 0$ : for $x = 1, y = 1$ *(Delhi 2013C)*

**108.** Find the particular solution of the following differential equation given that $y = 0$ when $x = 1$ : $(x^2 + xy)\,dy = (x^2 + y^2)\,dx$ *(Delhi 2013C)*

**109.** Find the particular solution of the differential equation $(x - y)\frac{dy}{dx}=x+2y$, given that when $x = 1, y = 0$ *(AI 2013C)*

**110.** Find the particular solution of the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$, given that when $x = 1$, $y=\frac{\pi}{4}$. *(AI 2013C)*

**111.** Solve the following differential equation :
$$x\cos\left(\frac{y}{x}\right)(y\,dx+x\,dy)=y\sin\left(\frac{y}{x}\right)(xdy-ydx).$$ *(AI 2013C, 2010C)*

# Detailed Solutions

**1.** The given differential equation is

$$\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\} = 0 \quad \Rightarrow \quad 3.\left(\frac{dy}{dx}\right)^2 \cdot \frac{d^2y}{dx^2} = 0$$

Order = 2 and Degree = 1

∴ Order + Degree = 2 + 1 = 3

**2.** Order = 2, Degree = 2.

∴ Order + Degree = 2 + 2 = 4

**3.** Order = 2, Degree = 3

∴ Order + Degree = 2 + 3 = 5

**4.** Degree of the given differential equation is 1.

**5.** Degree of the given differential equation is 2.

**6.** Degree of the given differential equation is 3.

**7.** The degree of the differential equation is 1.

**8.** Degree of given differential equation is 2.

**9.** Degree of the given differential equation is 1.

**10.** We have, $y = 3\cos(\log x) + 4\sin(\log x)$ ...(i)

Differentiating (i) w.r.t. $x$, we get

$$\frac{dy}{dx} = -3\sin(\log x)\frac{1}{x} + 4\cos(\log x)\frac{1}{x}$$

$$\Rightarrow \quad x\frac{dy}{dx} = -3\sin(\log x) + 4\cos(\log x) \quad ...(ii)$$

Differentiating (ii) w.r.t. $x$, we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -3\cos(\log x)\frac{1}{x} - 4\sin(\log x)\frac{1}{x}$$

$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -(3\cos(\log x) + 4\sin(\log x))$$

$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

**11.** $v = \dfrac{A}{r} + B \Rightarrow \dfrac{dv}{dr} = -\dfrac{A}{r^2} \Rightarrow \dfrac{d^2v}{dr^2} = \dfrac{2A}{r^3}$

Now, $\dfrac{d^2v}{dr^2} \div \dfrac{dv}{dr} = \dfrac{2A}{r^3} \div \left(\dfrac{-A}{r^2}\right)$

$$\Rightarrow \quad \frac{d^2v}{dr^2} \div \frac{dv}{dr} = \frac{-2}{r} \quad \Rightarrow \quad \frac{d^2v}{dr^2} = \frac{-2}{r}.\frac{dv}{dr}$$

$\Rightarrow \quad \dfrac{d^2v}{dr^2} + \dfrac{2}{r}\dfrac{dv}{dr} = 0$ is the required D.E.

**12.** Here, $xy = C\cos x$ ...(i)

Differentiating (i) w.r.t. $x$, we get

$$1 \cdot y + x \cdot \frac{dy}{dx} = -C\sin x \quad ...(ii)$$

∴ Eliminating $C$ from (i) and (ii), we get

$\dfrac{y + x\dfrac{dy}{dx}}{xy} = -\dfrac{\sin x}{\cos x} \Rightarrow x.\dfrac{dy}{dx} + y = -xy\tan x,$ is the required differential equation.

**13.** Here, $y = mx$ ...(i)

Differentiating (i) w.r.t. $x$, we get

$$\frac{dy}{dx} = m \quad ...(ii)$$

Eliminating $m$ from (i) and (ii), we get

$y = x \cdot \dfrac{dy}{dx} \quad \Rightarrow x\dfrac{dy}{dx} - y = 0,$ is the required differential equation.

**14.** Here, $y = a\cos(x + b)$ ...(i)

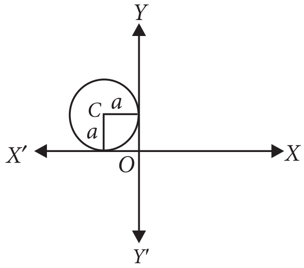
Differentiating (i) w.r.t. $x$, we get

$$\frac{dy}{dx} = -a\sin(x + b)$$

Again differentiating w.r.t. $x$, we get

$$\frac{d^2y}{dx^2} = -a\cos(x + b) \Rightarrow \frac{d^2y}{dx^2} = -y \Rightarrow \frac{d^2y}{dx^2} + y = 0.$$

**15.** The equation of the circles in II$^{nd}$ quadrant touching co-ordinate axes is




$(x + a)^2 + (y - a)^2 = a^2$ ...(i)

[Here $C$ is $(-a, a)$ and radius = $a$]

which has only one arbitrary constant $a$.

Differentiating (i) w.r.t. $x$, we get

$$2(x + a) + 2(y - a)\frac{dy}{dx} = 0 \Rightarrow a = \frac{x + yy'}{y' - 1}$$

Substituting for $a$ in (i), we get

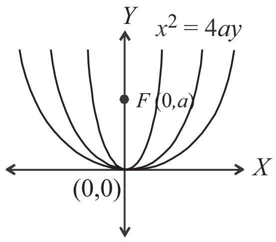
$$\left(x+\frac{x+y'y}{y'-1}\right)^2+\left(y-\frac{x+y'y}{y'-1}\right)^2=\left(\frac{x+y'y}{y'-1}\right)^2$$

$\Rightarrow [x(y'-1)+x+y'y]^2+[y(y'-1)-x-y'y]^2=(x+y'y)^2$

$\Rightarrow (x+y)^2(y')^2+(x+y)^2=(x+y'y)^2$

$\Rightarrow (x+y)^2\left[\left(\frac{dy}{dx}\right)^2+1\right]=\left(x+y\frac{dy}{dx}\right)^2$, is the required differential equation.

**16.**




Equation of parabola having vertex at origin and axis along positive $y$-axis is

$x^2 = 4ay$, where $a$ is the parameter. ...(i)

Differentiating (i) w.r.t. $x$, we get

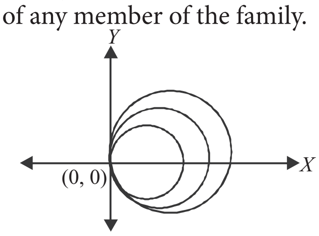
$2x = 4ay_1 \Rightarrow \frac{2x}{y_1} = 4a$ ...(ii)

Substituting the value of $4a$ from equation (ii) in equation (i), we get

$x^2=\frac{2x}{y_1}y \Rightarrow x^2y_1-2xy=0 \Rightarrow xy_1-2y=0,$

is the required differential equation.

**17.** Let $C$ denote the family of circles touching $y$-axis at the origin. Let $(a, 0)$ be the co-ordinates of the centre of any member of the family.




Therefore, equation of family $C$ is $(x-a)^2+y^2=a^2$

or $x^2+y^2=2ax$ ...(i)

where, $a$ is any arbitrary constant.

Differentiating (i) w.r.t. $x$, we get

$2x+2y\frac{dy}{dx}=2a \Rightarrow x+y\frac{dy}{dx}=a$ ...(ii)

Substituting the value of $a$ from (ii) in (i), we get

$$x^2+y^2=2x\left(x+y\frac{dy}{dx}\right)$$

$\Rightarrow x^2+y^2-2x^2=2xyy_1 \Rightarrow 2xyy_1+x^2=y^2$

which is the required differential equation.

**18.** Equation of circle touching $x$-axis at origin is

$x^2+(y-a)^2=a^2 \Rightarrow x^2+y^2-2ay=0$ ...(i)

Differentiating (i) with respect to $x$ we get

$2x+2yy'-2ay'=0 \Rightarrow a=\frac{x+yy'}{y'}$

Substituting the value of $a$ in eq. (i), we get

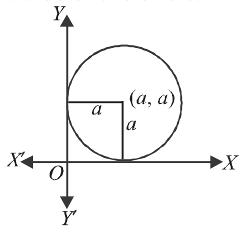
$$x^2+y^2-\frac{2(x+yy')}{y'}y=0$$

$\Rightarrow y'(x^2-y^2)=2xy \Rightarrow y'=\frac{2xy}{x^2-y^2}$

**19.** Let the equation of the family of circles which touch the coordinate axes in the first quadrant be

$(x-a)^2+(y-a)^2=a^2$ ...(i)

where $a$ is the radius of the circle.




*Refer to answer 15.*

**20.** The equation of family of ellipses is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(i)

Differentiating both sides of (i) w.r.t. $x$, we get

$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0 \Rightarrow \frac{y}{x}\left(\frac{dy}{dx}\right)=\frac{-b^2}{a^2}$ ...(ii)

Differentiating (i) both sides of (ii) w.r.t. $x$, we get

$$\left(\frac{y}{x}\right)\left(\frac{d^2y}{dx^2}\right)+\left(\frac{x\frac{dy}{dx}-y}{x^2}\right)\frac{dy}{dx}=0$$

$\Rightarrow xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$, is the required differential equation.

**21.** We have, $(x-a)^2+2y^2=a^2$

$\Rightarrow x^2+2y^2-2ax=0 \Rightarrow x^2+2y^2=2ax$ ...(i)

Differentiating (i) with respect to $x$, we get

$2x+4yy'=2a$

Substituting the value of $2a$ in (i), we get

$x^2+2y^2=(2x+4yy')x$

$\Rightarrow x^2+2y^2=2x^2+4xyy'$

$\Rightarrow 2y^2-x^2=4xyy' \Rightarrow x^2-2y^2+4xyy'=0$

**22.** We have, $y = A\cos 2x + B\sin 2x$ ...(i)

Differentiating (i) both sides w.r.t. $x$, we get

$$\frac{dy}{dx} = -2A\sin 2x + 2B\cos 2x \quad ...(ii)$$

Differentiating (ii) both sides w.r.t. $x$, we get

$$\frac{d^2y}{dx^2} = -4A\cos 2x - 4B\sin 2x$$

$$= -4(A\cos 2x + B\sin 2x)$$

$$\Rightarrow \frac{d^2y}{dx^2} + 4y = 0$$

**23.** We have, $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

or $\frac{dy}{dx} + \frac{1}{\sqrt{x}}y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{\sqrt{x}}, Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

$\therefore$ I.F. $= e^{\int P dx} \Rightarrow$ I.F. $= e^{\int \frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$

**24.** The given differential equation is

$$(1 + y^2) + (2xy - \cot y)\frac{dy}{dx} = 0$$

$$\Rightarrow (1 + y^2)\frac{dx}{dy} + 2xy - \cot y = 0$$

$$\Rightarrow \frac{dx}{dy} + \frac{2y}{1 + y^2}\cdot x = \frac{\cot y}{1 + y^2}$$

This is a linear differential equation of the form $\frac{dx}{dy} + Px = Q$, where, $P = \frac{2y}{1+y^2}$ and $Q = \frac{\cot y}{1+y^2}$

$\therefore$ I.F. $= e^{\int P\,dy} = e^{\int \frac{2y}{1+y^2}dy} = e^{\log(1+y^2)} = 1 + y^2$.

**25.** We have, $\frac{dy}{dx} = 2^{-y}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2^y} \Rightarrow 2^y\,dy = dx \quad ...(i)$$

Integrating both sides of (i), we get

$$\Rightarrow \frac{2^y}{\log 2} = x + C \Rightarrow 2^y = (C + x)\log 2$$

Taking log on both sides to the base 2, we get

$\Rightarrow \log_2 2^y = \log_2 [(C + x) \log 2]$

$\Rightarrow y = \log_2 [(C + x) \log 2]$

which is the required solution.

