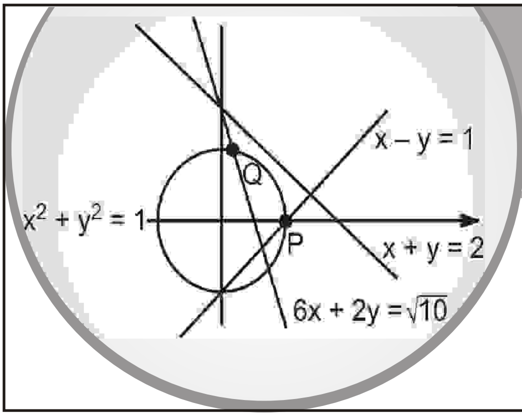


1

STRAIGHT LINE

KEY CONCEPTS

1. Distance Formula

The distance between the points (Ax_1, y_1) and $B(x_2, y_2)$ is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

2. Section Formula

If $P(x, y)$ divides the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$, then

$$x = \frac{mx_2 + nx_1}{m + n} \; ; \; y = \frac{my_2 + ny_1}{m + n}$$

If $\frac{m}{n}$ is positive, the division is internal, but if $\frac{m}{n}$ is negative, the division is external.

> **Note:** *If P divides AB internally in the ratio $m : n$ and Q divides AB externally in the ratio $m : n$, then P and Q are said to be harmonic conjugate to each other w.r.t. AB.*
>
> ***Mathematically;*** $\frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}$ *i.e., AP, AB and AQ are in H.P.*

3. Centroid and Incentre

If $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ are the vertices of triangle *ABC*, whose sides *BC, CA, AB* are of lengths *a, b, c* respectively, then the coordinates of the centroid are: $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

and the coordinates of the incentre are: $\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$

Note that incentre divides the angle bisectors in the ratio

$$(b + c) : a; \; (c + a) : b \text{ and } (a + b) : c.$$

REMEMBER:

(i) Orthocentre, centroid and circumcentre are always collinear and centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1.

(ii) In an isosceles triangle *G, O, I* and *C* lie on the same line.

4. Slope Formula

If θ is the angle at which a straight line is inclined to the positive direction of x-axis, and $0^\circ \le \theta < 180^\circ$, $\theta \ne 90^\circ$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m does not exist, but the line is parallel to the y-axis.

If $\theta = 0$, then $m = 0$ and the line is parallel to the x-axis.

If $A(x_1, y_1)$ and $B(x_2, y_2)$, $x_1 \ne x_2$, are points on a straight line, then the slope m of the line is given by: $m = \dfrac{y_1 - y_2}{x_1 - x_2}$.

5. Condition of Collinearity of Three Points-(Slope Form)

Points $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ are collinear if $\dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{y_2 - y_3}{x_2 - x_3}$

6. Equation of a Straight line in Various Forms

(i) Slope-intercept form: $y = mx + c$ is the equation of a straight line whose slope is m and which makes an intercept c on the y-axis.

(ii) Slope one point form: $y - y_1 = m(x - x_1)$ is the equation of a straight line whose slope is m and which passes through the point (x_1, y_1)

(iii) Parametric form: The equation of the line in parametric form is given by $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$ (say). Where 'r' is the distance of any point (x, y) on the line from the fixed point (x_1, y_1) on the line. r is positive if the point (x, y) is on the right of (x_1, y_1) and negative if (x, y) lies on the left of (x_1, y_1).

(iv) Two point form: $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ is the equation of a straight line which passes through the points (x_1, y_1) and (x_2, y_2).

(v) Intercept form: $\dfrac{x}{a} + \dfrac{y}{b} = 1$ is the equation of a straight line which makes intercepts a and b on OX and OY respectively.

(vi) Perpendicular form: $x\cos\alpha + y\sin\alpha = p$ is the equation of the straight line where the length of the perpendicular from the origin O on the line is p and this perpendicular makes angle α with positive side of x-axis.

(vii) General Form: $ax + by + c = 0$ is the equation of a straight line in the general form.

7. Position of The Point (x_1, y_1) Relative to the Line $ax + by + c = 0$

If $ax_1 + by_1 + c$ is of the same sign as c, then the point (x_1, y_1) lie on the origin side of $ax + by + c = 0$. But if the sign of $ax_1 + by_1 + c$ is opposite to that of c, the point (x_1, y_1) will lie on the non-origin side of $ax + by + c = 0$.

8. **The Ratio in Which A Given Line Divides the Line Segment Joining Two Points**

Let the given line $ax + by + c = 0$ divide the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$, then $\frac{m}{n} = -\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}$. If A and B are on the same side of the given line then $\frac{m}{n}$ is negative but if A and B are on opposite sides of the given line, then $\frac{m}{n}$ is positive.

9. **Length of Perpendicular From a Point on A line**

The length of perpendicular from $P(x_1, y_1)$ on $ax + by + c = 0$ is $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.

10. **Angle Between Two Straight Lines in Terms of Their Slopes**

If m_1 and m_2 are the slopes of two intersecting straight lines $(m_1 m_2 \neq -1)$ and θ is the acute angle between them, then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.

Note: *Let m_1, m_2, m_3 are the slopes of three lines $L_1 = 0; L_2 = 0; L_3 = 0$ where $m_1 > m_2 > m_3$ then the interior angles of the ΔABC found by these lines are given by,*

$$\tan A = \frac{m_1 - m_2}{1 + m_1 m_2}; \quad \tan B = \frac{m_2 - m_3}{1 + m_2 m_3} \text{ and } \tan C = \frac{m_3 - m_1}{1 + m_3 m_1}$$

11. **Parallel Lines**

(i) When two straight lines are parallel their slopes are equal. Thus any line parallel to $ax + by + c = 0$ is of the type $ax + by + k = 0$. Where k is a parameter.

(ii) The distance between two parallel lines with equations $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.

Note that the coefficients of x and y in both the equations must be same.

(iii) The area of the parallelogram $= \frac{p_1 p_2}{\sin\theta}$, where p_1 and p_2 are distances between two pairs of opposite sides and θ is the angle between any two adjacent sides. Note that area of the parallelogram bounded by the lines $y = m_1 x + c_1, y = m_1 x + c_2$ and $y = m_2 x + d_1, y = m_2 x + d_2$ is given by $\left|\frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2}\right|$.

12. **Perpendicular Lines**

(i) When two lines of slopes m_1 and m_2 are at right angles, the product of their slopes is -1, *i.e.*, $m_1 m_2 = -1$. Thus any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$, where k is any parameter.

(ii) Straight lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are at right angles if and only if $aa' + bb' = 0$.

13. **Equations of straight lines through (x_1, y_1) making angle α with $y = mx + c$ are**

$(y - y_1) = \tan(\theta - \alpha)(x - x_1)$ and $(y - y_1) = \tan(\theta + \alpha)(x - x_1)$, where $\tan\theta = m$.

14. Condition of Concurrency

Three lines $a_1x \quad b_1y \quad c_1 \quad 0$, $a_2x \quad b_2y \quad c_2 \quad 0$ and $a_3x \quad b_3y \quad c_3 \quad 0$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad 0$. **Alternatively:** If three constants A,B and C can be found such that $A(a_1x \quad b_1y \quad c_1) \quad B(a_2x \quad b_2y \quad c_2) \quad C(a_3x \quad b_3y \quad c_3) \quad 0$, then the three straight lines are concurrent.

15. Area of A Triangle

If $(x_i, y_i), i \quad 1,2,3$ are the vertices of a triangle, then its area is equal to $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$, provided the vertices are considered in the counter clockwise sense. The above formula will give a ( ) ve are a if the vertices $(x_i, y_i), i \quad 1,2,3$ are placed in the clockwise sense.

16. Condition of Collinearity of Three Points-(Area Form)

The points $(x_i, y_i), i \quad 1,2,3$ are collinear if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.

17. The Equation of A Family of Straight Lines Passing Through The Points of Intersection of Two Given Lines

The equation of a family of lines passing through the point of intersection of $a_1x \quad b_1y \quad c_1 \quad 0$ and $a_2x \quad b_2y \quad c_2 \quad 0$ is given by $(a_1x \quad b_1y \quad c_1) \quad k(a_2x \quad b_2y \quad c_2) \quad 0$, where k is an arbitrary real number.

> **Note:** *If* $u_1 \quad ax \quad by \quad c, u_2 \quad a\ x \quad b\ y \quad d, u_3 \quad ax \quad by \quad c\ , u_4 \quad a\ x \quad b\ y \quad d'$
> *then,* $u_1 \quad 0;\ u_2 \quad 0;\ u_3 \quad 0;\ u_4 \quad 0$ *form a parallelogram*
> $u_2u_3 \quad u_1u_4 \quad 0$ *represents the diagonal BD.*

Proof: Since it is the first degree equation in x and y therefore it is a straight line. Secondly point B satisfies the equation because the co-ordinates of B satisfy $u_2 \quad 0$ and $u_1 \quad 0$.

Similarly for the point D. Hence the result.

On the similar lines $u_1u_2 \quad u_3u_4 \quad 0$ represents the diagonal AC.

> **Note:** *The diagonal AC is also given by* $u_1 \quad u_4 \quad 0$ *and* $u_2 \quad u_3 \quad 0$, *if the two equations are identical for some and .*
> *[For getting the values of and compare the coefficients of x,y and the constant terms.]*

18. Bisectors of The Angles Between Two Lines

(i) Equations of the bisectors of angles between the lines $ax \quad by \quad c \quad 0$ and $a'x \quad b'y \quad c' \quad 0$ $(ab' \quad a'b)$ are: $\frac{ax \quad by \quad c}{\sqrt{a^2 \quad b^2}} \qquad \frac{a'x \quad b'y \quad c'}{\sqrt{a'^2 \quad b'^2}}$

(ii) To discriminate between the acute angle bisector and the obtuse angle bisector

If be the angle between one of the lines and one of the bisectors, find tan .

If $|\tan \ | \quad 1$, then $2 \quad 90$ so that this bisector is the acute angle bisector.

If $|\tan \ | \quad 1$, then we get the bisector to be the obtuse angle bisector.

(iii) To discriminate between the bisector of the angle containing the origin and that of the angle not containing the origin. Rewrite the equations, $ax \quad by \quad c \quad 0$ and $a'x \quad b'y \quad c' \quad 0$ such that the constant terms c, c' are positive. Then;

$\frac{ax \quad by \quad c}{\sqrt{a^2 \quad b^2}} \qquad \frac{a'x \quad b'y \quad c'}{\sqrt{a'^2 \; b'^2}}$ gives the equation of the bisector of the angle containing the origin and $\frac{ax \quad by \quad c}{\sqrt{a^2 \quad b^2}} \qquad \frac{a'x \quad b'y \quad c'}{\sqrt{a'^2 \; b'^2}}$ gives the equation of the bisector of the angle not containing the origin.

(iv) To discriminate between acute angle bisector and obtuse angle bisector proceed as follows, write $ax \quad by \quad c \quad 0$ and $a'x \quad b'y \quad c' \quad 0$ such that constant terms are positive.

If $aa' \quad bb' \quad 0$, then the angle between the lines that contains the origin is acute and the equation of the bisector of this acute angle is $\frac{ax \quad by \quad c}{\sqrt{a^2 \quad b^2}} \qquad \frac{a'x \quad b'y \quad c'}{\sqrt{a'^2 \; b'^2}}$

Therefore $\frac{ax \quad by \quad c}{\sqrt{a^2 \quad b^2}} \qquad \frac{a'x \quad b'y \quad c'}{\sqrt{a'^2 \; b'^2}}$ is the equation of other bisector.

If, however, $aa' \quad bb' \quad 0$, then the angle between the lines that contains the origin is obtuse and the equation of the bisector of this obtuse angle is:

$\frac{ax \quad by \quad c}{\sqrt{a^2 \quad b^2}} \qquad \frac{a'x \quad b'y \quad c'}{\sqrt{a'^2 \; b'^2}}$; therefore $\frac{ax \quad by \quad c}{\sqrt{a^2 \quad b^2}} \qquad \frac{a'x \quad b'y \quad c'}{\sqrt{a'^2 \; b'^2}}$ is the equation of other bisector.

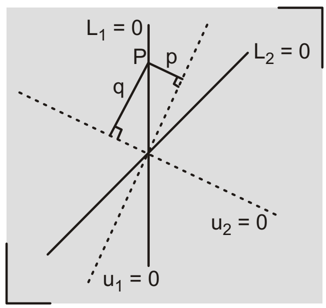
(v) Another way of identifying an acute and obtuse angle bisector is as follows:

Let $L_1 \quad 0$ and $L_2 \quad 0$ are the given lines and $u_1 \quad 0$ and $u_2 \quad 0$ are the bisectors between $L_1 \quad 0$ and $L_2 \quad 0$. Take a point P on any one of the lines $L_1 \quad 0$ or $L_2 \quad 0$ and drop perpendicular on $u_1 \quad 0$ and $u_2 \quad 0$ as shown. If,

$|p| \quad |q| \qquad u_1$ is the acute angle bisector.

$|p| \quad |q| \qquad u_1$ is the obtuse angle bisector.

$|p| \quad |q| \qquad$ the lines L_1 and L_2 are perpendicular.



Note: *Equation of straight lines passing through $P(x_1, y_1)$ and equally inclined with the lines $a_1x \quad b_1y \quad c_1 \quad 0$ and $a_2x \quad b_2y \quad c_2 \quad 0$ are those which are parallel to the bisectors between these two lines and passing through the point P.*

19. A Pair of Straight Lines Through Origin

(i) A homogeneous equation of degree two of the type $ax^2 \quad 2hxy \quad by^2 \quad 0$ always represents a pair of straight lines passing through the origin and if:

(a) $h^2 \quad ab \qquad$ lines are real and distinct.

(b) $h^2 \quad ab \qquad$ lines are coincident.

(c) $h^2 \quad ab \qquad$ lines are imaginary with real point of intersection *i.e.,* (0, 0)

(ii) If $y \quad m_1 x$ and $y \quad m_2 x$ be the two equations represented by $ax^2 \quad 2hxy \quad by^2 \quad 0$, then; $m_1 \quad m_2 \quad \frac{2h}{b}$ and $m_1 m_2 \quad \frac{a}{b}$.

(iii) If is the acute angle between the pair of straight lines represented by,

then; tan $\left|\frac{2\sqrt{h^2 \quad ab}}{a \quad b}\right|$

The condition that these lines are:

(a) At right angles to each other is $a \quad b \quad 0$. *i.e.,* coefficient of x^2 coefficient of $y^2 \quad 0$.

(b) Coincident is $h^2 \quad ab$.

(c) Equally inclined to the axis of x is $h \quad 0$. *i.e.,* coefficient of $xy \quad 0$.

Note: *A homogeneous equation of degree n represents n straight lines passing through origin.*

20. General Equation of Second Degree Representing A Pair Of Straight Lines

(i) $ax^2 \quad 2hxy \quad by^2 \quad 2gx \quad 2fy \quad c \quad 0$ represents a pair of straight lines if:

$$abc \quad 2fgh \quad af^2 \quad bg^2 \quad ch^2 \quad 0, \text{ i.e., if} \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \quad 0$$

(ii) The angle between the two lines representing by a general equation is the same as that between the two lines represented by its homogeneous part only.

21. The joint equation of a pair of straight lines joining origin to the points of intersection of the line given by

$$lx \quad my \quad n \quad 0 \qquad \text{....(i)}$$

and 2nd degree curve: $ax^2 \quad 2hxy \quad by^2 \quad 2gx \quad 2fy \quad c \quad 0$ ...(ii)

is $ax^2 \quad 2hxy \quad by^2 \quad 2gx \frac{lx \quad my}{n} \quad 2fy \frac{lx \quad my}{n} \quad c \left(\frac{lx \quad my}{n}\right)^2 \quad 0$ ...(iii)

(iii) is obtained by homogenizing (ii) with the help of (i), by writing (i) in the form: $\frac{lx \quad my}{n} \quad 1$.

22. The equation to the straight lines bisecting the angle between the straight lines,

$ax^2 \quad 2hxy \quad by^2 \quad 0$ is $\frac{x^2 \quad y^2}{a \quad b} \quad \frac{xy}{h}$.

23. The product of the perpendiculars, dropped from (x_1, y_1) to the pair of lines represented by the equation, $ax^2 \quad 2hxy \quad by^2 \quad 0$ is $\frac{ax_1^2 \quad 2hx_1y_1 \quad by_1^2}{\sqrt{(a \quad b)^2 \quad 4h^2}}$.

24. Any second degree curve through the four point of intersection of $f(xy) \quad 0$ and $xy \quad 0$ is given by $f(xy) \quad xy \quad 0$ where $f(xy) \quad 0$ is also a second degree curve.

25. Reflection of a Point About Line

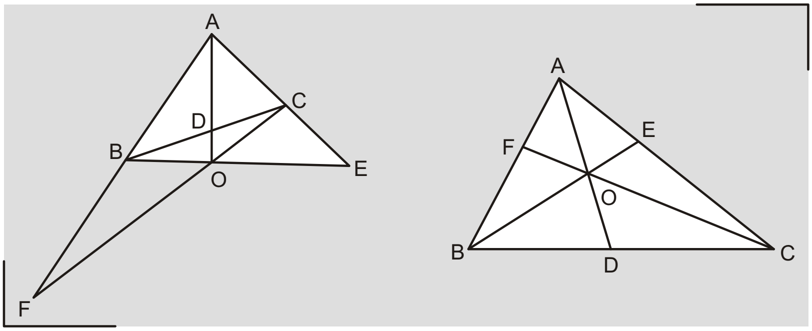
(i) Foot of the perpendicular from a point (x_1, y_1) on the line is

$$\frac{x \quad x_1}{a} \quad \frac{y \quad y_1}{b} \quad \frac{ax_1 \quad by_1 \quad c}{a^2 \quad b^2}$$

(ii) The image of a point (x_1, y_1) about the line $ax \quad by \quad c \quad 0$

$$\frac{x \quad x_1}{a} \quad \frac{y \quad y_1}{b} \quad 2\frac{ax_1 \quad by_1 \quad c}{a^2 \quad b^2}$$

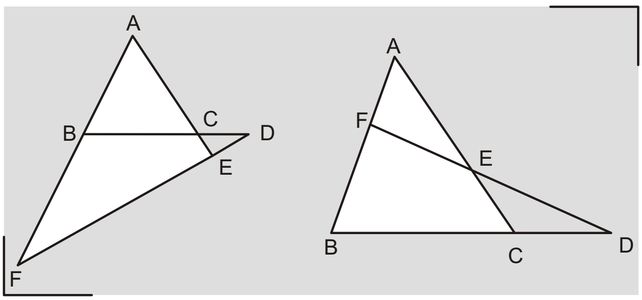
26. Cevas Theorem



If the lines joining a point O to the vertices of ABC meet the opposite sides in D, E, F respectively, then

$$\frac{BD}{DC} \quad \frac{CE}{EA} \quad \frac{AF}{FB} \quad 1$$

27. Menelaus Theorem



If points D, E, F on the sides BC, CA and AB (suitably extended) of ABC are collinear then

$$\frac{BD}{DC} \quad \frac{CE}{EA} \quad \frac{AF}{FB} \quad 1$$

Note: *Either all 3 points lie on the extended line segments or one lies on the extended line and the other two within the line segments.*

EXERCISE 1

Only One Choice is Correct:

1. If the straight lines joining the origin and the points of intersection of the curve $5x^2 \quad 12xy \quad 6y^2 \quad 4x \quad 2y \quad 3 \quad 0$ and $x \quad ky \quad 1 \quad 0$ are equally inclined to the x-axis then the value of k is :

(a) 1 (b) 1

(c) 2 (d) 3

2. Drawn from the origin are two mutually perpendicular straight lines forming an isosceles together with the straight line, $2x \quad y \quad a$. Then the area of triangle is :

(a) $\frac{a^2}{2}$ (b) $\frac{a^2}{3}$

(c) $\frac{a^2}{5}$ (d) None

3. Equation of bisector of the angle between two lines $3x \quad 4y \quad 12 \quad 0$ and $12x \quad 5y \quad 7 \quad 0$ which contains point $(\quad 1, 4)$ in its region is :

(a) $21x \quad 27y \quad 121 \quad 0$ (b) $21x \quad 27y \quad 121 \quad 0$

(c) $21x \quad 27y \quad 191 \quad 0$ (d) $\frac{3x \quad 4y \quad 12}{5} \quad \frac{12x \quad 5y \quad 7}{13}$

4. The point $(a^2, a \quad 1)$ lies in the angle between the lines $3x \quad y \quad 1 \quad 0$ and $x \quad 2y \quad 5 \quad 0$ containing the origin if :

(a) $a \quad (\quad 3, 0) \cup \quad \frac{1}{3}, 1$ (b) $a \quad (\quad , 3) \cup \quad \frac{1}{3}, 1$

(c) $a \quad 3, \frac{1}{3}$ (d) $a \quad \frac{1}{3},$

5. A ray of light through $(2, 1)$ is reflected at a pint A on the y-axis and then passes through the point $(5, 3)$. Then co-ordinates of A are :

(a) $0, \frac{11}{7}$ (b) $0, \frac{5}{11}$

(c) $0, \frac{11}{5}$ (d) $0, \frac{3}{5}$

6. The combined equation of the pair of lines through $(3, \quad 2)$ and parallel to the lines $x^2 \quad 4xy \quad 3y^2 \quad 0$ is :

(a) $x^2 \quad 4xy \quad 3y^2 \quad 14x \quad 24y \quad 45 \quad 0$ (b) $x^2 \quad 4xy \quad 3y^2 \quad 14x \quad 24y \quad 45 \quad 0$

(c) $x^2 \quad 4xy \quad 3y^2 \quad 14x \quad 24y \quad 45 \quad 0$ (d) $x^2 \quad 4xy \quad 3y^2 \quad 14x \quad 24y \quad 45 \quad 0$

7. If $(\;\; 2,6)$ is the image of the point $(4,2)$ with respect to the line $L \quad 0$, then L is equal to :

(a) $3x \quad 2y \quad 11 \quad 0$ (b) $2x \quad 3y \quad 11 \quad 0$

(c) $3x \quad 2y \quad 5 \quad 0$ (d) $6x \quad 4y \quad 1 \quad 0$

8. A man starts from the point $P(\;\; 3,4)$ and reaches point $Q\,(0,1)$ touching x axis at R such that $PR \quad RQ$ is minimum, then the point R is :

(a) $(3/5,0)$ (b) $(\;\; 3/5,0)$

(c) $(\;\; 2/5,0)$ (d) $(\;\; 2,0)$

9. The equation of line segment AB is $y \quad x$. If A & B lie on same side of line mirror $2x \quad y \quad 1$, then the equation of image of AB with respect to line mirror $2x \quad y \quad 1$ is :

(a) $y \quad 7x \quad 5$ (b) $y \quad 7x \quad 6$

(c) $y \quad 3x \quad 7$ (d) $y \quad 6x \quad 5$

10. If $\frac{a}{\sqrt{bc}} \quad 2 \quad \sqrt{\frac{b}{c}} \quad \sqrt{\frac{c}{b}}$ where $a,b,c \quad 0$, then family of lines $\sqrt{a}\,x \quad \sqrt{b}\,y \quad \sqrt{c} \quad 0$ passes through the point:

(a) $(1,1)$ (b) $(1, \;\; 2)$

(c) $(\;\; 1,2)$ (d) $(\;\; 1,1)$

11. The perpendicular distance p_1, p_2, p_3 of points $(a^2,2a),(ab,a \quad b),(b^2,2b)$ respectively from straight line $x \quad y\tan \quad \tan^2 \quad 0$ are in:

(a) AP (b) GP

(c) HP (d) AGP

12. ABC is a variable triangle such that A is $(1,2)$ B and C lie on $y \quad x$ (where is variable), then locus of the orthocentre of triangle ABC is :

(a) $(x \quad 1)^2 \quad y^2 \quad 4$ (b) $x \quad y \quad 3$

(c) $2x \quad y \quad 0$ (d) $x \quad 2y \quad 0$

13. The line $2x \quad y \quad 4$ meet x-axis at A and y-axis at B. The perpendicular bisector of AB meets the horizontal line through $(0, \;\; 1)$ at C. Let G be the centroid of ABC. The perpendicular distance from G to AB equals

(a) $\sqrt{5}$ (b) $\frac{\sqrt{5}}{3}$

(c) $2\sqrt{5}$ (d) $3\sqrt{5}$

14. Let ABC be a triangle. Let A be the point $(1,2)$, $y \quad x$ is the perpendicular bisector of AB and $x \quad 2y \quad 1 \quad 0$ is the angle bisector of angle C. If the equation of BC is given by $ax \quad by \quad 5 \quad 0$, then the value of $a \quad b$ is :

(a) 1 (b) 2

(c) 3 (d) 4

15. $I(1,0)$ is the centre of in circle of triangle ABC, the equation of BI is $x \quad 1 \quad 0$ and equation of CI is $x \quad y \quad 1 \quad 0$, then angle BAC is :

(a) $\frac{}{4}$ (b) $\frac{}{3}$

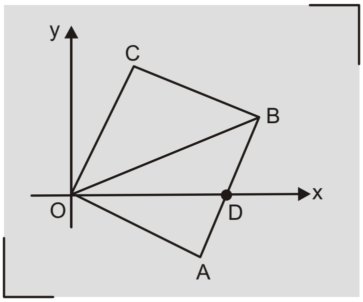
(c) $\frac{}{2}$ (d) $\frac{2}{3}$

16. If the points where the lines $3x \quad 2y \quad 12 \quad 0$ and $x \quad ky \quad 3 \quad 0$ intersect both the coordinate axes are concyclic, then number of possible real values of k is :

(a) 1 (b) 2

(c) 3 (d) 4

17. In the figure shown, $OABC$ is a rectangle with dimensions $OA \quad 3$ units and $OC \quad 4$ units. If $AD \quad 1.5$ units then slope of diagonal OB will be :



(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{2}}{\sqrt{3}}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

18. In a $\quad ABC$, the equations of right bisectors of sides AB and CA are $3x \quad 4y \quad 20$ and $8x \quad 6y \quad 65$ respectively. If the vertex A be $(10, 10)$, then the area of $\quad ABC$ will be :