**26.** We have, $\frac{dy}{dx} = x^3 e^{-2y} \Rightarrow e^{2y}\,dy = x^3 dx$

On integrating, we get $\frac{e^{2y}}{2} = \frac{x^4}{4} + C'$

$\Rightarrow 2e^{2y} = x^4 + C$, where $C = 4C'$

**27.** We have, $x \cos y\, dy = (xe^x \log x + e^x)\, dx$

$$\cos y\,dy = \left(\frac{xe^x \log x + e^x}{x}\right)dx$$

$$\int \cos y\,dy = \int\left(e^x \log x + \frac{e^x}{x}\right)dx$$

$$\sin y = e^x \log x - \int \frac{1}{x}\cdot e^x dx + \int \frac{e^x}{x}dx$$

$\sin y = e^x \log x + C.$

**28.** We have, $\sec^2 y \tan x\, dy = -\tan y\, dx$

$$\frac{\sec^2 y}{\tan y}dy = -\frac{dx}{\tan x} \Rightarrow \int \frac{\sec^2 y}{\tan y}dy = -\int \cot x\,dx$$

[Put $\tan y = t \Rightarrow \sec^2 y\, dy = dt$]

$$\int \frac{dt}{t} = -\log|\sin x| + \log C$$

$\Rightarrow \log|t| = -\log|\sin x| + \log C$

$\Rightarrow \log|\tan y| + \log|\sin x| = \log C$

$\Rightarrow \log|\tan y \cdot \sin x| = \log C$

$\Rightarrow \tan y \sin x = C$

**29.** We have, $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$\sec^2 y \tan x\, dy = -\sec^2 x \tan y\, dx$

$$\Rightarrow \int \frac{\sec^2 y}{\tan y}dy = -\int \frac{\sec^2 x}{\tan x}dx$$

$\Rightarrow \log|\tan y| = -\log|\tan x| + \log C$

$\Rightarrow \log|\tan x| + \log|\tan y| = \log C$

$\Rightarrow \log|\tan x \tan y| = \log C$

$\therefore \tan x \tan y = C.$

**30.** We have, $y(1 - x^2)\frac{dy}{dx} = x(1 + y^2)$

$$\Rightarrow \frac{y}{(1+y^2)}dy = \frac{x}{1-x^2}dx \Rightarrow \frac{2y}{(1+y^2)}dy = \frac{2x}{1-x^2}dx$$

Integrating both sides, we get

$\log(1+y^2) = -\log(1-x^2) + \log C$

$\Rightarrow \log(1+y^2) + \log(1-x^2) = \log C$

$\Rightarrow \log(1-x^2)(1+y^2) = \log C$

$\Rightarrow (1-x^2)(1+y^2) = C$

**31.** We have, $(1+y^2)+(x-e^{\tan^{-1}y})\dfrac{dy}{dx}=0$

$\Rightarrow (x-e^{\tan^{-1}y})\dfrac{dy}{dx}=-(1+y^2)$

$\Rightarrow \dfrac{dx}{dy}=\dfrac{x-e^{\tan^{-1}y}}{-(1+y^2)} \Rightarrow \dfrac{dx}{dy}+\dfrac{1}{1+y^2}\cdot x=\dfrac{e^{\tan^{-1}y}}{1+y^2}$

This is a linear differential equation of the form $\dfrac{dx}{dy}+Px=Q$, where $P=\dfrac{1}{1+y^2}$ and $Q=\dfrac{e^{\tan^{-1}y}}{1+y^2}$

$\therefore$ I.F. $= e^{\int\frac{dy}{1+y^2}} = e^{\tan^{-1}y}$

$\therefore$ Solution is

$$x\cdot e^{\tan^{-1}y}=\int\frac{(e^{\tan^{-1}y})^2}{1+y^2}dy+C$$

$$=\int\frac{e^{2\tan^{-1}y}}{1+y^2}dy+C$$

$\Rightarrow x\cdot e^{\tan^{-1}y}=\dfrac{e^{2\tan^{-1}y}}{2}+C_1$

$\Rightarrow x=\dfrac{e^{\tan^{-1}y}}{2}+C_1e^{-\tan^{-1}y}$

**32.** We have, $(1-y^2)(1+\log x)\,dx+2xy\,dy=0$

$\Rightarrow (1-y^2)(1+\log x)\,dx=-2xy\,dy$

$\Rightarrow \dfrac{(1+\log x)}{x}dx=-\dfrac{2y}{1-y^2}dy$

On integrating both sides, we get

$\dfrac{(1+\log x)^2}{2}=\log|1-y^2|+C$

When $x=1$, $y=0$

$\therefore \dfrac{(1+\log 1)^2}{2}=\log(1)+C \Rightarrow C=\dfrac{1}{2}$

$\Rightarrow \dfrac{(1+\log x)^2}{2}=\log|1-y^2|+\dfrac{1}{2}$

$\Rightarrow (1+\log x)^2 = 2\log|1-y^2|+1$ is the required solution.

**33.** We have, $y+x\dfrac{dy}{dx}=x-y\dfrac{dy}{dx}$

$\Rightarrow x\dfrac{dy}{dx}+y\dfrac{dy}{dx}=x-y \Rightarrow \dfrac{dy}{dx}=\dfrac{x-y}{x+y}$ ...(i)

This is a linear homogeneous D.E.

$\therefore$ Put $y=vx \Rightarrow \dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$

$\therefore$ Equation (i) becomes

$v+x\dfrac{dv}{dx}=\dfrac{x-vx}{x+vx}=\dfrac{1-v}{1+v}$

$\Rightarrow x\dfrac{dv}{dx}=\dfrac{1-v}{1+v}-v=\dfrac{1-v-v^2-v}{1+v}=\dfrac{1-2v-v^2}{1+v}$

$\Rightarrow \dfrac{(1+v)}{v^2+2v-1}dv=-\dfrac{dx}{x}$

Integrating both sides, we get

$\dfrac{1}{2}\log|v^2+2v-1|=-\log|x|+\log C$

$\Rightarrow \dfrac{1}{2}\log|v^2+2v-1|+\log|x|=\log C$

$\Rightarrow \dfrac{1}{2}\log\left|\dfrac{y^2}{x^2}+\dfrac{2y}{x}-1\right|+\log|x|=\log C$

$\Rightarrow \log\left|\dfrac{y^2}{x^2}+\dfrac{2y}{x}-1\right|+2\log|x|=2\log C$

$\Rightarrow \log\left|\dfrac{y^2+2xy-x^2}{x^2}\times x^2\right|=\log C^2$

$\Rightarrow y^2+2xy-x^2=\pm C^2$

$\Rightarrow y^2+2xy-x^2=C_1$ (where $C_1=\pm C^2$)

**34.** We have, $y^2\,dx+(x^2-xy+y^2)\,dy=0$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{-y^2}{x^2-xy+y^2}$

This is homogeneous differential equation.

$\therefore$ Put $y=vx \Rightarrow \dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$, we get

$v+x\dfrac{dv}{dx}=\dfrac{-v^2x^2}{x^2-vx^2+v^2x^2}$

$\Rightarrow v+x\dfrac{dv}{dx}=\dfrac{-v^2}{1-v+v^2}$

$\Rightarrow \quad x\dfrac{dv}{dx} = \dfrac{-v^2}{1-v+v^2} - v$

$\Rightarrow \quad x\dfrac{dv}{dx} = \dfrac{-v-v^3}{1-v+v^2}$

$\Rightarrow \quad \dfrac{1-v+v^2}{v(1+v^2)}dv = -\dfrac{1}{x}dx$

Integrating both sides, we get

$\int \dfrac{1+v^2}{v(1+v^2)}dv - \int \dfrac{v}{v(1+v^2)}dv = -\int \dfrac{1}{x}dx$

$\Rightarrow \quad \int \dfrac{1}{v}dv - \int \dfrac{1}{1+v^2}dv = -\int \dfrac{1}{x}dx$

$\Rightarrow \quad \log|v| - \tan^{-1}v = -\log|x| + \log C$

$\Rightarrow \quad \log\left|\dfrac{vx}{C}\right| = \tan^{-1}v \qquad \Rightarrow \quad \left|\dfrac{vx}{C}\right| = e^{\tan^{-1}v}$

$\Rightarrow \quad |y| = Ce^{\tan^{-1}y/x}$ is the required solution.

**35.** We have, $(\cot^{-1}y + x)\, dy = (1 + y^2)dx$

$\Rightarrow \quad \dfrac{dx}{dy} = \dfrac{\cot^{-1}y + x}{1+y^2}$

$\Rightarrow \quad \dfrac{dx}{dy} + \left(-\dfrac{1}{1+y^2}\right)x = \dfrac{\cot^{-1}y}{1+y^2}$

This is a linear differential equation of the form $\dfrac{dx}{dy} + Px = Q$, where, $P = -\dfrac{1}{1+y^2}$ and $Q = \dfrac{\cot^{-1}y}{1+y^2}$

$\therefore \quad \text{I.F.} = e^{-\int \frac{1}{1+y^2}dy} = e^{\cot^{-1}y}$

$\therefore$ Solution is,

$xe^{\cot^{-1}y} = \int \dfrac{\cot^{-1}y}{(1+y^2)} e^{\cot^{-1}y}dy + C$

[Put $t = \cot^{-1}y \Rightarrow dt = -\dfrac{1}{1+y^2}dy$]

$xe^{\cot^{-1}y} = -\int te^t dt + C$

$\Rightarrow \quad xe^{\cot^{-1}y} = -e^t(t-1) + C$

$\Rightarrow \quad xe^{\cot^{-1}y} = e^{\cot^{-1}y}(1-\cot^{-1}y) + C$

**36.** We have, $\dfrac{dy}{dx} = \dfrac{x(2\log x + 1)}{\sin y + y\cos y}$

$\Rightarrow \quad (\sin y + y\cos y)dy = x(2\log x + 1)dx$

On integrating both sides, we get

$\Rightarrow \quad -\cos y + y\sin y - (-\cos y)$

$= 2\left[\log x \times \dfrac{x^2}{2} - \int \dfrac{1}{x} \times \dfrac{x^2}{2}dx\right] + \dfrac{x^2}{2} + C$

$\Rightarrow \quad y\sin y = x^2\log x - \dfrac{x^2}{2} + \dfrac{x^2}{2} + C$

$\Rightarrow \quad y\sin y = x^2\log x + C$

when $x = 1$, $y = \dfrac{\pi}{2}$

$\therefore \quad \dfrac{\pi}{2}\sin\dfrac{\pi}{2} = 1\cdot\log(1) + C \Rightarrow \dfrac{\pi}{2} = C$

$\therefore \quad y\sin y = x^2\log x + \pi/2$ is the required solution.

**37.** We have, $(x^2-1)\dfrac{dy}{dx} + 2xy = \dfrac{2}{x^2-1}, |x| \neq 1$

$\Rightarrow \quad \dfrac{dy}{dx} + \dfrac{2xy}{x^2-1} = \dfrac{2}{(x^2-1)^2}$

This is a linear differential equation of the form, $\dfrac{dy}{dx} + Py = Q$, where $P = \dfrac{2x}{x^2-1}$ and $Q = \dfrac{2}{(x^2-1)^2}$

$\therefore \quad \text{I.F.} = e^{\int Pdx} = e^{\int \frac{2x}{x^2-1}dx} = e^{\log(x^2-1)} = x^2-1$

Hence, solution of differential equation is given by

$y(x^2-1) = \int \dfrac{2(x^2-1)}{(x^2-1)^2}dx + C$

$\Rightarrow \quad y(x^2-1) = 2\int \dfrac{dx}{x^2-1} + C$

$\Rightarrow \quad y(x^2-1) = 2 \times \dfrac{1}{2}\log\left|\dfrac{x-1}{x+1}\right| + C$

$\Rightarrow \quad y(x^2-1) = \log\left|\dfrac{x-1}{x+1}\right| + C$

**38.** We have, $e^x\sqrt{1-y^2}\, dx + \dfrac{y}{x}dy = 0$

$x\, e^x dx + \dfrac{y}{\sqrt{1-y^2}}dy = 0$

Integrating both sides, we get

$\Rightarrow x \cdot e^x - \int 1.e^x dx - \dfrac{1}{2}\int \left(1-y^2\right)^{-\frac{1}{2}}(-2y)dy = C$

$$\Rightarrow xe^x - e^x - \frac{1}{2}\frac{(1-y^2)^{\frac{1}{2}}}{1/2} = C$$

$$\Rightarrow e^x(x-1) - \sqrt{1-y^2} = C$$

When $x = 0, y = 1, e^0(0-1) - \sqrt{1-1} = C \Rightarrow C = -1$

$\therefore\ e^x(x-1) - \sqrt{1-y^2} = -1$ is the required solution.

**39.** We have, $\operatorname{cosec} x \log y \dfrac{dy}{dx} + x^2y^2 = 0$

$$\frac{\log y}{y^2}dy + \frac{x^2}{\operatorname{cosec} x}dx = 0$$

Integrating both sides, we get

$$\int \frac{\log y}{y^2}dy + \int x^2 \sin x\, dx = 0$$

[Put $\log y = t \Rightarrow \dfrac{1}{y}dy = dt$ and $y = e^t$]

$$\Rightarrow \int t.e^{-t}dt + \int x^2 \sin x\, dx = C$$

$$\Rightarrow t.\frac{e^{-t}}{-1} - \int 1.\frac{e^{-t}}{-1}dt + x^2(-\cos x) - \int 2x(-\cos x)dx = C$$

$$\Rightarrow -te^{-t} - e^{-t} - x^2\cos x + 2x\sin x - 2\int 1.\sin x\, dx = C$$

$$\Rightarrow -\frac{1+\log y}{y} - x^2\cos x + 2x\sin x + 2\cos x = C$$

is the required solution.

**40.** We have, $\dfrac{dy}{dx} = 1 + x + y + xy$

$$\Rightarrow \frac{dy}{dx} = (1+x) + (1+x)y = (1+x)(1+y)$$

$$\Rightarrow \frac{dy}{1+y} = (1+x)dx$$

Integrating both sides, we get

$$\int \frac{dy}{1+y} = \int (1+x)dx + C$$

$$\Rightarrow \log(1+y) = x + \frac{x^2}{2} + C \qquad \text{...(i)}$$

When $x = 1, y = 0$

$\therefore\ \log 1 = 1 + \dfrac{1}{2} + C \Rightarrow C = -\dfrac{3}{2}$

$\therefore$ The particular solution of (i) is

$$\log(1+y) = x + \frac{x^2}{2} - \frac{3}{2}.$$

**41.** *Refer to answer 31.*

**42.** We have, $x(1 + y^2)\,dx - y(1 + x^2)\,dy = 0$

$$\Rightarrow \frac{x}{1+x^2}dx - \frac{y}{1+y^2}dy = 0$$

$$\Rightarrow \frac{2x}{1+x^2}dx = \frac{2y}{1+y^2}dy$$

Integrating both sides, we get

$\log(1 + y^2) = \log(1+ x^2) + \log C$

$\Rightarrow\ 1 + y^2 = C(1+ x^2)$

When $x = 0, y = 1\ \therefore\ 1 + 1 = C(1 + 0) \Rightarrow C = 2$

$\therefore\ 1 + y^2 = 2(1 + x^2)$ is the required particular solution.

**43.** We have, $\log\left(\dfrac{dy}{dx}\right) = 3x + 4y$

$$\frac{dy}{dx} = e^{3x+4y} = e^{3x}\cdot e^{4y} \Rightarrow e^{-4y}dy = e^{3x}dx$$

Integrating both sides, we get

$$\int e^{3x}dx - \int e^{-4y}dy = C \Rightarrow \frac{e^{3x}}{3} - \frac{e^{-4y}}{-4} = C$$

When $x = 0, y = 0$

$\therefore\ \dfrac{1}{3} + \dfrac{1}{4} = C \Rightarrow C = \dfrac{7}{12}$

$$\Rightarrow \frac{e^{3x}}{3} + \frac{e^{4y}}{4} = \frac{7}{12}$$

$\Rightarrow\ 4e^{3x} + 3e^{4y} = 7$ is the required particular solution.

**44.** We have, $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$

$\Rightarrow\ x^2(1 - y)dy + y^2(1 + x^2)dx = 0$

$$\Rightarrow \int \frac{(1-y)}{y^2}dy + \int \frac{(1+x^2)}{x^2}dx = 0$$

$$\Rightarrow \int\left(\frac{1}{y^2} - \frac{1}{y}\right)dy + \int\left(\frac{1}{x^2} + 1\right)dx = 0$$

$$\Rightarrow -\frac{1}{y} - \log|y| - \frac{1}{x} + x = C$$

$\Rightarrow\ -x - xy\log|y| - y + x^2y = C(xy) \qquad$ ...(i)

when $x = 1, y = 1$

$\therefore\ -(1) - (1)(1)\log|1| - (1) + (1)^2(1) = C(1)$

$\Rightarrow C = -1$

Equation (i) becomes

$x^2y = x + xy\log|y| + y - xy$

**45.** We have, $\frac{dy}{dx} + y\cot x = 2\cos x$

This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = \cot x$, $Q = 2\cos x$

$\therefore$ I.F. $= e^{\int \cot x\,dx} = e^{\log|\sin x|} = |\sin x|$

$\therefore$ $y|\sin x| = \int |\sin x|(2\cos x)dx$

$\Rightarrow y|\sin x| = \int \sin 2x dx$

$\Rightarrow y(\sin x) = -\frac{1}{2}\cos 2x + C$

when $x = \frac{\pi}{2}, y = 0$

$0(\sin\frac{\pi}{2}) = -\frac{1}{2}\cos 2\left(\frac{\pi}{2}\right) + C \Rightarrow C = -\frac{1}{2}$

$\therefore$ $y(\sin x) = -\frac{1}{2}\cos 2x - \frac{1}{2}$

*i.e.*, $2y \sin x + \cos 2x + 1 = 0$ is the required solution.

**46.** We have $\frac{dy}{dx} + 2y\tan x = \sin x$

It is linear differential equation of the form

$\frac{dy}{dx} + Py = Q$

where $P = 2\tan x$, and $Q = \sin x$

Now, I.F. $= e^{\int 2\tan x\; dx} = e^{2\log|\sec x|} = |\sec^2 x|$

$\therefore$ $y(\sec^2 x) = \int(\sec^2 x)(\sin x)dx$

$\Rightarrow y(\sec^2 x) = \int \sec x\tan x\,dx$

$\Rightarrow y(\sec^2 x) = \sec x + C$

when $x = \frac{\pi}{3}, y = 0$

$(0)[\sec^2(\pi/3)] = \sec(\pi/3) + C \Rightarrow C = -2$

$\therefore$ $y(\sec^2 x) = \sec x - 2$ *i.e.*, $y = \cos x - 2\cos^2 x$ is the required solution.

**47.** We have, $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$

$\Rightarrow \frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x^2}$

It is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{x\log x}, Q = \frac{2}{x^2}$

$\therefore$ I.F. $= e^{\int \frac{1}{x\log x}dx} = e^{\log\log x} = \log x$

$\therefore$ $y(\log x) = \int(\log x)\frac{2}{x^2}dx$

$\Rightarrow y(\log x) = \log x\int\frac{2}{x^2}dx - \int\left(\frac{d}{dx}(\log x)\int\frac{2}{x^2}dx\right)dx$

$\Rightarrow y(\log x) = \log x\left(-\frac{2}{x}\right) + \int\frac{2}{x^2}dx$

$\Rightarrow y(\log x) = \log x\left(-\frac{2}{x}\right) - \frac{2}{x} + C$

**48.** We have, $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$

$\Rightarrow \frac{dy}{1+y} = -\frac{\cos x}{2+\sin x}dx$

Integrating both sides, we get

$\log(y + 1) = -\log(2 + \sin x) + \log C$

$\Rightarrow \log(y+1) = \log\frac{C}{2+\sin x}$

$\Rightarrow y + 1 = C/(2 + \sin x) \Rightarrow (y + 1)(2 + \sin x) = C$

Given : $y(0) = 1 \Leftrightarrow x = 0, y = 1$

$\therefore$ $(1 + 1).(2 + \sin 0) = C \Rightarrow C = 4$

$(y + 1)(2 + \sin x) = 4$

$\Rightarrow y = \frac{4}{2+\sin x} - 1$ ...(i)

Put $x = \frac{\pi}{2}$ in it (i), $y\left(\frac{\pi}{2}\right) = \frac{4}{2+1} - 1 = \frac{1}{3}$.

**49.** We have, $(x - y)\frac{dy}{dx} = x + 2y$

$\Rightarrow \frac{dy}{dx} = \frac{x+2y}{x-y}$ ...(i)

This is a linear homogeneous differential equation.

$\therefore$ Put $y = vx \Rightarrow \frac{dy}{dx} = v.1 + x\frac{dv}{dx}$

$\therefore$ Equation (i) becomes

$v + x\frac{dv}{dx} = \frac{x+2vx}{x-vx} = \frac{1+2v}{1-v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+2v}{1-v} - v = \frac{1+v+v^2}{1-v}$

$\Rightarrow \frac{1-v}{1+v+v^2}dv = \frac{dx}{x}$

Integrating both sides, we get

$$\int \frac{-\frac{1}{2}(2v+1)+\frac{3}{2}}{v^2+v+1}dv = \log x + C$$

$$\Rightarrow -\frac{1}{2}\int\frac{2v+1}{v^2+v+1}dv + \frac{3}{2}\int\frac{dv}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$=\log x + C$$

$$\Rightarrow -\frac{1}{2}\log\left(v^2+v+1\right) + \frac{3}{2}\cdot\frac{1}{\sqrt{3}/2}\tan^{-1}\left[\frac{v+\frac{1}{2}}{\sqrt{3}/2}\right] = \log x + C$$

$$\Rightarrow -\frac{1}{2}\log\left(\frac{y^2}{x^2}+\frac{y}{x}+1\right) + \sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}\cdot x}\right) = \log x + C$$

$$\Rightarrow -\frac{1}{2}\log(y^2+xy+x^2) + \sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3}\cdot x}\right) = C.$$

**50.** We have, $x\frac{dy}{dx} - y + x\,\text{cosec}\left(\frac{y}{x}\right) = 0$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = -\text{cosec}\left(\frac{y}{x}\right) \quad \text{...(i)}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v.1 + x.\frac{dv}{dx}$

$\therefore$ Equation (i) becomes

$$v + x\frac{dv}{dx} - v = -\text{cosec}\,v \Rightarrow x\frac{dv}{dx} = -\text{cosec}\,v$$

$$\Rightarrow -\sin v\,dv = \frac{dx}{x}$$

Integrating both sides, we get $\cos v = \log x + C$

$$\Rightarrow \cos\left(\frac{y}{x}\right) = \log x + C$$

When $x = 1, y = 0$

$$\Rightarrow \cos\left(\frac{0}{1}\right) = \log 1 + C \Rightarrow C = 1$$

$$\therefore \cos\left(\frac{y}{x}\right) = \log x + 1$$

is the required particular solution.

**51.** We have, $x\frac{dy}{dx} + y = x\cos x + \sin x$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x}.y = \frac{x\cos x + \sin x}{x}$$

This is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{x}$, $Q = \frac{x\cos x + \sin x}{x}$

$$\therefore \quad \text{I.F.} = e^{\int P\,dx} = e^{\int\frac{1}{x}dx} = e^{\log x} = x.$$

$$\therefore \quad y\cdot x = \int\frac{x\cos x + \sin x}{x}\cdot x\,dx + C$$

$$\Rightarrow xy = \int x\cos x\,dx + \int \sin x\,dx + C$$

$$= x\cdot\sin x - \int 1\cdot\sin x\,dx + \int \sin x dx + C$$

$$= x\sin x + C$$

Given $y\left(\frac{\pi}{2}\right) = 1$

$$\therefore \quad \frac{\pi}{2}.1 = \frac{\pi}{2}\sin\frac{\pi}{2} + C \Rightarrow C = 0$$

$\therefore$ $xy = x\sin x \Rightarrow y = \sin x$ is the required solution.