(a) 14 (b) 21

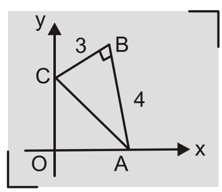
(c) 42 (d) 63

19. The least area of a quadrilateral with integral coordinates is :

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{3}{2}$ (d) 2

20. In the adjacent figure $\quad ABC$ is right angled at B. If $AB \quad 4$ and $BC \quad 3$ and side AC slides along the coordinate axes in such a way that 'B' always remains in the first quadrant, then B always lie on straight line :



(a) $y \quad x$ (b) $3y \quad 4x$

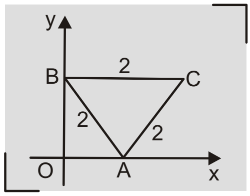
(c) $4y \quad 3x$ (d) $x \quad y \quad 0$

21. If the line $y \quad x$ is one of the angle bisector of the pair of lines $ax^2 \quad 2hxy \quad by^2 \quad 0$, then :

(a) $a \quad b \quad 0$ (b) $a \quad b \quad 0$

(c) $h \quad 0$ (d) $a \quad 2b \quad 0$

22. Adjacent figure represents a equilateral triangle ABC of side length 2 units. Locus of vertex C as the side AB slides along the coordinate axes is :



(a) $x^2 \quad y^2 \quad xy \quad 1 \quad 0$ (b) $x^2 \quad y^2 \quad xy\sqrt{3} \quad 1$

(c) $x^2 \quad y^2 \quad 1 \quad xy\sqrt{3}$ (d) $x^2 \quad y^2 \quad xy\sqrt{3} \quad 1 \quad 0$

23. Vertices of a variable triangle are $(3,4)$, $(5\cos \quad , 5\sin \quad)$ and $(5\sin \quad , \quad 5\cos \quad)$ where $\quad R$, then locus of its orthocenter is :

(a) $(x \quad y \quad 1)^2 \quad (x \quad y \quad 7)^2 \quad 100$ (b) $(x \quad y \quad 7)^2 \quad (x \quad y \quad 1)^2 \quad 100$

(c) $(x \quad y \quad 7)^2 \quad (x \quad y \quad 1)^2 \quad 100$ (d) $(x \quad y \quad 7)^2 \quad (x \quad y \quad 1)^2 \quad 100$

24. Consider the triangle OAB where $O \quad (0,0), B(3,4)$. If orthocenter of triangle is $H(1,4)$, then coordinates of ' A is :

(a) $0, \frac{15}{4}$ (b) $0, \frac{17}{4}$

(c) $0, \frac{21}{4}$ (d) $0, \frac{19}{4}$

25. On the portion of the straight line, $x \quad 2y \quad 4$ intercepted between the axes, a square is constructed on the side of the line away from the origin. Then the point of intersection of its diagonals has co-ordinates :

(a) $(2,3)$ (b) $(3,2)$

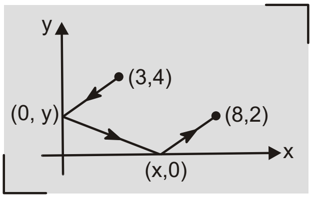
(c) $(3,3)$ (d) $(2,2)$

26. Through a point A on the x-axis a straight line is drawn parallel to y-axis so as to meet the pair of straight lines $ax^2 \quad 2hxy \quad by^2 \quad 0$ in B and C. If $AB \quad BC$ then :

(a) $h^2 \quad 4ab$ (b) $8h^2 \quad 9ab$

(c) $9h^2 \quad 8ab$ (d) $4h^2 \quad ab$

27. Suppose that a ray of light leaves the point $(3,4)$, reflects off the y-axis towards the x-axis, reflects off the x-axis, and finally arrives at the point $(8,2)$. The value of x, is :



(a) $x \quad 4\frac{1}{2}$ (b) $x \quad 4\frac{1}{3}$

(c) $x \quad 4\frac{2}{3}$ (d) $x \quad 5\frac{1}{3}$

28. Given a triangle whose vertices are at $(0,0), (4,4)$ and $(10,0)$. A square is drawn in it such that its base is on the x-axis and its two corners are on the 2 sides of the triangle. The area of the square is equal to :

(a) $\frac{400}{49}$ (b) $\frac{400}{25}$

(c) $\frac{625}{16}$ (d) $\frac{625}{49}$

29. A, B and C are points in the xy-plane such that $A(1,2)$; $B(5,6)$ and $AC \quad 3BC$. Then :

(a) ABC is a unique triangle

(b) There can be only two such triangles.

(c) No such triangle is possible

(d) There can be infinite number of such triangles.

30. A ray of light passing through the point $A(1,2)$ is reflected at a point B on the x-axis and then passes through $(5,3)$. Then the equation of AB is :

(a) $5x \quad 4y \quad 13$ (b) $5x \quad 4y \quad 3$

(c) $4x \quad 5y \quad 14$ (d) $4x \quad 5y \quad 6$

31. Vertices of a parallelogram $ABCD$ are $A(3,1), B(13,6), C(13,21)$ and $D(3,16)$. If a line passing through the origin divides the parallelogram into two congruent parts then the slope of the line is :

(a) $\frac{11}{12}$ (b) $\frac{11}{8}$

(c) $\frac{25}{8}$ (d) $\frac{13}{8}$

32. If the vertices P and Q of a triangle PQR are given by $(2,5)$ and $(4, \quad 11)$ respectively, and the point R moves along the line N: $9x \quad 7y \quad 4 \quad 0$, then the locus of the centroid of the triangle PQR is a straight line parallel to :

(a) PQ (b) QR

(c) RP (d) N

33. In a triangle ABC, if $A(2, \quad 1)$ and $7x \quad 10y \quad 1 \quad 0$ and $3x \quad 2y \quad 5 \quad 0$ are equations of an altitude and an angle bisector respectively drawn from B, then equation of BC is :

(a) $x \quad y \quad 1 \quad 0$
(b) $5x \quad y \quad 17 \quad 0$
(c) $4x \quad 9y \quad 30 \quad 0$
(d) $x \quad 5y \quad 7 \quad 0$

34. The image of the pair of lines represented by $ax^2 \quad 2hxy \quad by^2 \quad 0$ by the line mirror $y \quad 0$ is :

(a) $ax^2 \quad 2hxy \quad by^2 \quad 0$
(b) $bx^2 \quad 2hxy \quad ay^2 \quad 0$
(c) $bx^2 \quad 2hxy \quad ay^2 \quad 0$
(d) $ax^2 \quad 2hxy \quad by^2 \quad 0$

35. In an isosceles right angled triangle, a straight line drawn from the mid-point of one of equal sides to the opposite angle. It divides the angle into two parts, and (/4). Then tan and tan[(/4)] are equal to :

(a) $\frac{1}{2}, \frac{1}{3}$
(b) $\frac{1}{3}, \frac{1}{4}$
(c) $\frac{1}{5}, \frac{1}{6}$
(d) None of these

36. The line $(p \quad 2q)x \quad (p \quad 3q)y \quad p \quad q$, for different values of p and q, passes through the fixed point :

(a) $\frac{3}{2}, \frac{5}{2}$
(b) $\frac{2}{5}, \frac{2}{5}$
(c) $\frac{3}{5}, \frac{2}{5}$
(d) $\frac{2}{5}, \frac{3}{5}$

37. The orthocentre of a triangle whose vertices are $(0,0), (\sqrt{3},0)$ and $(0,\sqrt{6})$ is :

(a) $(2,1)$
(b) $(3,2)$
(c) $(4,1)$
(d) None of these

38. If the line $y \quad mx$ meets the lines $x \quad 2y \quad 1 \quad 0$ and $2x \quad y \quad 3 \quad 0$ at the same point, then m is equal to :

(a) 1
(b) 1
(c) 2
(d) 2

39. The distance of any point (x, y) from the origin is defined as $d \quad \max\{|x|, |y|\}$, then the distance of the common point for the family of lines $x(1 \quad) \quad y \quad 2 \quad \quad 0$ (being parameter) from origin is :

(a) 1
(b) 2
(c) $\sqrt{5}$
(d) 0

40. Let $ax \quad by \quad c \quad 0$ be a variable straight line, where a, b and c are $1^{st}, 3^{rd}$ and 7^{th} terms of some increasing A.P. Then the variable straight line always passes through a fixed point which lies on:

(a) $x^2 \quad y^2 \quad 13$ (b) $x^2 \quad y^2 \quad 5$

(c) $y^2 \quad 9x$ (d) $3x \quad 4y \quad 9$

41. Area of the triangle formed by the line $x \quad y \quad 3$ and angle bisector of the pair of straight lines $x^2 \quad y^2 \quad 2y \quad 1 \quad 0$ is :

(a) 2 sq. units (b) 4 sq. units

(c) 6 sq. units (d) 8 sq. units

42. The number of integral values of m, for which the x-coordinate of the point of intersection of the lines $3x \quad 4y \quad 9$ and $y \quad mx \quad 1$ is also an integer is :

(a) 2 (b) 0

(c) 4 (d) 1

43. A line passes through $(1,0)$. The slope of the line,for which its intercept between $y \quad x \quad 2$ and $y \quad x \quad 2$ subtends a right angle at the origin, is :

(a) $2/3$ (b) $3/2$

(c) 1 (d) None of these

44. A variable straight line passes through a fixed point (a,b) intersecting the coordinate axes at A & B. If 'O' is the origin, then the locus of centroid of triangle OAB is :

(a) $bx \quad ay \quad 3xy \quad 0$ (b) $bx \quad ay \quad 2xy \quad 0$

(c) $ax \quad by \quad 3xy \quad 0$ (d) $ax \quad by \quad 2xy \quad 0$

45. Two vertices of a triangle are $(5, \quad 1)$ and $(\quad 2,3)$. If orthocenter of the triangle is origin, then the co-ordinates of third vertex is :

(a) $(4,7)$ (b) $(3,7)$

(c) $(\quad 4, \quad 7)$ (d) $(4, \quad 7)$

46. The straight line $y \quad x \quad 2$ rotates about a point where it cuts the x-axis and becomes perpendicular to the straight line $ax \quad by \quad c \quad 0$. Then its equation is :

(a) $ax \quad by \quad 2a \quad 0$ (b) $ax \quad by \quad 2a \quad 0$

(c) $bx \quad ay \quad 2b \quad 0$ (d) $ay \quad bx \quad 2b \quad 0$

47. It is desired to construct a right angled triangle ABC ($C \quad /2$) in xy-plane so that its sides are parallel to co-ordinates axes and the medians through A and B lie on the lines $y \quad 3x \quad 1$ and $y \quad mx \quad 2$ respectively. The values of 'm' for which such a triangle is possible is/are :

(a) 12 (b) $3/4$

(c) $4/3$ (d) $1/12$

48. The medians AD and BE of a triangle ABC with vertices $A(0,b), B(0,0)$ and $C(a,0)$ are perpendicular to each other if :

(a) $b \quad \sqrt{2}\,a$ (b) $a \quad \sqrt{2}\,b$

(c) $b \quad \sqrt{3}\,a$ (d) $a \quad \sqrt{3}\,b$

49. The equations of the lines through $(\quad 1, \quad 1)$ and making angle 45 with the line $x \quad y \quad 0$ are given by :

(a) $x^2 \quad xy \quad x \quad y \quad 0$ (b) $xy \quad y^2 \quad x \quad y \quad 0$

(c) $xy \quad x \quad y \quad 0$ (d) $xy \quad x \quad y \quad 1 \quad 0$

50. The number of integral points inside the triangle made by the line $3x \quad 4y \quad 12 \quad 0$ with the coordinate axes which are equidistant from at least two sides is/are (an integral point is a point both of whose coordinates are integers):

(a) 1 (b) 2

(c) 3 (d) 4

51. If the lines $x \quad y \quad 1 \quad 0; 4x \quad 3y \quad 4 \quad 0$ and $x \quad y \qquad 0$, where $^2 \qquad ^2 \quad 2$, are concurrent then:

(a) 1, 1 (b) 1, 1

(c) 1, 1 (d) 1, 1

52. The straight line, $ax \quad by \quad 1$ makes with the curve $px^2 \quad 2axy \quad qy^2 \quad r$ a chord which subtends a right angle at the origin. Then:

(a) $r(a^2 \quad b^2) \quad p \quad q$ (b) $r(a^2 \quad p^2) \quad q \quad b$

(c) $r(b^2 \quad q^2) \quad p \quad a$ (d) none of these

53. Given the family of lines, $a(2x \quad y \quad 4) \quad b(x \quad 2y \quad 3) \quad 0$. Among the lines of the family, the number of lines situated at a distance of $\sqrt{10}$ from the point $M(2, \ 3))$ is:

(a) 0 (b) 1

(c) 2 (d)

54. *m, n* are integer with $0 \quad n \quad m$. A is the point (m, n) on the cartesian plane. B is the reflection of A in the line $y \quad x$. C is the reflection of B in the y-axis, D is the reflection of C in the x-axis and E is the reflection of D in the y-axis. The area of the pentagon $ABCDE$ is:

(a) $2m(m \quad n)$ (b) $m(m+3n)$

(c) $m(2m \quad 3n)$ (d) $2m(m \quad 3n)$

55. The area enclosed by the graphs of $|x \quad y| \quad 2$ and $|x| \quad 1$ is:

(a) 2 (b) 4

(c) 6 (d) 8

56. The ends of the base of an isosceles triangle are at (2, 0) and (0, 1) and the equation of one side is $x \quad 2$ then the orthocentre of the triangle is:

(a) $\frac{3}{4}, \frac{3}{2}$ (b) $\frac{5}{4}, 1$

(c) $\frac{3}{4}, 1$ (d) $\frac{4}{3}, \frac{7}{12}$

57. The equation of the pair of bisectors of the angles between two straight lines is, $12x^2 \quad 7xy \quad 12y^2 \quad 0$. If the equation of one line is $2y \quad x \quad 0$ then the equation of the other line is:

(a) $41x \quad 38y \quad 0$ (b) $11x \quad 2y \quad 0$

(c) $38x \quad 41y \quad 0$ (d) $11x \quad 2y \quad 0$

58. A piece of cheese is located at (12, 10) in a coordinate plane. A mouse is at (4, 2) and is running up the line $y \quad\quad 5x \quad 18$. At the point (a, b), the mouse starts getting farther from the cheese rather than closer to it. The value of $(a \quad b)$ is:

(a) 6 (b) 10

(c) 18 (d) 14

59. The equations of L_1 and L_2 are $y \quad mx$ and $y \quad nx$, respectively. Suppose L_1 make twice as large of an angle with the horizontal (measured counterclockwise from the positive x-axis) as does L_2 and that L_1 has 4 times the slope of L_2. If L_1 is not horizontal, then the value of the product (mn) equals:

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{2}}{2}$

(c) 2 (d) $\quad 2$

60. If L is the line whose equation is $ax \quad by \quad c$. Let M be the reflection of L through the y-axis, and let N be the reflection of L through the x-axis. Which of the following must be true about M and N for all choices of a, b and c ?

(a) The x-intercepts of M and N are equal.

(b) The y-intercepts of M and N are equal.

(c) The slopes of M and N are equal.

(d) The slopes of M and N are reciprocal.

61. The line $x \quad c$ cuts the triangle with corners $(0,0)$, $(1, 1)$ and $(9, 1)$ into two regions. For the area of the two regions to be the same c must be equal to :

(a) 5/2 (b) 3

(c) 7/2 (d) 3 or 15

62. The distance between the two parallel lines is 1 unit. A point 'A' is chosen to lie between the lines at a distance 'd' from one of them. Triangle ABC is equilateral with B on one line and C on the other parallel line. The length of the side of the equilateral triangle is:

(a) $\frac{2}{3}\sqrt{d^2 \quad d \quad 1}$ (b) $2\sqrt{\frac{d^2 \quad d \quad 1}{3}}$

(c) $2\sqrt{d^2 \quad d \quad 1}$ (d) $\sqrt{d^2 \quad d \quad 1}$

63. If m and b are real numbers and $mb > 0$, then the line whose equation is $y \quad mx \quad b$ cannot contain the point:

(a) (0, 2008) (b) (2008, 0)

(c) (0, 2008) (d) (20, – 100)

64. Given $A\ (0, 0)$ and $B\ (x, y)$ with $x \quad (0,1)$ and $y \quad 0$. Let the slope of the line AB equals m_1. Point C lies on the line $x \quad 1$ such that the slope of BC equals m_2 where $0 \quad m_2 \quad m_1$. If the area of the triangle ABC can be expressed as $(m_1 \quad m_2) f(x)$, then the largest possible value of $f(x)$ is:

(a) 1 (b) 1/2

(c) 1/4 (d) 1/8

65. What is the y-intercept of the line that is parallel to $y \quad 3x$, and which bisects the area of a rectangle with corners at (0, 0) (4, 0), (4, 2) and (0, 2) ?

(a) (0, –7) (b) (0, –6)

(c) (0, –5) (d) (0, –4)

66. The vertex of right angle of a right angled triangle lies on the straight line $2x \quad y \quad 10 \quad 0$ and the two other vertices, at points (2, –3) and (4, 1) then the area of triangle in sq. units is:

(a) $\sqrt{10}$ (b) 3

(c) $\frac{33}{5}$ (d) 11

67. Given $A \quad (1, 1)$ and AB is any line through it cutting the x-axis in B. If AC is perpendicular to AB and meets the y-axis in C, then the equation of locus of mid-point P of BC is:

(a) $x \quad y \quad 1$ (b) $x \quad y \quad 2$

(c) $x \quad y \quad 2xy$ (d) $2x \quad 2y \quad 1$

68. The number of possible straight lines, passing through (2, 3) and forming a triangle with coordinate axes, whose area is 12 sq. units, is:

(a) one (b) two

(c) three (d) four

69. Let $A \quad (3,2)$ and $B \quad (5,1)$. ABP is an equilateral triangle is constructed one the side of AB remote from the origin then the orthocentre of triangle ABP is:

(a) $4 \quad \frac{1}{2}\sqrt{3}, \frac{3}{2} \quad \sqrt{3}$ (b) $4 \quad \frac{1}{2}\sqrt{3}, \frac{3}{2} \quad \sqrt{3}$

(c) $4 \quad \frac{1}{6}\sqrt{3}, \frac{3}{2} \quad \frac{1}{3}\sqrt{3}$ (d) $4 \quad \frac{1}{6}\sqrt{3}, \frac{3}{2} \quad \frac{1}{3}\sqrt{3}$

70. If $P \quad \frac{1}{x_p}, p \ ; Q \quad \frac{1}{x_q}, q \ ; R \quad \frac{1}{x_r}, r$

where $x_k \quad 0$, denotes the k^{th} terms of a H.P. for $k \quad N$, then:

(a) ar. (PQR) $\frac{p^2q^2r^2}{2} \sqrt{(p \quad q)^2 \quad (q \quad r)^2 \quad (r \quad p)^2}$

(b) PQR is a right angled triangle

(c) the points P, Q, R are collinear

(d) None of these

ANSWERS

1.	(b)	**2.**	(c)	**3.**	(a)	**4.**	(a)	**5.**	(a)	**6.**	(c)	**7.**	(c)	**8.**	(b)	**9.**	(b)	**10.**	(d)
11.	(b)	**12.**	(b)	**13.**	(a)	**14.**	(b)	**15.**	(c)	**16.**	(b)	**17.**	(c)	**18.**	(c)	**19.**	(b)	**20.**	(b)
21.	(b)	**22.**	(c)	**23.**	(d)	**24.**	(d)	**25.**	(c)	**26.**	(b)	**27.**	(b)	**28.**	(a)	**29.**	(d)	**30.**	(a)
31.	(b)	**32.**	(d)	**33.**	(b)	**34.**	(d)	**35.**	(a)	**36.**	(d)	**37.**	(d)	**38.**	(b)	**39.**	(b)	**40.**	(a)
41.	(a)	**42.**	(a)	**43.**	(d)	**44.**	(a)	**45.**	(c)	**46.**	(d)	**47.**	(b)	**48.**	(b)	**49.**	(d)	**50.**	(a)
51.	(d)	**52.**	(a)	**53.**	(b)	**54.**	(b)	**55.**	(d)	**56.**	(b)	**57.**	(a)	**58.**	(b)	**59.**	(c)	**60.**	(c)
61.	(b)	**62.**	(b)	**63.**	(b)	**64.**	(d)	**65.**	(c)	**66.**	(b)	**67.**	(a)	**68.**	(c)	**69.**	(d)	**70.**	(c)

EXERCISE 2

One or More than One is/are Correct

1. Two sides of a triangle have the joint equation $(x \quad 3y \quad 2)(x \quad y \quad 2) \quad 0$, the third side which is variable always passes through the point $(\quad 5, \quad 1)$, then possible values of slope of third side such that origin is an interior point of triangle is/are:

   (a) $\frac{4}{3}$ (b) $\frac{2}{3}$

   (c) $\frac{1}{3}$ (d) $\frac{1}{6}$

2. The equations of lines passing through point (2, 3) and having an intercept of length 2 units between the lines $2x \quad y \quad 3$ and $2x \quad y \quad 5$ are:

   (a) $y \quad 3$ (b) $x \quad 2$

   (c) $y \quad x \quad 1$ (d) $4y \quad 3x \quad 18$

3. Two sides of a rhombus *ABCD* are parallel to lines $y \quad x \quad 2$ and $y \quad 7x \quad 3$. If the diagonals of the rhombus intersect at point (1, 2) and the vertex *A* is on the *y*-axis is, then the possible coordinates of *A* are:

   (a) $0, \frac{5}{2}$ (b) (0, 0)

   (c) (0, 5) (d) (0, 3)

4. Possible values of for which the point (cos , sin) lies inside the triangle formed by lines $x \quad y \quad 2$; $x \quad y \quad 1$ and $6x \quad 2y \quad \sqrt{10}$ are:

   (a) $\frac{\ }{8}$ (b) $\frac{\ }{4}$

   (c) $\frac{\ }{8}$ (d) $\frac{\ }{2}$

5. Two equal sides of an isosceles triangle are given by the equations $7x \quad y \quad 3 \quad 0$ and $x \quad y \quad 3 \quad 0$ and its third side passes through the point, $(1, \quad 10)$, then equations of the third side can be:

   (a) $x \quad 3y \quad 31 \quad 0$ (b) $y \quad 3x \quad 13 \quad 0$

   (c) $3x \quad y \quad 7 \quad 0$ (d) $y \quad 2x \quad 12 \quad 0$

6. All the points lying inside the triangle formed by the points (1, 3), (5, 6), and $(\quad 1, 2)$ satisfy:

   (a) $3x \quad 2y \quad 0$ (b) $2x \quad y \quad 1 \quad 0$

   (c) $\quad 2x \quad 11 \quad 0$ (d) $2x \quad 3y \quad 12 \quad 0$

7. The bisectors of angle between the straight lines $y \quad b \quad \frac{2m}{1 \quad m^2}(x \quad a)$ and $y \quad b \quad \frac{2m}{1 \quad m^2}(x \quad a)$ are:

(a) $(y \quad b)(m \quad m\,) \quad (x \quad a)(1 \quad m\,m\,) \quad 0$

(b) $(y \quad b)(m \quad m\,) \quad (x \quad a)(1 \quad m\,m\,) \quad 0$

(c) $(y \quad b)(1 \quad mm\,) \quad (x \quad a)(m \quad m\,) \quad 0$

(d) $(y \quad b)(1 \quad m\,m\,) \quad (x \quad a)(m \quad m\,) \quad 0$

8. Straight lines $2x \quad y \quad 5$ and $x \quad 2y \quad 3$ intersect at point A. Points B and C are chosen on these two lines such that $AB \quad AC$. Then the equation of a line BC passing through the point $(2,3)$ is:

(a) $3x \quad y \quad 3 \quad 0$

(b) $x \quad 3y \quad 11 \quad 0$

(c) $3x \quad y \quad 9 \quad 0$

(d) $x \quad 3y \quad 7 \quad 0$

9. The sides of a triangle are the straight lines $x \quad y \quad 1, 7y \quad x$ and $\sqrt{3}y \quad x \quad 0$. Then which of the following is an interior point of the triangle:

(a) circumcentre

(b) centroid

(c) incentre

(d) orthocentre

10. The x-coordinates of the vertices of a square of unit area are the roots of the equation $x^2 \quad 3|x| \quad 2 \quad 0$ and the y-coordinates of the vertices are the roots of equation $y^2 \quad 3y \quad 2 \quad 0$, then the possible vertices of the square is/are:

(a) (1, 1), (2, 1), (2, 2), (1, 2)

(b) (–1, 1), (–2, 1), (–2, 2), (–1, 2)

(c) (2, 1), (1, –1), (1, 2) (2, 2)

(d) (–2, 1), (–1, –1), (–1, 2), (–2, 2)

11. If one vertex of an equilateral triangle of side 'a' lies at origin and the other lies on the line $x \quad \sqrt{3}y \quad 0$, then the coordinates of the third vertex are:

(a) $(0, a)$

(b) $\frac{\sqrt{3}a}{2}, \frac{a}{2}$

(c) $(0, -a)$

(d) $\frac{\sqrt{3}a}{2}, \frac{a}{2}$

12. Line $\frac{x}{a} \quad \frac{y}{b} \quad 1$ cuts the coordinate axes at $A(a,0)$ and $B(0,b)$ and the line $\frac{x}{a'} \quad \frac{y}{b'} \quad 1$ at $A'(\ a',0)$ and $B'(0,\ b')$. If the points A, B, A', B' are concyclic then the orthocentre of the triangle ABA' is:

(a) (0, 0)

(b) $(0, b')$

(c) $0, \frac{aa'}{b}$

(d) $0, \frac{bb'}{a}$

13. The lines L_1 and L_2 denoted by $3x^2 \quad 10xy \quad 8y^2 \quad 14x \quad 22y \quad 15 \quad 0$ intersect at the point P and have gradients m_1 and m_2 respectively. The acute angle between them is . Which of following relations hold good:

(a) $m_1 \quad m_2 \quad \frac{5}{4}$

(b) $m_1 m_2 \quad \frac{3}{8}$

(c) $\quad \sin^{\ 1} \frac{2}{5\sqrt{5}}$

(d) sum of the abscissa and ordinate of point P is –1.