**52.** We have, $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x; x \neq 0$

$$\Rightarrow \cos\left(\frac{y}{x}\right).\frac{dy}{dx} = \left(\frac{y}{x}\right)\cos\left(\frac{y}{x}\right) + 1 \quad \text{...(i)}$$

This is a linear homogeneous differential equation

Put $y = vx \Rightarrow \frac{dy}{dx} = v\cdot 1 + x\cdot\frac{dv}{dx}$

Now (i) becomes

$$\cos v.\left[v + x\frac{dv}{dx}\right] = v\cos v + 1$$

$$\Rightarrow x\cos v\frac{dv}{dx} = 1 \Rightarrow \cos v\,dv = \frac{dx}{x}$$

Integrating both sides, we get

$\sin v = \log x + C \quad \Rightarrow \sin\left(\frac{y}{x}\right) = \log x + C$

is the required solution.

**53.** We have, $xy\frac{dy}{dx} = (x+2)(y+2)$

$$\Rightarrow \frac{y\,dy}{(y+2)} = \left(\frac{x+2}{x}\right)dx$$

$$\Rightarrow dy - \frac{2}{(y+2)}dy = dx + \frac{2}{x}dx$$

Integrating both sides, we get

$y - 2\log(y + 2) = x + 2\log x + C$

when $x = 1, y = -1$

So, $-1 - 2\log(-1 + 2) = 1 + 2\log 1 + C$

$\Rightarrow \quad C = -1 - 1 = -2$

So, we have $y - 2\log(y + 2) = x + 2\log x - 2$

$\Rightarrow \quad y - x + 2 = 2\log x(y + 2)$.

**54.** We have, $2x^2 \dfrac{dy}{dx} - 2xy + y^2 = 0$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} + \frac{y^2}{2x^2} = 0 \quad \text{...(i)}$$

This is a homogeneous linear differential equation

Put $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$\therefore$ (i) becomes

$$v + x\frac{dv}{dx} - v + \frac{v^2}{2} = 0 \Rightarrow x\frac{dv}{dx} = \frac{-v^2}{2}$$

$$\Rightarrow \frac{dv}{-v^2} = \frac{1}{2}\frac{dx}{x}$$

Integrating both sides, we get

$$\frac{1}{v} = \frac{1}{2}\log x + C_1 \Rightarrow \frac{x}{y} = \frac{1}{2}\log x + C_1$$

$$\Rightarrow \frac{2x}{y} = \log x + C. \text{ [where } C = 2C_1]$$

**55.** We have, $\dfrac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$

$$\therefore \frac{dy}{dx} = 1 + x^2 + y^2(1 + x^2)$$

$$= (1 + x^2) . (1 + y^2)$$

$$\Rightarrow \frac{dy}{1 + y^2} = (1 + x^2)dx$$

Integrating both sides, we get

$$\tan^{-1} y = x + \frac{x^3}{3} + C$$

when $x = 0, y = 1$

$$\tan^{-1} 1 = 0 + 0 + C \Rightarrow C = \frac{\pi}{4}$$

$$\therefore \tan^{-1} y = x + \frac{1}{3}x^3 + \frac{\pi}{4}$$

is the required particular solution.

**56.** We have, $(x + 1)\dfrac{dy}{dx} = 2e^{-y} - 1 \quad \text{...(i)}$

$$\Rightarrow \frac{dy}{2e^{-y} - 1} = \frac{dx}{x + 1} \qquad \Rightarrow \frac{e^y}{2 - e^y}dy = \frac{dx}{x + 1}$$

Integrating both sides, we get

$-\log(2 - e^y) = \log(x + 1) + C \quad \text{...(ii)}$

(Taking $2 - e^y = t$ on L.H.S.)

when $x = 0, y = 0$

$\therefore \quad -\log(2 - 1) = \log(0 + 1) + C \quad \Rightarrow C = 0$

$\therefore$ Eq. (ii) becomes

$-\log(2 - e^y) = \log(x + 1)$

$\Rightarrow \quad \log(x + 1)\log(2 - e^y) = 0$

$\Rightarrow \quad \log(x + 1)(2 - e^y) = 0 \quad \Rightarrow (x + 1)(2 - e^y) = 1$

is the required particular solution.

**57.** We have, $x(x^2 - 1)\dfrac{dy}{dx} = 1 \quad \text{...(i)}$

$$dy = \frac{1}{x(x^2 - 1)}dx$$

Integrating both sides, we get

$$\int dy = \int \frac{1}{x(x - 1)(x + 1)}dx + C$$

$$\Rightarrow y = \int\left[-\frac{1}{x} + \frac{1}{2}.\frac{1}{x - 1} + \frac{1}{2}.\frac{1}{x + 1}\right]dx + C$$

$$= -\log x + \frac{1}{2}\log(x - 1) + \frac{1}{2}\log(x + 1) + C \quad \text{...(ii)}$$

when $x = 2, y = 0$

$$\therefore 0 = -\log 2 + \frac{1}{2}\log 1 + \frac{1}{2}\log 3 + C$$

$$\Rightarrow C = \log 2 - \frac{1}{2}\log 3 = \frac{1}{2}\log\frac{4}{3}$$

$\therefore$ From (ii),

$$y = -\log x + \frac{1}{2}\log(x - 1) + \frac{1}{2}\log(x + 1) + \frac{1}{2}\log\frac{4}{3}$$

$= \dfrac{1}{2}\log\dfrac{4(x^2 - 1)}{3x^2}$ is the required particular solution

**58.** We have, $(1 + x^2)\,dy + 2xy\,dx = \cot x\,dx;\ x \neq 0$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1 + x^2}.y = \frac{\cot x}{1 + x^2} \quad \text{...(i)}$$

This is a linear differential equation of the form $\dfrac{dy}{dx} + Py = Q$, where $P = \dfrac{2x}{1 + x^2}, Q = \dfrac{\cot x}{1 + x^2}$

$$\therefore \quad \text{I.F.} = e^{\int P dx} = e^{\int \frac{2x}{1 + x^2}dx} = e^{\log(1 + x^2)} = 1 + x^2$$

$\therefore$ The solution of (i) is

$$y \cdot (1 + x^2) = \int \frac{\cot x}{1 + x^2} \cdot (1 + x^2)dx + C$$

$= \int \cot x\,dx + C = \log|\sin x| + C$

$\Rightarrow y = \dfrac{\log|\sin x| + C}{1+x^2}$ is the solution of the given differential equation.

**59.** We have $\dfrac{dy}{dx} + y\cot x = 4x \operatorname{cosec} x (x \neq 0)$ ...(i)

This is a linear differential equation of the form $\dfrac{dy}{dx} + Py = Q$,

where, $P = \cot x$, $Q = 4x \operatorname{cosec} x$

$\therefore$ I.F. $= e^{\int P\,dx} = e^{\int \cot x\,dx} = |\sin x|$

$\therefore$ The solution of (i) is

$y \cdot |\sin x| = \int 4x \operatorname{cosec} x \cdot |\sin x|\,dx + C = \int 4x\,dx + C$

$= 2x^2 + C$ ...(ii)

when $x = \dfrac{\pi}{2}, y = 0$

$\therefore$ From (ii), $0.\sin\dfrac{\pi}{2} = 2\cdot\left(\dfrac{\pi}{2}\right)^2 + C \Rightarrow C = -\dfrac{\pi^2}{2}$

$\Rightarrow y\cdot\sin x = 2x^2 - \dfrac{\pi^2}{2}$ is the required particular solution.

**60.** We have $x\dfrac{dy}{dx} - y + x\sin\left(\dfrac{y}{x}\right) = 0$

$\Rightarrow \dfrac{dy}{dx} - \dfrac{y}{x} + \sin\left(\dfrac{y}{x}\right) = 0$

This is a linear homogeneous differential equation.

Put $y = vx \Rightarrow \dfrac{dy}{dx} = v\cdot 1 + x\dfrac{dv}{dx}$

$\therefore v + x\dfrac{dv}{dx} - v + \sin v = 0$

$\Rightarrow x\dfrac{dv}{dx} + \sin v = 0 \Rightarrow \operatorname{cosec} v\,dv + \dfrac{dx}{x} = 0$

Integrating both sides, we get

$\log|\operatorname{cosec} v - \cot v| + \log x = \log C$

$\Rightarrow x(\operatorname{cosec} v - \cot v) = C$

$\Rightarrow x\left[\operatorname{cosec}\left(\dfrac{y}{x}\right) - \cot\left(\dfrac{y}{x}\right)\right] = C$

when $x = 2, y = \pi$

$\therefore 2\left[\operatorname{cosec}\dfrac{\pi}{2} - \cot\dfrac{\pi}{2}\right] = C \Rightarrow C = 2$

$\Rightarrow x\left[\operatorname{cosec}\left(\dfrac{y}{x}\right) - \cot\left(\dfrac{y}{x}\right)\right] = 2$

is the required particular solution.

**61.** We have, $\dfrac{dy}{dx} - y = \cos x$ ... (i)

This is a linear differential equation of the form $\dfrac{dy}{dx} + Py = Q$, where $P = -1$, $Q = \cos x$

$\therefore$ I.F. $= e^{\int P dx} = e^{-x}$

Hence the solution of (i) is

$y.e^{-x} = \int e^{-x}.\cos x\,dx + C$

$\int e^{-x}\cos x\,dx = -e^{-x}\cos x + e^{-x}\sin x - \int e^{-x}\cos x\,dx$

$= \dfrac{1}{2}e^{-x}(\sin x - \cos x)$

Thus the required solution of (i) is

$y.e^{-x} = \dfrac{1}{2}e^{-x}(\sin x - \cos x) + C$ ...(ii)

Putting, $x = 0$, $y = 1$ in (ii), we get

$1\cdot e^0 = \dfrac{1}{2}e^0(\sin 0 - \cos 0) + C$

$\Rightarrow 1 = \dfrac{1}{2}(-1) + C \Rightarrow C = \dfrac{3}{2}$

From (ii), $y = \dfrac{1}{2}(\sin x - \cos x) + \dfrac{3}{2}e^x$

is the required particular solution.

**62.** We have, $x\dfrac{dy}{dx} + 2y = x^2, (x \neq 0)$

$\Rightarrow \dfrac{dy}{dx} + \dfrac{2}{x}.y = x$ ...(i)

This is linear differential equation of the form $\dfrac{dy}{dx} + Py = Q$, where $P = \dfrac{2}{x}, Q = x$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int 2/x\,dx} = e^{2\log x} = e^{\log x^2} = x^2$

$\therefore$ The solution of (i) is

$y\cdot x^2 = \int x\cdot x^2 dx + C = \int x^3\,dx + C = \dfrac{x^4}{4} + C$

$\Rightarrow y = \dfrac{x^2}{4} + C\,x^{-2}$ ... (ii)

When $x = 2$, $y = 1$ in (ii), we get

$\therefore 1 = \dfrac{2^2}{4} + \dfrac{C}{2^2} = 1 + \dfrac{C}{4} \Rightarrow \dfrac{C}{4} = 0 \Rightarrow C = 0$

$y = \dfrac{x^2}{4}$ is the required particular solution.