14. The area of triangle ABC is 20 cm^2. The coordinates of vertex A are (–5, 0) and B are (3, 0). The vertex C lies on the line $x \quad y \quad 2$. The coordinates of C are:

(a) (5, 3)
(b) (–3, –5)
(c) (–5, –7)
(d) (7, 5)

15. Let $B(1,-3)$ and $D(0,4)$ represent two vertices of rhombus $ABCD$ in (x,y) plane, then coordinates of vertex A if $\quad BAD \quad 60$ can be equal to:

(a) $\frac{1 \quad 7\sqrt{3}}{2}, \frac{1 \quad \sqrt{3}}{2}$
(b) $\frac{1 \quad 7\sqrt{3}}{2}, \frac{1 \quad \sqrt{3}}{2}$
(c) $\frac{1 \quad 7\sqrt{3}}{2}, \frac{1 \quad \sqrt{3}}{2}$
(d) $\frac{1 \quad 7\sqrt{3}}{2}, \frac{1 \quad \sqrt{3}}{2}$

16. Let $L_1 : 3x \quad 4y \quad 1$ and $L_2 : 5x \quad 12y \quad 2 \quad 0$ be two given lines. Let image of every point on L_1 with respect to a line L lies on L_2 then possible equation of L can be:

(a) $14x \quad 112y \quad 23 \quad 0$
(b) $64x \quad 8y \quad 3 \quad 0$
(c) $11x \quad 4y \quad 0$
(d) $52y \quad 45x \quad 7$

17. Let $A(1,1)$ and $B(3, 3)$ be two fixed points and P be a variable point such that area of $\quad PAB$ remains constant equal to 1 for all position of P, then locus of P is given by:

(a) $2y \quad 2x \quad 1$
(b) $2y \quad 2x \quad 1$
(c) $y \quad x \quad 1$
(d) $y \quad x \quad 1$

18. If one diagonal of a square is the portion of line $\frac{x}{a} \quad \frac{y}{b} \quad 1$ intercepted by the axes, then the extremities of the other diagonal of the square are:

(a) $\frac{a \quad b}{2}, \frac{a \quad b}{2}$
(b) $\frac{a \quad b}{2}, \frac{a \quad b}{2}$
(c) $\frac{a \quad b}{2}, \frac{b \quad a}{2}$
(d) $\frac{a \quad b}{2}, \frac{b \quad a}{2}$

19. Two straight lines u 0 and v 0 passes through the origin and the angle between them is $\tan^{1}\frac{7}{9}$. If the ratio of slopes of v 0 and u 0 is $\frac{9}{2}$, then their equations are:

(a) y $3x$ and $3y$ $2x$

(b) $2y$ $3x$ and $3y$ x

(c) y $3x$ 0 and $3y$ $2x$ 0

(d) $2y$ $3x$ 0 and $3y$ x 0

20. The points $A(0,0)$, $B(\cos\ ,\sin\ )$ and $C(\cos\ ,\sin\ )$ are the vertices of a right angled triangle if:

(a) $\sin \frac{\quad}{2}\ \frac{1}{\sqrt{2}}$

(b) $\cos \frac{\quad}{2}\ \frac{1}{\sqrt{2}}$

(c) $\cos \frac{\quad}{2}\ \frac{1}{\sqrt{2}}$

(d) $\sin \frac{\quad}{2}\ \frac{1}{\sqrt{2}}$

21. $ABCD$ is rectangle with $A(\ 1,2)$, $B(3,7)$ and $AB:BC$ $4:3$. If d is the distance of origin from the intersection point of diagonals of rectangle, then possible values of $[d]$ is/are (where [ ] denote greatest integer function)

(a) 3

(b) 4

(c) 5

(d) 6

22. A straight line L drawn through the point $A(1,2)$ intersects the line x y 4 at a distance of $\frac{\sqrt{6}}{3}$ units from A. The angle made by L with positive direction of x-axis can be :

(a) $\frac{\quad}{12}$

(b) $\frac{\quad}{6}$

(c) $\frac{\quad}{3}$

(d) —

23. Let x_1 and y_1 be the roots of x^2 $8x$ 2009 0; x_2 and y_2 be the roots of $3x^2$ $24x$ 2010 0 and x_3 and y_3 be the roots of $9x^2$ $72x$ 2011 0. The points $A(x_1, y_2)$ $B(x_2, y_2)$ and $C(x_3, y_3)$:

(a) can not lie on a circle

(b) form a triangle of area 2 sq. units

(c) form a right-angled triangle

(d) are collinear

ANSWERS

1.	(b, c, d)	**2.**	(b, d)	**3.**	(a, b)	**4.**	(a, b, c)	**5.**	(a, c)	**6.**	(a, b, c)
7.	(a, d)	**8.**	(a, b)	**9.**	(b, c)	**10.**	(a, b)	**11.**	(a, b, c, d)	**12.**	(b, c)
13.	(b, c, d)	**14.**	(b, d)	**15.**	(a, b)	**16.**	(a, b)	**17.**	(c, d)	**18.**	(a, c)
19.	(a, b, c, d)	**20.**	(a, b, c, d)	**21.**	(b, d)	**22.**	(a, d)	**23.**	(a, d)		

EXERCISE 3

Comprehension: (1)

The base of an isosceles triangle is equal to 4, the base angle is equal to 45 . A straight line cuts the extension of the base at a point *M* at the angle and bisects the lateral side of the triangle which is nearest to *M*.

1. The area of quadrilateral which the straight line cuts off from the given triangle is:

(a) $\frac{3 \quad \tan}{1 \quad \tan}$ (b) $\frac{3 \quad 2\tan}{1 \quad \tan}$

(c) $\frac{3 \quad \tan}{1 \quad \tan}$ (d) $\frac{3 \quad 5\tan}{1 \quad \tan}$

2. The possible range of values in which area of quadrilateral which straight line cuts off from the given triangle lie in:

(a) $\frac{5}{2}, \frac{7}{2}$ (b) $(4, 5)$

(c) $4, \frac{9}{2}$ (d) $(3, 4)$

3. The length of portion of straight line inside the triangle may lie in the range:

(a) $(2, 4)$ (b) $\frac{3}{2}, \sqrt{3}$

(c) $(\sqrt{2}, 2)$ (d) $(\sqrt{2}, \sqrt{3})$

Comprehension: (2)

Let *ABCD* is a square with sides of unit length. Points *E* and *F* are taken on sides *AB* and *AD* respectively so that *AE* *AF*. Let *P* be any point inside the square *ABCD*.

1. The maximum possible area of quadrilateral *CDFE* is :

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{3}{8}$ (d) $\frac{5}{8}$

2. The value of $(PA)^2 \quad (PB)^2 \quad (PC)^2 \quad (PD)^2$ is equal to:

(a) 3 (b) 2

(c) 1 (d) 0

3. Let a line passing through point *A* divides the square *ABCD* into two parts so that area of one portion is double the other, then the length of portion of line inside the square is:

(a) $\frac{\sqrt{10}}{3}$ (b) $\frac{\sqrt{11}}{3}$

(c) $\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{13}}{3}$

Comprehension: (3)

Consider a trapezoid *ABCD,* one of whose non parallel sides *AB* which is 8cm long is perpendicular to the base. The base *BC* and *AD* of trapezoid are 6cm and 10cm in lengths respectively. Let L_1, L_2, L_3, L_4 represent the lines AB, BC, CD and *DA* respectively and $d(P, L)$ denote the perpendicular distance of point *P* from line *L*.

1. Find the area of region inside the trapezoid *ABCD* in which the point *Q* can lie satisfying $d(Q, L_4) \quad d(Q, L_3)$:

(a) $3(3\sqrt{5} \quad \sqrt{3})$ (b) $24(\sqrt{3} \quad 1)$

(c) $4(5 \quad \sqrt{5})$ (d) $25(\sqrt{5} \quad 1)$

2. Distance of the point *R* lying on line *AD* from vertex *A* so that perimeter of triangle *RBC* is minimum is:

(a) 2 (b) 3

(c) 4 (d) 5

3. The maximum possible area of rectangle inscribed in the trapezoid so that one of its sides lies on the larger base of trapezoid is:

(a) 36 (b) 54

(c) 42 (d) 48

Comprehension: (4)

Consider a variable line '*L*' which passes through the point of intersection '*P*' of the lines $3x \quad 4y \quad 12 \quad 0$ and $x \quad 2y \quad 5 \quad 0$ meeting the coordinate axes at points *A* and *B*.

1. Locus of the middle point of the segment *AB* has the equation:

(a) $3x \quad 4y \quad 4xy$ (b) $3x \quad 4y \quad 3xy$

(c) $4x \quad 3y \quad 4xy$ (d) $4x \quad 3y \quad 3xy$

2. Locus of the feet of the perpendicular from the origin on the variable line '*L*' has the equation:

(a) $2(x^2 \quad y^2) \quad 3x \quad 4y \quad 0$ (b) $2(x^2 \quad y^2) \quad 4x \quad 3y \quad 0$

(c) $x^2 \quad y \quad 2x \quad y \quad 0$

(d) $x^2 \quad y^2 \quad x \quad 2y \quad 0$

3. Locus of the centroid of the variable triangle *OAB* has the equation (where '*O*' is the origin):

(a) $3x \quad 4y \quad 6xy \quad 0$

(b) $4x \quad 3y \quad 6xy \quad 0$

(c) $3x \quad 4y \quad 6xy \quad 0$

(d) $4x \quad 3y \quad 6xy \quad 0$

Comprehension: (5)

Consider 3 non-collinear points $A(9,3)$, $B(7,-1)$ and $C(1,-1)$. Let $P(a,b)$ be the centre and R is the radius of circle 'S' passing through points A, B, C. Also $H(\bar{x}, \bar{y})$ are the coordinates of the orthocentre of triangle ABC whose area be denoted by .

1. If D, E and F are the middle points of BC, CA and AB respectively then the area of the triangle DEF is :

(a) 12

(b) 6

(c) 4

(d) 3

2. The value of $a \quad b \quad R$ equals:

(a) 3

(b) 12

(c) 13

(d) none of these

3. The ordered pair $(\bar{x}, \bar{y})$ is:

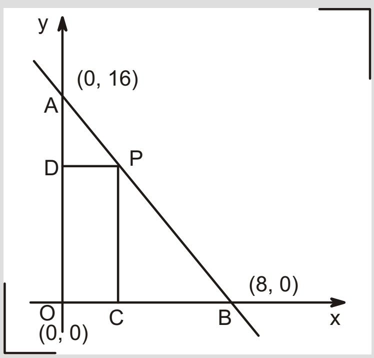
(a) (9, 6)

(b) (–9, 6)

(c) (9, –5)

(d) (9, 5)

Comprehension: (6)



In the diagram, a line is drawn through the points $A(0,16)$ and $B(8,0)$. Point P is chosen in the first quadrant on the line through A and B. Points C and D are chosen on the x and y axis respectively, so that $PDOC$ is a rectangle.

1. Perpendicular distance of the line *AB* from the point (2, 2) is :

(a) $\sqrt{4}$ (b) $\sqrt{10}$

(c) $\sqrt{20}$ (d) $\sqrt{50}$

2. The sum of the coordinates of the point *P* if *PDOC* is a square is:

(a) $\frac{32}{3}$ (b) $\frac{16}{3}$

(c) 16 (d) 11

3. Number of possible ordered pair(s) (x, y) of all positions of point *P* on *AB* so that area of the rectangle *PDOC* is 30 sq. units is:

(a) 3 (b) 2

(c) 1 (d) 0

Comprehension: (7)

Let a and b be the lengths of the legs of a right triangle with following properties

(a) All 3 sides of the triangle are integers.

(b) The perimeter of the triangle is numerically equal to area of the triangle, it is given that a b.

1. The number of ordered pairs (a, b) will be :

(a) 1 (b) 2

(c) 3 (d) 4

2. Maximum possible perimeter of the triangle is :

(a) 27 (b) 28

(c) 29 (d) 30

3. Minimum possible area of the triangle is :

(a) 24 (b) 25

(c) 26 (d) 27

Comprehension: (8)

Let A $(0,0)$, B $(5,0)$, C $(5,3)$ and D $(0,3)$ are the vertices of rectangle $ABCD$. If P is a variable point lying inside the rectangle $ABCD$ and $d(P, L)$ denote perpendicular distance of point P from line L.

1. If $d(P, AB)$ $\min\{d(P, BC), d(P, CD), d(P, AD)\}$, then area of the region in which P lies is :

(a) $\frac{17}{4}$ (b) $\frac{19}{4}$

(c) $\frac{21}{4}$ (d) $\frac{23}{4}$

2. If $d(P,AB)\quad \max\{d(P,BC),\ d(P,CD),\ d(P,AD)\}$, then area of the region in which P lies is :

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

3. If $d(P,AB)\quad \frac{3}{2}\ ^{2}\quad d(P,AD)^{2}\quad 1$, then area of region in which P lies is :

(a) 15 2 (b) 10 $\frac{\ }{2}$ (c) 15 (d) 15 $\frac{\ }{2}$

Comprehension: (9)

The equation of an altitude of an equilateral triangle is $\sqrt{3}\,x\quad y\quad 2\sqrt{3}$ and one of its vertices is $(3,\sqrt{3})$ then

1. The possible number of triangles is :

(a) 1 (b) 2 (c) 3 (d) 4

2. Which of the following can't be the vertex of the triangle :

(a) $(0,0)$ (b) $(0,2\sqrt{3})$ (c) $(3,\ \ \sqrt{3})$ (d) None of these

3. Which of the following can be the possible orthocentre of the triangle :

(a) $(1,\sqrt{3})$ (b) $(0,\sqrt{3})$ (c) $(0,2)$ (d) None of these

Comprehension: (10)

Given point $A(6,30)$ and point $B(24,6)$, equation of line AB is $4x\quad 3y\quad 114$. Point $P(0,\ \ )$ is a point on y-axis such that $0\qquad 38$ and point $Q(0,k)$ is a point on y-axis such that $k\quad 38$.

1. For all positions of point P, angle APB is maximum when point P is :

(a) $(0,12)$ (b) $(0,15)$ (c) $(0,18)$ (d) $(0,21)$

2. The maximum value of angle APB is :

(a) $\frac{\ }{3}$ (b) $\frac{\ }{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

3. For all position of point Q, angle AQB is maximum when point Q is :

(a) $(0,54)$ (b) $(0,58)$ (c) $(0,60)$ (d) None of these

Comprehension: (11)

***OACB* is a square on *x-y* plane where *O* is the origin. A line through *A* intersects the diagonal *OC* at *D* internally, side *OB* at *E* internally and side *CB* at *F* externally. Given that $AD:DE\quad 4:3$, $AD\quad 5$ units and the square lies completely in first quadrant.**

1. The area of square will be :

(a) 36 (b) 42 (c) 49 (d) 82

2. The abscissa of F will be :

(a) $\frac{8}{3}$ (b) $\frac{7}{3}$ (c) $\frac{5}{3}$ (d) $\frac{4}{3}$

3. Let O be the reflection of O along AD. The equation of circumcircle of $AO\ E$ will be :

(a) $x^2 \quad y^2 \quad 7x \quad 21y \quad 0$ (b) $4(x^2 \quad y^2) \quad 7x \quad 21y \quad 0$

(c) $4(x^2 \quad y^2 \quad 7x) \quad 21y \quad 0$ (d) $x^2 \quad y^2 \quad 21x \quad 7y \quad 0$

Comprehension: (12)

A variable line 'L' is drawn through $O(0,0)$ to meet the lines $L_1 : y \quad x \quad 10 \quad 0$ and $L_2 : y \quad x \quad 20 \quad 0$ at points A and B respectively. A point P is taken on line 'L'.

1. If $\frac{2}{OP} \quad \frac{1}{OA} \quad \frac{1}{OB}$, then locus of P is :

(a) $3x \quad 3y \quad 40$ (b) $3x \quad 3y \quad 40 \quad 0$ (c) $3x \quad 3y \quad 40$ (d) $3y \quad 3x \quad 40$

2. If $OP^2 \quad (OA)(OB)$, then locus of P is :

(a) $(y \quad x)^2 \quad 100$ (b) $(y \quad x)^2 \quad 50$ (c) $(y \quad x)^2 \quad 200$ (d) $(y \quad x)^2 \quad 250$

3. If $\frac{1}{OP^2} \quad \frac{1}{(OA)^2} \quad \frac{1}{(OB)^2}$, then locus of P is :

(a) $(y \quad x)^2 \quad 80$ (b) $(y \quad x)^2 \quad 100$ (c) $(y \quad x)^2 \quad 144$ (d) $(y \quad x)^2 \quad 400$

Comprehension: (13)

P is an interior point of triangle ABC. AP, BP, CP when produced meet the sides at D, E, F respectively. If $BD \quad 2DC$ and $AE \quad 3EC$, then

1. $AP : PD$

(a) 5 : 6 (b) 6 : 5 (c) 8 : 3 (d) 9 : 2

2. $BP : PE$

(a) 5 : 6 (b) 6 : 5 (c) 8 : 3 (d) 7 : 4

3. $CP : PF$

(a) 5 : 6 (b) 6 : 5 (c) 7 : 4 (d) 8 : 3

Comprehension: (14)

A ray of light travelling along the line *OP* (*O* being origin) is reflected by the line mirror $2x \quad 3y \quad 1 \quad 0$, the point of incidence being $P(1,1)$. The reflected ray, travelling along *PQ* is again reflected by the line mirror $2x \quad 3y \quad 1 \quad 0$, the point of incidence being *Q*, from *Q* ray move along *QR*, where *R* lies on the line $2x \quad 3y \quad 1 \quad 0$

1. The equation of *QR* is:

(a) $13x \quad 13y \quad 20$ (b) $13x \quad 13y \quad 20 \quad 0$ (c) $y \quad x \quad 1$ (d) $13x \quad 13y \quad 17 \quad 0$

2. The ordinate of point *R* is:

(a) $\frac{73}{13}$ (b) $\frac{53}{13}$ (c) $\frac{23}{13}$ (d) 1

ANSWERS

Comprehension-1:	1.	(d)	2.	(d)	3.	(c)
Comprehension-2:	1.	(d)	2.	(d)	3.	(d)
Comprehension-3:	1.	(d)	2.	(b)	3.	(d)
Comprehension-4:	1.	(a)	2.	(b)	3.	(c)
Comprehension-5:	1.	(d)	2.	(b)	3.	(c)
Comprehension-6:	1.	(c)	2.	(a)	3.	(b)
Comprehension-7:	1.	(b)	2.	(d)	3.	(a)
Comprehension-8:	1.	(c)	2.	(d)	3.	(d)
Comprehension-9:	1.	(b)	2.	(d)	3.	(a)
Comprehension-10:	1.	(c)	2.	(b)	3.	(b)
Comprehension-11:	1.	(c)	2.	(b)	3.	(c)
Comprehension-12:	1.	(d)	2.	(c)	3.	(a)
Comprehension-13:	1.	(d)	2.	(c)	3.	(a)
Comprehension-14:	1.	(a)	2.	(b)		

EXERCISE 4

Assertion and Reason

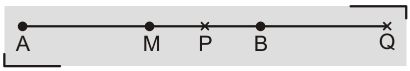
(a) Statement -1 is true , statement-2 is true and statement-2 is correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

1. A line segment AB is divided internally and externally in the same ratio at P and Q respectively and M is the midpoint of AB.



Statement-1: MP, MB, MQ are in G.P.

because

Statement-2: AP, AB and AQ are in H.P.

2. Given a $\quad ABC$ whose vertices are $A(x_1, y_1)$; $B(x_2, y_2)$; $C(x_3, y_3)$. Let there exists a point $P(a, b)$ such that $6a \quad 2x_1 \quad x_2 \quad 3x_3$; $6b \quad 2y_1 \quad y_2 \quad 3y_3$.

Statement-1: Area of triangle PBC must be less than area of $\quad ABC$.

because

Statement-2: P lies inside the triangle ABC.

3. Let $A(x_1, y_1)$ and $B(x_2, y_2)$ are two fixed points in x-y plane. Let us construct a line passing through 'A' at a perpendicular distance 'P' from B in the same plane, then

Statement-1: It is possible that no such line exist.

because

Statement-2: If $P \quad AB$, then no lines can be drawn through A at perpendicular distance 'P' from B.

4. Let 'P' denote the perimeter of $\quad ABC$. If M is a point in the interior of $\quad ABC$, then

Statement-1: $MA \quad MB \quad MC \quad P$

because

Statement-2: $MA \quad MB \quad AC \quad BC$

5. Let $a_1x \quad b_1y \quad c_1 \quad 0$,$a_2x \quad b_2y \quad c_2 \quad 0$ and $a_3x \quad b_3y \quad c_3 \quad 0$ represent three lines L_1, L_2 and L_3 respectively

Statement-1: If L_1, L_2, L_3 are concurrent, then

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad 0$$

because

Statement-2: If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad 0$, then lines L_1, L_2, L_3 must be concurrent at point whose x-y coordinates are finite numbers.

6. Consider a pair of lines represented by $ax^2 \quad 2hxy \quad by^2 \quad 0$ where a, b, h are real numbers and $h^2 \quad ab$, then

Statement-1: If $a \quad b \quad 2h \quad 0$, then one line of the pair $ax^2 \quad 2hxy \quad by^2 \quad 0$ bisects the angle between coordinate axes in first and third quadrants.

because

Statement-2: If $ax \quad y(2h \quad a) \quad 0$ is a factor of $ax^2 \quad 2hxy \quad by^2 \quad 0$, then $b \quad 2h \quad a \quad 0$

7. Statement-1: If a, b, c are variables such that $3a \quad 2b \quad 4c \quad 0$, then family of lines given by $ax \quad by \quad c \quad 0$ passes through a fixed point $\frac{3}{4}, \frac{1}{2}$.

because

Statement-2: The equation $ax \quad by \quad c \quad 0$ will represent a family of lines passing through a fixed point if there exists a linear relation between a, b and c.

8. Statement-1: The area of triangle formed by points A (20, 22), B (21, 24) and C (22, 23) is same as the area of triangle formed by points P (0, 0), Q (1, 2), R (2, 1).

because

Statement-2: The area of triangle is invariant with respect to the translation of the coordinate axes.

9. Statement-1: The equation $2xy \quad 3x \quad 4y \quad 12$ does not represent a line pair.

because

Statement-2: A general equation of degree two in which coefficient of $x^2 \quad 0$ and coefficient of $y^2 \quad 0$ and coefficient of $xy \quad 0$ can not represent a line pair.

10. Let points A, B, C are represented by $(a\cos_{\ i}, a\sin_{\ i}), i \quad 1,2,3$; and $\cos(_1 \quad _2) \quad \cos(_2 \quad _3) \quad \cos(_3 \quad _1) \quad \frac{3}{2}$; then

Statement-1: Orthocentre of ABC is at origin.

because

Statement-2: ABC is equilateral triangle.

ANSWERS

1.	(a)	**2.**	(a)	**3.**	(c)	**4.**	(a)	**5.**	(c)	**6.**	(a)	**7.**	(a)	**8.**	(a)	**9.**	(c)	**10.**	(a)

EXERCISE 5

Match the Columns:

1. Let $D(0,\sqrt{3})$, $E(1,0)$, $F(\ \ 1, 0)$ be the feet of perpendiculars dropped from vertices A,B,C to opposite sides BC, CA, AB respectively of triangle ABC

	Column-I		Column-II
(a)	The ratio of the inradius of $\ ABC$ to the inradius of $\ DEF$ is	**(p)**	2
(b)	Let 'H' be the orthocentre of $\ ABC$, then the greatest integer which is less than or equal to square of the length AH is	**(q)**	3
(c)	The square of the sum of ordinates of points A, B and C is	**(r)**	4
(d)	The length of side AB of $\ ABC$ is	**(s)**	5

2.

	Column-I		Column-II
(a)	If the lines $x \quad 2ay \quad a \quad 0, \quad x \quad 3by \quad b \quad 0, \quad x \quad 4cy \quad c \quad 0,$ where $a, b, c, \quad R$ are concurrent, then a,b,c are in	**(p)**	A.P.
(b)	The points with coordinates $(2a, 3a), (3b, 2b)$ (c, c) where $a,b,c,$ $\ R$ are collinear, then a,b,c are in	**(q)**	G.P.
(c)	If lines $ax \quad 2y \quad 1 \quad 0$, $bx \quad 3y \quad 1 \quad 0$ and $cx \quad 4y \quad 1 \quad 0$ where $a,b,c \ \ R$ passes through the same point, then a,b,c are in	**(r)**	H.P.
(d)	Let a,b,c be distinct non- negative real numbers. If the lines $ax \quad ay \quad c \quad 0, x \quad 1 \quad 0, cx \quad cy \quad b \quad 0$ pass through the same point then a,c,b are in	**(s)**	neither A.P. nor G.P. nor H.P.

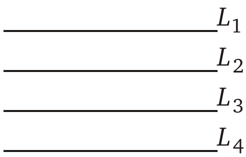
3.

	Column-I		Column-II
(a)	If a,b,c are in A.P., then lines $ax \quad by \quad c \quad 0$ are concurrent at	**(p)**	(–4, –7)
(b)	A point on the line $x \quad y \quad 4$ which lies at a unit distance from the line $4x \quad 3y \quad 10$ is	**(q)**	(–7, 11)
(c)	Orthocentre of triangle made by lines $x \quad y \quad 1, x \quad y \quad 3 \quad 0,$ $2x \quad y \quad 7$ is	**(r)**	(1, –2)
(d)	Two vertices of a triangle are (5, –1) and (2, 3). If orthocentre is the origin then coordinates of the third vertex are	**(s)**	(–1, 2)

4.

	Column-I		Column-II
(a)	The number of integral values of 'a' for which point (a, a^2) lies completely inside the triangle formed by lines $x \quad 0$, $y \quad 0$, $2y \quad x \quad 3$.	**(p)**	0
(b)	The number of values of a of the form $\frac{K}{3}$ where $K \quad I$ so that point (a, a^2) lies between the lines $x \quad y \quad 2$ and $4x \quad 4y \quad 3 \quad 0$	**(q)**	1
(c)	The reflection of point $(t \quad 1, 2t \quad 2)$ in a line is $(2t \quad 1, t)$ then the slope of line is	**(r)**	2
(d)	In a triangle ABC, the bisector of angles B and C lie along the lines $y \quad x$ and $y \quad 0$. If A is (1, 2) then $\sqrt{10}\ d(A, BC)$ equals (where $d(A, BC)$ denotes the perpendicular distance of A from BC.)	**(s)**	4

5. Given four parallel lines L_1, L_2, L_3 and L_4 as shown in figure. Let d_{ij} denote the perpendicular distance between lines L_i and L_j $i, j \quad \{1,2,3,4\}$. Let P be a point, sum of whose perpendicular distances from four lines is K, also $d_{12} \quad d_{23} \quad d_{34}$ Then the complete locus of point P.



	Column-I		Column-II
(a)	If $K \quad d_{12} \quad 2d_{23} \quad d_{34}$	**(p)**	Not possible
(b)	If $\quad K \quad d_{12} \quad 2d_{23} \quad d_{34} \quad 2$, where $0 \quad\quad d_{12}$	**(q)**	Entire region between the lines L_2 and L_3
(c)	If $\quad K \quad d_{12} \quad 2d_{23} \quad d_{34} \quad 2$ where $0 \quad\quad d_{34}$	**(r)**	Entire region between the lines L_1 and L_2
(d)	If $K \quad d_{12} \quad 2d_{23} \quad d_{34}$	**(s)**	Entire region between the lines L_1 and L_2 and between L_3 and L_4

6.

	Column-I		Column-II
(a)	If $P\ 1 \quad \frac{t}{\sqrt{2}}, 2 \quad \frac{t}{\sqrt{2}}$ be any point on a line then value of t for which the point P lies between parallel lines $x \quad 2y \quad 1$ and $2x \quad 4y \quad 15$ is	**(p)**	(1, 2)
(b)	If the point $(2x_1 \quad x_2 \quad t(x_2 \quad x_1), 2y_1 \quad y_2 + t(y_2 \quad y_1)$ divides the join of (x_1, y_1) and (x_2, y_2) internally, then	**(q)**	$\frac{\sqrt{13} \quad 1}{2}, \quad 1$ $1, \frac{\sqrt{13} \quad 1}{2}$
(c)	If the point (1, t) always remains in the interior of the triangle formed by the lines $y \quad x, y \quad 0$ and $x \quad y \quad 4$, then	**(r)**	$\frac{4\sqrt{2}}{3}, \frac{5\sqrt{2}}{6}$
(d)	Set of values of 't' for which the point $P(t, t^2 \quad 2)$ lies inside the triangle formed by lines $x \quad y \quad 1$, $y \quad x \quad 1$ and $y \quad 1$ is	**(s)**	(0, 1)

7. Vertex A of the ABC is at origin. The equation of medians through B and C are $15x \quad 4y \quad 240 \quad 0$ and $15x \quad 52y \quad 240 \quad 0$ respectively.

	Column-I		Column-II
(a)	The coordinates of incenter of ABC are	**(p)**	$\frac{56}{3}, 10$
(b)	The coordinates of centroid of ABC are	**(q)**	$(21, 12)$
(c)	The coordinates of excenter opposite to vertex C of ABC are	**(r)**	$(12, 21)$
(d)	The coordinates of orthocenter of ABC are	**(s)**	$(\ 4, 7)$
		(t)	$(0, 63)$

ANSWERS

1. a p; b s; c q; d r

2. a r; b s; c p; d q

3. a r; b q; c s; d p

4. a p; b r; c q; d s

5. a q; b r; c s; d p

6. a r; b p; c s; d q

7. a q; b p; c s; d t

EXERCISE 6

Subjective Problems

1. $P(3,1)$, $Q(6,5)$ and $R(x,y)$ are three points such that angle PRQ is right angle and the area of PRQ is 7, then number of such points R is.

2. The number of integral values of a for which the point $P(a^2, a)$ lies in the region corresponding to the acute angle between the lines $2y \quad x$ and $4y \quad x$ is.

3. The number of integral values of b for which the origin and the point $(1,1)$ lie on the same side of straight line $a^2x \quad aby \quad 1 \quad 0$ for $a \quad R \quad \{0\}$ is.

4. If the pair of lines $6x^2 \qquad xy \quad 3y^2 \quad 24x \quad 3y \qquad 0$ intersect on x-axis, then find the value of 20 .

5. If n_1 is the number of points on the line $3x \quad 4y \quad 5$ which is at distance of $1 \quad \sin^2 \quad$ units from $(2,3)$ and n_2 denotes the number of points on the line $3x \quad 4y \quad 5$ which is at distance of $\sec^2 \qquad 2\,\text{cosec}^2$ units from $(1,3)$, then find the sum of roots of equations $n_2x^2 \quad 6x \quad n_1 \quad 0$.

6. In a ABC, the vertex A is $(1,1)$ and orthocenter is $(2,4)$. If the sides AB and BC are members of the family of straight lines $ax \quad by \quad c \quad 0$. Where a, b, c are in A.P. then the coordinates of vertex C are (h,k). Find the value of $2h \quad 12k$.

7. Let P be any point on the line $x \quad y \quad 3 \quad 0$ and A be a fixed point $(3,4)$. If the family of lines given by the equation $(3\sec \qquad 5\,\text{cosec}\ )\,x \quad (7\sec \qquad 3\,\text{cosec}\ )\,y \quad 11(\sec \qquad \text{cosec}\ ) \quad 0$ are concurrent at a point B for all permissible values of and maximum value of $|PA \quad PB| \quad 2\sqrt{2n}\ (n \quad N)$, then find the value of n.

8. There exists two ordered triplets (a_1, b_1, c_1) and (a_2, b_2, c_2) for (a,b,c) for which the equation $4x^2 \quad 4xy \quad ay^2 \quad bx \quad cy \quad 1 \quad 0$ represents a pair of identical straight lines in x-y plane. Find the value of $a_1 \quad b_1 \quad c_1 \quad a_2 \quad b_2 \quad c_2$.

9. Each side of a square is of length 4 units. The center of the square is at $(3,7)$ and one of the diagonals is parallel to the line $y \quad x$. If the vertices of the square be $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) then find the value of $\max(y_1, y_2, y_3, y_4) \quad \min(x_1, x_2, x_3, x_4)$.

10. The base of an isosceles triangle is the intercept made by the line $x \quad 2y \quad 4$ with the coordinate axes. If the equations of the equal sides be $x \quad 4$ and $y \quad mx \quad c$ then find the value of $8m \quad c$.

11. The slope of one of lines given by $ax^2 \quad 2hxy \quad by^2 \quad 0$ be the square of the slope of the other, if $ab(a \quad b) \qquad abh \qquad h^3 \quad 0$, then is equals.

12. The slopes of three sides of a triangle ABC are 1, 2,3 respectively. If the orthocentre of triangle ABC is origin, then the locus of its centroid is $y \quad \dfrac{a}{b}x$ where a, b are relatively prime then $b \quad a$ is equal to.

13. The equation of a line through the mid point of the sides AB and AD of rhombus $ABCD$, whose one diagonal is $3x \quad 4y \quad 5 \quad 0$ and one vertex is $A(3,1)$ is $ax \quad by \quad c \quad 0$. Find the absolute value of $(a \quad b \quad c)$ where a, b, c are integers expressed in lowest form.

14. If there a real value of   for which the image of point $(\; , \quad 1)$ by the line mirror $3x \quad y \quad 6$ is the point $(^{2} \quad 1, \;)$? Then find .

15. Straight line L_1 is parallel to the bisector of first and third quadrant, forms a triangle of area 2 square units with coordinate axis in second quadrant. Line L_2 passes through (1,1) and has positive x and y intercepts. L_2 makes a triangle of minimum are with coordinate axes. The area of the triangle formed by L_1, L_2 and x-axis is of form $\frac{p}{q}$ where p and q are relatively natural numbers. Find $|p \quad q|$.

16. Consider two lines $L_1 \quad x \quad y \quad 0$ and $L_2 \quad x \quad y \quad 0$ and a moving point $P(x, y)$. Let $d(P, L_1), i \quad 1,2$ represents the perpendicular distance of the point P from L_1. If point P moves in certain region R in such a way that $\displaystyle \sum_{i \; 1}^{2} d(P, L_1) \quad [2,4]$. Let the area of region R is A, then find $\frac{A}{4}$.

17. In a $\; ABC$, $A \quad (\; , \;)$, $B(1,2)$, $C(2,3)$ and point A lies on line $y \quad 2x \quad 3$, where $\; , \quad I$. If the area of $\; ABC$ be such that area of triangle lies in interval [2,3). Find the number of all possible coordinates of A.

18. Consider $\; ABC$ with $A(m, \quad m \quad 1)$, $B(\quad 1,0)$, $C(l, l \quad 1)$ is such that a line of slope 2, drawn through centroid of $\; ABC$ meets the circumcentre of $\; ABC$ on y-axis, then find the value of $l \quad m$.

19. A variable line L_1 cuts $y \quad 3x \quad 1$ and $y \quad 2x \quad 3$ at points P_1 and P_2. If the locus of midpoints of P_1 and P_2 is line L_2 with undefined slope where slope of L_1 is constant. If slope of L_1 is $\frac{p}{q}$, where p, q are coprime natural number, then find $p \quad q$.

20. Let A, B, C lies on lines $y \quad x$, $y \quad 2x$ and $y \quad 3x$ respectively. Also AB passes through fixed point $(1,0)$, BC Passes through fixed point $(0, \quad 1)$, then AC also passes through fixed point (h, k), find the value of $h \quad k$.

ANSWERS

1.	0	**2.**	1	**3.**	3	**4.**	6	**5.**	3	**6.**	14	**7.**	5	**8.**	2	**9.**	8	**10.**	8
11.	2	**12.**	7	**13.**	1	**14.**	2	**15.**	3	**16.**	6	**17.**	4	**18.**	0	**19.**	3	**20.**	0

EXERCISE 7

1. Let *PQR* be a right angled isosceles triangle, right angled at $P\,(2, 1)$. If the equation of the line *QR* is $2x \quad y \quad 3$, then the equation representing the pair of lines *PQ* and *PR* is: **[IIT-JEE 1999]**

(a) $3x^2 \quad 3y^2 \quad 8xy \quad 20x \quad 10y \quad 25 \quad 0$ (b) $3x^2 \quad 3y^2 \quad 8xy \quad 20x \quad 10y \quad 25 \quad 0$

(c) $3x^2 \quad 3y^2 \quad 8xy \quad 10x \quad 15y \quad 20 \quad 0$ (d) $3x^2 \quad 3y^2 \quad 8xy \quad 10x \quad 15y \quad 20 \quad 0$

2. The equation of two equal sides *AB* and *AC* of an isosceles triangle *ABC* are $x \quad y \quad 5$ and $7x \quad y \quad 3$ respectively. Find the equations of the side *BC* if the area of the triangle of *ABC* is 5 units. **[REE 1999]**

3. (A) The incentre of the triangle with vertices $(1, \sqrt{3}), (0, 0)$ and $(2, 0)$ is:

(a) $1, \frac{\sqrt{3}}{2}$ (b) $\frac{2}{3}, \frac{1}{\sqrt{3}}$ (c) $\frac{2}{3}, \frac{\sqrt{3}}{2}$ (d) $1, \frac{1}{\sqrt{3}}$

(B) Let *PS* be the median of the triangle with vertices, $P(2, 2), Q(6, \quad 1)$ and $R(7, 3)$. The equation of the line passing through $(1, \quad 1)$ and parallel to *PS* is:

[IIT-JEE (Screening) 2000]

(a) $2x \quad 9y \quad 7 \quad 0$ (b) $2x \quad 9y \quad 11 \quad 0$

(c) $2x \quad 9y \quad 11 \quad 0$ (d) $2x \quad 9y \quad 7 \quad 0$

(C) For points $P \quad (x_1, y_1)$ and $Q \quad (x_2, y_2)$ of the co-ordinate plane, a new distance $d(P, Q)$ is defined by $d(P, Q) \quad |x_1 \quad x_2| \quad |y_1 \quad y_2|$. Let $O \quad (0, 0)$ and $A \quad (3, 2)$. Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from *O* and *A* consists of the union of a line segment of finite length and an infinite ray. Sketch this set in a labelled diagram. **[IIT-JEE (Mains) 2000]**

4. Find the position of point $(4, 1)$ after it undergoes the following transformations successively.

(i) Reflection about the line, $y \quad x \quad 1$.

(ii) Translation by one unit along *x*-axis in the positive direction.

(iii) Rotation through an angle $\quad /4$ about the origin in the anticlockwise direction.

[REE (Mains) 2000]

5. (A) Area of the parallelogram formed by the lines $y \quad mx, y \quad mx \quad 1, y \quad nx$ and $y \quad nx \quad 1$ equals:

(a) $\frac{|m \quad n|}{(m \quad n)^2}$ (b) $\frac{2}{|m \quad n|}$ (c) $\frac{1}{|m \quad n|}$ (d) $\frac{1}{|m \quad n|}$

(B) The number of integer values of m, for which the *x* co-ordinate of the point of intersection of the lines $3x \quad 4y \quad 9$ and $y \quad mx \quad 1$ is also an integer, is:

[IIT-JEE (Screening) 2001]

(a) 2 (b) 0 (c) 4 (d) 1

6. (A) Let $P \quad (\ 1, 0), Q \quad (0, 0)$ and $R \quad (3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is:

(a) $\frac{\sqrt{3}}{2}x \quad y \quad 0$ (b) $x \quad \sqrt{3}y \quad 0$ (c) $\sqrt{3}x \quad y \quad 0$ (d) $x \quad \frac{\sqrt{3}}{2}y \quad 0$

(B) A straight line through the origin O meets the parallel lines $4x \quad 2y \quad 9$ and $2x \quad y \quad 6 \quad 0$ at points P and Q respectively. Then the point O divides the segment PQ in the ratio:

(a) $1:2$ (b) $3:4$ (c) $2:1$ (d) $4:3$

(C) The area bounded by the curves $y \quad |x| \quad 1$ and $y \quad\quad |x| \quad 1$ is:

[IIT-JEE (Screening) 2002]

(a) 1 (b) 2 (c) $2\sqrt{2}$ (d) 4

(D) A straight line L through the origin meets the line $x \quad y \quad 1$ and $x \quad y \quad 3$ at P and Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn, parallel to $2x \quad y \quad 5$ and $3x \quad y \quad 5$ respectively. Lines L_1 and L_2 intersect at R. Show that the locus of R, as L varies, is a straight line. **[IIT-JEE (Mains) 2002]**

(E) A straight line L with negative slope passes through the point $(8, 2)$ and cuts the positive co-ordinates axes at points P and Q. Find the absolute minimum value of $OP \quad OQ$, as L varies, where O is the origin. **[IIT-JEE (Mains) 2002]**

7. The area bounded by the angle bisectors of the lines $x^2 \quad y^2 \quad 2y \quad 1$ and the line $x \quad y \quad 3$, is:

[IIT-JEE (Screening) 2004]

(a) 2 (b) 3 (c) 4 (d) 6

8. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through $P(h, k)$ with the lines $y \quad x$ and $x \quad y \quad 2$ is $4h^2$. Find the locus of the point P.

[IIT-JEE (Mains) 2005]

9. (A) Let $O(0, 0), P(3, 4), Q(6, 0)$ be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The co-ordinates of R are:

(a) $(4/3, 3)$ (b) $(3, 2/3)$ (c) $(3, 4/3)$ (d) $(4/3, 2/3)$

(B) Lines $L_1 : y \quad x \quad 0$ and $L_2 : 2x \quad y \quad 0$ intersect the line $L_3 : y \quad 2 \quad 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1: The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$

because

Statement-2: In any triangle, bisector of an angle divides the triangle into two similar triangles.

(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true. **[IIT-JEE 2007]**

10. Consider the lines given by

$$L_1 \quad x \quad 3y \quad 5 \quad 0, \quad L_2 \quad 3x \quad ky \quad 1 \quad 0, \quad L_3 \quad 5x \quad 2y \quad 12 \quad 0$$

Match the statements/expressions in **Column-I** with the statements/expressions in **Column-II** and indicate your answer by darkening the appropriate bubbles in the 4 4 matrix given in OMR. **[IIT-JEE 2008]**

	Column-I		Column-II
(a)	L_1, L_2, L_3 are concurrent, if	**(p)**	$k \quad 9$
(b)	One of L_1, L_2, L_3 is parallel to at least one of the other two, if	**(q)**	$k \quad \frac{6}{5}$
(c)	L_1, L_2, L_3 form a triangle, if	**(r)**	$k \quad \frac{5}{6}$
(d)	L_1, L_2, L_3 do not form a triangle, if	**(s)**	$k \quad 5$

11. The locus of the orthocentre of the triangle formed by the lines $(1 \quad p)x \quad py \quad p(1 \quad p) \quad 0$, $(1 \quad q)x \quad qy \quad q(1 \quad q) \quad 0$, and $y \quad 0$, where $p \quad q$, is: **[IIT 2009]**

(a) a hyperbola (b) a parabola (c) an ellipse (d) a straight line

12. A straight line L through the point $(3, \quad 2)$ is inclined at an angle 60 to the line $\sqrt{3}x \quad y \quad 1$. If L also intersects the x-axis, then the equation of L is: **[IIT 2011]**

(a) $y \quad \sqrt{3}x \quad 2 \quad 3\sqrt{3} \quad 0$ (b) $y \quad \sqrt{3}x \quad 2 \quad 3\sqrt{3} \quad 0$

(c) $\sqrt{3}y \quad x \quad 3 \quad 2\sqrt{3} \quad 0$ (d) $\sqrt{3}y \quad x \quad 3 \quad 2\sqrt{3} \quad 0$

13. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as $(0, 1), (1, 1)$ and $(1, 0)$ is : **[IIT-JEE (Mains) 2013]**

(a) $1 \quad \sqrt{2}$ (b) $2 \quad \sqrt{2}$ (c) $2 \quad \sqrt{2}$ (d) $1 \quad \sqrt{2}$

14. A ray of light along $x \quad \sqrt{3}y \quad \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is : **[IIT-JEE (Mains) 2013]**

(a) $\sqrt{3}y \quad x \quad 1$ (b) $y \quad x \quad \sqrt{3}$ (c) $\sqrt{3}y \quad x \quad \sqrt{3}$ (d) $y \quad \sqrt{3}x \quad \sqrt{3}$

15. For $a \quad b \quad c \quad 0$, the distance between $(1, 1)$ and the point of intersection of the lines $ax \quad by \quad c \quad 0$ and $bx \quad ay \quad c \quad 0$ is less than $2\sqrt{2}$. Then : **[IIT-JEE (Advance) 2013]**

(a) $a \quad b \quad c \quad 0$ (b) $a \quad b \quad c \quad 0$ (c) $a \quad b \quad c \quad 0$ (d) $a \quad b \quad c \quad 0$

ANSWERS

1. b **2.** $x \quad 3y \quad 21 \quad 0, x \quad 3y \quad 1 \quad 0, 3x \quad y \quad 12, 3x \quad y \quad 2$

3. **(A)** d; **(B)** d **4.** $(4, 1) \quad (2, 3) \quad (3, 3) \quad (0, 3\sqrt{2})$ **5.** **(A)** d; **(B)** a

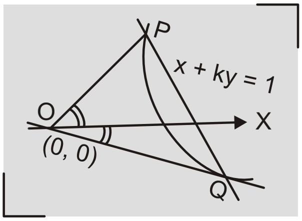
6. **(A)** c; **(B)** b; **(C)** b; **(D)** $x \quad 3y \quad 5 \quad 0$; **(E)** 18 **7.** a **8.** $y \quad 2x \quad 1, y \quad 2x \quad 1$

9. **(A)** c; **(B)** c **10.** (a) s; (b) p, q; (c) r; (d) p, q, s

11. d **12.** b **13.** c **14.** c **15.** a, c

Only One Choice is Correct:

1. (b) Equation of pair OP and OQ is obtained by homogenising.



Equation of pair OP and OQ is

$$5x^2 \quad 12xy \quad 6y^2 \quad (4x \quad 2y)(x \quad ky)$$

$$3(x \quad ky)^2 \quad 0$$

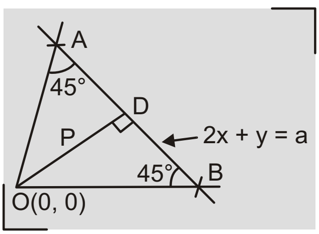
OP and OQ are equally inclined to x-axis.

Coefficient of xy $\quad 0$

$12 \quad 4k \quad 2 \quad 6k \quad 0$

$k \quad 1$

2. (c) $OD \quad AD \quad BD$



$OD \quad P \quad \dfrac{|a|}{\sqrt{5}}$

Area of $\quad OAB \quad \dfrac{1}{2}(2P)P \quad P^2$

Area $\quad \dfrac{a^2}{5}$

3. (a)

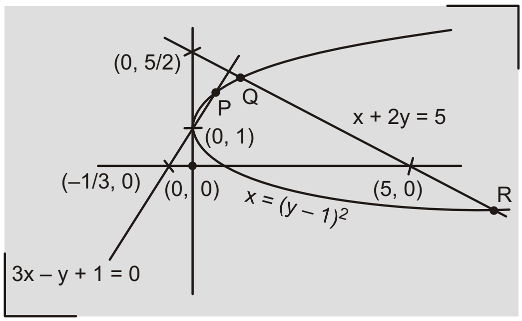
$\because [3(\quad 1) \quad 4(4) \quad 12][12(\quad 1) \quad 5(4) \quad 7] \quad 0$

Equation of bisector containing $(\quad 1, 4)$ in its region is

$$\frac{3x \quad 4y \quad 12}{5} \quad \frac{12x \quad 5y \quad 7}{13}$$

$21x \quad 27y \quad 121 \quad 0$

4. (a) Intersection of $x \quad (y \quad 1)^2$ with lines are



$(y \quad 1)^2 \quad \dfrac{y \quad 1}{3} \qquad y \quad 1,\ y \quad \dfrac{4}{3}$

$P \quad \dfrac{1}{9}, \dfrac{4}{3}$

$(y \quad 1)^2 \quad 5 \quad 2y \qquad y^2 \quad 4 \qquad y \qquad 2$

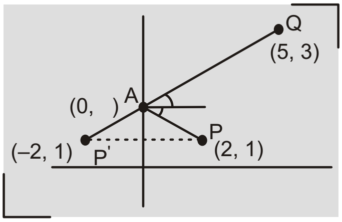
$Q \quad (1, 2),\ R \quad (9, \quad 2)$

$a \quad 1 \quad (\quad 2, 1) \qquad \dfrac{4}{3}, 2\ ;$

$a \quad (\quad 3, 0) \qquad \dfrac{1}{3}, 1$

5. (a) Equating slopes of PA and PQ.

$$\frac{1}{2} \quad \frac{3 \quad 1}{7}$$



$$\frac{11}{7} \qquad A \quad 0, \frac{11}{7}$$

6. (c) $(x \quad 3y)(x \quad y) \quad 0$

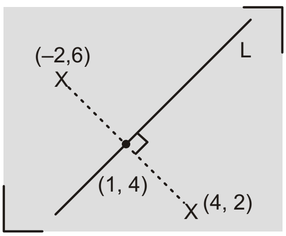
Equation of line parallel to line $x \quad 3y \quad 0$ and passing through $(3, \quad 2)$ is $L_1 \quad x \quad 3y \quad 9$

Similarly equation of line, parallel to line $x \quad y \quad 0$ and passing through $(3, \quad 2)$ is $L_2 \quad x \quad y \quad 5$

Equation of pair L_1 and L_2 is $(x \quad 3y \quad 9)(x \quad y \quad 5) \quad 0$

$x^2 \quad 4xy \quad 3y^2 \quad 14x \quad 24y \quad 45 \quad 0$

7. (c) Equation of L is



$$y \quad 4 \quad \frac{3}{2}(x \quad 1)$$

$2y \quad 8 \quad 3x \quad 3$

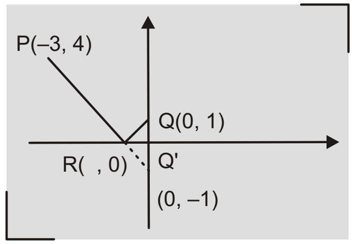
$3x \quad 2y \quad 5 \quad 0$

8. (b) $(PR \quad RQ)_{min} \quad (PR \quad RQ\ )_{min} \quad PQ$

$$\frac{0 \quad 4}{\quad 3} \quad \frac{4 \quad 1}{3 \quad 0}$$

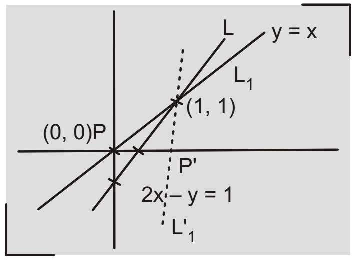
$$\frac{3}{5}$$

$$R \qquad \frac{3}{5}, 0$$



9. (b) Image of $(0,0)$ w.r.t. L lies on L_1

Image of $(0,0)$ w.r.t. L is



$$\frac{x \quad 0}{2} \quad \frac{y \quad 0}{1} \qquad 2 \quad \frac{1}{5}$$

$$P \qquad (x, y) \qquad \frac{4}{5}, \frac{2}{5}$$

Equation of L_1 which passes through $(1,1)$ and $\frac{4}{5}, \frac{2}{5}$ is

$$y \quad 1 \quad \frac{\frac{2}{5} \quad 1}{\frac{4}{5} \quad 1}(x \quad 1) \qquad \frac{7}{1}(x \quad 1)$$

$$y \quad 7x \quad 6$$

10. (d) $a \quad 2\sqrt{bc} \quad b \quad c$

$(\sqrt{b} \quad \sqrt{c})^2 \quad (\sqrt{a})^2 \quad 0$

$\sqrt{b} \quad \sqrt{c} \quad \sqrt{a} \quad 0$

or $\quad \sqrt{b} \quad \sqrt{c} \quad \sqrt{a} \quad 0$ (rejected)

$\sqrt{b} \quad \sqrt{c} \quad \sqrt{a} \quad 0$

$\sqrt{a}\,x \quad \sqrt{b}\,y \quad \sqrt{c} \quad 0$ passes through fixed point $(\quad 1, 1)$.