**63.** We have, $\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x, (x \neq 0)$

This is a linear D.E of the form $\frac{dy}{dx} + Py = Q$

where $P = \cot x$, $Q = 2x + x^2 \cot x$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int \cot x \, dx} = e^{\log|\sin x|} = \sin x$

$\Rightarrow \quad y \cdot \sin x = \int (2x + x^2 \cdot \cot x)\sin x \, dx + C$

$= 2\int x \sin x \, dx + \int x^2 \cos x \, dx + C$

$= 2\int x \sin x \, dx + x^2 \sin x - \int 2x \sin x \, dx + C$

$= x^2 \sin x + C$

When $x = \frac{\pi}{2}, y = 0$

$\therefore \quad 0 \cdot \sin\frac{\pi}{2} = \left(\frac{\pi}{2}\right)^2 \cdot \sin\frac{\pi}{2} + C$

$\Rightarrow 0 = \frac{\pi^2}{4} + C \quad \Rightarrow C = -\frac{\pi^2}{4}$

$\Rightarrow \quad y \cdot \sin x = x^2 \sin x - \frac{\pi^2}{4}$

$\Rightarrow y = x^2 - \frac{\pi^2}{4}\operatorname{cosec} x$

**64.** We have, $\frac{dy}{dx} + \sec x . y = \tan x, \left(0 \le x < \frac{\pi}{2}\right)$ ... (i)

This is a linear D.E. of the form $\frac{dy}{dx} + Py = Q$,
where $P = \sec x$, $Q = \tan x$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int \sec x \, dx}$

$= e^{\log|\sec x + \tan x|} = |\sec x + \tan x|$

$\therefore$ The solution of (i) is

$y \cdot |\sec x + \tan x|$

$= \int |\sec x + \tan x| \cdot \tan x \, dx + C$

$= \int |\sec x \tan x + \tan^2 x| dx + C$

$= \int |\sec x \tan x| dx + \int |\sec^2 x - 1| \, dx + C$

$= \sec x + \tan x - x + C$

**65.** We have, $x\frac{dy}{dx} + y - x + xy\cot x = 0, (x \neq 0)$

$\Rightarrow x\frac{dy}{dx} + (1 + x\cot x).\, y = x$

$\Rightarrow \frac{dy}{dx} + \frac{1 + x\cot x}{x}.y = 1$ ...(i)

This is linear D.E. of the form $\frac{dy}{dx} + Py = Q$

where $P = \frac{1 + x\cot x}{x} = \frac{1}{x} + \cot x, Q = 1$

$\therefore$ Now I.F. $= e^{\int P dx} = e^{\log(x \sin x)} = x \sin x$

$\therefore$ The solution. of (i) is

$y \cdot x\sin x = \int 1 \cdot x \sin x \, dx + C$

$= x(-\cos x) + \int 1 \cdot \cos x \, dx + C$

$\Rightarrow \quad x\, y\sin x = -x\cos x + \sin x + C$

**66.** We have, $\cos^2 x \cdot \frac{dy}{dx} + y = \tan x$

Dividing by $\cos^2 x$ on both sides, we get

$\frac{dy}{dx} + \frac{y}{\cos^2 x} = \frac{\tan x}{\cos^2 x}$

This is a linear D.E. of the form

$\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{\cos^2 x} = \sec^2 x, Q = \frac{\tan x}{\cos^2 x}$

I.F. $= e^{\int P dx} = e^{\int \sec^2 x . dx} = e^{\tan x}$

$y.e^{\tan x} = \int e^{\tan x} \times \tan x \times \sec^2 x . dx$

Put $\tan x = t \Rightarrow \sec^2 x \cdot dx = dt$

$y \times e^{\tan x} = \int e^t \times t \cdot dt$

$\Rightarrow \quad ye^{\tan x} = t\int e^t \, dt - \int\left[\frac{d}{dt}(t)\int e^t \, dt\right] \cdot dt$

$\Rightarrow \quad y\, e^{\tan x} = t \times e^t - e^t + C$

$\Rightarrow \quad y\, e^{\tan x} = \tan x\, e^{\tan x} - e^{\tan x} + C$

$\Rightarrow \quad y\, e^{\tan x} = e^{\tan x}[\tan x - 1] + C$

$\Rightarrow \quad y = (\tan x - 1) + Ce^{-\tan x}$

**67.** We have, $(1 + y^2)(1 + \log x)dx + xdy = 0$

$\Rightarrow \quad (1 + y^2)(1 + \log x)dx = -xdy$

$\Rightarrow \quad \left(\frac{1 + \log x}{x}\right)dx = \frac{-dy}{1 + y^2}$

Integrating both sides, we get

$\frac{(1 + \log x)^2}{2} = -\tan^{-1} y + C$

$\Rightarrow \quad \frac{(1 + \log x)^2}{2} + \tan^{-1} y = C$

which is the required general solution.

**68.** We have, $e^x \tan y dx + (1 - e^x) \sec^2 y\, dy = 0$

$\Rightarrow$ $e^x \tan y dx = -(1 - e^x) \sec^2 y\, dy$

$$\Rightarrow \frac{e^x}{1-e^x}dx = \frac{-\sec^2 y}{\tan y}dy \quad ...(1)$$

Integrating both sides, we get

$-\log|1-e^x| = -\log|\tan y| - \log C$

$\Rightarrow$ $1 - e^x = C\tan y$

**69.** We have, $xdy + (y - x^3)dx = 0$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = x^2$$

This is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{x}$, $Q = x^2$

$\therefore$ I.F. $= e^{\int \frac{1}{x}dx} = e^{\log x} = x.$

$\therefore$ The solution of differential equation is

$$y \cdot x = \int x^2 \cdot x\, dx + C$$

$$\Rightarrow yx = \frac{x^4}{4} + C \Rightarrow y = \frac{x^3}{4} + \frac{C}{x}.$$

**70.** We have, $xdy - (y + 2x^2)dx = 0$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = 2x.$$

This is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$, where $P = \frac{-1}{x}$, $Q = 2x$

$\therefore$ I.F. $= e^{-\int \frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$

$\therefore$ The solution of differential equation is

$$y\left(\frac{1}{x}\right) = \int 2x \frac{1}{x}\, dx + C$$

$$\Rightarrow \frac{y}{x} = 2x + C \Rightarrow y = 2x^2 + Cx$$

**71.** *Refer to answer 70.*

**72.** We have, $x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$

$$\Rightarrow x\frac{dy}{dx} = y + \sqrt{x^2 + y^2} \Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2} \quad ...(i)$$

This is a linear homogeneous differential equation.

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ Eq. (i) becomes

$$v + x\frac{dv}{dx} = v + \sqrt{1+v^2} \Rightarrow x\frac{dv}{dx} = \sqrt{1+v^2}$$

$$\Rightarrow \frac{dx}{x} = \frac{dv}{\sqrt{1+v^2}} \Rightarrow \int \frac{dx}{x} = \int \frac{dv}{\sqrt{1+v^2}}$$

$$\Rightarrow \log x + \log C_1 = \log|v + \sqrt{1+v^2}|$$

$$\Rightarrow \log x + \log C_1 = \log\left|\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}\right|$$

$$\Rightarrow \log C_1 x = \log|y + \sqrt{x^2 + y^2}| - \log x$$

$$\Rightarrow \pm C_1 x^2 = y + \sqrt{x^2 + y^2}$$

$$\Rightarrow Cx^2 = y + \sqrt{x^2 + y^2} \quad [\text{where } C = \pm C_1]$$

which is the required general solution.

**73.** We have, $[x\sin^2\left(\frac{y}{x}\right) - y]dx + x\,dy = 0$

$$\Rightarrow \sin^2\left(\frac{y}{x}\right) - \frac{y}{x} + \frac{dy}{dx} = 0 \quad ...(i)$$

This is a linear homogeneous differential equation

$\therefore$ Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (i) becomes

$$\sin^2 v - v + v + x\frac{dv}{dx} = 0$$

$$\Rightarrow x\frac{dv}{dx} + \sin^2 v = 0 \Rightarrow \operatorname{cosec}^2 v dv + \frac{dx}{x} = 0$$

Integrating both sides, we get

$-\cot v + \log x = C \Rightarrow -\cot\left(\frac{y}{x}\right) + \log x = C$

is the required solution.

**74.** We have, $\frac{dy}{dx} = y\tan x \Rightarrow \frac{dy}{y} = \tan x\, dx$

Integrating both sides, we get

$$\int \frac{dy}{y} = \int \tan x\, dx \Rightarrow \log y = \log|\sec x| + C$$

When $x = 0, y = 1 \Rightarrow \log 1 = \log(\sec 0) + C$

$\Rightarrow$ $C = 0$

$\therefore$ $\log y = \log|\sec x| \Rightarrow y = \sec x.$

which is required particular solution.

**75.** We have, $(x^2+1)\dfrac{dy}{dx}+2xy=\sqrt{x^2+4}$

$$\Rightarrow \quad \frac{dy}{dx}+\frac{2xy}{x^2+1}=\frac{\sqrt{x^2+4}}{x^2+1}$$

This is linear differential equation of the form $\dfrac{dy}{dx}+Py=Q$, where $P=\dfrac{2x}{x^2+1}$ and $Q=\dfrac{\sqrt{x^2+4}}{x^2+1}$

$\therefore$ I.F. $=e^{\int\frac{2x}{x^2+1}dx}=e^{\log|x^2+1|}=x^2+1$

The solution of the given differential equation is

$$y\cdot(x^2+1)=\int\frac{\sqrt{x^2+4}}{x^2+1}(x^2+1)\,dx+C$$

$$\Rightarrow \quad (x^2+1)y=\frac{x}{2}\sqrt{x^2+4}+2\log\left|x+\sqrt{x^2+4}\right|+C$$

**76.** We have, $(x^3+x^2+x+1)\dfrac{dy}{dx}=2x^2+x$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{2x^2+x}{x^3+x^2+x+1}$$

$$\Rightarrow \quad dy=\frac{2x^2+x}{x^2(x+1)+1(x+1)}dx \qquad ...(i)$$

Integrating (i) both sides, we get

$$\int dy=\int\frac{2x^2+x}{(x^2+1)(x+1)}dx$$

$$\Rightarrow \quad y=\int\frac{2x^2+x}{(x^2+1)(x+1)}dx$$

Now, let $\dfrac{2x^2+x}{(x^2+1)(x+1)}=\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+1}$

$\Rightarrow \quad 2x^2+x=A(x^2+1)+(Bx+C)(x+1)$

Comparing coefficients of $x$, we get

$1=B+C$ ...(ii)

Comparing constant terms, we get

$0=A+C\Rightarrow C=-A$ ...(iii)

Comparing coefficients of $x^2$, we get $2=A+B$... (iv)

Now solving (ii) & (iii), we get $-A+B=1$ ...(v)

Solving (iv) & (v), we get $2B=3 \Rightarrow B=3/2$

Substituting the value of $B$ in (2) & (5), we get

$A=1/2$ and $C=-1/2$

$$\therefore \quad y=\int\left[\frac{\frac{1}{2}}{x+1}+\frac{\frac{3}{2}x-\frac{1}{2}}{x^2+1}\right]dx$$

$$\Rightarrow \quad y=\frac{1}{2}\log(x+1)+\frac{3}{4}\int\frac{2x}{x^2+1}dx-\frac{1}{2}\int\frac{dx}{x^2+1}$$

$$\Rightarrow \quad y=\frac{1}{2}\log(x+1)+\frac{3}{4}\log(x^2+1)-\frac{1}{2}\tan^{-1}x+C$$

which is the required general solution.

**77.** We have, $y\,dx+x\log\left(\dfrac{y}{x}\right)dy-2x\,dy=0$

$$\Rightarrow \quad \frac{y}{x}+\log\left(\frac{y}{x}\right)\frac{dy}{dx}-2\frac{dy}{dx}=0$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{y/x}{2-\log(y/x)} \qquad ...(1)$$

This is a homogeneous differential equation.

$\therefore$ Put $y=vx \Rightarrow \dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$, then (1) becomes

$$v+x\frac{dv}{dx}=\frac{v}{2-\log v}$$

$$\Rightarrow \quad x\frac{dv}{dx}=\frac{v}{2-\log v}-v \Rightarrow \frac{(2-\log v)dv}{v\log v-v}=\frac{dx}{x}$$

$$\Rightarrow \quad \frac{1-(\log v-1)}{v(\log v-1)}dv=\frac{dx}{x}$$

Integrating both sides, we get

$$\int\left(\frac{1}{v(\log v-1)}-\frac{1}{v}\right)dv=\int\frac{dx}{x}$$

$$\Rightarrow \quad \log|\log v-1|-\log|v|=\log|x|+C_1$$

$$\Rightarrow \quad \log\left|\frac{\log v-1}{v\,x}\right|=C_1 \quad \Rightarrow \quad \left|\frac{\log v-1}{v\,x}\right|=e^{C_1}$$

$$\Rightarrow \quad \frac{\log v-1}{vx}=\pm e^{C_1}=C(\text{say})$$

$$\Rightarrow \quad \frac{\log\frac{y}{x}-1}{y}=C \Rightarrow \log\frac{y}{x}-1=Cy$$

$$\Rightarrow \quad \log\frac{y}{x}=1+Cy$$

which is the required general solution.

**78.** We have, $\frac{dy}{dx}+\frac{2xy}{x^2-1}=\frac{1}{(x^2-1)^2}$

This is a linear differential equation of the form $\frac{dy}{dx}+Py=Q$, where $P=\frac{2x}{x^2-1}, Q=\frac{1}{(x^2-1)^2}$

$\text{I.F.}=e^{\int P dx}=e^{\int\frac{2x}{x^2-1}dx}=e^{\log|x^2-1|}=x^2-1$

Required solution is

$y(x^2-1)=\int\frac{1}{(x^2-1)^2}.(x^2-1)dx+C$

$y(x^2-1)=\int\frac{dx}{x^2-1}+C \Rightarrow y(x^2-1)=\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+C$

$\therefore \quad y=\frac{1}{2(x^2-1)}\log\left|\frac{x-1}{x+1}\right|+\frac{C}{x^2-1}$

**79.** We have,

$xy\log\left(\frac{y}{x}\right)dx+\left[y^2-x^2\log\left(\frac{y}{x}\right)\right]dy=0$

$\Rightarrow \quad \frac{dy}{dx}=\frac{xy\log\left(\frac{y}{x}\right)}{x^2\log\left(\frac{y}{x}\right)-y^2}.$

This is a homogeneous linear differential equation.

Put $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore \quad v+x\frac{dv}{dx}=\frac{x\cdot vx\log v}{x^2\log v-v^2x^2}=\frac{v\log v}{\log v-v^2}$

$\Rightarrow \quad x\frac{dv}{dx}=\frac{v\log v}{\log v-v^2}-v=\frac{v^3}{\log v-v^2}$

$\Rightarrow \quad \frac{\log v-v^2}{v^3}dv=\frac{dx}{x}$

$\Rightarrow \quad v^{-3}\log v dv-\frac{1}{v}dv=\frac{dx}{x}$

Integrating both sides, we get

$\log v\cdot\frac{v^{-2}}{-2}-\int\frac{1}{v}\cdot\frac{v^{-2}}{-2}dv-\log v=\log x+C$

$\Rightarrow \quad -\frac{\log v}{2v^2}+\frac{1}{2}\int v^{-3}dv-\log v=\log x+C$

$\Rightarrow \quad -\frac{\log v}{2v^2}-\frac{1}{4v^2}-\log v=\log x+C$

$\Rightarrow \quad \frac{-1}{v^2}\left[\frac{\log v}{2}+\frac{1}{4}\right]-\log vx=C$

$\Rightarrow \quad \frac{x^2}{y^2}\left[\frac{\log\frac{y}{x}}{2}+\frac{1}{4}\right]+\log y=-C$

$\Rightarrow \quad x^2\left[2\log\left(\frac{y}{x}\right)+1\right]+4y^2\log y=-4Cy^2$

$\Rightarrow \quad x^2\left[2\log\left(\frac{y}{x}\right)+1\right]+4y^2(\log y+C)=0$

**80.** We have, $x\log x\frac{dy}{dx}+y=2\log x$

$\Rightarrow \quad \frac{dy}{dx}+\frac{y}{x\log x}=\frac{2\log x}{x\log x} \Rightarrow \frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x}$

This is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P=\frac{1}{x\log x}, Q=\frac{2}{x}$

*Refer to answer 47.*

**81.** We have, $\frac{dy}{dx}+y=\cos x-\sin x$, which is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P=1$, $Q=\cos x-\sin x$

$\therefore \quad \text{I.F.}=e^{\int dx}=e^x$

The solution of given differential equation is

$ye^x=\int e^x(\cos x-\sin x)dx+C$

$\Rightarrow \quad ye^x=e^x\cos x+C \Rightarrow y=\cos x+Ce^{-x}$

**82.** We have $(1+x^2)\frac{dy}{dx}+y=\tan^{-1}x$

$\Rightarrow \quad \frac{dy}{dx}+\frac{y}{1+x^2}=\frac{\tan^{-1}x}{(1+x^2)}.$

This is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P=\frac{1}{1+x^2}, Q=\frac{\tan^{-1}x}{1+x^2}$

$\therefore \quad \text{I.F.}=e^{\int\frac{1}{1+x^2}dx}=e^{\tan^{-1}x}$

Solution of given differential equation is

$y\cdot e^{\tan^{-1}x}=\int\frac{\tan^{-1}x}{1+x^2}e^{\tan^{-1}x}dx+C$

$\Rightarrow \quad ye^t=\int te^t dt+C \qquad$ [Putting $\tan^{-1}x=t$]

$\Rightarrow \quad ye^t=e^t(t-1)+C \Rightarrow y=(t-1)+Ce^{-t}$

$\Rightarrow \quad y=(\tan^{-1}x-1)+Ce^{-\tan^{-1}x}$

**83.** We have, $x\dfrac{dy}{dx} = y - x\tan\left(\dfrac{y}{x}\right)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{y - x\tan\dfrac{y}{x}}{x}$

This is a homogeneous differential equation.

Put $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$\therefore \quad v + x\dfrac{dv}{dx} = \dfrac{vx - x\tan v}{x} = v - \tan v$

$\Rightarrow \quad \dfrac{xdv}{dx} = -\tan v \Rightarrow \cot v\, dv = -\dfrac{1}{x}\, dx$

Integrating both sides, we get

$\log|\sin v| = -\log x + \log C$

$\Rightarrow \log\left|\sin\dfrac{y}{x}\right| + \log x = \log C$

$\Rightarrow \log\left(x\sin\dfrac{y}{x}\right) = \log C \Rightarrow x\sin\dfrac{y}{x} = C$

**84.** We have, $(x^2 - y^2)dx + 2xy\, dy = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{y^2 - x^2}{2xy} = \dfrac{\left(\dfrac{y}{x}\right)^2 - 1}{2\left(\dfrac{y}{x}\right)}$ ...(i)

This is a homogeneous differential equation.

Put $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$\therefore$ (i) becomes

$v + x\dfrac{dv}{dx} = \dfrac{v^2 - 1}{2v} \Rightarrow x\dfrac{dv}{dx} = \dfrac{v^2 - 1}{2v} - v$

$\Rightarrow x\dfrac{dv}{dx} = -\dfrac{1 + v^2}{2v} \Rightarrow \dfrac{2vdv}{v^2 + 1} = -\dfrac{dx}{x}$

Integrating both sides, we get

$\log|v^2 + 1| = -\log|x| + C_1 \Rightarrow \log|(v^2 + 1)x| = C_1$

$\Rightarrow \log\left|\dfrac{y^2 + x^2}{x}\right| = C_1 \Rightarrow \dfrac{x^2 + y^2}{x} = \pm e^{C_1} = C$ (say)

$\Rightarrow x^2 + y^2 = Cx$

When $x = 1, y = 1 \Rightarrow 1 + 1 = C \Rightarrow C = 2 \Rightarrow x^2 + y^2 = 2x$ which is the required solution.

**85.** We have, $\dfrac{dy}{dx} = \dfrac{x(2y - x)}{x(2y + x)} \Rightarrow \dfrac{dy}{dx} = \dfrac{2y - x}{2y + x}$

This is a linear homogeneous differential equation

$\therefore$ Put $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$\therefore \quad v + x\dfrac{dv}{dx} = \dfrac{2vx - x}{2vx + x} \Rightarrow x\dfrac{dv}{dx} = \dfrac{2v - 1}{2v + 1} - v$

$\Rightarrow \quad x\dfrac{dv}{dx} = \dfrac{2v - 1 - 2v^2 - v}{2v + 1} = \dfrac{v - 1 - 2v^2}{2v + 1}$

$\Rightarrow \quad \dfrac{2v + 1}{2v^2 - v + 1}dv = -\dfrac{dx}{x}$

Integrating both sides, we get

$\dfrac{1}{2}\displaystyle\int \dfrac{4v + 2}{2v^2 - v + 1}dv = -\int\dfrac{dx}{x}$

$\Rightarrow \quad \dfrac{1}{2}\displaystyle\int\dfrac{4v - 1}{2v^2 - v + 1}\,dv + \dfrac{3}{2}\int\dfrac{dv}{2v^2 - v + 1} = -\log x + C$

$\Rightarrow \quad \dfrac{1}{2}\log|2v^2 - v + 1| + \dfrac{3}{4}\displaystyle\int\dfrac{dv}{v^2 - \dfrac{v}{2} + \dfrac{1}{16} + \dfrac{1}{2} - \dfrac{1}{16}} = -\log x + C$

$\Rightarrow \quad \dfrac{1}{2}\log|2v^2 - v + 1| + \dfrac{3}{4}\displaystyle\int\dfrac{dv}{\left(v - \dfrac{1}{4}\right)^2 + \dfrac{7}{16}} = -\log x + C$

$\Rightarrow \quad \dfrac{1}{2}\log|2v^2 - v + 1| + \dfrac{3}{\sqrt{7}}\tan^{-1}\left(\dfrac{4v - 1}{\sqrt{7}}\right) = -\log x + C$

$\Rightarrow \quad \dfrac{1}{2}\log\left|\dfrac{2y^2 - xy + x^2}{x^2}\right| + \dfrac{3}{\sqrt{7}}\tan^{-1}\left(\dfrac{4y - x}{\sqrt{7}x}\right) = -\log x + C$

$\Rightarrow \quad \dfrac{1}{2}\log\left|\dfrac{2y^2 - xy + x^2}{x^2}\right| + \log x + \dfrac{3}{\sqrt{7}}\tan^{-1}\left(\dfrac{4y - x}{\sqrt{7}\,x}\right) = C$

$\Rightarrow \quad \dfrac{1}{2}\log\left(\dfrac{2y^2 - xy + x^2}{x^2}\right)(x^2) + \dfrac{3}{\sqrt{7}}\tan^{-1}\left(\dfrac{4y - x}{\sqrt{7}x}\right) = C$

$\Rightarrow \quad \dfrac{1}{2}\log\left|2y^2 - xy + x^2\right| + \dfrac{3}{\sqrt{7}}\tan^{-1}\left(\dfrac{4y - x}{\sqrt{7}x}\right) = C$

When $x = 1, y = 1$

$\therefore \dfrac{1}{2}\log 2 + \dfrac{3}{\sqrt{7}}\tan^{-1}\dfrac{3}{\sqrt{7}} = C$

$\therefore$ The solution of the given equation is

$\dfrac{1}{2}\log|2y^2 - xy + x^2| + \dfrac{3}{\sqrt{7}}\tan^{-1}\left(\dfrac{4y - x}{\sqrt{7}x}\right) = \dfrac{1}{2}\log 2 + \dfrac{3}{\sqrt{7}}\tan^{-1}\dfrac{3}{\sqrt{7}}$

**86.** *Refer to answer 46.*

**87.** We have, $x^2\frac{dy}{dx} = 2xy + y^2 \Rightarrow \frac{dy}{dx} = \frac{2xy+y^2}{x^2}$

This is a homogeneous differential equation.