11. (b) $p_1 \quad \dfrac{|a^2 \quad 2a\tan \quad \tan^2 \quad |}{|\sec \quad |}$

$\dfrac{(a \quad \tan \quad)^2}{|\sec \quad |}$

$p_3 \quad \dfrac{(b \quad \tan \quad)^2}{|\sec \quad |}$

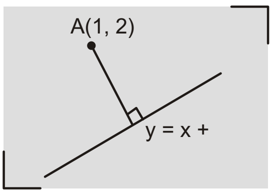
$p_2 \quad \dfrac{|ab \quad (a \quad b)\tan \quad \tan^2 \quad |}{|\sec \quad |}$

$\dfrac{|(a \quad \tan \quad)(b \quad \tan \quad)}{|\sec \quad |}$

$p_2^2 \quad p_1 p_3$

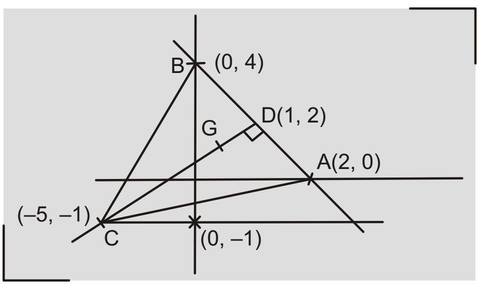
12. (b) Equation of BC varies

Orthocentre will always lie on line perpendicular to $y \quad x \qquad$ passing through $A(1,2)$.



Locus of orthocentre 'H' is $x \quad y \quad 3$.

13. (a) Slope of CD is $\dfrac{1}{2} \quad C \quad (\ 5, \ \ 1)$

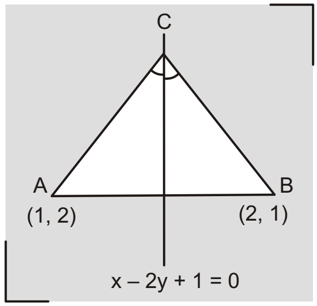


Perpendicular distance from G to AB

$\dfrac{1}{3}$ (Perpendicular distance from C to AB)

$\dfrac{1}{3} \quad \dfrac{|\ 10 \quad 1 \quad 4|}{\sqrt{4 \quad 1}} \quad \sqrt{5}$

14. (b) Image of A say A w.r.t. $x \quad 2y \quad 1 \quad 0$ lies on BC.



$\dfrac{x \quad 1}{1} \quad \dfrac{y \quad 2}{2} \quad 2\dfrac{(1 \quad 4 \quad 1)}{1 \quad 2^2} \quad \dfrac{4}{5}$

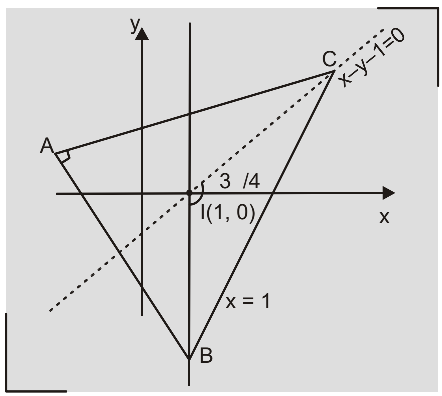
$A \quad \dfrac{9}{5}, \dfrac{2}{5}$

Equation of BC joining $A \quad \dfrac{9}{5}, \dfrac{2}{5}$ and $B(2,1)$ is

$y \quad 1 \quad \dfrac{1 \quad \dfrac{2}{5}}{2 \quad \dfrac{9}{5}}(x \quad 2) \quad \dfrac{3}{1}(x \quad 2)$

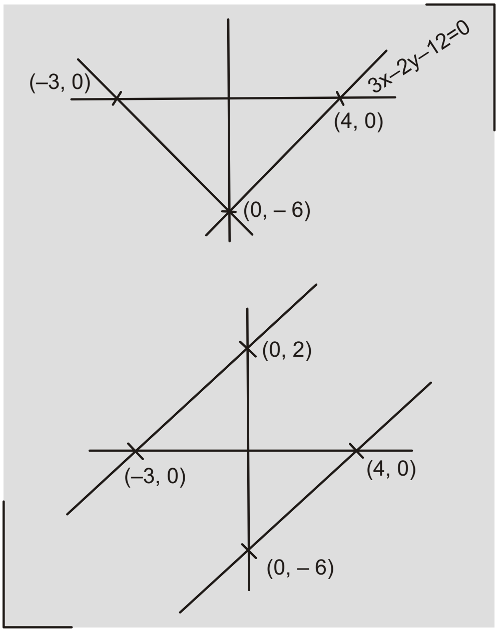
$3x \quad y \quad 5 \quad 0 \qquad a \quad b \quad 3 \quad 1 \quad 2$

15. (c) $BIC \quad \dfrac{3}{4} \quad \dfrac{\ }{2} \quad \dfrac{A}{2} \qquad A \quad \dfrac{\ }{2}$

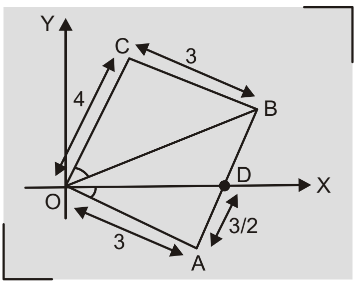


Hence, $BAC \quad \frac{}{2}$

16. (b) Possible cases are shown below



17. (c) $\tan \quad \frac{3}{4}, \tan \quad \frac{3}{2(3)} \quad \frac{1}{2}$



slope of $OB \quad \tan \quad \frac{}{2} \quad (\quad) \quad \cot(\quad)$

$$\frac{1 \quad \frac{3}{4} \frac{1}{2}}{\frac{3}{4} \quad \frac{1}{2}} \quad \frac{1}{2}$$

18. (c) Let perpendicular bisector of AB is $3x \quad 4y \quad 20 \quad 0$ and perpendicular bisector of AC is $8x \quad 6y \quad 65 \quad 0.$

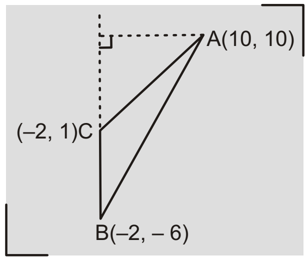
Image of A w.r.t. $3x \quad 4y \quad 20 \quad 0$ is B and image of A w.r.t. $8x \quad 6y \quad 65 \quad 0$ is C.

For B , $\frac{x \quad 10}{3} \quad \frac{y \quad 10}{4} \quad 2 \quad \frac{30 \quad 40 \quad 20}{25}$

$B \quad (\; 2, \; 6)$

For C , $\frac{x \quad 10}{8} \quad \frac{y \quad 10}{6} \quad 2 \quad \frac{80 \quad 60 \quad 65}{100}$

$C \quad (\; 2, 1)$

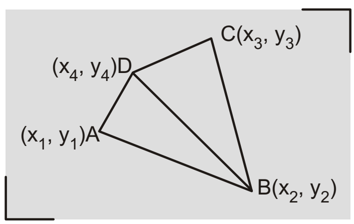


Area of $ABC \quad \frac{1}{2}(10 \quad 2)(1 \quad 6) \quad 42$

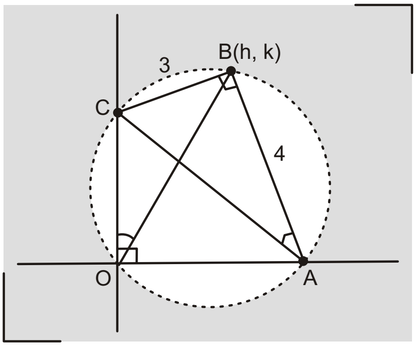
19. (b) Area of $ABCD$ mod of

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_4 & y_4 & 1 \end{vmatrix} \quad \text{mod of } \frac{1}{2}\begin{vmatrix} x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{vmatrix}$$

$$\frac{1}{2}(1) \quad \frac{1}{2}(1) \quad 1$$



20. (b) tan $\frac{3}{4}$ (from Fig.)



Slope of OB tan $\frac{}{2}$ cot

Locus of (h, k) is $\frac{k}{h} \quad \frac{4}{3}$

$3y \quad 4x$

21. (b) Lines make complementary angles with X-axis $m_1 m_2 \quad 1 \quad \frac{a}{b} \quad 1$

22. (c) $h \quad 2\cos \quad 2\cos(120 \quad)$

$\cos \quad \sqrt{3}\sin$ ...(1)

$k \quad 0 \quad 2\sin(120 \quad)$

$\sqrt{3}\cos \quad \sin$ ....(2)

squarring and adding eqs. (1) and (2), we get

$h^2 \quad k^2 \quad 4 \quad 4\sqrt{3}\sin \quad \cos$

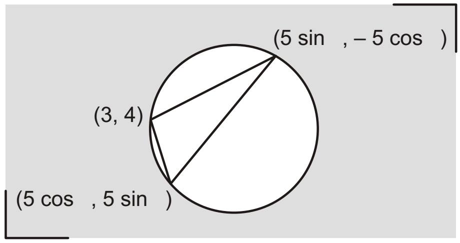
Multiplying eqs. (1) and (2), we get

$hk \quad \sqrt{3} \quad 4\sin \quad \cos$

$(h^2 \quad k^2) \quad 4 \quad \sqrt{3}hk \quad 3$

$x^2 \quad y^2 \quad \sqrt{3}\,xy \quad 1$ is the required locus.

23. (d) By observation, it is clear that 3 vertices of triangle lie on circle $x^2 \quad y^2 \quad 25$.



Centroid

$G \quad \frac{5\sin \quad 5\cos \quad 3}{3}, \frac{5\sin \quad 5\cos \quad 4}{3}$

Circumcentre $O \quad (0,0)$

Orthocentre, $H \quad (h, k)$

$3 \quad \frac{5\sin \quad 5\cos \quad 3}{3} \quad 2(0) \quad h$

$5\sin \quad 5\cos \quad 3$ ...(1)

$3 \quad \frac{5\sin \quad 5\cos \quad 4}{3} \quad 2(0) \quad k$

$5\sin \quad 5\cos \quad 4$ ...(2)

By eq. (1) eq. (2),

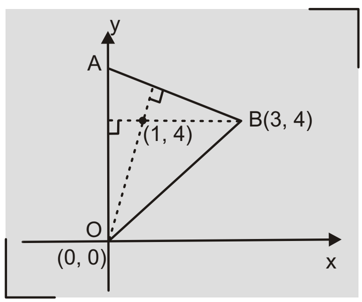
$h \quad k \quad 7 \quad 10\sin$

By eq. (1) eq. (2),

$h \quad k \quad 1 \quad 10\cos$

$(x \quad y \quad 7)^2 \quad (x \quad y \quad 1)^2 \quad 100$ is the required locus.

24. (d) $BH \quad OA \quad A$ lies on y-axis equation of AH is :



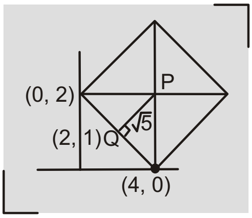
$y \quad 4 \quad \frac{3}{4}(x \quad 1)$

Put $x \quad 0,$

$$y \quad 4 \quad \frac{3}{4} \quad \frac{19}{4}$$

$$A \quad 0, \frac{19}{4}$$

25. (c) Slope of $PQ \quad 2 \quad \tan$



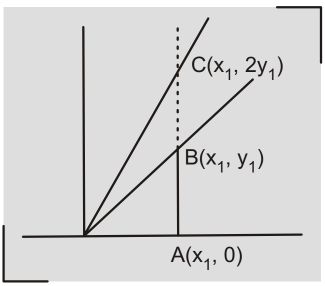
$$P \quad 2 \quad \sqrt{5}\ \frac{1}{\sqrt{5}}, 1 \quad \sqrt{5}\ \frac{2}{\sqrt{5}}$$

$$(3, 3)$$

26. (b) Put $x \quad x_1$ in equation of pair of lines

$$by^2 \quad 2hx_1 y \quad ax_1^2 \quad 0 \begin{matrix} y_1 \\ 2y_1 \end{matrix}$$

$$3y_1 \quad \frac{2hx_1}{b}, 2y_1^2 \quad \frac{ax_1^2}{b}$$

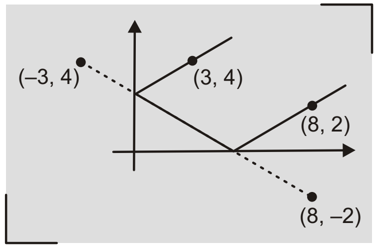


$$\frac{9y_1^2}{2y_1^2} \quad \frac{4h^2}{b^2} \frac{x_1^2 b}{ax_1^2}$$

$$\frac{9}{2} \quad \frac{4h^2}{ab} \qquad 9ab \quad 8h^2$$

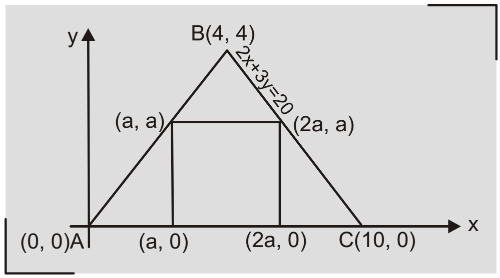
27. (b) $\frac{0 \quad 2}{x \quad 8} \quad \frac{4 \quad 2}{3 \quad 8} \quad \frac{6}{11}$

$$x \quad \frac{13}{3}$$



28. (a) $(2a, a)$ lies on $2x \quad 3y \quad 20$

So $4a \quad 3a \quad 20$



$$a \quad \frac{20}{7}$$

$$\text{Area} \quad \frac{400}{49}$$

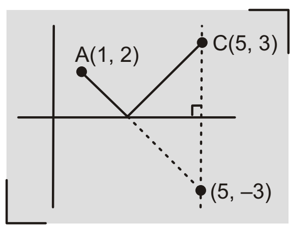
29. (d) Locus of C is a circle.

infinite $\quad ABC$ can be formed

30. (a) Reflection of C with x-axis $\quad (5, \quad 3)$

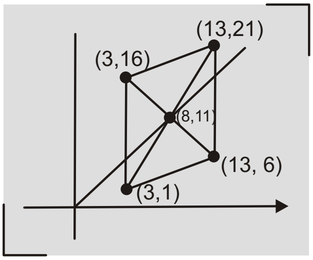
Equation of AB is $y \quad 2 \quad \frac{5}{4}(x \quad 1)$

$$5x \quad 4y \quad 13$$

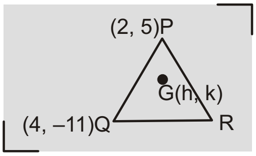


31. (b) Line passes through $(0,0)$ and $(8,11)$

its equation is $y \quad \frac{11}{8}x$



32. (d) $R \quad (3h \quad 6, 3k \quad 6)$



R lies on $9x \quad 7y \quad 4 \quad 0$.

$9(3h \quad 6) \quad 7(3k \quad 6) \quad 4 \quad 0$

$(9h \quad 7k)3 \quad 8 \quad 0$

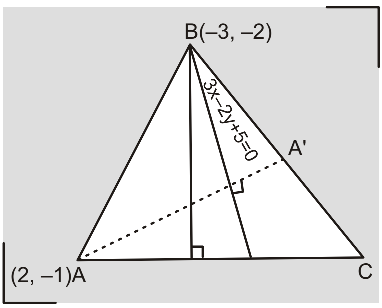
Which is parallel to N.

33. (b) Image of $A(2, \quad 1)$ w.r.t.

$3x \quad 2y \quad 5 \quad 0, A$ is given by

$$\frac{x \quad 2}{3} \quad \frac{y \quad 1}{2} \quad 2 \frac{(6 \quad 2 \quad 5)}{13} \quad 2$$

$A \quad (\ 4, 3)$



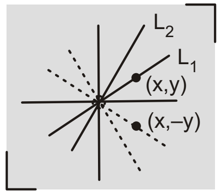
Equation of BC is

$$y \quad 3 \quad \frac{3 \quad 2}{4 \quad 3}(x \quad 4)$$

$5x \quad y \quad 17 \quad 0$

34. (d) $ax^2 \quad by^2 \quad 2hxy \quad 0$

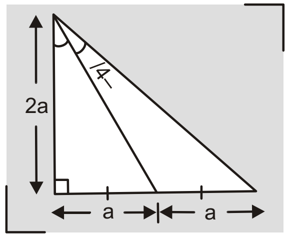
Put $x \quad X, y \quad Y$, in the equation of pair L_1 and L_2.



$aX^2 \quad b(\ Y)^2 \quad 2h(X)(\ Y) \quad 0$

$aX^2 \quad bY^2 \quad 2hXY \quad 0$ is the required equation.

35. (a)



$$\tan \quad \frac{a}{2a} \quad \frac{1}{2}$$

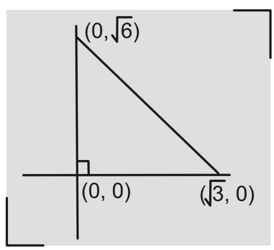
$$\tan \quad \overline{4} \qquad \frac{1 \quad 1/2}{1 \quad 1/2} \quad \frac{1}{3}$$

36. (d) $p(x \quad y \quad 1) \quad q(2x \quad 3y \quad 1) \quad 0$

$$(x \quad y \quad 1) \quad \frac{q}{p}(2x \quad 3y \quad 1) \quad 0$$

Always passes through intersection of $x \quad y \quad 1 \quad 0$ and $2x \quad 3y \quad 1 \quad 0$, which is $\frac{2}{5}, \frac{3}{5}$.

37. (d) Orthocentre is $(0,0)$.



38. (b) $y \quad mx \quad 0, x \quad 2y \quad 1 \quad 0$ and $2x \quad y \quad 3 \quad 0$ are concurrent

$$\begin{vmatrix} m & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} \quad 0 \quad 5m \quad 5 \qquad m \qquad 1$$

39. (b) $(x \quad 2) \qquad (x \quad y \quad 1) \quad 0$

Fixed point, $P \quad (\ 2, 1)$

Distance of P from origin $\quad 2$

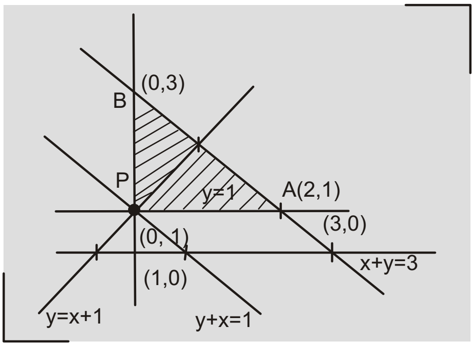
40. (a) $ax \quad (a \quad 2d)\, y \quad (a \quad 6d) \quad 0$

$a(x \quad y \quad 1) \quad 2d(y \quad 3) \quad 0$

$(x \quad y \quad 1) \quad \frac{2d}{a}(y \quad 3) \quad 0$

Fixed point $\quad (2, \quad 3)$ which lies on $x^2 \quad y^2 \quad 13$

41. (a) PAB is the required triangle .



Area of $\quad PAB \quad \frac{1}{2} \quad 2 \quad 2 \quad 2$

42. (a) For point of intersection,

$3x \quad 4mx \quad 4 \quad 9$

$x \quad \frac{5}{3 \quad 4m}$

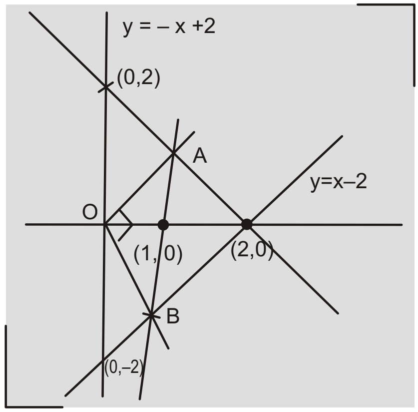
$3 \quad 4m \qquad 1, \quad 5$

$m \qquad 1/2\,,\ 1, 1/2\,,\ \ 2$

Number of integral values of $m \quad 2$

43. (d) Let the line be $y \quad m\,(x \quad 1)$

Equation of pair of lines is $(y \quad x \quad 2)(y \quad x \quad 2) \quad 0$



$y^2 \quad x^2 \quad 4x \quad 4 \quad 0$

Equation of pair OA and OB is obtained by homogenisation given by

$$y^2 \quad x^2 \quad 4x \ \frac{mx \quad y}{m} \quad 4 \ \frac{mx \quad y}{m}^2 \quad 0$$

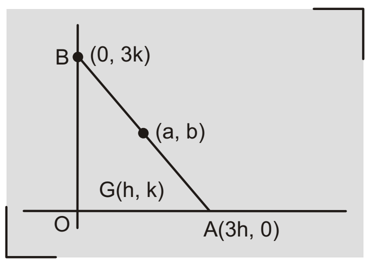
$\because \quad OA \quad OB$

Coefficient of $x^2 \quad$ Coefficient of $y^2 \quad 0$

$1 \quad 1 \quad 4 \quad 4 \quad \frac{4}{m^2} \quad 0 \qquad m$

Line is given by $x \quad 1$.

44. (a) Equation of AB is $\frac{x}{h} \quad \frac{y}{k} \quad 3$



$\because \quad AB$ passes through (a, b)

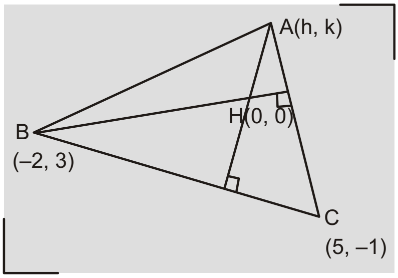
$\frac{a}{h} \quad \frac{b}{k} \quad 3$

$bx \quad ay \quad 3xy$ is the required locus.

45. (c) Slope of AH $\quad \frac{k}{h} \quad \frac{7}{4}$

Slope of AC $\quad \frac{k \quad 1}{h \quad 5} \quad \frac{2}{3}$

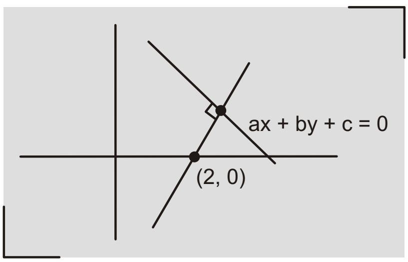
So, $3 \quad \frac{7}{4}h \quad 1 \quad 2(h \quad 5)$



$h \quad 4, k \quad 7$

$A \quad (\quad 4, \quad 7)$

46. (d) Equation of line after rotation becomes

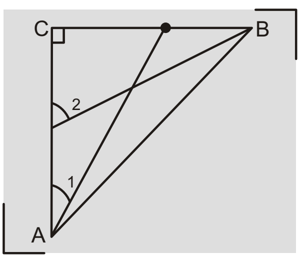


$bx \quad ay \quad c$

$\because$ It passes through $(2,0)$. $\quad c \quad 2b$

Equation of line is $bx \quad ay \quad 2b$.

47. (b) $\tan \quad _1 \quad \frac{BC}{2AC}$



$\tan \quad _2 \quad \frac{BC}{(AC/2)} \quad \frac{2BC}{AC}$

$\frac{\tan \quad _1}{\tan \quad _2} \quad \frac{1}{4}$

Case-I : If $m \quad 3$,

$\tan \quad _1 \quad \frac{1}{3}$ and $\tan \quad _2 \quad \frac{1}{m}$

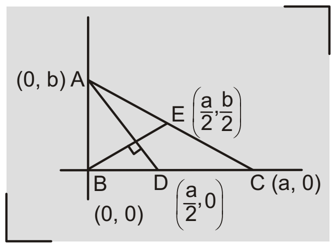
$\frac{m}{3} \quad \frac{1}{4} \qquad m \quad \frac{3}{4}$

Case-II : If $\qquad m \quad 3$,

$\tan \quad _1 \quad \frac{1}{m}$ and $\tan \quad _2 \quad \frac{1}{3}$

$\frac{3}{m} \quad \frac{1}{4} \qquad m \quad 12$

48. (b) $AD \quad BE$

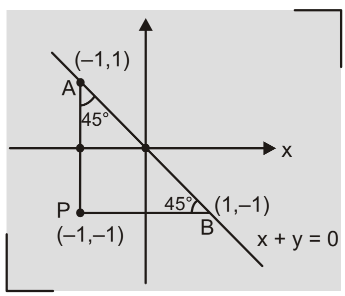


$\frac{b}{a} \quad \frac{b}{(a/2)} \qquad 1$

$2b^2 \quad a^2$

$a \qquad \sqrt{2}\, b$

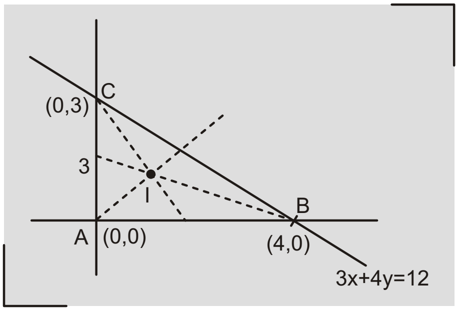
49. (d) Equation of pair PA and PB is



$(x \quad 1)(y \quad 1) \quad 0$

$xy \quad x \quad y \quad 1 \quad 0$

50. (a) Point P must lie on at least one of the angle bisectors.



Incentre,

$$I \quad \frac{4(0) \quad 3(4) \quad 5(0)}{4 \quad 3 \quad 5}, \frac{4(3) \quad 3(0) \quad 5(0)}{4 \quad 3 \quad 5}$$

$I \quad (1,1)$

$P \quad (1,1)$ only

51. (d) Lines are

$x \quad y \quad 1 \quad 0; 4x \quad 3y \quad 4 \quad 0$

and $\quad x \quad y \quad 0$

where, $\quad ^2 \quad ^2 \quad 2$

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & & \end{vmatrix} \quad 0$$

$1(3 \quad 4 \ ) \quad 1(4 \quad 4) \quad 1(4 \quad 3)$

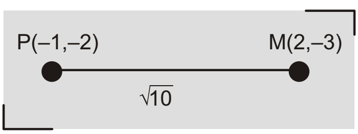
$3 \quad 4 \quad 4 \quad 4 \quad 4 \quad 3$

$1 \quad 0 \quad 1$

1

52. (a) Homogeneous equation of the curve with line. Coefficient of x^2 coefficient of $y^2 \quad 0$

53. (b) The point of intersection of the two lines are $(\ 1, \ 2)$

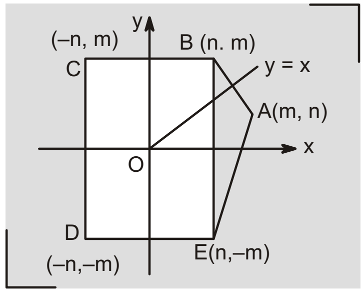


Distance $PM \quad \sqrt{10}$

Hence the required line is one which passes through $(\ 1, \ 2)$ and is to P.M.