Put, $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{2x\cdot vx + v^2x^2}{x^2} = 2v + v^2$

$\Rightarrow \quad x\frac{dv}{dx} = v^2 + v \Rightarrow \frac{dv}{v(v+1)} = \frac{dx}{x} \Rightarrow \frac{dv}{v} - \frac{dv}{v+1} = \frac{dx}{x}$

Integrating both sides, we get

$\log v - \log(v + 1) = \log x + \log C$

$\Rightarrow \quad \log\left(\frac{v}{v+1}\right) = \log Cx \Rightarrow \frac{y}{x+y} = Cx$

$\Rightarrow \quad y = Cx(x + y).$

**88.** We have, $(1 + e^{2x})dy + (1 + y^2)e^x\,dx = 0$

$\Rightarrow \quad \frac{dy}{1+y^2} + \frac{e^x}{1+e^{2x}}dx = 0$

$\Rightarrow \quad \frac{dy}{1+y^2} + \frac{dt}{1+t^2} = 0$ [putting $e^x = t \Rightarrow e^x dx = dt$]

Integrating both sides, we get

$\tan^{-1}y + \tan^{-1}t = C \Rightarrow \tan^{-1}y + \tan^{-1}e^x = C$

**89.** We have, $\frac{dy}{dx} + 2y = 6e^x$ which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = 2$, $Q = 6e^x$

$\therefore \quad$ I.F. $= e^{\int 2dx} = e^{2x}$

$\therefore \quad$ The solution of given differential equation is

$ye^{2x} = \int 6e^x \cdot e^{2x}dx + C$

$\Rightarrow \quad ye^{2x} = \int 6e^{3x}dx + C \quad \Rightarrow ye^{2x} = \frac{6e^{3x}}{3} + C$

$\Rightarrow y = 2e^x + Ce^{-2x}$

**90.** We have, $x\cos y\,dy = (xe^x \log x + e^x)dx$

$\Rightarrow \quad \cos y\,dy = \left(e^x \log x + \frac{e^x}{x}\right)dx$

$\Rightarrow \quad \int \cos y\,dy = \int e^x\left(\log x + \frac{1}{x}\right)dx$

$\Rightarrow \quad \sin y = e^x \log x + C$

**91.** We have, $4\frac{dy}{dx} + 8y = 5e^{-3x} \Rightarrow \frac{dy}{dx} + 2y = \frac{5}{4}e^{-3x}$

which a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = 2$, $Q = \frac{5}{4}e^{-3x}$

$\therefore \quad$ I.F. $= e^{\int 2dx} = e^{2x}$

The solution of differential equation is

$ye^{2x} = \int \frac{5}{4}e^{-3x}e^{2x}dx + C \qquad \Rightarrow ye^{2x} = \frac{5}{4}\int e^{-x}dx + C$

$\Rightarrow \quad ye^{2x} = \frac{-5}{4}e^{-x} + C \Rightarrow y = \frac{-5}{4}e^{-3x} + Ce^{-2x}$

**92.** We have, $(\tan^{-1}y - x)dy = (1 + y^2)dx$

$\Rightarrow \quad \frac{dx}{dy} = \frac{\tan^{-1}y - x}{1+y^2}$

$\Rightarrow \quad \frac{dx}{dy} + \frac{1}{1+y^2}\cdot x = \frac{\tan^{-1}y}{1+y^2}$

This is a linear differential equation of the form $\frac{dx}{dy} + Px = Q$, where $P = \frac{1}{1+y^2}$ and $Q = \frac{\tan^{-1}y}{1+y^2}$

*Refer to answer 82.*

**93.** We have, $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$

This is a homogeneous linear differential equation

$\therefore \quad$ Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \quad v + x\frac{dv}{dx} = \frac{x\cdot vx}{x^2+v^2x^2}$

$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{v}{1+v^2} \quad \Rightarrow \quad x\frac{dv}{dx} = \frac{v}{1+v^2} - v$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{-v^3}{1+v^2} \quad \Rightarrow \quad \frac{dx}{x} = -\left(\frac{1+v^2}{v^3}\right)dv$

Integrating both sides, we get

$\int \frac{dx}{x} = -\int v^{-3}dv - \int \frac{1}{v}dv$

$\Rightarrow \log x = \frac{1}{2v^2} - \log v + C$

$\Rightarrow \log x = \frac{x^2}{2y^2} - \log y + \log x + C$

$\Rightarrow \log y = \frac{x^2}{2y^2} + C$

When $y = 1$, $x = 0 \Rightarrow \log 1 = 0 + C \Rightarrow C = 0$

$\therefore \quad$ Particular solution is $y = e^{\frac{x^2}{2y^2}}$

**94.** We have, $\frac{dy}{dx}=\frac{y^2}{xy-x^2}=\frac{y^2/x^2}{(xy-x^2)/x^2}$ ... (i)

$\Rightarrow$ It is a homogeneous differential equation

$\therefore$ Put $y = vx \Rightarrow \frac{dy}{dx}=v\cdot 1+x\frac{dv}{dx}$

$\therefore$ Equation (i) becomes

$v+x\frac{dv}{dx}=\frac{v^2}{v-1} \Rightarrow x\frac{dv}{dx}=\frac{v^2}{v-1}-v=\frac{v}{v-1}$

$\Rightarrow \frac{v-1}{v}dv=\frac{dx}{x} \Rightarrow \left(1-\frac{1}{v}\right)dv=\frac{dx}{x}$

Integrating, we get

$v - \log v = \log x + C \Rightarrow v = \log vx + C$

$\Rightarrow \frac{y}{x}=\log y+C$

$\Rightarrow y = x(\log y + C)$ is the required solution.

**95.** *Refer to answer 92.*

We get the solution as

$x = \tan^{-1} y - 1 + Ce^{-\tan^{-1}y}$ ...(i)

Now, putting $x = 1$, $y = 0$ in (i), we get

$1 = \tan^{-1} 0 - 1 + Ce^{-\tan^{-1} 0} \Rightarrow C = 2$

So, required particular solution is

$x = \tan^{-1} y - 1 + 2e - \tan^{-1} y.$

**96.** We have,

$\left[y-x\cos\left(\frac{y}{x}\right)\right]dy+\left[y\cos\left(\frac{y}{x}\right)-2x\sin\left(\frac{y}{x}\right)\right]dx=0$

$\Rightarrow \frac{dy}{dx}=\frac{2x\sin\left(\frac{y}{x}\right)-y\cos\left(\frac{y}{x}\right)}{y-x\cos\left(\frac{y}{x}\right)}$

$\Rightarrow \frac{dy}{dx}=\frac{2\sin\left(\frac{y}{x}\right)-\frac{y}{x}\cos\left(\frac{y}{x}\right)}{\frac{y}{x}-\cos\left(\frac{y}{x}\right)}$ ...(i)

This is a homogeneous differential equation.

$\therefore$ Put $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore$ Equation (i) becomes,

$v+x\frac{dv}{dx}=\frac{2\sin v-v\cos v}{v-\cos v} \Rightarrow x\frac{dv}{dx}=\frac{2\sin v-v\cos v}{v-\cos v}-v$

$\Rightarrow x\frac{dv}{dx}=\frac{2\sin v-v^2}{v-\cos v} \Rightarrow \frac{v-\cos v}{2\sin v-v^2}dv=\frac{dx}{x}$

$\Rightarrow \frac{-1}{2}\frac{(2\cos v-2v)}{2\sin v-v^2}dv=\frac{dx}{x}$

Integrating both sides, we get

$\frac{-1}{2}\log(2\sin v-v^2)=\log x+C_1$

$\Rightarrow \log x^2+2C_1+\log\left(2\sin\frac{y}{x}-\frac{y^2}{x^2}\right)=0$

$\Rightarrow \log\left[x^2\left(2\sin\frac{y}{x}-\frac{y^2}{x^2}\right)\right]=-2C_1$

$\Rightarrow 2x^2\sin\frac{y}{x}-y^2=e^{-2C_1}=C$ (say)

which is the required solution.

**97.** We have, $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{-\sqrt{(1+x^2)(1+y^2)}}{xy}$

$\Rightarrow \int\frac{y}{\sqrt{1+y^2}}dy=-\int\frac{\sqrt{1+x^2}}{x^2}xdx$

$\Rightarrow \frac{1}{2}\int\frac{2y}{\sqrt{1+y^2}}dy=-\int\frac{v^2}{v^2-1}dv$

[putting $1 + x^2 = v^2 \Rightarrow 2xdx = 2vdv$]

$\Rightarrow \sqrt{1+y^2}=-\int\left(1+\frac{1}{v^2-1}\right)dv$

$\Rightarrow \sqrt{1+y^2}=-v-\frac{1}{2}\log\left|\frac{v-1}{v+1}\right|+C$

$\Rightarrow \sqrt{1+y^2}+\sqrt{1+x^2}+\frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right|=C$

**98.** *Refer to answer 65.*

The required solution

$y\cdot x\sin x = x(-\cos x)+\sin x+C$ ...(i)

Putting $x=\frac{\pi}{2}, y=0$ in (i), we get

$\therefore\ 0=-\frac{\pi}{2}\cos\frac{\pi}{2}+\sin\frac{\pi}{2}+C \Rightarrow C=-1$

$xy\sin x = \sin x - x\cos x - 1$ is the required particular solution.

**99.** We have, $x^2\,dy + (xy + y^2)\,dx = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{xy + y^2}{x^2} \quad ...(i)$$

This is a homogeneous linear differential equation

$\therefore$ Put $y = vx \Rightarrow \dfrac{dy}{dx} = v \cdot 1 + x \cdot \dfrac{dv}{dx}$

$\therefore$ (i) becomes $v + x\dfrac{dv}{dx} = -\dfrac{x \cdot vx + v^2x^2}{x^2}$

$$\Rightarrow x\frac{dv}{dx} = -(2v + v^2)$$

Separating the variables, we get

$$\frac{dv}{2v + v^2} + \frac{dx}{x} = 0 \qquad \frac{dv}{v(v+2)} + \frac{dx}{x} = 0$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{v} - \frac{1}{v+2}\right]dv + \frac{dx}{x} = 0$$

Integrating, we get

$$\frac{1}{2}\left[\log v - \log(v+2)\right] + \log x = \log C$$

$$\Rightarrow \log\left(\frac{v}{v+2}\right) + 2\log x = \log C$$

$$\Rightarrow \log\left(\frac{v}{v+2}\right) + \log x^2 = \log C$$

$$\Rightarrow \log\left(\frac{vx^2}{v+2}\right) = \log C \quad \Rightarrow \frac{vx^2}{v+2} = C$$

$$\Rightarrow \frac{\frac{y}{x}.x^2}{\frac{y}{x}+2} = C \Rightarrow x^2y = C\,(2x + y) \quad ...(ii)$$

Putting $x = 1, y = 1$ in (ii), we get

$$1^2.1 = C(2 \cdot 1 + 1) \Rightarrow C = \frac{1}{3}$$

$\therefore$ The required particular solution is

$$3x^2y = 2x + y \Leftrightarrow y = \frac{2x}{3x^2 - 1}$$

**100.** We have, $\left(x\sin^2\left(\dfrac{y}{x}\right) - y\right)dx + x\,dy = 0$

$$\Rightarrow \frac{dy}{dx} + \sin^2\left(\frac{y}{x}\right) - \frac{y}{x} = 0 \quad ...(i)$$

Put $y = vx \Rightarrow \dfrac{dy}{dx} = v \cdot 1 + x\dfrac{dv}{dx}$ $\therefore$ (i) becomes

$$v + x\frac{dv}{dx} + \sin^2 v - v = 0$$

$$\Rightarrow x\frac{dv}{dx} + \sin^2 v = 0 \quad \Rightarrow \text{cosec}^2 v\,dv + \frac{dx}{x} = 0$$

Integrating both sides, we get

$$\int \text{cosec}^2 v\,dv + \int \frac{dx}{x} = C$$

$$\Rightarrow -\cot v + \log x = C \quad \Rightarrow -\cot\left(\frac{y}{x}\right) + \log x = C \quad ...(ii)$$

Put $x = 1, y = \pi/4$ in (ii), we get

$$-\cot\frac{\pi}{4} + \log 1 = C \quad \Rightarrow C = -1$$

$\therefore -\cot\left(\dfrac{y}{x}\right) + \log x + 1 = 0$ is the required particular solution.