B

54 (b) Area of rectangle $BCDE \quad 4mn$



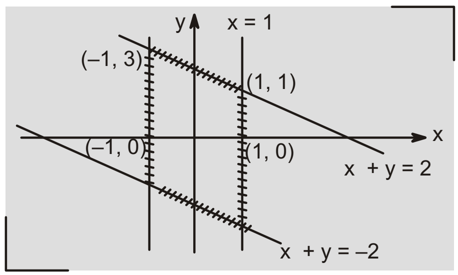
$$\text{Area of } \quad ABE \quad \frac{2m(m \quad n)}{2}$$

$m^2 \quad mn$

area of pentagon $\quad 4mn \quad m^2 \quad mn$

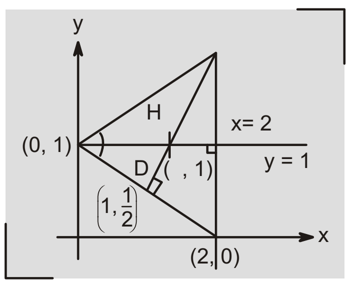
$m^2 \quad 3mn$

55. (d) Figure is a parallelogram



$$\text{Area} \quad 2 \quad \frac{1 \quad 3}{2}.2 \quad 8$$

56. (b)



Slope of $HD \quad 2$

$$1\ \frac{\frac{1}{2}}{1}\quad 2 \qquad\qquad \frac{5}{4}$$

$$H\quad \frac{5}{4},1$$

57. (a) Image of point $(2,1)$ lying on $2y \quad x \quad 0$ w.r.t. $4x \quad 3y \quad 0$ is

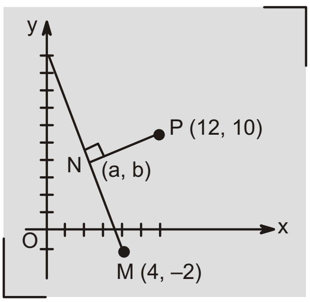
$$\frac{x \quad 2}{4} \quad \frac{y \quad 1}{3} \quad 2\frac{8 \quad 3}{25}$$

$$(x,y) \quad \frac{38}{25}, \quad \frac{41}{25}$$

Other line passes through $(0,0)$ and $\frac{38}{25}, \quad \frac{41}{25}$ and is given by

$$y \quad \frac{41}{38}x$$

58. (b) (a,b) is the foot of ar of (12, 10) on the line $y \quad 5x \quad 18$



$$\frac{a \quad 12}{5} \quad \frac{b \quad 10}{1} \qquad \frac{10 \quad 5(12) \quad 18}{26}$$

$$(a,b) \quad (2,8)$$

59. (c) Let $m \quad \tan 2 \quad$ and $\quad n \quad \tan$

and $m \quad 4n$ and $m \quad 0,$

$\tan 2 \quad 0;$ $\qquad\qquad 0$

$$\tan 2 \quad 4\tan$$

$$\frac{2\tan}{1 \quad \tan^2} \quad 4\tan$$

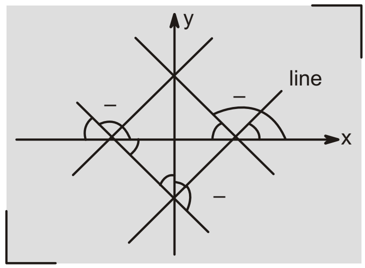
$$1 \quad 2 \quad 2\tan^2$$

$$2\tan^2 \quad 1 \qquad \tan^2$$

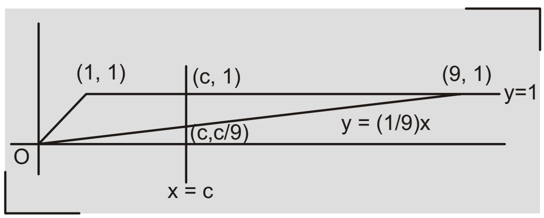
$$mn \quad \tan 2 \quad \tan \quad \frac{2\tan^2}{1 \quad \tan^2}$$

$$\frac{1}{1 \quad (1/2)} \quad 2$$

60. (c) Reflecting a graph over the x-axis results in the line M whose equation is $ax \quad by \quad c$, while a reflection through the y-axis results in the line N whose equation is $ax \quad by \quad c$. Both clearly have slope equal to a/b (from, say, the slope-intercept form of the equation.)



61. (b)

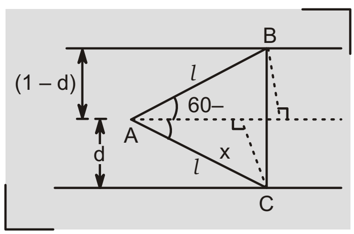


$$2\ \frac{1}{2}\ 1\ \frac{c}{9}(9 \quad c)\ \frac{1}{2}\ 8\ 1$$

$$c \quad 3$$

or $c \quad 15$ which is not possible

62. (b)



$$l\sin \qquad d \qquad\qquad ...(1)$$

$l\sin(60 \quad) \quad l\frac{\sqrt{3}}{2}\cos \quad \frac{l}{2}\sin$

$$l\cos \quad \frac{\frac{1 \quad d}{2 \quad d}}{\sqrt{3}} \quad ...(2)$$

Squaring and adding Eqns. (1) and (2)

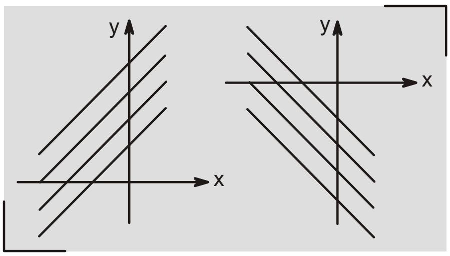
$$l^2 \quad d^2 \quad \frac{4 \quad d^2 \quad 4d}{33} \quad \frac{4}{3}(d^2 \quad d \quad 1)$$

$$l \quad 2\sqrt{\frac{d^2 \quad d \quad 1}{3}}$$

63. (b) $mb \quad 0 \qquad m \quad 0$

and $b \quad 0$ or $m \quad 0$ and $b \quad 0$

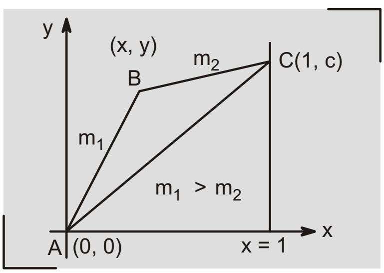
Hence possible lines are as shown



In both the cases x intercept cannot be +ve

(b)

64. (d) Let the coordinates of C be $(1, c)$



$$m_2 \quad \frac{c \quad y}{1 \quad x}; \quad m_2 \quad \frac{c \quad m_1 x}{1 \quad x}$$

$$m_2 \quad m_2 x \quad c \quad m_1 x$$

$$(m_1 \quad m_2)x \quad c \quad m_2$$

$$c \quad (m_1 \quad m_2)x \quad m_2 \quad ...(1)$$

now area of ABC $\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x & m_1 x & 1 \\ 1 & c & 1 \end{vmatrix}$

$$\frac{1}{2}[cx \quad m_1 x]$$

$$\frac{1}{2}|[((m_1 \quad m_2)x \quad m_2)x \quad m_1 x]|$$

$$\frac{1}{2}|[(m_1 \quad m_2)x^2 \quad m_2 x \quad m_1 x]|$$

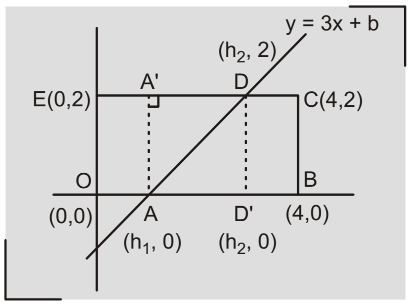
$$\frac{1}{2}(m_1 \quad m_2)(x \quad x^2)$$

$[\because x \quad x^2$ in $(0,1]$

Hence, $f(x) \quad \frac{1}{2}(x \quad x^2)$;

$f(x)]_{max} \quad \frac{1}{8}$ when $x \quad \frac{1}{2}$

65. (c)

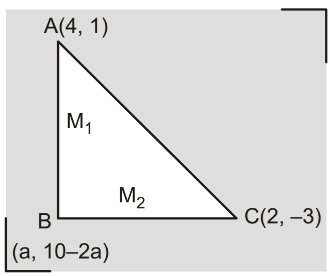


Line will pass through $(2, 1)$

Eqn. of line is $y \quad 3x \quad 5$

y-intercept is $(0, \quad 5)$

66. (b) $M_1 M_2 \quad 1$



$$\frac{9 \quad 2a}{a \quad 4} \quad \frac{13 \quad 2a}{a \quad 2} \quad 1$$

$117 \quad 26a \quad 18a \quad 4a^2 \quad (a^2 \quad 6a \quad 8)$

$5a^2 \quad 50a \quad 125 \quad 0$

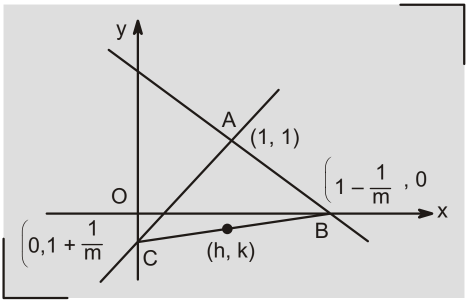
so B is (5, 0)

so area $\frac{1}{2} AB \quad AC \quad \frac{1}{2}\sqrt{2} \quad 3\sqrt{2} \quad 3$

67. (a) $y \quad 1 \quad m(x \quad 1)$

$y \quad 1 \quad \frac{1}{m}(x \quad 1)$

$2h \quad 1 \quad \frac{1}{m}$



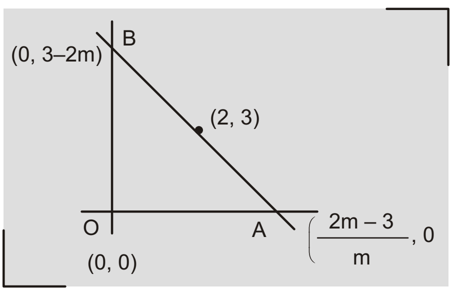
$2k \quad 1 \quad . \frac{1}{m}$

locus is $\overline{\underline{x \quad y \quad 1}}$

68. (c) Equation of any line through (2, 3) is

$y \quad 3 \quad m(x \quad 2)$

$y \quad mx \quad 2m \quad 3$



with the help of the fig. area of $OAB \quad 12$

i.e., $\frac{1}{2} \; \frac{2m \quad 3}{m} \; (3 \quad 2m) \quad 12$

taking + sign we get $(2m \quad 3)^2 \quad 0$

this gives one value of $m \quad 3/2$

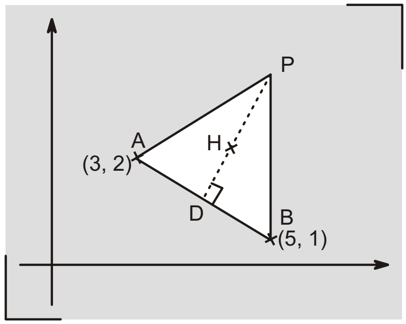
taking negative sign we get

$4m^2 \quad 36m \quad 9 \quad 0(D \quad 0)$

quadratic in m gives 2 values of m

3 st. lines are possible.

69. (d)



$PD \quad AB \sin 60 \quad \sqrt{5} \; \frac{\sqrt{3}}{2}$

$HD \quad \frac{1}{3} \; \frac{\sqrt{5}\sqrt{3}}{2} \quad \frac{\sqrt{5}}{2\sqrt{3}}$

Slope of $HD \quad 2, D \quad 4, \frac{3}{2}$

Using parametric, form

$H \quad 4 \quad \frac{\sqrt{5}}{2\sqrt{3}} \quad \frac{1}{\sqrt{5}}, \frac{3}{2} \quad \frac{\sqrt{5}}{2\sqrt{3}} \quad \frac{2}{\sqrt{5}}$

$H \quad 4 \quad \frac{\sqrt{3}}{6}, \frac{3}{2} \quad \frac{\sqrt{3}}{3}$

70. (c) $\frac{1}{x_p} \quad a \quad (p \quad 1)d;$

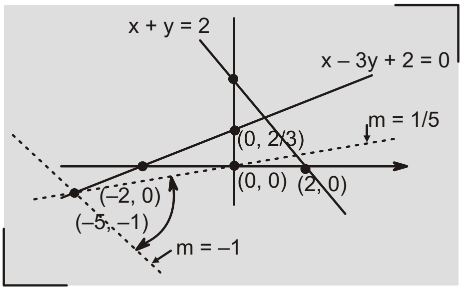
$\frac{1}{x_q} \quad a \quad (q \quad 1)d; \quad \frac{1}{x_r} \quad a \quad (r \quad 1)d$

$$\begin{vmatrix} a & (p & 1)d & p & 1 \\ a & (q & 1)d & q & 1 \\ a & (r & 1)d & r & 1 \end{vmatrix} \quad 0$$

One or More than One is/are Correct

1. (b, c, d)

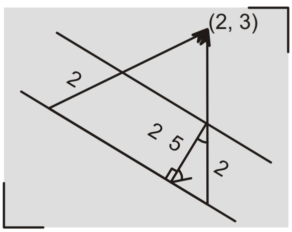
$m \quad 1, \frac{1}{5}$ for origin to lie inside the triangle



2. (b, d)

Slope of line is $m \quad \tan(\quad)$

$\tan \quad \frac{1}{2} . \tan \quad 2$



$$m \quad \frac{\frac{1}{2} \quad 2}{1 \quad \frac{1}{2} \quad 2}$$

$$m \quad \frac{\frac{1}{2} \quad 2}{1 \quad \frac{1}{2} \quad 2} \quad \frac{3}{4}$$

eqn. of lines are

$x \quad 2$ and $y \quad 3 \quad \frac{3}{4}(x \quad 2)$

$x \quad 2$ and $4y \quad 3x \quad 18$

3. (a, b)

The diagonal of rhombus is parallel to angle bisector of given lines

$$\frac{y \quad x \quad 2}{\sqrt{2}} \qquad \frac{y \quad 7x \quad 3}{5\sqrt{2}}$$

$4y \quad 2x \quad 7 \quad 0, 6y \quad 12x \quad 13 \quad 0$

Diagonals are $2y \quad x \quad 5$

and $2x \quad y \quad 0$

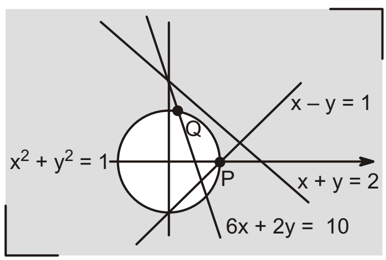
Possible coordinates of A are $0, \frac{5}{2}$ or (0, 0)

4. (a, b, c, d)

For Q:

$6\cos \quad 2\sin \quad \sqrt{10}$

$\tan^{1} 3 \quad , \sin(\quad) \quad \frac{1}{2}$



$$\frac{5}{6}$$

$$\frac{5}{6} \quad \tan^{1} 3$$

$$0, \frac{5}{6} \quad \tan^{1} 3$$

5. (a, c)

3rd side will be parallel to angle bisectors of given lines,

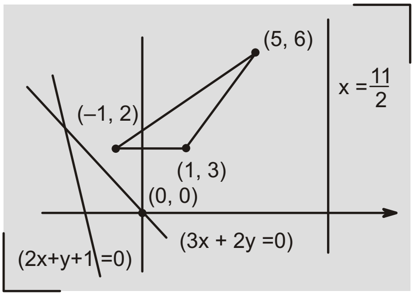
$$\frac{7x \quad y \quad 3}{5\sqrt{2}} \qquad \frac{x \quad y \quad 3}{\sqrt{2}}$$

$2x \quad 6y \quad c$

3rd side will be given by

$x \quad 3y \quad 31$ or $3x \quad y \quad 7$

6. (a, b, c)



7. (a, d)

$m \quad \tan \quad _1, m' \quad \tan \quad _2$

inclination of given lines are $2 \quad _1$ and $2 \quad _2$

inclination of angle bisectors $_1 \qquad _2$ or

$_1 \qquad _2 \qquad \overline{}\, 2$

slopes will be $\dfrac{m \quad m'}{1 \quad mm'}, \dfrac{mm' \quad 1}{m \quad m}$

Eqn. of bisectors are

$$y \quad b \quad \frac{(m \quad m')}{(1 \quad mm')}(x \quad a),$$

$$y \quad b \quad \frac{(mm' \quad 1)}{(m \quad m')}(x \quad a)$$

8. (a, b)

BC is parallel to angle bisector of lines $2x \quad y \quad 5$ and $x \quad 2y \quad 3$

$$\frac{2x \quad y \quad 5}{\sqrt{5}} \qquad \frac{x \quad 2y \quad 3}{\sqrt{5}} \qquad x \quad 3y \quad c$$

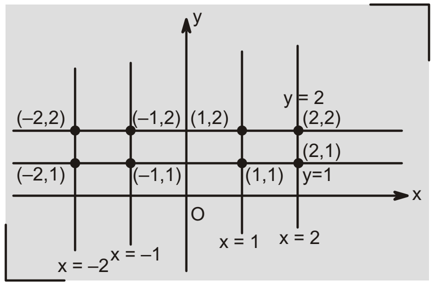
Eqn. of *BC* can be $x \quad 3y \quad 11$

or $3x \quad y \quad 3$

9. (b, c)

Triangle formed by lines is obtuse.

10. (a, b)

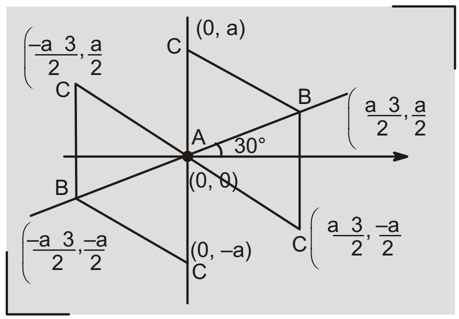


Possible squares have vertices

$(\quad 2, 1), (\quad 2, 2), (\quad 1, 1), (\quad 1, 2)$

or (1, 1), (1, 2), (2, 1), (2, 2)

11. (a, b, c, d)

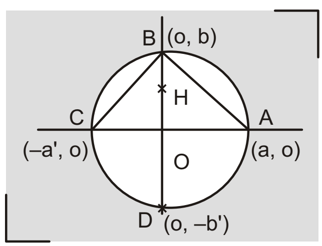


Possible ordinates of *C* are (0, a), (0, –*a*),

$$\frac{a\sqrt{3}}{2}, \frac{a}{2} \quad , \quad \frac{a\sqrt{3}}{2}, \frac{a}{2}$$

12. (b, c)

H is the image of *D*



$H \quad (0, b')$

Applying power of 'O'

$$aa' \quad bb' \qquad b' \quad \frac{aa'}{b}$$

13. (b, c)

$$(3x \quad 4y \quad 5)(x \quad 2y \quad 3) \quad 0$$

$$m_1 \quad \frac{3}{4}, m_2 \quad \frac{1}{2} \qquad m_1 m_2 \quad \frac{3}{8}$$

$$P \quad (1,-2)$$

$$\tan \quad \frac{\frac{1}{2} \quad \frac{3}{4}}{1 \quad \frac{3}{8}} \quad \frac{2}{11}$$

$$\sin \quad \frac{2}{5\sqrt{5}}$$

14. (b, d)

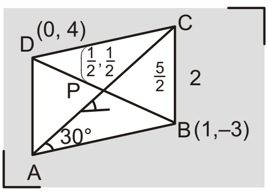
$$\text{Let } C \quad (\ , \qquad 2)$$

$$\frac{1}{2} \quad 8 \quad | \qquad 2| \quad 20$$

$$2 \quad 5, \qquad 7, \quad 3$$

$$C \quad (7, 5), (\ 3, \ 5)$$

15. (a, b)



$$\tan \quad \frac{1}{7}, \sin \qquad \frac{1}{5\sqrt{2}}, \cos \quad \frac{7}{5\sqrt{2}}$$

$$AP \quad \frac{5}{2}\sqrt{2}\cot 30 \qquad \frac{5}{2}\sqrt{2}\sqrt{3}$$

$$A \quad \frac{1}{2} \quad \frac{5\sqrt{2}\sqrt{3}}{2} \frac{7}{5\sqrt{2}}, \frac{1}{2} \quad \frac{5\sqrt{2}\sqrt{3}}{2} \frac{1}{5\sqrt{2}}$$

$$\text{or, } \frac{1}{2} \quad \frac{5\sqrt{2}\sqrt{3}}{2} \frac{7}{5\sqrt{2}}, \frac{1}{2} \quad \frac{5\sqrt{2}\sqrt{3}}{2} \frac{1}{5\sqrt{2}}$$

$$A \quad \frac{1 \quad 7\sqrt{3}}{2}, \frac{1 \quad \sqrt{3}}{2}$$

$$\text{or} \quad \frac{1 \quad 7\sqrt{3}}{2}, \frac{1 \quad \sqrt{3}}{2}$$

16. (a, b)

L must be angle bisector of L_1 and L_2

L is given by

$$\frac{3x \quad 4y \quad 1}{5} \qquad \frac{5x \quad 12y \quad 2}{13}$$

$$14x \quad 112y \quad 23 \quad 0,$$

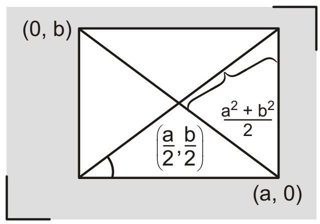
$$64x \quad 8y \quad 3 \quad 0$$

17. (c, d)

P will be on line parallel to $y \quad x$ at a perpendicular distance of $\frac{1}{\sqrt{2}}$

locus of P will be $y \quad x \quad 1$ or $y \quad x \quad 1$

18. (a, c)



Other vertices are

$$\frac{a}{2} \quad \frac{\sqrt{a^2 \quad b^2}}{2} \frac{b}{\sqrt{a^2 \quad b^2}},$$

$$\frac{b}{2} \quad \frac{\sqrt{a^2 \quad b^2}}{2} \frac{a}{\sqrt{a^2 \quad b^2}}$$

$$\text{and} \quad \frac{a}{2} \quad \frac{\sqrt{a^2 \quad b^2}}{2} \frac{b}{\sqrt{a^2 \quad b^2}},$$

$$\frac{b}{2} \quad \frac{\sqrt{a^2 \quad b^2}}{2} \frac{a}{\sqrt{a^2 \quad b^2}}$$

$$\frac{a \quad b}{2}, \frac{a \quad b}{2} \quad \text{and} \quad \frac{a \quad b}{2}, \frac{b \quad a}{2}$$

19. (a, b, c, d)

$$\frac{m_1}{m_2} \quad \frac{9}{2}, \quad \frac{m_1 \quad m_2}{1 \quad m_1 m_2} \qquad \frac{7}{9}$$

$$9m_2^2 \quad 9m_2 \quad 2 \quad 0,$$

$$9m_2^2 \quad 9m_2 \quad 2 \quad 0$$

$$m_2 \quad \frac{2}{3}, \frac{1}{3} m_2 \quad \frac{2}{3}, \frac{1}{3}$$

$$m_2 \quad \frac{2}{3}, m_1 \quad 3$$

$$y \quad 3x, 3y \quad 2x$$

$$m_2 \quad \frac{1}{3}, m_1 \quad \frac{3}{2}$$

$$3y \quad x, 2y \quad 3x$$

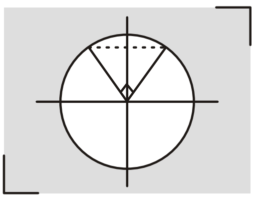
$$m_2 \quad \frac{2}{3}, m_1 \quad 3$$

$$3x \quad y \quad 0, 2x \quad 3y \quad 0$$

$$m_2 \quad \frac{1}{3}, m_1 \quad \frac{3}{2}$$

$$x \quad 3y \quad 0, 3x \quad 2y \quad 0$$

20. (a, b, c, d)



$$\frac{}{2}, \quad \frac{3}{2}$$

$$\frac{}{2} \qquad \frac{}{4}, \quad \frac{3}{4}$$

$$\cos \frac{}{2} \qquad \frac{1}{\sqrt{2}}, \quad \frac{1}{\sqrt{2}}$$

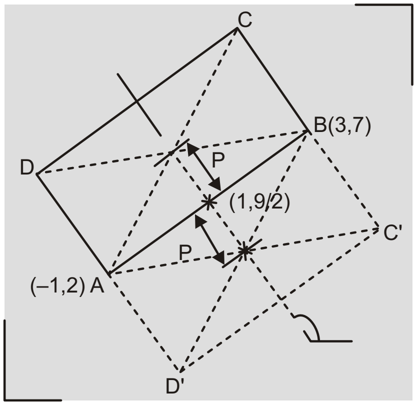
$$\sin \frac{}{2} \qquad \frac{1}{\sqrt{2}}$$

21. (b, d)

$$BC \quad \frac{3}{4} AB \quad \frac{3}{4}\sqrt{41}$$

$$P \quad \frac{1}{2} BC \quad \frac{3}{8}\sqrt{41}$$

$$\tan \qquad \frac{4}{5}$$



Intersection point of diagonals can be

$$1 \quad \frac{3}{8}\sqrt{41} \quad \frac{5}{\sqrt{41}}, \frac{9}{2} \quad \frac{3}{8}\sqrt{41} \quad \frac{4}{\sqrt{41}},$$

$$\text{or} \quad 1 \quad \frac{3}{8}\sqrt{41} \quad \frac{5}{\sqrt{41}}, \frac{9}{2} \quad \frac{3}{8}\sqrt{41} \quad \frac{4}{\sqrt{41}}$$

$$i.e. \quad \frac{7}{8}, 6 \quad \text{or} \quad \frac{23}{8}, 3$$

$$d \quad \sqrt{\frac{49}{64} \quad 36} \quad \text{or} \quad \sqrt{\frac{529}{64} \quad 9}$$

Hence, $[d] \quad 6$ and 4

22. (a, d)

$$\sin \quad \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{6}}{3}} \quad \frac{1}{2} \qquad \frac{}{6}$$

Angle made by L with positive x-axis can be

$\frac{}{4}\quad\frac{}{6}$ and $\frac{}{4}\quad\frac{}{6}$

$\frac{}{12}$ and $\frac{5}{12}$

23. (a, d)

$$D \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

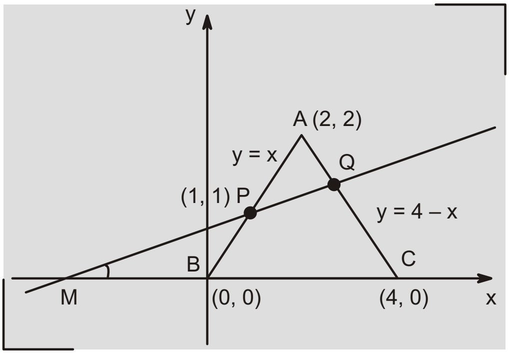
Applying $C_1 \quad C_1 \quad C_2$, we get,

$$D \begin{vmatrix} x_1 & y_1 & y_1 & 1 \\ x_2 & y_2 & y_2 & 1 \\ x_3 & y_3 & y_3 & 1 \end{vmatrix} \quad \begin{vmatrix} 8 & y_1 & 1 \\ 8 & y_2 & 1 \\ 8 & y_3 & 1 \end{vmatrix} \quad 0$$

Comprehension: (1)

Equation of line PM:

$$y \quad 1 \quad \tan \quad (x \quad 1)$$



Intersection point 'Q' of AC and MP

$$4 \quad x \quad 1 \quad \tan \quad (x \quad 1)$$

$$Q \quad \frac{3 \quad \tan}{1 \quad \tan}, \frac{1 \quad 3\tan}{1 \quad \tan}$$

Area of $\quad APQ$ = modulus of

$$\frac{1}{2}\begin{vmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ & & 1 \\ \frac{3 \quad \tan}{1 \quad \tan} & \frac{1 \quad 3\tan}{1 \quad \tan} & 1 \end{vmatrix} \quad \frac{1 \quad \tan}{1 \quad \tan}$$

1. (d) Area of quadrilateral $BPQC$,

$$A \quad \frac{1}{2} \quad 4 \quad 2 \quad \frac{1 \quad \tan}{1 \quad \tan}$$

$$\frac{3 \quad 5\tan}{1 \quad \tan}$$

2. (d) $A \quad 5 \quad \frac{}{1 \quad \tan}$

$$0, \frac{}{4} \qquad 1 \quad \tan \qquad (1,2)$$

$$A \quad (3,4)$$

3. (c)

$$(PQ)^2 \quad \frac{3 \quad \tan}{1 \quad \tan} \quad 1^{\;2} \quad \frac{1 \quad 3\tan}{1 \quad \tan} \quad 1^{\;2}$$

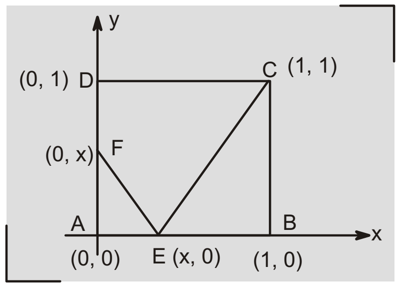
$$\frac{4}{(\sin \quad \cos \;)^2}$$

$$\frac{4}{1 \quad \sin 2}, \qquad 0, \frac{}{4}$$

$$(PQ)^2 \quad (2,4) \quad PQ \quad (\sqrt{2}, 2)$$

Comprehension: (2)

1. (d)



Area of $CDFE$,

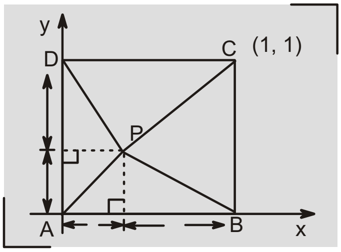
$$A(x) \quad 1 \quad \frac{1}{2}x^2 \quad \frac{1}{2}(1 \quad x)$$

$$\frac{1 \quad x \quad x^2}{2}$$

$$A_{\max} \quad A \quad \frac{1}{2} \quad \frac{1 \quad \frac{1}{2} \quad \frac{1}{4}}{2} \quad \frac{5}{8}$$

at $\quad x \quad \frac{1}{2}$

2. (d)



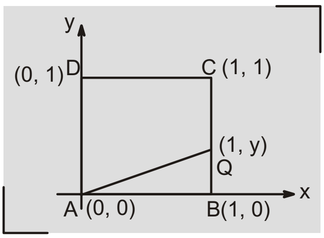
$$(PA)^2 \quad (PB)^2 \quad (PC)^2 \quad (PD)^2$$

$$(\ ^2 \quad ^2)\ (\ ^2 \quad ^2)$$

$$(\ ^2 \quad ^2)\ (\ ^2 \quad ^2)$$

$$0$$

3. (d)



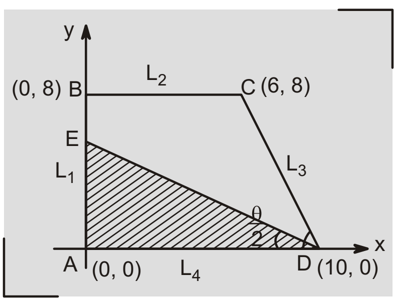
$$\frac{1}{2}y(1) \quad \frac{1}{3}(1)$$

$$y \quad \frac{2}{3}$$

$$L_{AQ} \quad \sqrt{(1)^2 \quad \frac{2}{3}^{\,2}} \quad \frac{\sqrt{13}}{3}$$

Comprehension: (3)

1. (d)



$$\tan \quad \frac{2\tan\frac{}{2}}{1 \quad \tan^2\frac{}{2}} \quad 2$$

$$\tan\frac{}{2} \quad \frac{\sqrt{5} \quad 1}{2}$$

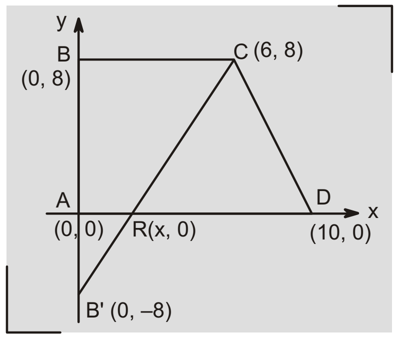
$$AE \quad 10\tan\frac{}{2} \quad 5(\sqrt{5} \quad 1)$$

Area of region required = shaded area

$$\frac{1}{2} \quad 10 \quad 5(\sqrt{5} \quad 1)$$

$$25(\sqrt{5} \quad 1)$$

2. (b)



$$(\text{Perimeter of } \quad RBC)_{\min}$$

$$(RB \quad RC)_{\min} \quad BC$$

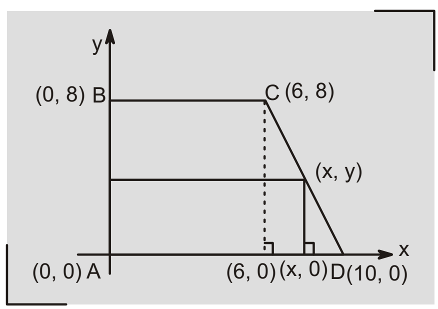
$$(RB' \quad RC)_{\min} \quad BC$$

$$B'C \quad BC$$

$$\frac{8}{6 \quad x} \quad \frac{16}{6} \qquad x \quad 3$$

$$R \quad (3, 0)$$

3. (d)



$$\frac{y}{8} \quad \frac{10 \quad x}{10 \quad 6} \qquad y \quad 2(10 \quad x)$$

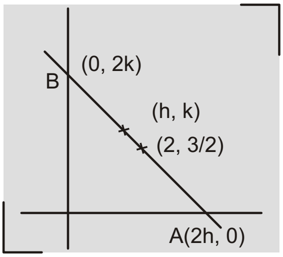
Area of rectangle, $A(x) \quad 2x(10 \quad x)$

$$x \quad [6, 10]$$

$(A)_{max} \quad A(6) \quad 2 \quad 6 \quad (10 \quad 6) \quad 48$

Comprehension: (4)

1. (a) Intersection point of lines is $2, \frac{3}{2}$,



Equation, of AB is

$$\frac{x}{2h} \quad \frac{y}{2k} \quad 1$$

put $2, \frac{3}{2} \qquad \frac{1}{h} \quad \frac{3}{4k} \quad 1$

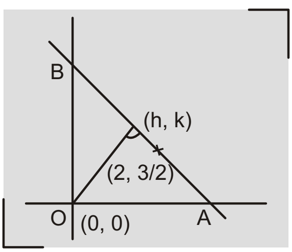
$$4k \quad 3h \quad 4hk$$

$$3x \quad 4y \quad 4xy$$

2. (b) Equation of AB is

$$hx \quad ky \quad h^2 \quad k^2$$

Put $2, \frac{3}{2}$



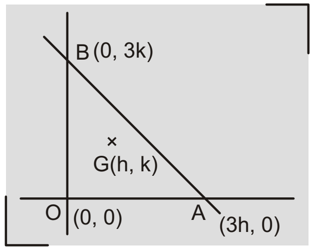
$$2h \quad \frac{3k}{2} \quad h^2 \quad k^2$$

$$2(x^2 \quad y^2) \quad 4x \quad 3y \quad 0$$

3. (c) Equation of AB is

$$\frac{x}{3h} \quad \frac{y}{3k} \quad 1$$

put $2, \frac{3}{2}$



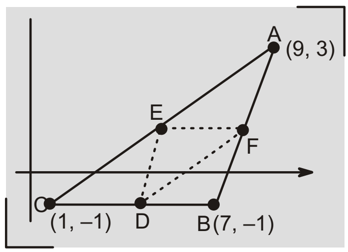
$$\frac{2}{3h} \quad \frac{1}{2k} \quad 1$$

$$4k \quad 3h \quad 6hk$$

$$3x \quad 4y \quad 6xy \quad 0$$

Comprehension: (5)

1. (d) Area of $DEF \quad \frac{1}{4}$ area of ABC



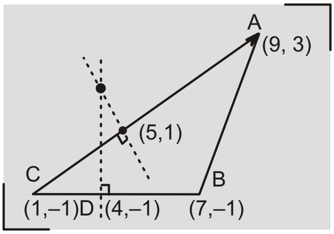
$$\frac{1}{4} \quad \frac{1}{2} \quad 6 \quad 4 \quad 3$$

2. (b) Equation of perpendicular bisector of AC is

$$y \quad 1 \quad 2(x \quad 5)$$

$$y \quad 2x \quad 11$$

perpendicular bisector of BC is



$x \quad 4$

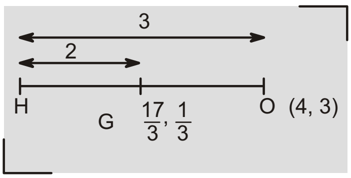
Circumcentre $\quad (4,3)$

$$R \quad \sqrt{(4 \quad 1)^2 \quad (3 \quad 1)^2} \quad 5$$

$a \quad b \quad R \quad 4 \quad 3 \quad 5 \quad 12$

3. (c) O circumcentre, G centroid

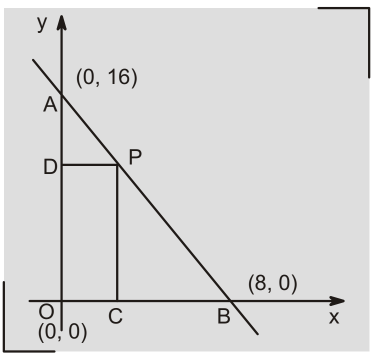
$$H \quad 3\ \frac{17}{3} \quad 2(4),3\ \frac{1}{3} \quad 2(3)$$



$H \quad (9, -5)$

Comprehension: (6)

1. (c) Equation of AB is

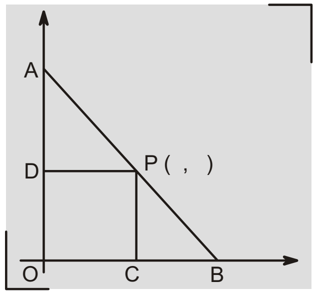


$$\frac{x}{8} \quad \frac{y}{16} \quad 1 \qquad 2x \quad y \quad 16$$

perpendicular distance of (2, 2) from AB

$$\frac{|4 \quad 2 \quad 16|}{\sqrt{5}} \quad \frac{10}{\sqrt{5}} \quad 2\sqrt{5} \quad \sqrt{20}$$

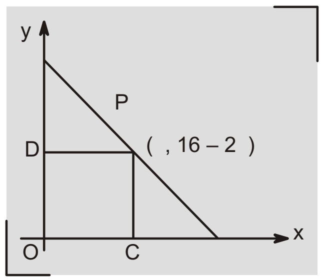
2. (a) Put (,) to eqn. of AB



$$2 \qquad 16 \qquad \frac{16}{3}$$

Sum of coordinates of $P \quad 2 \quad \frac{32}{3}$

3. (b)



Area of rectangle $PDOC$

$(16 \quad 2\ ) \quad 30$

$^2 \quad 8 \quad 15 \quad 0$

$(\quad 3)(\quad 5) \quad 0$

$3,5 \qquad P \quad (3, 10), (5, 6)$

Comprehension: (7)

$$a \quad b \quad \sqrt{a^2 \quad b^2} \quad \frac{1}{2}ab$$

$(2a \quad 2b \quad ab)^2 \quad 4(a^2 \quad b^2)$

$8ab \quad a^2b^2 \quad 4ab(a \quad b) \quad 0$

$ab \quad 4a \quad 4b \quad 8 \quad 0$

$(a \quad 4)(b \quad 4) \quad 8 \quad 2 \quad 4,\ 1 \quad 8$

$(a,b) \quad (6,8)$ or $(5,12)$

1. (b) No. of ordered pairs (a,b) 2

2. (d) For maximum perimeter,

a 5, b 12   Maximum perimeter

5   12   13   30

3. (a) For minimum area, a 6, b 8

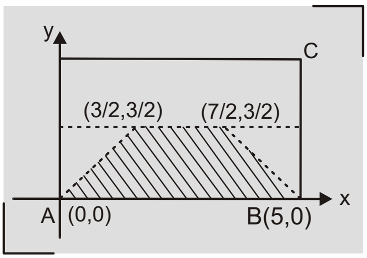
Minimum area $\frac{1}{2}$ 6   8   24

Comprehension: (8)

1. (c) $d(P,AB)$ $d(P,BC)$

$d(P,AB)$ $d(P,CD)$

$d(P,AB)$ $d(P,AD)$



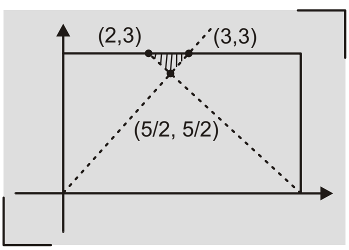
P lies in region as shown

Area $\frac{1}{2}$ $\frac{3}{2}$ (5   2) $\frac{21}{4}$

2. (d) $d(P,AB)$ $d(P,BC)$

$d(P,AB)$ $d(P,CD)$

$d(P,AB)$ $d(P,AD)$

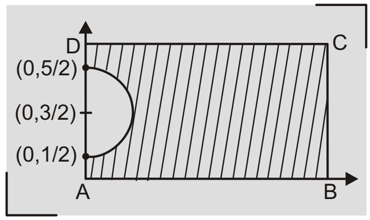


P lies in region as shown

Area $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{4}$

3. (d) x^2 y $\frac{3}{2}$ ^{2} 1

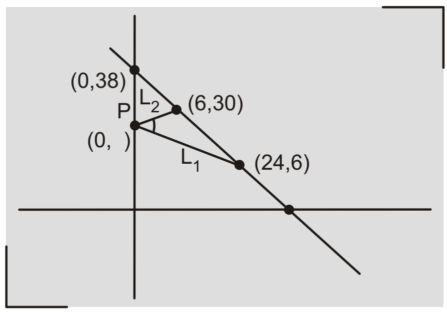
P lies in region as shown



Area   5   3 $\frac{(1)^2}{2}$ 15 $\frac{}{2}$

Comprehension: (10)

m_{L_1} $\frac{6}{24}$, m_{L_2} $\frac{30}{6}$



tan $\frac{m_{L_2} \quad m_{L_1}}{1 \quad m_{L_2} m_{L_1}}$ $\frac{\frac{30}{6} \quad \frac{6}{24}}{1 \quad \frac{30}{6} \quad \frac{6}{24}}$

$\frac{(38 \quad)18}{(\quad 18)^2}$

$\frac{d}{d}$(tan) $\frac{18(\quad 18)(\quad 58)}{(\quad 18)^4}$

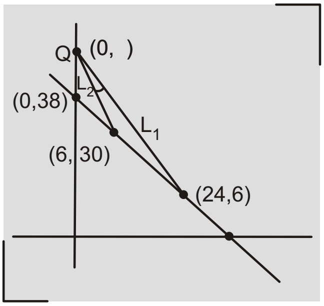
1. (c) tan   is maximum at   18.

2. (b) (tan)$_{max}$ $\frac{}{2}$

3. (b) $\tan \quad \frac{m_{L_1} \quad m_{L_2}}{1 \quad m_{L_1} m_{L_2}} \quad \frac{(\quad 38)18}{(\quad 18)^2}$

$\frac{d}{d}(\tan \quad) \quad \frac{18(18 \quad)(\quad 58)}{(\quad 18)^4}$

tan is maximum at 58



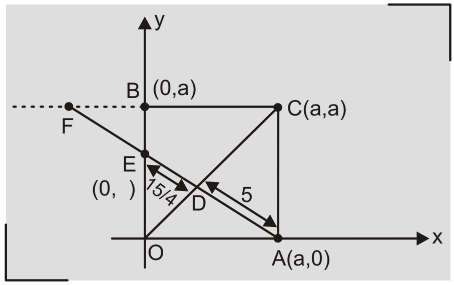
Comprehension: (11)

$ODE \sim \quad CDA$

$\frac{}{a} \quad \frac{15/4}{5} \qquad \frac{3}{4}a$

$E \quad 0, \frac{3}{4}a$

Similarly $BFE \sim \quad CFA$,



$\frac{BF}{CF} \quad \frac{BE}{AC} \quad \frac{a/4}{a}$

$BF \quad \frac{1}{4}(a \quad BF)$ or $BF \quad \frac{a}{3}$

$F \quad \frac{a}{3}, a$

$AE \quad \sqrt{a^2 \quad \frac{3}{4}a}^{\,2} \quad \frac{5}{4}a$

$5 \quad \frac{15}{4} \quad \frac{35}{4} \quad a \quad 7$

1. (c) Area of square $a^2 \quad 49$

2. (b) $F \quad \frac{a}{3}, a \quad \frac{7}{3}, 7$

3. (c) Circle circumscribing AOE has AE as diameter.

Equation of circle is

$x(x \quad 7) \quad y \quad y \quad \frac{3}{4}(7) \quad 0$

$4(x^2 \quad y^2 \quad 7x) \quad 21y \quad 0$

Assertion and Reason

1. (A) P and Q are harmonic conjugates,

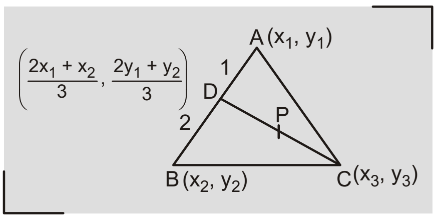
AP, AB, AQ are in H.P.

$$\frac{2}{AB} \quad \frac{1}{AP} \quad \frac{1}{AQ}$$

$$\frac{2}{2(BM)} \quad \frac{1}{BM \quad MP} \quad \frac{1}{BM \quad MQ}$$

$$(BM)^2 \quad (PM)(QM)$$

2. (A)



$$P \quad \frac{\frac{2x_1 \quad x_2}{3} \quad x_3}{2}, \frac{\frac{2y_1 \quad y_2}{3} \quad y_3}{2}$$

$$\frac{2x_1 \quad x_2 \quad 3x_3}{6}, \frac{2y_1 \quad y_2 \quad 3y_3}{6} .$$

P lies in side the ABC

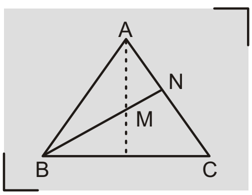
Area of PBC < area of ABC

3. (C) If $P \quad AB$, 2 lines exist

$P \quad AB$, 1 line exist

$P \quad AB$, no lines exist.

4. (A)



$$AC \quad BC \quad AN \quad NC \quad BC \quad AN \quad BN$$

$$AN \quad BM \quad MN \quad AM \quad BM$$

$$AM \quad BM \quad AC \quad BC \qquad (1)$$

Similarly $BM \quad CM \quad AB \quad AC \qquad (2)$

and $CM \quad AM \quad AB \quad BC \qquad (3)$

Adding (1), (2) and (3)

$$AM \quad BM \quad CM \quad AB \quad BC \quad CA \quad P$$

5. (C) Let lines be

$$x \quad 2y \quad 3 \quad 0$$
$$x \quad 2y \quad 4 \quad 0$$
$$x \quad 2y \quad 5 \quad 0$$

$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 1 & 2 & 5 \end{vmatrix}$ 0, while lines are not concurrent.

6. (A) $\frac{y}{x} \quad m, bm^2 \quad 2hm \quad a \quad 0$

$a \quad b \quad 2h \quad 0 \qquad m \quad 1$ is a root.

If $by^2 \quad 2hxy \quad ax^2 \quad [ax \quad y(2h \quad a)]$
$(x \quad y)$

equating from both sides coefficient of y^2

$$b \quad (2h \quad a)$$

7. (A) $3a \quad 2b \quad 4c \quad 0 \qquad \frac{3a}{4} \quad \frac{b}{2} \quad C \quad 0$

$ax \quad by \quad c \quad 0$ passes through fixed point $\frac{3}{4}, \frac{1}{2}$

Let $\quad pa \quad qb \quad rc \quad 0$

$$\frac{p}{r}a \quad \frac{q}{r}b \quad c \quad 0$$

line $ax \quad by \quad c \quad 0$ passes through fixed point $\frac{p}{r}, \frac{q}{r}$

8. (A) Shift origin to (20, 22)

9. (C) $2xy \quad 3x \quad 4y \quad 6 \quad 0$

$(x \quad 2)(2y \quad 3) \quad 0$

10. (A) $[2(\cos \ _1 \cos \ _2 \quad \cos \ _2 \cos \ _3$

$\cos \ _3 \cos \ _1) \quad \cos^2 \ _1 \quad \cos^2 \ _2$

$\cos^2 \ _3]$

$[2(\sin \ _1 \sin \ _2 \quad \sin \ _2 \sin \ _3$

$\sin \ _3 \sin \ _1) \quad \sin^2 \ _1 \quad \sin^2 \ _2$

$\sin^2 \ _3] \quad 0$

$(\cos \ _1 \quad \cos \ _2 \quad \cos \ _3)^2$

$(\sin \ _1 \quad \sin \ _2 \quad \sin \ _3)^2 \quad 0$

$\cos \ _1 \quad \cos \ _2 \quad \cos \ _3 \quad 0$

and $\sin \ _1 \quad \sin \ _2 \quad \sin \ _3 \quad 0$

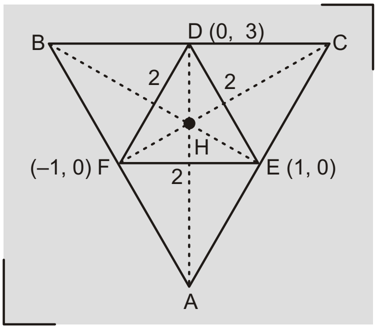
Centroid and circumcentre of *ABC* are at origin.

ABC is equilateral

Orthocentre of *ABC* is also origin.

Match the Columns:

1. a-p, b-s, c-q, d-r



Point A, B, C are excentres w.r.t. to DEF,

$$A \quad \frac{2(0) \quad 2(1) \quad 2(\ 1)}{2 \quad 2 \quad 2},$$

$$\frac{2(\sqrt{3}) \quad 2(0) \quad 2(0)}{2 \quad 2 \quad 2}$$

$$B \quad \frac{2(1) \quad 2(\ 1) \quad 2(0)}{2 \quad 2 \quad 2},$$

$$\frac{2(0) \quad 2(0) \quad 2(\sqrt{3})}{2 \quad 2 \quad 2} \quad (\ 2, \sqrt{3})$$

$$C \quad \frac{2(\ 1) \quad 2(1) \quad 2(0)}{2 \quad 2 \quad 2},$$

$$\frac{2(0) \quad 2(0) \quad 2(\sqrt{3})}{2 \quad 2 \quad 2} \quad (2, \sqrt{3})$$

Orthocentre H of ABC is the incentre of DEF

$$H \quad 0, \frac{1}{\sqrt{3}}$$

(a) In radius of ABC,

$$r_{ABC} \quad \overline{S} \quad \frac{\frac{\sqrt{3}}{4}(4)^2}{\frac{4 \quad 4 \quad 4}{2}}$$

$$\frac{\sqrt{3}(4)}{6} \quad \frac{2}{\sqrt{3}}$$

In radius of DEF,

$$r_{DEF} \quad \frac{\frac{\sqrt{3}}{4}(2)^2}{\frac{2 \quad 2 \quad 2}{2}} \quad \frac{1}{\sqrt{3}}.$$

$$\frac{r_{ABC}}{r_{DEF}} \quad 2$$

(b) $(AH)^2 \quad \frac{1}{\sqrt{3}} \quad \sqrt{3}^{\ 2} \quad \frac{16}{3}$

$$[(AH)^2] \quad 5$$

(c) $(y_A \quad y_B \quad y_C)^2 \quad (\sqrt{3})^2 \quad 3$

(d) $AB \quad 4$

2. a-r, b-s, c-p, d-q

(a) $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} \quad 0$

$$bc \quad 2a(c \quad b) \quad a(4c \quad 3b)$$

$$bc \quad 2ac \quad ab$$

$$\frac{2}{b} \quad \frac{1}{a} \quad \frac{1}{c}$$

(b) $\begin{vmatrix} 2a & 3a & 1 \\ 3b & 2b & 1 \\ c & c & 1 \end{vmatrix} \quad 0 \quad bc \quad ac \quad 5ab \quad 0$

(c) $\begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix} \quad 0$

$$4b \quad 3c \quad 4a \quad 2c \quad 3a \quad 2b$$

$$2b \quad a \quad c \qquad a \quad c \quad 2b$$

(d) $\begin{vmatrix} a & a & c \\ c & c & b \\ 1 & 0 & 1 \end{vmatrix} \quad 0 \quad ab \quad c^2 \quad 0$

$c^2 \quad ab$

3. a-r, b-q, c-s, d-p

(a) $a \quad c \quad 2b \qquad a \quad c \quad 2b \quad 0$

$ax \quad by \quad c \quad 0$ passes through fixed point (1, –2).