**101.** We have, $\dfrac{dy}{dx} - 3y\cot x = \sin 2x$ ...(i)

This is a linear differential equation of the form $\dfrac{dy}{dx} + Py = Q$

where $P = -3\cot x$, $Q = \sin 2x$

$\therefore$ I.F. $= e^{\int P dx} = e^{-3\int \cot x dx}$

$= e^{-3\log|\sin x|} = |\sin^{-3} x|$

$$\therefore \quad y \cdot \sin^{-3} x = \int \sin 2x \cdot \sin^{-3} x\,dx + C$$

$$\Rightarrow \frac{y}{\sin^3 x} = \int \frac{2\sin x\cos x}{\sin^3 x}dx + C$$

$$= \int \frac{2\cos x}{\sin^2 x}dx + C \quad (\text{Put } \sin x = t \Rightarrow \cos x\,dx = dt)$$

$$= 2\int \frac{dt}{t^2} + C = -\frac{2}{t} + C = -\frac{2}{\sin x} + C$$

$$\Rightarrow y = -2\sin^2 x + C\sin^3 x \quad ...(ii)$$

Put $x = \dfrac{\pi}{2}, y = 2$ in (ii), we get

$\therefore$ $2 = -2\cdot 1 + C \cdot 1 \Rightarrow C = 4$

$y = 4\sin^3 x - 2\sin^2 x$ is the required particular solution.

**102.** We have, $x\dfrac{dy}{dx}\sin\left(\dfrac{y}{x}\right) + x - y\sin\left(\dfrac{y}{x}\right) = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{y\sin\left(\frac{y}{x}\right) - x}{x\sin\left(\frac{y}{x}\right)} = \frac{\frac{y}{x}\sin\left(\frac{y}{x}\right) - 1}{\sin\left(\frac{y}{x}\right)} \quad ...(i)$$

This is a linear homogeneous differential equation

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\therefore$ (i) becomes

$v + x\frac{dv}{dx} = \frac{v\sin v - 1}{\sin v}$ $\Rightarrow$ $x\frac{dv}{dx} = \frac{v\sin v - 1}{\sin v} - v$

$\Rightarrow$ $x\frac{dv}{dx} = -\frac{1}{\sin v}$ $\Rightarrow$ $\sin v dv = -\frac{dx}{x}$

Integrating both sides, we get

$-\cos v = -\log x + C$

$\Rightarrow$ $-\cos\left(\frac{y}{x}\right) + \log x = C$ ...(ii)

Put $y = \frac{\pi}{2}$, $x = 1$ in (ii), we get

$\Rightarrow$ $-\cos\left(\frac{\pi}{2}\right) + \log 1 = C$ $\Rightarrow$ $C = 0$

$\Rightarrow \log x = \cos\left(\frac{y}{x}\right)$ is the required solution.

**103.** We have, $2y e^{x/y} dx + (y - 2x e^{x/y})dy = 0$

$\frac{dx}{dy} = \frac{2x e^{x/y} - y}{2y e^{x/y}} = \frac{x}{y} - \frac{1}{2e^{x/y}}$. ...(i)

This is homogenous linear differential equation.

$\therefore$ Put $\frac{x}{y} = t \Rightarrow x = yt$ $\Rightarrow \frac{dx}{dy} = 1\cdot t + y\cdot\frac{dt}{dy}$

$\therefore$ (i) becomes

$t + y\frac{dt}{dy} = t - \frac{1}{2e^t}$ $\Rightarrow 2e^t\, dt + \frac{dy}{y} = 0$

Integrating both sides, we get

$2e^t + \log y = C$ $\Rightarrow$ $2e^{x/y} + \log y = C$ ...(ii)

Putting $x = 0$, $y = 1$ in (ii) we get $C = 2$

$2e^{x/y} + \log y = 2$, is the required particular solution.

**104.** We have, $(x e^{x/y} + y)\, dx = x\, dy$

$\frac{dx}{dy} = \frac{x}{x e^{x/y} + y} = \frac{x/y}{\frac{x}{y}e^{x/y} + 1}$ ...(i)

This is a homogeneous linear differential equation

$\therefore$ Put $x = vy$ $\Rightarrow \frac{dx}{dy} = v + y\cdot\frac{dv}{dy}$

$\therefore$ (i) becomes

$v + y\frac{dv}{dy} = \frac{v}{ve^v + 1}$ $\Rightarrow y\frac{dv}{dy} = \frac{v}{ve^v + 1} - v = \frac{-v^2e^v}{ve^v + 1}$

$\Rightarrow \frac{ve^v + 1}{v^2e^v}dv = -\frac{dy}{y}$ $\Rightarrow \left(\frac{1}{v} + \frac{1}{v^2e^v}\right)dv = -\frac{dy}{y}$

Integrating both sides, we get

$\log v + \int\frac{1}{v^2e^v}dv = -\log y + C$

$\Rightarrow \log vy + \int\frac{dv}{v^2e^v} = C$ $\Rightarrow \log x + \int\frac{dv}{v^2e^v} = C$

which can't be integrated further.

**105.** *Refer to answer 92.*

We have, $x = \tan^{-1}y - 1 + Ce^{-\tan^{-1}y}$ ...(i)

Putting $x = 0$, $y = 0$ in (i), we get

$\therefore$ $0 = 0 - 1 + Ce^0$ $\Rightarrow$ $C = 1$

$x = \tan^{-1}y - 1 + e^{-\tan^{-1}y}$, is the required particular solution.

**106.** We have,

$\frac{dx}{dy} + x\cot y = 2y + y^2\cot y, (y \neq 0)$ ... (i)

This is a linear differential equation of the form $\frac{dx}{dy} + Py = Q$, where $P = \cot y$, $Q = 2y + y^2\cot y$

I.F. $= e^{\int P dy} = e^{\int \cot y\, dy} = e^{\log\sin y} = \sin y$.

$\therefore$ The required solution is

$$x\cdot\sin y = \int(2y + y^2\cot y)\sin y\, dy + C$$

$$= \int 2y\sin y\, dy + \int y^2\cos y\, dy + C$$

$$= \int 2y\sin y dy + \int[y^2\sin y - \int 2y\sin y dy] + C$$

$= y^2\sin y + C$ ...(ii)

Putting $y = \frac{\pi}{2}, x = 0$ in (ii), we get

$\therefore$ $0 = \frac{\pi^2}{4}\sin\frac{\pi}{2} + C$ $\Rightarrow$ $C = -\frac{\pi^2}{4}$

Hence the required particular solution of (i) is $x\sin y = y^2\sin y - \frac{\pi^2}{4}$.

**107.** We have, $(3xy + y^2)\, dx + (x^2 + xy)\, dy = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{3xy + y^2}{x^2 + xy}$ ...(i)

This is a homogeneous linear differential equation

Put $y = vx \Rightarrow \frac{dy}{dx} = v\cdot 1 + x\cdot\frac{dv}{dx}$

$\therefore$ (i) becomes

$v + x\frac{dv}{dx} = -\frac{3x\cdot vx + v^2x^2}{x^2 + x\cdot vx} = -\frac{3v + v^2}{1 + v}$

$\Rightarrow x\frac{dv}{dx} = -v - \frac{3v + v^2}{1 + v} = \frac{-2v^2 - 4v}{v + 1}$

$= \frac{v + 1}{2v^2 + 4v}dv + \frac{dx}{x} = 0$

Integrating both sides, we get

$$\frac{1}{4}\log|2v^2+4v|+\log|x|=\log C'$$

$$\Rightarrow \left(2v^2+4v\right)^{\frac{1}{4}} x = C' \quad \Rightarrow \left(\frac{2y^2}{x^2}+\frac{4y}{x}\right)^{\frac{1}{4}} x = C'$$

$$\Rightarrow \left(2x^2y^2+4x^3y\right)^{\frac{1}{4}} = C'$$

$\Rightarrow$ $2x^2y^2 + 4x^3y = C$ [where $C = (C')^4$] ...(ii)

Put $x = 1, y = 1$ in (ii), we get $\Rightarrow C = 6$

Hence $2x^2y^2 + 4x^3y = 6 \Rightarrow x^2y^2 + 2x^3y = 3$

is the required particular solution.

**108.** We have, $(x^2 + xy)\, dy = (x^2 + y^2)\, dx$

$$\Rightarrow \frac{dy}{dx}=\frac{x^2+y^2}{x^2+xy} \quad \text{... (i)}$$

This is a homogeneous linear differential equation

$\therefore$ Put $y = vx \Rightarrow \frac{dy}{dx} = v\cdot 1 + x\frac{dv}{dx}$

$\therefore$ (i) becomes

$$v+x\frac{dv}{dx}=\frac{x^2+v^2x^2}{x^2+x\cdot vx}=\frac{1+v^2}{1+v}$$

$$\Rightarrow x\frac{dv}{dx}=\frac{1+v^2}{1+v}-v=\frac{1-v}{1+v} \quad \Rightarrow \frac{1+v}{1-v}dv=\frac{dx}{x}$$

Integrating both sides, we get

$$\Rightarrow \int\left[-1+\frac{2}{1-v}\right]dt=\log x+C$$

$\Rightarrow$ $-v - 2\log(1 - v) = \log x + C$

$$\Rightarrow \frac{-y}{x}-2\log\left(1-\frac{y}{x}\right)=\log x+C \quad \text{...(ii)}$$

Putting $x = 1, y = 0$ in (ii), we get $\Rightarrow$ $C = 0$

$$\Rightarrow \frac{-y}{x}-2\left[\log(x-y)-\log x\right]=\log x$$

$$\Rightarrow \frac{-y}{x}-2\log(x-y)+\log x=0$$

is the required particular solution.

**109.** *Refer to answer 49.*

The general solution is

$$\log x + C = \frac{-1}{2}\log\left(\frac{y^2}{x^2}+\frac{y}{x}+1\right) + \sqrt{3}\tan^{-1}\left(\frac{2y}{x}+1\right)/\sqrt{3} \quad \text{...(i)}$$

Putting $x = 1, y = 0$ in (i), we get

$$0+C=-\frac{1}{2}\log(0+0+1)+\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \Rightarrow C=\frac{\pi}{2\sqrt{3}}$$

$$\therefore \quad \log x+\frac{\pi}{2\sqrt{3}}=-\frac{1}{2}[\log(y^2+xy+x^2)-\log x^2] + \sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3}x}\right)$$

$$\Rightarrow \frac{\pi}{2\sqrt{3}}=-\frac{1}{2}\log(x^2+xy+y^2)+\sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3}x}\right)$$

**110.** *Refer to answer 52.*

The general solution is $\sin\frac{y}{x}=\log x+C$ ...(i)

Putting $x = 1, y = \frac{\pi}{4}$ in (i), we get

$\therefore \sin\frac{\pi}{4}=\log 1+C \Rightarrow C=\frac{1}{\sqrt{2}}$

$\sin\left(\frac{y}{x}\right)=\log x+\frac{1}{\sqrt{2}}$, is the required particular solution.

**111.** We have,

$$x\cos\left(\frac{y}{x}\right)(ydx+xdy)=y\sin\left(\frac{y}{x}\right)(xdy-ydx)$$

$$\Rightarrow \cos\left(\frac{y}{x}\right)\left(y+x\frac{dy}{dx}\right)=\frac{y}{x}\sin\left(\frac{y}{x}\right)\left(x\frac{dy}{dx}-y\right) \quad \text{...(i)}$$

Putting $y = vx \Rightarrow \frac{dy}{dx}=v\cdot 1+x\frac{dv}{dx}$

$\therefore$ (i) becomes

$$\cos v\left[vx+x\left(v+x\frac{dv}{dx}\right)\right]=v\sin v\left[x\left(v+x\frac{dv}{dx}\right)-vx\right]$$

$$\Rightarrow \cos v\left(2v+x\frac{dv}{dx}\right)=v\sin v\cdot x\frac{dv}{dx}$$

$$\Rightarrow 2v\cos v=x(v\sin v-\cos v)\frac{dv}{dx}$$

$$\Rightarrow \frac{v\sin v-\cos v}{v\cos v}dv=2\frac{dx}{x} \Rightarrow \left(\tan v-\frac{1}{v}\right)dv=2\frac{dx}{x}$$

Integrating both sides, we get

$\log|\sec v| - \log v = 2\log x + \log C$

$$\Rightarrow \log\frac{|\sec v|}{v}=\log Cx^2 \quad \Rightarrow |\sec v|=Cvx^2$$

$$\Rightarrow \sec\left(\frac{y}{x}\right)=Cxy$$