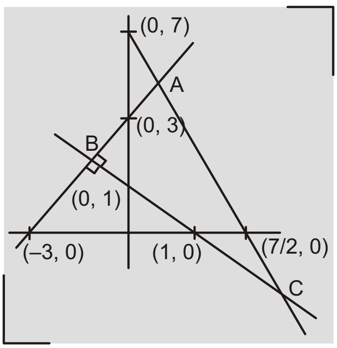
(b) Perpendicular distance of $P(\ ,4 \quad)$ from $4x \quad 3y \quad 10$

$\frac{|4 \quad 3(4 \quad) \quad 10|}{5} \quad 1$

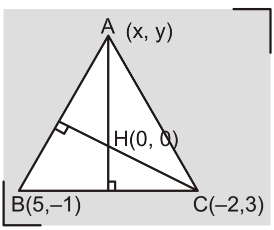
$| \quad 2| \quad 5 \qquad 2 \quad 5 \quad 3, \ 7$

$P \quad (3,\ 1), (\ 7,\ 11)$

(c) Point B $(\ 1,2)$



(d) $\frac{y}{x} \quad \frac{7}{4}$



$\frac{y \quad 1}{x \quad 5} \quad \frac{2}{3}$

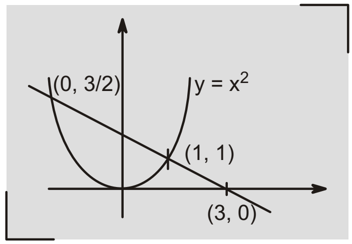
$\frac{21}{4}x \quad 2x \quad 13$

$x \quad 4, \ y \quad 7$

4. a-p, b-r, c-q, d-s

(a) $2a^2 \quad a \quad 3 \quad 0$

$(2a \quad 3)(a \quad 1) \quad 0$

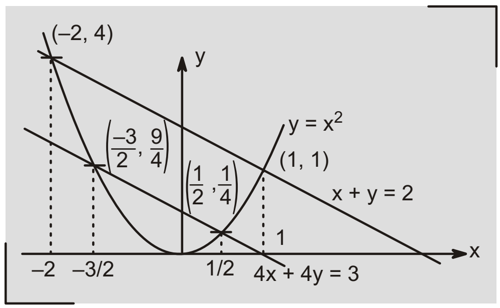


$a \quad (0,1)$

No. of integral values of $a \quad 0$

(b) $a^2 \quad a \quad 2 \quad 0$

$(a \quad 2)(a \quad 1) \quad 0 \qquad a \quad 2,1$



$4a^2 \quad 4a \quad 3 \quad 0$

$(2a \quad 1)(2a \quad 3) \quad 0$

$a \quad \frac{1}{2}, \ \frac{3}{2}$

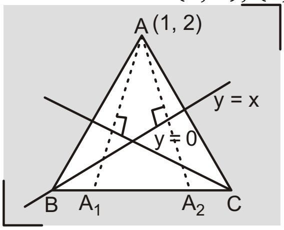
$a \qquad 2, \frac{3}{2} \qquad \frac{1}{2}, 1$

values of a of form $\frac{K}{3}$ are $\frac{5}{3}, \frac{2}{3}$

(c) Slope of line joining $(t \quad 1, 2t \quad 2)$ and $(2t \quad 1, t)$ is $\frac{2t \quad 2 \quad t}{t \quad 1 \quad 2t \quad 1} \quad 1$

slope of perpendicular bisectors of points is 1.

(d) Images of A w.r.t. $y \quad x$ and $y \quad 0$ lies on BC which are (2, 1), (1, –2)



Equation of BC is $y \quad 3x \quad 5$

Perpendicular distance of A from

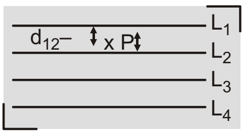
$$BC \quad \frac{|3 \quad 2 \quad 5|}{\sqrt{10}}$$

$$d(A, BC) \quad \frac{4}{\sqrt{10}}$$

$$\sqrt{10}\, d(A, BC) \quad 4$$

5. a-q, b-r, c-s, d-p

If P lies between L_1 and L_2, then



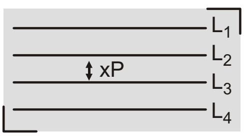
$K \quad (d_{12} \quad) \qquad (d_{23} \quad)$

$(d_{34} \quad d_{23}$

$d_{12} \quad 2d_{23} \quad d_{34} \quad 2$

$(0, d_{12})$

If P lies between L_2 and L_3, then

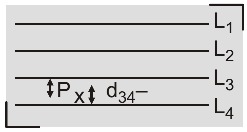


$K \quad d_{12} \quad (d_{23} \quad) \quad (\ ) \quad (d_{34} \quad)$

$(d_{23} \quad)$

$d_{12} \quad 2d_{23} \quad d_{34}$

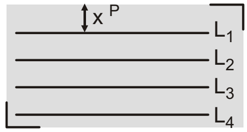
If P lies between L_3 and L_4, then



$K \quad d_{12} \quad d_{23} \qquad d_{23} \qquad d_{34}$

$d_{12} \quad 2d_{23} \quad d_{34} \quad 2 \qquad (0, d_{34})$

If P lies above L_1

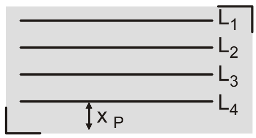


$K \qquad (d_{12} \quad) \quad (d_{23} \quad d_{12} \quad)$

$(d_{34} \quad d_{23} \quad d_{12} \quad)$

$K \quad 2d_{12} \quad 2d_{23} \quad d_{34} \quad 4 \quad , \qquad 0$

If P lies below L_4



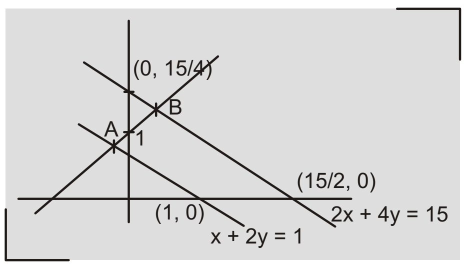
$K \quad (d_{12} \quad d_{23} \quad d_{34} \quad)$

$(d_{23} \quad d_{34} \quad) \quad (d_{34} \quad)$

$K \quad d_{12} \quad 2d_{23} \quad 2d_{34} \quad 4 \quad , \qquad 0$

6. a-r, b-p, c-s, d-q

(a) $y \quad x \quad 1$



$$A \quad \frac{1}{3}, \frac{2}{3}, \ B \quad \frac{11}{6}, \frac{17}{6}$$

$$1 \quad \frac{t}{\sqrt{2}} \qquad \frac{1}{3}, \frac{11}{6}$$

$t \quad \frac{4\sqrt{2}}{3}, \frac{5\sqrt{2}}{6}$

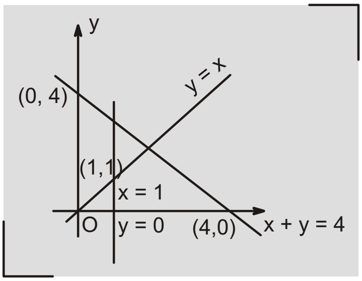
(b) $P((2 \quad t)x_1 \quad (t \quad 1)x_2, (2 \quad t)y_1 \quad (t \quad 1)y_2)$

divides (x_1, y_1), (x_2, y_2) internally in ratio $(t \quad 1) : (2 \quad t)$

$(t \quad 1)(2 \quad t) \quad 0 \qquad t \quad (1,2)$

(c) $t \quad (0,1)$

(see from figure)

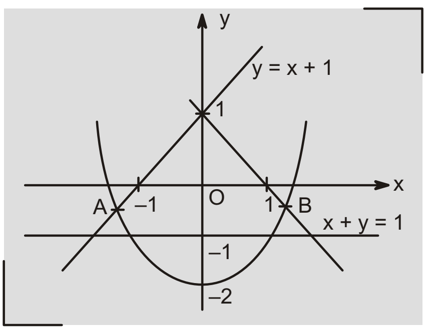


(d) for A:

$x^2 \quad 2 \quad x \quad 1$

$x^2 \quad x \quad 3 \quad 0$

$x \quad \frac{1 \quad \sqrt{13}}{2}$

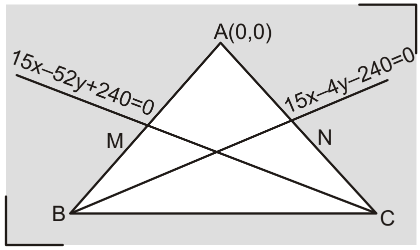


$x \quad \frac{1 \quad \sqrt{13}}{2}$

similarly for B, $x \quad \frac{\sqrt{13} \quad 1}{2}$

$t \quad \frac{1 \quad \sqrt{13}}{2}, \ 1 \qquad 1, \frac{\sqrt{13} \quad 1}{2}$

7.



Let $B \quad x_1, \frac{15x_1 \quad 240}{4}$

and $C \quad x_2, \frac{15x_2 \quad 240}{52}$

Midpoints of AB and AC are $M \quad \frac{x_1}{2}, \frac{15x_1 \quad 240}{8}$

and $N \quad \frac{x_2}{2}, \frac{15x_2 \quad 240}{104}$

lie on $15x \quad 25y \quad 240 \quad 0$ and $15x \quad 4y \quad 240 \quad 0$ respectively

$x_1 \quad 20$ and $x_2 \quad 36$

$B(20,15)$ and $C(36,15)$

$a \quad BC \quad 16$, $b \quad CA \quad 39$

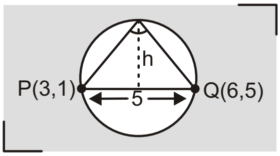
and $c \quad AB \quad 25$

incentre $I(21,12)$, centroid $G \quad \frac{56}{3}, 10$,

excentre opposite to $C, I_3 (\quad 4,7)$ and orthocentre is $(0,63)$.

Subjective Problems

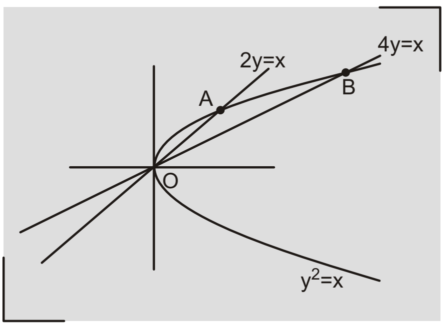
1. (0) $\frac{1}{2}\quad 5 \quad h \quad 7 \quad h \quad \frac{14}{5}$



But h_{max} Radius of circle with diameter $PQ \quad \frac{5}{2} \quad \frac{14}{5}$

No such triangle exists.

2. (1)



$2y \quad y^2$

$y \quad 0,2 \quad A \quad (4,2)$

$4y \quad y^2$

$y \quad 0,4 \quad B \quad (16,4)$

$a \quad (2,4)$

Number of integral values of a 1

3. (3) $(a^2 \quad ab \quad 1)(1) \quad 0 \qquad a \quad R$

$D \quad 0 \quad b^2 \quad 4 \quad 0 \qquad b \quad (\ 2,2)$

4. (6) $6x^2 \quad xy \quad 3y^2 \quad 24x \quad 3y \qquad 0$

Intersection point of lines lies on x-axis.

Put $y \quad 0,$

$6x^2 \quad 24x \qquad 0$ must have equal real roots

$D \quad 0 \quad (24)^2 \quad 24 \quad 0 \qquad 24$

Apply condition of pair of lines,

i.e., $abc \quad 2fgh \quad af^2 \quad bg^2 \quad ch^2 \quad 0$

$6(\ 3) \quad (\quad)(\ 12)\frac{3}{2} \quad 6 \quad \frac{3}{2}^{\ 2}$

$(\ 3)(12)^2 \qquad \frac{\ }{2}^{\ 2} \quad 0$

$18(24) \quad 18 \quad \frac{27}{2}$

$3(144) \quad 6 \quad ^2 \quad 0$

$6 \quad ^2 \quad 18 \quad \frac{27}{2} \quad 0$

$4 \quad ^2 \quad 12 \quad 9 \quad 0$

$3/2$ Hence, $20 \qquad 6$

5. (3) Perpendicular distance of (2,3) from line $3x \quad 4y \quad 5 \quad 0,$

$P_1 \quad \frac{|3(2) \quad 4(3) \quad 5|}{5} \quad \frac{13}{5}$

$\because \quad 1 \quad \sin^2 \quad P_1 \qquad R \qquad n_1 \quad 0$

Perpendicular distance of (1,3) from line $3x \quad 4y \quad 5 \quad 0,$

$P_2 \quad \frac{|3 \quad 12 \quad 5|}{5} \quad 2$

$\because \quad \sec^2 \quad 2\,\text{cosec}^2 \quad 3 \quad P_2 \qquad R$

$n_2 \quad 2$

Equation becomes, $2x^2 \quad 6x \quad 0$

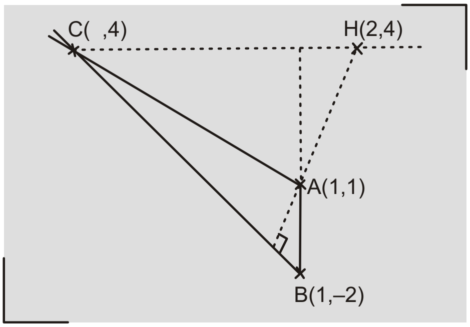
$x \quad 0,3$

Hence, sum of roots 3

6. (14) $a \quad c \quad 2b \quad 0$

$ax \quad by \quad c \quad 0$ passes through fixed point $(1, \quad 2)$.

$B \quad (1, \quad 2)$



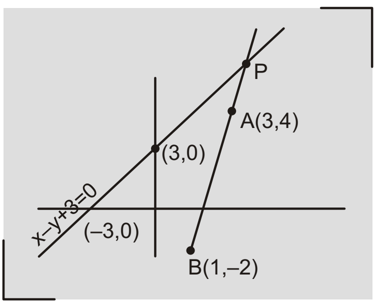
Slope of $BC \quad \frac{2 \quad 1}{4 \quad 1} \quad \frac{4 \quad 2}{\quad 1}$

$1 \quad 18 \quad\quad 17$

$C \quad (\quad 17, 4) \quad (h, k)$

$2h \quad 12k \quad 34 \quad 48 \quad 14$

7. (5) $(3x \quad 7y \quad 11)\sec \quad (5x \quad 3y \quad 11)$ cosec $\quad 0$ passes through intersection of $3x \quad 7y \quad 11 \quad 0$ and $5x \quad 3y \quad 11 \quad 0$ permissible values of given by $(1, \quad 2)$



$B \quad (1, \quad 2)$

It is clear that $|PA \quad PB| \quad AB$

$|PA \quad PB|_{max} \quad AB \quad \sqrt{6^2 \quad 2^2}$

$2\sqrt{10} \quad n \quad 5$

8. (2) By observation, directly equation must be

$(4x^2 \quad 4xy \quad y^2) \quad 2(2x \quad y) \quad 1 \quad 0$

or $4x^2 \quad 4xy \quad y^2 \quad 2(2x \quad y) \quad 1 \quad 0$

$(2x \quad y \quad 1)^2 \quad 0$ or $(2x \quad y \quad 1)^2 \quad 0$

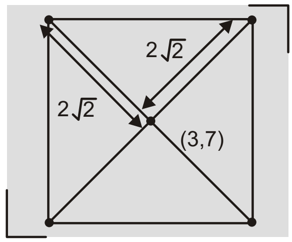
i.e., $4x^2 \quad 4xy \quad y^2 \quad 4x \quad 2y \quad 1 \quad 0$

or $4x^2 \quad 4xy \quad y^2 \quad 4x \quad 2y \quad 1 \quad 0$

$a_1 \quad b_1 \quad c_1 \quad a_2 \quad b_2 \quad c_2 \quad 1 \quad 4 \quad 2$

$1 \quad 4 \quad 2 \quad 2$

9. (8) Vertices of square are



$3 \quad 2\sqrt{2} \quad \frac{1}{\sqrt{2}}, 7 \quad 2\sqrt{2} \quad \frac{1}{\sqrt{2}}$,

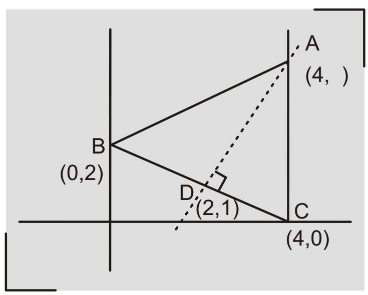
$3 \quad 2\sqrt{2} \quad \frac{1}{\sqrt{2}}, 7 \quad 2\sqrt{2} \quad \frac{1}{\sqrt{2}}$

i.e., (5,9), (1,5), (1,9), (5,5)

$\max(y_1, y_2, y_3, y_4) \quad \min$

$(x_1, x_2, x_3, x_4) \quad 9 \quad 1 \quad 8$

10. (8) Slope of $AD \quad \frac{1}{4 \quad 2} \quad 2 \quad\quad 5$



Equation of AB is

$$y \quad 2 \quad \frac{5 \quad 2}{4 \quad 0}(x \quad 0)$$

$$4y \quad 8 \quad 3x$$

$$y \quad \frac{3}{4}x \quad 2$$

$$8m \quad c \quad 8 \quad \frac{3}{4} \quad 2 \quad 8$$

11. (2) $a \quad 2hm \quad bm^2 \quad 0$ (m_1, m_1^2)

where $m \quad \frac{y}{x} \quad m_1 \quad m_1^2 \quad \frac{2h}{b}; m_1^3 \quad \frac{a}{b}$

$$\frac{8h^3}{b^3} \quad (m_1 \quad m_1^2)^3 \quad m_1^3 \quad m_1^6$$

$$3m_1^3(m_1 \quad m_1^2)$$

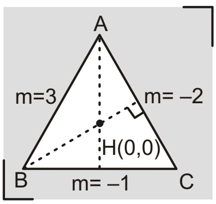
$$\frac{a}{b} \quad \frac{a^2}{b^2} \quad \frac{3a}{b} \quad \frac{2h}{b}$$

$$\frac{8h^3}{b^3} \quad \frac{ab \quad a^2 \quad 6ah}{b^2}$$

$$ab(a \quad b) \quad 6abh \quad 8h^3 \quad 0$$

$$8 \quad 6 \quad 2$$

12. (7) Let slopes of BC, CA, AB be $1, \; 2, 3$ respectively.



Slope of $AH \quad 1 \quad A \quad (x_1, x_1)$

Slope of $BH \quad \frac{1}{2} \quad B \quad x_2, \frac{1}{2}x_2$

Slope of $CH \quad \frac{1}{3} \quad C \quad x_3, \frac{x_3}{3}$

$\because$ Slope of $BC \quad \frac{\frac{x_2}{2} \quad \frac{x_3}{3}}{x_2 \quad x_3} \quad \frac{3x_2 \quad 2x_3}{6(x_2 \quad x_3)}$

$$1 \quad 9x_2 \quad 4x_3$$

$\because$ Slope of $CA \quad \frac{x_1 \quad \frac{x_3}{3}}{x_1 \quad x_3} \quad 2$

$$\frac{3x_1 \quad x_3}{3(x_1 \quad x_3)} \quad 9x_1 \quad 5x_3$$

$$G \quad (h,k) \quad \frac{x_1 \quad x_2 \quad x_3}{3}, \frac{x_1 \quad \frac{x_2}{2} \quad \frac{x_3}{3}}{3}$$

$$3h \quad x_3 \quad \frac{5x_3}{9} \quad \frac{4x_3}{9} \quad 2x_3 \qquad ....(1)$$

$$3k \quad \frac{5x_3}{9} \quad \frac{2x_3}{9} \quad \frac{x_3}{3} \quad \frac{4x_3}{9} \qquad ....(2)$$

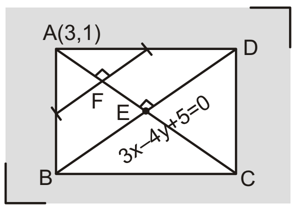
From eqs. (1) and (2), we get $\frac{k}{h} \quad \frac{2}{9} \quad y \quad \frac{2}{9}x; \; b \quad a \quad 9 \quad 2 \quad 7$

13. (1) Point E

$$\frac{x \quad 3}{3} \quad \frac{y \quad 1}{4} \quad \frac{9 \quad 4 \quad 5}{25}$$

$$\frac{2}{5} \quad E \quad \frac{9}{5}, \frac{13}{5}$$

Line, L passing through midpoint of sides AB and AD passes through F.



$$F \quad \frac{3 \quad \frac{9}{5}}{2}, \frac{\frac{13}{5} \quad 1}{2} \quad \frac{12}{5}, \frac{9}{5}$$

Equation of line L is

$$3x \quad 4y \quad \frac{36}{5} \quad \frac{36}{5} \quad 0$$

$$3x \quad 4y \quad 0 \quad |a \quad b \quad c| \; |3 \quad 4| \quad 1$$

14. (2) Image of (, 1) w.r.t.

$3x \quad y \quad 6 \quad 0$ is

$$\frac{x}{3} \quad \frac{y \quad (\quad 1)}{1}$$

$$2 \; \frac{3 \qquad 1 \quad 6}{9 \quad 1} \qquad \frac{2 \qquad 1}{5}$$

$$(x, y) \quad \frac{11 \quad 3}{5}, \frac{7 \quad 4}{5}$$

$$(\;^{2} \quad 1, \;) \qquad \frac{7 \quad 4}{5}$$

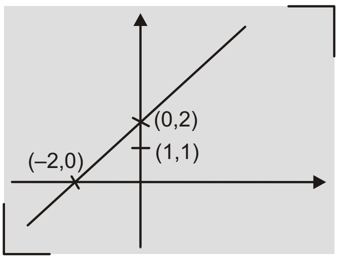
2

15. (3) $L_1 \quad y \quad 2 \quad x$

$$L_2 \quad \frac{x}{a} \quad \frac{y}{b} \quad 1$$

Put (1, 1),

$$\frac{1}{a} \quad \frac{1}{b} \quad 1$$



2

$$\frac{1}{2}ab \quad \frac{1}{2} \; \frac{2}{\frac{1}{a} \quad \frac{1}{b}} \qquad 2$$

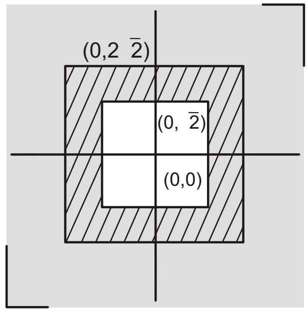
2 is minimum when $a \quad b \quad 2$

$L_2 \quad x \quad y \quad 2$

Area of $ABC \quad \frac{1}{2} \quad 4 \quad 2 \quad 4$

$|p \quad q| \quad 4 \quad 1 \quad 3$

16. (6) $\frac{|x \quad y|}{\sqrt{2}} \quad \frac{|y \quad x|}{\sqrt{2}} \quad [2, 4]$



If $y \quad x$ and $y \quad x \quad 0$

$y \quad [\sqrt{2}, 2\sqrt{2}\,]$

If $y \quad x$ and $y \quad x \quad 0$

$y \quad [\; 2\sqrt{2}, \quad \sqrt{2}\,]$

If $y \quad x$ and $y \quad x \quad 0$

$x \quad [\; 2\sqrt{2}, \quad \sqrt{2}\,]$

If $y \quad x$ and $y \quad x \quad 0$

$x \quad [\sqrt{2}, 2\sqrt{2}\,]$

Area of region

$(4\sqrt{2})^2 \quad (2\sqrt{2})^2 \quad 32 \quad 8 \quad 24$

17. (4) Area Mod of

$$\frac{1}{2}\begin{vmatrix} & 2 & 3 & 1 \\ 1 & 2 & & 1 \\ 2 & 3 & & 1 \end{vmatrix} \quad \frac{1}{2}| \qquad 2 \qquad 3 \quad 1|$$

$$\frac{| \qquad 2|}{2} \quad [2, 3)$$

$(\;\; 8, \;\; 6] \quad [2, 4)$

$(\; , \;) \quad (\;\; 7, \;\; 11), (\;\; 6, \;\; 9), (2, 7), (3, 9)$

Number of possible co-ordinates of $A \quad 4$

18. (0) $l \quad m \quad 0$

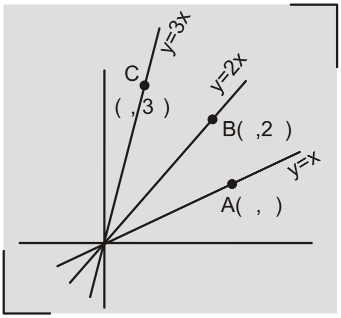
19. (3) $\frac{p}{q} \quad \frac{1}{2}; \qquad p \quad q \quad 3$

20. (0) Slope of $AB \quad \frac{\quad}{1} \quad \frac{2}{1}$

$$\frac{2}{1}$$

Slope of BC $\frac{2 \quad 1}{} \frac{3 \quad 1}{}$

$\frac{}{1}$



Equation of AC is

$$y \quad \frac{2}{\quad 1} \quad \frac{\frac{3}{1} \quad \frac{2}{1}}{\frac{}{1} \quad \frac{2}{1}} \quad x \quad \frac{2}{\quad 1}$$

$(x \quad y) \quad (3y \quad 5x \quad 4) \quad 0$

which passes through fixed point $\frac{1}{2}, \frac{1}{2}$

$h \quad k \quad 0$